Although the new measurements became legal and obligatory on 7 April 1795, it was not until 1840 when we could say that they were systematically applied throughout France. The adoption of the metric system by other nations was even slower. An international convention in Paris in 1875 helped to pave the road toward universal usage, but it took another century before most nations in the world decided to follow suit. Today, the United States is the only industrialized country which does not conform to the metric system. To be fair, in 1988 President Reagan signed an act that specifies the metric system as the preferred system of measurement for trade and commerce. Many individual institutions long ago declared their support for the meter. The American Institute of Architects' policy dates back to the 1940s and the American Geographical Society used the metric system on its maps from the very first one it published in 1852. But resistance to change is prevalent. In 1992, a survey showed that American land surveyors rejected the metric system by a 60 to 40 vote. To combat this attitude, the United States Department of Commerce announced a new Metric Transition Plan on April 12, 1994, and the American Congress on Surveying and Mapping stepped up its campaign for metric conversion. Perhaps it is really "just a matter of time" – give or take a century – before the whole world will finally have a universal system.

An uncommon used block of six stamps cancelled on the first day of issue. (Scott no. 732)

Tick Tock Earth

Our member Sandra Lach Arlinghaus, who has brought us several interesting articles – the nost recent being "The Ends Of The Earth" in our March,1994 issue, now goes scientific-ballistic on us to present a mathematical model with respect to how time relates to the earth's movement. At least that's what I think it does – it requires you to closely follow the explanation of a theory of the Greek geometrician Euclid – but she does show how it all works and includes some good illustrations. What does it have to do with maps, which is what we are all about? You will have to think three-dimensionally to understand that the precise determination of longitude is connected to time and it is a prerequisite to modern navigation and mapping. Try it.

–MDL, Ed.

TICK TOCK, EARTH

by Sandra Lach Arlinghaus

To see a World in a Grain of Sand
And a Heaven in a Wild Flower,
Hold Infinity in the palm of your hand
And Eternity in an hour.

William Blake, "Auguries of Innocence"

Portolan charts, with radial lines emanating from principal points of a compass rose, are eye-catching backdrops for maps on stamps (Figure 1). These navigational charts that date from about 1300 (Pisan chart) are derived from entries in pilot books that list courses, anchorage, and ports. The network of line segments linking the centers of converging rays were apparently used by seamen to design routes from port to port, not unlike the general strategy of modern seamen. Directional information is communicated on these charts with only minimal use of text; thus, the visual impact of the stamp is heightened. The simple elegance of portolan stamps offers an unusually attractive opportunity to let the mind wander in the various realms behind the philatelic surface – perhaps in previously uncharted directions.

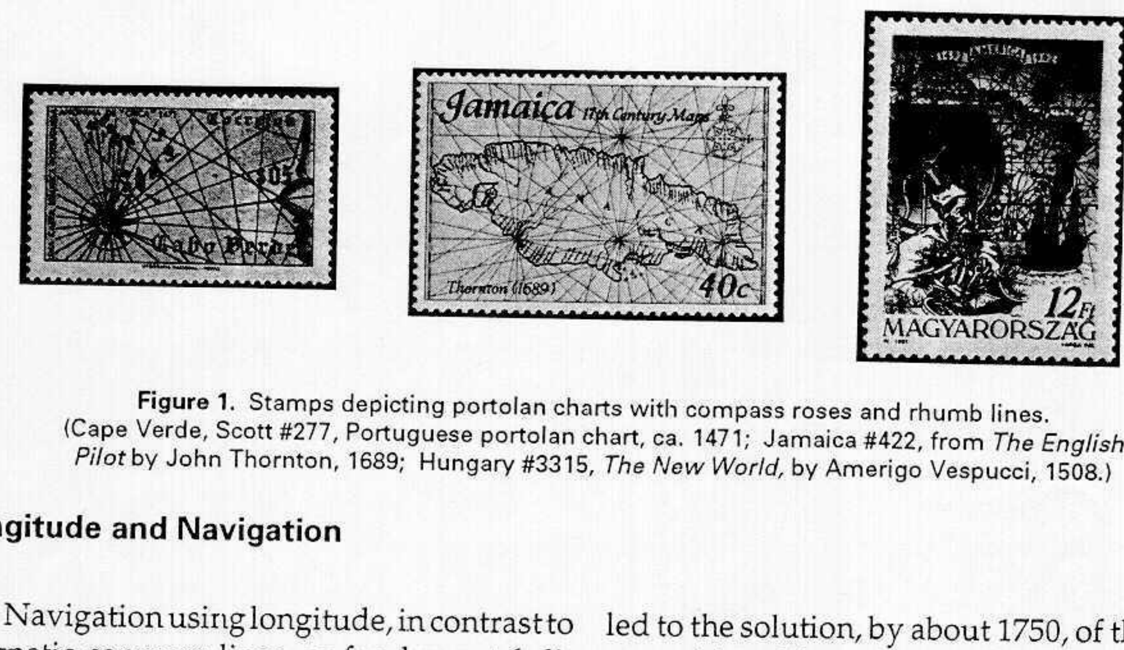

Figure 1. Stamps depicting portolan charts with compass roses and rhumb lines. (Cape Verde, Scott #277, Portuguese portolan chart, ca. 1471; Jamaica #422, from *The English Pilot* by John Thornton, 1689; Hungary #3315, *The New World,* by Amerigo Vespucci, 1508.)

Longitude and Navigation

Navigation using longitude, in contrast to magnetic compass lines, as fundamental directional elements required a connection between time and longitude. Clocks that were portable and that measured a standard hour (rather than a varying or "temporary" hour) led to the solution, by about 1750, of the 2000-year old problem of measuring longitude at sea.[2*] As early as 1530, Gemma Frisius understood the theory of how to measure longitude at sea using a clock, but he had no clock suited to that task.[2] Christian Huygens' application

* Number refers to the list of references.

of Galileo's discoveries of physical laws governing the motion of pendulum led to the pendulum clock by 1656. These clocks were quite accurate on land, but they were difficult to transport and certainly were not precise on a turbulent sea.[2,5] The use of Robert Hooke's principle that the force exerted by a spring is directly proportional to the spring's stretched length minus its length at rest permitted John Harrison (carpenter and clock maker) to construct a sequence of clocks with springs, rather than pendulums, as regulatory mechanisms (Figure 2).[2,5] These were portable, and by 1756 Harrison and his supporters had proven them accurate at sea to within three seconds a day over a period of six weeks.[2] Thus, by the middle of the eighteenth century, all the equipment necessary to measure longitude at sea was available. Clocks based on Harrison's construction permitted continuing exploration westward into the New World, and they formed the basis for navigational fixes until the development of radio and atomic clocks in the twentieth century.[3]

Figure 2. John Harrison's Marine Timekeeper No. 4, 1759.

Traditional clock faces exhibit a pattern of twelve numerals evenly spaced in a "clockwise" orientation around a circle; This clock face may be used as a model of the relationship between longitude and time. Many contemporary watches and clocks use a digital display to track time. The distinction between these two types of clock faces, made by those who market clocks, is that digital provides discrete tracking of the time while analogue produces a continuous display.[1] Beyond this commercial interpretation, however, the word "analogue" means "something that is similar to something else"[1] – to what else is an analogue clock similar? One natural answer is a sundial, the forerunner of the mechanical clock.[3] Physical evidence from sundials of both the Northern and the Southern Hemispheres suggests two types of dial: the horizontally mounted face frequently found in gardens and the vertically mounted dial often embedded in building walls. In the Northern Hemisphere (north of 23.5° North latitude, Tropic of Cancer), horizontal faces require clockwise orientation of numerals to record the time and, vertical dials need a counterclockwise arrangement. In the Southern Hemisphere (south of 23.5° South latitude, Tropic of Capricorn) the opposite holds: horizontal dials need counterclockwise orientation of numerals and, vertical dials require a clockwise pattern. The reversal in orientation of numerals, which distinguishes a horizontal from a vertical sundial of the same hemisphere, is a result of the switch in position of the background on which the shadow is cast. Between the Tropics, the northern hemisphere orientation is appropriate as long as the direct ray of the sun is overhead south of the dial, whiles the southern hemisphere orientation is applicable when the direct ray is north of the dial. The clockwise orientation of the numerals on the analogue clocks commonly in use today corresponds to that of a northern hemisphere garden sundial, or equivalently, to a southern hemisphere wall mounted model, although occasionally novelty models of a clock face appear with counterclockwise orientation of the numerals.[8] In contrast to analogue clocks, however, the sundial is not portable, and it records hours of varying length depending on the season and on the latitude.[7] The sundial is not the sort of clock that might be linked with longitude for navigational purposes. It is also, only superficially, related to the 12-hour analogue clock face.

At a deeper level, the twelve-hour analogue clock face serves directly as a structural replica of the relationship between longitude and time. This fact is not immediately apparent; lack of clarity arises from the simultaneous partitioning of:

the clock face into twelve equal central angles each containing 30 degrees of angular measure and each representing one hour of elapsed time; and,

the equatorial diametral plane of the earth into twenty-four equal central angles each containing 15 degrees of angular measure (longitude) and each representing one hour of elapsed time.

If one supposes the center of the analogue clock face (with 12 numerals) to be superimposed on the center of the earth, within the equatorial diametral plane, then the partitions do not mesh, and this clock is ***not*** a structural model of this relationship. *However, if this natural, but unnecessary, supposition of coincident clock and earth center is discarded, and a theorem from Euclidean geometry invoked, then this twelve hour clock face is not only a precise model of the relationship of longitude to time but it is the most efficient one as well.*[1]

Clocks and Longitude: Direct correspondence of central angles

Represent the Earth as a sphere. Construct two orthogonal great circles, one representing the equator and the great circle containing the prime and the 180th meridians (Figure 3). Inscribe two circles with centers O_1 and O_2 in the equatorial diametral plane of the earth-sphere so that they are mutually tangent at the center of the earth-sphere, O. The centers O, O_1 and O_2 are collinear, and the diameter containing this segment may be positioned so that its end points lie on the meridians as well as on the equator. It is convenient, but not necessary, to view these end points as "12 noon" and "12 midnight."

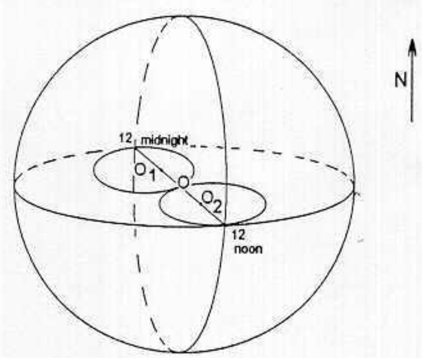

Figure 3. Earth-sphere with equator, prime meridian and 180th meridian; smaller circles, representing outlines of clock faces, are embedded in the diametral plane. The clock faces are centered at O_1 and O_2 and are mutually tangent at O, the center of the earth-sphere.

Because Figure 3 is a two-dimensional view of a three-dimensional solid, the great circles are represented as ellipses for proper perspective. Figure 4 shows only the equatorial plane from Figure 3; thus, the equator is represented as a circle. Let the positions P and Q represent two consecutive numerals on the perimeter of an analogue clock. Let P′ and Q′ be the points where the lines OP and OQ intersect the equator; the center O is used to project P to P′ and Q to Q′. The central angle in the clock face, angle (PO_1Q), has measure 30 (360÷12) degrees. The following Theorem of Euclid will align the central clock face angle measuring one hour with the central earth angle measuring one hour.

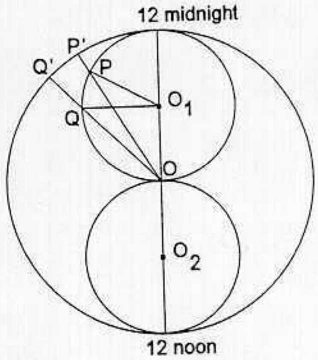

Figure 4. A one-hour central angle (PO1Q) of clock face, of measure 30°, corresponds using a Theorem of Euclid to a one-hour central angle (P'OQ') of the earth-sphere, of measure 15° of longitude.

<u>*Theorem of Euclid*</u>

In a circle the central angle is double the angle at the perimeter when the rays forming the angles meet the perimeter in the same two points,[4] *so that* $\angle POQ = 2(\angle PO'Q)$ *in Figure 5.*

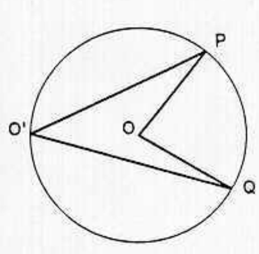

Figure 5. Theorem of Euclid, angle (POQ) is twice angle (PO'Q)

The angle (P'OQ'), a central angle in the equator (Figure 4), measures some amount of longitude as angular distance between meridians. The Theorem of Euclid shows that the measured amount is one-half that of angle (PO1Q) or 30÷2 = 15 degrees of longitude; that is, angle (PO_1Q) = 30 degrees is twice angle (POQ) and this latter angle is the same as angle (P'OQ') or 15 degrees. Thus, this Theorem of Euclid shows that when a clock face is embedded in the earth-sphere's diametral plane so that the clock perimeter, rather than its center, passes through the earth-sphere's center, one hour on the clock (a 30-degree angle) corresponds to one hour on the earth-sphere (15 degrees of longitude).

The choice of O, as a point from which to project the arc PQ on the clock perimeter to the arc P'Q' on the equator was critical. Indeed, had a different position, R, been chosen as a center of projection, this association of time and longitude, using a 12-hour clock face would not have been possible. Had the center of projection been outside the clock face centered on O_1, the projected arc on the equator would be less than 15 degrees. Had the center of projection been within the clock face centered on O_1, the projected arc on the equator would be greater than 15 degrees. Therefore, this style of position for a center of projection does not convert time measured on the clock face to time measured by shifts in longitude resulting from the rotation of the earth on its axis.

The two-circle clock face configuration provides exactly the needed association and it does so in a unique manner. More than two circular clock faces would not provide the required correspondence because more than two circular clock faces would not provide the required point. A single circle configuration cannot provide the correct association between time and longitude for a twelve-hour clock face. Thus, this two-circle configuration is the most efficient model of the relation of time to longitude.

Use of longitude to number clock faces

In the previous section points from clock faces were projected to the earth-sphere. In this section, the projection direction is reversed, in order to number the clock faces in a manner that is consistent with the relation of the earth to the sun. Clock-face numerals will coincide with meridian positions appropriate to the natural ordering established by the rotation of the earth on its polar axis (Figure 6).

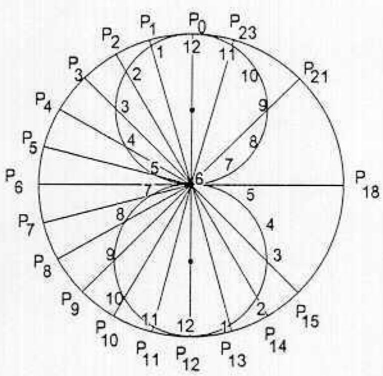

Figure 6. Longitude, and earth-sun relations, are used to determine numerals on clock faces. With midnight assigned the initial position, P_0, on the 180th meridian, the value of P_1, at an angle of 15° of longitude (one hour of rotation time) from the initial position, assigns a numeral of 1 to the corresponding clock position, which is 1 hour (30°) from 12.

As an initial position, join O to 12 midnight, and call this position P_0 on the earth. After one hour, position P_0 will have rotated 15° to position P_1. Label the corresponding position on the clock face with the numeral 1. From the Theorem of Euclid, the spacing between 12 and 1 on the clock face will be such that $\angle P_0O_1P_1 = 30°$, as required. After another hour, P_1 will have rotated through 15° to P_2. The corresponding clock position will be labeled 2 and it will be in the correct location, again because of the Theorem of Euclid. Continue this procedure, shifting to the clock face centered at O_2, once 6, the pivotal position corresponding to P_6, has been reached. Thus, after another hour, P_6 will have rotated to P_7 and the corresponding position of the clock face centered on O_2 will be labeled 7. After 13 hours the point P_0 will have rotated to P_{13}, through 195° of longitude, and two alternate labeling strategies arise.

1) Label the corresponding clock position 13. Continuing this labeling would produce a scheme requiring 24 different positions for numerals on the associated clock faces.

2) The position P_{13} is antipodal in the sphere to the position P_1. Identify (glue together, abstractly) the labels of the associated antipodal clock positions. So, label the clock position corresponding to P_{13} with the antipodal numeral 1. Continuing this process requires only 12 different numerals on the clock faces to describe 24 different positions for P_0. To distinguish the time position P_{13} from that at its antipodal point P_1, two conventions are in use:

a) read the time at P_{13} to be 13:00, as is done by the U.S. military and in continental Western Europe;

b) label the time at P_{13} as 1:00 after the sun's position, or 1:00 p.m. as is done in the United States. Positions in the hemisphere preceding arrival at noon meridian are assigned a.m. suffixes to distinguish them from the times at their antipodal positions in the hemisphere succeeding arrival at the noon meridian.

Either a twelve-hour clock or a 24-hour clock may be used as an analogue model for the relationship between longitude and time that follows the natural ordering created by the rotation of the earth on its axis. Certainly the use of twelve distinct numerals, rather than twenty-four, is more efficient on small clock faces, such as wrist watches, and anywhere reduction of clutter of symbols is significant.

The orientation of the numerals on these clock faces, that look a bit like an opened pocket watch, when viewed from above from the north are "backward" according to our clockwise numbering traditions. When viewed from a southern hemisphere perspective, however, the clockwise orientation emerges; indeed, there is a perspective problem much like the one associated with reading astronomical star charts. Alternatively, one might wonder what sorts of conditions have led northern hemisphere inhabitants to wear southern hemisphere analogue watches – a case for global perspective, perhaps, but certainly as Cipolla has noted, a case for "clock and culture."

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Great Britain #1059, the Greenwich Prime Meridian, now a universally accepted standard reference for longitude and time.

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