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Technical Appendix to

LABOUR SUPPLY: THE ROLES OF HUMAN CAPITAL AND THE EXTENSIVE MARGIN

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Appendix A. Social Security Benefits

Actual Social Security (SS) benefits are calculated in the following steps.

A.1. *Step 1: Calculate Average Indexed Monthly Earnings (AIME)*

In the actual rule (which we simplify somewhat in the text), Social Security taxable earnings (SSTE) are first indexed (adjusted) to reflect real dollar values by using national average wages for the year the beneficiary reaches age 60. The SSTE is also capped by a maximum value and this cap has increased over time (see Table A1).

Once indexed, the 35 highest adjusted SSTE years are selected to compute AIME. If the beneficiary continues to work after applying for SS benefits, AIME is recomputed (and may increase if income from the additional year is higher than one of the previous selected 35 years).

Table A1
Maximum Social Security Taxable Earnings

Year	Amount ($)	Year	Amount ($)	Year	Amount ($)
1937–50	3,000	1986	42,000	2006	94,200
1951–4	3,600	1987	43,800	2007	97,500
1955–8	4,200	1988	45,000	2008	102,000
1959–65	4,800	1989	48,000	2009	106,800
1966–7	6,600	1990	51,300	2010	106,800
1968–71	7,800	1991	53,400	2011	106,800
1972	9,000	1992	55,500	2012	110,100
1973	10,800	1993	57,600		
1974	13,200	1994	60,600		
1975	14,100	1995	61,200		
1976	15,300	1996	62,700		
1977	16,500	1997	65,400		
1978	17,700	1998	68,400		
1979	22,900	1999	72,600		
1980	25,900	2000	76,200		
1981	29,700	2001	80,400		
1982	32,400	2002	84,900		
1983	35,700	2003	87,000		
1984	37,800	2004	87,900		
1985	39,600	2005	90,000		

In the model, since we use the real dollars for all years, the approximate AIME is not adjusted. The maximum SSTE is based on Table A1 but converted to 1999 dollars for all years.[1] Everyone is assumed to be born in 1934. Hence, for example, the maximum SSTE for a dropout is based on years 1934+16 to 1934+74.

A.2. *Step 2: Calculate Primary Insurance Amount (PIA)*

PIA is a piecewise linear function of AIME with three bend points:

$$\text{PIA} = 0.9 \times \text{bendpoint1} + 0.32(\text{bendpoint2} - \text{bendpoint1}) + 0.15 \times (\text{AIME-bendpoint2}).$$

The bend points are determined by the eligible year (the year that the beneficiary reaches age 62). See Table A2 for a list of bend points by year.

Table A2

Bend Points in PIA Formula (in $)

Year first eligible (year reaches age 62)	bendpoint1 ($)	bendpoint2 ($)
1979	180	1,085
1980	194	1,171
1981	211	1,274
1982	230	1,388
1983	254	1,528
1984	267	1,612
1985	280	1,691
1986	297	1,790
1987	310	1,866
1988	319	1,922
1989	339	2,044
1990	356	2,145
1991	370	2,230
1992	387	2,333
1993	401	2,420
1994	422	2,545
1995	426	2,567
1996	437	2,635
1997	455	2,741
1998	477	2,875
1999	505	3,043
2000	531	3,202
2001	561	3,381
2002	592	3,567
2003	606	3,653
2004	612	3,689
2005	627	3,779
2006	656	3,955
2007	680	4,100
2008	711	4,288
2009	744	4,483
2010	761	4,586
2011	749	4,517
2012	767	4,624

[1] Note that these caps were increased over time in nominal terms but they did not always increase in every year. When the nominal caps were fixed (e.g. 1968–71), the real dollar caps used in the model for that period are the average over those years.

A.3. *Step 3: Calculate Benefit Deduction for Early Retirement or Credit for Delayed Retirement*

Everyone is eligible for the SS benefit at age 62, but the 'normal' retirement age gradually increases over time. Benefits are reduced for those who claim prior to their normal retirement age. There is a credit for those who delay claiming their benefit up to age 70 (see Table A3). In the simulated data, based on the 1934 cohort, the normal retirement age is 65. In the actual SS rule, benefits are also adjusted by a cost-of-living factor over time. We did not incorporate this factor as we use real dollars in all years.

Table A3
'Normal' Retirement Age

Year of birth	Normal retirement age (NRA)	Penalty for each year of retirement before NRA (percent of PIA)	Credit for each year of delayed retirement after NRA (percent of PIA)
1924	65	6.67	3
1925–6	65	6.67	3.5
1927–8	65	6.67	4
1929–30	65	6.67	4.5
1931–2	65	6.67	5
1933–4	65	6.67	5.5
1935–6	65	6.67	6
1937	65	6.67	6.5
1938	65, 2 months	6.58	6.5
1939	65, 4 months	6.50	7
1940	65, 6 months	6.43	7
1941	65, 8 months	6.36	7.5
1942	65, 10 months	6.30	7.5
1943–54	66	6.25	8
1955	66, 2 months	6.20	8
1956	66, 4 months	6.15	8
1957	66, 6 months	6.11	8
1958	66, 8 months	6.07	8
1959	66, 10 months	6.03	8
1960 and later	67	6.00	8

Appendix B. Tax on Social Security Benefits and the Earnings Test

The Social Security earnings test and tax on Social Security Benefit are separate rules, although some people view the earnings test as a tax. The earnings test applies to beneficiaries who continue to work (after claiming benefits) and who earn above a threshold. It is refunded at a certain age that has varied over time (see below). The SS tax applies to beneficiaries whose total income net of a certain exemption exceeds a threshold.

B.1. *The Social Security Earnings Test*

For beneficiaries who continue to work and have earnings above a threshold, a portion of benefits may be withheld until the person reaches a certain age (which has varied over time). There are two withholding rates and two thresholds (exempt amounts). The higher rate (50%) and lower threshold apply to beneficiaries who receive benefits before reaching their normal retirement age. The lower rate (33.33%) and higher threshold apply after the beneficiaries reach their normal retirement age. The amount of benefits withheld, $BW(w_t h_t, t, SSinc^*, t_B)$ in (15) in the text, is calculated based on the following formula:

$$BW = \begin{cases} \min\{0.5(w_t h_t - exempt_{low}), SSinc^*\} & \text{if } w_t h_t \geq exempt_{low} \text{ and } t < t_R^N \\ \min\{0.33(w_t h_t - exempt_{high}), SSinc^*\} & \text{if } w_t h_t \geq exempt_{high} \text{ and } t_R^N \leq t \leq t^{NoET} \\ 0 & \text{otherwise,} \end{cases}$$

where $w_t h_t$ is wage income, t is age, $SSinc^*$ is the person's Social Security benefit, t_R^N is the 'normal' retirement age which depends on birth year, and $exempt_{low}$ and $exempt_{high}$ are the low and high exemption amounts given in Table B1. The variable t^{NoET} denotes the age when the earnings test no longer applies. The rules have been changed over time. t^{NoET} was 71 in 1983 or earlier, 69 from 1984 to 1999, and $t^{NoET} = t_R^N$ since 2000.

After the beneficiary reaches age t^{NoET}, the withheld benefit is credited back by adjusting the PIA in a way that is roughly actuarially fair. However, it is believed that many beneficiaries do not understand this rule and treat the earnings test as a tax on earnings. In our model, the parameter θ in (15) captures the perceived fairness of the refund.

Table B1

PIA Exempt Amounts (in $)

	Exempt amount	
Year	Low amount	High amount
1975	2,520	2,520
1976	2,760	2,760
1977	3,000	3,000
1978	3,240	4,000
1979	3,480	4,500
1980	3,720	5,000
1981	4,080	5,500
1982	4,440	6,000
1983	4,920	6,600
1984	5,160	6,960
1985	5,400	7,320
1986	5,760	7,800
1987	6,000	8,160
1988	6,120	8,400
1989	6,480	8,880
1990	6,840	9,360
1991	7,080	9,720
1992	7,440	10,200
1993	7,680	10,560
1994	8,040	11,160
1995	8,160	11,280
1996	8,280	12,500
1997	8,640	13,500
1998	9,120	14,500
1999	9,600	15,500
2000	10,080	17,000
2001	10,680	25,000
2002	11,280	30,000
2003	11,520	30,720
2004	11,640	31,080
2005	12,000	31,800
2006	12,480	33,240
2007	12,960	34,440
2008	13,560	36,120
2009	14,160	37,680
2010	14,160	37,680
2011	14,160	37,680
2012	14,640	38,880

B.2. *Tax on Social Security Benefits*

The tax on Social Security benefits has varied over time. Prior to 1984, benefits were not taxed. From 1984 to 1993, up to 50% of benefits could be included in taxable income if 'provisional income' exceeded a threshold. Since 1993, up to 85% of benefits can be included in taxable income if provisional income exceeds either of two statutory thresholds. Provisional income, *provinc*, is defined as follows:

$$\begin{aligned} provinc = {} & \text{total income} + \text{certain otherwise tax-exempt income} \\ & - \text{deduction (education, moving expenses etc)} + 50\% \text{ of SS benefit.} \end{aligned}$$

Taxable Social Security income ($tSSinc_t$) is then determined by the following formula:

$$tSSinc_t = \begin{cases} 0 & \text{if } provinc \leq T_1 \\ \min\{0.5SSinc^*, 0.5(provinc - T_1)\} & \text{if } T_1 < provinc \leq T_2 \\ \min\{0.85SSinc^*, 0.85(provinc - T_2) \\ \quad +\min(0.5SSinc^*, 0.5(T_2 - T_1))\} & \text{if } provinc > T_2, \end{cases}$$

where the two statutory thresholds, T_1 and T_2, are 25,000 and 34,000 for single, and 32,000 and 44,000 for married couples, respectively. In the model, we use the thresholds for singles. Note that, unlike the PIA bend points, the thresholds T_1 and T_2 are not indexed for inflation. Therefore, it is expected that more people will be affected by these thresholds over time.

Appendix C. Probability of Receiving a Private Pension

We use the data from the Health and Retirement Study (HRS) to estimate the probability of receiving a private pension (16). The sample includes male respondents aged 55–90 in the 1992–2010 waves. The indicator of whether the respondent receives a pension is derived from whether the respondent reported positive pension or annuity income. Because the HRS collected data every 2 years, the lagged pension variable is derived from the previous wave data (i.e. we assume that $dpen_{t-1} = dpen_{t-2}$).

The same sample is used to estimate the average annual pension income for each education group. The figures are converted to 1999 dollars. Conditional on reporting pension income, average annual pensions are \$8,922, \$14,887 and \$23,565 for dropout, high school and college types.

Table C1

Estimates of (16) – *Logit Model for Private Pensions*

		Coefficient estimates	SE
Intercept	q_1	−65.414	8.3489
Age	q_2	2.418	0.3485
Age^2	q_3	−0.030	0.0048
Age^3	q_4	0.000128	0.000022
I(high school)	q_5	5.573	2.5169
Age × I(high school)	q_6	−0.130	0.0711
Age^2 × I(high school)	q_7	0.000776	0.000497
I(college)	q_8	3.369	2.9690
Age × I(college)	q_9	−0.078	0.0848
Age^2 × I(college)	q_{10}	0.000491	0.000599
$dpen_{(t-1)}$	q_{11}	3.387	0.0317

Appendix D. Medical Costs by Age

Medical costs by age are estimated from the Medical Expenditure Panel Survey (MEPS). The sample includes male respondents in the 1996–2009 waves aged 21 or over.

Table D1

Estimates of (18) – *Medical Costs by Age*

		Coefficient est.	SE
Intercept	c_1	105.26	72.35
Age	c_2	−3.92	3.89
Age^2	c_3	0.24	0.05
I(age ≥ 65)	c_4	−169.47	44.85

Appendix E. Estimation of the Tax Function

We use CPS data from 2005 to 2006 to fit the regression in (21):

$$\ln(Tax_t) = -3.9543 + 1.2263 \times \ln(TI_t)$$
$$(0.0624) \quad (0.00063).$$

Here *Tax* is the sum of Federal and State income tax liability, converted to 1999 dollars. The variable *TI*, taxable income, consists of adjusted gross income minus allowable itemised deductions (or a standard allowance amount) and exemptions for the taxpayer and his or her dependents. The numbers in the parentheses are the standard errors.

The CPS tax-related variables are not determined by direct questioning of respondents. Rather, values for these variables come from the Census Bureau's tax model, which simulates individual tax returns to produce estimates of Federal, State, and payroll taxes. The model incorporates information from non-CPS sources, such as the Internal Revenue Service's Statistics of Income series, the American Housing Survey, and the State Tax Handbook.

The sample includes males with positive total income and tax filing status as single. We use 2005–6 data because some of tax variables are only available from 2005 onwards. The regression provides a good fit to the CPS data (R^2 = 0.936) (Figure E1).

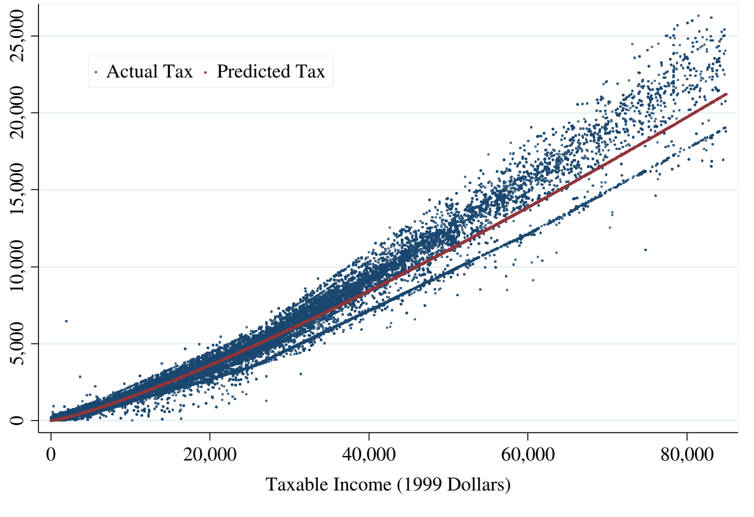

Fig. E1. *Scatter Plot of Actual and Predicted Taxes Versus Taxable Income* (1999$)

Appendix F. Calculation of Compensation for Tax Changes

In experiments with a permanent compensated tax change at t', we assume that the change is a surprise at t' but the agent anticipates the new tax regime to last until T. Between period t' and T, the agent also knows he will receive an exogenous (non-taxable) compensation amount, x. Let S_t denote a set of state variables where $S_t = \{A_t, k_t, AIME_{t-1}, p_{t-1}, dpen_{t-1}, ss_{t-1}, age_{ss}\}$. The state variables vary across t (see Section 3 in the paper for more details).

In all experiments, we first simulate behaviour under the baseline situation in two steps:

(*i*) Use backward induction to solve the optimisation problem from T: $t_0(e)$ under the baseline situation (tax^0) and save $EV(S_t \mid tax^0)$ for each grid point.

(*ii*) Simulate the behaviour from $t_0(e)$ to T given the initial condition A_0, k_0, p_0 for $i = 1, \ldots, N$ for this baseline situation where N is the number of simulated workers at $t_0(e)$.

To simulate the effect of a compensated tax change, we next:

(*iii*) Calculate the annual compensation amount, x, for a permanent tax change at t' using information from step (*ii*) (see details below). x is calculated for each education group.

(*iv*) Resolve the optimisation problem (similar to step ***i***) except now an agent faces a new tax scheme and receives an exogenous (non-taxable) income amount, x, from T: t'. The $EV(S_t|tax^1, x)$ for all grid points are saved.

(*v*) Simulate the behaviour from $t_0(e)$ to T given the initial condition A_0, k_0, p_0 for all i under the new tax regime. For a surprise tax change, the agents use $EV(S_t \mid tax^0)\}$ in their optimisation from $t_0(e) : t' - 1$. Once at t', they expect the new tax regime to be permanent, and they use $EV(S_t \mid tax^1, x)\}$ in their optimisation. They also receive the compensation, x, from t' onward.

Calculation of compensated income (based on the Slutsky approach)

(*a*) From step (*ii*), save optimal hours worked $\{h^*_{it}(.|tax^0)\})\}$ and the total tax payment, $tax(w_{it}h^*_{it}, .|tax^0)$. The tax payment depends on labour income and other sources of income (see (21) in the paper).

(*b*) Calculate the tax payment under the new tax situation, holding hours of work fixed at baseline levels: $tax(w_{it}h^*_{it}, .|tax^1)$.

(*c*) Calculate year-by-year income differences between the baseline and the new tax regime (with hours of work held fixed). We denote these differences by Δ_{it}, where:

$$\Delta_{it} = tax(w_{it}h^*_{it}, .|tax^1) - tax(w_{it}h^*_{it}, .|tax^0) \text{ for all } i, t.$$

(*d*) In order to avoid having the compensation stream become a new person-specific state variable (which would greatly slow down solution of the DP problem), we find an average compensation stream that is common to all agents (conditional on their education). We found that tailoring the compensation to observed income streams did not make much difference, presumably because most heterogeneity of income (within type) is unpredictable. To proceed, for each year t, we calculate the average income differences for people who are alive in that period:

$$E_i[\Delta_t] = \sum_{i=1}^{N_t} \Delta_{i,t}/N_t \text{ where } N_t \text{ is the number of people alive at } t.$$

(*e*) Calculate the present value of $E_i[\Delta_t]$, discounted to the period of the tax change t':

$$PV[E\Delta_t] = \frac{E_i[\Delta_t]}{(1+r)^{(t-t')}}.$$

(*f*) Calculate the expected present value of the total compensation from t' to T, taking into account the survival probabilities:

$$E[PV] = E_i[\Delta_t] + s_{t'+1}\frac{E_i[\Delta_{t'+1}]}{(1+r)} + s_{t'+2}\frac{E_i[\Delta_{t'+2}]}{(1+r)^2} + \ldots + s_{T-1}\frac{E_i[\Delta'_{T-1}]}{(1+r)^{T-1-t'}},$$

where $s_{t'+t} = s_{t'+t|t} \ldots s_{t'+t|t+t-1}$ is the probability of surviving to $t' + t$.

(*g*) Construct a *constant* stream of compensation that has the same present value as $E[PV]$, i.e., solve for x in:

$$E[PV] = x + s_{t'+1}\frac{x}{(1+r)} + s_{t'+2}\frac{x}{(1+r)^2} + \ldots + s_{T-1}\frac{x}{(1+r)^{T-1-t'}}.$$

Rearranging yields:

$$x = E[PV]\left(1 + \frac{s_{t'+1}}{1+r} + \frac{s_{t'+2}}{1+r^2} + \ldots + \frac{s_{T-1}}{1+r^{T-1-t'}}\right)^{-1}.$$

The compensations (x) for the flat tax rate experiment when implemented at $t = t_0(e)$ are \$671, \$1,209, \$2,041 for the dropout, HS and college groups, respectively. For the experiment where we changed the intercept (slope) of the tax function, the x are \$423 (\$451), \$1,215 (\$1,199) and \$2,314 (\$2,794) for the dropout, HS and college groups, respectively.

Appendix G. Calculating Elasticities for Changes in the Tax Function Intercept and Slope

The tax function used in our article is as follows:

$$Tax_{i,t} = \exp[a^0 + b^0 \times \ln(TI_{i,t})].$$

At the baseline situation, this is given by (21):

$$Tax_{i,t} = \exp[-3.9543 + 1.2263 \times \ln(TI_{i,t})].$$

G.1. *Changes in the Tax Function Intercept*

In one simulation, we increase the constant term in the tax function (21) from −3.9543 to −3.7048. This corresponds to a 28% increase in the tax rate on taxable income, where:

$$\text{tax rate} = \frac{Tax_{i,t}}{TI_{i,t}} = \tau_{i,t}.$$

By changing a^0, $\tau_{i,t}$ is increased by the same percentage for all levels of $TI_{i,t}$:

$$\%\Delta\tau_{i,t} = \exp(a^1 - a^0) - 1.$$

Labour supply elasticities are typically reported with respect to percentage changes in the after-tax rate $(1 - \tau)$, rather than changes in tax rates (τ). Note that a $\delta\%$ increase in the tax rate corresponds to a $-\delta\tau/(1 - \tau)\%$ decrease in the after-tax rate:

$$\%\Delta(1-\tau) = -\tau/(1-\tau) \times \%\Delta\tau. \tag{G.1}$$

Although $\%\Delta\tau$ is constant for all i, t, $\%\Delta(1 - \tau)$ depends on τ which varies across i and t. We consider three different approaches to calculate this quantity:

G.1.1. *Approach 1*

To calculate the elasticity of lifetime hours changes with respect to $(1 - \tau)$, we need:

$$\%\Delta hours/\%\Delta(1-\tau).$$

The numerator is calculated in a usual way:

$$\%\Delta hours = \frac{\sum_i \sum_{t=t_0(e)}^{74} hours_{i,t}^1 - \sum_i \sum_{t=t_0(e)}^{74} hours_{i,t}^0}{\sum_n \sum_{t=t_0(e)}^{74} hours_{i,t}^0}.$$

The denominator $\%\Delta(1-\tau)$ can be estimated as follows:

$$\%\Delta(1-\tau) = -\overline{\tau}/\overline{(1-\tau)} \times \%\Delta\tau,$$

where

$$\overline{\tau} = \frac{\sum_{t=t_0(e)}^{74} \sum_{Nt} \tau_{i,t}^0}{\sum_{t=t_0(e)}^{T} Nt} \text{ and } \overline{(1-\tau)} = \frac{\sum_{t=t_0(e)}^{74} \sum_{Nt}(1-\tau_{i,t}^0)}{\sum_{t=t_0(e)}^{T} Nt} \tag{G.2}$$

and N_t denotes the number of people living in each period; 74 is the last working age.

For the first experiment in Section 6.2, (G.2) gives Marshallian elasticities of 0.58, 0.39 and 0.35 for dropout, high school and college workers, and Hicks elasticities of 1.17, 0.82 and 0.87 respectively.

G.1.2. *Approach 2*

A second approach is to calculate the average of the factor $\tau/(1-\tau)$ directly as:

$$\overline{\left[\frac{\overline{\tau}}{(1-\tau)}\right]} = \frac{\sum_{t=t_0(e)}^{74} \sum_{Nt} \left[\tau_{i,t}^0 / \left(1-\tau_{i,t}^0\right)\right]}{\sum_{t=t_0(e)}^{T} N_t}. \tag{G.3}$$

Equation (G.3) gives Marshallian elasticities of 0.56, 0.38 and 0.34 for dropout, high school and college workers, and Hicks elasticities of 1.12, 0.78 and 0.83 respectively.

G.1.3. *Approach 3*

An alternative approach is to calculate the elasticity for each person and each period, and then average over people and time. For each person, we still need to calculate his average tax rate because tax rates also vary across t. For instance:

$$\varepsilon_{i,t} = \%\Delta hours_{i,t} / \%\Delta\left(1-\tau_{i,t}\right).$$

A naïve application of this approach will not work because some people do not work in some periods (causing any percentage changes in hours from zero to positive to be infinite).

Instead, we consider calculating aggregate elasticities by each age and then taking a weighted average by hours at the baseline situation for the lifetime elasticity:

$$\varepsilon_{lifetime} = \sum_{t=t_0(e)}^{74} w_t \varepsilon_t, \tag{G.4}$$

where $w_t = hours_t^0 / \sum_t hours_t^0$:

$$\varepsilon_t = \%\Delta hours_t / \%\Delta(1-\tau)_t$$

$$\%\Delta hours_t = \frac{\sum_i hours_{i,t}^1 - \sum_i hours_{i,t}^0}{\sum_i hours_{i,t}^0}, \quad \%\Delta(1-\tau)_t = -\overline{\left[\frac{\tau}{(1-\tau)}\right]}_t \times \%\Delta\tau$$

and

$$\overline{\left(\frac{\tau}{(1-\tau)}\right)}_t = \frac{\sum_{Nt} \left[\tau_{i,t}^0/(1-\tau_{i,t}^0)\right]}{N_t}, t = 16/18/22 \ldots 74.$$

Equation (G.4) gives Marshallian elasticities of 0.50, 0.38 and 0.35 for dropout, high school and college workers and Hicks elasticities of 1.08, 0.78 and 0.85 respectively.

Notice that elasticities calculated using the three methods are very similar.

G.2. *Changes in the Tax Function Slope*

For a change in the slope, b^0, the expression $\%\Delta(1-\tau) = -\tau/(1-\tau) \times \%\Delta\tau$ still holds. Here, however, $\%\Delta\tau$ is no longer a constant. If we increase the slope to b^1, $\tau_{i,t}$ is increased proportionally more for people with higher taxable income:

$$\%\Delta\tau_{i,t} = TI_{i,t}^{(b^1-b^0)} - 1. \tag{G.5}$$

We can consider three approaches to calculate $\%(1-\tau)$, similar to the case of changing the intercept, but incorporating the fact that $\%\Delta\tau$ varies across i and t.

G.2.1. *Approach 1*

$$\%\Delta(1-\tau) = -\overline{\tau}/\overline{(1-\tau)} \times \overline{\%\Delta\tau}.$$

The terms $\overline{\tau}$ and $\overline{(1-\tau)}$ are given in (G.2), and:

$$\overline{\%\Delta\tau} = \frac{\sum_{t=t_0(e)}^{74}\sum_{N_t}\left(TI_{i,t}^{(b^1-b^0)} - 1\right)}{\sum_{t=t_0(e)Nt}^{T}}.$$

For the second experiment in Section 6.2, this approach gives Marshallian elasticities of 0.71, 0.47 and 0.43 for dropout, high school and college workers and Hicks elasticities of 1.34, 0.93 and 1.03 respectively.

G.2.2. *Approach 2*

A second approach is to calculate the average of the factor $\left(\frac{\tau}{1-\tau}\right) \times \%\Delta\tau$ directly as:

$$\overline{\left[\frac{\tau}{(1-\tau)}\right]_t} = \frac{\sum_{N_t}\left[\frac{\tau_{i,t}^0}{(1-\tau_{i,t}^0)}\right] \times \left[TI_{i,t}^{b^1-b^0} - 1\right]}{N_t}.$$

This approach gives Marshallian elasticities of 0.66, 0.44 and 0.41 for dropout, high school and college workers and Hicks elasticities of 1.24, 0.87 and 0.97 respectively.

G.2.3. *Approach 3*

A third approach is similar to (G.4) where the only change is the term $\%\Delta(1-\tau)_t$:

$$\%\Delta(1-\tau)_t = -\overline{\left[\frac{\tau}{(1-\tau)}\right]\%\Delta\tau_t} = \frac{\sum_{N_t}\frac{\tau_{i,t}^0}{(1-\tau_{i,t}^0)} \times \left[TI_{i,t}{}^{(b^1-b^0)} - 1\right]}{N_t}.$$

This approach gives Marshallian elasticities of 0.60, 0.46 and 0.43 for dropout, high school and college workers and Hicks elasticities of 1.24, 0.87 and 0.99 respectively.

Notice again that elasticities calculated using the three methods are very similar.