Ph 230 Particle Physics (R.P. Feynman) 10/1/68 LECTURE
TWF 11:00 am

Remarkable that there are no applications of high energy physics -- either to technology or in nature. No application to any other sciences. What role do they play? Seem at this stage to be irrelevant to the universe.

Best book Gasiorowicz.

Will only concentrate on theories which can reproduce experimental data to some degree.

—o—

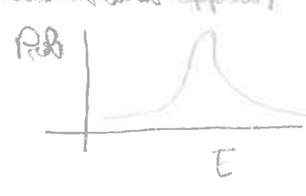
General Beliefs

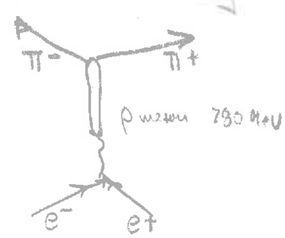
i.) general ideas of q.m. and relativity are correct [such as superposition of states, conservation laws, etc.]. Some people think q.m. + rel. demand a field theory and imply TCP. [Feynman thinks something else is needed -- like causality]

ii.) TCP and locality

iii.) nature can be separated into different types of interactions (E+M, weak, strong, gravitation). Different interactions are independent of one another. [Heisenberg doesn't believe this.] [but CP violation]

iv.) resonances in intermediate states appear





Width to ρ-state ⇒ unstable.

Theoretical Discoveries of Importance

i.) Electrodynamics -- current operator $j_\mu(x,t)$

weak interactions j_μ^V, j_μ^A

ii.) strong interactions

approx. SU_3 symmetry

exact current commutation laws ($SU_3 \times SU_3$)

approx. Goldberger Treiman (PCAC) -- the way a pion interacts can be expressed in terms of one of the j_μ.

$\rho, \varphi, \omega, K^*$ coupled via j_μ

Regge trajectories

i.) puts multiplets together

ii.) explicit energy dependence of various very high energy collisions ($E \to \infty$)

iii.) extension of amplitudes by analytic continuation beyond physical values of variables (dispersion relations)

10/2/68 LECTURE

Really only worried about scattering experiments, and in particular about the asymptotic ingoing and outgoing state.

Initial state: $(p_i') = (p_1', p_2', p_3', \dots)$ $\qquad P_i' = \sum p_i'$ $\qquad$ (4 vectors throughout)

Final state: $(p_f) = (p_1, \dots)$ $\qquad P_f = \sum_f p_f$

Amplitude that if we start with state i, we end up with state f is the scattering matrix S_{fi}

$$S_{fi} = \underbrace{\delta_{fi}}_{\text{no interaction}} - (2\pi)^4 i\, \underbrace{\delta^4\Big(\sum_f p_f - \sum_i p_i\Big) T_{fi}}_{\text{interaction (+ conservation law)}}$$

Then

$$\underset{i\to f}{\text{Rate}} = \left(\prod_i \frac{1}{2\epsilon_i}\right)(2\pi)^4\, \delta^4\Big(\sum p_f - \sum p_i\Big)\, |T_{fi}|^2 \left[\prod_f 2\pi\, \delta(p_f^2 - m_f^2)\, \frac{d^4 p_f}{(2\pi)^4}\right]$$

Now notice two examples which demonstrate invariance

i.) one particle disintegrates, ϵ_1

$$\text{Rate} = \frac{1}{\tau} = \frac{1}{\epsilon_1} \text{ invariant}$$

ii.) 2 particles colliding

$$\text{Rate} = \sigma |v_1 - v_2|$$

Can show $\quad 2\epsilon_1 2\epsilon_2 |v_1 - v_2| = 4\sqrt{(p_1 p_2)^2 - p_1^2 p_2^2}$

Hence again invariant.

An incoming scalar particle wave fcn is normalized to 1

vector " " e_μ is normalized $\quad e_\mu e_\mu = -1$

spinor u $\quad \not{p} u = m u, \quad (\bar{u} u) = 2m$

$[a_\mu b_\mu \equiv a_4 b_4 - \vec{a}\cdot\vec{b}]$

Electrodynamics:

Propagators:

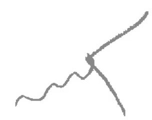
electron or muon $\qquad \dfrac{i}{\not p - m + i\epsilon}$

spin zero $\qquad \dfrac{i}{p^2 - m^2 + i\epsilon}$

vector $\qquad -i\,\dfrac{\delta_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}}{p^2 - m^2 + i\epsilon}$

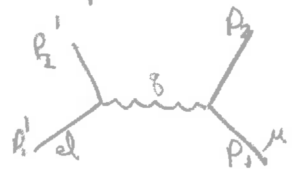
photon $\qquad -i\,\dfrac{\delta_{\mu\nu}}{p^2 + i\epsilon}$

Coupling for electrodynamics



$$-i\sqrt{4\pi e^2}\,\gamma_\mu$$

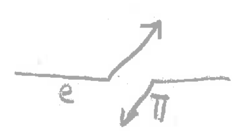
Example



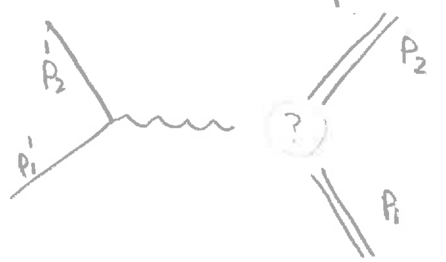
$$T = 4\pi e^2 \frac{1}{q^2}\left(\bar u_2' \gamma_\mu u_1'\right)\left(\bar u_2 \gamma_\mu u_1\right)$$

What about strong interactions? How do we find T? Don't know how. First consider an intermediate step. Electrons interacting with strongly interacting particles -- e.g. protons.

Electron-Pion elastic scattering



Assume one photon process -- but how do we handle photon pion coupling



Propose

vector

$$T = -i\,\frac{4\pi e^2}{q^2}\,(\bar{U}_2'\,\gamma_\mu\,U_1')\,(\text{pion}_2\,|\,j_\mu\,|\,\text{pion}_1)$$

All we know is that $(\text{pion}_2|j_\mu|\text{pion}_1)$ is a vector depending on P_1, P_2, q.

or

$$(\text{pion}_2|j_\mu|\text{pion}_1) = P_{1\mu}\,A + P_{2\mu}\,B$$

where A is invariant fn of P_1 and P_2. Only invariants are $P_1^2, P_2^2, P_1\cdot P_2$
But $P_1^2 = P_2^2 = m_\pi^2$. But $q^2 = P_1^2 + P_2^2 - 2P_1\cdot P_2 = 2(m^2 - P_1 P_2)$. Hence $A = A(q^2)$.
Frequently written as

$$P_{1\mu}\,A(q^2) + P_{2\mu}\,B(q^2) = (P_{1\mu}+P_{2\mu})\,C(q^2) + (P_{1\mu}-P_{2\mu})\,D(q^2)$$

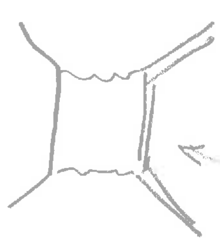
form factors

In fact can show $D(q^2)=0$. Hence

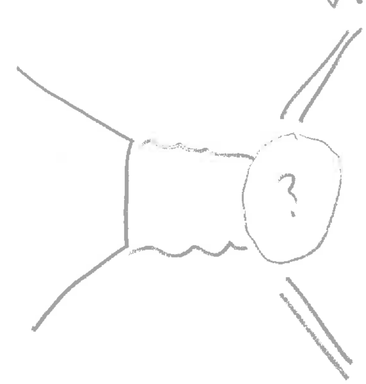
$$(\text{pion}_2|j_\mu|\text{pion}_1) = (P_{1\mu}+P_{2\mu})\,F_\pi(q^2)$$

In a certain sense, the form factor $F_\pi(q^2)$ describes the pion charge distribution.

Not Suppose we had to calculate the second order effect,



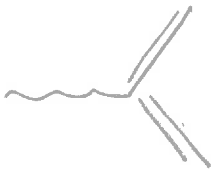
but rather



since we don't know that pion remains a pion between photon interactions

Hence in this sense, the usual perturbation theory doesn't work.

PROBLEM: Electron Proton Scattering



$\langle \text{proton}_2 | j_\mu | \text{proton}_1 \rangle = \langle \bar{u}_2 \; \gamma_\mu \; u_1)$

what do we put in here

Show this can written as

$$(\bar{u}_2 | \gamma_\mu F_1(q^2) + \tfrac{1}{2}(\not{q}\gamma_\mu - \gamma_\mu \not{q}) F_2(q^2) | u_1)$$

Now show

$$\sigma = \frac{e^4}{4E^2 \sin^4 \theta/2} \frac{\cos^2 \theta/2}{1 + \frac{2E}{M}\sin^2 \theta/2} \left\{ F_1^2 - \frac{q^2}{4M^2} |4M^2 F_2^2 + 2(F_1 + 2MF_2)|^2 \tan^2 \theta/2 \right\}$$

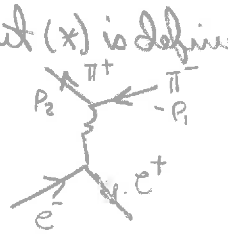
Now recall 10/4/68 LECTURE

$$\langle \text{pion} | j_\mu | \text{pion} \rangle = (P_{1\mu} + P_{2\mu})\, C(q^2)$$

Now want Fourier inversion

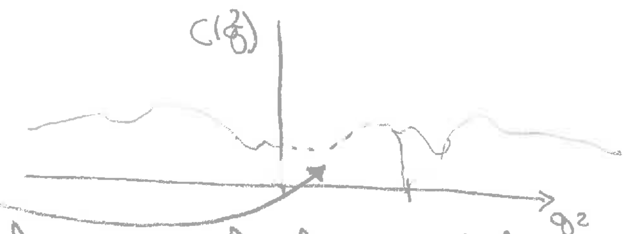
$$j(x) = \int C(q^2)\, e^{iq\cdot x}\, d^4q$$

But (*) is defined only for $q^2 < 0$. Hence $j(x)$ is not defined. Consider a pair production reaction. Then get



$$C(q^2) \qquad q^2 > 4m_\pi^2$$

But these aren't connected.
Need a "analytic continuation"
through the unphysical
region



Hence the idea of a "local" operator $j(x)$ is closely connected with analytic continuation and dispersion relations.

Electron - Proton Scattering:

$$\langle \text{proton}_2 | j_\mu | \text{proton}_1 \rangle = (\bar{u}_2 \gamma_\mu u_1) A(q^2) + (\bar{u}_2 (\not{P}_2 \gamma_\nu) u_1)\left[P_{2\mu} B(q^2) + P_{1\mu} C(q^2)\right] + \cdots$$

$\not{p}_1$; $\not{p}_1 \not{p}_2 \cdots$

Actually the only scalar operator is $(\bar{u}_2 u_1)$ since $\not{p}_1 u_1 = m^2 u_1 \cdots$
Same idea for vectors. Also use $q_\mu j_\mu = 0$. Hence in standard form

$$\langle \text{proton}_2 | j_\mu | \text{proton}_1 \rangle = (\bar{u}_2 \gamma_\mu u_1)\left[F_1(q^2) + 2M F_2(q^2)\right] - (P_{1\mu} + P_{2\mu})(\bar{u}_2 u_1) F_2(q^2)$$

$$= \langle \bar{u}_2 | F_1(q^2)\gamma_\mu + \tfrac{1}{2}(\not{q}\gamma_\mu - \gamma_\mu \not{q}) F_2(q^2) | u_1 \rangle$$

Actually the form factors are sometimes given as

$$G_E(q^2) = F_1 + \frac{q^2}{2M} F_2 \qquad \text{electric form factor} \qquad G_E(0) = 1$$

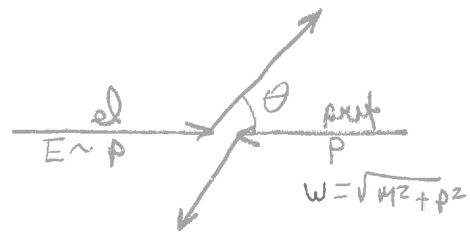
$$G_M(q^2) = F_1 + 2MF_2 \qquad \text{magnetic form factor} \qquad G_M(0) = 2.79$$

Now can plug back into find

$$T = \frac{4\pi e^2}{q^2} \left(\bar{u}_2' \gamma_\mu \bar{u}_1'\right) \left(\bar{u}_2 \left| (F_1 + 2MF_2)\gamma_\mu - (P_{1\mu} + P_{3\mu}) \underline{\underline{I}} \right| u_1\right)$$

Now

Rate $= \sigma V$



10/8/68 LECTURE

1.) Find the cross-section for electron-proton scattering of $\langle prot_2 | J_\mu | prot_1 \rangle = \bar{u}_2 \left(\gamma_\mu F_1 + \frac{1}{2} (\not{q} \gamma_\mu - \gamma_\mu \not{q}) F_2 \right) u_1$

2.) Find a.) the form of matrix element of a scalar particle which disintegrates to 2γ

b.) " " " " " $\Lambda \to p + \pi$ (no cons. of parity)

3.) For $\pi + p_1 \to p_2 + \pi$ write the amplitude as $u_{2\,proton} (M)\, u_{1\,proton}$ and find an expression for M with least no. of functions

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Notation: $\gamma_t = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$; $\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$; $i\gamma_x \gamma_y = \sigma_z = \begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}$; $\gamma_5 = i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Note: γ_5 is a pseudovector. Also $\epsilon_{\mu\nu\sigma\rho}$ is a pseudoscalar.

Example of Pseudoscalar Particle Decay: $\pi \to 2\gamma$

$k_1 + k_2 = p$

Argues that matrix must have form

$$e_{1\mu} e_{2\nu} \epsilon_{\mu\nu\sigma\rho} k_{1\sigma} k_{2\rho} \underbrace{A(\text{invariants})}_{\text{const.} = a}$$

Can verify that this satisfies gauge invariance.

Now finish with EM interactions for awhile.

WEAK INTERACTIONS

$n \to p + e^- + \bar\nu$ or $p + e^- \to n + \bar\nu$

$\mu^- \to e^- + \bar\nu + \nu'$

$\pi^- \to \mu^- + \bar\nu_\mu$

$\mu^- + p \to n + \nu'$

$\pi^- \to e^- + \bar\nu_e$

$K \to \mu + \nu$

$\Lambda \to p + e^- + \nu$, $\Lambda \to p + \mu + \nu$

all involve ν and are very slow $O(10^{-5})$

weak decays or β decays

$\Lambda \to p + \pi$, $\overset{\Theta}{K} \to 2\pi$, $\overset{\tau}{K} \to 3\pi$, etc. -- violate strangeness, hence can't be strong. Since of $O(10^{-5})$, assume them to be weak interactions

Also violate parity is violated [$K \to 2\pi$, $K \to 3\pi$ decays. Old θ-τ ~~theory~~ — mentions M. Bloch incident -- first suggestion of parity violation. Push crazy ideas only when you're young.]

Fermi proposed we just write an amplitude for this process.

$$n \to p + e^- + \bar\nu$$

Played the same game we've been playing

$$(\bar u_n u_p)(\bar u_\nu u_e)$$

or

$$(\bar u_n \gamma_\mu u_p)(\bar u_\nu \gamma_\mu u_e)$$

$$\sigma_{\mu\nu} \qquad \sigma_{\mu\nu}$$

$$\gamma_5\gamma_\mu \qquad \gamma_5\gamma_\mu$$

Some linear combination of these. But might have violation of parity, so add in

$$+ \ (\bar{u}_n \gamma_5 u_p)(\bar{u}_\nu u_e)$$
$$+$$

Now experiments indicate the present form. Landau suggested ν spin only to left. Define a projection operator

$$a = \frac{1+i\gamma_5}{2} \qquad a^2 = a \qquad \left[\bar{a} = \frac{1-i\gamma_5}{a}, \ a\bar{a} = 0\right]$$

Then use

$$\left(\frac{1+i\gamma_5}{2}\right) u_\nu$$

Try

$$(\overline{a u_n} \gamma_\mu a u_p)(\overline{a u_\nu} \gamma_\mu a u_e)$$
$$(\bar{u}_\nu \bar{a} \gamma_\mu a u_e)$$
$$(\bar{u}_2 \gamma_\mu a u_e)$$

[Rule: every wave fn gets an a in front of it]

This knocks out all but vector coupling.

10/9/68 LECTURE

Notation: $A+B \to C+D$

$(\bar{C}A)_\mu(\bar{D}B)_\mu$ means (at least for "ideal" particles

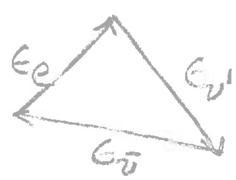
$$(\bar{C}A)_\mu = \left(\bar{u}_c \gamma_\mu \left(\frac{1+i\gamma_5}{2}\right) u_a\right) \quad \text{etc.}$$

Consider $\mu^- \to e^- + \nu' + \bar{\nu}$

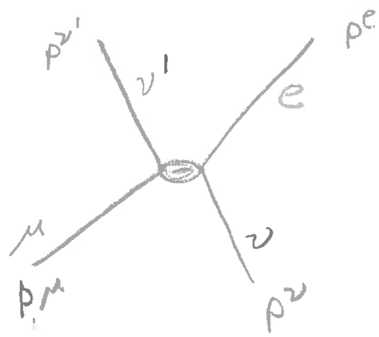
Write amplitude as $(\bar{e}\nu)_\nu(\bar{\nu}'\mu)_\mu$ or a VA coupling

$$G\sqrt{8}\left(\bar{u}_e \left(\gamma_\alpha\left(\frac{1+i\gamma_5}{2}\right)u_\nu\right)\left(\bar{u}_{\nu'}\,\gamma_\alpha\left(\frac{1+i\gamma_5}{2}\right)u_\mu\right)\right.$$

This is what we think β-decay looks like (at least today). To illustrate this, Feynman will work it out in detail



$$\epsilon_e + \epsilon_{\nu'} + \epsilon_{\bar{\nu}} = \mu$$



$$p^\mu + p^\nu = p^e + p^{\nu'}$$

$$\text{Rate} = \frac{1}{\tau} = \frac{1}{2m_\mu}\,\frac{2\pi\,\delta(\epsilon_\nu + \epsilon_{\bar{\nu}} + \epsilon_e - m_\mu)}{(2\epsilon_e)(2\epsilon_\nu)(2\epsilon_{\nu'})}\,|T|^2\,\frac{d^3p_e\,d^3p_\mu}{(2\pi)^6}$$

$$|T|^2 = G^2 8\left(\bar{u}_\mu \gamma_\beta\left(\frac{1+i\gamma_5}{2}\right)u_{\nu'}\right)\left(\bar{u}_\nu\left(\frac{1-i\gamma_5}{2}\right)\gamma_\beta u_e\right)\left(\bar{u}_e\gamma_\alpha\left(\frac{1+i\gamma_5}{2}\right)u_\nu\right)\left(\bar{u}_{\nu'}\gamma_\alpha\left(\frac{1+i\gamma_5}{2}\right)u_\mu\right)$$

$$\gamma_\beta\left(\frac{1+i\gamma_5}{2}\right)$$

Now stick in spinors for various cases. We'll calculate for total unpolarized decay. Hence sum over all spin states. Now use trick

$$\sum_{\substack{\text{spins}\\ \text{of } e}} (\cdots\cdots u_e)(\bar{u}_e \cdots\cdots) = \cdots\cdots (\not{p}_e + m_e) \cdots\cdots$$

↑ neglect mass of electron

Also

$$\sum (\bar{u}_\nu A u_\nu) = Sp\left[(\not{p}_\nu + \overset{0}{m})A\right]$$

Then

$$|T|^2 = 4G^2 Sp\left[\not{p}_\mu \gamma_\beta \left(\frac{\cancel{1+i\gamma_5}}{\cancel{2}}\right) \not{p}_e \gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right)\right] Sp\left[\not{p}_\nu \gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right)(\not{p}_\mu + \cancel{m_\mu}) \gamma_\beta \left(\frac{1+i\gamma_5}{2}\right)\right]$$

Now

$$Sp\left[\not{p}_\mu \gamma_\beta \not{p}_e \gamma_\alpha (1+i\gamma_5)\right] = Sp\left[\not{p}_\mu \gamma_\beta \not{p}_e \gamma_\alpha\right] + i\, Sp\left[\not{p}_\nu \gamma_\beta \not{p}_e \gamma_\alpha \gamma_5\right]$$

[~~Fact~~:

$$\frac{1}{4} Sp\left[(\not{p}_1 - m_1)(\not{p}_2 + m_2)(\not{p}_3 - m_3)(\not{p}_4 - m_4)\right]$$

$$= (p_1 \cdot p_2 - m_1 m_2)(p_3 \cdot p_4 - m_3 m_4) + (p_2 \cdot p_3 - m_2 m_3)(p_1 \cdot p_4 - m_1 m_2)$$
$$- (p_1 \cdot p_3 - m_1 m_3)(p_2 \cdot p_4 - m_2 m_4)$$

Another Fact:

$$\frac{1}{4} Sp\left[\not{a} \not{b} \not{c} \not{d} \gamma_5\right] = -\epsilon_{\nu\mu\sigma\rho}\, a_\mu b_\nu c_\sigma d_\rho$$

$\epsilon_{txyz} = +1$]

Hence

$$Sp\left[\not{p}_\mu \gamma_\beta \not{p}_e \gamma_\alpha\right] = p_\beta^\nu p_\alpha^e + p_\beta^e p_\alpha^\nu - (p^\nu \cdot p^e)\,\delta_{\alpha\beta}$$

$$Sp\left[\not{p}_\nu \gamma_\beta \not{p}_e \gamma_\alpha \gamma_5\right] = -\epsilon_{\lambda\beta\sigma\alpha}\, p_\lambda^\nu p_\sigma^e$$

Finally

$$|T|^2 = 16G^2(p^\mu \cdot p^\nu)(p^e \cdot p^{\nu'})$$

Now recall 10/11/63 LECTURE

$$\text{Rate} = \frac{1}{2m_\mu} 2\pi\,\delta(\epsilon_{\bar\nu}+\epsilon_{\nu'}+\epsilon_e - m_\mu)\,T^2\;\frac{\overset{\epsilon_e^2 d\epsilon_e d\Omega_e}{d^3P_e}}{(2\pi)^3}\;\frac{\overset{\epsilon_{\bar\nu}^2 d\epsilon_{\bar\nu} d\Omega_{\bar\nu}}{d^3P_{\bar\nu}}}{(2\pi)^3}\;\frac{1}{2\epsilon_e 2\epsilon_\nu 2\epsilon_{\nu'}}$$

Now consistent with our neglect of m_e, $2p^e \cdot p^{\nu'} = m_\mu^2 + 2(p^\mu \cdot p^\nu)$

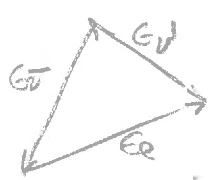
Hence $= m_\mu^2 - 2\mu\,\epsilon_\nu$

$$|T|^2 = 16G^2(m_\nu\,\epsilon^\nu)(m_\mu^2 - 2m_\mu\,\epsilon^\nu)$$

or

$$\text{Rate} = (\#)\,G^2\delta(\epsilon_{\bar\nu}+\epsilon_{\nu'}+\epsilon_e - m_\mu)\,m_\mu^2\,\epsilon_{\bar\nu}\,(m_\mu - 2\epsilon_{\bar\nu})\underbrace{\epsilon_e^2 d\epsilon_e\, d\Omega_e\;\epsilon_{\bar\nu}^2 d\epsilon_{\bar\nu}\, d\Omega_{\bar\nu}}_{d\epsilon_e\, d\epsilon_{\bar\nu}\, d\epsilon_{\nu'}}$$

where we have used



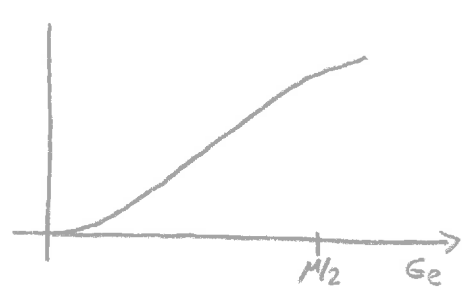
$$\epsilon_{\nu'}^2 = \epsilon_{\bar\nu}^2 + \epsilon_e^2 - 2\epsilon_{\bar\nu}\,\epsilon_e\cos\theta_{\nu e}$$

$$\therefore\; d\Omega_\nu = 2\pi\, d(\cos\theta_{\nu e}) = \frac{2\pi\,\epsilon_{\nu'}\, d\epsilon_{\nu'}}{\epsilon_{\bar\nu}\,\epsilon_e}$$

Now from geometry $\epsilon_{\bar\nu} = \frac{m_\mu}{2} - \epsilon_e$ to $\frac{m_\mu}{2}$ as integration limits on $\epsilon_{\bar\nu}$

Finally get

$$\text{Rate} = \frac{G^2 m_\mu}{24\pi^3}(3m_\mu - 4\epsilon_e)\,\epsilon_e^2\, d\epsilon_e$$



Hence total rate is

$$R_T = \frac{G^2 m_\mu^5}{192\,\pi^3} = \frac{1}{\tau_\mu}$$

Now $\tau_\mu = 2.212 \pm .001 \times 10^{-6}$ sec. But what is G?
Find

$$G M_p^2 = 1.0233 \pm .0004 \times 10^{-5}$$

[To get good agreement with experimental spectrum, we must correct for electromagnetic effects

primary secondary]

Such radiative corrections  are very difficult -- and vary from experiment to experiment. Whenever a charge moves, there is possibility of photon emission.

PROBLEM: $n \to p + e + \nu$

Assume coupling is $\frac{G}{\sqrt{2}} C \left(\bar{u}_p \gamma_\alpha (1 + g_a i \gamma_5) u_n\right) \left(\bar{u}_e \gamma_\alpha (1 + i\gamma_5) u_\nu\right)$

[2 fudge factors correspond to fact that strong interactions modify coupling.]

[Answer: if $m_e = 0$, Rate $= \frac{2 G^2 W^5 c^2}{30 \pi^3} \left(1 + \frac{3 g_a^2}{4}\right)$]

$W = M_n - M_p \ll M_p$

Now what about other types of weak interactions

Notation:

$$(\bar{A}\,B)_\alpha\,(\bar{C}\,D)_\alpha = \left(\bar{u}_A \gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right) u_B\right)\left(\bar{u}_C \gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right) u_D\right)$$

We have just discussed $\quad (\bar{\nu}'\mu)(\bar{e}\nu)$

Also must have "$(\bar{n}\,p)_\alpha$" $(\bar{e}\nu)_\alpha$. The "$(\bar{n}p)_\alpha$" part involves strong interactions.

Other types of disintegration like $\pi^- \rightarrow e + \bar{\nu}$. Do we need yet another coupling for this. No. Imagine

$$\pi^- \xrightarrow[\text{strong}]{\text{virtual}} \bar{p},\, n \xrightarrow{\text{weak}} (e+\nu) + p$$

annihilate

Back to our discussion of the various weak interaction couplings

10/15/69 LECTURE

$(\bar{\nu}\mu)(\bar{e}\nu)$ μ-decay

"$(\bar{p}n)$"$(\bar{e}\nu)$ β-decay

"$(\bar{p}n)$"$(\bar{\mu}\nu_\mu)$

"$(\bar{p}\Lambda)$"$(\bar{e}\nu)$ [violates strangeness $\Lambda \to p + e + \nu$
$\Delta S = -1$, $\Delta Q = -1$ -- "$\Delta S = \Delta Q$ strong current"]
[on strong particle]

"$(\bar{p}\Lambda)$"$(\bar{\mu}\nu_\mu)$ [$\Delta S = -\Delta Q$ -- but never been found]

[so far no need for a $\Delta S = 2$, so this seems to be all we need]
But what about reactions like

$$\Lambda \to p + \pi^-, \; K \to 2\pi, \; \ldots$$

These are slow and violate parity -- hence must involve weak interactions without leptons

"$(\bar{p}\Lambda)$" "$(\bar{n}p)$" $\Delta S = \pm 1$

These couplings explain all observed weak interactions (except CP violation).

Current-Current Hypothesis

Perhaps we could rewrite these as

$$\sqrt{8}\, G\, \bar{J}_\mu J_\mu$$

where

$$J_\mu = (\bar{e}\nu_e) + (\bar{\mu}\nu_\mu) + \text{``}(\bar{n}p)\text{''} + \text{``}(\bar{\Lambda}p)\text{''}$$

$$\bar{J}_\mu = (\bar{\nu}e) + (\bar{\nu}\mu) + \text{``}(\bar{p}n)\text{''} + \text{``}(\bar{p}\Lambda)\text{''}$$

These are called the "charge current", and the cross-terms of $\bar{J}_\mu J_\mu$ give our previous list. What are coefficients of the components of J_μ? Also what about diagonal terms, e.g.

$$(\bar{\nu}e)(\bar{e}\nu) \Rightarrow \quad \nu + e \to e + \nu$$

or $\quad e^- + e^+ \to \nu + \bar{\nu}$

[no lab. evidence. But astronomers are worried about existence of $e^- + e^+ \to \nu + \bar{\nu}$ because of implications, since energy carried by $\nu + \bar{\nu}$ leaks out of stars. Don't know yet.]

Also

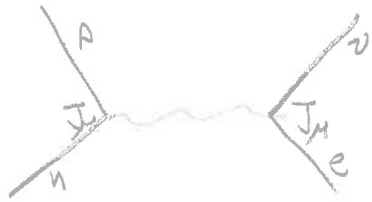
$$\text{``}(\bar{n}p)\text{''}\text{``}(\bar{p}n)\text{''} \Rightarrow \quad n + p \to p + n$$

[Differs from strong $np \to pn$ because of parity violation]

This current-current coupling suggests an analogy with electrodynamics

Perhaps there is a vector particle which causes the coupling. Then if W_μ is analogue of A_μ, coupling is

$$\sqrt{4\pi e_w^2}\, W_\mu J_\mu$$

Then 

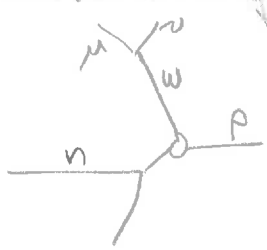
$q = p_1 - p_2$

$$4\pi e_w^2 \bar{J}_\mu \frac{\delta_{\mu\nu} - \frac{q_\mu q_\nu}{m_w}}{q^2 - m_w^2} J_\nu$$

Now suppose $|q| \ll M_w$

$$\sim \frac{4\pi e_w^2}{-M_w^2} J_\mu J_\nu$$

Hence $\sqrt{8}\, G = \frac{4\pi e_w^2}{M_w^2}$ ‡. This theory predicts that if you hit something hard enough, you might free w-meson.



Still haven't seen this yet.

‡ Here $m_w \gg m_p$ -- quite large.

Write 10/16/68 LECTURE

$$J_\mu = (\bar{e}\nu)_\mu + (\bar{\mu}\nu_\mu)_\mu + ``(\bar{n}p)"_\mu + ``(\Lambda p)"_\mu$$

$$\equiv j_\mu^{e\nu} + j_\mu^{\mu\nu} + c\, j_\mu^{\Delta S=0} + s\, j_\mu^{\Delta S = \Delta Q = 1}$$

Can also decompose

$$j_\mu = j_\mu^V + j_\mu^A$$

In analogy with before, can write general coupling for $n \to p + \bar{e} + \bar{\nu}$

$$\langle \text{proton}_2 | j_\mu^{\Delta S=0} | \text{neutron}_1 \rangle = (\bar{u}_{p_2} | \gamma_\mu V_1(q^2) + \tfrac{1}{2}(\not{q}\gamma_\mu - \gamma_\mu \not{q}) + i q_\mu V_3(q^2)$$
$$+ i\gamma_\mu\gamma_5 A_1(q^2) + \tfrac{1}{2}(\not{q}\gamma_\mu - \gamma_\mu\not{q})\gamma_5 A_2(q^2) + i q_\mu \gamma_5 A_3(q^2) | u_{n_1})$$

Now for β-decay, q is very small [20 Mev << 1 Gev]
Hence for nuclear physics, let $q \to 0$

$$c\left(\bar{u}_{p_2}(\gamma_\mu \underbrace{V_1(0)}_{g_V} + i\gamma_\mu\gamma_5 \underbrace{A_1(0)}_{g_A})\, u_{n_1}\right)\left(\bar{u}_e \gamma_\mu (1 + i\gamma_5) u_\nu\right) \frac{G}{\sqrt{2}}$$

Now choose $g_V = 1$ [explain this later]. Hence find

$$c\left(\bar{u}_{p_2} \Big| \gamma_\mu \frac{(1 + i g_A \gamma_5)}{2} \Big| u_{n_1}\right)$$

In the non-relativistic form, we find this reduces to 2 components:

$(\bar{u}_p \mid u_n)$ scalar Fermi

and

$(\bar{u}_p\, \vec{\sigma}\, u_n)$ vector Gamow-Teller

For a nucleus

$$(\text{nucleus}'|\sum_i \tau_+^i|\text{nucleus})$$

or

$$(\text{nucleus}'|\sum_i \vec{\sigma}_i \tau_+^i|\text{nucleus})$$

Idea is to now use nuclear decay to measure g_A and C.
Quite complicated in general except for two cases in which nuclear wave fns are known

(1) neutron decay

(2) mirror nuclei $\qquad O^{14}\ (6n\ 8p) \rightarrow N^{14}\ (7n\ 7p)$

We find

$$\begin{aligned} C = .980 &\pm .002 \quad \text{experimentally} \\ &\pm .005 \quad \text{uncertainty in radiative corrections} \\ &\pm <.002 \quad \text{wave function overlap} \\ = .980 &\pm .008 \end{aligned}$$

$$g_A = 1.23 \pm .01$$

[From current theory of strong interactions

$g_A = 1.24 \pm .03$ (mixed)]

REVIEW OF SU(3) 10/18/68 LECTURE

Lipkin Lie Groups for Pedestrians
Gasiorowicz 16, 17, 18

Recall for nucleons, can define

$$T^+ = a_p^+ a_n$$

$$T^- = a_n^+ a_p$$

$$T_3 = \frac{1}{2}[T^+, T^-] = \frac{1}{2}[a_p^+ a_n - a_n^+ a_p]$$

also

$$[T_3, T_-] = -T_- \qquad [T_3, T_-] = 2T_+$$

These commutation relations are same as angular momentum — hence these serve as infinitesimal generators of group SU(2) ~ O(3)

When we introduce strangeness, we now have 3 objects (p, n, Λ) and find SU(3). Also 2 quantum numbers

Recall weak coupling $\frac{1}{\sqrt{2}} G \bar{J}_\mu J_\mu$ where 10/22/63 LECTURE

$$J_\mu = (\bar{\nu} e) + (\bar{\nu} \mu) + c\left(j^{\Delta S = 0}_{\text{strong}}\right) + s\left(j^{\Delta S = \Delta Q = 1}_{\text{strong}}\right)$$

To test whether the coefficients of $(\bar{\nu} e)$ and $(\bar{\nu}\mu)$ are equal, we study

$$\pi^- \to e + \bar{\nu} \quad , \quad \pi^- \to \mu + \bar{\nu}$$

First note

$$(0|j_\mu^{\Delta S = 0}|\pi) = (0|j_\mu^{\text{vector}}|\pi) + (0|j_\mu^{\text{axial}}|\pi)$$

$$= \cancel{p_\mu \, ?}^{\,0} + p_\mu \sqrt{2} f_\pi$$

Hence

$$T = \frac{Gc}{\sqrt{2}} (\bar{u}_\mu \gamma_\mu (1 + i\gamma_5) u_\nu) \sqrt{2} f_\pi p_\mu$$

Hence $$\text{Rate} = \frac{c^2 G^2 m_\pi^3 f_\pi^2}{2\pi} \left(\frac{\mu^2}{m_\pi^2}\right) \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

$$\Rightarrow f_\pi = .96\, m_\pi^3$$

Get a similar expression for $\pi^- \to e + \bar{\nu}$. Find

$$\frac{\text{Rate}_{e\nu}}{\text{Rate}_{\mu\nu}} = \frac{m_e^2}{m_\mu^2} \left(\frac{1 - m_e^2/m_\pi^2}{1 - m_\mu^2/m_\pi^2}\right)^2 \longrightarrow 13.6 \times 10^{-5} \quad \text{[after EM corrections]}$$

Experiment within 2%. Hence coupling of $(\bar{\nu} e)$, $(\bar{\nu}\mu)$ appears to be equal. [Universal Fermi coupling hypothesis]

Here's an argument that [illegible] vector part of weak interaction current is very similar to electrodynamics -- isotopic spin operator in z direction?

$$(\bar{p}|j^V_\mu|n) = (\bar{u}_p|\gamma_\mu(F_1^p(q^2)-F_1^n(q^2)) + \frac{1}{2}(\not{q}\gamma_\mu - \gamma_\mu\not{q})(F_2^p(q^2)-F_2^n(q^2))|u_n)$$

Use isotopic spin concepts for $j^{\Delta S=0}_{strong}$. Use SU(3) for $(j^{\Delta S=\Delta Q=1}_{strong})$.

Examples

$$\pi^- \to \pi^0 + e^- + \bar{\nu}$$

$$(\pi^0|j^V_\mu|\pi^-) = \sqrt{2}(P_{1\mu} + P_{2\mu})$$

$$K^- \to \pi^0 + e^- + \bar{\nu}$$

$$(\pi^0|j^V_\mu|K^-) = (?)(P_{1\mu} + P_{2\mu})$$

ν_e, e and ν_μ, μ are exactly the same -- except for mass. Very deep mystery. Nobody has any idea why. 4 state neutrino?

10/23/68 LECTURE

Last time we identified $J_\mu^{\Delta S=0}$ with T_+.
Now what is an analogy for $J_\mu^{\Delta S=\Delta Q=1}$. Use SU(3)
Recall

			Strangeness	Hypercharge
	n	p	0	+1
Σ^-	Σ^0 Λ^0	Σ^+	−1	0
	Ξ^-	Ξ^0	−2	−1

Guess that just as isospin $T_+ \to J_\mu^{\Delta S=0}$, there is an analogous transformation in SU(3) $\to J_\mu^{\Delta S=\Delta Q=1}$,

---

REVIEW OF SU(3) [a la Feynman]

Consider 3 objects (quarks) A, B, & C with q.n. T and S

	$T=-\frac{1}{2}$	$T=+\frac{1}{2}$
$Y=0$	B	A
$Y=-1$	C	

	$\bar{C}$	$Y=+1$
$-\bar{A}$	$\bar{B}$	$Y=0$
$T=-\frac{1}{2}$	$T=+\frac{1}{2}$	

Now study "rotations". There is an invariant

$$\frac{1}{\sqrt{3}}(\bar{A}A+\bar{B}B+\bar{C}C)$$

There are 8 other combinations

$$\overset{n}{\bar{C}B} \qquad \overset{p}{\bar{C}A}$$

$$\overset{\sigma^-}{-\bar{A}B} \qquad \overset{\sigma^0}{\tfrac{1}{\sqrt{2}}(\bar{B}B - \bar{A}A)} \qquad \overset{\sigma^+}{\bar{B}A}$$

$$\underset{\lambda}{\tfrac{1}{\sqrt{6}}(\bar{A}A + \bar{B}B - 2\bar{C}C)}$$

$$\underset{\xi^-}{-\bar{A}C} \qquad \underset{\xi^0}{\bar{B}C}$$

For charges, choose $B=0$, $A=+1$, $C=0$

Now	n, p	σ	λ	ξ, ξ	1
nucleon octet	N, P	Σ	Λ	Ξ	
pion octet	$K^+ K^0$	π	η	$\bar{K}^- \bar{K}^0$	η'
vector meson	K^{*+}	ρ	$\frac{\omega}{\varphi}$	$\bar{K}^*$	$\frac{\varphi}{\omega}$

assign masses $A, B = 0$, $C \to c$, $\bar{C} \to d$. Find

N	d	$+x$
Σ	0	$+x$
Ξ	c	$+x$
Λ	$\frac{2}{3}(c+d)$	$+x$

$$\Rightarrow \quad \frac{N+\Xi}{2} = \frac{3\Lambda + \Sigma}{4}$$

mass law (works very well for nucleon mass)

However the mass law for pion octet works for square of masses anyhow, don't take it too seriously.

Most striking success of SU(3) was in the patterns -- in particular the prediction of the Ω^-. Feynman thinks SU(3) is indeed at the basis of strong interactions.

10/25/68 LECTURE

$$\begin{array}{ccc} & \bar{C}B \qquad\qquad \bar{C}A & \\ -\bar{A}B & \frac{1}{\sqrt{3}}(\bar{B}B-\bar{A}A) & \bar{B}A \\ & \frac{1}{\sqrt{6}}(\bar{A}A+\bar{B}B-2\bar{C}C) & \\ & -\bar{A}C \qquad\qquad (\bar{B}C) & \end{array} \Bigg\} \text{ mass } \mu_8$$

singlet: $\frac{1}{\sqrt{3}}(\bar{A}A+\bar{B}B+\bar{C}C)$ } mass μ_1

PROBLEMS:

1.) Find mass laws if $\mu_1 + \mu_8$ mass operator is mass C + mass $\bar{C}$
2.) Study multiplets generated by 3 quarks (no anti-quarks)
 AAA, ABA, ... [1, 8, 8, 10]
3.) SU(6) -- now include spin
 A↑ B↑ C↑ A↓ B↓ C↓
 3 Quarks in symmetric state
4.) Predict relations among magnetic moments of hyperons via SU(3)
5.) Prove $m_p - m_n + (m_{\sigma^-} - m_{\sigma^+}) = m_{\xi^-} - m_{\xi^0}$ $= -1.3 + .77 = 6.5$

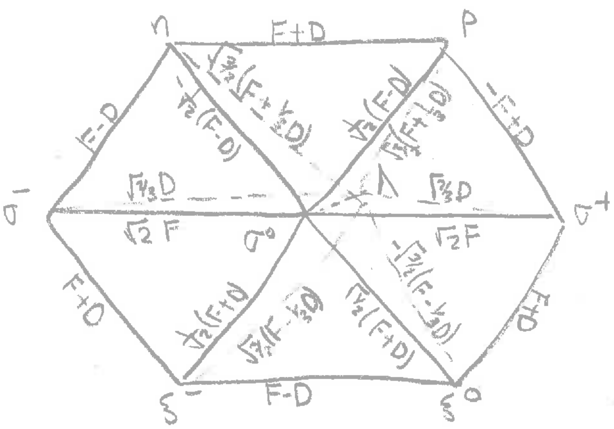
Can interpret each member of octet as operators

$$\bar{C}A \to \lambda_p \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} C \\ 0 \\ 0 \end{pmatrix} \Rightarrow \lambda_p = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

These matrices are generators of transformation group.

Mentions $8 \times 8 = 1 + 8 + 8 + 10 + \overline{10} + 27$

Find $(\rho_j | M_k | N_i) = F f_{ijk} + D d_{ijk}$



λ

$\sqrt{3/2}(F-\frac{1}{3}D)$

σ's $\sqrt{2/3}\,D$

Λ $-\sqrt{2/3}\,D$

$-\sqrt{3/2}(F+\frac{1}{3}D)$

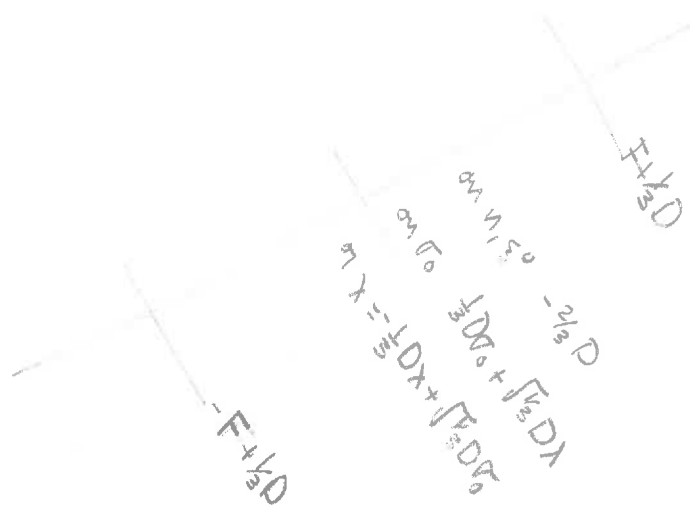
$-\frac{1}{\sqrt2}(F+D)$ $\sigma_0 \leftrightarrow \lambda$ $\frac{1}{\sqrt2}(F+D)$

$\sqrt{2/3}D$

σ_0 $-\sqrt2 F$ $\sqrt2 F$

$-\frac{1}{\sqrt2}(F-D)$ $\frac{1}{\sqrt2}(F-D)$

For electrodynamics, it is useful to use $\frac{1}{\sqrt2}\sigma_0 + \frac{1}{\sqrt6}\lambda$



Predicts

$\mu_p = \mu_{\sigma^+} = F + \frac{1}{3}D$ magnetic moments

$\mu_n = \mu_{\xi^0} = -\frac{2}{3}D$

$\mu_{\sigma_0} = \frac{1}{3}D$

$\mu_\Lambda = -\frac{1}{3}D$ $\mu_{\xi^-} = \mu_{\sigma^-} = -F + \frac{1}{3}D$

10/29/68 LECTURE

Assume electric current operator is a member of an octet.

$$(p|\, j_\mu(q)\,|n) = \gamma_\mu F_1^{\,v}(q^2) - \tfrac{1}{2}(\gamma_\mu \not{q} - \not{q}\gamma_\mu) F_2^{\,v}(q^2)$$

Can represent F_1 and F_2 in terms of $f(q^2)$ and $d(q^2)$

Now return to subject of weak interactions. We have ~~discussed~~

$$J_\mu^{\Delta S=0} \rightarrow J_\mu^{T=+1} \quad \text{for vector part.}$$

For axial vector part, assume again J_μ^A is octet --e.g.

$$(\Sigma^0|\, J_\mu^A\,|\Sigma^+) = \sqrt{2}\, F_A$$

$$(n|\, J_\mu^A\,|p) = F_A + D_A$$

In the limit $q^2 \rightarrow 0$

$$(B|\, J_\mu^{\Delta S=0}\,|A)$$

	Vector g_V		Axial g_A
$n \rightarrow p + e + \nu$	$F_V + D_V$	$\Rightarrow 1$)	$F_A + D_A = g_A = 1.23 \pm .01$
$\Sigma^+ \rightarrow \Sigma^0 + e^+ + \nu$	$\sqrt{2}\, F_V$	$\sqrt{2}$)	$\sqrt{2}\, F_A$
$\Sigma^- \rightarrow \Sigma^0 + e^- + \bar{\nu}$	$\sqrt{2}\, F_V$	$\sqrt{2}$)	$\sqrt{2}\, F_A$
$\Sigma^- \rightarrow \Lambda + e^- + \nu$	$-\sqrt{2/3}\, D_V$	0)	$-\sqrt{2/3}\, D_A$
$\Sigma^+ \rightarrow \Lambda + \cdots$	$+\sqrt{2/3}\, D_V$	0)	$+\sqrt{2/3}\, D_A$
$\Xi^- \rightarrow \Xi^0$	$F_V - D_V$	1)	$F_A - D_A$

Recall

$$\text{Rate} = \frac{\text{branching ratio}}{\text{lifetime}} = \frac{2G^2W^5}{30\pi^3}c$$

$$W = \frac{M_A^2 - M_B^2}{2M_A}$$

$$= .430 \times 10^7/\text{sec} \left(\frac{W}{100\,\text{MeV}}\right)^5 c$$

$$c = 1 + 2\epsilon + \frac{4}{7}\epsilon^2$$

$$\epsilon = \frac{M_A - M_B}{M_A + M_B}$$

Can calculate

	Branch Ratio	UFI
$\Lambda \to p + e^- + \nu$	$.88 \pm .15 \times 10^{-3}$	1.44×10^{-2}
$\Sigma^+ \to \Lambda + e^+ + \nu$	$2.2 \pm 0.7 \times 10^{-5}$	$.70 \times 10^{-4}$
$\Sigma^- \to \Lambda + e^- + \nu$	$.66 \pm .11 \times 10^{-4}$	2.36×10^{-4} *
$\Sigma^- \to n + e^- + \nu$	$1.25 \pm .17 \times 10^{-3}$	5.62×10^{-2}

Hence from *

$$\frac{3}{4}\left(\sqrt{\tfrac{2}{3}}\,D_A c\right)^2 = \frac{.66}{2.36} = .28 \pm .05$$

$$\Rightarrow D_A = .66 \pm .06$$

Now what about $J_\mu^{\Delta S = \Delta Q = 1}$. Cabibbo suggested that $J_\mu^{\Delta S=1}$ is also a member of an octet -- the same octet as for $J_\mu^{\Delta S=0}$.

	Vector	Axial
$\Lambda \to p + e^- + \nu$	$\sqrt{3/2}\,(F_V + \tfrac{1}{3} D_V) \to \sqrt{3/2}$	$\sqrt{3/2}\,(F_A + \tfrac{1}{3} D_A)$
$\Sigma^0 \to p + e^- + \nu$	$\sqrt{1/2}$	$\sqrt{1/2}\,(F_A - D_A)$
$\Xi^- \to \Sigma^0$	$\sqrt{1/2}$	$\sqrt{1/2}\,(F_A + D_A)$
$\Xi^- \to \Lambda$		
$\Xi^0 \to \Sigma^+$		
$\Sigma^- \to n$	1	$F_A - D_A$

But we know D_A from our work before, so can compare. Find you're doomed to failure. -- But wait. Cabibbo proposed we multiply all rates by s. [from $K \to \pi + e + \nu$, find $s = .23 \pm .04$. Now find $D_A = .77 \pm .03$.

Leptonic decays of hyperons seem to fit fairly well. Predicts $c^2 + s^2 = 1$. Find $(.98 \pm .01)^2 + (.26 \pm .02)^2 = 1.03 \pm .02$

Cabibbo suggested that we use model $\begin{matrix} B^0 & A^+ \\ C^0 & \cdot \end{matrix}$

10/30/68 LECTURE

$J = (e\nu) + (\mu\nu) + (\bar{A}^+ | \gamma_\mu (1 + i\gamma_5) | B')$

$B' = (\cos\theta_c) B + (\sin\theta_c) C$

Further questions about weak interactions

1.) why μ & e

2.) why just $(\bar{A}^+ | \gamma_\mu (1+i\gamma_5) | B')$. Any other terms?

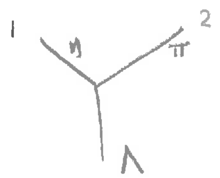
3.) $1 - i\gamma_5$? — they renormalized to $1 + i1.23\,\gamma_5$? [Parity violation permits definition of particle & antiparticle]

Theory of weak interactions is still in a phenomenological stage. It works, but doesn't predict. [Stresses importance of models—like quarks.]

Consequences of rules for non-leptic decays
[crossterm $(J_\mu^{\Delta S=0})(J_\mu^{\Delta S=\Delta Q})$]

11/1/63 LECTURE

Can get $\Delta S=1$, $\Delta I = \frac{1}{2}, \frac{3}{2}$. By a miracle, the rates for $\Delta I = \frac{1}{2}$ are several hundred times greater than $\Delta I = \frac{3}{2}$. No one really knows why.



$$(\bar{u}_2 (A - i\gamma_5 B) u_1)$$

$$\text{Rate} = \frac{Q}{16\pi m_1^2}\left(|s|^2 + |p|^2\right)$$

$$s = \sqrt{(E_2+m_2)2m_1}\, A$$

$$p = \frac{Q\sqrt{2m_1}}{\sqrt{E_2+m}}\, B$$

Can actually "measure" A & B

A, B in units $10^5\ \text{sec}^{-1/2}$

	A	B/10
$\Lambda \to p + \pi^-$	1.55 ± .02	1.10 ± .05
$\Sigma^+ \to n + \pi^-$	−.01 ± .03	1.90 ± .03
$\Sigma^+ \to P + \pi^0$	1.36 ± .15	−1.36 ± .15
$\Sigma^- \to n + \pi^-$	1.86 ± .02	−.015 ± .04
$\Xi^- \to \Lambda + \pi^-$	2.02 ± .03	.66 ± .06

K^0 DISINTEGRATION

11/5/68 LECTURE

$$K^0 \to \pi^+ + \pi^-$$
$$\pi^0 + \pi^0$$

Test $\Delta I = \frac{1}{2}$ rule $\left[J_{\lambda,\ \Delta I=\frac{1}{2}}^{\Delta S=1} \right]$. Now for 2π's, find

	$I_z = 2$	$I_z = 1$	$I_z = 0$
$I = 2$	$\pi^+\pi^+$	$\frac{1}{\sqrt{2}}(\pi^+\pi^0 + \pi^0\pi^+)$	$\frac{1}{\sqrt{6}}(\pi^+\pi^- + \pi^-\pi^+ - 2\pi^0\pi^0)$
~~$I = 1$~~		~~$\frac{1}{\sqrt{2}}(\pi^+\pi^0 - \pi^0\pi^+)$~~	~~$\frac{1}{\sqrt{2}}(\pi^+\pi^- - \pi^-\pi^+)$~~ Bose statistics
$I = 0$			$\frac{1}{\sqrt{3}}(\pi^+\pi^- + \pi^-\pi^+ + \pi^0\pi^0)$

Now $\Delta I = \frac{1}{2} \Rightarrow I = 0$. Hence rates

	Rate Predicted	Rate measured
$K^0 \to \pi^+ + \pi^-$	2	1
$K^0 \to \pi^0 + \pi^0$	1	
$K^+ \to \pi^+ + \pi^0$	0	1/600

Hence some amplitude for $\Delta I = \frac{3}{2}$

11/6/68 LECTURE

Have $\bar{K}^0$ [S=+1] and K^0 [S=+1] since

$$p + n \to \Lambda + p + K^0$$
$$\to \Xi^0 + p + \bar{K}^0$$

Because of failure of cons. of strangeness in weak decays

$$K^0 \to \pi^+ + \pi^- \qquad \text{amplitude } a$$

$$\bar{K}^0 \to \pi^+ + \pi^- \qquad \text{"} \quad a$$

Hence $|K_1^0\rangle \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) = \frac{1}{\sqrt{2}}(a+a) = \sqrt{2}a$ to go into 2π's

$$|K_2^0\rangle \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) = \frac{1}{\sqrt{2}}(a-a) = 0$$

Hence Gell-Mann & Pais predicted half of K^0's would not decay
To analyze, use as base states $|K_1^0\rangle$ and $|K_2^0\rangle$ [hence we have assumed C is good. But it isn't for β-decay. Now use CP invariant]

Now since $K^0 \to \pi^- + e^+ + \nu$
$\bar{K}^0 \to \pi^+ + e^- + \nu$

Hence $|K_2^0\rangle$ will eventually decay -- but at a different rate. -- in fact a rate 500 times slower. Say $|K_1^0\rangle \to \gamma_1, m_1,$ $|K_2^0\rangle \to \gamma_2, m_2$. Then

$$\text{Prob. of } \bar{K}_0 = \frac{1}{4}\left[e^{-\gamma_1 \tau} + e^{-\gamma_2 \tau} - 2e^{-\left(\frac{\gamma_1+\gamma_2}{2}\right)\tau}\cos(m_1-m_2)\tau\right]$$

Get an oscillation in space. (can see this)

Find a mass difference $m_1 - m_2 = -.47 \pm .02 \ \gamma_1$
Reasonable since amplitude for virtual process

$$K^0 \leftrightarrow \pi^+ + \pi^- \leftrightarrow \bar{K}^0$$

Second order effect in G.

CP invariance ?: Experimenters found $|K_2^0\rangle$ can go into $\pi^- + \pi^+$. This violates CP.

$$K_2^0 \rightarrow \pi^+ + \pi^- \qquad \text{with branching ratio of } 0.2\%$$

Lots of proposals on this. But transmission experiments show throw out some of these [K_2^0 does indeed go into π's.]

11/13/68 LECTURE

$$\eta_{+-} = \frac{a^L_{+-}}{a^S_{+-}} = \epsilon + \epsilon' + \epsilon''$$

$$\eta_{00} = \epsilon - 2\epsilon' + \epsilon''$$

$$\epsilon'' = \frac{A_0 - B_0}{2A_0} \; ; \quad \epsilon' = \frac{1}{2\sqrt{2}} e^{i(\delta_0 - \delta_2)} \frac{(A_2 - B_2)}{A_0} \; ; \; \epsilon = \frac{-ih_{12}}{\frac{\gamma_S - \gamma_L}{2} + i(m_S - m_L)}$$

$$h_{12} = \langle K_1^0 | h | K_2^0 \rangle = \frac{1}{2}(h_{00} - h_{0\bar{0}} + h_{\bar{0}0} - h_{\bar{0}\bar{0}})$$

$$h_{0\bar{0}} = \langle K_0 | h | \bar{K}_0 \rangle \quad \text{etc.}$$

Consider now a perturbation H_{wk}

$$h_{ij} = \langle K_i | H_{wk} | K_j \rangle + \sum_{\text{all } n} \rho \frac{\langle n | H_{wk} | K_j \rangle^* \langle n | H_{wk} | K_i \rangle}{m_K - m_n + i\epsilon}$$

Then

$$i h^I_{ij} \equiv i\,\mathrm{Im}\{h_{ij}\} = -i\pi \sum_F \rho_F \langle F | H_{wk} | K_j \rangle^* \langle F | H_{wk} | K_i \rangle$$

Possible Models of CP Violations

1.) Very tiny external potential -- analogue of electricity for baryons. $\Rightarrow$ rate $= f(E)$. But it isn't.

2.) Scalar field $\quad m_{K_0} - m_{\bar{K}_0} = \delta \gamma_s \quad \Rightarrow h_{12} = \frac{1}{2} \delta \gamma_s$

$\Rightarrow \quad \eta_{+-} = \epsilon = -i \delta \sqrt{2}\, e^{i(45°)}$ wrong phase

3.) $K^0 \to \pi^- + \mu^+ + \nu$

$\bar{K}^0 \to \pi^+ + \mu^- + \bar{\nu}$ Assume rates $(K^0 \to \mu) - (\bar{K}^0 \to \mu) = r \gamma_s$

$\therefore h_{12} = -\frac{i}{4} r \gamma_s$

$\Rightarrow \eta_{+-} = \epsilon = -\frac{r\sqrt{2}}{4} e^{+i(45°)}$ phase OK, but magnitude wrong

4) CP violation entirely due to π's.

Several classes

① environments are unsymmetrical

② something wrong with Q.M. [principles of interference fails after too many phases] K_L decay is beautiful test of Q.M. -- but it fails. Hence maybe something wrong with Q.M., not CP.

③

11/15/63 LECTURE

CP violation in weak interaction
i.) but not in 2π decay
ii.) or in 2π-decay

Indirect violation (strong interactions ?) (electromagnetics)

What about other symmetries T or CPT?
As far as RF knows, it is impossible to have a theory which is "natural" and violates CPT.
relativity + q.m. + locality ⇒ CPT
But nobody has given a counter example.

If CPT OK, then T must be violated.

T VIOLATION:

Let "a" be the name of a state with vector $|a\rangle$
For each state, a time reversed state named "Ta" and $|Ta\rangle$
[not $T|a\rangle$.]
Note if $|c\rangle = \alpha|a\rangle + \beta|b\rangle$
then
$$|Tc\rangle = \alpha^*|Ta\rangle + \beta^*|Tb\rangle$$

If time reversal is valid

$$\langle Ta|Tb\rangle = \langle b|a\rangle$$

Can show $|TTb\rangle = \pm |b\rangle$ [nothing to do with relativity.]

± 1 might be proof of Fermi ; Bose statistics.

11/19/63 LECTURE

We found two cases

$$|TTa\rangle = +|a\rangle \qquad (\text{if } \exists\ Ta = a)$$

$$|TTa\rangle = -|a\rangle \qquad (\text{if } \nexists\ Ta = a)$$

If $TT = +1$, can find a base n so that $S_{mn} = S_{nm}$ (symmetric)
[Can make basis ~~out~~ of states which remain same under time reversal. Can adjust phases so $|Tn\rangle = |n\rangle$.
Then since

$$\langle b|S|a\rangle = \langle Ta|S|Tb\rangle \Rightarrow S \text{ symmetric}]$$

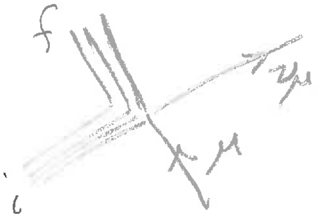
This means H is real (since it is already Hermitian).

In this sense, time reversal determines the phases.

CHARACTER OF STRONG INTERACTIONS

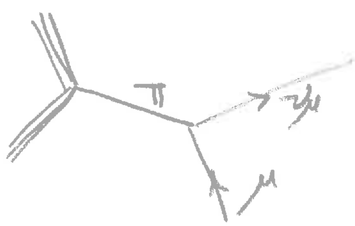
EXAMPLE: (Goldberger-Treiman relation)

First demonstrate general theory of poles in amplitudes.
Consider capture of μ by a nucleus ($\Delta S=0$)



$$\langle f | \overset{\Delta S=0}{j_\mu^V + j_\mu^A} | i \rangle \quad \langle \bar{\nu} | \gamma_\mu (1+i\gamma_5) | \nu \rangle$$

Will just discuss axial current. $\langle f | j_\mu^A(q) | i \rangle$
Maybe the μ becomes a π.



$$\langle f | \text{amp to emit a } \pi | i \rangle \frac{1}{q^2 - m_\pi^2} \langle \pi | J_\mu^A | 0 \rangle \langle \bar{\nu} | \gamma_\mu (1+i\gamma_5) | \nu \rangle$$

or

$$\langle f | j_\mu^A(q) | i \rangle = \langle f | i, \pi \rangle \frac{1}{q^2 - m_\pi^2} \overbrace{\langle \pi | J_\mu^A | 0 \rangle}^{\sqrt{2} f_\pi q_\mu} + \text{other terms}$$

—o—

How can we make this more precise. Use concept of <u>poles</u>:

$$\lim_{q^2 \to m_\pi^2} (q^2 - m_\pi^2) \langle f | j_\mu^A(q) | i \rangle = \langle f | i, \pi \rangle \sqrt{2} f_\pi q_\mu$$

This is a common feature of amplitudes. Not a result of perturbation theory or field theory — but just the physical statement of high possibility of π production for correct $q^2 \to m_\pi^2$.

11/20/68 LECTURE

EXAMPLE: $\quad A+B \to p+C \qquad D+p \to E$

Over all $\quad A+B+D \to C+E$

Then S matrix will have to have a pole near $p^2 = m_p{}^2$

$$\langle C+E_{out} | A+B+D_{in} \rangle \sim \sum_{n_p} \frac{1}{p^2 - m_p{}^2 + i\epsilon} \langle C_{out} n_p | AB_{in} \rangle \langle E_{out} | n_p D_{in} \rangle$$

$$p = p_A + p_B - p_C$$

PROBLEM: Formulate need for such poles as a property of the S matrix. Try to include everything which might be physically obervable. [Never been done. May be key to S matrix theory.

[F. doesn't think S matrix is way to formulate theory however. Probably a simpler underlying structure. Mentions stability of Ω.] S matrix is a many-body problem -- hence must get to guts of problem itself. Maybe a Hamiltonian doesn't exist -- but surely something simpler does.

11/22/68

Return now to pion pole in axial current

$$\langle f| j_\mu^A |i\rangle = \frac{\langle \pi | j_\mu^A | 0\rangle \langle f\pi | i\rangle}{q^2 - m_\pi^2 + i\epsilon} + \text{other terms}$$

Now assume this form is OK for $0<q^2<m_\pi^2$ -- almost. False

$$\langle f| q_\mu j_\mu^A|i\rangle = \frac{\sqrt{2} f_\pi m_\pi^2 \langle f\pi|i\rangle}{q^2-m_\pi^2} + \text{other}$$

$$= \frac{\sqrt{2} f_\pi m_\pi^2 \langle f\pi|i\rangle}{q^2-m_\pi^2}\,\xi(q^2)$$

Assume $\xi(m_\pi^2) = 1$

$\longrightarrow \xi(q^2)$ slowly varying

$\xi(0) \sim \xi(m_\pi^2)$

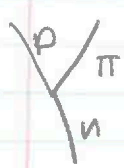
Now

$$\langle p|q_\mu j_\mu^A|n\rangle\Big|_{q^2=0} = \frac{\sqrt{2}\, f_\pi \cancel{m_\pi^2}\,\langle p\pi|n\rangle\, \xi(0)}{-\cancel{m_\pi^2}}$$

Recall

$$\langle p|j_\mu^A|n\rangle = (\bar{u}_p|\gamma_\mu\gamma_5 F_1^A(q^2) + q_\mu\gamma_5 F_2^A(q^2)|u_n)$$

$$\sim g_A(\bar{u}_p|\gamma_\mu\gamma_5|u_n)$$

What about $\langle p\pi|n\rangle$? 

can't occur. But can use virtual π process

Find determine $\sqrt{4\pi}\, g_{nn\pi}(\bar{u}_p|\gamma_5|u_n)$ and can

$$g^2_{nn\pi} = 14.4 \pm .4$$

Hence

$$g_A\,(\bar{u}_p|\not{q}\gamma_5|u_n) = \sqrt{2}\,f_\pi\,\xi(0)\sqrt{4\pi}\,g_{nn\pi}\,(\bar{u}_p|\gamma_5|u_n)$$

Now $(\bar{u}_p(\not{p}_2-\not{p}_1)\gamma_5 u_n) = (m_2+m_1)(\bar{u}_p\gamma_5 u_n) = 2m\,(\bar{u}_p\gamma_5 u_n)$

Hence

$$2m\,g_A = -\sqrt{2}\,f_\pi\sqrt{4\pi}\,g_{nn\pi}$$

Goldberger-Treiman relation

Relates 3 experimental constants: $g_A, f_\pi, g_{nn\pi}$. Find

$$f_\pi = .87\ m_\pi^2$$

$$= .96\ m_\pi^2 \quad \text{from decay}$$

Seems very significant. But 9% error also is significant. Mentions gradient coupling $\gamma_5\not{q}$.

Originally derived by field theory -- but needs too many assumption. This way is much better.

11/27/68 LECTURE

Now another "modern" (doesn't involve field theory) derivation of Goldberger-Treiman relation due to Nambu:

$$\langle p | j_\mu^A | n \rangle = \bar{u}_p \left(\gamma_5 \gamma_\mu F_1^A(q^2) + q_\mu \gamma_5 F_2^A(q^2) \right) u_n$$

Nambu proposed that just as divergence of j_μ^V is conserved, $q_\mu j_\mu^A$ is essentially conserved. This yields

$$2M F_1(q^2) + q^2 F_2(q^2) = 0$$

Hence Nambu proposed

$$F_2(q^2) = -\frac{2M F_1(q^2)}{q^2}$$

or

$$\langle p | j_\mu^A | n \rangle = \bar{u}_p \left(\left(\gamma_5 \gamma_\mu - \frac{2m\, q_\mu \gamma_5}{q^2} \right) F_1(q^2) \right) u_n$$

Now in real world, $m_\pi \neq 0$. Hence use $q^2 \to q^2 - m_\pi^2$ which leads to G-T relation.

Is this $[q_\mu j_\mu^A = 0]$ a new symmetry? Gell-Mann thinks so. Feynman thinks not.

PROPERTIES OF CURRENTS

First, the ~~electromagnetic~~ current which is a vector current

$$\langle p | j_\mu^V | p \rangle = \bar{u}_p | \gamma_\mu F_1^V(q^2) + \tfrac{1}{2}(\not{q}\gamma_\mu - \gamma_\mu \not{q}) F_2^V(q^2) | u_p$$

Argue that for small q^2, $F_1(q^2)$ is related to charge distribution

$$F_1(Q^2) = \int e^{i\vec{Q}\cdot\vec{R}} \rho(\vec{R})\, d^3\vec{R}$$

Argue that

$$F_1(q^2) = \int_{2\mu}^{\infty} \frac{G(\lambda)\, d\lambda}{(\lambda^2 - q^2 + i\epsilon)}$$

11/27/68 LECTURE

Recall we have found

$$F(t) = \int_{3\mu}^{\infty} \frac{G(x)\,dx}{t - x^2 + i\epsilon} \qquad t \equiv +q^2 = -Q^2 \qquad (*)$$

Want to extend this to $q^2 > 4m^2$
Need concept of analytic continuation.
But where is $F(t)$ analytic?

Can try to determine the analyticity by old-fashioned field (perturbation) theory on simple examples. Not legit.
[Mentions Drell & Zachariasen, "EM Structure of Nucleon."]

$$\left[\frac{1}{x+i\epsilon} = \frac{x-i\epsilon}{x^2+\epsilon^2} = \frac{x}{x^2+\epsilon^2} - i\frac{\epsilon}{x^2+\epsilon^2} = P\frac{1}{x} - i\pi\delta(x) \right]$$

The eqn (*) is the only thing we know about form factors.
Can't even prove (*).

Ph 230a

Lecture #1

The course will meet Tuesday, Wednesday, & Friday at 11:00. A better title might be "Present attempts to Understand High Energy Phenomena." The situation in high energy physics today resembles the situation the old quantum mechanics was in just before Schroedinger's discovery. There were many bits, pieces, & rules one had to know. This course will bear a closer resemblance to a course you might have taken in the old quantum mechanics (had you been a student then) than to the course you actually did take, in which the various rules were derived from Schroedinger's equation.

There is an excellent book by S. Gasiorowicz; Elementary Particle Physics, which you should begin reading. However, the growth of the field since 1966, when the book was published, is such that 2/3 of the lecture material will not be found in the book.

Before going over the main topics which will be discussed, we will review several general beliefs, which are assumed in most theoretical work, though usually unstated in texts or courses. Most are already familiar to you.

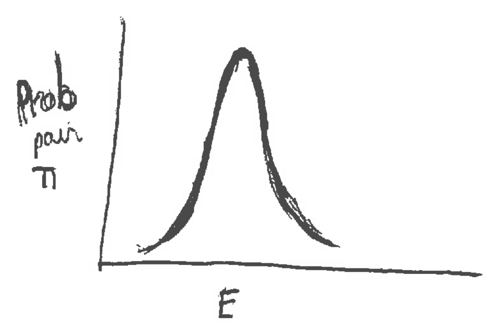
These are (1) conservation of energy, angular momentum, etc.
(2) the general character of quantum mechanics i.e. superposition of amplitudes (summing over intermediate states) etc.
(3) relativity
(4) four types of interactions: (1) strong, (2) electromagnetic, (3) weak, and (4) gravitational. People assume they can be treated as separate interactions, i.e. they can be studied by themselves (although they may occur together in a real physical process.) The interactions are characterized mainly by the order of magnitude of their speeds and their violation of certain quantum numbers. (There is one process which disagrees with the existence of only three interactions (st., wk., & em.) — the non-invariance of CP in the $K_L^0 \to 2\pi$ decay. CP should be conserved in all these according to our present understanding. No one knows the answer.)

You're already familiar with one theory whic combines (2) & (3), namely Q.E.D., & you've probably seen others of that type. Local field theories of this type may not be the only ones possible under restriction (1), (2) & (3). However, whatever extra is added to (2) & (3) to make field theory as we know it also produces the C.P.T. theorem. No violations of this theorem are known.

(4) may be new to you. Charts will be handed out listing known particles & how they interact. For instance electrons have weak & electromagnetic interactions. Pions have strong, electromagnetic, & weak interactions. An interaction between π's & e's must take place by means of electromagnetism or weak interactions.

Here the interaction takes place by means of an intermediate state photon. However simple perturbation theory applied to the coupling of the photon to the pion isn't valid, because when the two pions are close together, the interact strongly, & could for example form a $\rho\bar{\rho}$ which could interact strongly to again form a $\pi^+\pi^-$. The blob at the vertx is complicated. However we can assume that the interaction at the vertex takes place through a ρ meson intermediate state.

When the energy of the 2π's is near that of the resonance, there will be an increase in the amplitude.



This illostrates a situation often occurring when strongly interacting particles are involved. Namely, that although perturbation theory is of no help, we can often pick intermediate states which are important enough to give reasonable answers.

The interaction of strongly interacting particles with particles which do not have strong interactions (γ, ν, e, etc.) are governed by matrix elements of currents. Here it is the electromagnetic current matrix element $\langle 1 J_{em} | \pi^+\pi^- \rangle$ which determines $\langle \gamma | \pi^+\pi^- \rangle$. For purposes of analysis in strong interactions, we can ignore the existence of particles which don't interact strongly. However, the correct theory of strong interactions will also tell us the matrix elements of the currents, enabling us to calculate the interactions between strongly & nonstrongly interacting particles. So much for generalities.

The main topics to be covered in this course are:

(1) SU(3)

Although the spectrum of states does not exhibit exact SU(3) symmetry, Gell-Mann has proposed that SU(3) is valid at small distances & times; that is the currents which we mentioned satisfy

$SU(3) \times SU(3)$ commutation relations at equal times.

Several successful attempts have been made to connect matrix elements of currents with the matrix elements of strongly interacting particles. We just saw an example of this type of thing when we assumed the matrix element of J_{em} between 2 pions was dominated by the existence of a ρ meson.

(2) Regge poles

Regge trajectories group particles of differing angular momentum, but equal quantum numbers such as charge, parity, I spin, & make a connection between the existence of these particles & the very high energy behaviour of scattering amplitudes.

(3) Analyticity

This is the idea that amplitudes can be continued analytically away from physical values of their arguments, & that only physically meaningful singularities exist in the complex plane. For instance a bound state would mean a pole in the complex plane at an (energy)^2 equal to the mass of the bound state.

Lecture # 2. Week # 1.

In practice, all measurements in high energy physics are made by scattering experiments. The quantity of interest, the scattering matrix element S_{fi}, is the probability amplitude that a certain initial configuration of particles in state i (usually specified as independent particles with various momenta and spins at $t=-\infty$) ends up as a different configuration of particles in state f (specified as indep. particles at $t=+\infty$). The scattering matrix element may be written:

$$S_{fi} = \delta_{fi} - (2\pi)^4 i \, \delta^4\left(\sum_f p_f - \sum_i p_i\right) T_{fi}$$

Other people use slightly different normalizations, but the factors $(2\pi)^4 i \, \delta^4(\sum_f p_f - \sum_i p_i)$ are common to all. The rate can now be expressed as:

$$\text{Rate} = \prod_i \frac{1}{2E_i} (2\pi)^4 \delta^4\left(\sum_f p_f - \sum_i p_i\right) |T_{fi}|^2 \prod_f 2\pi \, \delta(p_f^2 - m_f^2) \frac{d^4 p_f}{(2\pi)^4}$$

note that $\prod_f 2\pi \, \delta(p_f^2 - m_f^2) \frac{d^4 p_f}{(2\pi)^4} = \prod_f \frac{d^3 p_f}{2E_f (2\pi)^3}$

$$\text{also, Rate} = \prod_i \frac{1}{2E_i} 2\pi \, \delta\left(\sum_f E_f - \sum_i E_i\right) |T_{fi}|^2 \prod_{\substack{\text{all } f \\ \text{but } 1}} \frac{d^3 p_f}{2E_f (2\pi)^3}$$

For a one particle disintegration,

$$\text{Rate} = \underset{\nwarrow \text{lifetime}}{\frac{1}{\tau}} = \frac{1}{E_1} \times \text{invariant} \quad \left(\text{because } \frac{\tau}{\tau_{\text{proper}}} = \frac{E}{m}\right)$$

For a two particle collision,

$$\text{Rate} = \sigma \cdot |\vec{v}_1 - \vec{v}_2| \quad \text{where} \quad \sigma = \text{cross section}$$
$$|\vec{v}_1 - \vec{v}_2| = \text{magnitude of relative velocity}$$

Exercise: Show that $2E_1 2E_2 |\vec{v}_1 - \vec{v}_2| = 4\sqrt{(p_1 \cdot p_2)^2 - p_1^2 p_2^2}$ i.e. an invariant
(Note that this implies σ is an invariant.)

Exercise: Show that the phase space factor $\left[(2\pi)^4 \delta^4(\Sigma_f p_f - \Sigma_i p_i) \prod_f \frac{d^3 p_f}{2E_f (2\pi)^3}\right]$ reduces to: (a) $\frac{1}{4(2\pi)^2} \frac{p_1^3 \, d\Omega_1}{E p_1^2 - E_1 (\vec{P} \cdot \vec{p}_1)}$ for a two particle final state

where E = total energy, $\vec{P}$ = total momentum
E_1 = energy of final particle 1, $\vec{p}_1$ = mom. of final particle 1
$d\Omega_1$ = solid angle of final particle 1 etc.

(b) $\frac{1}{8E_1 (2\pi)^5} \frac{p_2^3 \, p_1^2 \, dp_1 \, d\Omega_1 \, d\Omega_2}{p_2^2 (E - E_1) - E_2 \, \vec{p}_2 \cdot (\vec{P} - \vec{p}_1)}$ for a three particle final state

No assumption has been made yet about perturbation theory. It will be assumed that the student is familiar with the laws of quantum electrodynamics (Feynman rules) and knows how to represent a spin ½ particle with a four component spinor and a spin one particle by a vector.

Rules for Feynman Diagrams for QED

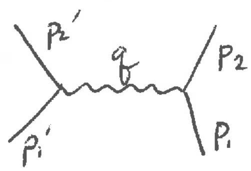
normalization: scalar 1
vector $e_\mu \cdot e_\mu = -1$ (spacelike polarization)
spinor $\bar{u} u = 2m$, $\not{p} u = m u$

spin ½ propagator $\frac{i}{\not{p}-m+i\epsilon}$

spin 0 propagator $\frac{i}{p^2-m^2+i\epsilon}$

spin 1 propagator $-i\,\frac{\delta_{\mu\nu}-\frac{p_\mu p_\nu}{m^2}}{p^2-m^2+i\epsilon}$ (photon $-i\,\frac{\delta_{\mu\nu}}{p^2+i\epsilon}$)

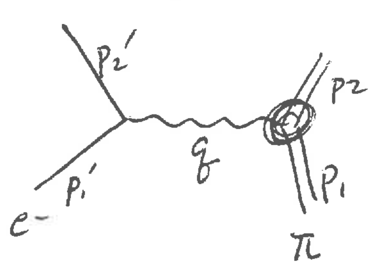
Coupling of spin ½ particle to photon $-i\sqrt{4\pi e^2}\,\gamma_\mu$ $e^2 = 1/137$ $\hbar = c = 1$

e.g. 

$T = -i\,4\pi e^2\,\frac{1}{q^2}\,(\bar{u}_2'\gamma_\mu u_1)(\bar{u}_2\gamma_\mu u_1)$

But what about the strong interactions? The whole problem is to find the T, but we don't know anything except a few bits and pieces. The rest of the course will be to explain what we do know about T.

We will demonstrate a type of reasoning frequently used in determining the form of the interaction of photons with strongly interacting particles. One example is electron pion scattering (which hasn't been done experimentally since the π disintegrates too quickly).



$e^-\pi \to e^-\pi$

We will assume that the whole interaction in first order is determined by the virtual photon. We know the amplitude for an electron to emit a photon from QED. We also know the amplitude for the photon to propagate into the vicinity of the pion. However, we don't know the amplitude for the pion to absorb a photon of momentum q. We can't use the coupling to an elemental spin zero particle, because the pion interacts strongly and is not a simple point charge (i.e. sometimes it is an $n\bar{n}$  , etc.).

We can write T as follows: $T = -i\,\frac{4\pi e^2}{q^2}\,\bar{u}_2'\gamma_\mu u_1'\langle \text{pion}2|j_\mu|\text{pion}1\rangle$

$\langle \text{pion } 2|j_\mu|\text{pion } 1\rangle$ is the amplitude for a pion to absorb a photon of polarization μ. It is a 4-vector which depends on p_1 & p_2.

$\therefore \langle \text{pion } 2|j_\mu|\text{pion } 1\rangle = A\,p_{1\mu} + B\,p_{2\mu}$ where A, B are invariant functions of p_1 & p_2

The invariant combinations of p_1 & p_2 are $p_1^2 = p_2^2 = m_\pi^2$ and $p_1\cdot p_2$, and only $p_1\cdot p_2$ can vary.

Note that $q = p_1 - p_2$ so $q^2 = p_2^2 - 2p_1\cdot p_2 + p_1^2 = 2m_\pi^2 - 2p_1\cdot p_2$

$\therefore \langle \text{pion } 2|j_\mu|\text{pion } 1\rangle = p_{1\mu}\,A(q^2) + p_{2\mu}\,B(q^2)$

or $= (p_{1\mu} + p_{2\mu})\,C(q^2) + (p_{1\mu} - p_{2\mu})\,D(q^2)$

For light, we can go further. The electromagnetic current is conserved which implies that $q_\mu j_\mu = 0$, or

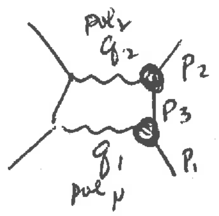
$q_\mu \langle \text{pion } 2|j_\mu|\text{pion } 1\rangle = 0 = 0 + q^2 D(q^2)$

$\therefore D(q^2) \equiv 0$

$\therefore \langle \text{pion } 2 | j_\mu | \text{pion } 1 \rangle = (p_{1\mu} + p_{2\mu}) F_\pi(q^2)$

$F_\pi(q^2)$ is called the pion form factor. If the pion were a point charge, $F_\pi(q^2) \equiv 1$. So it sort of represents the fourier transform of the charge distribution for low q^2. the pion has a unit charge; and if we measure it with a d.c. field, i.e. $q^2 \sim 0$, we see that $F_\pi(0) = 1$ (the e has already been factored out).

It is quite amazing that we have reduced everything to one unknown function. However, it is <u>not</u> true that the second order is:



$$\frac{1}{q^2}\,\frac{1}{q_1^2}\,\frac{1}{p_3^2 - m^2}\,(p_{2\nu} + p_{3\nu}) F_\pi(q_2^2)\,(p_{1\mu} + p_{3\mu}) F_\pi(q_1^2)$$

the intermediary p_3 is not a simple physical pion.

the right answer is: $\langle \text{pion } 2 | \text{bubble}_{\mu\nu} | \text{pion } 1 \rangle = V_{\mu\nu}(q_1, q_2; p_1, p_2)$
It cannot be supposed V is known when A is known.

Exercise: Do the same thing for the proton.

Ans. $\langle \text{proton } 2 | j_\mu | \text{proton } 1 \rangle = \bar{u}_2 \left(F_1(q^2) + \tfrac{1}{2}(\not{q}\gamma_\mu - \gamma_\mu \not{q}) F_2(q^2) \right) u_1$
this is the most general form.

Also, calculate the collision cross section ~~in terms~~ of e^- and p.

Ans. $$\sigma = \frac{e^4}{4E^2 \sin^4\frac{\theta}{2}} \; \frac{\cos^2\frac{\theta}{2}}{1+\frac{2E}{m}\sin^2\frac{\theta}{2}} \left\{ F_1^2 - \frac{q^2}{4m^2}\left[4m^2F_2^2 + 2(F_1+2mF_2)^2\tan^2\frac{\theta}{2}\right]\right\}$$

The electron mass has been neglected in comparision to the proton mass. What is the answer if the proton and neutron were point charges?

Ph 230a Lecture #3 Week 1

Assignment: Read Chs. 1+3 of Gasiorowicz. Do problems 1+2 of Ch. 3.

The question of current conservation came up this lecture. Gauge invariance, or current conservation, is necessary in order that the electromagnetic field $A^\mu(x)$ contain only the effects of physical, transverse photons. In quantizing the free E.M. field, we can eliminate all but transverse photons by choosing the gauge $\varphi = A^0 = 0$, $\tilde{\nabla} \cdot \tilde{A} = 0$. This is the radiation gauge familiar from classical E&M. Such a quantization is not covariant since a Lorentz transformation to a new frame does not give us the radiation gauge in the new frame.

Exercise: Make a Lorentz transformation to a new frame. Show that in the new frame A^μ differs from radiation gauge by a gauge term. Find the form of the gauge term for an infinitesimal Lorentz transformation.

Therefore if we quantize the E.M. field in such a way that only physical photons appear, then covariance implies one man's results using radiation gauge must be the same as another man's radiation gauge. Therefore all observables must be independent of choice of gauge.

Exercise: Show for $\langle 2 | \int d^3x \, A^\mu(x) J_\mu(x) | 1 \rangle$ that gauge invariance implies $\frac{\partial J^\mu(x)}{\partial x^\mu} = 0$
(Hint: integrate by parts.)
Show $\frac{\partial}{\partial x^\mu} \langle 2 | J^\mu(x) | 1 \rangle = e^{i(p_2 - p_1)x} q^\mu \langle 2 | J_\mu(0) | 1 \rangle$

We will give a discussion of the physical nature of the photon in order to show why all this gauge business is necessary.

Massless particles

The concept of spin is really only good for massive particles, where spin is just the angular momentum in its rest frame. This definition is obviously no good for massless particles. The thing that replaces spin is helicity, which is spin along the direction of motion. $hel = \frac{\vec{J} \cdot \vec{p}}{|\vec{p}|}$

If you start out with a massless helicity λ particle, no transformation of coordinate systems will ever make it look any different, that is change its helicity

(except for parity, under which $\lambda \to -\lambda$)
This is not the case for a massive particle, which we could catch up to, rotate a little, then speed up again, thus changing its helicity. Let us examine what happens if we make a Lorentz transformation of an $m=0$ particle.

$$\Lambda \, | p, 0, 0, p, \lambda \rangle \qquad \begin{array}{l} \text{momentum } p \text{ along } z \text{ axis} \\ \text{spin } \lambda \;\; " \;\; z \text{ axis} \\ p > 0 \end{array}$$

$$\doteq (R \, L_{z \, |k| \leftarrow p}) \, (R \, L_{z \, |k| \leftarrow p})^{-1} \Lambda \, | p, 0, 0, p \, \lambda \rangle$$

$\Lambda p = k$, $L_{z \, |k| \leftarrow p}$ is a L.T. along the z axis taking $p \to |\vec{k}|$. R is a rotation of $(k, 0\, 0\, |\vec{k}|)$ into $\vec{k}$. Therefore $(R L_{z \, |k| \leftarrow p})^{-1} \Lambda \, (p, 0, 0, p) = (p, 0, 0, p)$

The most general form of $(R \, L_{z \, |k| \leftarrow p})^{-1} \Lambda$ is

$$\Lambda^{\mu}{}_{\nu} = \begin{bmatrix} 1 + \frac{|z|^2}{2} & x\cos\varphi + y\sin\varphi & x\sin\varphi - y\cos\varphi & -\frac{|z|^2}{2} \\ x & \cos\varphi & \sin\varphi & -x \\ -y & -\sin\varphi & \cos\varphi & y \\ \frac{|z|^2}{2} & -x\cos\varphi - y\sin\varphi & -x\sin\varphi + y\cos\varphi & 1 - \frac{|z|^2}{2} \end{bmatrix}$$

$|z|^2 = x^2 + y^2 \qquad z = 0$

gives

$$\begin{bmatrix} 1 & & & \\ & \cos\varphi & \sin\varphi & \\ & -\sin\varphi & \cos\varphi & \\ & & & 1 \end{bmatrix}$$

which just rotates the particle about its z axis, leaving ρ, & λ unchanged.

$\varphi = 0$ gives

$$\begin{bmatrix} 1+\frac{|z|^2}{2} & x & -y & -\frac{|z|^2}{2} \\ x & 1 & 0 & -x \\ -y & 0 & 1 & y \\ \frac{|z|^2}{2} & -x & y & 1-\frac{|z|^2}{2} \end{bmatrix}$$

We'll call these $R(\varphi)$ & Z.

Show that $R(\varphi)(Z) = (RZ)\,R(\varphi)$ (RZ is just a rotation of z through φ in the x, y plane.)

So
$$R(\varphi)\, Z\, |p\,o\,o\,p\,\lambda\rangle = RZ\, R(\varphi)\, |p\,o\,o\,p\,\lambda\rangle$$
$$R(\varphi)\, Z\, |p\,o\,o\,p\,\lambda\rangle = e^{i\lambda\varphi}\, RZ\, |p\,o\,o\,p\,\lambda\rangle$$

Suppose $Z\,|p\,o\,o\,p\,\lambda\rangle = |p\,o\,o\,p\ \lambda + \lambda_1(z)\rangle$

then $RZ\,|p\,o\,o\,p\,\lambda\rangle = |p\,o\,o\,p\ \lambda + \lambda_1(Rz)\rangle$

then $e^{i\lambda_1(z)} = 1$

so $\lambda_1(z) = 0$

Therefore $(R_{\hat{k}\leftarrow\hat{p}} L_z)^{-1} \Lambda$ leaves λ & POOP unchange

L_z certainly leaves λ (which is here spin along z axis) unchanged.
R leaves λ unchanged (hel. is invariant under rotations.)

Therefore a zero mass helicity 1 particle looks like a zero mass helicity 1 particle no matter what proper Lorentz transformation you make. It is perfectly possible, on grounds of Lorentz invariance only, to have a world in which only one kind of photon existed. Of coure this world wouldn't be invariant under parity.

We've gone through this so you'll understand that a hel +1 (-1) photon, which is unchanging, can never be represented by a 4 vector, which is a frame dependent quantity.

We could attempt to let $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\i\\0\end{pmatrix}$ be a polarizati vector for hel +1 photon moving along the

z axis. $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-i\\0\end{pmatrix}$ could represent hel. (-1).
This could be okay if we stuck to rotations. We could then say:

$$A^\mu(x) = \int \frac{d^3k}{2k} \, \epsilon(k,\lambda)\left[a(k,\lambda)\, e^{-ikx} + a^+(k,\lambda)\, e^{ikx}\right]$$

$\lambda = 1, 2$

$a(k,\lambda)$ destroys photons of polarization λ. $\lambda = 1, 2$ correspond to 2 degrees of linear polarization. ((0100) & (0010) for $k = (k, 0\,0\,k)$)

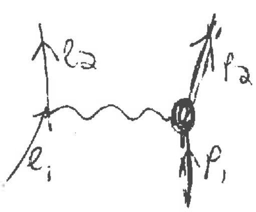
This is fine, as long as we don't make a Lorentz transformation to a new frame. Exercise: Show that under a Lorentz transformation $A^\mu(x)$ of this form transforms into an A^μ of this form plus $\frac{\partial}{\partial x_\mu}\chi$. (Show that we pick up stuff which behaves like hel. = 0)

~~If a man in another frame used our $A^\mu(x)$ it's going its goin.~~

The freedom of gauge simply allows us to eliminate the extra crud which accumalates if we make a Lorentz transformation to a new frame. ~~At this~~ This extra crud ~~likes~~ behaves as if it had helicity zero. (which is unphysical.)

Now that you understand why we imposed the restriction $q^\mu \langle 2 | J_\mu(0) | 1 \rangle = 0$, complete the problem assigned in Lecture 2.

Returning to our discussion of the pion form factor

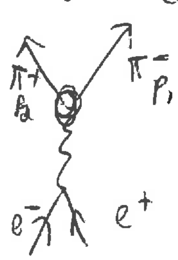


$$\langle \text{pion}_2 | J_\mu(0) | \text{pion } 1 \rangle = (P_1 + P_2)_\mu \, C(q^2)$$

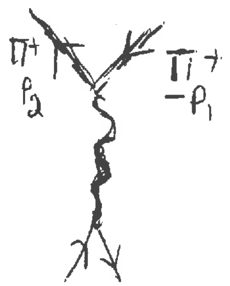
Show that $q^2 = (P_2 - P_1)^2 \leq 0$
(Hint: Stay in c.m.)

$\therefore$ $C(q^2)$ is defined only for $q^2 \leq 0$.

Consider the problem



Drawing the arrows as we would in Q.E.D.:



The outgoing π^- becomes an incoming π^+ of negative energy.

$$\left\langle \pi_2^{(+p_2)} \middle| J_\mu(0) \middle| \text{pion}, -p_1 \right\rangle$$

Show that physical q^2 for this process is $q^2 = (p_2 - (-p_1))^2 \geq 4m^2$

We only measure $C(q^2)$ for $q^2 \leq 0$ & $q^2 \geq 4m^2$ but we somehow believe they must be the same function. Of course this doesn't mean much unless $C(q^2)$ is some kind of nice function. We will learn later when we study idea 3, that the ideas of causality, & locality are very closely related to analyticity, dispersion relations, & in particular to $C(q^2)$ being an analytic function. (Of course we must know about its singularities.)

Returning to ep scattering, we'll run through how you find the most general form factor form, although you should try to do it yourself & calculate the cross-section.

$$\langle \text{Proton}_2 | J_\mu(0) | \text{proton}_1 \rangle = (\bar{u}_2 (\gamma_\mu) u_1)\, A(q^2)$$

$$+ (\bar{u}_2\, p_{1\nu} \gamma^\nu u_1)\, p_{2\mu} B(q^2)$$

$$+ p_{2\mu}\, C(q^2)$$

The only things behaving like vectors are $p_{i\mu}$ & γ^μ; terms like $(\bar{u}_2\, p_\nu \gamma^\nu u_1)\, p_\mu$ can always be simplified by using $\not{p}\, u = m\, u$.

$$(\bar{u}_2 \gamma_\mu u_1) A(q^2) + (\bar{u}_2 u_1)\left(p_{2\mu} B(q^2) + p_{1\mu} C(q^2)\right)$$

is the most general form.
Show that current conservation requires $B(q^2) = C(q^2)$.

This gives

$$\langle P_2 | J_\mu(0) | P_1 \rangle = (\bar{u}_2 \gamma_\mu u_1)\left(F_1(q^2) + 2M F_2(q^2)\right)$$

$$- (p_{1\mu} + p_{2\mu})\, \bar{u}_2 u_1\, F_2(q^2)$$

Show this can be written in the form

$$\bar{u}_2\left[\left(F_1(q^2) + \tfrac{1}{2}\left(\not{q}\,\gamma_\mu - \gamma_\mu \not{q}\right) F_2(q^2)\right]u_1\right.$$

For evaluating x sections it turns out that

$$G_E(q^2) = F_1 + \frac{q^2}{2M} F_2$$

$$G_M(q^2) = F_1 + 2MF_2$$ are more convenient.

Using the interaction $A^\mu J_\mu$, & letting the proton interact ~~ing~~ with a weak external ~~electro~~magnetic field, you might try to show that the coupling looks like $\mu \cdot B$ where μ is the magnetic moment & turns out to be $G_M(0)$. Experimentally $G_M(0) = 2.79\,\mu_0$.

$G_E(0) = F_1(0) = 1$ just as in the case of the pion, & just means the proton also has one unit of charge.

Lecture #1. Week # 2.

* Problems: (1) Find the cross-section for electron-proton
to be handed in
scattering if $\langle \text{proton}2 | j_\mu | \text{proton}1 \rangle = \bar{u}_2(\gamma_\mu F_1 + \frac{1}{2}(q\gamma_\mu - \gamma_\mu q)F_2)$
(2) Find a) the form of the matrix element if a scalar particle disintegrates to 2γ (e.m. int.)
b) the form of the matrix element if Λ disintegrates to $p + \pi^-$ (weak int., i.e. parity may be violated.) Also, find the angular dist. from a polarized Λ.
(3) For $\pi + p_1 \rightarrow \pi + p_2$ (strong int.), write the amplitude as $\bar{u}_2 (M) u_1$.

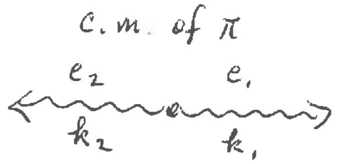
Notation: $\gamma_t = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$ $\sigma_z = i\gamma_x\gamma_y = \begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}$

$$\gamma_5 = \gamma_t \gamma_x \gamma_y \gamma_z = \frac{1}{4!}\epsilon_{\mu\nu\sigma\lambda}\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\lambda = i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$\epsilon_{\mu\nu\sigma\lambda}$ is the covariant totally antisymmetric tensor (i.e. $\epsilon_{0123} = 1$ and antisymmetric with respect to interchange of any two indicies.)

The 16 combinations of γ matrices are:
$$1,\ \gamma_\mu,\ \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu],\ \gamma_5,\ \gamma_5\gamma_\mu.$$
γ_5 is a pseudoscalar (i.e. $\bar{\psi}(x)\gamma_5\psi(x)$ is a pseudoscalar). Likewise, $\gamma_\mu\gamma_5$ is a pseudovector.

As an example, let us find the matrix element for the electromagnetic decay $\pi^0 \to \gamma + \gamma$.



π is a pseudoscalar spin-zero particle

$p = k_1 + k_2$

The principle of superposition tells us that the amplitude must be linear in e_1 and e_2, the polarization vectors of the two photons (we must be able to calculate the amplitude for a polarization vector $\frac{1}{\sqrt{\alpha^2+\beta^2}}(\alpha \vec{e}_x + \beta \vec{e}_y)$ by adding $\frac{1}{\sqrt{\alpha^2+\beta^2}}(\alpha \cdot \text{amp for } \vec{e}_x + \beta \cdot \text{amp for } \vec{e}_y)$).
Also, the amplitude must be a pseudoscalar (remembering that we actually multiply this amplitude by a quantity representing the pseudoscalar pion which in magnitude is 1 but changes sign on inversion.)
The amplitude must be of the form:

$\epsilon_{1\mu} \epsilon_{2\nu} \mathcal{G}_{\mu\nu}$ where $\mathcal{G}_{\mu\nu}$ can be constructed from the momenta in the problem

$\mathcal{G}_{\mu\nu} = k_{1\mu} k_{2\nu}$ won't work because of transversely polarized photons, i.e. $\epsilon \cdot k = 0$ (equivalently, gauge invariance requires invariance under $A_\mu \to A_\mu + \partial_\mu \chi$, or $e_\mu \to e_\mu + \alpha k_\mu$ in momentum space). $\mathcal{G}_{\mu\nu} = k_{2\mu} k_{1\nu}$ won't work either because parity requires antisymmetry with respect to k_1 & k_2.
However, $\mathcal{G}_{\mu\nu} = \epsilon_{\mu\nu\sigma\rho} k_{1\sigma} k_{2\rho}$ has the desired properties.

$$\therefore \mathcal{M} = \epsilon_{\mu\nu\sigma\rho}\, \epsilon_{1\mu}\, \epsilon_{2\nu}\, k_{1\sigma}\, k_{2\rho}\; A\,(\text{invariants})$$

The invariants must be constructed out of k_1 and k_2.

$k_1^2 = k_2^2 = 0$, $k_1 \cdot k_2 = \text{constant}$

$$\therefore \mathcal{M} = \epsilon_{\mu\nu\sigma\rho}\, \epsilon_{1\mu}\, \epsilon_{2\nu}\, k_{1\sigma}\, k_{2\rho}\; A$$ constant of order e^2 relative to strong decay matrix elements

$\mathcal{M}$ is gauge invariant because replacing $\epsilon_{1\mu}$ by $k_{1\mu}$ yields zero.
[Show that for $\epsilon_{1\mu} = (0, \vec{\epsilon}_1)$, $\epsilon_{2\mu} = (0, \vec{\epsilon}_2)$,

$$\mathcal{M} = A\, k_0\, \vec{k} \cdot (\vec{\epsilon}_1 \times \vec{\epsilon}_2) \quad \text{where } k_0 = k_{10} = k_{20}\,,\ \vec{k} = \vec{k}_1 = -\vec{k}_2$$

which implies that the polarization vectors of the two photons will tend to be perpendicular. What if the pion were a scalar particle? Could this be used as a test for the pseudoscalar nature of the pion?]

Weak Interactions

The chapter in Gasiorowicz on the Weak Interactions, starting pg. 495, are very good and well worth reading; and by all means, everyone should read R.P. Feynman and M. Gell-Mann, Phy. Rev 109, 193 (1958).

Examples: β decay $\quad n \to p + e^- + \bar{\nu}_e$

K capture $p + e^- \rightarrow n + \nu_e$ (same process as β-decay except differen physical region for momenta)

μ decay $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

μ capture $\mu^- + p \rightarrow n + \nu_\mu$

π decay $\pi^- \rightarrow e^- + \bar{\nu}_e$

The reactions are characterized by a slow rate (actually the matrix elements should be compared since phase space affects the rate.) The matrix elements are reduced by approximately 10^{-5} from those of the strong interactions. The above reactions are either leptonic or semi-leptonic, i.e. they involve the leptons μ, ν_μ, e, ν_e. These processes are governed by the conservation of leptons.

	L_e	L_μ
e^-	1	
e^+	-1	
ν_e	1	
$\bar{\nu}_e$	-1	
μ^-		1
μ^+		-1
ν_μ		1
$\bar{\nu}_\mu$		-1

Other slow reactions (strange decays and non-leptonic decays) are:

$K^+ \rightarrow \mu^+ + \nu_\mu$, $\Lambda \rightarrow p + e^- + \bar{\nu}_e$, $\Lambda \rightarrow p + \pi^-$, $K \rightarrow 2\pi$, $K \rightarrow 3\pi$

The last two decays gave one of the first indications of parity violation. If parity were conserved, $K \to 2\pi$ would imply K parity is +1 (spin of K is zero), while $K \to 3\pi$ would imply K parity is −1 (parity $= (-1)^3(-1)^\ell(-1)^\ell = -1$ where ℓ = relative ang. mom. of two of the pions). Originally, it was thought there were two particles τ and θ. An experimentalist named Block was the first to suggest that possibly τ and θ were the same particle and perhaps parity was violated. In 1956, Lee and Yang pointed out that no experiment had been done which tested whether parity was violated in the weak interactions. they proposed several experiments to test parity conservation; one of which was to measure the correlation of the electron's momentum and the polarization of the nucleus in the β-decay of a polarized nucleus. this would measure $p_e \cdot J$ which is a pseudoscalar quantity, and such a term in the matrix element would mean the interaction was not parity invariant. the experiment was done, and it showed that parity was not conserved.

The most general form of the beta decay matrix element can be written (with no gradient coupling):

$$\sum_i C_i(\bar{u}_p \Gamma_i u_n)(\bar{u}_e \Gamma_i u_\nu) + \sum_i C_i^*(\bar{u}_n \Gamma_i u_p)(\bar{u}_\nu \Gamma_i u_e)$$

$$+ \sum_i C_i'(\bar{u}_p \Gamma_i u_n)(\bar{u}_e \Gamma_i \gamma_5 u_\nu) + \sum_i C_i'^*(\bar{u}_n \Gamma_i u_p)(\bar{u}_\nu \Gamma_i \gamma_5 u_e)$$

[20 constants, or 10 if CP is conserved]

It was discovered that the massless spin $\frac{1}{2}$ neutrinos have only one type of helicity (negative for the neutrinos and positive for the anti-neutrinos). This two component neutrino theory requires parity not be conserved since space inversion changes the sign of helicity. (Remember that for a massless particle, the helicity cannot be changed by proper Lorentz transformations.) It was proposed by Feynman and Gell-Mann (and others independently) that all spinors in the weak int. appear as $\left(\frac{1+i\gamma_5}{2}\right)u$ where $\frac{1+i\gamma_5}{2}$ is the negative helicity projection operator for a massless particle. Thus, as $E/m \to \infty$, particles in weak int. tend to have their spin opposite to their momentum and vice-versa for antiparticles. This then requires the weak interaction to be of the form:

$$(\overline{au_p}\,\gamma_\mu\, a u_p) = \bar{u}_p\, a^+ \gamma_\mu\, a\, u_p = \bar{u}_p\, \gamma_\mu\, a\, u_p$$

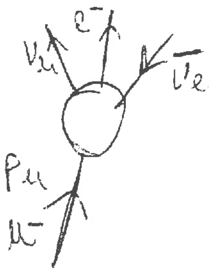
where $a = \left(\frac{1+i\gamma_5}{2}\right)$

Thus, for beta decay $n \to p\, e^- \bar{\nu}_e$,

$$\bar{u}_p\, \gamma_\mu \left(\frac{1+i\gamma_5}{2}\right) u_n\ \bar{u}_e\, \gamma_\mu \left(\frac{1+i\gamma_5}{2}\right) v_\nu$$

Lecture #2, Week #2

We shall calculate the rate for a μ to decay into $e^- + \nu_\mu + \bar{\nu}_e$ as an example of a weak interaction.



$$T = G\sqrt{8}\left(\bar{u}_e\,\gamma_\beta\left(\frac{1+i\gamma_5}{2}\right)v_{\bar{\nu}}\right)\left(\bar{u}_{\nu'}\,\gamma_\beta\left(\frac{1+i\gamma_5}{2}\right)u_\mu\right)$$

In this form, all wave functions have been multiplied by $\frac{(1+i\gamma_5)}{2}$. As $p/c \to 1$ particles have helicity $-\frac{v}{c}$, & antiparticles have helicity $+\frac{v}{c}$.

$$\text{Rate} = \frac{1}{\tau} = \frac{1}{2m_\mu}\,2\pi\,\delta(E_\nu+E_{\nu'}+E_e-m_\mu)\,|T|^2\,\frac{d^3p_e\,d^3p_\nu}{2(2\pi)^6\,2E_e\,2E_\nu E_{\nu'}}$$

$$\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5 \qquad \bar{\gamma}_\mu = \gamma_\mu \qquad \bar{\gamma}_5 = \gamma_5$$

$$\bar{\gamma}_\mu \equiv \gamma_t\,\gamma_\mu^{\dagger}\,\gamma_t$$

We shall calculate the total rate, averaging over initial spins & summing over final spins.

$\therefore$ we want $\frac{1}{2}\sum_{\substack{\text{all}\\ \text{polarizations}}} |T|^2$

For a review of trace tricks used in calculating $\sum_{pol.} |T|^2$, see Feynman, Q.E.D. pgs 112, 113; or Gasiorowicz, pgs. 149, 150. Our normalization is $\sum_{spins} u\bar{u} = \not{p} + m$.

$$|T|^2 = 8G^2 \left(\bar{u}_\mu \gamma_\beta \left(\frac{1+i\gamma_5}{2}\right) u_{\nu'}\right)\left(\bar{u}_{\nu'} \gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right) u_\mu\right)$$

$$\times \left(\bar{u}_e \gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right) v_{\bar{\nu}}\right)\left(\bar{v}_{\bar{\nu}} \gamma_\beta \left(\frac{1+i\gamma_5}{2}\right) u_e\right)$$

$$\frac{1}{2}\sum_{pol} |T|^2 = 4G^2 \, Tr\left[(\not{p}_\mu + m_\mu)\gamma_\beta \left(\frac{1+i\gamma_5}{2}\right) \not{p}_{\nu'} \gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right)\right] \times$$

$$Tr\left[(\not{p}_e + m_e)\gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right)(-\not{p}_{\bar{\nu}})\gamma_\beta \left(\frac{1+i\gamma_5}{2}\right)\right]$$

$$= 4G^2 \, Tr\left[(\not{p}_\mu + m_\mu)\gamma_\beta \not{p}_{\nu'} \gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right)\right] Tr\left[\not{p}_e \gamma_\alpha \not{p}_{\bar{\nu}} \gamma_\beta \left(\frac{1+i\gamma_5}{2}\right)\right]$$

Since $\left(\frac{1+i\gamma_5}{2}\right)^2 = \frac{1+i\gamma_5}{2}$ and neglecting m_e

$$= G^2 \, Tr\left[\not{p}_\nu \gamma_\beta \not{p}_{\nu'} \gamma_\alpha (1+i\gamma_5)\right] Tr\left[\not{p}_e \gamma_\alpha \not{p}_{\bar{\nu}} \gamma_\beta (1+i\gamma_5)\right]$$

Since $Tr\left(m_\mu \gamma_\beta \not{p}_{\nu'} \gamma_\alpha (1+i\gamma_5)\right) = 0$

$$Tr\left[\not{p}_\mu \gamma_\beta \not{p}_{\nu'} \gamma_\alpha\right] = 4\left[p^\mu_\beta p^{\nu'}_\alpha + p^\mu_\alpha p^{\nu'}_\beta - p^\mu \cdot p^{\nu'} \delta_{\alpha\beta}\right]$$

$$Tr\left[\not{p}_e \gamma_\alpha \not{p}_{\bar{\nu}} \gamma_\beta\right] = 4\left[p^e_\alpha \cdot p^{\bar{\nu}}_\beta + p^e_\beta p^{\bar{\nu}}_\alpha - p^e \cdot p^{\bar{\nu}} \delta_{\alpha\beta}\right]$$

$$\mathrm{Tr}\left[\not{p}_\mu \gamma_\beta \not{p}_{\nu'} \gamma_\alpha i \gamma_5\right] = -4i\, \epsilon_{\beta\alpha\sigma\lambda}\, p^\mu_\sigma\, p^{\nu'}_\lambda$$

$$\mathrm{Tr}\left[\not{p}_e \gamma_\alpha \not{p}_{\bar\nu} \gamma_\beta i \gamma_5\right] = -4i\, \epsilon_{\alpha\beta\sigma\lambda}\, p^e_\sigma\, p^{\bar\nu}_\lambda$$

$$= 4i\, \epsilon_{\beta\alpha\sigma\lambda}\, p^e_\sigma\, p^{\bar\nu}_\lambda$$

$$\tfrac{1}{2}\sum |T|^2 = 16\, G^2\left[p^\mu_\beta p^{\nu'}_\alpha + p^\mu_\alpha p^{\nu'}_\beta - p^\mu\cdot p^{\nu'}\delta_{\alpha\beta} - i\, \epsilon_{\beta\alpha\sigma\lambda}\, p^\mu_\sigma\, p^{\nu'}_\lambda\right] \times$$
$$\left[p^e_\beta p^{\bar\nu}_\alpha + p^e_\alpha p^{\bar\nu}_\beta - p^e\cdot p^{\bar\nu}\delta_{\alpha\beta} + i\, \epsilon_{\beta\alpha\chi\varphi}\, p^e_\chi\, p^{\bar\nu}_\varphi\right]$$

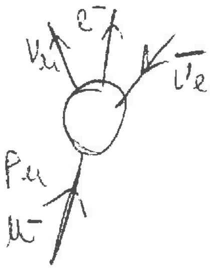
$$= 16\, G^2\Big[2\, p^\mu\cdot p^e\; p^{\nu'}\cdot p^{\bar\nu} + 2\, p^\mu\cdot p^{\bar\nu}\; p^{\bar\nu}\cdot p^e - \cancel{4\, p^e\cdot p^{\bar\nu}\; p^\mu\cdot p^{\nu'}}$$
$$+ 4\, \cancel{p^e\cdot p^{\bar\nu}\; p^\mu\cdot p^{\nu'}} + \underbrace{\epsilon_{\beta\alpha\sigma\lambda}\, p^\mu_\sigma\, p^{\nu'}_\lambda\, \epsilon_{\beta\alpha\chi\varphi}\, p^e_\chi\, p^{\bar\nu}_\varphi}_{= -2\, p^\mu\cdot p^e\; p^{\nu'}\cdot p^{\bar\nu} + 2\, p^\mu\cdot p^{\bar\nu}\; p^{\nu'}\cdot p^e}\Big]$$

$$\epsilon_{\beta\alpha\sigma\lambda}\,\epsilon_{\beta\alpha\chi\varphi} = -2\left(\delta_{\sigma\chi}\delta_{\lambda\varphi} - \delta_{\sigma\varphi}\delta_{\lambda\chi}\right)$$

$$= 64\, G^2\, p^\mu\cdot p^{\bar\nu}\; p^{\nu'}\cdot p^e$$

Lecture #2, Week #2

We shall calculate the rate for a μ to decay into $e^- + \nu_\mu + \bar{\nu}_e$ as an example of a weak interaction.



$$T = G\sqrt{8}\left(\bar{u}_e \gamma_\beta \left(\frac{1+i\gamma_5}{2}\right) v_{\bar{\nu}}\right)\left(\bar{u}_{\nu}\gamma_\beta\left(\frac{1+i\gamma_5}{2}\right)u_\mu\right)$$

In this form, all wave functions have been multiplied by $\frac{(1+i\gamma_5)}{2}$. As $v/c \to 1$ particles have helicity $-v/c$, & antiparticle have helicity $+v/c$.

$$Rate = \frac{1}{\tau} = \frac{1}{2m_\mu} 2\pi\,\delta(E_{\bar{\nu}}+E_{\nu}+E_e-m_\mu)\frac{|T|^2\, d^3p_e\, d^3p}{2(2\pi)^6\, 2E_e\, 2E_{\bar{\nu}} E}$$

$$\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5 \qquad \bar{\gamma}_\mu = \gamma_\mu \qquad \bar{\gamma}_5 = \gamma_5$$

$$\bar{\gamma}_\mu \equiv \gamma_t \gamma_\mu^{\dagger}\gamma_t$$

We shall calculate the total rate, averaging over initial spins & summing over final spins.

$\therefore$ we want $\frac{1}{2}\sum_{\substack{\text{all}\\ \text{polarizations}}} |T|^2$

For a review of trace tricks used in calculating $\sum_{pol.} |T|^2$, see Feynman, Q.E.D. pgs 112, 11[...] or Gasiorowicz, pgs. 149, 150. Our normalizat[...] is $\sum_{spins} u\bar{u} = \not{p} + m$.

$$|T|^2 = 8G^2 \left(\bar{u}_\mu \gamma_\beta \left(\frac{1+i\gamma_5}{2}\right) u_{\nu'}\right)\left(\bar{u}_{\nu'} \gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right) u_\mu\right)$$

$$\times \left(\bar{u}_e \gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right) v_{\bar{\nu}}\right)\left(\bar{v}_{\bar{\nu}} \gamma_\beta \left(\frac{1+i\gamma_5}{2}\right) u_e\right)$$

$$\frac{1}{2}\sum_{pol} |T|^2 = 4G^2 \,\mathrm{Tr}\left[(\not{p}_\mu + m_\mu)\gamma_\beta \left(\frac{1+i\gamma_5}{2}\right) \not{p}_{\nu'} \gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right)\right] \times$$

$$\mathrm{Tr}\left[(\not{p}_e + m_e)\gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right)(\not{p}_{\bar{\nu}})\gamma_\beta \left(\frac{1+i\gamma_5}{2}\right)\right]$$

$$= 4G^2 \,\mathrm{Tr}\left[(\not{p}_\mu + m_\mu)\gamma_\beta \not{p}_{\nu'} \gamma_\alpha \left(\frac{1+i\gamma_5}{2}\right)\right] \mathrm{Tr}\left[\not{p}_e \gamma_\alpha \not{p}_{\bar{\nu}} \gamma_\beta \left(\frac{1+i\gamma_5}{2}\right)\right]$$

Since $\left(\frac{1+i\gamma_5}{2}\right)^2 = \frac{1+i\gamma_5}{2}$ and neglecting m_e

$$= G^2 \,\mathrm{Tr}\left[\not{p}_\nu \gamma_\beta \not{p}_{\nu'} \gamma_\alpha (1+i\gamma_5)\right] \mathrm{Tr}\left[\not{p}_e \gamma_\alpha \not{p}_{\bar{\nu}} \gamma_\beta (1+i\gamma_5)\right]$$

Since $\mathrm{Tr}\left(m_\mu \gamma_\beta \not{p}_{\nu'} \gamma_\alpha (1+i\gamma_5)\right) = 0$

$$\mathrm{Tr}\left[\not{p}_\mu \gamma_\beta \not{p}_{\nu'} \gamma_\alpha\right] = 4\left[p^\mu_\beta p^{\nu'}_\alpha + p^\mu_\alpha p^{\nu'}_\beta - p^\mu \cdot p^{\nu'} \delta_{\alpha\beta}\right]$$

$$\mathrm{Tr}\left[\not{p}_e \gamma_\alpha \not{p}_{\bar{\nu}} \gamma_\beta\right] = 4\left[p^e_\alpha \cdot p^{\bar{\nu}}_\beta + p^e_\beta p^{\bar{\nu}}_\alpha - p^e \cdot p^{\bar{\nu}} \delta_{\alpha\beta}\right]$$

$$Tr\left[\not p_{\mu}\, \gamma_{\beta}\, \not p_{\nu'}\, \gamma_{\alpha}\, i\gamma_5\right] = -4i\, \epsilon_{\beta\alpha\sigma\lambda}\, p^{\mu}_{\sigma}\, p^{\nu'}_{\lambda}$$

$$Tr\left[\not p_{e}\, \gamma_{\alpha}\, \not p_{\bar\nu}\, \gamma_{\beta}\, i\gamma_5\right] = -4i\, \epsilon_{\alpha\beta\sigma\lambda}\, p^{e}_{\sigma}\, p^{\bar\nu}_{\lambda}$$

$$= 4i\, \epsilon_{\beta\alpha\sigma\lambda}\, p^{e}_{\sigma}\, p^{\bar\nu}_{\lambda}$$

$$\tfrac{1}{2}\sum |T|^2 = 16\, G^2 \Big[p^{\mu}_{\beta} p^{\nu'}_{\alpha} + p^{\mu}_{\alpha} p^{\nu'}_{\beta} - p^{\mu}\cdot p^{\nu'}\, \delta_{\alpha\beta} - i\, \epsilon_{\beta\alpha\sigma\lambda}\, p^{\mu}_{\sigma}\, p^{\nu'}_{\lambda}\Big] \times$$
$$\Big[p^{e}_{\beta} p^{\bar\nu}_{\alpha} + p^{e}_{\alpha} p^{\bar\nu}_{\beta} - p^{e}\cdot p^{\bar\nu}\, \delta_{\alpha\beta} + i\, \epsilon_{\beta\alpha\chi\varphi}\, p^{e}_{\chi}\, p^{\bar\nu}$$

$$= 16\, G^2 \Big[2\, p^{\nu}\cdot p^{e}\; p^{\nu'}\cdot p^{\bar\nu} + 2\, p^{\mu}\cdot p^{\bar\nu}\; p^{\bar\nu}\cdot p^{e} - \cancel{4\, p^{e}\cdot p^{\bar\nu}\; p^{\mu}\cdot p^{\nu'}}$$
$$+ 4\, \cancel{p^{e}\cdot p^{\bar\nu}\; p^{\mu}\cdot p^{\nu'}} + \underbrace{\epsilon_{\beta\alpha\sigma\lambda}\, p^{\mu}_{\sigma}\, p^{\nu'}_{\lambda}\, \epsilon_{\beta\alpha\chi\varphi}\, p^{e}_{\chi}\, p}_{= -2\, p^{\mu}\cdot p^{e}\; p^{\nu'}\cdot p^{\bar\nu} + 2\, p^{\mu}\cdot p^{\bar\nu}\; p}$$

$$\epsilon_{\beta\alpha\sigma\lambda}\, \epsilon_{\beta\alpha\chi\varphi} = -2\left(\delta_{\sigma\chi}\delta_{\lambda\varphi} - \delta_{\sigma\varphi}\delta_{\lambda\chi}\right)$$

$$= 64\, G^2\, p^{\mu}\cdot p^{\bar\nu}\; p^{\nu'}\cdot p^{e}$$

Ph 230a

Lecture #3. Week #2.

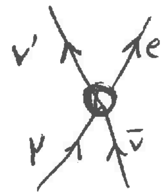
Corrections : (1) L#2. W#1. pg 1. ℓ 16.

$$\text{Rate} = \Big(\prod_i \frac{1}{2E_i}\Big)\Big(\prod_f \frac{1}{2E_f}\Big)\, 2\pi\,\delta\Big(\sum_f E_f - \sum_i E_i\Big)\, |T_{fi}|^2 \prod_{\substack{\text{all } f\\ \text{but } 1}} \frac{d^3p_f}{(2\pi)^3}$$

(2) L#1. W#2. pg 1.

$$\gamma_5 = \gamma_x\gamma_y\gamma_z\gamma_t = -\gamma_t\gamma_x\gamma_y\gamma_z = -\frac{1}{4!}\epsilon_{\mu\nu\sigma\lambda}\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\lambda = i\,(\,0 \;\; 1 \ldots$$

Continuation of the muon decay rate calculation:



$P^\mu = (m_\mu, 0)$

$$\text{Rate} = \frac{1}{2m_\mu}\, 2\pi\,\delta(E_{\bar\nu} + E_{\nu'} + E_e - m_\mu)\, |T|^2 \frac{d^3p_e}{(2\pi)^3}\frac{d^3p_{\bar\nu}}{(2\pi)^3}\frac{1}{8E_eE_{\bar\nu}E_{\nu'}}.$$

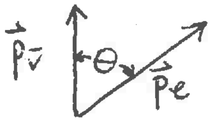
$$|T|^2 = 64\,G^2\,(\underline{P}^\mu\cdot\underline{P}^{\bar\nu})(\underline{P}^{\nu'}\cdot\underline{P}^e)$$

The three particle final state phase space may be taken from L#2. W#1. pg 2. (Be sure you can derive it!)

$$\text{Rate} = \frac{1}{2m_\mu}\,|T|^2\,\frac{1}{8E_e(2\pi)^5}\;\frac{p_{\bar\nu}^3\,p_e^2\,dp_e\,d\Omega_e\,d\Omega_{\bar\nu}}{p_{\bar\nu}^2(m_\mu - E_e) + p_{\bar\nu}(\vec p_{\bar\nu}\cdot\vec p_e)}$$

(note: the rate will be evaluated slightly differently from in class.)

We may choose our axis in the direction of $\vec p_{\bar\nu}$ (π is spinless, so no preferred direction). Define θ as the angle between $\vec p_{\bar\nu}$ and $\vec p_e$, i.e.



then $\displaystyle\iint d\Omega_e\, d\Omega_{\bar\nu} = \int_{-1}^{1} 4\pi\cdot 2\pi\, d(\cos\theta)$

$$\therefore \text{Rate} = \frac{1}{64 m_\mu \pi^3} \iint_{-1}^{1} \frac{p_{\bar\nu}\, p_e \, d(\cos\theta)\, dp_e}{m_\mu - p_e + p_e\cos\theta} \, |T|^2$$

the mass of the e^- has been neglect

$$P^\mu = P^e + P^{\bar\nu} + P^{\nu'} \Rightarrow (P^{\nu'})^2 = 0 = (P^e + P^{\bar\nu} - P^\mu)^2$$

$$= m_\mu^2 + 2 p_e p_{\bar\nu}(1-\cos\theta) - 2 p_{\bar\nu} m_\mu - 2 p_e m_\mu$$

$$\therefore \quad p_{\bar\nu} = \frac{m_\mu (m_\mu - 2 p_e)}{2(m_\mu - p_e + p_e\cos\theta)} \qquad \text{where } P^e = (p_e, \vec{p}_e),\ P^{\bar\nu} = (p_{\bar\nu}, \vec{p}_{\bar\nu})$$

$$|T|^2 = 64 G^2 (P^\mu \cdot P^{\bar\nu})(P^{\nu'} \cdot P^e) = 64 G^2 m_\mu p_{\bar\nu}\, P^{\nu'} \cdot P^e$$

$$P^e \cdot P^{\nu'} = P^e \cdot (P^\mu - P^{\bar\nu} - P^e) = m_\mu p_e - p_e p_{\bar\nu}(1-\cos\theta)$$

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$$\left\{\frac{m_\mu(m_\mu - 2p_e)}{2(m_\mu - p_e + p_e\cos\theta)}\right\} \left\{\frac{p_e\, dp_e\, d(\cos\theta)}{m_\mu - p_e + p_e\cos\theta}\right\}$$

$$= \frac{G^2}{8\pi^3} \int dp_e\, m_\mu^4 p_e^2 (m_\mu - 2p_e)^2 \int_{-1}^{1} \frac{(1+\cos\theta)}{(m_\mu - p_e + p_e\cos\theta)^4} d(\cos\theta)$$

$$\int_{-1}^{1} \frac{1+x}{(m_\mu - p_e + p_e x)^4} dx = \frac{2}{3} \frac{(3m_\mu - 4p_e)}{m_\mu^3 (m_\mu - 2p_e)^2}$$

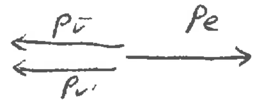
$$\therefore \text{Rate} = \frac{G^2}{8\pi^3} \int dp_e\, m_\mu^4 p_e^2 \cancel{(m_\mu - 2p_e)^2}\, \frac{2}{3} \frac{(3m_\mu - 4p_e)}{m_\mu^3 \cancel{(m_\mu - 2p_e)^2}}$$

$$\therefore \text{Rate} = \frac{G^2}{12\pi^3}\int m_\mu\, p_e^2\,(3m_\mu - 4p_e)\,dp_e$$

The spectrum is proportional to $p_e^2(3m_\mu - 4p_e)$. Small corrections to the spectrum from electromagnetism (of order e^2) need to be made before comparison with experiment. the electromagnetic corrections come from graphs of the type:

(There exists a technical problem in the analysis of the experimental data. The detectors cannot detect a photon with a wavelength greater than a certain maximum, say λ_m; so that the experimental apparatus measures $\mu \to e\nu\nu$ plus $\mu \to e\nu\nu\gamma$ for E_γ up to hc/λ_m.) The theoretically predicted spectrum agrees very well with the experimental spectrum (see pg 530, Gasiorowicz).

We must integrate over the electron's momentum to obtain the total rate for μ decay (the partial mode $\mu \to e\nu\nu$ accounts for nearly 100% of muon decays). the electron's momentum is a maximum when the neutrinos come out opposite to the electron, i.e.



$\underline{p}^\mu = \underline{p}^e + \underline{p}^{\nu'} + \underline{p}^{\bar\nu} \Rightarrow$ $p_e = p_{\nu'} + p_{\bar\nu}$

$0 = m_\mu^2 + (\cancel{2p_{\nu'}\cdot p_{\bar\nu}} - \cancel{2p_e\cdot p_{\bar\nu}}) - 2m_\mu p_{\nu'} - 2m_\mu p_{\bar\nu}$

$$\therefore\ p_e = p_{\nu'} + p_e = \frac{m_\mu}{2}$$

i.e. $0 \le p_e \le \frac{1}{2}m_\mu$

$$\text{Total Rate} = \frac{1}{\tau_{\leftarrow \tau\ \text{lifetime}}} = \frac{G^2}{12\pi^3}\int_0^{\frac{1}{2}m_\mu} m_\mu p_e^2(3m_\mu - 4p_e)\,dp_e$$

$$= \frac{G^2 m_\mu^5}{192\pi^3} \Rightarrow \tau = \frac{192\pi^3}{G^2 m_\mu^5}$$

Experimentally, $\tau = 2.1983 \pm .0008 \times 10^{-6}$

which implies $G M_p^2 \simeq 1.02 \times 10^{-5}$ (M_p is the mass of the proton)

* Problem: Find the rate for beta decay $n \to p e \bar{\nu}_e$ using

$$\frac{cG}{\sqrt{2}}\left(\bar{u}_p \gamma_\alpha (1 + g_A i\gamma_5) u_N\right)\left(\bar{u}_e \gamma_\alpha (1 + i\gamma_5) v_\nu\right)$$

as the matrix element. Do not neglect the mass of the electron, but do neglect $m_n - m_p$ in comparison to m_p. (check: put $m_e = 0$ and you should get $\frac{2G^2}{30\pi^3} W^5 c^2 \left(\frac{1+3g_A^2}{4}\right)$ where $W = m_n - m_p$)

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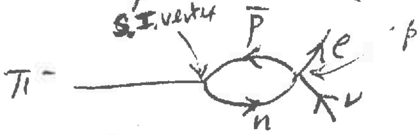

is a possible graph leading to the decay

$(\bar{p}n)_\mu$ includes both a vector and an axial vector part (since parity violation indicates an interference of the two terms).

Ph 230a

Lecture #3. Week #2.

Corrections: (1) L#2. W#1. pg 1. ℓ 16.

$$\text{Rate} = \left(\prod_i \frac{1}{2E_i}\right)\left(\prod_f \frac{1}{2E_f}\right) 2\pi\,\delta\left(\sum_f E_f - \sum_i E_i\right) |T_{fi}|^2 \prod_{\substack{\text{all } f \\ \text{but } 1}} \frac{d^3p_f}{(2\pi)^3}$$

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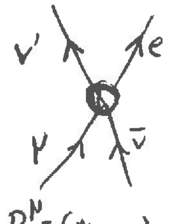
$$\gamma_5 = \gamma_x\gamma_y\gamma_z\gamma_t = -\gamma_t\gamma_x\gamma_y\gamma_z = -\frac{1}{4!}\epsilon_{\mu\nu\sigma\lambda}\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\lambda = i\left(\begin{smallmatrix}0 & \\ 1 & \end{smallmatrix}\right.$$

Continuation of the muon decay rate calculation:



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$$|T|^2 = 64\,G^2\,(\underline{P}^\mu \cdot \underline{P}^{\bar\nu})(\underline{P}^{\nu'} \cdot \underline{P}^e)$$

the three particle final state phase space may be taken from L#2. W#1. pg 2. (Be sure you can derive it!)

$$\text{Rate} = \frac{1}{2m_\mu}|T|^2 \frac{1}{8E_e(2\pi)^5}\,\frac{p_{\bar\nu}^3\,p_e^2\,dp_e\,d\Omega_e\,d\Omega_{\bar\nu}}{p_{\bar\nu}^2(m_\mu - E_e) + p_{\bar\nu}(\vec p_{\bar\nu}\cdot\vec p_e)}$$

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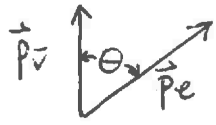

then $$\iint d\Omega_e\,d\Omega_{\bar\nu} = \int_{-1}^{1} 4\pi\cdot 2\pi\,d(\cos\theta)$$

$$\therefore \text{Rate} = \frac{1}{64 m_\mu \pi^3} \iint_{-1}^{1} \frac{p_{\bar\nu}\, p_e\, d(\cos\theta)\, dp_e}{m_\mu - p_e + p_e \cos\theta} |T|^2$$

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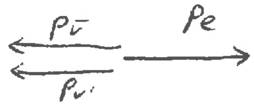

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i.e. $0 \le p_e \le \frac{1}{2}m_\mu$

$$\text{Total Rate} = \frac{1}{\tau} = \frac{G^2}{12\pi^3}\int_0^{\frac{1}{2}m_\mu} m_\mu\, p_e^2 (3m_\mu - 4p_e)\, dp_e$$
$\tau \leftarrow$ lifetime

$$= \frac{G^2 m_\mu^5}{192\pi^3} \Rightarrow \tau = \frac{192\pi^3}{G^2 m_\mu^5}$$

Experimentally, $\tau = 2.1983 \pm .0008 \times 10^{-6}$

which implies $G M_p^2 \simeq 1.02 \times 10^{-5}$ (M_p is the mass of the proton)

* Problem: Find the rate for beta decay $n \to p e \bar{\nu}_e$ using

$$\frac{cG}{\sqrt{2}} (\bar{u}_p \gamma_\alpha (1 + g_A i \gamma_5) u_N)(\bar{u}_e \gamma_\alpha (1 + i\gamma_5) v_\nu)$$

as the matrix element. Do not neglect the mass of the electron, but do neglect $m_n - m_p$ in comparison to m_p. (check: put $m_e = 0$ and you should get $\frac{2G^2}{30\pi^3} W^5 c^2 \left(\frac{1+3g_A^2}{4}\right)$ where $W = m_n - m_p$)

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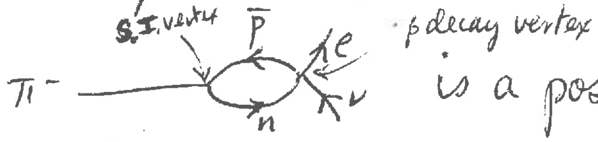


is a possible graph leading to the decay

$(\bar{p}n)_\mu$ includes both a vector and an axial vector part (since parity violation indicates an interference of the two terms).

Ph230a

Lecture #1, week #3

Last time we said $n \to p + e + \bar{\nu}$ was described by $\frac{g_C}{\sqrt{2}} (\bar{u}_p \gamma_\alpha (1 + g_a i \gamma_5) u_N)(\bar{u}_e \gamma_\alpha (1 + i\gamma_5) u_\nu)$.

This means $\pi^- \to \pi^0 + e + \bar{\nu}$ via $\pi^- \to n + \bar{p} \to p + \bar{p} + e + \to \pi^0 + e + \bar{\nu}$. There should, in fact, be a whole class of decays in which an electron-antineutrino current is emitted, carrying away one unit of charge, & there is no change in strangeness of the strongly interacting particles. The change in state of the strongly interacting particles can be described by the matrix elements of a current, i.e. $\langle b | J_\mu^{\Delta S = 0\ \Delta Q = 1} | a \rangle$. J_μ has a vector & an axial vector part. For $a = n$ $b = p$, the form is given above.

In $\Lambda \to p + e + \bar{\nu}$, the electron-neutrino current carries away one unit of charge as usual, but the strangeness increases by one.
(others of this type: $K^- \to \pi^0 + e + \bar{\nu}$ $K^0 \to \pi^+ + e^- + \bar{\nu}$)
Such decays are characterized by $\Delta S = \Delta Q = \pm 1$.
So we have another current: $J_\mu^{\Delta S = \Delta Q = 1}$

What about terms such that $\Delta S = -\Delta Q$?

The experimental absence of $K^0 \to \pi^- + e^+ + \nu$, or $\Sigma^+ \to n + e^+ + \nu$ is a strong argument against there being any $\Delta S = -\Delta Q = 1$ type terms. ~~(Note for the curre~~
(Note: if J_μ creates certain quantum numbers, J_μ^+ will create the "anti" quantum numbers.)

Again, there seems to be no $\Delta S = 2$ terms, such as $\Xi^- \to n + e^- + \bar{\nu}$.
You might try to show that the absence of $|\Delta S| = 2$ terms implies for $|\Delta S| = 1$ terms, $\Delta S = \Delta Q$.

The absence of $\Lambda \to N + \nu + \bar{\nu}$, indicates that there are no $\Delta S = 1$, $\Delta Q = 0$ terms.
The lack of $\Delta Q = 0$ terms in general is a result of the seemingly correct rule that in weak interactions, leptons are always produced in the combination lepton-leptonantineutrino.

We can write symbolically the types of weak interactions we've discussed.

$(\bar{\nu}\mu)(\bar{e}\nu)$
$(\bar{P}N)(\bar{e}\nu)$
$(\bar{P}N)(\bar{\mu}\nu)$
$(\bar{P}\Lambda)(\bar{e}\nu)$
$(\bar{P}\Lambda)(\bar{\mu}\nu)$

In addition, we have to discuss the structure of non-leptonic decays,

$\Lambda \to p + \pi$

$K \to 2\pi$

$K \to 3\pi$

The absence of $\Delta S = 2$ terms here, i.e. $\Xi^- \to \pi + n$, means, as in the leptonic decays, that $|\Delta S| = 1$ terms are of the form $\Delta S = \Delta Q = \pm 1$. (Can you show this?) We can therefore take into account non-leptonic decays by including terms like "$(\bar{P}\Lambda)$" "$(\bar{N}P)$".

You may notice that weak interactions corresponding to almost all combinations of the currents on our list occur. The hypothesis has been made that in fact they all do occur. That is, the weak interactions can be described by $G J_\mu^+ J_\mu$

$$J_\mu = \bar{e}\nu + \bar{\mu}\nu + \text{“}\bar{n}p\text{”} + \text{“}\bar{\Lambda}p\text{”}$$
$$J_\mu^+ = \bar{\nu}e + \bar{\nu}\mu + \text{“}\bar{p}n\text{”} + \text{“}\bar{p}\Lambda\text{”}$$

$$J_\mu = J_\mu^{\text{lepton}} + J_\mu^{\Delta S=0} + J_\mu^{\Delta S=-1}$$

All weak interactions (except CP violation) can be explained by this form.
We have yet to discuss the relative sizes of the 4 different contributions.
Note that while there are 4 unknown coefficients, there are 10 possible interactions so we should get predictions involving

$e^+ + e^- \rightarrow \nu + \bar{\nu}$

& $Np \rightarrow NP$ (with parity violation)

The former reaction ~~h~~ is very difficult to observe & hasn't been seen. However its existence or nonexistence is very important to an understanding of collapsing stars.
~~If the process occurs, then the small size of~~

Neutrinos interact so weakly, that

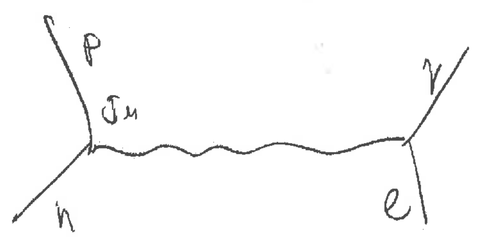
if the process occurs, they will escape from the star, carrying energy. Escaping energy causes contraction, contraction increases e^+e^- density, hence more $\nu+\bar{\nu}$'s are formed, more energy escapes, more contraction, (boom?)

~~In principle it should~~

Although the non-parity violating part of the weak $NP \to PN$ interaction is swamped by the strong interactions, we should expect to be able to see the parity violating part of the weak interaction in a nuclear physics experiment. There is evidence that it exists.

The form of the weak interaction: $\sqrt{8}G J_\mu^\dagger J_\mu$ is reminiscent of QED, where there is a current at each vertex, & a photon propagating across, $\Box^2 A_\mu = J_\mu$ & the interaction is $J^\mu A_\mu$.

We could try writing the weak interac as $W^\mu J_\mu$, with a massive vector meson W propagating,



$$(\Box^2 + m_\omega^2) = J_\omega^\mu$$

Using spin one perturbation theory:

$$(\sqrt{4\pi e_\omega^2})^2 \; J_\mu^+ \left[\frac{\delta_{\mu\nu} - \frac{q_\mu q_\nu}{m_\omega^2}}{q^2 - m_\omega^2} \right] J_\nu$$

The lack of any $\frac{1}{q^2}$ dependence in the weak interactions ~~[illegible]~~ means $M_\omega \gg |q|$, so that what we see is

$$\approx \frac{4\pi e_q^2 \, J_\mu^+ J_\mu}{-M^2}$$

$$\sqrt{8}\, G = \frac{4\pi e_\omega^2}{M_\omega^2}$$

if $M^2 \omega$ is very large, then ~~there~~ should be a large probability ~~for~~ of W meson production. ~~the~~ Lack of evidence ~~of~~ for W meson production in present experiments implies that, if it exists, $M_\omega > 2$ Bev.

Lecture # 2. Week # 3.

We have determined the weak interaction to be of the following form:

$$\frac{G}{\sqrt{2}} J_\mu^+ J_\mu$$

constants

$$\text{where} \quad J_\mu = (\bar{e}\,\nu_e)_\mu + (\bar{\mu}\,\nu_\mu)_\mu + c\,\text{"}(\bar{n}\,p)\text{"}_\mu + s\,\text{"}(\bar{\Lambda}\,p)\text{"}_\mu$$
$$= j_\mu^{e\nu} + j_\mu^{\mu\nu} + c\, j_\mu^{\Delta S=0} + s\, j_\mu^{\Delta S=-1}$$

$$(\bar{e}\,\nu_e) = \bar{u}_e \gamma_\mu (1 + i\gamma_5) u_\nu \quad \text{and} \quad (\bar{\mu}\,\nu_\mu) = \bar{u}_\mu \gamma_\mu (1 + i\gamma_5) u_\nu$$

Let us examine the non-strangeness changing hadronic current. For example in neutron β-decay, the matrix element of $j_\mu^{\Delta S=0}$ may be written in general:

$$\langle \text{proton } 2 |\, c\, j_\mu^{\Delta S=0} \,|\, \text{neutron } 1 \rangle = c\, \bar{u}_p(p_2) [\overbrace{\gamma_\mu V_1(q^2) + \tfrac{1}{2}(\not{q}\gamma_\mu - \gamma_\mu \not{q}) V_2(q^2) + i q_\mu V_3(q^2)}^{\text{vector part}}$$
$$\underbrace{+ i\gamma_\mu \gamma_5 A_1(q^2) + \tfrac{1}{2}(\not{q}\gamma_\mu - \gamma_\mu \not{q})\gamma_5 A_2(q^2) + i q_\mu \gamma_5 A_3(q^2)}_{\text{axial vector part}}] u_n$$

There is both a vector and an axial vector part (as for the leptons) since parity is violated. The form factors depend on $q^2 = (p_2 - p_1)^2$ and are unknown if we are ignorant of the strong interactions. The form factors may be chosen real if CP is conserved.

In most cases of nuclear β decay, the ~~total~~ maximum momentum is less than 20 mev, so that the q dependence may be neglected. If we let $q \to 0$ in the neutron β-decay matrix element, we have:

$$c \; \bar{u}_p(p_2)\left[\gamma_\mu g_V + i\gamma_\mu\gamma_5 g_A\right]u_n(p_1) \; \bar{u}_e \gamma_\mu (1+i\gamma_5)u_\nu \; \frac{G}{\sqrt{2}}$$

where we have defined $g_V = V_1(0)$ and $g_A = A_1(0)$. Experimental evidence has shown that $c \times g_V$ is very close to one. Originally, the idea of universality of the weak interactions (i.e. that all weak interactions have the same coupling constant G) suggested that $c \equiv 1$ and g_V very close to but not quite one. However, we shall see that there are good reasons for proposing $g_V \equiv 1$ and c not quite one. The numbers g_A and g_V just include the strong interaction renormalization effects which do not enter in the lepton current (since the leptons do not interact strongly).

Since the proton has very little momentum in the neutron rest frame, we may make a non-relativistic reduction of the spinors:

$$\bar{u}_p(p_2)\,\gamma_t\, u_n(p_1) \simeq \bar{u}_p u_n \quad , \quad \bar{u}_p \gamma_i u_n \simeq 0 \quad \text{for } i = x,y,z$$

$$\bar{u}_p\, i\gamma_i\gamma_5\, u_n \simeq \bar{u}_p \sigma_i u_n \quad \text{for } i = x,y,z \quad , \quad \bar{u}_p\, i\gamma_t\gamma_5\, u_n \simeq 0$$

$$\therefore M.E. = (\bar{u}_p u_n)(\bar{u}_e \gamma_4 (1+i\gamma_5) u_\nu) + \bar{u}_p \sigma_i u_n \bar{u}_e \gamma_i (1+i\gamma_5) u_\nu$$

Transitions taking place via the first term are called "Fermi" transitions ($\Delta l = 0$) and via the second term are called "Gammow-Teller" transitions ($\Delta l = \pm 1$ except no $0 \to 0$). The current $j^{\Delta S=0}$ has changed a neutron into a proton — similar to the raising operator τ_+ for isospin. The matrix element in the case of nuclear β-decay is much more difficult to determine (since nuclear wave functions are poorly known). There are two easy cases: (1) $n \to p e \nu$ (i.e. no nuclear physics) and (2) two nuclei ~~having~~ with different no. of protons and neutrons having the same wave function and one decaying into the other (possible since nuclear forces isospin independent, e.g. O^{14} 6 n 8 p. Not all states of O^{14} and N^{14} are the same, ~~for they~~ the corresponding states must have the same symmetry.

N^{14} 7 n 7 p

$O^{14} \to N^{14*} + e + \bar{\nu}_e$

From O^{14} decay, we can deduce:

$$c = .980 \pm .002 \quad \left(\begin{array}{c}\text{when electromagnetic corrections} \\ \text{are taken into account}\end{array}\right)$$

$\pm .002$ is the experimental error. There is also a $\pm .005$ uncertainty in c from the uncertainity in the energy separation of the two nuclear states (from e.m radiative corrections), and a $\pm .002$ uncertainty from e.m. corrections to the wave functions.

therefore, we can say that:

$$c \simeq .981 \pm .008$$

c is close to one, but it looks as if it is not exactly one. the current value of g_A is:

$$g_A = 1.23 \pm .01$$

One of the greatest theoretical achievements in recent years has been the beautiful calculation of this number from strong interaction physics. Adler and Weisberger have computed g_A to be:

$$g_A = 1.24 \pm .03$$ — quite remarkable agreement.

Lecture #1, Week #4

We have the form $\frac{G}{\sqrt{2}} J_\mu^{+} J_\mu$ for the weak interactions, with

$$J_\mu = (\bar{e}\nu) + (\bar{\mu}\nu) + c\,(J^{\Delta S=0}) + s\,(J^{\Delta S=\Delta Q=-1})$$

The same coefficient appears in front of $\bar{e}\nu$ & $(\bar{\mu}\nu)$; we can check this against pion leptonic decays.

$$\pi^- \to e + \bar{\nu}_e$$
$$\to \mu + \bar{\nu}_\mu$$

$$\langle 0 | J_\mu^{\Delta S=0} | \pi \rangle = \langle 0 | J_\mu^{\text{vector } \Delta S=0} + J_\mu^{\text{Axial} \, \Delta S=0} | \pi \rangle$$

$$= A\, p_{\pi\mu}$$

$p_{\pi\mu}$ is a vector. π is pseudoscalar & therefore $\langle 0 | J_\mu^{\text{vector}} | \pi \rangle$ transforms like an axial vector, while $\langle 0 | J_\mu^{\text{Axial}} | \pi \rangle$ " " vector, & so only J_μ^{Axial} contributes.

A is usually defined as $\sqrt{2} f_\pi$.

$$T = \frac{GC}{\sqrt{2}} \left(\bar{u}_e \gamma_\mu (1 + i\gamma_5) V_{\bar{\nu}} \right) \sqrt{2} f_\pi p_\mu$$

$$= \frac{GC}{\sqrt{2}} \left(\bar{u}_e \not{p}_\pi (1 + i\gamma_5) V_{\bar{\nu}} \right) \sqrt{2} f_\pi$$

$$|T|^2 = G^2 C^2 f_\pi^2 \, m_e^2 \, (p_{\bar\nu} \cdot p_e) \, 4$$

$$(p_e \cdot p_{\bar\nu}) = \frac{m_\pi^2 - m_e^2}{2}$$

$$\Gamma = \frac{1}{2M_\pi} |T|^2 \frac{1}{4(2\pi)^2} \frac{p_\nu^3 \, d\Omega_\nu}{E_\pi \tilde p_\nu^2 - E_\nu(\tilde p_\pi \cdot \tilde p_\nu)}$$

$p_\pi = 0$

$E_\pi = m_\pi$

$$p_\nu = \frac{m_\pi^2 - m_e^2}{2m_\pi}$$

$$\Gamma = \frac{m_\pi^3 G^2 C^2 f_\pi^2 \left(\frac{m_e^2}{m_\pi^2}\right)}{8\pi} \left(1 - \frac{m_e^2}{m_\pi^2}\right)^2$$

$$f_\pi = .96 \, m_\pi^3$$

Sometimes written $F_\pi = \dfrac{CG f_\pi}{\sqrt{4\pi} \, m_\pi^2}$

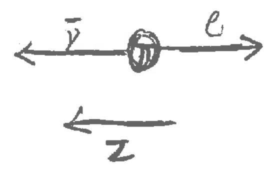
$$\frac{\Gamma(\pi \to e\bar\nu)}{\Gamma(\pi \to \mu\bar\nu)} = \left(\frac{m_e^2}{m_\mu^2}\right) \frac{\left(1 - \frac{m_e^2}{m_\pi^2}\right)^2}{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2} = 13.6 \times 10^{-5}$$

With equal coefficients for $(\bar e\nu)$ & $(\bar\mu\nu)$ the rate ratio agrees with experiment to within 2%. Therefore the coefficients are equal to within 1%.

Phase space favors $\pi \to e\nu$, get $\dfrac{\Gamma(\pi \to \mu\bar\nu)}{\Gamma(\pi \to e\bar\nu)} \gg 1$

Why?

The decay looks like:



$\bar{\nu}$ has positive helicity; e must also have positive helicity in order that $J_z = 0$.

The electron would like to have negative helicity as $\frac{E}{m_e} \to \infty$ because of $1+i\gamma_5$. $\frac{E_e}{m_e} \approx \frac{m_\pi}{2m_e}$

The electron is highly relativistic, while the much heavier muon is not so relativistic. The electron decay is greatly suppressed.

Return to beta decay:

(Suggestion: Read pp. 573-576 of Gas.)

$$c J_\mu^{\Delta S=0} = c\left(g_V(0)\, \bar{u}_p \gamma_\mu u_n + g_A(0)\, \bar{u}_p\, i\, \gamma_\mu \gamma_5 u_n\right)$$

$c\, g_V(0) = .980$

Let's try setting $g_V(0) = 1$.

$q^2 = 0 \qquad \langle p | J^{+\,\text{vector}\,\Delta S=0} | n \rangle = \bar{u}_p \gamma_\mu u_n$

In terms of fields $J_\mu^{+} \overset{\text{vector } \Delta S=0}{=} \bar{\Psi}_P \gamma_\mu \psi_N$ $(q^2=0)$

It's ~~be~~ an Ispin raising operator.

The corresponding I spin lowering operator is $\bar{\Psi}_N \gamma_\mu \psi_P$

The I_3 member of the isotopic triplet I^+, I^-, I_3 is $\frac{1}{\sqrt{2}}(\bar{\Psi}_P \gamma_\mu \psi_P - \bar{\Psi}_N \gamma_\mu \psi_N)$

We can also form an isoscalar operator

$$I_0 = \frac{1}{\sqrt{2}} (\bar{\Psi}_p \gamma_\mu \psi_p + \bar{\Psi}_N \gamma_\mu \psi_n)$$

(Note on + & - sign:)

($\begin{pmatrix}\psi_p \\ \psi_n\end{pmatrix}$ transforms according to $e^{-iT_i \cdot \theta}\begin{pmatrix}\psi_p \\ \psi_n\end{pmatrix}$ $(T_i^+ = T_i)$

the adjoint fields therefore transform according to $e^{iT_i^* \cdot \theta}\begin{pmatrix}\bar{\Psi}_p \\ \bar{\Psi}_n\end{pmatrix}$ which is equivalent

to $e^{-iT_i\theta}\begin{pmatrix}-\bar{\Psi}_n \\ \bar{\Psi}_p\end{pmatrix}$)

The $q^2=0$ electromagnetic current ~~[illegible]~~ is just the charge measuring operator

$Q = I_3 + \frac{1}{2}Y. = \frac{1}{\sqrt{2}}(I_3 + I_0)$ here.

At $q^2=0$ The Isovector part of the electro-magnetic current, & the $\Delta S=0$ weak curre belong to the same isotopic triplet of operators.

Let's suppose that in fact the $\Delta S=0$ weak vector current & the isotopic vector part of the electromagnetic current ~~are the same~~ belong to the same Isotopic triplet, that $J_\mu{}^{I=1}_{em}$ is just I_3 (or I_z) $J_\mu^{\text{vector } \Delta S=0}$ is $I^- = \frac{1}{\sqrt{2}}(I_x - iI_y)$. That is, suppose we really have only one vector current & depending on which direction we point it in isotopic spin space we get various combinations of $J_\mu{}^{I=1}_{em}$ & $J_\mu^{\text{vector } \Delta S=0}$.

The hint to try such a thing comes from the remarkable equality of the muon decay constant G & the neutron decay constant $G_C g_V(0)$, in spite of the fact that the latter contains the effects of strong interaction renormalization, while the former (obviously) doesn't. In electromagnetism the electron coupling constant e, & the $q^2=0$ proton coupling constant $eF(0)$ are also equal again in spite of the fact that $F(0)$ contains strong interaction renormalization effects. If we assumed that we're dealing with the <u>same</u> vector operator in the $\Delta S=0$

weak interactions (except for an isotopic spin rotation) & in the e.m. interactions ($I=1$ part) then ~~and~~ the $\Delta S=0$ weak vector current would be unrenormalized at $q^2=0$, just as the em current is unrenormalized at $q^2 =$
Why should we expect that the same vector current appears in both? Maybe the world is made of some type of ideal particles, & God can only form 1 vector current from the coordinates of these particles.
This is sometimes called the conserved vector current theory because $\partial_\mu J^\mu = 0$.

$$q \neq 0 \quad \langle P | J_\mu^{\,V\,\Delta S=0} | n \rangle = \Big(\bar{u}_p \big(\gamma_\mu (F_1^{\,P}(q^2) - F_1^{\,N}(q^2))$$

$$+ \tfrac{1}{2} (\not{q} \gamma_\mu - \gamma_\mu \not{q}) (F_2^{\,P}(q^2) - F_2^{\,n}(q^2)) \big| u_n \Big)$$

($q^2 \to 0$ in F_1, F_2)

$$= \Big(\bar{u}_p \big| \gamma_\mu + \tfrac{1}{2} (\not{q} \gamma_\mu - \gamma_\mu \not{q}) \Big(\frac{\mu_P}{2M} - \frac{\mu_N}{2M} \Big) \big| u_n \Big)$$

This can be tested by comparing the spectrum of e^+ in the decay of N^{12} ($T_3 = 1$) to C_{12} ($T=0$), to the spectrum of e^- in the decay of B^{12} ($T_3=-1$) to C_{12}. The third member of this isotopic triplet is C^{12*} ($T_3 = 0$) which decays by

γ emission to C^{12}, primarily by a magnetic dipole transition, & gives a value for the transition magnetic moment. The experimental results agree quite well with the CVC theory. (See Gas. pp 576 - 578, picture on 578.)

Generalize the idea of I vector current pointing in different directions in Isotopic spin space, to SU(3) space. Then fill in the blank:

$$\pi^- \to \pi^0 + e^- + \bar{\nu}$$

$$\langle \pi^0 | J_\mu^{V\,\Delta S=0} | \pi^- \rangle = \sqrt{2}\,(p_{1\mu} + p_{2\mu})$$

$$K^- \to \pi^0 + e^- + \bar{\nu}$$

$$\langle \pi^0 | J_\mu^{\Delta S = \Delta Q = 1} | K^- \rangle = \underline{\quad ? \quad}\,(p_{1\mu} + p_{2\mu})$$

(Calculate Rate & find coefficients.)

(Assuming SU(3) symmetry $(m_K^2 = m_\pi^2)$ & $q^2 = 0$)

[illegible]

Summary: Weak interaction is product of currents. Current has some pieces; $\bar{e}\nu$ & $\bar{\mu}\nu$ enter equally. (coeff = 1)

Vector hadronic $\Delta S = 0$ part is isotopic density. (coeff = C)

Lectures # 2 and 3. Week #4.

Correction: L#3. W#2. pg 1. ln 12.

We may choose our axis in the direction of $\vec{p}_\nu$ (summing over polarizations implies no preferred direction). Define ....

$$J_\mu = (\bar{\nu}_e e)_\mu + (\bar{\nu}_\mu \mu)_\mu + c\,(j^{\Delta S=0})_\mu + s\,(j^{\Delta S=\Delta Q=1})_\mu$$

(sometimes Prof. Feynman writes J_μ as the adjoint of the above — however, it makes no difference since the interaction is $\frac{G}{\sqrt{2}} J_\mu^\dagger J_\mu$)

We have seen that the vector part $j^{\Delta S=0}$ is just the I-spin lowering component of the isotopic spin current (which is a conserved current). This conserved vector current hypothesis (CVC) makes plausible the fact that the strong interactions do not appear to renormalize the non-strangeness changing vector coupling. The vector part of the strangeness changing current ($j_V^{\Delta S=1}$), which changes a K^- into a π^0 (see problem in last lecture), is not a conserved current since $m_K \neq m_\pi$ (see pg 583 in Gasiorowicz). However, in the limit of SU(3) symmetry, the vector part of $j^{\Delta S=1}$ could be a conserved current and perhaps could be identified with V^+ just as $j_V^{\Delta S=0}$

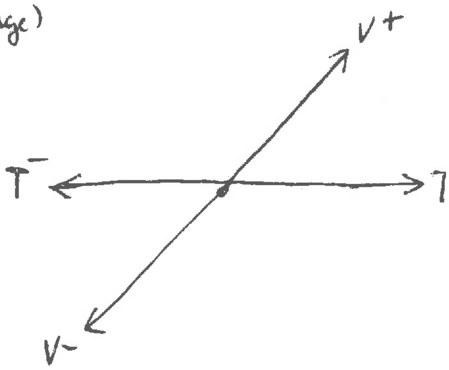
is identified with T^+.

n $\quad$ p $\qquad$ $+1$ $\quad Y$ (hypercharge)

Σ^- $\quad$ Σ^0 Λ $\quad$ Σ^+ $\qquad$ 0

Ξ^- $\quad$ Ξ^0 $\qquad$ -1

$\frac{1}{2}^+$ baryon octet



V^+ could change a Λ into a p, Ξ^0 into a Σ^+, etc.

However, from the problem in the last lecture ($K^- \to \pi^0 e\nu$ decay using $\langle \pi^0 | s\, j_V^{\Delta S=1} | K^- \rangle = \frac{s}{\sqrt{2}}(P_{1\mu} + P_{2\mu})$ in the limit $q^2 \to 0$), s is found to be much smaller than expected, i.e. the rates are off. Universality does not seem to work. However, Cabibbo was able to resurrect universality in a new form which could explain this discrepancy. Before explaining Cabibbo's idea, we shall review SU(3) in a way different from the usual.

In SU(2), if we understand spin ½, we can understand everything by building the other representations out of the ½. We can do the same in SU(3) using three quarks (forget the quark spin and just think of it as an object for exploiting the SU(3) symmetry).

B $\quad$ A $\qquad$ $\bar{C}$ $\qquad$ Y $+1$

C $\qquad$ $-\bar{A}$ $\quad$ $\bar{B}$ $\qquad$ 0 — remember the minus sign

quarks $\qquad$ antiquarks $\qquad$ -1

If we form $q\bar{q}$, we have nine states ($\underset{\sim}{3} \times \underset{\sim}{3} = \underset{\sim}{8} + \underset{\sim}{1}$)

	$\overset{n}{\bar{C}B}$		$\overset{p}{\bar{C}A}$	
$\overset{\sigma^-}{-\bar{A}B}$		$\overset{\sigma^0}{\frac{1}{\sqrt{2}}(\bar{B}B - \bar{A}A)}$ $\underset{\lambda}{\frac{1}{\sqrt{6}}(\bar{A}A + \bar{B}B - 2\bar{C}C)}$	$\overset{\sigma^+}{\bar{B}A}$	$\frac{1}{\sqrt{3}}(\bar{A}A + \bar{B}B + \bar{C}C)$
	$\underset{\xi^-}{-\bar{A}C}$		$\underset{\xi^0}{\bar{B}C}$	
		$\underset{\sim}{8}$		$\underset{\sim}{1}$

n, p, σ, ξ represent the members of a general octet (not just the $\frac{1}{2}^+$ baryon octet). Since we are only dealing with transformation properties, the ~~quarks and~~ antiquarks need not be the antiparticles of the quarks (so that the n need not be the antiparticle of the ξ^0).

	$\underbrace{n, p \quad \sigma \quad \lambda \quad \xi}$ (8)				1
	n, p	σ	λ	ξ	$\underset{\sim}{1}$
nucleon octet ($\frac{1}{2}$+)	N, P	Σ	Λ	Ξ	
pion octet (0^-)	K^0, K^+	π	η	$K^- \bar{K}^0$	η_0
vector meson octet (1^-)	K^{*0}, K^{*+}	ρ	ω φ	$K^{*-} \bar{K}^{*0}$	ω φ

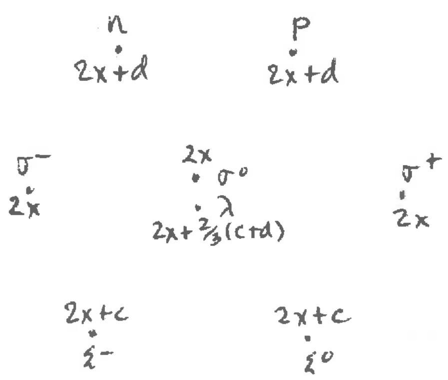
SU(3) is not an exact symmetry of the strong interactions — as is easily seen by the fact that the masses are not degenerate within an SU(3) multiplet. However, the masses are the same within each I-spin multiplet (since isospin is an exact symmetry). By assuming something about the symmetry breaking, we can perhaps relate the broken masses within a particular representation. For example, let us assume that the quark masses are split as follows:

$$m_A = m_B = m_{\bar{A}} = m_{\bar{B}} = x$$

$$m_C = x + c \quad , \quad m_{\bar{C}} = x + d$$

i.e. the mass of the strange quark is shift
(d=c if $\bar{C}$ is the antiparticle of c)

Then the masses in the octet are:



note: λ is no longer an eigenstate of the mass operator

$$\therefore \quad \frac{m_n + m_\xi}{2} = 2x + \frac{1}{2}(d+c) = \frac{3m_\lambda + m_\sigma}{4}$$

or in the $\frac{1}{2}^+$ octet, $\quad \dfrac{m_N + m_\Xi}{2} = \dfrac{3m_\Lambda + m_\Sigma}{4}$ the famous mass relation! (which agrees very well: 1129 mev ≅ 1135 m

The equal spacing formula for the baryon decuplet is also satisfied. For the pseudoscalar meson octet, the mass formula is: $m_K = \dfrac{3m_\eta + m_\pi}{4}$ (496 mev $\mp$ 446 mev)

Here the formula does not work as well, but if it is written in terms of the squares of the masses:

$$m_K^2 = \frac{3m_\eta^2 + m_\pi^2}{4} \quad (.246\ GEV^2 \simeq .231\ GEV^2)$$

it also agrees with experiment.

Why should we use m^2 for the mesons and m for the baryons? No one really knows, but it is often said the reason is that the meson propagator is in terms of m^2, i.e. $\frac{1}{q^2 - m^2}$, while the baryon propagator is in terms of m, i.e. $\frac{1}{\not{p} - m}$. However, this is a fuzzy argument which is not very good if examined closely. We have also forgotten the second order corrections. Why should they be small? Perhaps the second order corrections for the mesons are large but partially offset by using m^2 rather than m in the formula. It becomes apparent that the mass formulas are not strong evidence for belief in SU(3). The most striking predictions from SU(3) (and why it is generally believed) are the patterns i.e. the multiplets with the same spin and parity. The existence and properties of Ω^- and Ξ^* were predicted from holes in the patterns. SU(3) will certainly have to be reckoned with in an understanding of high energy physics.

* Problems:

(1) Suppose the mass of the octet is μ_8 and the singlet μ_1 ($\mu_1 \neq \mu_8$) before mass splitting. Find the mass laws and the mixing between the singlet and octet if the mass operator is proportional to (mass c + mass $\bar{c}$)? Use the 1^- octet and singlet ($K^*, \rho, \omega, \varphi$) (or the 0^- octet and singlet) as an example.

(2) Invent a different system and study the multiplets (such as $\underset{\sim}{q}\underset{\sim}{q}\underset{\sim}{q}$ i.e. $\underset{\sim}{3} \times \underset{\sim}{3} \times \underset{\sim}{3} = \overset{\text{Sym}}{\underset{\sim}{10}} + \underset{\sim}{8} + \underset{\sim}{8} + \overset{\text{anti-Sym}}{\underset{\sim}{1}}$ note: ABA not same as BAA)

(3) Consider quarks as objects of spin A↑ B↑ C↑ A↓ B↓ C↓ and examine this enlarged group (SU(6)) even though it seems strange that spin should be mixed in with quality.

(4) Predict relations among the magnetic moments of the hyperons supposing SU(3) exact. e.g. $\mu_n = \frac{1}{2}\mu_\Lambda$

(5) Prove the following relation concerning electromagnetic mass differences of the baryon octet

$$(m_p - m_n) + (m_{\Sigma^-} - m_{\Sigma^+}) = m_{\Xi^-} - m_{\Xi^0} \quad (-1.3 + 7.7 \simeq 6.5)$$

We can consider the octet diagram of states as an octet diagram of operators (i.e. $\bar{C}A$ means annihilate A and create C)

for instance, $\lambda_p \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} C \\ 0 \\ 0 \end{pmatrix} \Rightarrow \lambda_p = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\lambda_\lambda = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$, etc. - - - $\quad \lambda_q = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

The eight operators of the $\underset{\sim}{8}$ (forgetting the identity from the $\underset{\sim}{1}$) are generators of the group which transform ABC. If the basis A, B, C is transformed, then the generators move around (transform) like an octet (just as τ_+, τ_-, τ_3; or τ_1, τ_2, τ_3; transform like a vector). Thus, the generators form an octet type irreducible tensor. The real generators may be abstracted from this 3x3 representation by the commutation relations.

$$[\tfrac{1}{2}\lambda_i, \tfrac{1}{2}\lambda_j] = i \sum_k f_{ijk} (\tfrac{1}{2}\lambda_k)$$ ← traceless so unit matrix excluded (λ_1 to λ_8 are just combinations of λ_p;

see Gasiorowicz pg 261

$$\therefore [F_i, F_j] = i \sum_k f_{ijk} F_k$$ where the F's are the abstract generators

$$\{\lambda_i, \lambda_j\} = \lambda_i \lambda_j + \lambda_j \lambda_i = \tfrac{2}{3}\delta_{ij} 1 + 2 d_{ijk} \lambda_k$$ (only true for 3x3 rep)

Two octets can be combined <u>two</u> ways to get another octet.

$$\underset{\sim}{8} \times \underset{\sim}{8} = \underbrace{\underset{\sim}{1} + \underset{\sim}{8} + \underset{\sim}{27}}_{\text{sym}} + \underbrace{\underset{\sim}{10} + \underset{\sim}{\overline{10}} + \underset{\sim}{8}}_{\text{antisym}}$$

(sym means sym under interchange of the two octets as in SU(2), $\tfrac{1}{2} \times \tfrac{1}{2} = \underset{\text{anti-sym}}{0} + \underset{\text{sym}}{1}$

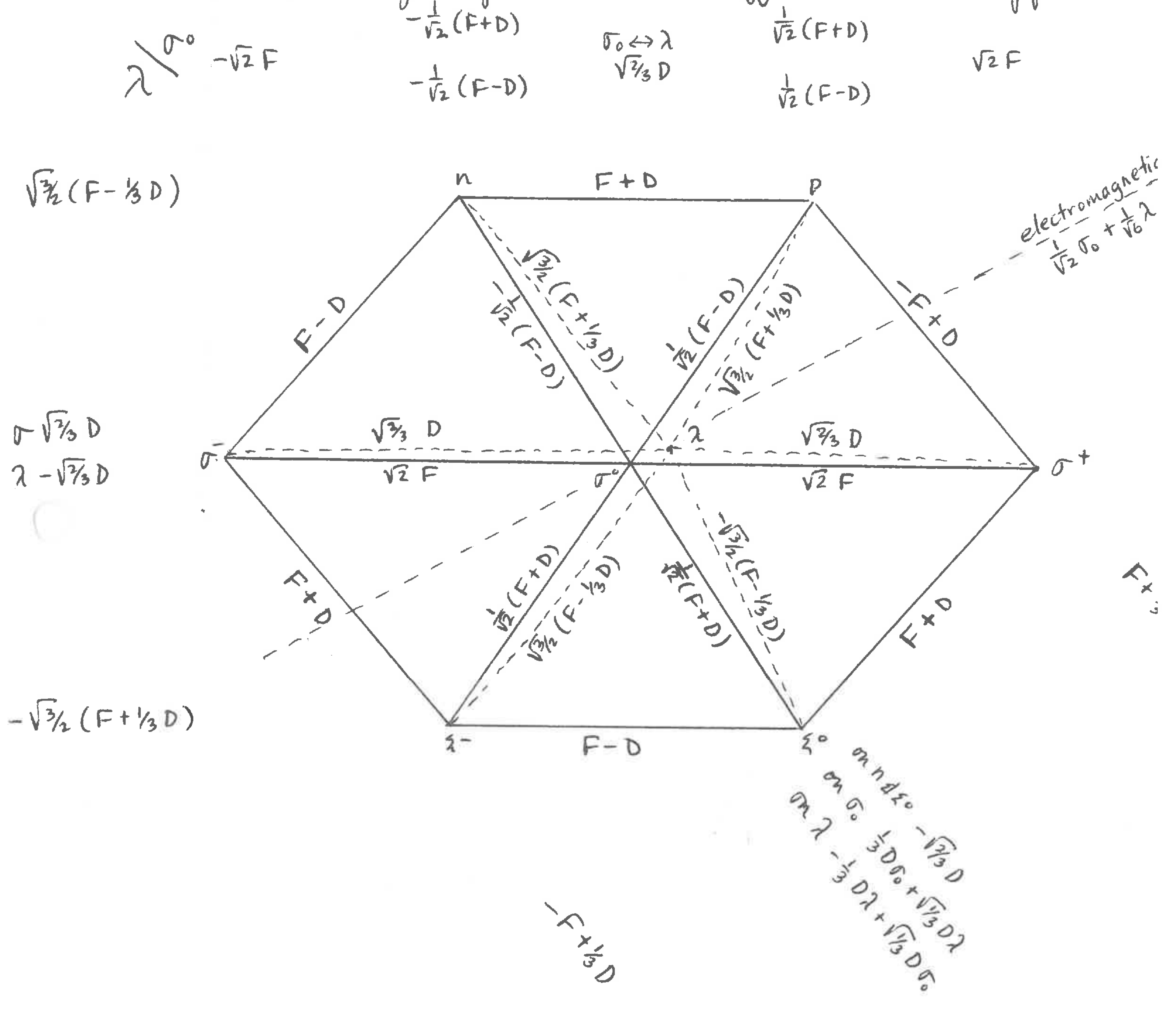
This is a situation not encountered in SU(2) ($1 \times 1 = 0 + 1 + 2$ i.e. the 1 occurs only once in the product). Consequently, the generalization of the Wigner-Eckart theorem for an octet operator between two states of an octet (which is the most important practical case) reads:

↙ refers to particular state in multiplet

$$\langle \underset{\sim}{8}, k \,|\, \text{Octet Operator}, j \,|\, \underset{\sim}{8}, j\rangle = i f_{ijk} F + i d_{ijk} D$$

need two arbitrary constants

the following diagram lists the coefficients in a handy form.



If the electromagnetic current is assumed to be a member of an octet, then it transforms as $\frac{1}{\sqrt{2}}\sigma_0 + \frac{1}{\sqrt{6}}\lambda$ (why?). Therefore, the slanted line gives the coefficients for $\langle 8_i | j_{em} | 8_j \rangle$. the charge (form factors evaluated at $q^2=0$) corresponds to $F=1$, $D=0$. See if you can derive the relations for the magnetic moments. ($\mu_p = \mu_{\sigma^+} = F+\frac{1}{3}D$, etc.)

Ph230a

Lecture # 2 Week # 5

If you assume quarks have spin ½ & make totally symmetric 3q wave functions you'll find you can make 8 (spin ½) & 10 (3/2) which are observed baryon multiplets.

Note: When you paste 3 quarks together they form a third rank tensor under SU(3) rotations. Just like putting 3 spin ½ particles together. You can take combinations which have definite symmetry properties with respect to interchange of the indices & form irreducible combinations. Symmetry properties of tensor indices are preserved under transformations. Suppose $T_{\alpha\beta\gamma\ldots}$ is symmetric with respect to α & β. Transform: $D_{\alpha'}{}^{\alpha} D_{\beta'}{}^{\beta} \cdots T_{\alpha\beta}$

$$= D_{\beta'}{}^{\beta} D_{\alpha'}{}^{\alpha} \cdots T_{\beta\alpha}$$

still symmetric in α', & β'.

In forming irreducible 3q multiplets you just ~~put~~ ~~take~~ combinations with as much symmetry or ~~anti~~ antisymmetry among the indices as you can squeeze in, then separate them accordingly. ~~...~~ To show you don't have to be a group ~~theory~~ whiz I'll ~~...~~ do 8 (½).

First take the spin wave fcn $(\sigma_A{}^i \sigma_B{}^j \sigma_C{}^k - \sigma_A{}^j \sigma_B{}^i \sigma_C{}^k)$ its antisymmetric in A & B & behaves just like σ^k. (σ_k stands for a number, not a σ matrix.)

For the quark wave function, you can take an antisymmetric combination of A & B:

$(q_A^\alpha q_B^\beta - q_A^\beta q_B^\alpha)\, q_C^\gamma$ there's more we can do, namely symmetrize in α, γ by adding

$(q_A^\gamma q_B^\beta - q_A^\beta q_B^\gamma)\, q_C^\alpha$, but that's all.

~~That's it for the choice, although~~

$$[q_A^\alpha q_B^\beta q_C^\gamma - q_A^\beta q_B^\alpha q_C^\gamma + q_A^\gamma q_B^\beta q_C^\alpha - q_A^\beta q_B^\gamma q_C^\alpha](\sigma_A^i \sigma_B^j \sigma_C^k - \sigma_A^j \sigma_B^i \sigma_C$$

Now we just add to this the five other ~~[illegible]~~ orderings of A, B, C in order to make a totally symmetric state:

$$[q_A^\beta q_B^\alpha q_C^\gamma - q_A^\alpha q_B^\beta q_C^\gamma + q_A^\gamma q_B^\alpha q_C^\beta - q_A^\alpha q_B^\gamma q_C^\beta](\sigma_A^j \sigma_B^i \sigma_C^k - \sigma_A^i \sigma_B^j \sigma_C^k)$$

$$[q_A^\alpha q_B^\gamma q_C^\beta - q_A^\gamma q_B^\alpha q_C^\beta + q_A^\beta q_B^\gamma q_C^\alpha - q_A^\gamma q_B^\beta q_C^\alpha](\sigma_A^i \sigma_B^k \sigma_C^j - \sigma_A^k \sigma_B^i \sigma_C^j)$$

$$[q_A^\gamma q_B^\alpha q_C^\beta - q_A^\alpha q_B^\gamma q_C^\beta + q_A^\beta q_B^\alpha q_C^\gamma - q_A^\alpha q_B^\beta q_C^\gamma](\sigma_A^k \sigma_B^i \sigma_C^j - \sigma_A^i \sigma_B^k \sigma_C^j)$$

$$[q_A^\gamma q_B^\beta q_C^\alpha - q_A^\beta q_B^\gamma q_C^\alpha + q_A^\alpha q_B^\beta q_C^\gamma - q_A^\beta q_B^\alpha q_C^\gamma](\sigma_A^k \sigma_B^j \sigma_C^i - \sigma_A^j \sigma_B^k \sigma_C^i)$$

$$[q_A^\beta q_B^\gamma q_C^\alpha - q_A^\gamma q_B^\beta q_C^\alpha + q_A^\alpha q_B^\gamma q_C^\beta - q_A^\gamma q_B^\alpha q_C^\beta](\sigma_A^j \sigma_B^k \sigma_C^i - \sigma_A^k \sigma_B^j \sigma_C^i)$$

Anyway, you don't have to be a group theory whiz to get the results.

Anything you do with SU(3) symmetry by using quarks, can later be expressed in terms of the SU alone.

Recall that our old mass law was that the mass (m^2 for mesons) is M_0 plus a constant times the number of strange quarks & antiquarks.

Redefine the quark masses:

$$\begin{array}{ccc} A & B & -1a \\ & & \\ & C & 2a \end{array}$$

Then $\lambda_8 = \frac{1}{\sqrt{6}} (\bar{A}A + \bar{B}B - 2\bar{c}c)$ measures the mass splitting.

Letting $m = m_0 + a\lambda_8$ & using Colglazier's table from lecture #3, week #4, we get for the nucleon octet,

$$\left.\begin{aligned} m_p &= m_0 + \sqrt{\tfrac{3}{2}}\,(F - \tfrac{1}{3}D)a \\ m_\Sigma &= m_0 + \sqrt{\tfrac{2}{3}}\;Da \\ m_\Lambda &= m_0 - \sqrt{\tfrac{2}{3}}\;Da \\ m_\Xi &= m_0 - \sqrt{\tfrac{3}{2}}\,(F + \tfrac{1}{3}D)a \end{aligned}\right\} \Rightarrow \frac{3m_\Lambda + m_\Sigma}{4} = \frac{m_N + m_\Xi}{2}$$

$D/F = -.31$ for this case.

Giving the mass operator a term (which acts) like the

SU(3) generator λ gives the same mass splitting within any SU(3) multiplet that we'd get by counting the number of strange & nonstrange quarks.

The electromagnetic & $\Delta S=0$ weak charges involve I_1, I_2, I_3 & Y, which commute with λ_8 are still conserved when we include a λ_8 type term in the strong interaction Hamiltonian. In the limit of SU(3) symmetry it was said that the $\Delta S=\Delta Q=1$ charge was V^+ member of the octet. $[V^+, T^+]=0$ also $[\lambda_8, T^+]=$ Therefore we don't expect the effect of of λ_8 or V^+ to change $[V^+, T^+]=0$ This can be tested by comparing $K^+ \to \pi^0 e\nu$ to $K^0 \to \pi^- e^+ + \nu$.

The $\Delta I = \frac{1}{2}$ nature of the strangeness changing current can be tested in

$$K^+ \to \pi^0 + e^+ + \nu$$
$$K^0 \to \pi^- + e^+ + \nu$$

We want $\langle \pi | \Delta_{I=\frac{1}{2}} | K \rangle$ since we're just combining 3 I spins to make a scalar, we can put the π on the other side & make the $I=\frac{1}{2}$ $K\pi$ state $= \sqrt{2}\,(K^+\pi^0 - K^0\pi^+)$.

$\therefore$ Rates $\dfrac{K^+ \to \pi^0 e\nu}{K^0 \to \pi^- e\nu} = \frac{1}{2}$

Cabibbo's form of universality says the hadronic weak current is a superposition of 2 vectors pointing in different directions in SU(3) space. It's the sum of a current of length $\cos\theta$ in the $\Delta S=0$ direction & a current of length $\sin\theta$ in the $\Delta S=1$ direction.

Suppose that the weak interaction for the triplet

$B \quad A^+$

C

is

$$\underbrace{c\bar{A}^+ \gamma_\mu (1+\gamma_5) B}_{\Delta S=0} + \underbrace{s\bar{A}^+ \gamma_\mu (1+\gamma_5) C}_{\Delta S=1}$$

Think of $B' \equiv B\cos\theta + C\sin\theta$ as the state $\bar{A}^+$ couples to in the weak interaction. Before SU(3) symmetry breaking God chose B' as the particle to couple to A^+.

This type of thinking forms the unpublishable underpinnings of the Cabibbo theory.

The undepinnings may not be right, but the results are right.

Where do we go from here?

Nobody has the slightest idea about e, μ. That is nobody can predict the e, μ mass ratio in a theory that predicts other numbers also.

It's curious that only one extra term: $(A^+|\gamma_\mu(1+i\gamma_5)|B')$ must be added to the electron & muon currents in the weak interactions. Is there some relationship between e^- & A^+, & an analogous relation between ν & B'? Is there some relationship between the strangeness & the muon? There are lots of things to play with.

We've classified fermions as particles or antiparticles depending on whether they ~~have~~ $\underbrace{}_{\text{enter the W.I's with}}$ a $1+i\gamma_5$ or a $1-i\gamma_5$.

The e^- is a particle.

$G_A = 1.23$ We say it was 1 deep in the guts somewhere, but the strong interactions renormalize it. So baryons are particles in the sense of e^-. Maybe originally it was $1-i\gamma_5$ for baryons, but renormalization changed it. Adler-Weisberger's calculation doesn't give the sign. It's still possible to have a theory where nucleons have $1-i\gamma_5$.

In the nonleptic decays, $\Lambda \to N+\pi$, it's hard to test whether you have some $\Delta I = 3/2$, along with $\Delta I = \frac{1}{2}$. You can't ~~[illegible]~~ check it too accurately because the SU(3) symmetry isn't exact.

Physics 230a

Lecture #1 Week # 6

Correction: In lecture #2, Week # 5, I put into the notes that the mass-splitting operator gave the same result you'd get by counting quarks. However, for the baryons $D \neq 1$, & therefore the mass splitting term, which transforms like a λ_8 type member of an octet, is not exactly equal to the SU(3) generator λ_8: i.e. $\frac{1}{\sqrt{6}}(\bar{A}A + \bar{B}B - 2\bar{C}C)$, which does count quarks, (mass differences) & is pure F.

Problem #1 was to try out the mass law on the pseudoscalar & pseudovector nonets. Think of the nonets as $q\bar{q}$ in $l=0$, $S=0$ (p.s.) or $S=1$ (p.v.) states.

Assume that the p.v. singlet & octet were originally degenerate. Note SU(3) symmetry does not require that this be so since the singlet & octet aren't connected by an SU(3) rotation.

We're assuming then that all the $q\bar{q}$ $l=0$ $S=1$ states have the same spatial wave function.

The singlet is $\lambda_0 = \frac{1}{\sqrt{3}}(\bar{A}A + \bar{B}B + \bar{C}C)$

The $I=0$ member of the octet is $\lambda_8 = \frac{1}{\sqrt{6}}(\bar{A}A + \bar{B}B - 2\bar{C}C)$

Now assume that the mass splitting is ~~the~~ operator λ_8 (quark counter). Then the mass matrix (actually m^2 for mesons) is

$$\begin{pmatrix} \langle \lambda_8 | \lambda_8' | \lambda_8 \rangle & \langle \lambda_8 | \lambda_8' | \lambda_0 \rangle \\ \langle \lambda_0 | \lambda_8' | \lambda_0 \rangle & \langle \lambda_0 | \lambda_8' | \lambda_0 \rangle \end{pmatrix}$$

$$\langle \lambda_8 | \lambda_8' | \lambda_8 \rangle = \langle \tfrac{1}{\sqrt{6}} (\bar{A}A + \bar{B}B - 2\bar{C}C) | \lambda_8 | \tfrac{1}{\sqrt{6}} (\bar{A}A + \bar{B}B - 2\bar{C}C) \rangle$$

$$= \frac{1}{6\sqrt{6}} \langle \bar{A}A + \bar{B}B - 2\bar{C}C | (2\bar{A}A + 2\bar{B}B + 8\bar{C}C) \rangle$$

$$= -2/\sqrt{6}$$

similarly $\langle \lambda_8 | \lambda_8' | \lambda_8 \rangle = 2/\sqrt{3}$

$$\langle \lambda_0 | \lambda_8' | \lambda_0 \rangle = 0$$

The eigenvectors of $\begin{array}{c|cc} & \lambda_8 & \lambda_0 \\ \lambda_8 & -2/\sqrt{6} & 2/\sqrt{3} \\ \lambda_0 & 2/\sqrt{3} & 0 \end{array}$ are $\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix}$ & $\sqrt{\tfrac{2}{3}}\begin{pmatrix} 1 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

Therefore $\frac{1}{\sqrt{2}} (\bar{A}A + \bar{B}B)$ & $\bar{C}C$ are the eigenstates of the mass operator. These are the physical states ω_9 & φ.

When we try this with the pseudoscalar nonet, it doesn't work. The assumption of degeneracy doesn't work for the ps nonet. If the singlet and octet spatial wave functions are not the same to begin with, (they're not for ps) then there is no way of relating $\langle \lambda_8 | \lambda_8' | \lambda_8 \rangle$ to $\langle \lambda_8 | \lambda_8' | \lambda_0 \rangle$. Within multiplet i.e. within the p.s. octet, the mass splitting formula still works. It seems to always work within SU(3) multiplets. (Note here we let $D=0$, while we needed some D for the baryons.)

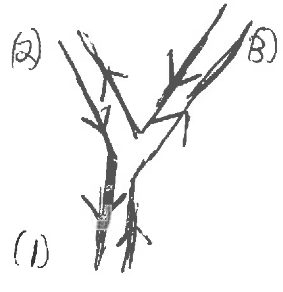
For the p.v.'s you get $m^2_\omega = m^2_\rho$

$$m^2_K - m^2_\rho = m^2_\phi - m^2_{K^*}$$

which checks pretty well.

φ & ω have a suprising difference, which can be understood in terms of the above. φ decays mostly into $2K$'s, while ω never does. ω decays to $\rho\pi$, while φ hardly ever does.

There's a famous rule of George Zweig's for decay of 1 meson into 2 others: you draw the following type of pictures:



The roads are just the mesons. Drawing all possible pictures of this type is equivalent to calculating the C.G. coefficients for the decay. Say (1) is a φ, then it contains a c & a $\bar{c}$, therefore (2) & (3) must each contain either c or $\bar{c}$, for the decay to occur.

Using this rule, plus the structure of φ & ω gotten from symmetry breaking, you can account for differences between φ & ω in decay & production processes.

Note: degeneracy of singlet & octet is more than SU(3). You have to include spin of quarks so each quark has 6 components (3 for quark spin, times two for spin ½). Then you make $q\bar{q}$ multiplets of SU(6), corresponding to rotations of the 6 components of the quark. If you want to ~~try~~ try this you might think of the reason why the ps nonet isn't degenerate but the pv is. (SU(6) symmetry)

Recall that the test of the $\Delta I = \frac{1}{2}$ nature of $J_u^{\Delta S=\Delta Q=1}$ was that in

$$K^+ \to \pi^0 + e^+ + \nu$$

$$K^0 \to \pi^- + e^- + \nu$$

The ratio of rates is $2/1$.

Colglazier looked at the new data in the Rosenfeld tables & discovered the numbers are

1.66 ± .08 for electrons

1.85 ± .15 " muons.

If this is right, things are really confused.

The best example of the dominance of $\Delta I = \frac{1}{2}$ in strange decays, are the $K \to \pi\pi$ type decay. The current-current form of the interaction, here: $J_\mu^{\Delta I = 1} J^{\mu\, \Delta I = \frac{1}{2}}$ does not require ~~that~~ this be so (unlike the previous case.)

The 2π's are in an $L=0$ state. Bose statistics implies $I = 0$ or 2.

	$I_z = 2$	$I_z = 1$	$I_z = 0$
$I=2$	$\pi^+\pi^+$	$\frac{1}{\sqrt{2}}(\pi^+\pi^0 + \pi^0\pi^+)$	$\frac{1}{\sqrt{6}}\left[2(\pi^0\pi^0) + \pi^+\pi^- + \pi^-\pi^+\right]$
$I=1$		$\frac{1}{\sqrt{2}}(\pi^+\pi^0 - \pi^0\pi^+)$	$\frac{1}{\sqrt{2}}(\pi^+\pi^- - \pi^-\pi^+)$
$I=0$			$\frac{1}{\sqrt{3}}(\pi^+\pi^- + \pi^-\pi^+ - \pi^0\pi^0)$

If the decays were pure $\Delta I = \frac{1}{2}$:

factor from C.G. coefficient in rate

$$K^0 \to \pi^+ + \pi^- \qquad 2$$
$$\to \pi^0 + \pi^0 \qquad 1$$
$$K^+ \to \pi^+ + \pi^0 \qquad 0$$

If there's an amount a of $\Delta I = \frac{3}{2}$:

$$K^0 \to \pi^+ + \pi^- \qquad \tfrac{2}{3}\left(1 + \tfrac{a}{\sqrt{2}}\right)^2$$
$$\pi^0 + \pi^0 \qquad \tfrac{1}{3}\left(1 - \sqrt{2}\,a\right)^2$$
$$K^+ \to \pi^+ + \pi^0 \qquad a^2$$

Experimentally, Rate $\frac{K_0 \to \pi^+ \pi^0}{K^+ \to \pi^+ \pi^0} \approx 500$

$$\Rightarrow \quad 500 = \frac{2/3\left(1 + \sqrt{2}\, a + \frac{a^2}{2}\right)}{a^2}$$

$$|a| \approx .04$$

$$\text{Rate} \quad \frac{K_0 \to \pi^0 + \pi^0}{K_0 \to \text{all } \pi\text{'s}} = \frac{1}{3}\left(\frac{(1-\sqrt{2}\,a)^2}{1+a^2}\right)$$

we don't know the phase of a, so this could be between .29 & .38; for $a = 0$ its .333.

Experimentally the number is .316, so there is some $\Delta I = 3/2$.

$\Delta I = 3/2$ is small for non-leptic decays. People have proposed that there are products of neutral hadron currents to be added to the current current interaction, & arranged so that $\Delta I = 1/2$. (there are no neutral leptic currents observed.) So far this has been an empty hypothesis since it doesn't tell us anything more than what it was cooked up to do. Also, you have to account for the small amount of $K^+ \to \pi^+ + \pi^0$ ($\Delta I = 3/2$).

Ph 230 a

Lecture #2. Week #6.

Correction: L#3 W#5 pg 1

$j^{\Delta S=1}$ transforms under isospin like $I = \frac{1}{2}$; $j^{\Delta S=0}$ transforms like $I = 1$.

There are two neutral K-mesons, denoted by K^0 and $\bar{K}^0$, which are eigenstates of the strong interaction. The K^0 has strangeness $+1$ and belongs to an isodoublet with the K^+; the $\bar{K}^0$ has strangeness -1 and belongs to an isodoublet with the K^-. The K^0 is seen in associated production with other strange particles e.g. $p + n \to p + \overset{S=-1}{\Lambda} + \overset{S=+1}{K^0}$; while the $\bar{K}^0$ can be seen in associated production with just the K^0 e.g. $p + n \to p + n + K^0 + \bar{K}^0$. Gell-Mann and Pais pointed out that the K^0 and $\bar{K}^0$ ~~$\bar{K}^0$ are degenerate, and that they~~ are distinct particles in the strong interactions where strangeness is a conserved quantum number. However, strangess is violated in the weak interactions. Therefore, the K^0 and $\bar{K}^0$ can convert into one another by a second order weak process like $K^0 \to \pi^+\pi^- \to \bar{K}^0$ (Hence, K^0 & $\bar{K}^0$ are no longer eigenstates of the total Hamiltonian)

Quantum Mechanics then says that there is a linear combination of K° and $\bar{K}^\circ$ which decays into $\pi^+\pi^-$ while the orthogonal combination doesn't. The two linear combinations are eigenstates of the mass operator; so that each is characterized by its own decay rate (there are no transitions between the two). Let us analyze the neutral kaons supposing C is conserved.

	Amplitude
$K^\circ \rightarrow \pi^+\pi^-$	a
$\bar{K}^\circ \rightarrow \pi^+\pi^-$	a

can choose phases so that this is true

$\therefore$ if $|K_1^\circ\rangle = \frac{1}{\sqrt{2}}(|K^\circ\rangle + |\bar{K}^\circ\rangle)$, then $\text{amp}(K_1^\circ \rightarrow \pi^+\pi^-) = \sqrt{2}\,a$

$|K_2^\circ\rangle = \frac{1}{\sqrt{2}}(|K^\circ\rangle - |\bar{K}^\circ\rangle)$, then $\text{amp}(K_2^\circ \rightarrow \pi^+\pi^-) = 0$

(note: here $C|K^\circ\rangle = |\bar{K}^\circ\rangle$ so that $C|K_1^\circ\rangle = +|K_1^\circ\rangle$, $C|K_2^\circ\rangle = -|K_2^\circ\rangle$, $C|\pi^+\pi^-{}^{\ell=0}\rangle = +|\pi^+\pi^-{}^{\ell=0}\rangle$)

This particle is very peculiar — although its funny properties are just a consequence of Quantum Mechanics (superposition). For strong interaction production, the convenient base states are the strangeness eigenstates K° & $\bar{K}^\circ$. For the weak interactions, the convenient base states are the mass operator eigenstates K_1° & K_2°. The existence of two neutral K-mesons states with definite mass and definite lifetime does not depend on charge conjugation invariance — which, of course, is violated in the weak interactions. We could repeat the analysis ~~assuming~~ assuming CP conservation (the current current theory $\frac{G}{\sqrt{2}} j^{+} j$ is CP conserving); so that, with redefining phases $\ni$ $CP|K^0\rangle = |\bar{K}^0\rangle$, we have.

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad |K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

$$\therefore K_1^0 \to \pi^+\pi^- \text{ while } K_2^0 \not\to \pi^+\pi^- \qquad \left(\begin{array}{l} CP|K_1^0\rangle = +|K_1^0\rangle \\ CP|K_2^0\rangle = -|K_2^0\rangle \\ CP|\pi^+\pi^-_{\ell=0}\rangle = +|\pi^+\pi^-_{\ell=0}\rangle \end{array}\right)$$

Since the K_1^0 and K_2^0 have definite masses and lifetimes, we can write:

$$\frac{d\psi_1(\tau)}{d\tau} = -\left(\frac{\gamma_1}{2} + i m_1\right)\psi_1(\tau) \qquad \frac{d\psi_2(\tau)}{d\tau} = -\left(\frac{\gamma_2}{2} + i m_2\right)\psi_2(\tau)$$

$$\therefore \psi_1(\tau) = e^{-\gamma_1\tau/2} e^{-i m_1 \tau}\psi_1(0) \quad \& \quad \psi_2(\tau) = e^{-\gamma_2\tau/2} e^{-i m_2 \tau}\psi_2(0)$$

Suppose that we produce a K^0 and ask for the probability of a $\bar{K}^0$ at time τ:

$$\text{initial state} \quad |K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1^0\rangle + |K_2^0\rangle\right) \Rightarrow \psi_1(0) = \frac{1}{\sqrt{2}}, \ \psi_2(0) = \frac{1}{\sqrt{2}}$$

$$\text{amp of } \bar{K}^0 \text{ at time } \tau = \frac{1}{\sqrt{2}}\left(\psi_1(\tau) - \psi_2(\tau)\right) = \frac{1}{2}\left(e^{-\gamma_1\tau/2} e^{-i m_1\tau} - e^{-\gamma_2\tau/2} e^{-i m_2\tau}\right)$$

$\therefore$ probability of a $\bar{K}^\circ = \frac{1}{4}[e^{-\gamma_1\tau} + e^{-\gamma_2\tau} - 2e^{-(\gamma_1+\gamma_2)\tau/2}\cos(m_1-m_2)\tau]$

thus, the beam acquires a $\bar{K}^\circ$ ~~amplitude~~ component which oscillates in strength with distance from the source. (see graph on pg 552 of Gasiorowicz). the $\bar{K}^\circ$ can be observed by looking for:

$$\bar{K}^\circ + p \to \pi + \Lambda \quad \text{, etc.}$$

which cannot result from a K° beam ($K^\circ + p \to K^\circ + p$). the interference (which implies $m_1 \neq m_2$) has been observed with the result:

$$m_1 - m_2 = -.47 \pm .02 \times \frac{\hbar}{\tau_1} \quad (\text{very small i.e.} \sim 3.6 \cdot 10^{-12}\,\text{mev})$$

the interference would have been difficult to measure if Δm were not on the order of $1/\tau_1$. But this size is reasonable since the amplitude for $K^\circ \to \pi^+\pi^- \to \bar{K}^\circ$ should be of order G^2, i.e. Δm should be of order G^2, and the K_1° lifetime (amplitude squared) also of order G^2. the mass difference is a second order effect of the weak interaction. Because of the existence of degenerate neutral kaons, we are able to measure a mass difference of 10^{-12} mev!

But we have only a partial understanding — for we have no theory to predict this mass difference. In fact, all second order calculations from the current current theory diverge.

Could the mass difference arise from a $\Delta S = -\Delta Q$ current i.e. $j^{\Delta S=-\Delta Q}\, j^{\Delta S=\Delta Q}$ so that $\Delta S = 2$ is possible and $K^0 \to \bar{K}^0$ with strength G. $\Delta m \sim \frac{1}{\tau}$, on the order of G^2 seems to eliminate this possibility (unless $j^{\Delta S=-\Delta Q} \sim \frac{1}{500} j^{\Delta S=\Delta Q}$ but we are just sticking in the result).

The size of the mass difference seems to eliminate the possibility that matter and anti-matter repel each other (so that the K^0 & $\bar{K}^0$ would have a different gravitational potential). Any calculation of this effect yields a mass difference orders of magnitude larger than that observed. (Carry out the analysis seeing if the fact that $\Delta m = m_{K_1^0} - m_{K_2^0}$ while K^0 & $\bar{K}^0$ have the different grav. potentials causes any problems.)

We have predicted that the K_2^0 cannot decay into two pions. Experimentally, pion pairs (a very small fraction) have been seen far down the beam where only K_2^0 remains.

$$\frac{\Gamma(K_2^0 \to \pi^+\pi^-)}{\Gamma} = .0015 \pm .0001$$

This is a rather devastating blow — for it demands that CP be violated (remember that our theory of the weak interaction is CP conserving). The violation of CP alters very slightly our discussion of the neutral kaons. (the mass operator eigenstates — call them K^0_S and K^0_L — are no longer exactly K^0_1 and K^0_2.) The experimentors were done very carefully to minimize the possibility of K^0_1 being regenerated and decaying into two pions. For example, a K^0_2 passing through matter regenerates a coherent K^0_1 amplitude.

$$K^0_2 \longrightarrow \left| \;\text{MATTER}\; \right| \begin{array}{l} \rightarrow K^0(1+f) \\ \rightarrow \bar{K}^0(1+\bar{f}) \end{array} \longrightarrow \begin{array}{l} \text{mostly } K^0_2 \text{ but} \\ \text{some } K^0_1 \end{array}$$

the K^0 and $\bar{K}^0$ interact differently passing through matter (strong interactions)

So if there were any gas in the experimenter's vacuum, some K^0_1's would be regenerated which would subsequently decay into $\pi^+\pi^-$. However, the experimenters were careful to minimize this effect.

Lecture #2. Week #6.

Correction: L#3. W#5. pg 1

$j^{\Delta S=1}$ transforms under isospin like $I=\frac{1}{2}$; $j^{\Delta S=0}$ transforms like $I=1$.

There are two neutral K-mesons, denoted by K° and $\bar{K}^\circ$, which are eigenstates of the strong interaction. The K° has strangeness $+1$ and belongs to an isodoublet with the K^+; the $\bar{K}^\circ$ has strangeness -1 and belongs to an isodoublet with the K^-. The K° is seen in associated production with other strange particles e.g. $p+n \rightarrow p + \overset{S=-1}{\Lambda} + \overset{S=+1}{K^\circ}$; while the $\bar{K}^\circ$ can be seen in associated production with just the K° e.g. $p+n \rightarrow p+n+K^\circ+\bar{K}^\circ$. Gell-Mann and Pais pointed out that the K° and $\bar{K}^\circ$ ~~$\bar{K}^\circ$ are degenerate, and that they~~ are distinct particles in the strong interactions where strangeness is a conserved quantum number. However, strangess is violated in the weak interactions. therefore, the K° and $\bar{K}^\circ$ can convert into one another by a second order weak process like $K^\circ \rightarrow \pi^+\pi^- \rightarrow \bar{K}^\circ$

(Hence, K° & $\bar{K}^\circ$ are no longer eigenstates of the total Hamiltonian)

Quantum Mechanics then says that there is a linear combination of K^0 and $\bar{K}^0$ which decays into $\pi^+\pi^-$ while the orthogonal combination doesn't. The two linear combinations are eigenstates of the mass operator; so that each is characterized by its own decay rate (there are no transitions between the two). Let us analyze the neutral kaons supposing C is conserved.

	Amplitude	
$K^0 \rightarrow \pi^+\pi^-$	a	can choose phases so that this is true
$\bar{K}^0 \rightarrow \pi^+\pi^-$	a	

$\therefore$ if $|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$, then $\text{amp}(K_1^0 \rightarrow \pi^+\pi^-) = \sqrt{2}\,a$

$|K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$, then $\text{amp}(K_2^0 \rightarrow \pi^+\pi^-) = 0$

(note: here $C|K^0\rangle = |\bar{K}^0\rangle$ so that $C|K_1^0\rangle = +|K_1^0\rangle$, $C|K_2^0\rangle = -|K_2^0\rangle$, $C|{}^{\ell=0}_{\pi^+\pi^-}\rangle = +|{}^{\ell=0}_{\pi^+\pi^-}\rangle$)

This particle is very peculiar — although its funny properties are just a consequence of Quantum Mechanics (superposition). For strong interaction production, the convenient base states are the strangeness eigenstates K^0 & $\bar{K}^0$. For the weak interactions, the convenient base states are the mass operator eigenstates K_1^0 & K_2^0. The existence of two neutral K-meson states with definite mass and definite lifetime does not depend on charge conjugation invariance — which, of course, is violated in the weak interactions. We could repeat the analysis ~~assuming~~ assuming CP conservation (the current current theory $\frac{G}{\sqrt{2}} J^{\dagger} J$ is CP conserving); so that, with redefining phases $\ni$ $CP|K^0\rangle = |\bar{K}^0\rangle$, we have:

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad |K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

$$\therefore K_1^0 \rightarrow \pi^+\pi^- \quad \text{while} \quad K_2^0 \not\rightarrow \pi^+\pi^- \qquad \left(\begin{array}{l} CP|K_1^0\rangle = +|K_1^0\rangle \\ CP|K_2^0\rangle = -|K_2^0\rangle \\ CP|\pi^+\pi^-{}^{\ell=0}\rangle = +|\pi^+\pi^-{}^{\ell=0}\rangle \end{array}\right)$$

Since the K_1^0 and K_2^0 have definite masses and lifetimes, we can write:

$$\frac{d\psi_1(\tau)}{d\tau} = -\left(\frac{\gamma_1}{2} + i m_1\right)\psi_1(\tau) \qquad \frac{d\psi_2(\tau)}{d\tau} = -\left(\frac{\gamma_2}{2} + i m_2\right)\psi_2(\tau)$$

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Ph 230 a

Lecture # 1. Week # 7.

review: $\eta_{+-} = \dfrac{\text{amp that } K_L^0 \to \pi^+\pi^-}{\text{amp that } K_S^0 \to \pi^+\pi^-} = \dfrac{a_{+-}^L}{a_{+-}^S} = |\eta_{+-}| e^{i\phi_{+-}}$

(Feynman mentioned that the phase of ϕ_{+-} has been recently measured avoiding regeneration complications).

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad |K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

$$|K_S^0\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle\right] \simeq |K_1^0\rangle + \epsilon|K_2^0\rangle$$

$$|K_L^0\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle\right] \simeq |K_2^0\rangle + \epsilon|K_1^0\rangle$$

note: $|K_S^0\rangle$ and $|K_L^0\rangle$ are <u>not</u> orthogonal.

$\langle K_S^0 | K_L^0\rangle \simeq 2\,\mathrm{Re}\,\epsilon$ i.e. a measure of CP non-conservation

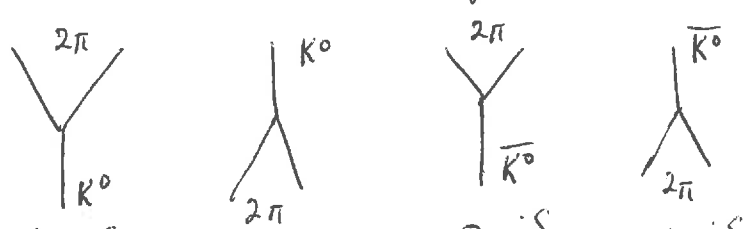
From the last lecture, we expect that:

$$\eta_{+-} = \underset{\substack{\uparrow \\ \text{from a cockeyed wave function} \\ \text{i.e. part of } K_L^0 \text{ is } \epsilon K_1^0 \text{ and } K_1^0 \to \pi^+\pi^-}}{\epsilon} + \text{"something else"}$$

← from CP non-conservation i.e. K_2^0 can decay directly into $\pi^+\pi^-$

this separation is only one way of thinking, for both terms are actually interlocked.

References: T. Wu & Yang, Phy. Rev. Letters 13, pg 380 (1964).
Lee and Wu, Annual Review of Nuclear Science, vol 16, pg 511 (1966)
(this last reference is an excellent source for all phenomenological aspects of K decay).



$A e^{i\delta}$ $A^* e^{i\delta}$ $B e^{i\delta}$ $B^* e^{i\delta}$

note: $A_0 e^{i\delta} = \langle \left(\begin{smallmatrix} I=0 \\ 2\pi \end{smallmatrix}\right)^{out} | K^o \, in \rangle$

where $\left\{\begin{smallmatrix} in \\ out \end{smallmatrix}\right\}$ represents a state that is asymptotically a free particle state with the required quantum numbers at $\left\{\begin{smallmatrix} t=-\infty \\ t=+\infty \end{smallmatrix}\right\}$. (see pg 94 of Gasiorowicz for a clear explanation).

$$\langle K^o out | \left(\begin{smallmatrix} I=0 \\ 2\pi \end{smallmatrix}\right) in \rangle^* = \langle \left(\begin{smallmatrix} I=0 \\ 2\pi \end{smallmatrix}\right) in | K^o out \rangle$$

but $\langle K^o out | \left(\begin{smallmatrix} I=0 \\ 2\pi \end{smallmatrix}\right) in \rangle^* \neq \langle \left(\begin{smallmatrix} I=0 \\ 2\pi \end{smallmatrix}\right) out | K^o in \rangle$

If the system is invariant under time reversal, then

all τ means is to reverse the spins and momenta

$$\langle \left(\begin{smallmatrix} I=0 \\ 2\pi \end{smallmatrix}\right) out | K^o in \rangle = \langle \tau(K^o) \, out | \tau \left(\begin{smallmatrix} I=0 \\ 2\pi \end{smallmatrix}\right) in \rangle$$

$$= \langle K^o \, out | \left(\begin{smallmatrix} I=0 \\ 2\pi \end{smallmatrix}\right) in \rangle \Rightarrow A_0 e^{i\delta_0} = A_0^* e^{i\delta_0}$$

the strong and em Hamiltonians are invariant under the unitary transformation generated by e^{iSX} where S is the strangeness operator and X is an arbitrary real number, i.e. the relative phase between the strange and non-strange particles — or between the $|K^0\rangle$ and $|\bar{K}^0\rangle$ — is not an observable quantity and can be arbitrarily chosen. A convenient choice that we shall later use is to set the amplitude A_0 real. (note: if time reversal is violated, $B_0 \neq B_0^*$ even though we chose $A_0 = A_0^*$ from the phase ambiguity).

Let us review what the various symmetries tell us about the amplitudes:

CP. $Ae^{i\delta} = Be^{i\delta}$, $A^*e^{i\delta} = B^*e^{i\delta}$

i.e. $A = B$

(note: if we include the phase ambiguity, we have: $Ae^{i\delta} = Be^{i\delta}e^{iX}$, $A^*e^{i\delta} = B^*e^{i\delta}e^{iX}$ $\Rightarrow e^{iX} = \pm 1$)

T: $Ae^{i\delta} = A^*e^{i\delta}$, $Be^{i\delta} = B^*e^{i\delta}$

i.e. $A = A^*$, $B = B^*$

CPT: $Ae^{i\delta} = B^*e^{i\delta}$, $A^*e^{i\delta} = Be^{i\delta}$

i.e. $A = B^*$

We shall now evaluate η_{+-} and η_{00} from the machinery that we have developed.

let $|K^0_?\rangle = \alpha|K^0\rangle + \beta|\bar{K}^0\rangle$ i.e. K^0_1 $\alpha = \frac{1}{\sqrt{2}}, \beta = \frac{1}{\sqrt{2}}$
K^0_L $\alpha = \frac{1+\epsilon}{\sqrt{2}}, \beta = -\frac{1-\epsilon}{\sqrt{2}}$

amp. for $K^0_?$ to decay into 2π's : $I=0$ $\alpha A_0 e^{i\delta_0} + \beta B_0 e^{i\delta_0}$
$I=2$ $\alpha A_2 e^{i\delta_2} + \beta B_2 e^{i\delta_2}$

note: $\left|{I=0 \atop \pi\pi}\right\rangle = \frac{1}{\sqrt{3}}(\pi^+\pi^- + \pi^-\pi^+ - \pi^0\pi^0)$
$\left|{I=2 \atop \pi\pi}\right\rangle = \frac{1}{\sqrt{6}}(\pi^+\pi^- + \pi^-\pi^+ + 2\pi^0\pi^0)$

We don't know the π-π scattering phase shifts (which in this case are at a c.m. energy equal to m_K) — but this is a problem of strong interaction physics.

amp for $K^0_?$ to decay into $\pi^+\pi^-$:

$$\frac{1}{\sqrt{3}}\left[\alpha\sqrt{2}A_0 e^{i\delta_0} + \beta\sqrt{2}B_0 e^{i\delta_0} + \alpha A_2 e^{i\delta_2} + \beta B_2 e^{i\delta_2}\right]$$

amp for $K^0_?$ to decay into $\pi^0\pi^0$:

$$\frac{1}{\sqrt{3}}\left(-\alpha A_0 e^{i\delta_0} - \beta B_0 e^{i\delta_0} + \sqrt{2}\alpha A_2 e^{i\delta_2} + \sqrt{2}\beta B_2 e^{i\delta_2}\right)$$

$$\eta_{+-} = \frac{\langle\pi^+\pi^-|K^0_L\rangle}{\langle\pi^+\pi^-|K^0_S\rangle}$$

the numerator is small compared to the denominator;

So the denominator does not need to be too accurate. Therefore, we can take $\alpha \sim \frac{1}{\sqrt{2}}$, $\beta \sim \frac{1}{\sqrt{2}}$, $A_0 \sim B_0$, $A_2 \sim B_2$ ($\begin{smallmatrix} A_0 = B_0 \\ A_2 = B_2 \end{smallmatrix}$ if CP conserved) for $\langle \pi^+\pi^- | K_S^0 \rangle$. also, $A_2 \ll A_0$ and $B_2 \ll B_0$ from the $\Delta I = \frac{1}{2}$ rule.

$$\therefore \langle \pi^+\pi^- | K_S^0 \rangle = a_{+-}^S \simeq \frac{2}{\sqrt{3}} A_0 e^{i\delta_0}$$

likewise, $\langle \pi^+\pi^- | K_L^0 \rangle = a_{+-}^L \simeq \frac{2}{\sqrt{3}} A_0 e^{i\delta_0} \epsilon + \frac{1}{\sqrt{3}}\left[(A_0 - B_0)e^{i\delta_0} + \frac{(A_2 - B_2)}{\sqrt{2}} e^{i\delta_2}\right]$

$$\therefore \eta_{+-} = \frac{a_{+-}^L}{a_{+-}^S} = \epsilon + \epsilon' + \epsilon''$$

where $\epsilon = \epsilon$, $\epsilon' = \frac{1}{2\sqrt{2}} \frac{A_2 - B_2}{A_0} e^{i(\delta_2 - \delta_0)}$, $\epsilon'' = \frac{A_0 - B_0}{2A_0}$

$$\eta_{00} = \epsilon - 2\epsilon' + \epsilon''$$

$$CPT \Rightarrow A_0 = B_0^*, \quad A_2 = B_2^*$$

We can choose the arbitrary phase so that $A_0 = A_0^*$.

then $A_0 \overset{\text{phase conv.}}{=} A_0^*$, $A_0 \overset{CPT}{=} B_0^* \Rightarrow A_0 = B_0$

$\therefore \epsilon'' = 0$ if CPT is conserved (so ϵ'' is a measure of CPT violation)

Also from CPT, $\epsilon' = \frac{i}{\sqrt{2}} \frac{\mathrm{Im}\, A_2}{A_0} e^{i(\delta_2 - \delta_0)}$

If CP is valid, $A = B$ and $\epsilon = \epsilon' = 0$ as expected. As mentioned earlier, we do not know the $\pi\pi$ scattering phase shifts.

* Problem: According to the $\Delta S = \Delta Q$ rule,

$$K_0 \rightarrow \pi^- e^+ \nu \qquad \bar{K}^0 \rightarrow \pi^+ e^- \nu$$
$$\not\rightarrow \pi^+ e^- \nu \qquad \not\rightarrow \pi^- e^+ \nu$$

Suppose $\Delta S = \Delta Q$ is not exactly right, so that K^0 decays to $\pi^+ e^- \nu$ and $\bar{K}^0$ decays to $\pi^- e^+ \nu$ both with amp. $x \ll 1$. Assume form factors are not important (which is better for $\pi e \nu$ than $\pi \mu \nu$), so that x is a constant.

Prove: $$\frac{\text{Rate } K_0^L \rightarrow \pi^- e^+ \nu}{\text{Rate } K_0^L \rightarrow \pi^+ e^- \nu} = 1 + \overbrace{4 \, \mathrm{Re}\, \epsilon \, \frac{1 - |x|^2}{|1 - x|^2}}^{4w}$$

If $x = 0$, this experiment can be used to measure the real part of ϵ i.e. providing another demonstration of CP violation. The experimental results are:

$$W_\mu = -4.0 \pm 1.3 \cdot 10^{-3}$$
$$W_e = -2.2 \pm 0.4 \cdot 10^{-3}$$

(remember $|\eta_{+-}| = 1.9 \cdot 10^{-3}$)

Both numbers should be the same if $x = 0$. So there is a definite asymmetry between matter and antimatter.

We have yet to write an expression for ϵ. Let $\psi_2(t)$ be the K_L^0 amplitude at time t. Then:

$$\frac{dc_1}{dt} = -\left(\frac{\gamma_1}{2} + i m_1\right) c_1(t) + i h_{11}\, c_1(t) - i h_{12}\, c_2(t)$$

can be absorbed into m's

$$\frac{dc_2}{dt} = -\left(\frac{\gamma_2}{2} + i m_2\right) c_2(t) + i h_{22}\, c_2(t) - i h_{12}\, c_1(t)$$

where $c_1(t) = K_1$ amplitude, $c_2(t) = K_2$ amplitude, and h_{12} is $\langle K_1^0 |$ whatever it is that does it $| K_2^0 \rangle$

Since K_L^0 has definite mass and lifetime, let:

$$c_1 = k_1\, e^{-\left(\frac{\gamma_L}{2} + i m_L\right)t} \quad , \quad c_2 = k_2\, e^{-\left(\gamma_L/2 + i m_L\right)t}$$

then
$$-\left(\frac{\gamma_L}{2} + i m_L\right) k_1 = -\left(\frac{\gamma_1}{2} + i m_1\right) k_1 - i h_{12}\, k_2$$
$$-\left(\frac{\gamma_L}{2} + i m_L\right) k_2 = -\left(\frac{\gamma_2}{2} + i m_2\right) k_2 - i h_{21}\, k_1$$

$$\therefore\ i h_{12} k_2 = \left[\frac{(\gamma_L - \gamma_1)}{2} + i(m_L - m_1)\right] k_1 = \left[\frac{(\gamma_L - \gamma_S)}{2} + i(m_L - m_S)\right] \text{ using } \begin{matrix}\gamma_1 \simeq \gamma_S \\ \gamma_2 \simeq \gamma_L\end{matrix}$$

try $k_2 = 1$, $k_1 = \epsilon$ since $|K_L^0\rangle = |K_2^0\rangle + \epsilon\, |K_1^0\rangle$, then

$$\epsilon = \frac{i h_{12}}{\frac{(\gamma_L - \gamma_S)}{2} + i(m_L - m_S)} \simeq \frac{-i h_{12}}{\gamma_S} \sqrt{2}\, e^{i 45°} \quad \text{since } m_L - m_S \simeq \tfrac{1}{2}\gamma_S$$

To find ϵ, we need h_{12} — and to find h_{12}, we need a theory.

Lecture # 3. Week # 7.

Let us review some of the ideas on the origin of CP violation:

(1) in the weak interaction

a. but not in $\pi\pi$ decays

must be in one of 3π, $\pi\mu\nu$, $\pi e\nu$, $\pi\pi\gamma$ — but it seems that none of these can produce an effect as large as observed (e.g. see $\pi\mu\nu$ calculation in last lecture)

b. in $\pi\pi$ decay

(2) indirect

a. in the strong interaction

b. in the em interaction (proposed by Lee who also suggested experiments which eventually eliminated this proposal)

(3) others

e.g. suppose the weak interaction violates CP largely, but only in second order, etc.

We are trying to make a theory which will predict the effect of CP violation elsewhere. However, there is <u>no</u> convincing theory up to now.

CP - violated
T - so far no violation known
CPT - can't find a good theory which violates CPT
(the combination of Quantum Mechanics, Relativity, and Local Operators)

We shall examine T symmetry in more detail. The properties of T are not so familiar because T is an antiunitary operator.

Let $|a\rangle$ be the ket vector of the state "a". For every state, there exists a time reversed state whose ket vector we shall call $|Ta\rangle$ (<u>not</u> $T|a\rangle$)

e.g. $|a\rangle$ electron with momentum p, spin $+z$
$|Ta\rangle$ electron with momentum $-p$, spin $-z$

If $|c\rangle = \alpha|a\rangle + \beta|b\rangle$, then $|Tc\rangle = \alpha^*|Ta\rangle + \beta^*|Tb\rangle$. (the antilinear property)

We can see the reason for this odd property in the following example:

let $|x\rangle$ be the state of an electron at x
$|p\rangle$ be the state of an electron with momentum p

$|Tx\rangle$ must be equal to $|x\rangle$ and $|Tp\rangle$ must be equal to $|-p\rangle$ (at least within a phase)

Since $|p\rangle = \int e^{ipx}\,|x\rangle\,dx$, we have:

$$|Tp\rangle = |-p\rangle = \int e^{-ipx}\,|x\rangle = \int e^{-ipx}\,|Tx\rangle$$

It can be shown that there are only two possible superpositions:

$$|Tc\rangle = \alpha\,|Ta\rangle + \beta\,|Tb\rangle$$
$$|Tc\rangle = \alpha^*\,|Ta\rangle + \beta^*\,|Tb\rangle$$

What about the idea of a positron being an electron moving backward in time? there is an ambiguity in the definition of operators. the operator which relates a positron to an electron moving backward in time is CPT.

Suppose that a system is invariant under time reversal (the system is the whole world; so that nothing is external.) then the arbitrary phases can always be chosen so that:

$$\langle b, t_2 \,|\, a, t_1 \rangle = \langle Ta, -t_1 \,|\, Tb, -t_2 \rangle$$

this is the statement of time reversibility. Note that the following was assumed

$$\langle b, t_2 | a, t_1 \rangle = \langle b, t_2 - t_1 | a, 0 \rangle$$

i.e. only the interval — and not the absolute time — is important. (note: We have used the Heisenberg representation where the ket $|a, t_1\rangle$ defines a state for all time by specifying its properties at time t_1. In particular, $|a^{out}\rangle$ equals $|a, +\infty\rangle$. the familiar evolution operator $U(t_2, t_1)$ is defined as follows:

$$\langle b t_2 | a t_1 \rangle = \langle b t_1 | U(t_2, t_1) | a t_1 \rangle \qquad \text{So that}$$

$$\langle b t_2 | a t_1 \rangle = \langle b, t_2 - t_1 | a, 0 \rangle \Rightarrow U(t_2, t_1) = U(t_2 - t_1)$$

therefore, in the Schrodinger representation,

$$\langle b | \Psi(t_2) \rangle = \sum_a \langle b | U(t_2 - t_1) | a \rangle \langle a | \Psi(t_1) \rangle$$

$|a\rangle_{Sch.} \equiv |a, 0\rangle_{Heis.}$) Actually there are two possibilities for the statement of time reversibility:

$$\langle b, t_2 | a, t_1 \rangle = \langle Ta, -t_1 | Tb, -t_2 \rangle^* \quad \text{OR} \quad \langle b, t_2 | a, t_1 \rangle = \langle Ta, -t_1 | Tb, -t_2 \rangle$$

The second one is correct which is related to the choice
$|Tc\rangle = \alpha^* |Ta\rangle + \beta^* |Tb\rangle$

Let us time reverse a state twice.

$$|TTb\rangle = e^{i\varphi} |b\rangle$$

note: φ is a univer~~sable~~sal phase for a set of interferable states. However, the choice of phase is independent for states with different quantum numbers such as charge and strangeness.

$e^{i\varphi} = \pm 1$ Pf: $\langle Ta | b\rangle = e^{-i\varphi} \langle Ta | TTb\rangle = e^{-i\varphi} \langle Tb | a\rangle$

$\therefore \langle b | a\rangle = \langle Ta | Tb\rangle = e^{-i\varphi} \langle TTb | a\rangle = e^{-2i\varphi} \langle b | a\rangle$

$\therefore e^{2i\varphi} = 1 \Rightarrow e^{i\varphi} = \pm 1$

(There is a weakness in the proof - find & correct it.)

e.g. spin ½

phase here makes no difference

$|T(+z)\rangle = |-z\rangle$

$|T(-z)\rangle = e^{i\lambda} |+z\rangle$ $\quad |+x\rangle = \frac{1}{\sqrt{2}} (|+z\rangle + |-z\rangle)$

$|T(+x)\rangle = e^{i\mu} |-x\rangle$ $\quad |-x\rangle = \frac{1}{\sqrt{2}} (|+z\rangle - |-z\rangle)$

$$\therefore |T(+x)\rangle = \frac{1}{\sqrt{2}} (|T(+z)\rangle + |T(-z)\rangle)$$
$$= \frac{1}{\sqrt{2}} (|-z\rangle + e^{i\lambda} |+z\rangle)$$
$$\therefore |-x\rangle = \frac{1}{\sqrt{2}} (e^{i(\lambda-\mu)} |+z\rangle + e^{-i\mu} |-z\rangle \Rightarrow e^{-i\mu} = -1, e^{i\lambda} = -1$$

$\therefore$ if $|T(+z)\rangle = |-z\rangle$, then $|T(-z)\rangle = -|+z\rangle$ which implies

$$|TT(+z)\rangle = -|+z\rangle$$

The funny minus sign is also seen is assigning isospin to the nucleon and antinucleon doublet:

$$\begin{pmatrix} p \\ n \end{pmatrix} \quad \& \quad \begin{pmatrix} \bar{n} \\ -\bar{p} \end{pmatrix}$$

The sign involves half-integral representations (not 2-state systems) — and it is related to the Bose and Fermi statistics.

$TT = -1$ for fermions, $TT = +1$ for bosons

However, relativity is also needed to prove Bose and Fermi statistics — but maybe we can understand the relative sign in the statistics by the curious sign in TT.

* Prove: Necessary and Sufficient Condition for $TT = \begin{cases} +1 \\ -1 \end{cases}$
If there exists a state which is its own time reversal, i.e. $|Ta\rangle = |a\rangle$, then $TT = +1$ and vice versa.

With spin ½, it is impossible to find such a state. The $J_z = 0$ state of an integral spin representation is necessarily its own time reversal.

Elementary Particles (Heusch) 10/7/68 LECTURE

References:

Sichtenberg: Meson & Baryon Spectroscopy (Springer, 1965)
Williams: Introduction to Elementary Particles (Academic Press) -- old
Levi-Setti: Elementary Particles (U. Chicago) -- old, cheap
Frazer: Elementary Particles
Ioaiorowicz: Elementary Particles (Wiley 1966) -- excellent ($15)
Sakurai: Invariance Principles & Elementary Particles -- old
Källén: Elementary Particles (Addison-Wesley)
Nishijima: Fundamental Particles (Benj.) -- old

What is an "elementary particle"? Even particles like protons are composed of many constituents. So merely a name for sub-nuclear particles.

What about electron? To best of our knowledge, e^- is a point particle -- no structure -- no characteristic radius.

Mentions bootstrap theory. Also quark multiplet theory.

Particles	Calculable Example	Size	Energy	Main Part of Wave fcn	Force	Complication	Correction by
Atoms	H	10^{-8} cm	~eV	nucleus + electrons	photon exchange	many electron problem	e^+e^- pair nuclear structure ~1%
Nuclei	D	$10^{-13}A^{1/3}$	10^6 eV	$Zp+(A-Z)n$	meson exchange	many nucleon	~10%
"elementary" particles	none	SIP $\lesssim 10^{-13}$ cm WIP ~ no structure	~10^3 eV	?	π exchange K exchange ρ exchange	~~strong~~ many particle problem	~50%

Relations with other areas of physics

10/9/68 LECTURE

Atomic Physics -- very little connection

Nuclear Physics -- nuclear forces are mediated by π's
β-decay

Astrophysics -- high density star $\Delta E \sim m_\pi$
synchrotron acceleration
ν emission

Cosmic ray physics

Accelerators: e or p bremsstrahlung of e's limits use of synchrotron at high E -- hence use linear accelerator
[Heavier mass of p limits radiation loss via bremsstrahlung.]
Mentions clashing beam facilities [pp, e^-e^-, e^-e^+].

No uses (until we find stable quarks). (Quark bombs)

—o—

Elementary Particle Interactions

	σ_{tot} [cm^2]	$\frac{\sigma_{total}}{\pi \lambda\!\!\!^{-}{}^2}$ [coupling strength]	$\sim\sqrt{\sigma}$	τ [sec]	Range [cm]
Strong	10^{-26}	1	$\frac{f^2}{\hbar c} \sim 1$	10^{-23}	$\leq 10^{-13}$
Electromagnetic	10^{-30}	10^{-4}	$\frac{e^2}{\hbar c} \sim \frac{1}{137}$	10^{-18}	long $\sim \frac{1}{r}$
Weak	10^{-40}	10^{-14}	$\frac{g}{\sqrt{\hbar c}} \sim 10^{-7}$	10^{-9}	$\leq 10^{-13}$
Gravitational	10^{-104}	10^{-78}	$\frac{G m_p^2}{\hbar c} \sim 10^{-39}$	10^{55}	long $\sim \frac{1}{r}$

10/11/68 LECTURE

Certain particles interact only via one of these forces

$\gamma \rightarrow$ EM interaction

$\nu \rightarrow$ weak interaction

None of the strongly interacting particles (p, π, K, etc.) interact <u>solely</u> via strong interactions.

Actually the most dramatic distinction between the types of interactions involve conservation laws

Conservation Law	Strong	EM	Weak
$p(E, \vec{p})$	✓	✓	✓
$\vec{J}$	✓	✓	✓
Q	✓	✓	✓
B (baryon #)	✓	✓	✓
L (lepton #)	✓	✓	✓
P	✓	✓	NO
T	?	?	?
C	✓	?	?
CP	✓	✓	NO
CPT	✓	✓	✓
I (isospin)	✓	NO	NO
G (G parity)	✓	NO	NO
S or Y (strangeness or hypercharge)	✓	✓	NO

Because of the long range of the EM interaction, it can frequently compete with strong interactions. E.g., π^+ in a lead brick -- find <u>multiple scattering</u> in addition to strong interactions

Need multiple scattering corrections.

10/14/68 LECTURE

Comments on Data Sheets

i.) "Stable particle" means stable under strong interaction $[\tau > 10^{-20} s]$

True stable particles are

$\gamma, \nu_e, \nu_\mu, e^\pm, p$

Weak decaying $(\tau \sim 10^{-9} s)$: e.g., $\mu^\pm$

Electromagnetically decaying $(\tau \sim 10^{-16} s)$: e.g., $\pi^0 \to 2\gamma$

Usually we strike with stable particles (e^-, p) in accelerators, but actually all weakly decaying particles can be used in beams: e.g. μ, $\pi^\pm$ (LAMF), $K^{\pm,0}$ [also n, but hard to collimate]
Also $\bar{p}$ beams.

[Must keep these experimental limitations in mind]

In 200 Gev, may also get $\Lambda^0, \Sigma^{+0-}, \Xi^0$

ii.) Nomenclature

photons γ

leptons $\nu_{e,\mu}, e^\pm, \mu^\pm$ [structureless, point particles]

hadrons (strongly interacting) $\left\{ \begin{array}{ll} \text{mesons} & \pi^{\pm,0}, K^{\pm,0}, \ldots \\ \text{baryons} & p, n, \Lambda, \ldots \end{array} \right.$

iii.) Possible Targets:

a.) p [liquid H_2]: This is only really useful target since a well defined initial state [p at rest]. For, say, C, need corrections for proton motion in C nucleus; also binding & neutron interactions.

b.) n (deuteron): quite difficult.

c.) e (heavy nucleus like U^{235}): even more complicated

RELATIVISTIC KINEMATICS 10/18/68 LECTURE

Lorentz transformation: $x_1 = \gamma(x_1' + \beta x_0')$ $x_0 = ct$

$x_0 = \gamma(\beta x_1' + x_0')$ $[x_4 = ict]$

or

$x^\mu = (x^0, x^1, x^2, x^3)$ $g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

Lorentz invariant scalar product is $x^\mu x^\nu g_{\mu\nu} = (x^0)^2 - (\vec{x})^2$

Now since

$$x_\mu = g_{\mu\nu} x^\nu$$

$$x^\mu x_\nu = x \cdot x = x^2$$

Hence any transformation $(x \to x', y \to y')$ is Lorentz invariant iff the scalar product remains the same:

$$x' \cdot y' = x \cdot y$$

Convention: $\hbar = 1$, $c = 1$, $m_p = 1$

Then

Compton wavelength of proton

length unit $\frac{\hbar}{m_p c} = \lambda\!\!\!^{-}_p = 0.211 \times 10^{-13}$ cm

time unit $\frac{\hbar}{m_p c^2} = \frac{\lambda\!\!\!^{-}_p}{c} = 0.7 \times 10^{-24}$ sec

mass unit $m_p = 1.67 \times 10^{-24}$ gm

or

mass $\mu\,(m_p)$

length $\lambda_\mu = l_0 \frac{1}{\mu} \left[\frac{1}{m_p}\right]$

time $t_\mu = \# \left[\frac{1}{m_p}\right]$

$m_p \sim 1$ Gev $[10^9$ eV$]$

Useful 4-vectors:

$$x = (ct, \vec{x})$$

$$p = (E/c, \vec{p})$$

$$k = \left(\frac{\omega}{c}, \vec{k}\right)$$

$$j = (\rho c, \rho \vec{v})$$

$$v = (c\gamma, c\gamma\vec{\beta})$$

Consider a 2 particle system: $(m_1, \vec{p}_1)$, $(m_2, \vec{p}_2)$. What is energy in CM system? Can express this in terms of 3 invariants

$$p_1^2 = m_1^2, \quad p_2^2 = m_2^2, \quad p_1 \cdot p_2 \text{ or } (p_1+p_2)^2 \text{ or } (p_1-p_2)^2$$

As convention, in CM system x^* and p^*. By definition

$$\vec{p}_1^{\,*} + \vec{p}_2^{\,*} = 0$$

Also

$$E^{*2} = (E_1^* + E_2^*) = (p_1^* + p_2^*)^2 = (p_1+p_2)^2$$

Total mass $\quad M^2 = P^2 = (p_1+p_2)^2 = E^{*2}$

Hence $\quad (E_1+E_2)^2 - (\vec{p}_1+\vec{p}_2)^2 = \text{invariant}$

Also

$$\vec{\beta}_{CM} = \frac{\vec{P}}{E} = \frac{(\vec{p}_1+\vec{p}_2)}{(E_1+E_2)} \quad \left(\text{velocity of CM in lab}\right)$$

$$\gamma_{CM} = \frac{E}{M} = \frac{E_1+E_2}{\sqrt{(E_1+E_2)^2 - (\vec{p}_1+\vec{p}_2)^2}}$$

Verify: $E_1^* = \dfrac{M^2+(m_1^2-m_2^2)}{2M}$

$$E_2^* = \frac{M^2-(m_1^2-m_2^2)}{2M}$$

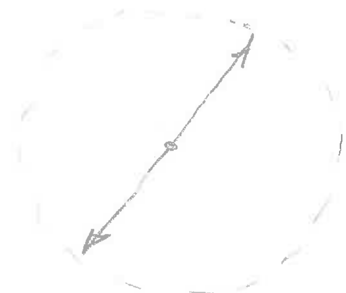
$$|\vec{P}_{1,2}^*|^2 = \frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}$$

$$|\vec{V}_{1,2}^*|^2 = \left(\frac{|p^*|}{E_i^*}\right)^2$$

All of these are invariant quantities, and are useful in decay $M \to m_1 + m_2$

EXAMPLE: Isotropic decay of a spinless particle, e.g. $\pi^0 \to 2\gamma$

CM



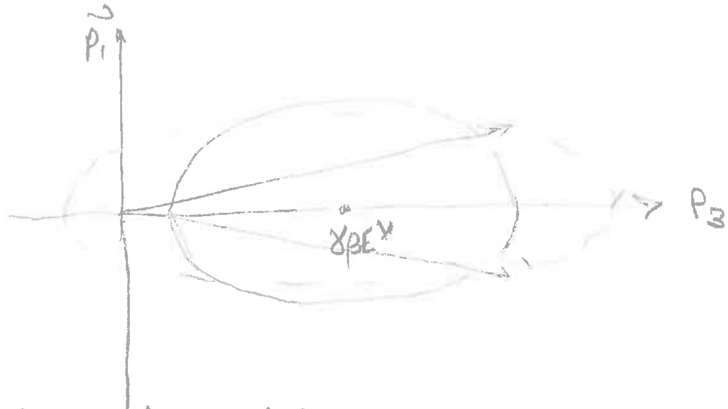
spherical decay distribution in CM

$|\vec{p}_1| = p_1 = p_1^*$

$p_3 = \gamma[p_3^* + \beta E^*]$

LAB Spheroid → ellipsoid

$a_1 = p^*$, $a_2 = p^*$, $a_3 = \gamma p^*$



$|\vec{P}_3|_{max} = \gamma(p^* + \beta E^*)$

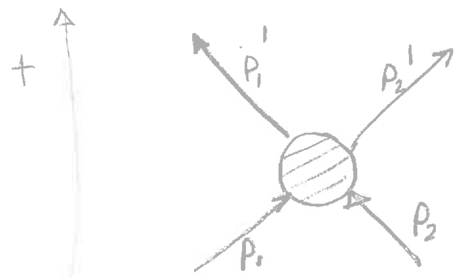
$|\vec{P}_3|_{min} = \gamma(p^* - \beta E^*)$

$|\vec{P}_3|_{cen} = \gamma\beta E^*$

Backward emitted decay products only if $v^* > \beta$

Elastic Scattering Kinematics

10/21/68 LECTURE



Disregard spin, isospin, etc.

Transition amplitude

$$T_{fi} = T(p_1, p_2, p_1', p_2')$$

First formulate problem in a Lorentz invariant form: 10 variables $p_1^2, p_2^2, \ldots$ But know

$$p_1^2 = p_1'^2 = m_1^2 \qquad p_2^2 = p_2'^2 = m_2^2$$

+ conservation laws $\Big\} \Rightarrow$ 2 independent invariants

[function of 2 complex variables

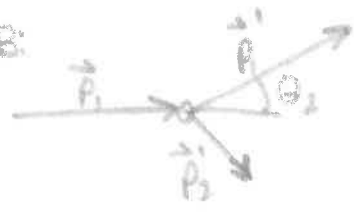
Take, for example, $p_1 p_2$ and $p_1 p_2'$

Lorentz frames: i.) Lab $\vec{p}_2 = 0$

ii.) CM: $\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2' = 0$

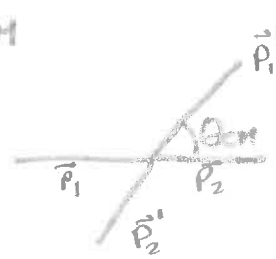
iii.) Brick wall (Breit): $\vec{p}_1 + \vec{p}_1' = 0 = \vec{p}_2 + \vec{p}_2'$

LAB:



$$E_1 = \frac{1}{m} p_1 p_2 \qquad E_1' = \frac{1}{m} p_2 p_1' \cdots$$

CM



total energy

$$s = (p_1 + p_2)^2 = (p_1' + p_2')^2 = E_{CM}^2$$

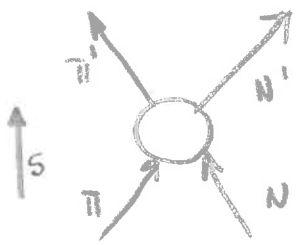
$$\cos\theta_{CM} = 1 + \frac{t}{2p^2}$$

momentum transfer

$$t = (p_1 - p_1')^2 = 2|\vec{p}|^2(\cos\theta_{CM} - 1)$$

Breit: $t = -2|\vec{p}_i|^2$

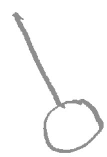
CROSSING



s channel

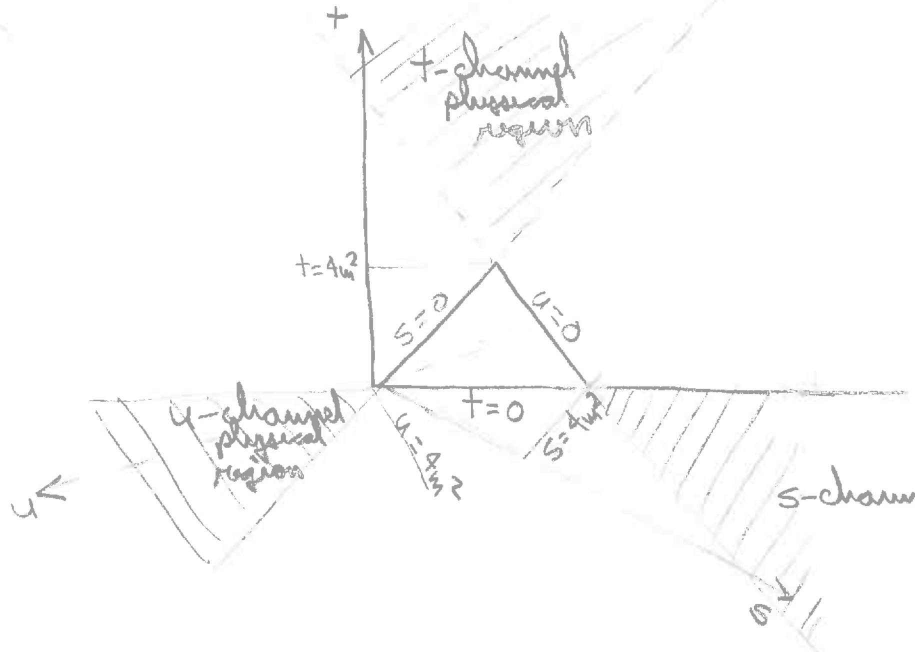
$\pi N \to \pi N$

$(\pi \bar{N} \to \pi \bar{N})$

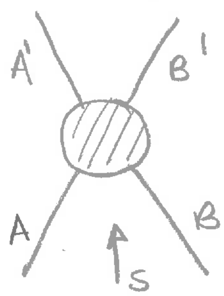


Consider equal masses.

10/23/68 LECTURE



Physical regions for s-channel scattering:



$$s = (2E^*)^2 = 4\left(m^2 + \vec{p}^{\,2}\right)$$

$$= (p_1 + p_2)^2$$

Can see

$$s \geq 4m^2$$

Also

$$t = 2\vec{p}^{\,2}(\cos\theta^* - 1) = (p_1' - p_1)^2$$

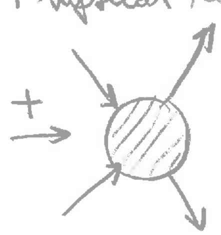
$$0 \geq t \geq -4\vec{p}^{\,2} = 4m^2 - s$$

Also use

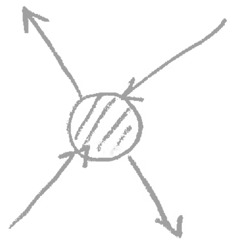
$$s + t + u = 4m^2$$

to sketch our regions above.

Physical region for t-channel: For equal masses, going from s to t causes no change in u



Physical region for u channel:



Now t does not change as $s \to u$
also $t \to u$, s does not change

Hence the idea is to regard $T = T(s,t,u)$ as a function of 2 complex variables (since s, t, & u are dependent), and to analytically continue T from one physical region to another.

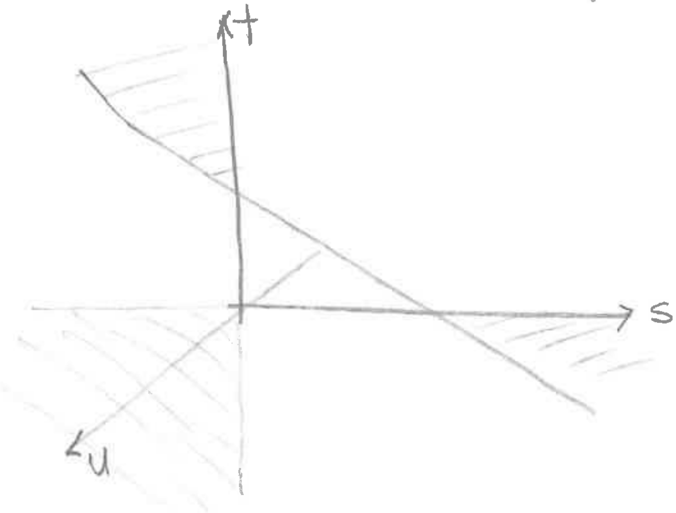
Hence as a summary of physical regions for equal masses

s-channel
$$\begin{cases} s \geq 4m^2 \\ t \leq 0 \\ u \leq 0 \end{cases}$$

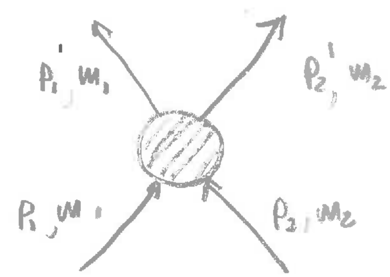
t-channel
$$\begin{cases} s \leq 0 \\ t \geq 4m^2 \\ u \leq 0 \end{cases}$$

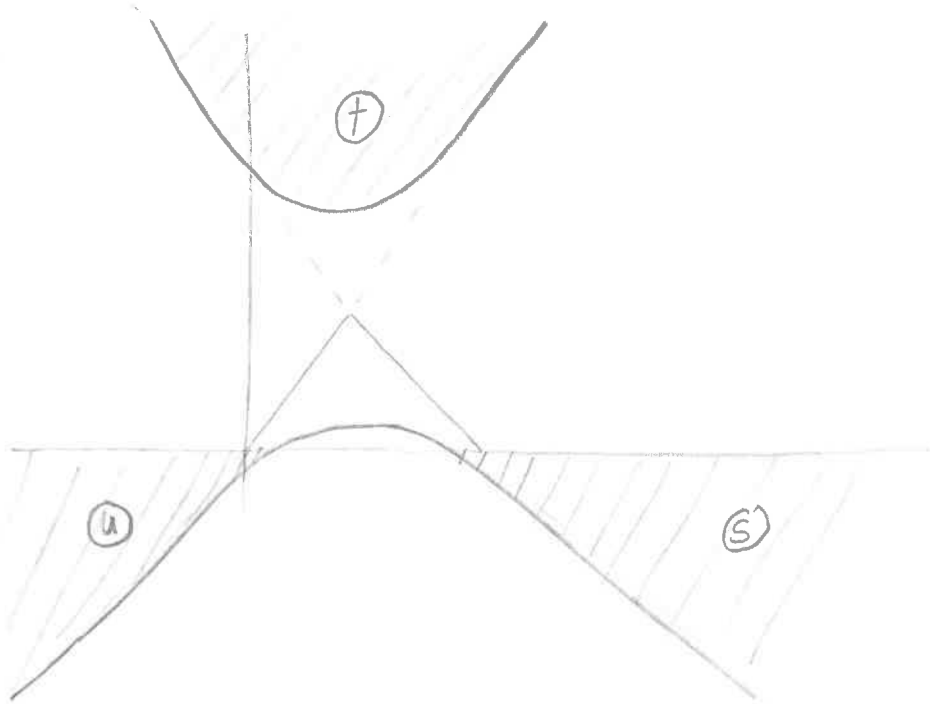
u-channel
$$\begin{cases} s \leq 0 \\ t \leq 0 \\ u \geq 4m^2 \end{cases}$$

Can redraw the diagram in a rectangular coord system



Now generalize to different masses.





PHASE SPACE FACTOR

Fermi's Golden Rule:

$$\text{Rate} \equiv W = \frac{2\pi}{\hbar}|M_{fi}|^2 \underbrace{\frac{dn}{dE}}_{\text{phase space factor}}$$

phase space factor [density of final states]

The factor $\frac{dn}{dE} = f(E, \text{individual particle masses in final state})$

Can also write

$$W = \frac{2\pi}{\hbar}|M'_{fi}|^2 \rho(E) \qquad \text{non invariant}$$

$$= \frac{2\pi}{\hbar}|M''_{fi}| R(E) \qquad \text{lorentz invariant}$$

Now Fermi ~~statistical theory~~ assumes $|M_{fi}| \neq fcn(\vec{p}_i)$
Hence $W, \vec{p}_f$ are functions of phase space alone.

Consider one particle system: $\vec{x}, \vec{p}$

In Q.M. we require

$$\Delta x_i \Delta p_i \geq 2\pi\hbar$$

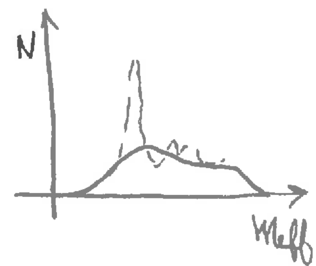
This defines an elementary cell in phase space as $(2\pi\hbar)^3$. Hence for one particle

$$N_1 = \frac{1}{(2\pi\hbar)^3}\int d^3x\, d^3p = \frac{V}{(2\pi\hbar)^3}\int d^3p$$

Recall Fermi's golden rule

$$W = (\text{const.})\underbrace{|\mathcal{M}|^2}_{\text{dynamical features}} \left.\frac{dN}{dE}\right\} \text{kinematical features}$$

We shall look for features superimposed on the phase space structure



For the example of a single particle, we found

$$N_1 = \frac{V}{(2\pi\hbar)^3}\int d^3\vec{p}$$

For one particle of mass $\leq m$, energy $\leq E$

$\therefore N_1 =$ # cells in a sphere $p_1^2+p_2^2+p_3^2 = E^2-m^2$ $=$ total # states

Now for a system of several particles

$$N_n = \frac{1}{(2\pi\hbar)^{3n}}\int_{3n}\cdots\int \prod_{i=1}^{n} d^3x_i\, d^3p_i$$

$$\therefore \quad \rho_n(E) = \frac{dN_n}{dE} = \int \prod_{i=1}^{n} d^3p_i\, \delta^3\Big(\sum_j^N \vec{p}_j - \vec{P}\Big)\, \delta\Big(\sum_{i=1}^{n} E_i - E\Big)$$

In the special case of 2 particles : m_1, m_2 final state masses

$\vec{p}_1, \vec{p}_2$ " " momenta

$$\therefore \rho_2(E) = \int d\vec{p}_1 \int d\vec{p}_2\, \delta(\vec{p}_1+\vec{p}_2)\, \delta(E_1+E_2-E)$$

Using $E_1 = \sqrt{m_1^2 + P^2}$, we find

$$\rho_2(E) = \frac{4\pi}{E} \frac{\sqrt{[E^2-(m_1-m_2)^2][E^2-(m_1+m_2)^2]}}{2E} \frac{E^4-(m_2^2-m_1^2)^2}{4E^2}$$

But remember this is still in non-relativistic notation. To get it into invariant form, we recall from field theory

$$R_n(E) = \int \prod_i^N \frac{d^3 p_i}{2E_i} \delta^3(\vec{p}_i - \vec{P}) \delta\left(\sum_i^N E_i - E\right)$$

$$= \rho(E) \prod_i^n \frac{1}{2E_i}$$

Now

$$\int d^4 p\, \delta(p^2 - m^2) \equiv \int d^3 p\, dE\, \delta(E - (p^2 + m^2)) \qquad \text{for } E > 0$$

$$= \int \frac{d^3 p}{2E} \qquad E = p^2 + m^2$$

Limit integration to $E, E_i > 0$. Hence

$$R_n(E) = \int \prod_{i=1}^n \left[d^4 p_i\, \delta(p_i^2 - m_i^2) \right] \delta^4\left(\sum_{i=1}^N p_i - P\right)$$

To examine this in more detail, can look at momentum spectrum for a particle in final state

$$\frac{dR_n}{dp_i}$$

As a special case

$$R_2 = \int \frac{d^3\vec{p}_1}{2E_1} \frac{d^3 p_2}{2E_2} \, \delta(\vec{p}_1 + \vec{p}_2) \, \delta(E_1 + E_2 - E)$$

$$= \frac{\pi}{E} \, \frac{\sqrt{[E^2 - (m_2 - m_1)^2][E^2 - (m_2 + m_1)^2]}}{2E}$$

10/30/68 LECTURE

To get the phase space for more than two particles in the final state, it is easiest to use a recursion formula

$$R_n(E, \vec{P}=0) = \int \frac{d|\vec{P}_n|}{2E_n} R_{n-1}(\epsilon, 0) \qquad \epsilon^2 = (E-E_n)^2 - \vec{P}_n^{\,2}$$

EXAMPLE: 3 particle phase space

$$R_3(E, 0) = \int_{P_{3\,min}=0}^{P_{3\,max}} \frac{d|\vec{P}_3|}{2E_3} R_3(\epsilon, 0)$$

$$P_{3\,max} = \frac{1}{2E}\sqrt{\left[E^2-(m_1+m_2-m_3)^2\right]\left[E^2-(m_1+m_2+m_3)^2\right]}$$

If we use $\cos\theta = \dfrac{\vec{P}_n \cdot \vec{P}_{n-1}}{|\vec{P}_n||\vec{P}_{n-1}|}$, then can find $\dfrac{dR_n}{d\cos\theta}$.

"Effective mass" of 2 particle system is

$$m_{12}^2 = (E_1+E_2)^2 - (\vec{P}_1+\vec{P}_2)^2 \quad \rightarrow \quad \frac{dR_2}{dm_{12}}$$

Can also find for 3 particles

$$m_{12}^2 = (E-E_3)^2 - \vec{P}_2^{\,2} = E^2 - 2EE_3 + m_3^2$$

$$\frac{dR_3}{dp^3} = \frac{\pi^2 P_3^2}{E_3} \frac{\left\{\left[E^2-2EE_3+m_3^2-(m_2+m_1)^2\right]\left[E^2-2EE_3+m_3^2-(m_2-m_1)^2\right]\right\}^{1/2}}{E^2-2EE_3+m_3^2}$$

Find

$$\frac{dR_3}{dm_{12}} = \frac{\pi^2}{2E^2 m_{12}} \left\{\left[m_{12}^2-(m_1+m_2)^2\right]\left[m_{12}^2-(m_1-m_2)^2\right]\left[E^2-(m_3+m_{12})^2\right]\left[E^2-(m_3-m_{12})^2\right]\right\}^{1/2}$$

Now can plot

$$\frac{dR_3}{dM_{12}} \equiv f(\mu^2)$$

Reaction final state: n particles
We want effective mass of $k < n$ particles in final state. $\}\ {}^{n}_{k}M^2$

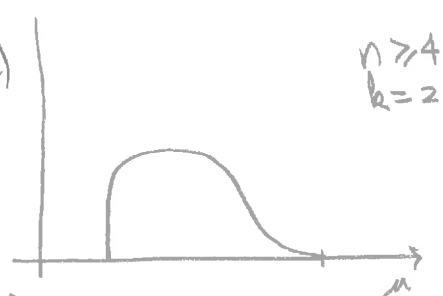
$$ {}^{n}_{k}M^2 = \left(\sum_{i=1}^{k} p_i\right)^2 = \left(P - \sum_{i=k+1}^{n} p_i\right)^2 $$

Want to calculate

$$ f(\mu^2) = \frac{dR_n}{d\left({}^{n}_{k}M^2\right)} $$

As another example

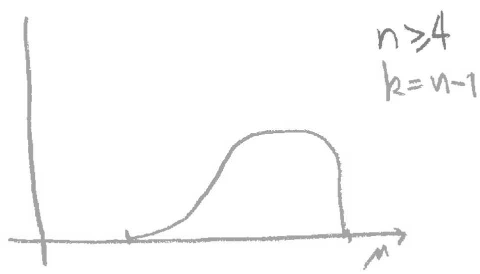
$f(\mu^2)$

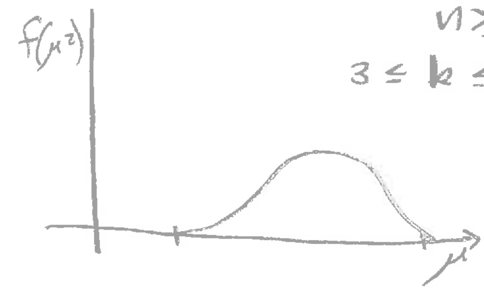


e.g. $pp \to pn\pi^+\pi^-\pi^0\pi^0K^+K^-$

Third typical shape

$f(\mu)$



Only one other shape 

$n \geq 5$

$3 \leq k \leq n-2$

These tell us what happens from kinematics only. The variations from this in the data must then be due to dynamics -- i.e. the matrix element $|M_{fi}|^2$. In general

$$f(\mu^2) = \int \left[\prod_{i=1}^{n} d^4p_i \; \delta(p_i^2 - m_i^2) \; \delta^4\left(\sum_{i=1}^{n} p_i - P\right) \; \delta\left({}^{n}_{k}M^2 - \mu^2\right) \right]$$

These graphs therefore allow you to get rid of the kinematics and look for things like resonances.
Also

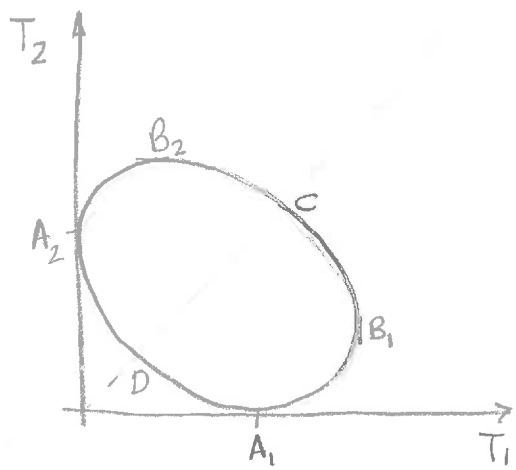
$$f(\mu^2) = R_k(\mu, 0, m_1, \cdots m_k)\, R_{n-k+1}(E, 0, m_{k+1} \cdots m_n)$$

DALITZ PLOTS

11/1/68 LECTURE

Three particles m_1, m_2, m_3 in final state
Final energies T_1, T_2, T_3 ; $T_1+T_2+T_3 = Q$

Now plot kinematic event in a plot. Note



$$E_3 = E - (E_1+E_2)$$

$$E_i = T_i + m_i$$

$$|\vec{P}_3|^2 = |\vec{P}_1|^2 + |\vec{P}_2|^2 + 2|\vec{P}_1||\vec{P}_2|\cos\theta_{12}$$

All possible final state events must lie in an ellipse given by

$$T_1 = \frac{V + a\sqrt{V^2 - uw}}{u}$$

where $u = B^2 - 2ET_2$

$a = \cos\theta_{12}$

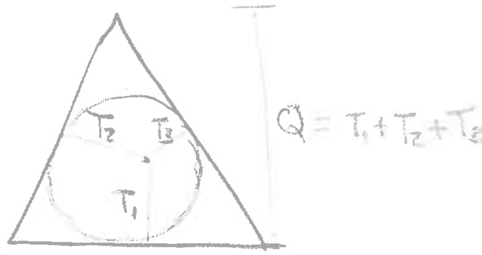
$$V = BC - (AB + C - 2m_1m_2)T_2 + CT_2^2$$

$$w = (C - AT_2^2)$$

$$A = E - m_1$$
$$B = E - m_2$$
$$C = \frac{1}{2}\left[(E - m_1 - m_2)^2 - m_3^2\right]$$

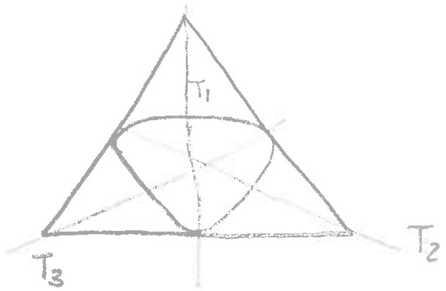
For 3 equal masses [e.g. $K^0 \to \pi^+ + \pi^- + \pi^0$
$\omega \to \pi^+ + \pi^- + \pi^0$]

nonrelativistic



[Dalitz, Proc. Phy. London A69, 527 (1956) nonrel.
Fabri Nuovo Cimento, 11 479 (1957)]

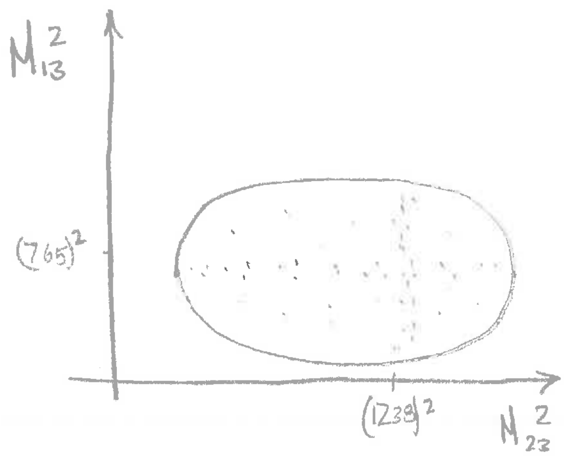
relativistic



Equal areas on a Dalitz plot $\Leftrightarrow$ equal probabilities in Lorentz invariant phase space

That is Lorentz invariant phase space predicts a uniform population of Dalitz plot. Any clustering is due to matrix element of interaction.

Another useful plot is



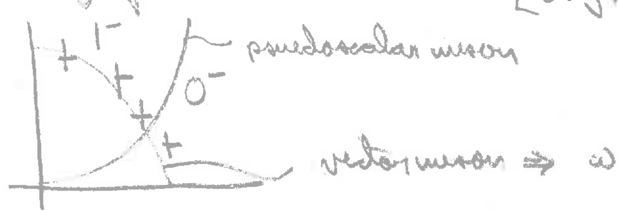
$\gamma p \rightarrow p\pi^+\pi^-$

[$\Delta^{++}(1238)$

$\rho^0 \rightarrow \pi^+\pi^-$ (765)

Conservation laws impose constraints on Dalitz plot uniformity ("modulation of phase space") Hence can use Dalitz plots to pick off quantum numbers. [e.g.

$\omega \rightarrow \pi^+\pi^-\pi^-$



CONSERVATION LAWS

CONSERVATION OF CHARGE Q

Seems very good. Additive quantum number

$$\sum_i Q_i = Q$$

Example: $pp \to pp$

$\not\to pn$

$\not\to pp\pi^-$

$\to pp\pi^0$

To estimate how good Q is, we look at stability of e^-. If Q were not good,

$e^- \to \gamma\nu$ or something

Can set a limit on conservation of charge

$\tau_e > 10^{17}$ years

Ph 231 PROBLEMS

Due: Nov. 4, 1968

1. (6 points) Consider the process

$$\pi^- p \rightarrow \pi^0 n \quad .$$

   (a) What are the corresponding reactions in the t- and u-channels? Which of these reactions are experimentally feasible?

   (b) What are the physical regions for these processes? (Assume $m_{\pi^-} = m_{\pi^0}$; $m_p = m_n$).

   (c) Draw a diagram and indicate, qualitatively, the experimentally accessible parts of the physical regions.

2. (4 points) Consider the decay

$$M \rightarrow \pi^+ \pi^- \pi^0 \quad .$$

   (a) What is the minimum value of M, M_a, such that two of the π's can form a ρ meson? (Let π_1, π_2 form a ρ, keep π_3 at rest.)

   (b) What is the minimum value of M, M_b, such that there can be two simultaneous ρ's?

   (c) What is the maximum value of M, M_c, for two ρ's?

   (d) At what value of M, M_d, will three ρ bands overlap?

Draw the Dalitz plots (m_{12}^2 vs. m_{23}^2) for these cases.

11/4/68 LECTURE

CONSERVATION OF BARYON NUMBER B

Again B is a simple additive quantum number

$B = +1$ $\quad n, p, \Sigma, \Lambda, \Xi, \cdots$

$B = -1$ $\quad \bar{n}, \bar{p}, \bar{\Sigma}, \bar{\Lambda}, \bar{\Xi}, \cdots$

$B = 2$ $\quad d$

$B = 238$ $\quad U^{238}$

$B = 0$ $\quad$ all leptons and mesons

Experimental test: Stability of proton (lightest particle with $B=1$) (Reines, et. al., Phys. Rev. 96, 1157 (1954)). Look for

$$p \to e^+ \nu \bar{\nu}$$

Find

$$\tau_p > 10^{21} \text{ yrs} \quad \text{(bound in hydrogen)}$$
$$> 10^{22} \text{ yrs} \quad \text{(bound in carbon)}$$

Hence B seems to be conserved. Maybe there is a formalism between B and Q. (Lee & Yang, Phys. Rev. 98, 1501, 1955). Maybe B also coupled to a vector field like Q. Then $p-p$ repulsion, $p-\bar{p}$ attraction. Must be weaker than gravity, since matter attracts. Cosmological observations seems to contradict Lee-Yang field [Sakurai, pp. 177-184]

CONSERVATION OF LEPTON NUMBER L

Prior to 1962 : $L = 1$ e^-, μ^-, ν

$L = -1$ $e^+, \mu^+, \bar{\nu}$

$L = 0$ all other particles

Is again a simple, additive quantum number.

$\Delta L = 1$ - nonconserving terms in weak interactions $< 10\%$

(Experiments are very hard to perform)

$\Delta L = 2$: $(Z, A) \rightarrow (Z+2, A) + 2e^-$ (no $\bar{\nu}$'s)

Example

$$(0^+ \text{ ground state})\ Ca^{48}_{20} \rightarrow (0^+ \text{ ground state})\ Ti^{48}_{22}$$

$$\tau > 10^{18} \text{ years} \quad (1956)$$

Nuclear reactor neutrinos

$$p \rightarrow n\, e^- \begin{pmatrix} \bar{\nu} \\ \nu \end{pmatrix} \qquad \rightarrow \begin{pmatrix} \bar{\nu} \\ \nu \end{pmatrix} + Cl^{37} \not\rightleftarrows A^{37} + e^-$$

In 1962 : $L_\mu \neq L_e$

Both independent, simple additive, conserved quantum numbers

	L	L_μ	L_e
e^-	1	0	1
μ^-	1	1	0
ν_e	1	0	1
ν_μ	1	1	0
e^+	-1	0	-1
μ^-	-1	-1	0
$\bar{\nu}_e$	-1	0	-1
$\bar{\nu}_\mu$	-1	-1	0

Brookhaven; CERN: $\nu_\mu Z \to \mu$ (no electrons)

$\nu_e Z \to e$ (no muons)

To do this, take a ν beam [$\pi \to \mu \bar{\nu}_\mu$, $K \to \pi e \bar{\nu}_e$]

Find 99.5% $\bar{\nu}_\mu$, 0.5% $\bar{\nu}_e$. Look for signature in spark chamber

Found <1% of events had showers $\Rightarrow e$

>99% of " " long, straight tracks $\Rightarrow \mu$

Functional Equations of Neutral Type 10/28/68

Examples: differential-difference equations

$$\dot{x}(t) + a\dot{x}(t-r) + bx(t) + cx(t-r) = 0$$

$$\dot{x}(t) + f[x(\tau(t))]\,\dot{x}(\tau(t)) + h[x(\omega(t))] = 0$$

$$t-r \leq \tau(t) \leq t, \quad t-r \leq \omega(t) \leq t$$

Example: $\dot{x}(t) + b\dot{x}(t-r) + ax(t) = 0$

$$a>0, \ |b|<1$$

Use Liapunov function $V(\varphi) = \varphi^2(0) + \frac{1}{a}\int_{-r}^{0} \dot{\varphi}^2(\theta)\,d\theta$

Find

$$\dot{V}(x_t) = -ax^2(t) - \frac{1-b^2}{a}\dot{x}^2(t-r) \leq 0 \qquad x_t(\theta) = x(t+\theta)$$

$$\Rightarrow x(t) \to 0 \text{ as } t \to \infty$$

~~Let~~ stuck on

$$x(t) - ax(t-\sqrt{2}) - bx(t-1) = 0$$

NUCLEAR STRUCTURE PHYSICS 10/1/68 LECTURE

Biedenharn

(M 2, Th 1)

Postulates

1.) Shell model works (but don't know why)

Outline

1.) Review of Angular Momentum Techniques 1

" Basic Shell Model

2.) Few Particle Problem (outside closed shell)

Theory of pairing, seniority, C.F.P., quasi-spin (BCS) 4

Empirical Data (Bohr-Mottelson lectures

Generalized quasi-spin

3.) Energy Spectra (Talmi) 2

4.) Collective Aspects 4

Deformed nuclei, Nilsson orbitals, Elliott model (SU_3), Empirical Data

5.) Isospin in Nuclear Physics (IAS) 4

6.) Survey Nuclear Reaction Theory 5

ANGULAR MOMENTUM IN THE SHELL THEORY   10/3/68 LECTURE

Why? ① physical
② deals with finite sets [shell model as mapping from a continuum to a discrete set]

I. Usual Treatment [Born-Jordan]

Heisenberg: $[p_i, x_j] = -i\hbar\delta_{ij}$

Classical mechanics: $\vec{L} = \vec{r} \times \vec{p}$

Hence $[L_i, L_j] = i\epsilon_{ijk} L_k$ as commutation relations for angular momentum

Now problem is to construct hermitian matrices $\vec{L}$ which satisfies such relations.
Introduced raising and lowering operators

$$L_\pm = L_x \pm i L_y$$

Answer: $\vec{J}^2 \to j(j+1)$ $\quad j = 0, \frac{1}{2}, 1, \dots$

$J_z \to m$ $\quad -j \le m \le j$

Why did spin ½ appear?

Real Form: $[J_+, J_-] = 2J_0$

$[J_0, J_\pm] = \pm J_\pm$

P. Jordan ZfP (1933?): Found he could make a model;

Model: use bosons or fermions (creation & destruction operators)

Boson: $[a_i, a_j^*]_{(-)} = \delta_{ij}$

Fermions $[f_i, f_j^*]_{(+)} = \delta_{ij}$

Now Schwinger (1952), "On Angular Momentum": made a mapping of angular momentum theory into the bosons.

Boson: a_i^* $\quad i = 1, 2$

Model: $J_+ \to a_1^* a_2$ $\qquad$ [a representation]

$$(J_+)^\dagger = J_- \to a_2^* a_1$$

Note

$$[J_+, J_-] \to [a_1^* a_2, a_2^* a_1] = a_1^* a_1 - a_2^* a_2 \to 2J_0$$

Now can show $[J_0, J_\pm] = J_\pm$

Model states: $|jm\rangle = N(a_1^*)^{j+m} (a^*)^{j-m} |0\rangle$

$$N = [(j+m)!(j-m)!]^{-1/2}$$

Now

$$J_z \to \tfrac{1}{2}(a_1^* a_1 - a_2^* a_2) \, |N(a_1^*)^{j+m} (a_2^*)^{j-m} |0\rangle \to m$$

$$\vec{J}^2 \to \tfrac{1}{2}(a_2^* a_1 a_1^* a_2 + a_1^* a_2 a_2^* a_1) + m^2 \to j(j+1)$$

Why does model work? Using bosons which are symmetric systems
To every "j" we associate 2j bosons

$$\underbrace{\overbrace{\boxed{1}\boxed{1}\boxed{1}\boxed{1}\boxed{1}\boxed{1}}^{j+m}\overbrace{\boxed{2}\boxed{2}\boxed{2}\boxed{2}\boxed{2}}^{j-m}}_{2j}$$

"Young frame"

Now use

$$|jm\rangle = \sqrt{\frac{2|m|!}{(j+m)!(j-m)!}}\,(a_1^* a_2^*)^{j-|m|}\;\big|\,|m|, m\rangle$$

$$= \eta\,(K_+)^{j-|m|}\;\big|\,|m|, m\rangle$$

$K_+ = a_1^* a_2^*$ $\qquad [K_+, K_-]_- = -2K_0$ $\Rightarrow$ quasispin operators

$K_- \equiv (K_+)^\dagger = a_1 a_2$

$[\vec{K}, J_z] = 0$

Antisymmetric Pairs of Bosons

Two sets of bosons $\{a_i\}, \{b_i\}$

$J_+ \to a_1^* a_2 + b_1^* b_2$

Define $a_{12}^* = (a_1^* b_2^* - a_2^* b_1^*) = \begin{vmatrix} a_1^* & a_2^* \\ b_1^* & b_2^* \end{vmatrix}$

$N = a_1^* a_1 + a_2^* a_2 + b_1^* b_1 + b_2^* b_2$

Angular Momentum

One way or another we can generate the ang. mom. matrices. Now rotations become

$$R = e^{-i\Theta \hat{n} \cdot \vec{J}_{op}}$$

Now

$$(|jm\rangle)' = R(\alpha\beta\gamma)\,|jm\rangle$$

$$= \sum_{j'm'} |j'm'\rangle\langle j'm'|\, R(\alpha\beta\gamma)\,|jm\rangle$$

$$= \sum_{m'} |jm'\rangle \underbrace{\langle jm'|\, R(\alpha\beta\gamma)\,|jm\rangle}_{\mathcal{D}^{(j)}_{m'm}(\alpha\beta\gamma)}$$

① unitary (conserves prob.)

② $R(\alpha\beta\gamma)\,R(\alpha'\beta'\gamma') = R(\alpha''\beta''\gamma'')$

Tensor Operators T_k^q

$$[J_i, T_k^q] = \sum_{q'} \langle kq'|J_i|kq\rangle\, T_k^{q'}$$ infinitesimal form

Can find

$$R_{op}(\alpha\beta\gamma)\left(T_k^q\right) = \sum_{q'} \mathcal{D}^{(k)}_{q'q}(\alpha\beta\gamma)\; T_k^{q'}$$

or

$$R_{op}(\alpha\beta\gamma)\left(|kq\rangle\right) = \sum \mathcal{D}(\alpha\beta\gamma)\,|kq\rangle$$

Wigner Eckart Theorem: If T_k^q is a tensor operator, then

$$\langle \alpha' j' m' | T_k^q | \alpha j m \rangle = \langle \alpha' j' \| T_k \| \alpha j \rangle \, C^{j\,k\,j'}_{m\,q\,m'}$$

Physics 236a
SPECIAL RELATIVITY
Fall Term, 1968

Kip S. Thorne
California Institute of Technology
October, 1968

SPECIAL RELATIVITY - OUTLINE

I. Elementary, Physical Introduction

(Objective: To develop physical intuition for special relativity, comparable to one's intuition for Newtonian mechanics.)

1. Inertial reference frames (TW-1.2; MTW-1.1)
2. The fundamental principle of special relativity (TW-1.3; MTW-1.3)
3. Events, coordinate systems, and observers (TW-1.4)
4. Invariance of the interval (TW-1.1, 1.5; MTW-1.3)
5. Simple Lorentz transformations (TW-1.8)
6. Spacetime diagrams (TW-1.6,1.7,1-exercise 48; MTW-1.6)
7. Fundamental phenomena of special relativity (TW-exercises of Chapter 1; MTW-1.6)
   - a. Simultaneity and the temporal order of events
   - b. Lorentz contraction
   - c. Time dilation
   - d. Rotation of rods
   - e. Rotation of directions of motion
   - f. The headlight effect
   - g. Doppler shift
8. Paradoxes (TW-exercises of Chapter 1)
   - a. Twin paradox
   - b. Pole-in-the-barn paradox
   - c. Space-war paradox
   - d. Thin-man-and-the-grid paradox
9. Accelerated observers

II Mathematical Formulation of Special Relativity

1. Tensor algebra (MTW-1.4)
2. Lorentz transformations (MTW-1.5)

III. Applications of Special Relativity

1. Particle kinematics (MTW-1.7, 1.8)

2. Hydrodynamics; the stress-energy tensor (MTW-1.9)
3. Conservation laws (MTW-1.10,1.11)
4. Angular momentum and spin (S-ST-7.5)
5. Statistical physics (no reference)
6. Electromagnetic theory (no reference)
7. Electromagnetic field interacting with charged perfect fluid (MTW-1.12)

IV. <u>Accelerated Observers in Special Relativity</u>

1. The observer's tetrad and measurements (MTW-1.13, 1.14).
2. Confinement of measurements to observer's world line (MTW-1.15)
3. Pound-Rebca experiment (MTW-1.16)

Physics 236

RELATIVITY
1968-69

Kip S. Thorne
California Institute of Technology
October 1968

PRELIMINARY COURSE OUTLINE

I. Special Relativity (Fall term; about 7 weeks)
Texts - TW; MTW Chapter 1.

II. Differential Geometry (Fall term; about 3 weeks)
Text - MTW Chapter 2.

III. Fundamentals of General Relativity (Winter term)
Text - MTW Chapters 3-8.

IV. Specialized Topics in General Relativity (Spring term)
Text - MTW Chapters 9 ff.

TEXTBOOKS (available from bookstore after October 6)

1. TW: E.F. Taylor and J.A. Wheeler, Spacetime Physics (W.H. Freeman and Company, San Francisco, 1966).

2.MTW: C.W. Misner, K.S. Thorne, J.A. Wheeler, General Relativity (Typewritten Manuscript, 1968-69).

GENERAL REFERENCES (On reserve in Millikan Library)

1. Tol. R.C. Tolman, Relativity, Thermodynamics, and Cosmology (Clarendon Press, Oxford, 1934).

2. WTh: J.A. Wheeler, Relativity (lectures delivered in the Princeton University graduate course, Physics 569-570, 1964-65); notes by K.S. Thorne.

3.RSSD: K.S. Thorne, "Relativistic Stellar Structure and Dynamics", 2nd half of vol. 3 of High Energy Astrophysics (Gordon & Breach, New York, 1967).

4. LL: Landau & Lifschitz, Classical Theory of Fields (Addison-Wesley, Reading, 1962).

5.S-ST: Synge, J.L., Relativity-The Special Theory (North Holland, Amsterdam, 1965).

6.S-GT: Synge, J.L., Relativity-The General Theory (North Holland, Amsterdam, 1960).

7. And: Anderson, J.L., Principles of Relativity Physics (Academic Press, New York, 1967).

8. RN : Robertson, H.P., and Noonan, T.W., Relativity and Cosmology (W.B. Saunders, Philadelphia, 1968).

9. ABS: Adler, R., Bazin, M., Schiffer, M., Introduction to General Relativity (McGraw-Hill, New York, 1965).

Ph 236 RELATIVITY KIP THORNE 10/1/68 LECTURE

Inertial Reference Frame: A reference frame is inertial if, relative to it, every test particle released from rest remains at rest, and every test particle initially in motion continues that motion without change in velocity.

Principle of Relativity

(1) All the laws of physics are the same in every inertial reference frame

(2) The laws of physics cannot provide a way to distinguish one inertial reference frame from another.

Comments

i) observer ≡ inertial ref. frame

ii.) inertial ref. frame is impossible to realize because of gravitational inhomogeneity (tidal gravitational forces). However we shall use it as a construct which frequently is a good approximation in a local region

iii.) General relativity takes into account this inhomogeneity -- that is, includes inhomogeneous gravitational fields. This is the only physical difference.

iv.) Another difference is in geometry

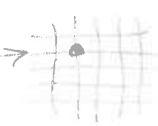
a.) special relativity: "flat" geometry $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$

b.) general relativity: curvature of spacetime

Introduction to SRT 10/2/68 LECTURE

i.) Events
ii.) Invariance of the interval
a.) $\perp$ distances
b.) Δs^2
iii.) Lorentz transformations
iv.) Space-time diagrams

Homework [Oct. 2-14]
1.) Prove invariance of $(\Delta s)^2$ for spacelike intervals
2.) Taylor & Wheeler, Ch. 1, 18, 19, 21, 22, 25, 26, 33-36, 44-46, 48, 50-54

Event: A point in space-time (x^1, x^2, x^3, t) 
In a different inertial frame this event will have different coordinates $(x^{1'}, x^{2'}, x^{3'}, t')$

Invariance of $\perp$ distances: principle of relativity demands this invariance since otherwise we would have a means to distinguish inertial frames.

Invariance of $(\Delta s)^2$: Consider 2 events

S: ① (t_1, x_1, y_1, z_1) ② (t_2, x_2, y_2, z_2) $\Big\}$ $(\Delta\tau)^2 \equiv -(\Delta s)^2 \equiv c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$

$\|$

S': ① (t_1', x_1', y_1', z_1') ② (t_2', x_2', y_2', z_2') $\Big\}$ $(\Delta\tau')^2 = -(\Delta s')^2 \equiv c^2(\Delta t')^2 - (\Delta x')^2 - (\Delta y')^2 - (\Delta z')^2$

Prove this for time like interval: $(\Delta\tau)^2 > 0$
[also null or light like $(\Delta\tau)^2 = 0$
space like interval $(\Delta\tau)^2 < 0$ or $(\Delta s)^2 > 0$]

For null like, since speed of light must be constant

$$\frac{\sqrt{(\Delta r)^2}}{\Delta t} = c = \frac{\sqrt{(\Delta r')^2}}{\Delta t'}$$