Active Safety Conflict Modeling (ASCOM)

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16. Abstract

A modeling tool is described for studying the nature and relative frequency of driving conflicts that are likely to be encountered when the innovation called Adaptive (or intelligent) Cruise Control (ACC) becomes a common automotive option. ACC constitutes an enhancement to a standard cruise control system which allows the subject vehicle to adapt to the speed of a preceding vehicle at an appropriate distance by controlling the engine and/or power train/ and/or brake. Thus, ACC-assisted drivers can merely set a desired speed and then permit the system to deal with the conflicts in longitudinal separation that arise when other vehicles are approached. However many design and operating variables can influence the quality of the conflict-resolution processes. It is further recognized that all the operating variables appear as stochastic variables each with its characteristic distribution and, perhaps, correlation to other variables. Thus, all forms of encroachment conflict are probabilistic in their manifestation throughout ones driving experience.

The stochastic nature of the problem weighs heavily on the configuration of a suitable modeling tool for assessing the occurrence and severity of conflicts likely to be encountered when driving under ACC control. Both highway geometric variables, distributed spatially along the road system, and inter-vehicular kinematic variables, distributed temporally across all driving exposure, determine the primary conditions under which the ACC system must manage encroachment conflicts. The conflicts, themselves, are ultimately of concern insofar as they pose at least a psychological stress and perhaps even a crash threat for the ACC operator or others in the traffic stream.

A so-called Active Safety Conflict Model (ASCOM) is presented as a design tool for predicting a variety of first-order conflicts expected to arise with ACC usage in normal traffic. This report addresses the background for such modeling followed by an overview of the ASCOM model structure and a description of a statistical regression procedure to reduce the computational cost associated with a Monte Carlo procedure. Empirical data available for representing the distributions of the condition variables are presented as are results from exercising ASCOM in two conflict scenarios.

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Active Safety Conflict Modeling (ASCOM)

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SUMMARY

A modeling tool is described for studying the nature and relative frequency of driving conflicts that are likely to be encountered when the innovation called Adaptive (or intelligent) Cruise Control (ACC) becomes a common automotive option. As defined in draft form by Working Group 14 of the ISO/TC204 Committee, ACC constitutes "an enhancement to a standard cruise control system which allows the subject vehicle to adapt to the speed of a preceding vehicle at an appropriate distance by controlling the engine and/or power train/ and/or brake...". Thus, ACC-assisted drivers can merely set a desired speed and then permit the system to deal with the conflicts in longitudinal separation that arise when other vehicles are approached.

This report describes the background for such modeling followed by an overview of the ASCOM model structure and a description of a statistical regression procedure to reduce the computational cost associated with a Monte Carlo procedure. Empirical data available for representing the distributions of the condition variables are presented as are results from exercising the model.

Monte Carlo model results are included for two conflict scenarios (closure from long range and lead vehicle decelerating) using highway speed and road geometry distributions from the states of Michigan and West Virginia. These results show the relative probability of conflict for the same ACC system using the two topographical descriptions for these two states. Additionally, the relative probability of conflict after implementing an automatic-transmission downshift, for more deceleration authority, is also presented.

To accelerate the computational turnaround regression-type "results modeling" and intelligent subsetting of the condition variables, are used in the model. Encouraging results were obtained with respect to the accuracy and efficiency gained from the regression fitting approach. Effort is required to estimate the best predictor in a design for accuracy as well as efficiency and future work is also required, of course, to implement each of the remaining four scenarios and to locate or develop empirical data sets as needed for representing the conflict phenomena in the longitudinal and lateral realm of highway driving.

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INTRODUCTION

A modeling tool is described for studying the nature and relative frequency of driving conflicts that are likely to be encountered when the innovation called Adaptive (or intelligent) Cruise Control (ACC) becomes a common automotive option. Virtually every major auto manufacturer in the world appears to be currently developing such a system, with products already available in Japan and market introduction expected elsewhere during the 1998 through 2003 time frame. As defined in draft form by Working Group 14 of the ISO/TC204 Committee (i.e., the international body currently working to develop an ACC standard), ACC constitutes "an enhancement to a standard cruise control system which allows the subject vehicle to adapt to the speed of a preceding vehicle at an appropriate distance by controlling the engine and/or power train/ and/or brake...". (Draft N61.2) Thus, ACC-assisted drivers can merely set a desired speed and then permit the system to deal with the conflicts in longitudinal separation that arise when other vehicles are approached. However many design and operating variables can influence the quality of the conflict-resolution processes. It is further recognized that all the operating variables appear as stochastic variables—each with its characteristic distribution and, perhaps, correlation to other variables. Thus, all forms of encroachment conflict are probabilistic in their manifestation throughout one's driving experience.

The stochastic nature of the problem weighs heavily on the configuration of a suitable modeling tool for assessing the occurrence and severity of conflicts likely to be encountered when driving under ACC control. Both highway geometric variables, distributed spatially along the road system, and inter-vehicular kinematic variables, distributed temporally across all driving exposure, determine the primary "conditions" under which the ACC system must manage encroachment conflicts. The conflicts, themselves, are ultimately of concern insofar as they pose at least a psychological stress and perhaps even a crash threat for the ACC operator or others in the traffic stream. Model-based assessment of such conflicts should help expedite the development of truly road-worthy ACC system designs while laying the basis for some portion of the safety-related specifications and standards.

A so-called "Active Safety Conflict Model" (ASCOM) is presented as a design tool for predicting a variety of first-order conflicts expected to arise with ACC usage in normal traffic. This report addresses the background for such modeling followed by an overview of the ASCOM model structure and a description of a statistical regression procedure to reduce the computational cost associated with a Monte Carlo procedure. Empirical data available for representing the distributions of the condition variables are presented as are results from exercising ASCOM in two conflict scenarios.

BACKGROUND

Monte Carlo simulations of rear-end crash warning and avoidance systems have been reported which predict the probabilistic occurrence of conflicts that merit a warning or control intervention. Farber has reported modeling a single scenario in which the following vehicle approaches a braking or stopped vehicle ahead (Farber, 1994). Wilson has reported the development of a model that includes a complex description of a rear-end crash avoidance system, affording the ability to study the basis for detailed performance specifications (Wilson, 1995). While both of these efforts have addressed the longitudinal domain of motion conflict, as we have, our work introduces a multiplicity of conflict scenarios, adds functional constraints imposed by roadway geometry, introduces regression modeling, and applies the conflict analysis to the context of ACC rather than collision warning and avoidance systems. Also, because ACC constitutes an automatic function on board the host vehicle, the host driver is not modeled in our work while the driver's response delays are central in the referenced studies.

Additional literature using Monte Carlo simulation has been produced by Young (1995). His work reported simulation of a lane-change warning system in which other vehicles are characterized by their ability to block the lane change actions of the host vehicle. Young's simulation was intended for supporting the development of performance specifications and represents the distributions of speed and gap conflict variables by derivation from police-reported crash data.

The work reported here strikes a similar theme of combined deterministic and probabilistic modeling, but in a design tool that embraces many conflict types actually seen from field studies using ACC prototype vehicles. Another difference of the work reported here is the use of a regression model to replace the dynamic model. This increases the efficiency of ASCOM as a design tool. The goal is to predict the conflict experience of a driver over the vehicle's useful lifetime, as a function of ACC designs.

ASCOM: A FUNCTIONAL DESCRIPTION

Modeling encroachment conflicts requires consideration of the different aspects of the driving environment. In addition to representing the dynamic and kinematic characteristics of the host vehicle, there is a need to consider the external influences, such as road grade and curvature. The major elements of ASCOM are shown in figure 1 including (1) the dynamic system model of the host vehicle and ACC system, (2) the definition of common conflict scenarios, (3) the predictor consisting of a Monte Carlo simulation model and a regression model, and (4) the effects of road geometry and traffic data on the vehicle model.

The Dynamic System Model

As discussed in the introduction, ACC can be thought of as an enhancement to the conventional cruise control system. By sensing the presence and measuring the range of a leading vehicle, the ACC can change the host vehicle speed to match the speed of the leading vehicle at a specified following distance (also referred to as the desired headway).

The elements and control strategy of one example system that has been implemented in this tool are shown in figure 2. These elements include (1) a Sensor, (2) a Headway Control Unit (HCU), (3) a Conventional Cruise Control (CCC) unit, (4) a powertrain model of the host vehicle, and (5) the vehicle dynamics of the host vehicle. The computational details of these elements are discussed in the subsequent sections.

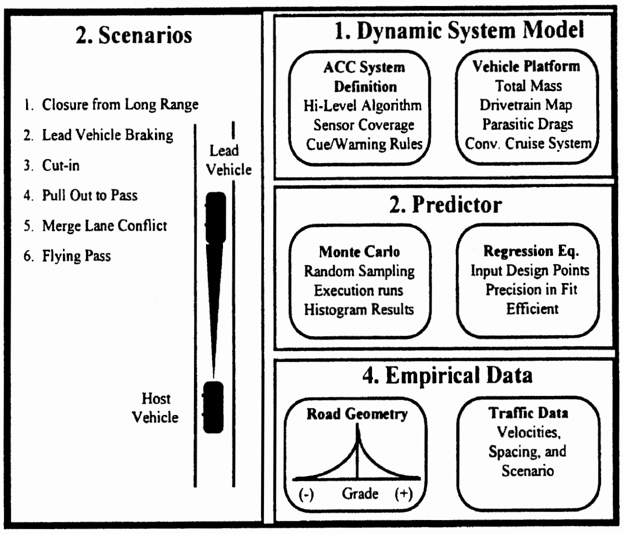


Figure 1. Overview of the ASCOM Environment

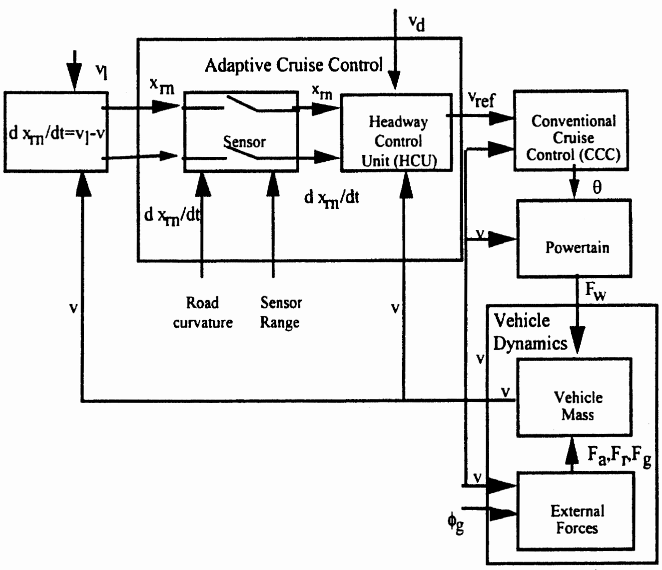


Figure 2. Diagram of the Dynamic System Model

The control sequence originates in the front mounted sensor of the host vehicle. The sensor is modeled firstly as a switch which is "thrown" if a lead vehicle is within its active sensing region. When a leading vehicle is present, the sensor measures the range, x_m, and range rate, dx_m/dt, relative to the leading vehicle. The HCU uses this information, along with the host velocity, v, and the driver's desired speed, v_d, to determine a reference velocity, v_{ref}, that meets the control objective of the ACC[1]. The conventional cruise control compares v to v_{ref} and adjusts the throttle position, θ, so as to minimize the difference between v and v_{ref}. As the throttle position changes, the engine and the associated powertrain effects are calculated and reduced to a representative force, F_w, that manifests itself at the vehicle wheels. This force along with the forces caused by grade, F_g, aerodynamic drag, F_a, and tire rolling resistance, F_r, are used in a vehicle dynamics calculation to yield a new host velocity.

Model of the Adaptive Cruise Control

An ACC system can be considered for purposes of predicting conflict kinematics as being composed of two major components: the Sensor and the HCU. Various sensors and control strategies are currently being modeled and tested. ASCOM has been designed such that the analysis of different ACC systems is possible. For the example presented in this report, we chose a design based on results of previous research conducted by Fancher (1994) at the University of Michigan Transportation Research Institute (UMTRI) in collaboration with Leica AG. The following discussion outlines our modeling of this particular ACC system.

Sensor

The ACC sensor is modeled to cover a fixed area in front of the host vehicle. This fixed area of coverage is referred to as the sensor view volume and is defined by an elevation angle, azimuth angle, and maximum sensor range. To account for road grade and curvature, the sensor algorithm performs a coordinate shift and rotation to calculate the elevation and azimuth angle between the host and the vehicle ahead. These angles are then compared to the corresponding sensor coverage arcs to determine if the lead vehicle is within the sensor view volume. A target is called active if the position of the lead vehicle lies within the sensor view volume, and the lead vehicle is traveling at a speed less than that of the host vehicle speed.

[1]Details of the ACC control objective and the determination of v_{ref} are discussed briefly in the section labeled "Model of the Adaptive Cruise Control". For complete details, see Fancher (1994).

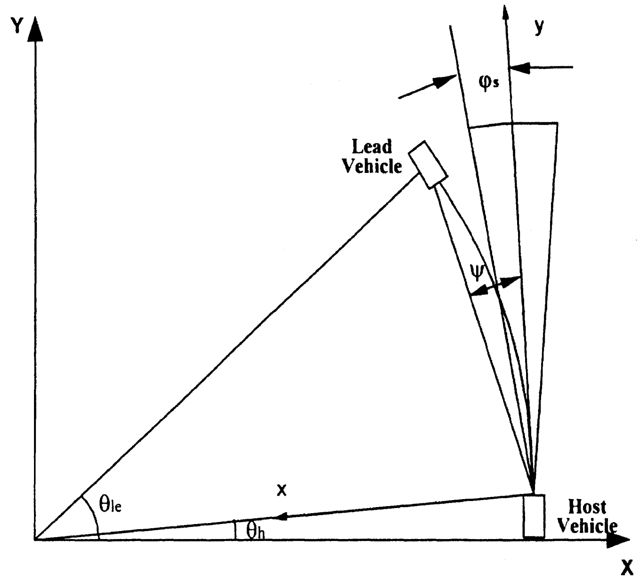


Figure 3. Lead Vehicle Acquisition Diagram

Figure 3 shows the lead and host vehicles along with a representation of the sensor view area in two dimensions. The lead vehicle will be acquired once it is inside the coverage arc, that is, the sensor angle, ϕ_s, is greater than the angle ψ shown in figure 3. Using simple geometric relations, the angle ψ is found to be, $\psi=(\theta_{le} - \theta_h)/2$, where θ_{le} and θ_h are determined by vehicle positions along defined tangent and constant radius paths (with no sideslip).

Headway Control Unit

The currently implemented HCU algorithm is restricted to administering throttle position adjustments only (i.e., the current control strategy relies only on the deceleration authority provided by a throttle-off condition). A feature for commanding forced transmission-gear downshifts is also implemented in the model. In order to describe the system's headway-control function and vehicle response under ACC control, plots of range-vs.-range rate relationships are employed (Fancher, 1994). Figure 4 shows a simplified view of the currently implemented HCU logic.

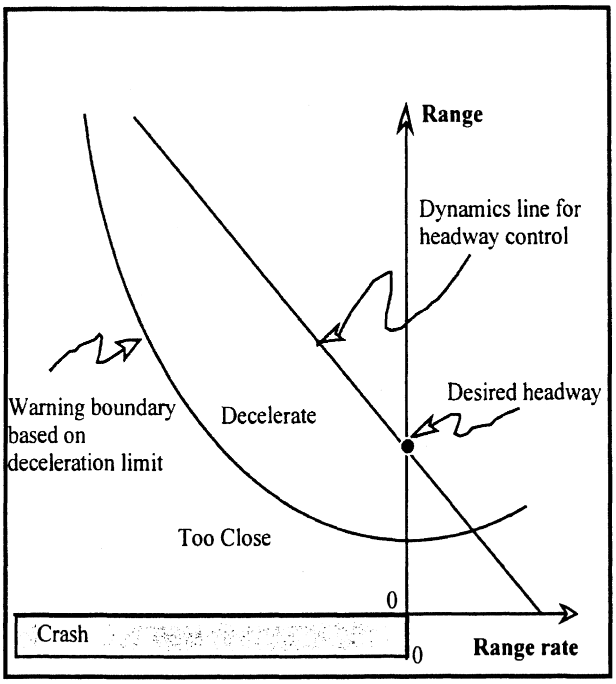


Figure 4. Range vs. Range Rate Diagram with Switching Logic

Each point in the diagrammed space consists of an ordinate value representing the range between the host and lead vehicles and an abscissa value representing the rate of closure between the vehicles. The ordinate intercept labeled *desired headway* corresponds to the range value at which the host vehicle is commanded to follow the target vehicle when the system is in a steady-state following mode:

$$x_d = v_l t_{ref} \quad (1)$$

where: x_d is the desired headway range,

v_l is the speed of the lead vehicle, and

t_{ref} is the reference headway time.

Note: x_d changes continually with lead-vehicle speed.

The line passing through the desired headway (labeled the *dynamics line*) is defined as:

$$x_d = x_m + \tau_c(dx_m/dt) \quad (2)$$

where: x_m is the range, and

τ_c is the slope of the *dynamics line* (derived from the level of deceleration authority available for headway control).

Note the following in figure 4:

1. In the first quadrant, the target vehicle has a greater speed than the host vehicle, such that no conflict develops.

2. In the third and fourth quadrants, the host and lead vehicles have collided.

3. The second quadrant is divided by the dynamics line. When the host vehicle is operating at points above this line, the HCU determines that there is no conflict (allowing the conventional cruise control to maintain the present set-speed.) At points below the dynamics line, the HCU identifies the conflict and begins to command a new reference-speed which would lead toward a response that follows the dynamics line down to x_d (i.e., the HCU attempts to satisfy equation 2 by typically decreasing the throttle angle and perhaps downshifting the transmission.)

4. Although not implemented in the example calculation, a warning boundary line may easily serve to indicate a region of the range-rate curve that may require driver intervention or a warning of an impending collision.

Model of the Conventional Cruise Control (CCC)

The CCC algorithm used in this work seeks to achieve a reference vehicle speed, v_{ref}, by manipulating the throttle position, θ. The implemented method uses a proportional plus integral linear control strategy given by:

$$\theta = K_P \, \varepsilon + K_I \qquad (3)$$

$$\varepsilon = v_{ref} - v \qquad (4)$$

where: θ is the throttle position,

v_{ref} is the reference speed as determined by the HCU,

K_P is the proportioning gain of the system,

K_I is the integration gain of the system, and

ε is the velocity error.

Model of the Powertrain

The ACC either commands a throttle position determined by the engagement rules of the conflict avoidance algorithm or allows the conventional controller to command throttle input aimed at maintaining the original set–speed. This information is input into the powertrain algorithm to produce the required forces on the body to accelerate or aid in decelerating the host vehicle. A simplified powertrain model has been implemented and is described in block form in figure 5.

Among the many simplifying assumptions, a few of the most notable include:

1. The engine speed used to determine the current engine torque, ω_0, is taken from the previous simulation step.

2. The torque delivered from the torque converter is transmitted directly to the tires after including the proper gear ratio, r, and torque efficiency loss, c.

3. A means for calculating the output speed of the torque converter, ω_t, (see the Appendix).

4. The remainder of this section briefly describes the way in which the powertrain response is calculated. The engine torque, T_e, is a function of the engine speed, ω_0, (taken from the previous simulation time step) and the throttle position, θ, commanded either from the ACC or the CCC (see figure A-3 in the Appendix).

$$T_e = f(\theta, \omega_0) \tag{5}$$

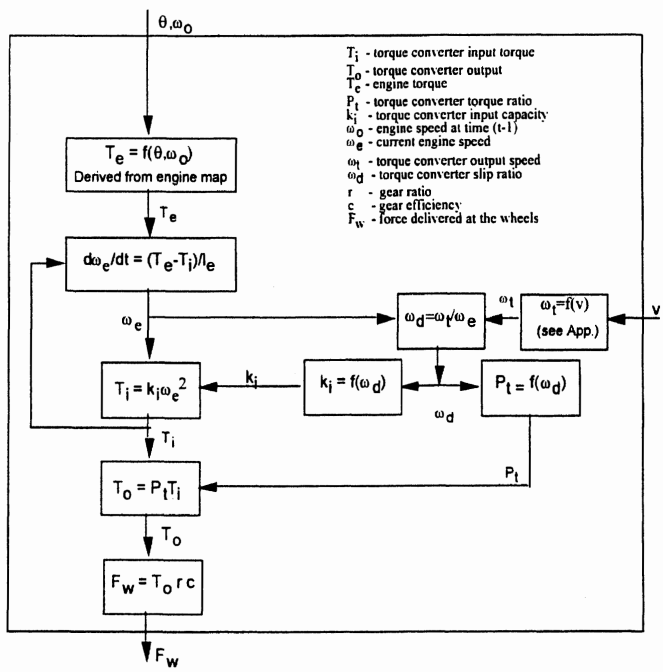


Figure 5. Powertrain Block Diagram

The engine torque, T_e, and input torque from the torque converter, T_i, are used to calculate the current engine acceleration (which in turn is integrated to compute the engine speed):

$$(d\omega_e/dt) = (T_e - T_i)/I_e \tag{6}$$

where: I_e is the engine rotational moment of inertia plus one half of the torque converter inertia, and

ω_e is the engine speed.

The torque converter input torque, T_i, is characterized by the torque converter input capacity, k_i, and the torque converter torque ratio, P_t (these functions are represented as tables A-3 and A-4 in the Appendix):

$$k_i = f_1(\omega_t/\omega_e) \tag{7}$$

$$P_t = f_2(\omega_t/\omega_e) \tag{8}$$

where: ω_t is the torque converter input speed.

Using k_i and P_t, the input and output torque are given by:

$$T_i = k_i\,\omega_e^2 \tag{9}$$

$$T_o = P_t T_i \tag{10}$$

The force delivered to the wheel from the drivetrain, F_w, is proportional to the output torque, T_o, a gear ratio, r, and losses due to gear efficiencies, c.

$$F_w = T_o\, r\, c \tag{11}$$

Model of the Vehicle Dynamics

The required dynamic behavior of the vehicle model depends on the ACC conflict being studied. Although a single robust model would be sufficient for all scenarios, in order to be practically useful, the ASCOM tool requires each simulation to be resolved in a fraction of real-time. Therefore, in order to maximize execution speed, the alternative types of ACC conflicts are each studied by means of vehicle models that are matched to the conflict in question. In order to facilitate the generation and integration of several models into the ASCOM software environment, an off-the-shelf modeling tool called AUTOSIM (Sayers, 1989) was used to generate the equations of motion.

The example ACC conflict scenarios that are presented here employ a model that is confined to longitudinal motion dynamics. The lateral and vertical aspects of motion induced by road geometries are dealt with only in terms of the sensor coverage along the curves. This model is represented mathematically by three ordinary differential equations that describe its kinematic and dynamic behavior. It is composed of a single point mass and has two degrees-of-freedom (one translational along the roadway and one rotational motion within the powertrain.) The forces acting to accelerate (and decelerate) the vehicle include those resulting from the powertrain dynamics, road grade, aerodynamic drag, and tire rolling resistance. The model is given by

$$dv/dt = 1/m(F_a + F_g + F_r + F_w) \quad (12)$$

where: F_a is the aerodynamic drag,

F_g is the grade force,

F_r is the rolling resistance,

F_w is the powertrain force,

v is the host vehicle velocity, and

m is the host vehicle mass.

The aerodynamic forces are given by,

$$F_a = -0.5\, C_D\, A \rho v^2 \quad (13)$$

where: A is the aerodynamic cross-section of the vehicle,

ρ is the air density, and

C_D is the drag coefficient.

Rolling resistance force, F_r, is assumed to be a function of vehicle speed (while also a function of several other variables, including tire load, which is assumed to be constant during the simulation). In the model this force is represented as a bilinear function of vehicle speed (table A-2 in the Appendix) for a midsize vehicle.

The forces resulting from road grades are simulated by changing the direction of the gravity vector to include the appropriate component parallel to the ground,

$$F_g = mg(\sin\phi_g) \quad (14)$$

where: ϕ_g is the grade angle between the horizontal plane and the road.

CONFLICT SCENARIOS

A conflict scenario is defined as a stereotypical interaction between an ACC-equipped vehicle and one or more others that (1) is provoked by conditions and/or driving behaviors normally appearing in the traffic environment, (2) is influenced by the ACC functionality to some degree, and (3) can pose an encroachment conflict for either the ACC driver or another nearby driver. Six such scenarios have been identified to date based upon driving experience with ACC prototypes in everyday freeway traffic in

Michigan (Sayer, 1995). Among the six, the first three have been implemented in the ASCOM simulation tool to date, and are described below in terms of a defined set of influential condition variables. The second three are known to exist based upon field experience but have not yet been codified within the model and involve a more speculative set of condition variables. The scenarios are as follows:

- *Closure from Long Range* - With the host vehicle approaching a slower-moving vehicle ahead, the ensuing headway closure may require a deceleration level and/or an extent of headway time that exceeds the system's ability to control. Conflict is in the form of a potential undershoot beyond the intended headway time and is influenced by roadway grade, horizontal curvature, the initial velocity of the ACC vehicle and the velocity difference between the two vehicles.
- *Lead Vehicle Braking* - With the host vehicle initially traveling at a steady and relatively short value of controlled headway, the preceding vehicle applies its brakes.. Conflict is characterized by an undershoot of the intended headway time. Although a range value of zero or less would imply a crash, it is believed that the results speak more to the severity and frequency of driver brake intervention events than to crash issues, per se. The influential conditions include road grade, initial velocity of the vehicle pair, and the severity/duration of deceleration by the preceding vehicle.
- *Cut In* - From an initial condition of steady speed keeping, a vehicle cuts in from the side at constant lateral and longitudinal velocity, terminating its lateral excursion upon reaching the center of the lane ahead of the host vehicle. The intensity of headway conflict is influenced by the road grade, the initial velocity of the ACC vehicle, and the initial range and range-rate values presented by the cut-in vehicle.
- *Merge-Lane Conflict* - With the host vehicle traveling in the right-hand through lane, another vehicle approaches along an entrance ramp and merges within the fixed location of the ramp terminus. While this conflict has not been fully defined to present, it is presumed to be influenced by the forward speed, rate of lateral movement, and point of through lane entry by the merging vehicle, the available length of the merge zone, the prevailing grade, and the initial speed of the host vehicle.
- *Flying Pass* - While in the process of approaching a slower-moving vehicle, the host driver steers into the left in an attempt to pass without slowing down. This scenario superimposes a passing transient onto the process of approaching—a sequence in which the ACC system may begin to decelerate, thus disrupting the rhythm of the intended flying pass. Performance is influenced by host vehicle speed, closing speed, the range at which passing commences, the lateral rate of passing movement, the grade and the road curvature.

- *Pull Out To Pass* - The host vehicle terminates a headway keeping episode by pulling out to pass. A conflict is posed by the delay in reaching a free stream speed, following commencement of the pass. The condition variables include the initial speed, the rate at which the passing maneuver proceeds, the final speed, and the grade.

IMPLEMENTATION OF TWO CONFLICT SCENARIOS

Implementation of the closure from long range (CLR) and lead vehicle braking (LVB) scenarios is described here as background for results that are presented later in the paper. In both cases, implementation has covered the relatively straightforward computation of the longitudinal conflict dynamics, mechanization of the Monte Carlo selection of condition variables for setting up each of the individual conflict simulation runs, and computation of a measure of merit from the output time histories that are produced. The merit scores are then compiled as cumulative distribution functions (CDF) for presentation of results. Concerning conditions variables, empirically derived distributions from which the Monte Carlo sampling is accomplished are presented later in the paper. A synopsis of the implementation of the two indicated scenarios follows.

Closure From Long Range

The CLR scenario is initialized with the host vehicle approaching the slower-moving target vehicle, both at constant initial speeds. As the closure transient proceeds, the ACC vehicle decelerates in order to arrive at its desired headway range (defined in these calculations, as the product of the target vehicle speed and a constant value of headway time, $T_h = 1.5$ seconds). The initial conditions for the scenario are defined as:

$$Ra \leq x_{Target} - x_{Host} \leq Rs$$

$$\dot{x}_{Host} > \dot{x}_{Target}$$

where: Ra is the range at which the ACC control logic causes a change in the commanded velocity,

Rs is the maximum range of the ACC sensor ($Ra \leq Rs$),

x_{Target} is the initial position of the preceding, or target vehicle,

x_{Host} is the initial position of the host vehicle, and

the velocity of the host vehicle is greater than the velocity of the target vehicle.

Grade and curve radius are held constant throughout the simulation. The performance measure, or merit score, is defined as the ratio of the minimum value of range to the desired value of range (which, in turn, is based upon the speed of the preceding vehicle).

Lead Vehicle Braking

In the LVB scenario, the host vehicle is initially at a selected, close, value of headway time behind the preceding vehicle. Then the target vehicle stimulates a conflict by braking at a given level of deceleration for a prescribed period of time. Deceleration levels and durations are drawn from empirical data distributions, as is the initial velocity selection. Road grade is held fixed throughout each single simulation. As above, the merit score is defined as the ratio of the minimum value of range to the desired headway.

MEASURES OF PERFORMANCE

Obviously, many candidate measures of performance can be imagined for characterizing the encroachment conflicts arising with ACC. The two conflict scenarios addressed in this paper are scored by use of a single "merit" measure, defined as:

$$\text{Merit} = \frac{R_{min}}{R_d} \tag{15}$$

where: R_{min} is the minimum value of range between the host and target vehicles, in the course of the conflict transient (Both positive and negative values of R_{min} are observed. While negative values nominally denote a crash, the quantity simply depicts conflict severity and thus the intensity of the braking intervention that is required), and

R_d is the system-determined value of intended headway range.

Merit performance is thus interpreted as follows:

- merit < 0.0 indicates that driver intervention was required simply to avoid a crash.
- 0.0 < merit < 1.0 indicates closure inside of the desired headway range, calling for driver intervention on a discretionary basis.
- merit near 1.0 indicates that the ACC controller achieved an essentially conflict-free response.

EMPIRICAL DATA COVERING THE STATISTICALLY DISTRIBUTED VARIABLES

Implementation of a Monte Carlo simulation of any of the conflict scenarios requires that empirical data be available documenting the probability distribution of each of the condition variables mentioned above. These variables roughly divide into geometric properties of the roadway environment and kinematic variables depicting the motion of vehicles in real traffic. Admittedly, additional environmental variables pertaining to wind conditions, surface friction, atmospheric obscuration, and so on, may also impact upon ACC operations but are not addressed in the first-order conflict scenarios presented here. Figure 6 below presents distributions of the variables that have been employed in computing ACC conflicts arising during closure from long range and lead vehicle braking scenarios.

Taking the respective sub figures left to right from the top, empirical distributions and data representations are employed as follows:

- Vehicular speeds were selected from the files of the so-called "REAMACS" data edited from New Mexico freeway data (Farber, 1994). Individual velocity values, V, are employed as initial speed conditions in both the CLR and LVB scenarios.
- In the CLR scenario, each value of V for the host vehicle is paired with an individual value of V for the target vehicle, although only in the negative range and range rate cases do vehicles actually close on one another.
- The probability of braking deceleration levels as seen in the third and fourth sub-figures represent the individual brake applications exhibited by 36 subjects in moderate freeway traffic (Sayer, 1995). The sampled data are represented by a probability distribution of the normalized deceleration and normalized deceleration as a function of brake application (i.e., duration) time. Because both deceleration level and brake application time determine the velocity profile of the target vehicle, they both are included in the model in the form of deceleration level and time-duration pairs. The model randomly selects from the deceleration distribution and then associates the selected value with a corresponding duration to derive the velocity profile of the target vehicle. (Please note that the authors consider these data to be only a preliminary indication of the deceleration-duration relationship. These findings may have been skewed by the odd nature of the route that included four freeway-to-freeway interchanges or exits in a 50 mile drive. A more authoritative empirical basis for this relationship is currently being developed.)
- Highway geometry representing the states of Michigan (i.e., flat terrain) and West Virginia (i.e., hilly terrain) were drawn from the primary system mileages in the Highway Performance Monitoring System (HPMS) database compiled by the Federal Highway Administration. Distributions of road grade are presented with equal probability assumed for uphill and downhill.
- Horizontal curvature values in HPMS were also selected from the primary system mileages in the two indicated states. While the grade severity influences ACC performance because of the represented limitations in deceleration authority, the influence of curvature derives from non steered, monobeam sensing which acquires vehicular targets at shortened range values on tight-radius curves.

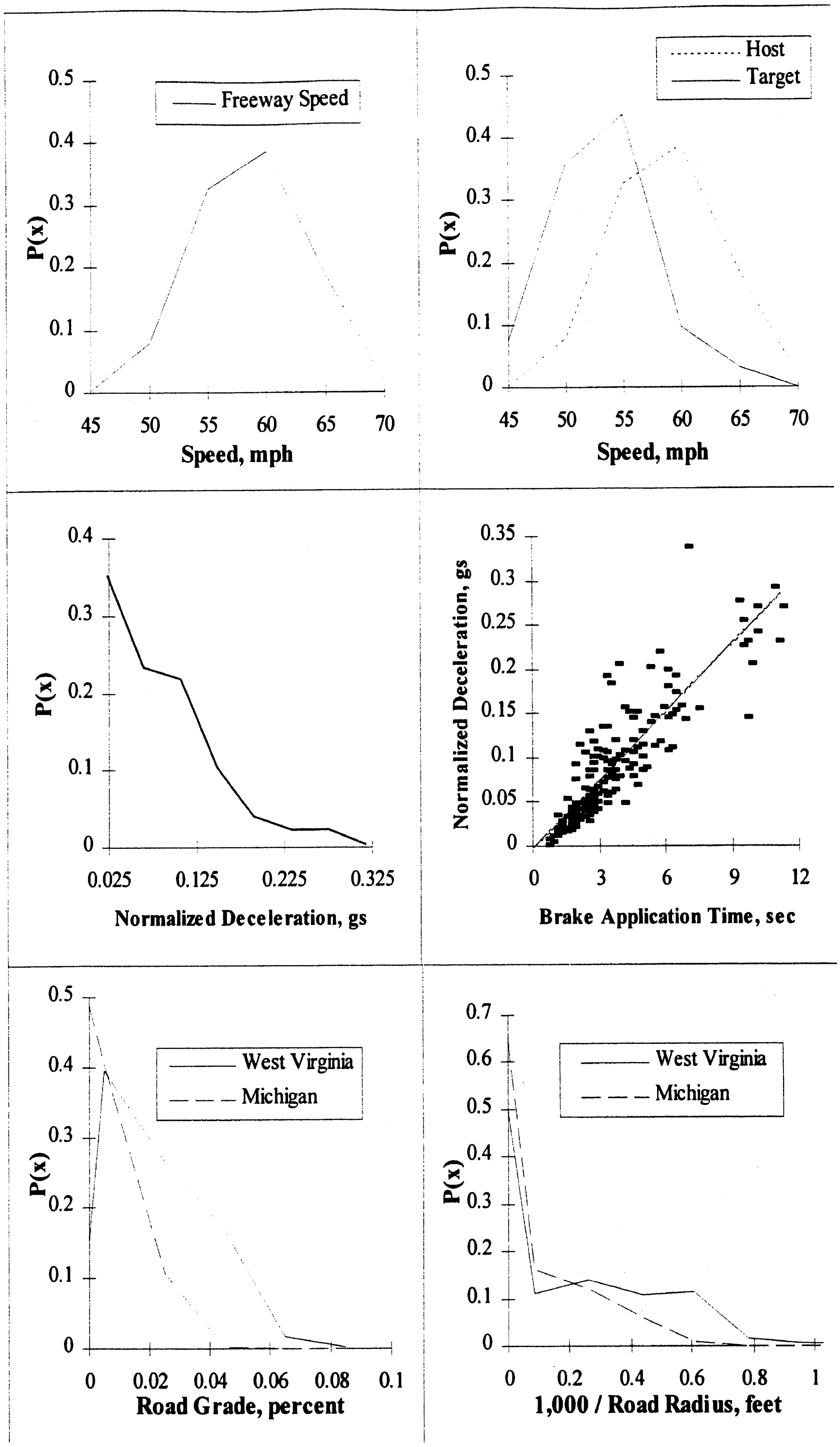


Figure 6. Empirical Data and Distributions

PREDICTORS: MODEL EXECUTION

Monte Carlo simulation

Monte Carlo simulation offers a classical means for predicting the cumulative or life-cycle experience of a physical system that will be operated under the joint probabilities of multiple condition variables and initialization values. The Monte Carlo method employed here assumes that the input variables are independent of one another. To compile a schedule of runs, random sampling is performed on empirically-measured distributions of each of the condition variables to create a set of N simulation runs. As the value of N grows, the results tend toward a representative distribution of the conflicts accruing over a long-term driving experience with the defined vehicle and ACC system. While more work is needed to calibrate ASCOM results relative to credible long-term driving exposures, run batches totalling a nominal N = 600 have been found to yield practicably asymptotic distributions of results within a single conflict scenario.

Monte Carlo Results for Closure from Long Range and Lead Vehicle Braking

Results from exercising the ASCOM tool in the two illustrative scenarios are presented here. Both the CLR and LVB scenarios were explored by conducting a nominal 600 runs under each of two versions of controller design (i.e., with or without the additional deceleration resulting from downshift of the host vehicle transmission) and two topographical descriptions for the road network.

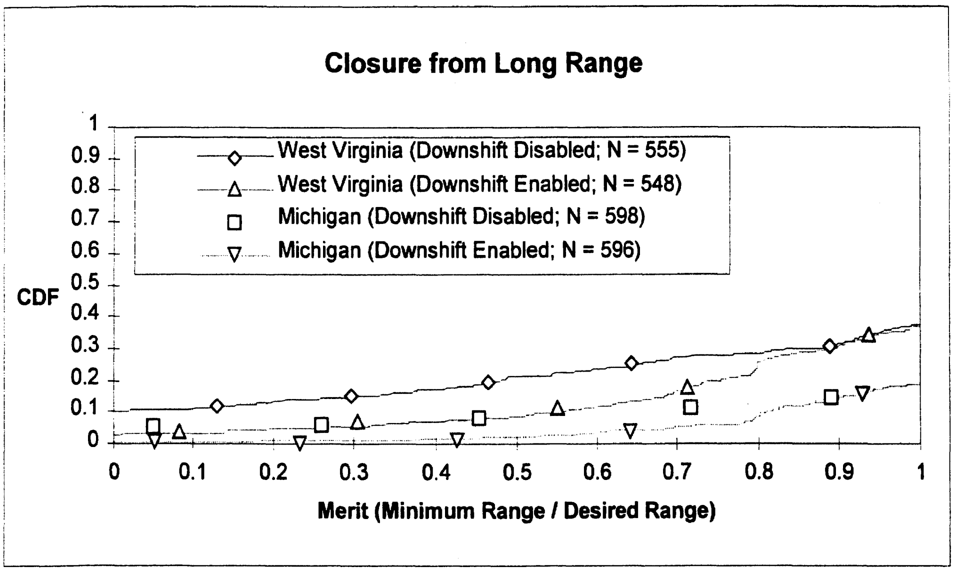


Figure 7. Results for the CLR Scenario Showing Predicted Conflict Distributions for ACC Usage in West Virginia and Michigan

The results plotted in figure 7 show cumulative (CDF) distributions for the CLR scenario over the merit score range from zero to one. A CDF value of 0.1 appearing at a merit value of 0.7, for example, would indicate that 10% of all CLR episodes occurring on the indicated system of roads would arrive at a minimum headway value equal to 70% of the intended reference value, barring driver intervention. CDFs are presented for the Michigan and West Virginia freeway networks, assuming equal driving exposure throughout all miles of each respective network. The number listed within the legend block indicate the number or runs, out of a total of 600, with steady-state initial conditions. For example, if the host vehicle did not have enough deceleration authority to hold its initial speed on negative grade road, then the simulation was not counted. Data are shown for two versions of ACC controller, one having throttle-only control (labeled "downshift-disabled" on the figure) whose deceleration authority is rated at 0.035 gs at 65 mph and a second whose throttle-plus-downshift control (labeled "downshift-enabled") yields a deceleration authority of 0.065 gs at 65 mph.

We see that ACC operation in Michigan (whose roads are characterized by mild grades and large-radius highway curves) encounters virtually zero-conflict CLR approaches (i.e. merit = 1) in approximately 80% of all episodes, with or without downshift capability. Cases in which the Michigan driver must brake to avoid simply crashing his ACC vehicle into the slower vehicle ahead (merit = zero or below) occur in less than 5% of all approaches without downshift and in less than 1% of approaches with it. (Early field experience without downshifting ACC control, by the way, indicates that drivers readily perceive crash-intervention needs and begin to brake well in advance of the critical headway zone (Sayer, 1995). Thus, the point of even noting the zero-level merit data is not to predict crash experience, per se, but rather to acknowledge the statistical occurrence of conflicts which tend to diminish the expected satisfaction of the ACC customer.)

Since the Michigan road system is quite flat, those CLR episodes that register low merit scores are typified by high closure speeds (rather than steep downgrades). Further, the downshift-enabled control feature produces a large fractional reduction in CDF levels at low merit scores. (Please note that the discontinuity around a merit of 0.8 derives from an implementation feature by which downshift engages when the anticipated final range falls below 80% of the desired range (Fancher, 1994).)

The corresponding West Virginia results show zero conflict in only about 64% of the closures and an absolute need to brake in approximately 10% of all closures without the downshifting controller. When downshift is enabled, improvements (lower CDF values) appear below the merit = 0.8 level at which downshift engagement prevails, leaving only 3% of closures that result in a zero-merit outcome. Clearly, the steep grades in this state and delayed target acquisition on its tighter highway curves cause much higher rates of conflict when considering a low-authority ACC controller and the non steered sensor such as was modeled here.

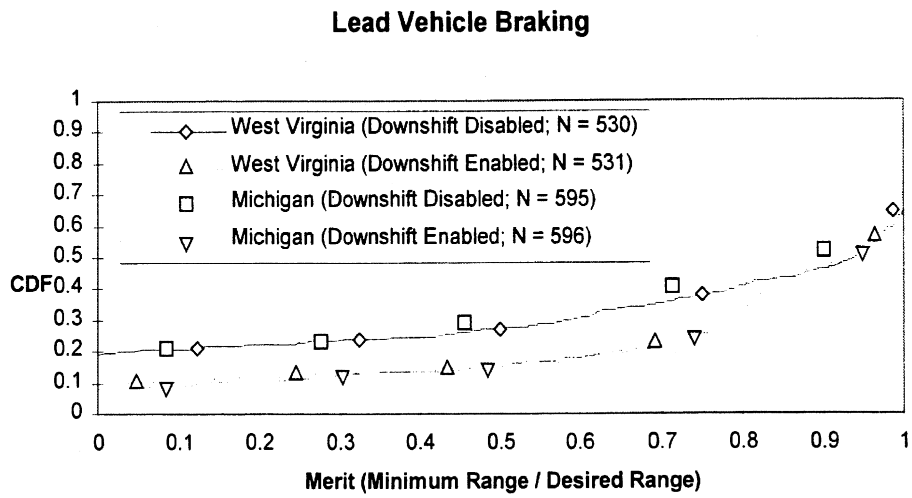


Figure 8. Results for the LVB Scenario of ACC Usage in West Virginia and Michigan

The set of cumulative distributions in figure 8, show that conflicts in the LVB scenario would require the driver to intervene relatively frequently with these ACC controllers, due to braking ahead. An estimate of the absolute frequency of intervention can be obtained by considering the original empirical source of freeway braking data used in these calculations. These data, measured during moderate, midday, traffic conditions as a kind of "average" case, showed that one brake application occurs in approximately every 8.2 miles of conventional driving. Assuming that all lead vehicles are human controlled and not ACC equipped, we can compute the absolute intervention rate by dividing the 8.2 miles per LVB episode by the CDF value measured at a supposed intervention merit threshold. For example, driver intervention to deal with a pending LVB merit level of 0.5 or below would occur approximately every 27 miles of ACC headway keeping in Michigan using a nondownshifting control system and, correspondingly, every 51 miles if a downshifting controller was in use. Further, if the host driver's behavior and traffic conditions resulted in ACC headway keeping for only, say, 50% of all driving miles, these respective LVB interventions would occur once in every 54 and 102 *total* miles of ACC operation.

We see that no-conflict responses (i.e., merit = 1) occur in 27% to 37% of LVB cases, over all four of the combinations of state highway topography and ACC controllers. Curiously, the Michigan cases with downshift disabled show CDF values above those of West Virginia throughout the center of the Merit range, revealing an anomaly of the LVB scenario definition. Namely, if the downgrade is so steep that the ACC controller cannot maintain the initial headway keeping mode of operation, the headway keeping assumption becomes invalid and the run is disqualified. As a result, West Virginia LVB cases are biased into the more benign, higher-merit range because the steep upgrades in this state are not counterbalanced by steep downgrades in the surviving set of Monte Carlo selections.

Cases in which brakes must be applied simply to avoid crashing the ACC vehicle number between 8% and 20% of all LVB episodes.

Regression Model

Monte Carlo simulations are, in general, computationally intensive. Results of the CLR scenario implemented in ASCOM for one host vehicle and one ACC configuration, such as those shown in figure 7, require 15 to 40 seconds on a typical desktop computer. Thus, in a Monte Carlo context where the simulation must be repeated approximately 600 times to get reasonable confidence limits on the merit score distributions, two to three hours of computation time are required for each scenario. If separate merit score distributions are required for all 50 states, then approximately 150 hours of simulation time are necessary. Thus, the efficiency of computation is important and a method to drastically reduce this computational burden has been developed in this work.

One solution is to replace the dynamic model of the host vehicle and ACC with a regression model. That is, a simple polynomial function is used to regress the output of a set of simulation runs to the inputs used for those simulation runs (Crary, 1995). In the example given, one set of condition variables contains numerical values for grade, curvature, absolute vehicle speed, and initial vehicle speed difference for CLR. The regression model can predict, with known statistical accuracy, the output of the simulation at untried input points. In addition the regression model, once derived, applies as long as the host vehicle and ACC system remain fixed. Therefore merit CDFs can be determined for as many different sets of condition variable distributions as one has. For example, one can compute fifty merit score distributions, one for each state in the U.S.. A final advantage the regression model provides is its ability to predict the sensitivity of the merit scores to the condition variables. This can be found by simply differentiating the regression model equation with respect to the condition variable of interest.

A piecewise fourth order regression model was used in the CLR scenario. The regression model is given by

$$\hat{Y}_{mrt} = f(\beta, cv_1, cv_2, cv_3, cv_4) \tag{16}$$

for $\hat{Y}_{mrt} > 1$, set $\hat{Y}_{mrt} = 1$

for $\hat{Y}_{mrt} < 0$, set $\hat{Y}_{mrt} = 0$

where:

$\hat{Y}_{mrt}$ is the merit score predicted by the regression model,

cv_1 is the inverse road grade condition variable,

cv_3 is the curvature condition variable,

cv_3 is the initial variable representing the speed difference between host and lead vehicle, and

cv_4 is the average speed of the two vehicles.

The betas, β_0-β_{57}, are found by the following standard minimum least squares procedure.

$$\beta = (X^T X)^{-1} X^T Y_{mrt} \tag{17}$$

where: $\beta = [\ \beta_0\ \beta_1\ \dots\ \beta_{57}\ ]$

X is the matrix of input condition variables such that:

$$X = \begin{bmatrix} cv_{11} & cv_{12} & \cdots & cv_{14} \\ cv_{21} & cv_{22} & \cdots & cv_{24} \\ \vdots & \vdots & \cdots & \vdots \\ cv_{m1} & cv_{m2} & \cdots & cv_{m4} \end{bmatrix}$$

where: cv_{ij} is the ith sample of the jth condition variable,

Y_{mrt} is a [mx1] vector of true merit scores found from the simulation.

The current design matrix, X_{mX4}, consists of m=625 input condition variable sets evenly distributed over the condition variable space. Several other more efficient designs, such as the Latin Hypercube and the I-Optimality statistical designs, were also considered for use in the condition variable selection process. The merit scores, Y_{mrt}, from this set of condition variables, resulting in less than 1 and greater than -1 (i.e., -$1<Y_{mrt}>1$) were regressed to the input variable sets to provide the betas of equation 18. To determine the goodness of fit of the regression model, an empirical integrated squared error term, e_{emp}, is used. The e_{emp} is a means of evaluating the "average" error of the regression with respect to the true values produced by the Monte Carlo simulation. This error equation is given by:

$$e_{emp} = \frac{1}{n}\{\sum[\hat{Y}_{mrt} - Y_{mrt}]^2\} \tag{18}$$

where: $\hat{Y}_{mrt}$ are predicted merits at input vector x (i.e., a row of matrix X) of the regression model,

Y_{mrt} are the actual merit values at input vector x given by Monte Carlo simulation,

n is the total number of runs.

Regression Results

Results from exercising the ASCOM tool in the illustrative scenario are presented here. The reader should note that the ACC implementation used in these calculations employed a throttle-only control provision, thus limiting the controller's range of deceleration authority to approximately 0.04 g. Clearly, this limitation has a major influence on the numerical results shown below.

The plotted results in figures 9 and 10 show the cumulative distribution function (CDF) over the range of merit scores from zero to one. Two issues will be addressed in discussing these results. Firstly, the confidence bands which bracket the simulation-based CDF results are addressed in light of the total number of Monte Carlo runs. Secondly, the utility of the CDF result obtained using regression equation 16, is discussed insofar as this estimation technique offers a practicable alternative to the computationally burdensome Monte Carlo simulation, itself.

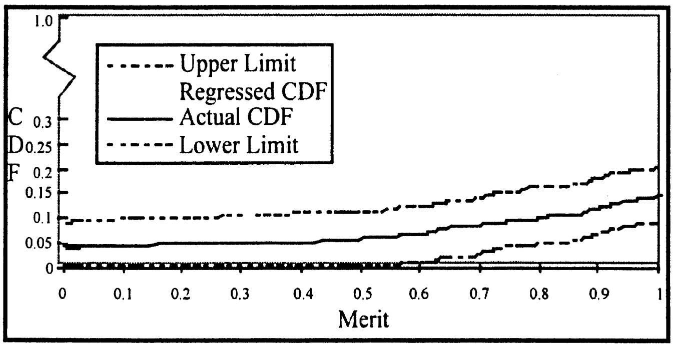


Figure 9. A comparison of results obtained from the fourth order regression equation (16) and the true Monte Carlo results for Michigan/CLR.

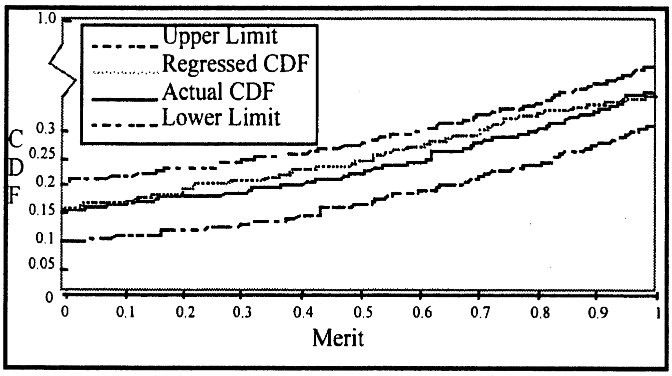


Figure 10. A comparison of results obtained from the fourth order regression equation (16) and the true Monte Carlo results for West Virginia/CLR.

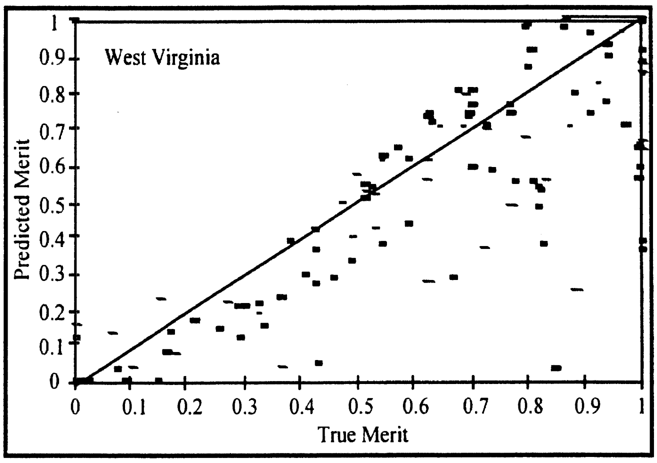
Bounds of Confidence

Figure 9 and 10 present upper and lower confidence bounds for the respective Michigan and West Virginia merit scores obtained using Monte Carlo simulation. As indicated earlier, the size of the confidence band is a function of the inverse square root of the number of Monte Carlo runs used to create each CDF plot. Based upon a 600-run group in each case, the upper and lower bounds delimit the range of results satisfying the 95% confidence level. The bounding limits show that we have 95% confidence that any point on the "real CDF" lies within a cumulative distribution band that is approximately 0.11 high.

Utility of Regression Results

The thin solid line in each CDF plot was produced by Monte Carlo execution of a fourth-order regression equation in thirty six terms corresponding to the CLR condition variables shown in equation 16. We see that the regression-derived results for Michigan in figure 9 and West Virginia in figure 10, lie within 0 to 0.03 of the CDF values obtained from simulation results. We note, firstly, that this form of the regression equation approximates the simulation-based CDF curve to a degree that appears suitable for first-order engineering design. In support of this contention, consider that a common application of the ASCOM tool may entail the comparison of design alternatives in terms of the CDF value that prevails at a chosen threshold value of the merit score. For example, we have already discussed the likely utility of finding the percentage of all CLR approaches falling below the merit value of 0.5 for which manual intervention is warranted. For the illustrated case, the simulation results show that 5% of all CLR approaches in Michigan fall below 0.5 compared with a corresponding regression-estimated result of 8%. Similarly, a simulation result of 22% of all CLR approaches in West Virginia falling below a merit of 0.5 compares with a regression estimate of 24%. Thus, we see that an adequate accurate fit may be obtained through a relatively simple (and thus computationally efficient) regression expression that relates condition variables to the performance metric.

As to the nature of errors encountered with a fourth-order regression expression, figure 11 presents scatterplots comparing regression and simulation-derived merit values for the Michigan and West Virginia road environments. The scatterplots show good correspondence of results in the range of merit values between zero and one (i.e., the range within which the regression equation was fitted to the simulation results). Ninety-eight percent of the points contained on the scatterplots are in good agreement, although not in a visually apparent way since the great majority overlay one another at the extreme upper right of the two plots (at the nominal coordinates, (1,1)). Thus, one can say that a relatively simple regression equation provides an accurate and efficient supplement to the ASCOM numerical simulation.



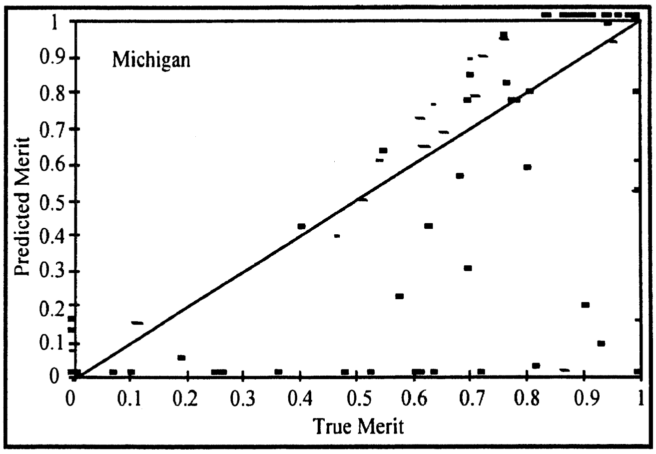


Figure 11. Scatter Plots Showing the Actual vs. the Predicted Merits for West Virginia and Michigan Respectively.

CONCLUDING REMARKS

The ASCOM tool offers a means of reaching an overall assessment of the general suitability of an ACC system design, in recognition of the variety of operational and topographical variables found in the highway environment. While it is true that the absolute validation of a tool of this kind would require extensive field work, the ability to consolidate the known influential variables within desktop evaluation is compelling. The fact that statistical distributions of these variables can begin to be represented with the crude data resources available today provides a means of making some useful level of design assessment, forthwith. The substantial number of new empirical measurement programs being initiated during the 1990s offers hope that in the next five to ten years, very persuasive estimations of certain major aspects of system performance, over a vehicle's lifetime, will be achievable (NHTSA, 1995).

Notwithstanding the inherent potential of an ASCOM approach for supporting the development of ACC and related active safety technologies, the speed of brute Monte Carlo implementations is too slow for an efficient desktop tool. Even with a 100 MHz computing platform, performing 600 runs under a typical ASCOM scenario requires more that 90 minutes of continuous computing. Obtaining a complete set of results for several geographic regions and/or driving behaviors would seem prohibitive for supporting engineering development. In order to dramatically accelerate the computational turnaround, remedial techniques have been undertaken, including regression-type "results modeling" and intelligent subsetting of the condition variables (to delete all the "benign" cases). Encouraging results have been obtained with respect to the accuracy and efficiency gained from the regression fitting approach. Further effort is required to estimate the best predictor in a design for accuracy as well as efficiency. Future work is also required, of course, to implement each of the remaining scenarios and to locate or develop empirical data sets as needed for representing the conflict phenomena.

ACKNOWLEDGMENTS

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APPENDIX

Vehicle Dynamics and Regression Models

Figure A-1 contains the engine map used to find the engine torque, T_e, given a throttle position, θ, and engine speed, ω_t. Tables A-1 to A-5 provide the numerical values for the equations 1 to 14.

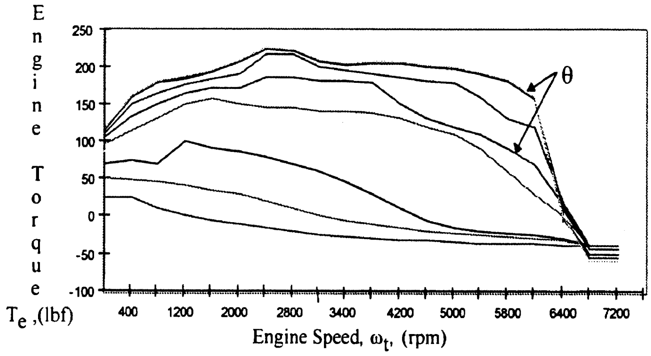

Figure A-1. Engine Torque vs. Engine Speed Given a Throttle Position

TABLE A-1:

Integral Coefficient for cruise control systems - Ki	.2
Proportional Coefficient for cruise control systems- Kp	.5
Mass (m)	3000 lbs
Maximum Sensor Range	500 ft
Aerodynamic Air Coefficient (C_D)	.343
Air Density (ρ)	.00236 slug/ft^3
Tire Radius (r_t)	12 in
Frontal Area (A)	20 ft^2
Gear Efficiencies (c)	.95
Gear Ratio (r)	3.7
Sensor beam azimuth angle ($2\phi_s$)	1.5 deg
Slope of Dynamics Line (τ_c)	11 sec

TABLE A-2: Speed (v) vs. Total Tire Rolling Resistance (F_r)

0 mph	0 lbf
1 mph	40 lbf
60 mph	45 lbf

TABLE A-3: Speed Ratio (ω_d) vs. Input Capacity (k_i, ft-lbf/rpm^2)

0	0.0000163
0.5	0.0000163
0.6	0.0000161
0.8	0.0000146
0.9	0.0000081
1	0.0000000
1.1	-0.0000081

1.2	-0.0000146
1.4	-0.0000161
1.5	-0.0000163
2	-0.0000163

TABLE A-4: Speed Ratio (ω_d) vs. Torque Ratio (P_t)

wd	Pt	wd	Pt
0	2.25	1.03	1.00
0.2	1.85	1.10	1.00
0.4	1.60	1.18	1.00
0.6	1.35	1.20	1.00
0.8	1.00	1.40	1.00
0.82	1.00	1.60	1.00
0.90	1.00	1.80	1.00
0.97	1.00	2.00	1.00

TABLE A-5: Gear vs. Gear Transmission Ratio Table

1	2.842
2	1.573
3	1
4	0.689

Determining ω_t:

$$\omega_r = v/2\pi r_t \qquad \textbf{(A-1)}$$

$$\omega_{ds} = \omega_r r \qquad \textbf{(A-2)}$$

$$\omega_t = \omega_{ds} R_{tran} \qquad \textbf{(A-3)}$$

where: ω_r rear axle speed

ω_{ds} driveshaft speed

r_t tire radius

r gear ratio

R_{tran} Transmission Ratio (see table A-5)

Finally, table A-6 gives the numerical beta values obtained by solving equation (19).

TABLE A-6: Betas of the Regression Equation

β_0	-98.9148	β_{20}	19260.19	β_{39}	-46812.4
β_1	0.002925	β_{21}	70.29227	β_{40}	-226.9
β_2	130.2885	β_{22}	-0.41141	β_{41}	8.621251
β_4	6.10185	β_{23}	-0.19091	β_{42}	1.506331
β_5	0.011587	β_{24}	0.001449	β_{43}	-5.56E-06
β_6	-1.74E-07	β_{25}	1.56E-06	β_{44}	-9.31E-09
β_7	0.014669	β_{26}	0.025392	β_{45}	-4.93E-06
β_8	-9.20E-05	β_{27}	1.15E-05	β_{46}	-5.21E-08
β_9	-7.55E-07	β_{28}	3.77E-05	β_{47}	8.55E-06
β_{10}	-3130.39	β_{29}	-2.20E-07	β_{48}	-2.42E-09
β_{11}	-3.59973	β_{30}	0.142667	β_{49}	-0.00036
β_{12}	-0.12286	β_{31}	-0.00054	β_{50}	-3.27E-07
β_{13}	-0.14132	β_{32}	-0.00028	β_{51}	-5.65E-07
β_{14}	-0.00072	β_{33}	4.38E-05	β_{52}	1.13E-05
β_{15}	0.00252	β_{34}	0.002348	β_{53}	8.87E-12
β_{16}	1.22E-11	β_{35}	-1.52E-16	β_{54}	1.40E-11
β_{17}	8.47E-07	β_{36}	-4.58E-11	β_{55}	-0.33049
β_{18}	4.70E-10	β_{37}	-9.84E-14	β_{56}	5.75E-06
β_{19}	2.05E-10	β_{38}	-9.41E-15	β_{57}	-1.03395