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Synthesis of feedforward/feedback control systems for nonlinear processes

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SYNTHESIS OF FEEDFORWARD/FEEDBACK CONTROL SYSTEMS FOR NONLINEAR PROCESSES

by

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of the requirements for the degree of
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“Στην μνημη του πατερα μου Στελιου ”

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The thesis is dedicated to the memory of my father, whose personality has influenced me in so many ways...

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ABSTRACT

SYNTHESIS OF FEEDFORWARD/FEEDBACK CONTROL SYSTEMS FOR NONLINEAR PROCESSES

by

Prodromos Daoutidis

Chairperson: Costas Kravaris

In this thesis, the unified problem of disturbance rejection and output tracking for general nonlinear processes is studied, using methods from differential geometry. An analysis framework is initially established, through a detailed study of the concept of relative order. The general problem of disturbance rejection and output tracking is formulated as a feedforward/feedback control problem, and is addressed first for single-input single-output processes and then for multiple-input multiple-output processes. Feedforward/state feedback laws are synthesized that completely eliminate the effect of measured disturbances on the controlled outputs and induce a well-characterized linear input/output behavior. A general feedforward/feedback control structure is developed that also accounts for modeling error and unmeasured disturbances. The developed control methodology is applied to composition control in a cascade of chemical reactors and to temperature and number average molecular weight control in a continuous polymerization reactor. On the basis of the proper-

ties of relative order and the controller synthesis results, the problem of synthesis of control configurations is also addressed. A general framework for the structural evaluation of alternative control configurations is developed, based on fundamental structural limitations in the control quality and structural coupling considerations. The developed evaluation framework is applied to the synthesis of control configurations in an evaporation unit, a continuous chemical reactor and a heat-exchanger network.

CHAPTER I

INTRODUCTION

1.1 Motivation

All physical systems are nonlinear. In the field of chemical engineering, process nonlinearities are the rule, rather than the exception. They arise mainly due to complex reaction mechanisms, the Arrhenius dependence of reaction rates on temperature, and thermodynamic and transport correlations. They manifest themselves in the static and the dynamic behavior of processes, in the form of multiple steady states, finite stability regions, parametric sensitivity, limit cycles, quasi-stochastic behavior, etc. Examples of chemical engineering processes with highly nonlinear behavior include polymerization reactions, high-purity distillation columns, bioprocesses, pH processes, etc. For control purposes, the traditional approach in dealing with nonlinearities involves the approximate linearization of a nonlinear process model around an operating steady state, followed by a linear controller design. The presence of strong nonlinearities, however, necessitates large robustness margins in the linear controller design, leading to degraded and, very often, unacceptable performance characteristics. The above difficulties are aggravated, whenever there exists a wide range of operating conditions or in the case of processes with a purely transient mode of operation, such as batch processes. Linear controller design based on linear time-invariant models for such cases may lead to unacceptable performance characteristics, even in the presence of mild nonlinearities.

In addition to their nonlinear nature, chemical processes are inherently multivariable and exhibit a highly interactive behavior. Coupled with the nonlinearities, the above issues lead to a formidable challenge in the field of process control: the development of a rigorous, yet practical, **nonlinear multivariable control** framework, able to systematically address the basic problems of regulation, set-point tracking and interactions. Meeting this challenge necessitates an appropriate mathematical and methodological framework, able to capture the fundamental nature of the control problem in nonlinear systems.

1.2 Scope and Objectives

During the 50's and the early 60's the control field was dominated by the controversy between the classical and the modern linear control theory. The classical theory was essentially limited to single-input single-output (SISO) systems described by linear differential equations with constant coefficients (or their corresponding Laplace transforms). The modern control theory adopted a state-space perspective and advocated the use of matrix algebra techniques for analysis and synthesis purposes. Even the matrix algebra framework, however, proved to be inadequate to provide transparent solutions to typical multiple-input multiple-output (MIMO) control problems like invertibility, noninteracting control, etc. In the late 60's, some new linear geometric tools were introduced, such as invariant and controllability subspaces (Basile and Marro, 1969, Wonham, 1970), and were used to understand and formulate precise solutions for the above problems. This led to the so called linear geometric control theory, the basic results of which can be found in the classical book by Wonham, 1979.

On the nonlinear front, during the 50's and 60's most of the research was concentrated on stability analysis, based on operator methods and functional analysis (Zames, 1966a,b). Although significant progress was made in studying nonlinear stability and feedback properties (Safonov, 1980, Desoer and Vidyasagar, 1975), this line of research had limited impact on controller synthesis. This was mainly due to the abstract nature of nonlinear operators, which made the derivation of analytical controller synthesis results almost prohibitive. Numerical control algorithms were proposed instead (Economou and Morari, 1985), within an explicit inversion control framework (Economou et al., 1986, Parrish and Brosilow, 1988). A breakthrough in nonlinear systems theory occured in the late 60's and early 70's, when concepts from differential geometry were used to study the accessibility property of nonlinear systems (Herman, 1963) and motivated further research on observability, controllability and realization theory (e.g., Lobry, 1970, Sussmann, 1972, 1977). The above early results provided meaningful nonlinear analogs of fundamental system-theoretic notions and motivated the, so called, **differential geometric** approach for the control of nonlinear systems. Research in this area has progressively evolved from the study of fundamental mathematical concepts to the point where basic nonlinear control problems can be systematically addressed and find explicit, general and elegant solutions. To this end, Lie Algebra has emerged as a powerful analog of matrix algebra, providing the necessary mathematical tools for the manipulation of nonlinear ordinary differential equations. Some of the most important results in this area include solutions to the problems of invertibility (Hirschorn, 1979a,b), exact state-space linearization (Jakubcsyk and Respondek, 1980, Su, 1982, Hunt et al., 1983a,b), input/output decoupling via static state feedback (Freund, 1975, Ha and Gilbert, 1986) and input/output linearization (Claude et al., 1983, Isidori and Ruberti, 1984, Kravaris and Chung, 1987, Kravaris and Soroush, 1990).

Limited research effort has been devoted, however, in studying explicitly the role of disturbance inputs for analysis and controller synthesis purposes, and addressing the problem of disturbance compensation and output tracking in a unified framework. The only available results are within the context of the disturbance decoupling problem (Hirschorn, 1981b, Isidori et al., 1981, Nijmeijer and van der Schaft, 1983, Moog and Glumineau, 1983) and the exact state-space linearization problem (Calvet and Arkun, 1988), and their application hinges upon extremely restrictive conditions. On the other hand, disturbance inputs arise naturally in practice, whenever input variables are subject to unpredictable variations. The majority of the control problems in continuous chemical engineering processes involves the regulation of output variables to desired steady state values despite the presence of disturbances. Important control problems in batch and semi-batch processes involve tracking of output profiles in the presence of disturbances. Moreover, measurements of the disturbance inputs are often available, allowing for significant improvement in the control quality, if properly incorporated in the controller synthesis.

Motivated by the above, this thesis studies the unified problem of **disturbance rejection and set-point tracking** for general nonlinear processes, within a feedforward/feedback controller synthesis framework. Natural implications of the solution of this problem in the synthesis of control configurations are also identified and studied. More specifically, the main objectives of the thesis are :

- The development of analysis tools for studying the role of disturbance inputs in nonlinear control
- The synthesis of feedforward/feedback controllers for multivariable nonlinear

processes

- The development of an evaluation framework for the selection of control configurations for multivariable nonlinear processes

The mathematical and methodological framework of the thesis lies within the differential geometric control approach. The emphasis, however, is on the development of explicit results, which are transparent from an analysis point of view and consistent with intuitive considerations. For this reason, the treatment is purely analytical, and is very often related to key structural characteristics of nonlinear systems.

In Chapter II, an analysis framework is developed, based on various formulations and interpretations of the concept of relative order. The general disturbance rejection and set-point tracking problem is addressed for SISO nonlinear processes in Chapter III. The key step to the solution of this problem is the synthesis of nonlinear feedforward/state feedback control laws that compensate completely for measurable disturbances and induce a linear input/output behavior. Emphasis is placed on interpreting the structure and the nature of the control laws. Closed-loop stability issues are addressed in detail. The application of the method is illustrated in a system of chemical reactors. Chapter IV generalizes the results of Chapter III in MIMO nonlinear processes. Closed-loop design considerations, including stability, performance and degree of coupling are studied in detail. A comparison with the classical disturbance decoupling problem is also included. The method is applied to a polymerization reaction system. Motivated by the controller synthesis results, Chapter V addresses the problem of selection of control configurations among a set of alternative ones. Structural evaluation guidelines are developed for this purpose, and a number of chemical engineering examples are studied to illustrate the proposed methodology. Finally, in Chapter VI, the main results of the thesis are summarized and future research directions are outlined.

CHAPTER II

RELATIVE ORDER: A FUNDAMENTAL ANALYSIS TOOL

2.1 Introduction

In this chapter, the main analysis tools of the thesis will be studied. In particular, following a brief discussion on the form of nonlinear systems studied in this thesis, the concept of relative order will be introduced in various forms. A graph-theoretic interpretation of relative order will establish its structural nature. Relative order will also be shown to quantify the notions of "direct effect" and "physical closeness" between input and output variables, and to provide a measure of sluggishness of the response of the output variables. The detailed study of the concept of relative order and its interpretations will establish an analysis framework, which will allow the controller synthesis results of Chapters III and IV to find transparent and intuitively appealing interpretations. The same analysis framework will provide the theoretical basis for studying the problem of selection of control configurations in Chapter V.

2.2 Preliminaries

The nonlinear processes (systems) considered are described by continuous-time state-space models of the form:

$$\begin{aligned} \dot{x} &= f(x) + \sum_{j=1}^{m} u_j(t) g_j(x) + \sum_{\kappa=1}^{p} d_\kappa(t) w_\kappa(x) \\ y_i &= h_i(x) \ , \ i = 1, \cdots, m \end{aligned} \tag{2.1}$$

where x denotes the vector of state variables, u_j denotes a manipulated input, d_κ denotes a disturbance input, and y_i denotes an output (to be controlled). It is assumed that $x \in X \subset \mathbb{R}^n$, where X is open and connected. Also, $u(t) = [u_1(t), \cdots, u_m(t)]^T \in \mathbb{R}^m$ and $d(t) = [d_1(t), \cdots, d_p(t)]^T \in \mathbb{R}^p$, $\forall t \in [0, \infty)$, and $y = [y_1, \cdots, y_m]^T \in \mathbb{R}^m$. The dependence of x and y on time t is suppressed throughout the thesis for notational simplicity. $f(x)$, $g_j(x)$, $w_\kappa(x)$ denote analytic vector fields on $\mathbb{R}^n$, and $h_i(x)$ denote analytic scalar fields on $\mathbb{R}^n$. Some of the results of the thesis will hold even under weaker smoothness assumptions on the above fields. If $u_j(t)$ and $d_\kappa(t)$ are piecewise constant functions, then there exists a unique solution of Eq.2.1, at least locally. Conditions that guarantee existence and uniqueness of the solutions of Eq.2.1 for more general input functions (e.g., piecewise continuous) can be found in standard nonlinear systems textbooks (e.g., Vidyasagar, 1978, Hirsch and Smale, 1974). Finally, it is assumed that the input and output variables in Eq.2.1 represent deviations from some nominal values. Then, $x_0 \in X$ will be a nominal equilibrium point (or steady state) for Eq.2.1 if $f(x_0) = 0$.

The following remarks should also be made with regard to the process model of Eq.2.1:

1. It describes general MIMO processes with an equal number of manipulated inputs and controlled outputs. SISO process models can be easily obtained by

setting $m = 1$.

2. It identifies and models explicitly the disturbance inputs d_κ. These disturbance inputs will be assumed to be measurable for controller synthesis purposes, although for analysis purposes this is irrelevant.

3. It has an affine and separable structure, i.e., disturbance inputs and manipulated inputs enter the dynamic equations linearly and separately. This structure corresponds to a broad class of practical situations and is especially convenient from a mathematical point of view. Treatment of more general process models is also possible, and is briefly discussed in some parts of the thesis.

4. The standard linear model description of the form:

$$\begin{aligned} \dot{x} &= Ax + \sum_{j=1}^{m} u_j(t) b_j + \sum_{\kappa=1}^{p} d_\kappa(t)\gamma_\kappa \\ y_i &= c_i x \ , \ i = 1, \cdots, m \end{aligned} \tag{2.2}$$

is easily recovered from Eq.2.1, for $f(x) = Ax$, $g_j(x) = b_j$, $w_\kappa(x) = \gamma_\kappa$ and $h_i(x) = c_i x$, where $A, b_j, \gamma_\kappa, c_i$ are matrices of appropriate dimension.

2.3 The concept of relative order

In this section, various formulations of the concept of relative order will be introduced. All the definitions, unless otherwise stated, refer to nonlinear systems in the form of Eq.2.1. For the definitions, as well as for the subsequent results, the standard Lie derivative notation will be used, which is explained in Appendix A. First, a standard concept of relative order for MIMO nonlinear systems will be reviewed (e.g., Ha and Gilbert, 1986, Kravaris and Soroush, 1990):

Definition 2.1: *The relative order r_i of the output y_i with respect to the manipulated input vector u is defined as the smallest integer for which there exists some*

$j \in \{1, 2, \cdots, m\}$ such that:

$$L_{g_j} L_f^{r_i - 1} h_i(x) \not\equiv 0 \tag{2.3}$$

for $x \in X$. If no such integer exists, $r_i = \infty$.

Proposition 2.1 (Isidori, 1989): *Consider the nonlinear system of Eq.2.1, and assume that each output y_i possesses a finite relative order r_i. Then, $(r_1 + \cdots + r_m) \leq n$.*

It will be assumed that each output y_i possesses a finite relative order r_i. This is a necessary condition for output controllability, since, otherwise, certain output variables would not be affected by any of the manipulated inputs.

Proposition 2.2: *Consider the nonlinear system of Eq.2.1 and assume that $d_\kappa = 0$, $\forall \kappa = 1, \cdots, p$. Then, r_i is the smallest-order derivative of y_i that explicitly depends on u.*

Proof: Based on Definition 2.1 and the assumptions of Proposition 2.2, the following expressions for the derivatives of the output y_i can be easily obtained:

$$\begin{aligned} y_i &= h_i(x) \\ \frac{dy_i}{dt} &= L_f h_i(x) \\ &\vdots \\ \frac{d^{r_i - 1} y_i}{dt^{r_i - 1}} &= L_f^{r_i - 1} h_i(x) \\ \frac{d^{r_i} y_i}{dt^{r_i}} &= L_f^{r_i} h_i(x) + \sum_{j=1}^{m} u_j(t) L_{g_j} L_f^{r_i - 1} h_i(x) \end{aligned} \tag{2.4}$$

which directly establish the validity of Proposition 2.2.

Note that the above concept of relative order relates a **single output variable** with the whole **manipulated input vector**. For analysis purposes, it is also meaningful to relate a **single output variable** with a **single manipulated input**. The concept of relative order between a single output variable and a single manipulated input for SISO systems is originally due to Hirschorn, 1979a. Definition 2.2 provides a natural generalization of this concept in a MIMO context:

Definition 2.2: *The relative order r_{ij} of the output y_i with respect to a manipulated input u_j is defined as the smallest integer for which:*

$$L_{g_j} L_f^{r_{ij}-1} h_i(x) \not\equiv 0 \tag{2.5}$$

for $x \in X$. If no such integer exists, $r_{ij} = \infty$.

Proposition 2.3: *Consider the nonlinear system of Eq.2.1. Then,*

$$r_i = min\ \{r_{i1}\ ,\ r_{i2}\ ,\ \cdots\ ,\ r_{im}\} \tag{2.6}$$

Proof: The above relation is a direct consequence of Definitions 2.1 and 2.2.

Based on Proposition 2.3, the relative orders r_i can be immediately identified, once the individual relative orders r_{ij} have been calculated.

In analogy with Definition 2.2, the relative order between an **output variable** and a **disturbance input** can be defined as follows (Daoutidis et al., 1990, Daoutidis and Kravaris, 1989):

Definition 2.3: *The relative order $\rho_{i\kappa}$ of the output y_i with respect to the disturbance input d_κ is defined as the smallest integer for which:*

$$L_{w_\kappa} L_f^{\rho_{i\kappa}-1} h_i(x) \not\equiv 0 \tag{2.7}$$

for $x \in X$. If no such integer exists, $\rho_{i\kappa} = \infty$.

Remark 2.1: Proposition 2.2 can be easily modified to establish that $\rho_{i\kappa}$ is the smallest-order derivative of the output y_i that explicitly depends on d_κ, assuming that all other input variables are equal to 0.

In what follows, unless otherwise stated, the term relative order will imply the relative order between an input/output pair (Definitions 2.2 and 2.3).

Remark 2.2: For the special case of a MIMO linear system of the form of Eq.2.2,

r_{ij} is the smallest integer for which:

$$c_i A^{r_{ij}-1} b_j \neq 0 \tag{2.8}$$

while $\rho_{i\kappa}$ is the smallest integer for which:

$$c_i A^{\rho_{i\kappa}-1} \gamma_\kappa \neq 0 \tag{2.9}$$

The quantities $c_i A^k b_j,\ k = 0, 1, \cdots$ are known as Markov parameters in linear systems theory (e.g., Kailath, 1980), and are used to characterize the input/output behavior of linear systems.

Furthermore, for a linear system, the relative order between any input/output pair is equal to the difference between the degrees of the denominator and the numerator polynomials of the corresponding transfer function.

Finally, if the transfer matrix between u and y, $G(s)$, has the matrix fraction form:

$$G(s) = N(s)[D(s)]^{-1}$$

where $N(s)$ and $D(s)$ are polynomial matrices and $D(s)$ is column reduced, then the relative order r_i is equal to the difference between the column degrees of the i-th column of $D(s)$ and the i-th column of $N(s)$.

2.4 Relative orders, graph-theory and the notion of "direct effect"

In this section, a graph-theoretic interpretation of relative order will be developed, which will provide intuitive insight on the concept and will also suggest an alternative way for its calculation. First, a brief review of notions from graph-theory will be given. The state-space model of Eq.2.1 can be associated with a directed graph (digraph), defined by a set of vertices (or nodes) and a set of edges as follows :

- The vertex set consists of the set of manipulated inputs $(u_1, \cdots, u_m)$, the set of disturbance inputs $(d_1, \cdots, d_p)$, the set of state variables $(x_1, \cdots, x_n)$ and the set of output variables $(y_1, \cdots, y_m)$.

- The set of edges consists of directed lines connecting two vertices according to the following rules:

  - If $\frac{\partial f_l(x)}{\partial x_k} \not\equiv 0, \quad k, l = 1, \cdots, n$, then there is an edge from x_k to x_l
  - If $g_{jl}(x) \not\equiv 0, \quad l = 1, \cdots, n$, then there is an edge from u_j to x_l
  - If $w_{\kappa l}(x) \not\equiv 0, \quad l = 1, \cdots, n$, then there is an edge from d_κ to x_l
  - If $\frac{\partial h_i(x)}{\partial x_k} \not\equiv 0, \quad k = 1, \cdots, n$, then there is an edge from x_k to y_i

  where $f_l(x), g_{jl}(x), w_{\kappa l}(x)$ denote the l-th element of the vector fields $f(x)$, $g_j(x)$ and $w_\kappa(x)$, respectively.

A **path** of a digraph is a particular directed sequence of some of its edges, such that the initial vertex of the succeeding edge is the final vertex of the preceding edge. The number of edges contained in a path is called the **length** of the path (for a detailed review of notions of graph theory see e.g., Ore, 1962).

It can be easily seen from the above rules that the digraph representation of a dynamic system contains much less information than its detailed state-space description. In particular, for nonlinear systems of the form of Eq.2.1, their digraph representation contains no information about:

1. The dependence of the vector fields g_j and w_κ on x
2. The exact functional dependence of the vector field f on x
3. The numerical values of the system parameters

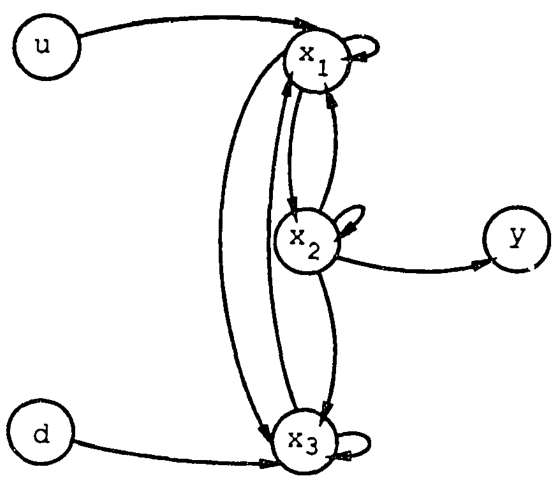

Figure 2.1: A typical digraph

In fact, a digraph representation contains only the pattern of interdependencies among the variables and is uniquely determined by them. This pattern of interdependencies can also be expressed through the notion of a structural model, associated with the well known notion of structural (or structured) matrices (e.g., Shields and Pearson, 1976). Figure 2.1 provides a typical illustration of a digraph corresponding to the class of dynamic systems with a structural model of the form:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, x_3) + u(t)g_1(x) \\ \dot{x}_2 &= f_2(x_1, x_2) \\ \dot{x}_3 &= f_3(x_1, x_2, x_3) + d(t)w_3(x) \\ y &= h(x_2) \end{aligned} \tag{2.10}$$

Applying Definitions 2.2 and 2.3 for the calculation of the relative orders between u and y and between d and y, one easily finds that $r = 2$ and $\rho = 3$. Referring to the digraph of the above system in Figure 2.1, it is also easily seen that the shortest path between u and y has length equal to 3, while the shortest path between d and y has length equal to 4. The above example suggests an interesting connection between relative orders and length of paths in a digraph. This connection will be rigorously established in Theorem 2.1 that follows, which generalizes a result by Kasinski and Levine, 1984. The proof of Theorem 2.1 is given in Appendix B.

Theorem 2.1: *Consider the nonlinear system of Eq.2.1 and its corresponding digraph. Let ℓ_{ij} and $\ell_{i\kappa}$ denote the lengths of the shortest paths connecting u_j and y_i, and d_κ and y_i, respectively. Also, let r_{ij} and $\rho_{i\kappa}$ be the relative orders between u_j and y_i, and d_κ and y_i, respectively. Then, the following relations hold generically: $r_{ij} = \ell_{ij} - 1$ and $\rho_{i\kappa} = \ell_{i\kappa} - 1$.*

Remark 2.4: By generically in the above theorem, it is meant that the result holds for all vector fields f, g_j, w_κ and all scalar fields h_i, except possibly for a "set of measure zero". Non-generic situations in the calculation of relative orders through the digraph may arise because of the specific nonlinear dependence of the vector and scalar fields on x.

A number of important observations arise from Theorem 2.1:

- Firstly, the result of Theorem 2.1 establishes that the generic calculation of relative orders for a process requires knoweledge of its structural model only, or equivalently its digraph, i.e., the lowest level of information about the process. This fact makes the relative order a generic analysis tool and establishes its structural nature.

- Furthermore, it is clear from the definition of a graph that, except from the edges connecting state and output vertices, every other edge denotes the effect of one variable on another through an integration step. Therefore, the result of Theorem 1 leads to a graph-theoretic interpretation of relative order as the number of integrations that an input has to go through before it affects an output, generalizing the well-known SISO result obtained through the Byrnes-Isidori normal form. In the above sense, relative order is a rigorous and meaningful measure of how **direct effect** an input variable has on an output variable. The above interpretation is also supported by the result of Proposition 2.2. Theorem 2.2 in the next section will illustrate how this notion of direct effect manifests itself in typical response characteristics.

- Finally, the result of Theorem 2.1 can be used to increase the efficiency of calculation of relative orders in a symbolic manipulation environment, especially for large-scale systems.

Remark 2.5: For linear systems, the existence of a finite relative order r_{ij} corresponds to the property of **accessibility** (Lin, 1974) of the output node y_i from the input node u_j. To denote accessibility of an output node from a disturbance node, the term **disturbability** has been used (Shah et al., 1977, Morari and Stephanopoulos, 1980), which obviously corresponds to a finite relative order between a disturbance input and an output.

2.5 Relative order as a measure of sluggishness

In this section, a rigorous interpretation of relative order will be provided as a **measure of sluggishness** of the response of a dynamic system. The main result is summarized in Theorem 2.2 that follows (the proof is given in Appendix B):

Theorem 2.2: *Consider the nonlinear system of Eq.2.1 at an initial condition $x(0) = x_0$, where x_0 is a nominal equilibrium point. Also, let r_{ij} denote the relative order of the output y_i with respect to the manipulated input u_j. Then, the initial response of the output y_i under a unit-step change at the input u_j can be approximated* *for small times t by:*

$$y_i(t) \cong L_{g_j} L_f^{r_{ij}-1} h_i(x_0) \frac{t^{r_{ij}}}{r_{ij}!} \tag{2.11}$$

Corollary 2.1: *Consider a SISO linear system of the form:*

$$\begin{aligned} \dot{x} &= Ax + u(t)b \\ y &= cx \end{aligned} \tag{2.12}$$

and let r denote the relative order of the output y with respect to the manipulated input u. Then, the initial response of the output y under a unit-step change at the input u can be approximated for small times t by:

$$y(t) \cong (cA^{r-1}b)\frac{t^r}{r!} \tag{2.13}$$

Remark 2.6: The result of Corollary 2.1 is already known and proven independently in standard linear control books (the independent proof is included in Appendix B for completeness).

The result of Theorem 2.2 establishes in a rigorous way that the relative order r_{ij} is a structural measure of how sluggish the response of the output y_i is for step shanges at the input u_j: the larger the relative order, the more sluggish the response is. More specifically (see Figure 2.2):

- $r_{ij} = 1$ implies that the initial slope of the response will be non-zero
- $r_{ij} = 2$ implies that the initial slope of the response will be zero, but its rate of change will be non-zero
- $r_{ij} > 2$ implies that the initial slope of the response as well as its rate of change will be zero, while a higher-order derivative of the slope will be non-zero if r_{ij} is finite

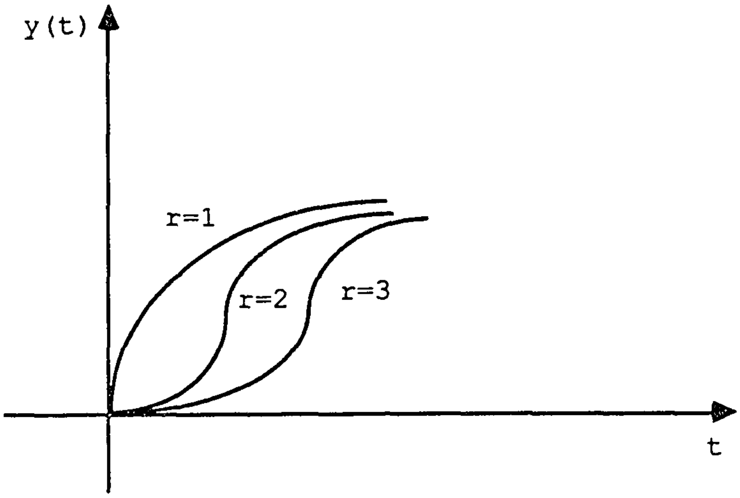

Figure 2.2: Relative order as a measure of sluggishness

The overall characteristics of the output response to an input change will also depend on:

- the time constant, which will determine how quickly the output will adjust to the input change, once it responds
- the steady state gain, which will determine the large time value of the output

As the time constant quantifies how "quick" the effect of an input variable is on an output variable and the static gain how "significant" this is, the relative order quantifies how "direct" this effect is.

Remark 2.7: A similar result to Theorem 2.2 can be obtained for the relative order $\rho_{i\kappa}$, as well as for r_i. Clearly, r_i is a measure of the sluggishness of the output y_i with respect to the manipulated input vector, i.e., a measure of the maximum sluggishness of the response of the output y_i with respect to any of the manipulated inputs.

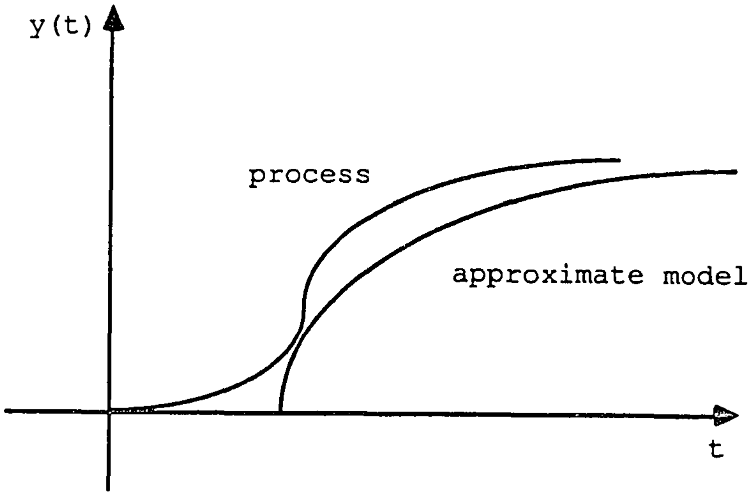

Figure 2.3: Typical step-response of a high-order process

2.6 Relative order, dead time and the notion of "physical closeness"

The analysis so far has indicated that the concept of relative order quantifies how "direct" the effect of an input variable is on an output variable and has demonstrated how this property affects the small-time response characteristics. In what follows, motivated by the previous discussion, the concept of relative order will be associated with apparent dead time, which has traditionally been used to capture small-time response characteristics. Consider a typical step response of the output of a process with dynamics higher than first-order (Figure 2.3). Along the lines of the above treatment and assuming negligible transportation delay (which is the most common case in a single processing unit), one can obtain a clear interpretation of the sigmoidal shape of the response: it is due to the presence of a higher than one relative order between the input and the output. When such a high-order process is approximated by a first-order lag plus dead time model, the neglected dynamics gives rise to the dead time, which is therefore an apparent but not real quantity; although it provides a useful indication of how responsive the output is, it has no physical significance or rigorous justification. On the other hand, without any response data and based purely on structural information, one can rigorously assess the qualitative feature of the initial part of the response through the concept of relative order. It should be clear, therefore, that relative order represents the structural analog of apparent dead time. This analogy becomes obvious in the context of discrete linear systems, where the pole excess of the pulse transfer function (i.e., the relative order) is exactly the time delay of the process.

The above analogy between relative order and apparent dead time leads to an interpretation of relative order as a measure of "physical closeness" between an input variable and an output variable. An especially appealing illustration of this interpretation can be obtained in the case of staged processes (e.g., distillation columns, cascades of chemical reactors, etc.). Consider, for example, the cascade of two continuous stirred tank reactors shown in Figure 2.4, where a second order reaction $A \longrightarrow B$ takes place. Under standard assumptions, the material and energy balances that describe the dynamic behavior of this process take the following form:

$$
\begin{aligned}
\frac{dc_{A1}}{dt} &= \frac{F}{V}(c_{A0} - c_{A1}) - k_0 e^{\left(-\frac{E}{RT_1}\right)} {c_{A1}}^2 \\
\frac{dc_{A2}}{dt} &= \frac{F}{V}(c_{A1} - c_{A2}) - k_0 e^{\left(-\frac{E}{RT_2}\right)} {c_{A2}}^2 \\
\frac{dT_1}{dt} &= \frac{F}{V}(T_0 - T_1) + \frac{(-\Delta H)}{\rho C_p} k_0 e^{\left(-\frac{E}{RT_1}\right)} {c_{A1}}^2 + \frac{1}{V\rho C_p} Q_1 \\
\frac{dT_2}{dt} &= \frac{F}{V}(T_1 - T_2) + \frac{(-\Delta H)}{\rho C_p} k_0 e^{\left(-\frac{E}{RT_2}\right)} {c_{A2}}^2 + \frac{1}{V\rho C_p} Q_2
\end{aligned}
\tag{2.14}
$$

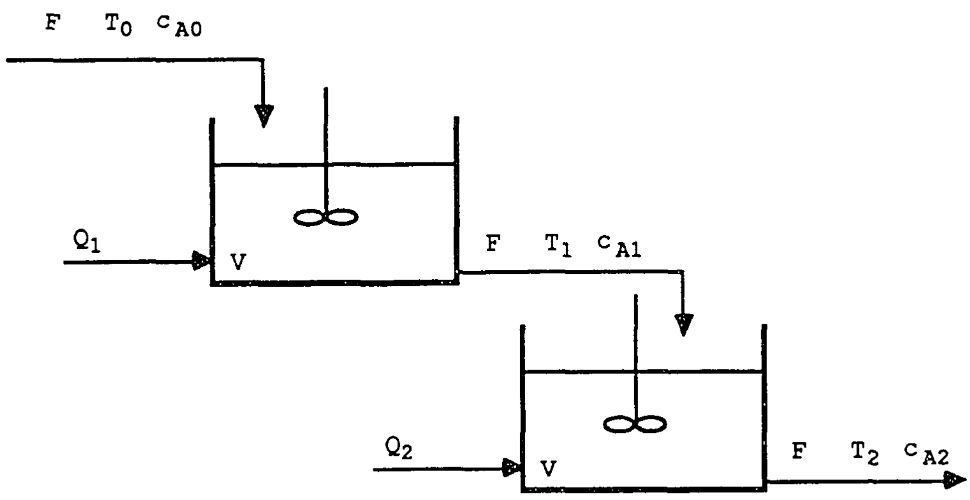

Figure 2.4: A cascade of two continuous stirred tank reactors

where

C_p	=	heat capacity
E	=	activation energy
F	=	volumetric flow rate
Q_1, Q_2	=	heat inputs to tanks 1 and 2
T_1, T_2	=	temperatures in tanks 1 and 2
T_0	=	inlet temperature
V	=	volume
$-\Delta H$	=	heat of reaction
c_{A1}, c_{A2}	=	molar concentration of A in tanks 1 and 2
c_{A0}	=	inlet molar concentration of A
k_0	=	Arrhenius frequency factor
ρ	=	density

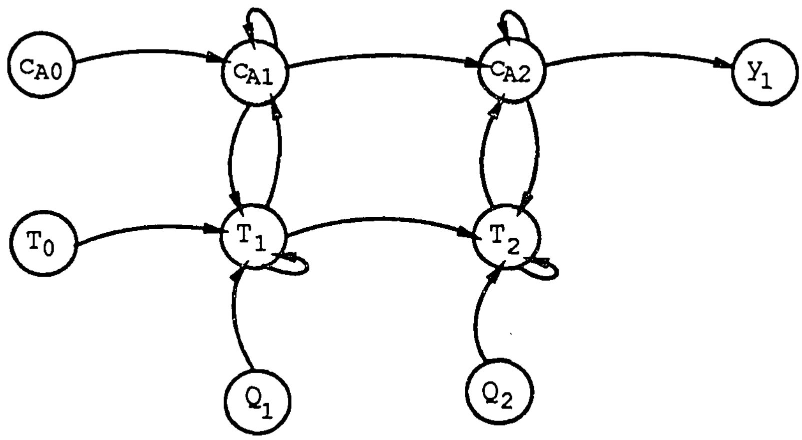

Figure 2.5: The digraph of the reactor cascade

From the dynamic model of Eq.2.14, one can easily obtain the digraph of the process, which is shown in Figure 2.5. Suppose that we wish to control the concentration at the exit of the second reactor, c_{A2}, and available manipulated inputs are the heat inputs to the reactors, Q_1 and Q_2. For notational consistency, set $y_1 = c_{A2} - c_{A2s}$ and $u_1 = Q_1 - Q_{1s}$, $u_2 = Q_2 - Q_{2s}$ for the alternative manipulated inputs, where the subscript s denotes a nominal steady state value. Based on the result of Theorem 2.1 and the digraph of Figure 2.5, we can easily calculate the corresponding relative orders which take the values: $r_{11} = 3$ and $r_{12} = 2$. Clearly, the smallest relative order corresponds to the heat input Q_2 which is "physically closer" to the controlled output and has a more "direct effect" on it than the heat input Q_1.

2.7 Notation

Roman Letters

d_κ	=	disturbance input
f, g_j, w_κ	=	vector fields
h_i	=	output scalar field
ℓ_{ij}	=	the lenght of the shortest path connecting u_j and y_i
$\ell_{i\kappa}$	=	the lenght of the shortest path connecting d_κ and y_i
r_i	=	relative order of the output y_i with respect to the manipulated input vector u
r_{ij}	=	relative order of the output y_i with respect to the manipulated input u_j
t	=	time
u_j	=	manipulated input
x	=	vector of state variables
y_i	=	output to be controlled

Greek Letters

$\rho_{i\kappa}$ = relative order of the output y_i with respect to the disturbance d_κ

Math Symbols

$\cong$	=	approximately equal to
$>$	=	graeter than
$\in$	=	belongs to
$\subset$	=	subset
$\forall$	=	for all
T	=	transpose
!	=	factorial
∞	=	infinity
min	=	minimum element
$\mathbb{R}$	=	real line
$\mathbb{R}^n$	=	$n-$dimensional Euclidean space

Acronyms

SISO	=	single-input single-output
MIMO	=	multiple-input multiple-output

CHAPTER III

FEEDFORWARD/FEEDBACK CONTROL OF SISO NONLINEAR PROCESSES

3.1 Introduction

In this chapter, the unified disturbance rejection and set-point tracking problem will be studied for SISO nonlinear processes. More specifically, the problem will be formulated in two-steps: a) a feedforward/state feedback synthesis step, and b) a linear controller design step. In the first step, feedforward/state feedback laws will be calculated that completely eliminate the effect of measurable disturbances on the output, and induce a well-characterized linear input/output behavior. The concept of relative order will arise naturally in the calculation of the control laws and the interpretation of their nature. Important stability notions for the closed-loop system will be studied. Finally, the developed feedforward/feedback control methodology will be applied to composition control in a system of three CSTR's in series.

3.2 Formulation of the feedforward/feedback control problem

SISO nonlinear processes will be considered with a state-space representation of the form:

$$\begin{aligned} \dot{x} &= f(x) + u(t)g(x) + \sum_{\kappa=1}^{p} d_{\kappa}(t) w_{\kappa}(x) \\ y &= h(x) \end{aligned} \tag{3.1}$$

where $x \in X$, $u(t) \in \mathbb{R}$ and $d(t) \in \mathbb{R}$, $\forall t \in [0, \infty)$, and $y \in \mathbb{R}$. A general control problem for processes of the above form, within a feedforward/feedback framework, can be stated as follows:

Calculate a feedforward/feedback control law of the form:

$$u = \mathcal{F}(x, y, d_{\kappa}, y_{sp})$$

where x, y and d_{κ} are measurements of the states, the output and the disturbances, and y_{sp} is the output set-point, which :

- *Rejects the effect of disturbance inputs on the output*
- *Enforces fast and smooth tracking of set-point changes*
- *Maintains stability in the closed-loop system*

Figure 3.1 provides a pictorial representation of the desired control structure.

Clearly, the above formulation of the problem is too general to allow analytical control laws to be derived. For this reason, the following two-step formulation of the problem is proposed, which will lead to a corresponding two-step control methodology:

- **Step 1:** *Calculate a feedforward/state feedback control law of the form:*

$$u = p(x) + q(x)v + q'(x)Q(x, d_{\kappa})$$

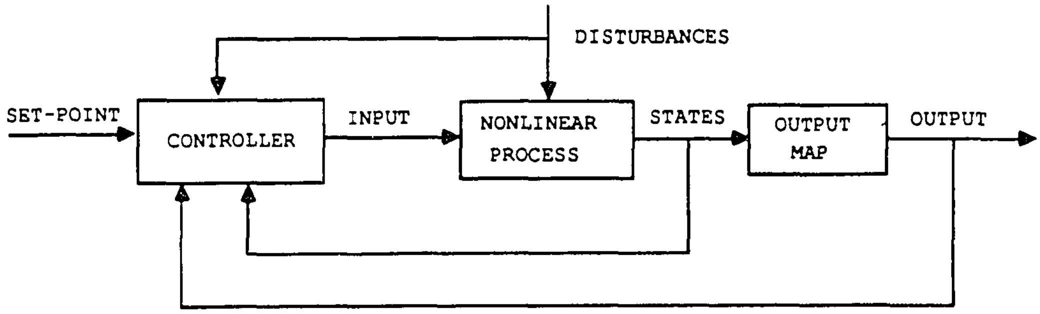

Figure 3.1: General feedforward/feedback control structure

where $p(x)$, $q(x)$ and $q'(x)$ are algebraic functions of the states, with $q(x)$ invertible on X, v is an external reference input and Q is a nonlinear operator that may include time derivatives, which:

- *Completely eliminates the effect of measured disturbances on the output*
- *Induces a linear input/output behavior between y and v of the form:*

$$\sum_k \beta_k \frac{d^k y}{dt^k} = v$$

where β_k are adjustable parameters

- **Step 2:** *Design a linear controller with integral action around the linear v/y loop, to achieve the desired servo and regulatory behavior, in the presence of unmeasured disturbances and/or modeling errors*

The overall control structure resulting from the above two-step methodology is depicted in Figure 3.2.

Step 1 will be referred to as the **feedforward/state feedback synthesis prob-**

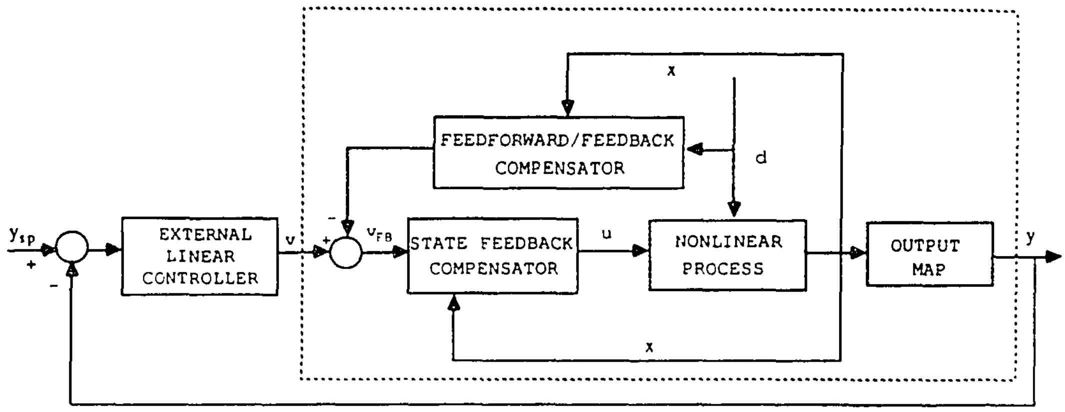

Figure 3.2: Proposed feedforward/feedback control structure

lem and its solution will be the main focus of the subsequent sections. Note that the control law requested in Step 1 can, in general, be a dynamic feedforward/state feedback law, including derivative action on both the state variabes and the disturbance inputs.

Step 2 will be referred to as the **linear controller design problem.** Given the available linear control design methods, the solution to this problem is straightforward and a brief discussion will be included in Section 3.6.

Remark 3.1: The requirement of complete elimination of disturbances in Step 1 is reasonable, since measurements of the disturbances are allowed to be incorporated in the control law. The requirement of linear input/output behavior in the same step is by no means restrictive. As will be shown later, a nonlinear input/output behavior can also be requested and easily achieved. It is the linear dynamics however, that we can better understand and for which we can more conveniently express performance specifications. Moreover, requesting a linear input/output behavior in Step 1, allows a straightforward incorporation of integral action in the control structure through the linear control loop of Step 2.

3.3 The feedforward/state feedback synthesis problem

Referring to nonlinear processes in the form of Eq.3.1, let r denote the relative order of the output y with respect to the manipulated input u, i.e., the smallest integer such that:

$$L_g L_f^{r-1} h(x) \not\equiv 0$$

for $x \in X$. Without loss of generality, it will be assumed that X does not contain any singular points, i.e., points $x \in \mathbb{R}^n$ for which $L_g L_f^{r-1} h(x) = 0$. In particular, as long as $L_g L_f^{r-1} h(x_0) \neq 0$, where x_0 is the nominal equilibrium point, by the continuity of $L_g L_f^{r-1} h(x)$ one can always redefine X as an open and connected set that contains x_0 and is such that $L_g L_f^{r-1} h(x) \neq 0$, $\forall x \in X$. Then, let ρ_κ denote the relative order of the output with respect to the disturbance input d_κ, i.e., the smallest integer for which:

$$L_{w_\kappa} L_f^{\rho_\kappa - 1} h(x) \not\equiv 0$$

for $x \in X$.

Based on the results of Chapter II, the above concepts of relative order capture the dynamic effect of the various input variables on the output variable. In particular, they represent a measure of how direct the effect of an input variable (manipulated or disturbance) is on the output variable, in the sense of the number of integrations that the input variable has to go through before it affects the output variable. Thus, by comparing the magnitudes of r and ρ_κ, one can determine which input, the manipulated u or the disturbance d_κ, has more direct effect on the process output. Motivated by the above, the following classification of the disturbances to the classes

$\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ is proposed:

$$\begin{aligned} d_\kappa \in \mathcal{A} &\iff \rho_\kappa > r \\ d_\kappa \in \mathcal{B} &\iff \rho_\kappa = r \\ d_\kappa \in \mathcal{C} &\iff \rho_\kappa < r \end{aligned} \tag{3.2}$$

Referring to the above classification:

- Disturbances that belong to class $\mathcal{A}$ have a less direct effect on the output compared with the effect of the manipulated input
- Disturbances that belong to class $\mathcal{B}$ have an equally direct effect on the output compared with the effect of the manipulated input
- Disturbances that belong to class $\mathcal{C}$ have a more direct effect on the output compared with the effect of the manipulated input

Given the competing nature of manipulated inputs and disturbance inputs from a control point of view, it is intuitively expected that the degree of difficulty of the regulatory control problem will be lower for disturbances that belong to class $\mathcal{A}$ and higher for disturbances that belong to classes $\mathcal{B}$ and $\mathcal{C}$. The above intuitive considerations will arise naturally in the solution of the feedforward/state feedback synthesis problem, which is given in Theorem 3.1 that follows. The proof can be found in Appendix C.

Theorem 3.1: *Consider the SISO nonlinear process described by Eq.3.1. Let r, ρ_κ denote the relative orders of the output y with respect to the manipulated input u and the disturbance inputs d_κ, respectively. Let also $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ denote the classes of disturbances defined in Eq.3.2. Then, a feedforward/state feedback law of the form:*

$$u = \left[\beta_r L_g L_f^{r-1} h(x)\right]^{-1} \left\{ v - \sum_{k=0}^{r} \beta_k L_f^k h(x) - \sum_{d_\kappa \in \mathcal{B}} \beta_r d_\kappa(t) L_{w_\kappa} L_f^{r-1} h(x) \right.$$

$$-\sum_{d_\kappa \in \mathcal{C}} \sum_{l=0}^{(r-\rho_\kappa)} \sum_{k=\rho_\kappa+l}^{r} \beta_k \frac{d^l}{dt^l} \left(d_\kappa(t) L_{w_\kappa} L_f^{k-l-1} h(x) \right) \Bigg\} \tag{3.3}$$

- *Completely eliminates the effect of the disturbances d_κ on y*

- *Induces the linear input/output behavior:*

$$\sum_{k=1}^{r} \beta_k \frac{d^k y}{dt^k} = v \tag{3.4}$$

where β_k are adjustable parameters

Remark 3.2: The feedforward/state feedback law of Eq.3.3 is composed of:

- a pure state feedback component, which is static in nature:

$$u = \frac{v_{FB} - \sum_{k=0}^{r} \beta_k L_f^k h(x)}{\beta_r L_g L_f^{r-1} h(x)} \tag{3.5}$$

- a feedforward/state feedback component, which in general involves anticipatory action:

$$v_{FB} = v - \sum_{d_\kappa \in \mathcal{B}} \beta_r d_\kappa(t) L_{w_\kappa} L_f^{r-1} h(x) - \sum_{d_\kappa \in \mathcal{C}} \sum_{l=0}^{(r-\rho_\kappa)} \sum_{k=\rho_\kappa+l}^{r} \beta_k \frac{d^l}{dt^l} \left(d_\kappa(t) L_{w_\kappa} L_f^{k-l-1} h(x) \right) \tag{3.6}$$

These two parts are clearly depicted as two separate compensators in the control structure of Figure 3.2.

Remark 3.3: The feedforward/state feedback component v_{FB} of Eq.3.6 contains explicit compensation terms for each disturbance that belongs to the classes $\mathcal{B}$ and $\mathcal{C}$. Note the distinct and intuitively consistent nature of these compensation terms. Disturbances that belong to class $\mathcal{B}$ require **static** feedforward/state feedback compensation. This is consistent with the argument that these disturbances have an equally direct effect on the output compared with the effect of the manipulated input. Disturbances that belong to class $\mathcal{C}$ require **dynamic** feedforward/state feedback compensation. This is consistent with the argument that these disturbances have a more direct effect on the output compared with the effect of the manipulated input. Finally, disturbances that belong to class $\mathcal{A}$ and have a less direct effect on the output than the manipulated input, do not require **any** feedforward compensation. Measurements of these disturbances are not used in the control law. All the useful information on how these disturbance change is captured in the state measurements. In fact, in the absence of disturbances that belong to classes $\mathcal{B}$ and $\mathcal{C}$, Eq.3.3 reduces to a static state feedback law of the form:

$$u = \frac{v - \sum\limits_{k=0}^{r} \beta_k L_f^k h(x)}{\beta_r L_g L_f^{r-1} h(x)}$$

and the control structure of Figure 3.2 reduces to the Globally Linearizing Control (GLC) structure (Kravaris and Chung, 1987).

Remark 3.4: In the presence of disturbances that belong to class $\mathcal{C}$ the control law of Eq.3.3 includes the time derivatives of state and disturbance dependent terms. For the implementation of these components of the control law, one would have to employ filtering of the data, in order to obtain approximations of the derivative terms.

Remark 3.5: Following a procedure similar to the one the proof of the Theorem 3.1, it is possible to calculate a control law that induces any desired nonlinear disturbance-free behavior of the form:

$$\frac{d^r y}{dt^r} = \Phi(y, \frac{dy}{dt}, \cdots, \frac{d^{r-1} y}{dt^{r-1}}, v) \tag{3.7}$$

Such an extension, however, does not seem particularly meaningful.

Remark 3.6: Consider the class of nonlinear processes of the form:

$$\dot{x} = f(x) + \phi(x, u(t), d^*(t)) g(x) + \sum_{\kappa=1}^{p} d_\kappa(t) w_\kappa(x) \tag{3.8}$$

where $\phi(x,u,d^*)$ is a scalar function solvable for u, and d^* is an additional vector of disturbances. This is a more general class of systems than the one described by Eq.3.1. The proposed methodology can be applied to the above class of systems by simply letting $\phi(x,u,d^*) = \mathcal{U}$, calculating $\mathcal{U}$ from Eq.3.3 and solving for the actual manipulated input u. In this way, compensation for the disturbances d^* is also possible.

Proposition 3.1 that follows provides a solution to the feedforward/state feedback synthesis problem for the special case of a linear process description.

Proposition 3.1 : *Consider a SISO linear system of the form:*

$$\begin{aligned} \dot{x} &= Ax + u(t)b + \sum_{\kappa=1}^{p} d_\kappa(t)\gamma_\kappa \\ y &= cx \end{aligned} \tag{3.9}$$

where A, b, γ_κ, c are matrices of appropriate dimensions. Let r, ρ_κ denote the relative orders of the output y with respect to the manipulated input u and the disturbance inputs d_κ, respectively. Let also $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ denote the classes of disturbances defined in Eq.3.2. Then, a feedforward/state feedback law of the form:

$$\begin{aligned} u = \left[\beta_r c A^{r-1} b\right]^{-1} \Bigg\{ v - \sum_{k=0}^{r} \beta_k c A^k x - \sum_{d_\kappa \in \mathcal{B}} \beta_r d_\kappa(t) c A^{r-1}\gamma_\kappa \\ - \sum_{d_\kappa \in \mathcal{C}} \sum_{l=0}^{(r-\rho_\kappa)} \sum_{k=\rho_\kappa+l}^{r} \beta_k c A^{k-l-1}\gamma_\kappa \frac{d^l}{dt^l}\left(d_\kappa(t)\right) \Bigg\} \end{aligned} \tag{3.10}$$

- *Completely eliminates the effect of the disturbances d_κ on y*

- *Induces the input/output behavior:*

$$\sum_{k=1}^{r} \beta_k \frac{d^k y}{dt^k} = v$$

where β_k are adjustable parameters

Proof: It is straightforward to verify that, for $f(x) = Ax$, $g(x) = b$, $w_\kappa(x) = \gamma_\kappa$ and $h(x) = cx$, the following relations hold:

$$L_f^k h(x) = cA^k x$$

$$L_g L_f^k h(x) = cA^k b$$

$$L_{w_\kappa} L_f^k h(x) = cA^k \gamma_\kappa$$

Substituting the above relations to the control law of Eq.3.3, Eq.3.10 is easily obtained.

The result of Proposition 3.1 lends itself to a number of important observations. More specifically:

- According to Proposition 3.1, the solution to the feedforward/state feedback synthesis problem for linear systems is a control law, which may be dynamic in the disturbance inputs, but is always **static** in the state variables. On the other hand, Eq.3.3 indicates that the corresponding solution for nonlinear systems is a control law which may be dynamic both in the state variables and the disturbance inputs. Therefore, from a theoretical point of view it would be very interesting to characterize the class of nonlinear systems for which a static in the states control law solves the posed synthesis problem.

- From a practical point of view, the computational effort required for the implementation of the dynamic components of the control law of Eq.3.3 increases considerably with their complexity (see also Remark 3.4). Clearly, a control law which is static in the states would be significantly easier to implement. It would simply require filtering of the disturbance measurements in order to suppress noise effects, and this can be easily achieved using an appropriate low-pass filter.

Motivated by the above considerations, the rest of this section will be devoted to obtaining a characterization of the class of nonlinear systems for which, a feedforward/**static** state feedback law of the form:

$$u = p(x) + q(x)v + Q'\left(x, d(t), d(t)^{(1)}, d(t)^{(2)}, \cdots\right) \tag{3.11}$$

where $p(x), q(x)$ are algebraic functions of the states with $q(x)$ invertible on X, and Q' is an algebraic function which is nonsingular under nominal conditions (i.e., remains finite when $d(t) = 0$), induces the input/output behavior of Eq.3.4 independently of the values of the disturbance inputs.

Without loss of generality and in order to simplify the notation, SISO nonlinear systems with a single disturbance (SISOSD) will be considered initially, i.e., systems of the form:

$$\begin{aligned} \dot{x} &= f(x) + u(t)g(x) + d(t)w(x) \\ y &= h(x) \end{aligned} \tag{3.12}$$

For such systems, the relative order of the output with respect to the manipulated input will be denoted by r, while the relative order of the output with respect to the disturbance input will be denoted by ρ. Then, according to Theorem 3.1, a control law which eliminates the effect of d on y and induces the input/output behavior of Eq.3.4, has the form:

$$u = \begin{cases} \dfrac{v - \sum\limits_{k=0}^{r} \beta_k L_f^k h(x)}{\beta_r L_g L_f^{r-1} h(x)}, & if\ \rho > r \\[2ex] \dfrac{v - \sum\limits_{k=0}^{r} \beta_k L_f^k h(x) - \beta_r d(t) L_w L_f^{r-1} h(x)}{\beta_r L_g L_f^{r-1} h(x)}, & if\ \rho = r \\[2ex] \dfrac{v - \sum\limits_{k=0}^{r} \beta_k L_f^k h(x) - \sum\limits_{l=0}^{(r-\rho)} \sum\limits_{k=\rho+l}^{r} \beta_k \dfrac{d^l}{dt^l}\left(d(t) L_w L_f^{k-l-1} h(x)\right)}{\beta_r L_g L_f^{r-1} h(x)}, & if\ \rho < r \end{cases} \tag{3.13}$$

Obviously, the case of $\rho \geq r$ is of no interest, since in this case the resulting control law does not require any dynamic compensation. Proposition 3.2 that follows provides necessary and sufficient conditions in order for a control law of the form of Eq.3.11 to induce the disturbance-free input/output behavior of Eq.3.4 in the case that $\rho < r$. The proof can be found in Appendix C.

Proposition 3.2: *Consider the nonlinear system of Eq.3.12. Let r and ρ denote the relative orders of the output y with respect to the manipulated input u and the disturbance input d, respectively, with $\rho < r$. Then, the conditions:*

$$L_g\phi_\ell(x, d(t)) \equiv 0, \quad \ell = 0, 1, \cdots, r - \rho - 1 \tag{3.14}$$

where

$$\phi_\ell(x, d(t)) = \sum_{\mu=0}^{\ell} L_f^{\ell-\mu}\left(d(t)L_w + \frac{\partial}{\partial t}\right)\left(L_f + d(t)L_w + \frac{\partial}{\partial t}\right)^{\mu} L_f^{\rho-1}h(x) \tag{3.15}$$

are necessary and sufficient in order for a feedforward/static state feedback law of the form of Eq.3.11 to:

- *Completely eliminate the effect of the disturbance d on y, and*
- *Induce the linear input/output behavior:*

$$\sum_{k=1}^{r} \beta_k \frac{d^k y}{dt^k} = v$$

  where β_k are adjustable parameters.

If these conditions are satisfied, the appropriate control law takes the form:

$$u = \left[\beta_r L_g L_f^{r-1}h(x)\right]^{-1}\left\{v - \sum_{k=0}^{r} \beta_k L_f^k h(x) \right.$$
$$\left. - \sum_{k=\rho}^{r} \beta_k \phi_{k-\rho}\left(x, d(t), d(t)^{(1)}, \cdots, d(t)^{(k-\rho)}\right)\right\} \tag{3.16}$$

Remark 3.7: The multiplication of the operators $\left(L_f + d(t)L_w + \frac{\partial}{\partial t}\right)$ is not associative. Therefore, the following convention is assumed in Eq.3.15:

$$\left(L_f + d(t)L_w + \tfrac{\partial}{\partial t}\right)^{\mu} L_f^{\rho-1}h(x) =$$
$$\left(L_f + d(t)L_w + \tfrac{\partial}{\partial t}\right)\left(\left(L_f + d(t)L_w + \tfrac{\partial}{\partial t}\right)^{\mu-1} L_f^{\rho-1}h(x)\right), \quad \mu = 1, 2, \cdots$$

Remark 3.8: The functions ϕ_ℓ defined in Eq.3.15 take the following form for various values of ℓ:

- $\phi_0\left(x, d(t)\right) = d(t)L_wL_f^{\rho-1}h(x)$

- $\phi_1\left(x, d(t), d(t)^{(1)}\right) =$
$$d(t)\left[L_fL_wL_f^{\rho-1}h(x) + L_wL_f^{\rho}h(x)\right]$$
$$+d(t)^2\left[L_w^2L_f^{\rho-1}h(x)\right] + d(t)^{(1)}\left[L_wL_f^{\rho-1}h(x)\right]$$

- $\phi_2\left(x, d(t), d(t)^{(1)}, d(t)^{(2)}\right) =$
$$d(t)\left[L_f^2L_wL_f^{\rho-1}h(x) + L_fL_wL_f^{\rho}h(x) + L_wL_f^{\rho+1}h(x)\right]$$
$$+d(t)^2\left[L_fL_w^2L_f^{\rho-1}h(x) + L_wL_fL_wL_f^{\rho-1}h(x) + L_w^2L_f^{\rho}h(x)\right]$$
$$+d(t)^3\left[L_w^3L_f^{\rho-1}h(x)\right]$$
$$+d(t)^{(1)}\left[L_wL_f^{\rho}h(x) + 2L_fL_wL_f^{\rho-1}h(x)\right]$$
$$+d(t)d(t)^{(1)}\left[3L_w^2L_f^{\rho-1}h(x)\right] + d(t)^{(2)}\left[L_wL_f^{\rho-1}h(x)\right]$$

- $\phi_3\left(x, d(t), d(t)^{(1)}, d(t)^{(2)}, d(t)^{(3)}\right) =$
$$d(t)\left[L_wL_f^{\rho+2}h(x) + L_fL_wL_f^{\rho+1}h(x) + L_f^2L_wL_f^{\rho}h(x) + L_f^3L_wL_f^{\rho-1}h(x)\right]$$
$$+d(t)^2\left[L_w^2L_f^{\rho+1}h(x) + L_wL_fL_wL_f^{\rho}h(x) + L_wL_f^2L_wL_f^{\rho-1}h(x) + L_fL_w^2L_f^{\rho}h(x)\right.$$
$$\left.+L_fL_wL_fL_wL_f^{\rho-1}h(x) + L_f^2L_w^2L_f^{\rho-1}h(x)\right]$$
$$+d(t)^3\left[L_w^3L_f^{\rho}h(x) + L_w^2L_fL_wL_f^{\rho-1}h(x) + L_wL_fL_w^2L_f^{\rho-1}h(x) + L_fL_w^3L_f^{\rho-1}h(x)\right]$$
$$+d(t)^4\left[L_w^4L_f^{\rho-1}h(x)\right]$$
$$+d(t)^{(1)}\left[L_wL_f^{\rho+1}h(x) + 2L_fL_wL_f^{\rho}h(x) + 3L_f^2L_wL_f^{\rho-1}h(x)\right]$$

$$+d(t)d(t)^{(1)}\left[3L_w^2L_f^{\rho}h(x)+4L_wL_fL_wL_f^{\rho-1}h(x)+5L_fL_w^2L_f^{\rho-1}h(x)\right]$$
$$+d(t)^2d(t)^{(1)}\left[6L_w^3L_f^{\rho-1}h(x)\right]+d(t)^{(1)}d(t)^{(1)}\left[3L_w^2L_f^{\rho-1}h(x)\right]$$
$$+d(t)^{(2)}\left[L_wL_f^{\rho}h(x)+3L_fL_wL_f^{\rho-1}h(x)\right]$$
$$+d(t)d(t)^{(2)}\left[4L_w^2L_f^{\rho-1}h(x)\right]+d(t)^{(3)}\left[L_wL_f^{\rho-1}h(x)\right]$$

- $\phi_4\left(x,d(t),d(t)^{(1)},d(t)^{(2)},d(t)^{(3)},d(t)^{(4)}\right)=$

$$d(t)\left[L_wL_f^{\rho+3}h(x)+L_fL_wL_f^{\rho+2}h(x)+L_f^2L_wL_f^{\rho+1}h(x)+L_f^3L_wL_f^{\rho}h(x)\right.$$
$$\left.+L_f^4L_wL_f^{\rho-1}h(x)\right]$$
$$+d(t)^2\left[L_w^2L_f^{\rho+2}h(x)+L_wL_fL_wL_f^{\rho+1}h(x)+L_wL_f^2L_wL_f^{\rho}h(x)+L_wL_f^3L_wL_f^{\rho-1}h(x)\right.$$
$$+L_fL_w^2L_f^{\rho+1}h(x)+L_fL_wL_fL_wL_f^{\rho}h(x)+L_f^2L_w^2L_f^{\rho}h(x)$$
$$\left.+L_fL_wL_f^2L_wL_f^{\rho-1}h(x)+L_f^2L_wL_fL_wL_f^{\rho-1}h(x)+L_f^3L_w^2L_f^{\rho-1}h(x)\right]$$
$$+d(t)^3\left[L_w^3L_f^{\rho+1}h(x)+L_w^2L_fL_wL_f^{\rho}h(x)+L_wL_fL_w^2L_f^{\rho}h(x)+L_fL_w^3L_f^{\rho}h(x)\right.$$
$$+L_w^2L_f^2L_wL_f^{\rho-1}h(x)+L_wL_fL_wL_fL_wL_f^{\rho-1}h(x)+L_wL_f^2L_w^2L_f^{\rho-1}h(x)$$
$$\left.+L_fL_w^2L_fL_wL_f^{\rho-1}h(x)+L_fL_wL_fL_w^2L_f^{\rho-1}h(x)+L_f^2L_w^3L_f^{\rho-1}h(x)\right]$$
$$+d(t)^4\left[L_w^4L_f^{\rho}h(x)+L_w^3L_fL_wL_f^{\rho-1}h(x)+L_wL_fL_w^3L_f^{\rho-1}h(x)+L_w^2L_fL_w^2L_f^{\rho-1}\right.$$
$$\left.+L_fL_w^4L_f^{\rho-1}h(x)\right]$$
$$+d(t)^5\left[L_w^5L_f^{\rho-1}h(x)\right]$$
$$+d(t)^{(1)}\left[L_wL_f^{\rho+2}h(x)+2L_fL_wL_f^{\rho+1}h(x)+3L_f^2L_wL_f^{\rho}h(x)+4L_f^3L_wL_f^{\rho-1}h(x)\right]$$
$$+d(t)d(t)^{(1)}\left[3L_w^2L_f^{\rho+1}h(x)+4L_wL_fL_wL_f^{\rho}h(x)+5L_fL_w^2L_f^{\rho}h(x)\right.$$
$$\left.+5L_wL_f^2L_wL_f^{\rho-1}h(x)+6L_fL_wL_fL_wL_f^{\rho-1}h(x)+7L_f^2L_w^2L_f^{\rho-1}h(x)\right]$$
$$+d(t)^2d(t)^{(1)}\left[6L_w^3L_f^{\rho}h(x)+7L_w^2L_fL_wL_f^{\rho-1}h(x)+8L_wL_fL_w^2L_f^{\rho-1}h(x)\right.$$
$$\left.+9L_fL_w^3L_f^{\rho-1}h(x)\right]$$
$$+d(t)^3d(t)^{(1)}\left[10L_w^4L_f^{\rho-1}h(x)\right]$$
$$+d(t)^{(1)}d(t)^{(1)}\left[3L_w^2L_f^{\rho}h(x)+4L_wL_fL_wL_f^{\rho-1}h(x)+8L_fL_w^2L_f^{\rho-1}h(x)\right]$$
$$+d(t)d(t)^{(1)}d(t)^{(1)}\left[15L_w^3L_f^{\rho-1}h(x)\right]$$
$$+d(t)^{(2)}\left[L_wL_f^{\rho+1}h(x)+3L_fL_wL_f^{\rho}h(x)+6L_f^2L_wL_f^{\rho-1}h(x)\right]$$

$$+d(t)d(t)^{(2)}\left[4L_w^2L_f^\rho h(x)+7L_wL_fL_wL_f^{\rho-1}h(x)+9L_fL_w^2L_f^{\rho-1}h(x)\right]$$
$$+d(t)^2d(t)^{(2)}\left[10L_w^3L_f^{\rho-1}h(x)\right]+d(t)^{(1)}d(t)^{(2)}\left[10L_w^2L_f^{\rho-1}h(x)\right]$$
$$+d(t)^{(3)}\left[L_wL_f^\rho h(x)+4L_fL_wL_f^{\rho-1}h(x)\right]$$
$$+d(t)d(t)^{(3)}\left[5L_w^2L_f^{\rho-1}h(x)\right]+d(t)^{(4)}\left[L_wL_f^{\rho-1}h(x)\right]$$

- etc.

Under nominal conditions (i.e., $d(t)=0$), the above functions vanish and the control law of Eq.3.16 reduces to a static state feedback law, as expected. Furthermore, for $d(t)$ a sufficiently smooth function of time, the control law of Eq.3.16 is nonsingular for all values of the disturbance.

Remark 3.9: It is important to note that the functions $\phi_\ell\left(x,d(t),d(t)^{(1)},\cdots,d(t)^{(\ell)}\right)$ are polynomial forms in $d(t)$ and its derivatives. Therefore, the conditions of Eq.3.14 should be interpreted in the sense that all the x-dependent coefficients of the polynomials must be identically equal to zero. More specifically, the pattern of the conditions develops as follows:

- $r-\rho=1$:

$$L_g\phi_0\left(x,d(t)\right)\equiv 0$$

which implies:

$$L_gL_wL_f^{\rho-1}h(x) = 0 \tag{3.17}$$

- $r-\rho=2$:

$$\begin{aligned}L_g\phi_0\left(x,d(t)\right) &\equiv 0\\ L_g\phi_1\left(x,d(t),d(t)^{(1)}\right) &\equiv 0\end{aligned}$$

which, in addition to Eq.3.17, also imply:

$$\begin{aligned}L_g\left(L_wL_f+L_fL_w\right)L_f^{\rho-1}h(x) &= 0\\ L_gL_w^2L_f^{\rho-1}h(x) &= 0\end{aligned} \tag{3.18}$$

- $r - \rho = 3$:

$$\begin{aligned} L_g \phi_0 \left(x, d(t)\right) &\equiv 0 \\ L_g \phi_1 \left(x, d(t), d(t)^{(1)}\right) &\equiv 0 \\ L_g \phi_2 \left(x, d(t), d(t)^{(1)}, d(t)^{(2)}\right) &\equiv 0 \end{aligned}$$

which, in addition to Eqs.3.17 and 3.18, also imply:

$$\begin{aligned} L_g \left(L_w L_f + 2 L_f L_w\right) L_f^{\rho-1} h(x) &= 0 \\ L_g \left(L_w^2 L_f + L_w L_f L_w + L_f L_w^2\right) L_f^{\rho-1} h(x) &= 0 \\ L_g \left(L_w L_f^2 + L_f L_w L_f + L_f^2 L_w\right) L_f^{\rho-1} h(x) &= 0 \\ L_g L_w^3 L_f^{\rho-1} h(x) &= 0 \end{aligned} \tag{3.19}$$

- $r - \rho = 4$:

$$\begin{aligned} L_g \phi_0 \left(x, d(t)\right) &\equiv 0 \\ L_g \phi_1 \left(x, d(t), d(t)^{(1)}\right) &\equiv 0 \\ L_g \phi_2 \left(x, d(t), d(t)^{(1)}, d(t)^{(2)}\right) &\equiv 0 \\ L_g \phi_3 \left(x, d(t), d(t)^{(1)}, d(t)^{(2)}, d(t)^{(3)}\right) &\equiv 0 \end{aligned}$$

which, in addition to Eqs.3.17, 3.18 and 3.19, also imply:

$$\begin{aligned} L_g \left(L_w L_f^3 + L_f L_w L_f^2 + L_f^2 L_w L_f + L_f^3 L_w\right) L_f^{\rho-1} h(x) &= 0 \\ L_g \left(L_w^2 L_f^2 + L_w L_f L_w L_f + L_f^2 L_w^2 + L_f L_w^2 L_f + L_w L_f^2 L_w \right. & \\ \left. + L_f L_w L_f L_w\right) L_f^{\rho-1} h(x) &= 0 \\ L_g \left(L_w^3 L_f + L_w^2 L_f L_w + L_w L_f L_w^2 + L_f L_w^3\right) L_f^{\rho-1} h(x) &= 0 \\ L_g \left(L_w L_f^2 + 2 L_f L_w L_f + 3 L_f^2 L_w\right) L_f^{\rho-1} h(x) &= 0 \\ L_g \left(3 L_w^2 L_f + 4 L_w L_f L_w + 5 L_f L_w^2\right) L_f^{\rho-1} h(x) &= 0 \\ L_g L_w^4 L_f^{\rho-1} h(x) &= 0 \end{aligned} \tag{3.20}$$

- etc.

Remark 3.10: Suppose that the conditions of Eq.3.14 are violated, and, in particular, let $r' < r$ be the smallest integer for which:

$$L_g\phi_{r'-\rho-1}\left(x, d(t), d(t)^{(1)}, \cdots, d(t)^{(r'-\rho-1)}\right) \not\equiv 0 \tag{3.21}$$

Then, it is straightforward to verify that a feedforward/static state feedback law of the form:

$$u = \left[\beta_{r'}L_g\phi_{r'-\rho-1}\left(x, d(t), d(t)^{(1)}, \cdots, d(t)^{(r'-\rho-1)}\right)\right]^{-1}$$
$$\left\{v - \sum_{k=0}^{r'} \beta_k L_f^k h(x) - \sum_{k=\rho}^{r'} \beta_k \phi_{k-\rho}\left(x, d(t), d(t)^{(1)}, \cdots, d(t)^{(k-\rho)}\right)\right\} \tag{3.22}$$

eliminates the effect of the disturbance on the output and induces the input/output behavior:

$$\sum_{k=0}^{r'} \beta_k \frac{d^k y}{dt^k} = v \tag{3.23}$$

The above suggests that, in principle, one can achieve elimination of the disturbance and an input/output behavior of order lower than r, using feedforward/static state feedback. It can be easily verified, however, that:

$$L_g\phi_{r'-\rho-1}\left(x, d(t), d(t)^{(1)}, \cdots, d(t)^{(r'-\rho-1)}\right) = L_g\left(L_f + d(t)L_w + \frac{\partial}{\partial t}\right)^{r'-\rho} L_f^{\rho-1}h(x) \tag{3.24}$$

which is equal to zero when $d(t) = 0$. In other words, under nominal conditions the control law of Eq.3.22 becomes singular, which makes such an approach meaningless.

Example 3.1: Consider the system:

$$\begin{aligned} \dot{x}_1 &= x_2^2 - c_1 + d(t)x_1x_3 \\ \dot{x}_2 &= x_3 - c_2 \\ \dot{x}_3 &= x_1 - c_3 + u(t) \\ y &= x_1 \end{aligned} \tag{3.25}$$

where c_1, c_2, c_3 are positive real numbers and $x \in \mathbb{R}^3$. Let $x_0 = (c_3, c_1^{0.5}, c_2)^T$ be the nominal equilibrium point (corresponding to $u = d = 0$). For the above system:

$$f(x) = \begin{bmatrix} x_2^2 - c_1 \\ x_3 - c_2 \\ x_1 - c_3 \end{bmatrix}, \; g(x) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; w(x) = \begin{bmatrix} x_1 x_3 \\ 0 \\ 0 \end{bmatrix}, \; h(x) = x_1 \qquad (3.26)$$

It can be easily verified that:

$$\begin{aligned} L_g h(x) &= L_g L_f h(x) = 0 \\ L_g L_f^2 h(x) &= 2x_2 \not\equiv 0 \end{aligned}$$

Consequently, the relative order of y with respect to u is $r = 3$. Let:

$$S = \left\{ x \in \mathbb{R}^3 : x_2 = 0 \right\}$$

Define a set $X \subset \mathbb{R}^3$ with the following properties:

- X is connected and open
- X contains the nominal equilibrium point x_0
- $X \bigcap S = \emptyset$

The set X will be considered to be the state space for Eq.3.25. Then,

$$L_w h(x) = x_1 x_3 \not\equiv 0$$

for $x \in X$. Consequently, the relative order of y with respect to d is $\rho = 1$. Clearly, $\rho < r$ and the control law:

$$u = \frac{1}{\beta_3 L_g L_f^2 h(x)} \left(v - \sum_{k=0}^{3} \beta_k L_f^k h(x) - \sum_{l=0}^{2} \sum_{k=1+l}^{3} \beta_k \frac{d^l}{dt^l} \left(d(t) L_w L_f^{k-l-1} h(x) \right) \right) \qquad (3.27)$$

enforces the input/output behavior:

$$\sum_{k=0}^{3} \beta_k \frac{d^k y}{dt^k} = v \qquad (3.28)$$

independently of the values of the disturbance $d(t)$.

It can be easily verified that the conditions of Eq.3.14 are not satisfied. More specifically, $r - \rho = 2$, and for $\ell = 0$:

$$L_g L_w h(x) = x_1 \not\equiv 0 \tag{3.29}$$

Therefore, dynamic feedforward/state feedback is necessary in order to achieve the input/output behavior of Eq.3.28 for all values of $d(t)$.

Theorem 3.2 that follows generalizes the result of Proposition 3.2 for general SISO nonlinear systems of the form of Eq.3.1. The proof of Theorem 3.2 is completely analogous to the one of Proposition 3.1 and is omitted for brevity.

Theorem 3.2 : *Consider the SISO nonlinear process described by Eq.3.1. Let r, ρ_κ denote the relative orders of the output y with respect to the manipulated input u and the disturbance inputs d_κ, respectively. Let also $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ denote the classes of disturbances defined in Eq.3.2 and assume that $\mathcal{C} \neq \emptyset$. Then, the conditions:*

$$L_g \phi_\ell (x, d(t)) \equiv 0, \quad \ell = 0, 1, \cdots, r - \rho_* - 1 \tag{3.30}$$

where

$$\phi_\ell (x, d(t)) = \sum_{\mu=0}^{\ell} L_f^{\ell-\mu} \left(\sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} + \frac{\partial}{\partial t} \right) \left(L_f + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} + \frac{\partial}{\partial t} \right)^{\mu} L_f^{\rho_*-1} h(x) \tag{3.31}$$

and

$$\rho_* = min \{\rho_1, \cdots, \rho_p\} \tag{3.32}$$

are necessary and sufficient in order for a feedforward/static state feedback law of the form of Eq.3.11 to:

- *Completely eliminate the effect of the disturbance d on y, and*

- *Induce the linear input/output behavior:*

$$\sum_{k=1}^{r} \beta_k \frac{d^k y}{dt^k} = v$$

where β_k are adjustable parameters.

If these conditions are satisfied, the appropriate control law takes the form:

$$u = \left[\beta_r L_g L_f^{r-1} h(x)\right]^{-1} \left\{ v - \sum_{k=0}^{r} \beta_k L_f^k h(x) - \sum_{d_\kappa \in \mathcal{B}} \beta_r d_\kappa(t) L_{w_\kappa} L_f^{r-1} h(x) \right.$$
$$\left. - \sum_{k=\rho_*}^{r} \beta_k \phi_{k-\rho_*} \left(x, d(t), d(t)^{(1)}, \cdots, d(t)^{(k-\rho_*)}\right) \right\} \tag{3.33}$$

Remark 3.11: The results of Theorems 3.1 and 3.2 and Proposition 3.1 illustrate the fundamental differences in the solution of the feedforward/state feedback synthesis problem between linear and nonlinear systems. In particular, for linear systems, the solution to the feedforward/state feedback synthesis problem is a control law which is **always** static in the states, and affine in the disturbances and their derivatives (Eq.3.10). On the other hand, it is only for a **specific** class of nonlinear systems (i.e., those satisfying the conditions of Eq.3.14) that the resulting control law is static in the states. Even for this class of systems, however, the control law is not affine in the disturbances and their derivatives, in general (Eq.3.33).

3.4 Closed-loop stability considerations

The purpose of this section is to develop a framework that will allow the characterization of the stability of the closed-loop system under the control law of Eq.3.3.

Clearly, the BIBO stability characteristics of the $v - y$ system depend on the location of the roots of the characteristic polynomial:

$$\beta_0 + \beta_1 s + \beta_2 s^2 + \cdots + \beta_r s^r = 0$$

Since the parameters β_k are adjustable, they can be chosen by the designer for input/output stability and fast dynamics. In the absence of disturbances, ISE-optimal response for step changes in v can be achieved as the roots of the above characteristic equation approach $-\infty$.

An interesting observation regarding the $v - y$ system is that the relative order between y and v is exactly equal to r, i.e., in loose temrs, relative order is preserved under the control law of Eq.3.3. Moreover, the order of the $v - y$ system is equal to r, where $r \leq n$, and the $v - y$ system has no zeros. This effective reduction in the order of the original system implies that part of the dynamics of the original system has become unobservable. Intuitively, the unobservable dynamics must be stable in some sense, in order for the closed-loop system to be well behaved. More precisely, in addition to input/output stability, **asymptotic stability of the states** of the unforced closed-loop system is also required. In the absence of disturbance inputs, this would be equivalent to the requirement of internal stability for the closed-loop system, i.e., the requirement of asymptotic stability of the states with respect to perturbations in the initial conditions. Conditions that guarantee the internal stability of the closed-loop system, in the context of input/output linearization, have been obtained based on the concepts of zero dynamics (Byrnes and Isidori, 1985) and forced zero dynamics (Kravaris, 1988). These concepts have provided meaningful generalizations of the notion of zeros in linear systems, in a nonlinear context. In the case of nonlinear systems with disturbances, a characterization of the asymptotic stability of the states must be more general, including stability with respect to the disturbance inputs, as well as the initial conditions. Such a characterization will be obtained through a concept analogous to the zero dynamics of a disturbance-free system.

At first, one must observe that when the system of Eq.3.1 is subject to the control law of Eq.3.3, the output dynamics of the unforced closed-loop system is governed by:

$$\sum_{k=0}^{r} \beta_k \frac{d^k y}{dt^k} = 0$$

under appropriate initial conditions. Thus, by choosing the adjustable parameters β_k so that closed-loop system is BIBO stable, any initial conditions of the states will generate exponentially decaying signals for the output y and its derivatives $\frac{dy}{dt}, \cdots, \frac{d^{r-1}y}{dt^{r-1}}$. Moreover, the output and its derivatives will get arbitrarily close to zero in finite time. Consequently, the asymptotic stability of the states (i.e., the stability as $t \rightarrow \infty$) of the unforced closed-loop system will depend, for all practical purposes, on the asymptotic stability characteristics of the dynamic system resulting when $y = \frac{dy}{dt} = \cdots = \frac{d^{r-1}y}{dt^{r-1}} = 0$. The above considerations motivate the definition of a concept of zero dynamics for a system described by Eq.3.1, as its dynamics when the output y is constrained to remain at zero for all times. The above concept of zero dynamics is consistent with the Byrnes-Isidori concept of zero dynamics for disturbance-free systems. In what follows, a normal form for systems of the form of Eq.3.1 will be introduced and will be used for the calculation of a representation of the zero dynamics for such systems.

Referring to SISO nonlinear systems of the form of Eq.3.1, consider the following nonlinear coordinate transformation (Byrnes and Isidori, 1985):

$$\zeta = T(x) = \begin{bmatrix} t_1(x) \\ \vdots \\ t_{n-r}(x) \\ h(x) \\ L_f h(x) \\ \vdots \\ L_f^{r-1} h(x) \end{bmatrix} \tag{3.34}$$

where:

- $t_1(x), \cdots, t_{n-r}(x), h(x), L_f h(x), \cdots, L_f^{r-2} h(x), L_f^{r-1} h(x)$ are linearly independent scalar fields
- $L_g t_i(x) = 0,\ i = 1, \cdots, (n-r)$

The above coordinate transformation has been used to obtain a normal-form description of disturbance-free SISO nonlinear systems (Byrnes and Isidori, 1985). In what follows, the same transformation will be used to develop a normal form for systems of the form of Eq.3.1, which include disturbance inputs.

Consider first the case where only disturbances that belong to classes $\mathcal{A}$ and $\mathcal{B}$ are present. Then, the system of Eq.3.1 under the coordinate transformation of Eq.3.34

becomes:

$$
\begin{aligned}
\dot{\zeta}_1 &= L_f t_1(\zeta) + \sum_{d_\kappa \in \mathcal{A},\mathcal{B}} d_\kappa(t) L_{w_\kappa} t_1(\zeta) \\
&\vdots \\
\dot{\zeta}_{n-r} &= L_f t_{n-r}(\zeta) + \sum_{d_\kappa \in \mathcal{A},\mathcal{B}} d_\kappa(t) L_{w_\kappa} t_{n-r}(\zeta) \\
\dot{\zeta}_{n-r+1} &= \zeta_{n-r+2} \\
&\vdots \\
\dot{\zeta}_{n-1} &= \zeta_n \\
\dot{\zeta}_n &= L_f^r h(\zeta) + u(t) L_g L_f^{r-1} h(\zeta) + \sum_{d_\kappa \in \mathcal{B}} d_\kappa(t) L_{w_\kappa} L_f^{r-1} h(\zeta) \\
y &= \zeta_{n-r+1}
\end{aligned}
\tag{3.35}
$$

where the (ζ) dependence in the right hand-side of the above equations implies that the corresponding expressions are evaluated at $x = T^{-1}(\zeta)$. In this new system representation, it is clear how the various disturbances enter the system and affect the output. In particular,

- Disturbances of class $\mathcal{A}$ enter the system only through the first $n - r$ state equations. The first $n - r$ state variables in turn affect the right-hand side of the last state equation and finally through a chain of r integrations, the output $y = \zeta_{n-r+1}$.

- The effect of disturbances of class $\mathcal{B}$ is similar, except that they also affect the right-hand side of the last state equation in a direct way.

Referring to the new system representation given by Eq.3.35, the conditions:

$$y = \frac{dy}{dt} = \cdots = \frac{d^{r-1}y}{dt^{r-1}} = 0$$

imply:

$$\zeta_{n-r+1} = 0 \Rightarrow \zeta_{n-r+2} = 0 \Rightarrow \cdots \Rightarrow \zeta_n = 0$$

Consequently, the zero dynamics of Eq.3.1 is given by the dynamic system:

$$\begin{aligned}
\dot{\zeta}_1 &= L_f t_1(\zeta_1,\cdots,\zeta_{n-r},0,\cdots,0) + \sum_{d_\kappa \in \mathcal{A},\mathcal{B}} d_\kappa(t) L_{w_\kappa} t_1(\zeta_1,\cdots,\zeta_{n-r},0,\cdots,0) \\
&\vdots \\
\dot{\zeta}_{n-r} &= L_f t_{n-r}(\zeta_1,\cdots,\zeta_{n-r},0,\cdots,0) + \sum_{d_\kappa \in \mathcal{A},\mathcal{B}} d_\kappa(t) L_{w_\kappa} t_{n-r}(\zeta_1,\cdots,\zeta_{n-r},0,\cdots,0)
\end{aligned} \tag{3.36}$$

The asymptotic stability characteristics of Eq.3.36 with respect to the disturbance inputs and the initial conditions, will then determine the asymptotic stability of the unforced closed-loop system.

Remark 3.12: In the coordinate system of Eq.3.34, the control law of Eq.3.3 takes the form:

$$u = \left[\beta_r L_g L_f^{r-1}(\zeta)\right]^{-1} \left\{ v - \sum_{k=0}^{r-1} \beta_k \zeta_{n-r+k+1} - \beta_r L_f^r h(\zeta) - \sum_{d_\kappa \in \mathcal{B}} \beta_r d_\kappa(t) L_{w_\kappa} L_f^{r-1} h(\zeta) \right\} \tag{3.37}$$

Substituting the above control law in Eq.3.35, the following closed-loop dynamics is obtained:

$$\begin{aligned}
\dot{\zeta}_1 &= L_f t_1(\zeta) + \sum_{d_\kappa \in \mathcal{A},\mathcal{B}} d_\kappa(t) L_{w_\kappa} t_1(\zeta) \\
&\vdots \\
\dot{\zeta}_{n-r} &= L_f t_{n-r}(\zeta) + \sum_{d_\kappa \in \mathcal{A},\mathcal{B}} d_\kappa(t) L_{w_\kappa} t_{n-r}(\zeta) \\
\dot{\zeta}_{n-r+1} &= \zeta_{n-r+2} \\
&\vdots \\
\dot{\zeta}_{n-1} &= \zeta_n \\
\dot{\zeta}_n &= \frac{1}{\beta_r}(v - \beta_0 \zeta_{n-r+1} - \beta_1 \zeta_{n-r+2} - \cdots - \beta_{r-1}\zeta_n) \\
y &= \zeta_{n-r+1}
\end{aligned} \tag{3.38}$$

Clearly, the first $n-r$ state variables which correspond to the zero dynamics, become unobservable in the $v-y$ system, whose dynamics depends only on the last r state variables and is not affected by the disturbance inputs.

In the general case where disturbances that belong to class $\mathcal{C}$ are also present, the system of Eq.3.1 under the coordinate transformation of Eq.3.34 takes the form:

$$\begin{aligned}
\dot{\zeta}_1 &= L_f t_1(\zeta) + \sum_{d_\kappa \kappa \in \mathcal{A},\mathcal{B},\mathcal{C}} d_\kappa(t) L_{w_\kappa} t_1(\zeta) \\
&\vdots \\
\dot{\zeta}_{n-r} &= L_f t_{n-r}(\zeta) + \sum_{d_\kappa \in \mathcal{A},\mathcal{B},\mathcal{C}} d_\kappa(t) L_{w_\kappa} t_{n-r}(\zeta) \\
\dot{\zeta}_{n-r+1} &= \zeta_{n-r+2} \\
&\vdots \\
\dot{\zeta}_{n-r+\rho_*-1} &= \zeta_{n-r+\rho_*} \\
\dot{\zeta}_{n-r+\rho_*} &= \zeta_{n-r+\rho_*+1} + \sum_{d_\kappa \in \mathcal{C}^{(1)}} d_\kappa(t) L_{w_\kappa} L_f^{\rho_*-1} h(\zeta) \\
\dot{\zeta}_{n-r+\rho_*+1} &= \zeta_{n-r+\rho_*+2} + \sum_{d_\kappa \in \mathcal{C}^{(1)} \cup \mathcal{C}^{(2)}} d_\kappa(t) L_{w_\kappa} L_f^{\rho_*} h(\zeta) \\
&\vdots \\
\dot{\zeta}_{n-1} &= \zeta_n + \sum_{d_\kappa \in \mathcal{C}} d_\kappa(t) L_{w_\kappa} L_f^{r-2} h(\zeta) \\
\dot{\zeta}_n &= L_f^r h(\zeta) + u(t) L_g L_f^{r-1} h(\zeta) + \sum_{d_\kappa \in \mathcal{B},\mathcal{C}} d_\kappa(t) L_{w_\kappa} L_f^{r-1} h(\zeta) \\
y &= \zeta_{n-r+1}
\end{aligned} \tag{3.39}$$

where ρ_* was defined in Eq.3.31, and $\mathcal{C}^{(1)} = \{d_\kappa \in \mathcal{C} : \rho_\kappa = \rho_*\}$, $\mathcal{C}^{(2)} = \{d_\kappa \in \mathcal{C} : \rho_\kappa = \rho_* + 1\}$, etc. In this system representation, it is easy to see that the effect of disturbances of class $\mathcal{C}$ on the output $y = \zeta_{n-r+1}$ is much more direct compared with that of the disturbances of classes $\mathcal{A}$ and $\mathcal{B}$. Disturbances of class $\mathcal{C}$ affect not only the last state equation but also some of the previous state equations, and therefore have to go through a smaller number of integrations before they affect the output y. Imposing the zero-output conditions in the above system representation, we obtain the zero dynamics of Eq.3.1 as the dynamic system:

$$
\begin{aligned}
\dot{\zeta}_1 &= L_f t_1(\zeta) + \sum_{d_\kappa \in \mathcal{A},\mathcal{B},\mathcal{C}} d_\kappa(t) L_{w_\kappa} t_1(\zeta) \\
&\vdots \\
\dot{\zeta}_{n-r} &= L_f t_{n-r}(\zeta) + \sum_{d_\kappa \in \mathcal{A},\mathcal{B},\mathcal{C}} d_\kappa(t) L_{w_\kappa} t_{n-r}(\zeta)
\end{aligned}
\tag{3.40}
$$

subject to the constraints:

$$
\begin{aligned}
\zeta_{n-r+1} &= 0 \\
\zeta_{n-r+2} &= 0 \\
&\vdots \\
\zeta_{n-r+\rho_*} &= 0 \\
\zeta_{n-r+\rho_*+1} &= -\sum_{d_\kappa \in \mathcal{C}^{(1)}} d_\kappa(t) L_{w_\kappa} L_f^{\rho_*-1} h(\zeta) \\
\zeta_{n-r+\rho_*+2} &= -\sum_{d_\kappa \in \mathcal{C}^{(2)}} d_\kappa(t) L_{w_\kappa} L_f^{\rho_*} h(\zeta) \\
&\quad -\sum_{d_\kappa \in \mathcal{C}^{(1)}} \left[d_\kappa(t) L_{w_\kappa} L_f^{\rho_*-1} h(\zeta) + \frac{d}{dt}\left(d_\kappa(t) L_{w_\kappa} L_f^{\rho_*-1} h(\zeta)\right)\right] \\
&\vdots \\
\zeta_n &= -\sum_{d_\kappa \in \mathcal{C}^{(r-\rho_*)}} d_\kappa(t) L_{w_\kappa} L_f^{r-2} h(\zeta) \\
&\quad -\sum_{d_\kappa \in \mathcal{C}^{(r-\rho_*-1)}} \left[d_\kappa(t) L_{w_\kappa} L_f^{r-2} h(\zeta) + \frac{d}{dt}\left(d_\kappa(t) L_{w_\kappa} L_f^{r-3} h(\zeta)\right)\right] - \cdots \\
&\quad -\sum_{d_\kappa \in \mathcal{C}^{(1)}} \left[d_\kappa(t) L_{w_\kappa} L_f^{r-2} h(\zeta) + \frac{d}{dt}\left(d_\kappa(t) L_{w_\kappa} L_f^{r-3} h(\zeta)\right) + \cdots \right. \\
&\quad \left. + \frac{d^{r-\rho_*-1}}{dt^{r-\rho_*-1}}\left(d_\kappa(t) L_{w_\kappa} L_f^{\rho_*-1} h(\zeta)\right)\right]
\end{aligned}
\tag{3.41}
$$

The asymptotic stability characteristics of the above dynamic system will determine the asymptotic stability of the closed-loop system under no external input. Obtaining algebraic expressions for the last $r - \rho_*$ state variables in Eq.3.41 in terms of $\zeta_1, \cdots, \zeta_{n-r}$ and d_κ and their derivatives may not always be possible. The order and the exact state-space realization of the zero dynamics will therefore depend on the specific form of the nonlinear process.

Remark 3.13: In the absence of disturbance inputs (i.e., when $d_\kappa = 0$, $\forall \kappa = 1, \cdots, p$) the zero dynamics defined previously reduces, as expected, to the standard Byrnes-Isidori concept of zero dynamics for disturbance-free systems, i.e., the dynamic system:

$$\begin{aligned} \dot{\zeta}_1 &= L_f t_1(\zeta_1, \cdots, \zeta_{n-r}, 0, \cdots, 0) \\ &\vdots \\ \dot{\zeta}_{n-r} &= L_f t_{n-r}(\zeta_1, \cdots, \zeta_{n-r}, 0, \cdots, 0) \end{aligned} \tag{3.42}$$

Whenever the above system is asymtotically stable, the system of Eq.3.1 is called **minimum-phase**, and the closed-loop system under the control law of Eq.3.3 (with $d_\kappa = 0$) is internally stable.

3.5 The linear controller design problem

The second step of the proposed methodology involves the design of a linear controller with integral action around the linear $v - y$ system, which will reject the effect of modeling errors and/or unmeasured disturbances. **For example**, one can use a PI controller:

$$v = K_c \left[(y_{sp} - y) + \frac{1}{\tau_I} \int_0^t (y_{sp} - y) \right] \tag{3.43}$$

in which case, the overall closed-loop BIBO stability and performance will depend on the location of the roots of the characteristic equation:

$$(\beta_0 + K_c) + \frac{K_c}{\tau_I s} + \beta_1 s + \beta_2 s^2 + \cdots + \beta_r s^r = 0 \tag{3.44}$$

In general, one can choose a linear controller with transfer function:

$$\frac{v(s)}{y_{sp}(s) - y(s)} = \frac{\beta_0 + \beta_1 s + \cdots + \beta_r s^r}{(\epsilon s + 1)^r - 1} \tag{3.45}$$

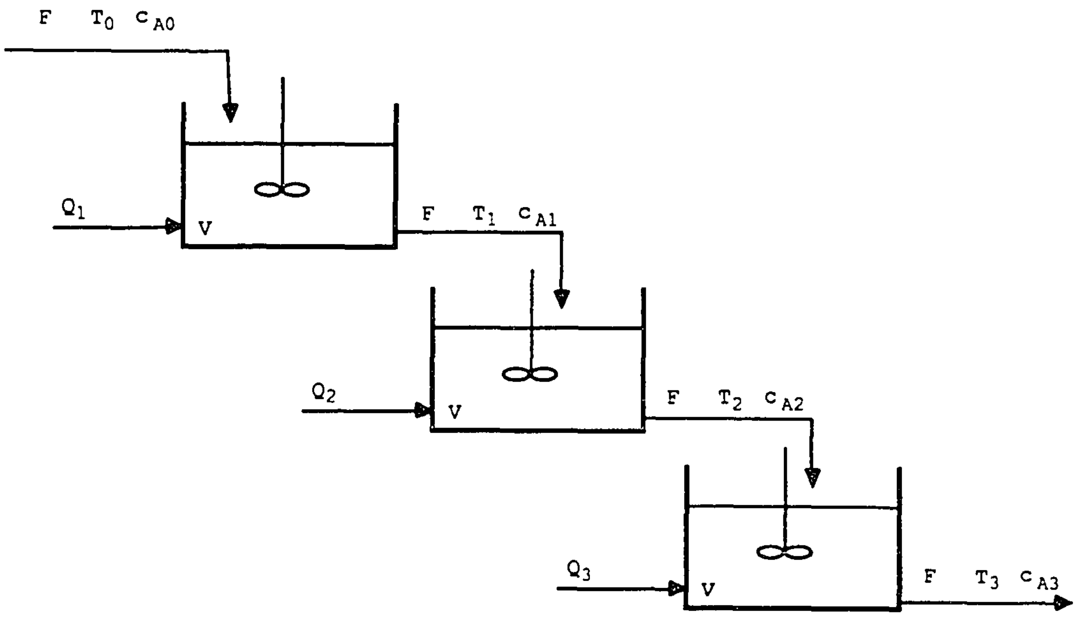

Figure 3.3: A cascade of three continuous stirred tank reactors

in order to induce a critically damped closed-loop response with the transfer function:

$$\frac{y(s)}{y_{sp}(s)} = \frac{1}{(\epsilon s + 1)^r} \tag{3.46}$$

3.6 Application of the control methodology to a cascade of chemical reactors

In this section, the developed control methodology will be applied to composition control of a system of 3 CSTR's in series, where a second order reaction $A \longrightarrow B$ takes place. Figure 3.3 provides a schematic description of the process under consideration.

It is desired to maintain the composition of the stream leaving the last reactor constant, despite fluctuations in the feed temperature and/or composition.

Three cases are examined:

1. Case 1: The major disturbances are the inlet concentration and composition and the manipulated input is the heat input in the third vessel.

2. Case 2: The major disturbance is the inlet temperature and the manipulated input is the heat input in the first vessel.

3. Case 3: The major disturbance is the inlet concentration and the manipulated input is the heat input in the first vessel.

Although Cases 2 and 3 could be examined together, they are examined separately for methodological reasons. It is assumed that both heating and cooling of the reactors is possible. It is also assumed that the inlet and intermediate flowrates, as well as the reactor volumes, are equal and remain constant during the operation. The dynamic equations of the system are the mass and energy balances for each reactor and the form they take under the previous assumptions is :

$$\begin{aligned}
V\frac{dc_{A1}}{dt} &= F(c_{A0}-c_{A1}) - Vk_1{c_{A1}}^2 \\
V\frac{dc_{A2}}{dt} &= F(c_{A1}-c_{A2}) - Vk_2{c_{A2}}^2 \\
V\frac{dc_{A3}}{dt} &= F(c_{A2}-c_{A3}) - Vk_3{c_{A3}}^2 \\
V\rho c_p\frac{dT_1}{dt} &= F\rho c_p(T_0-T_1) + V(-\Delta H)k_1{c_{A1}}^2 + Q_1 \\
V\rho c_p\frac{dT_2}{dt} &= F\rho c_p(T_1-T_2) + V(-\Delta H)k_2{c_{A2}}^2 + Q_2 \\
V\rho c_p\frac{dT_3}{dt} &= F\rho c_p(T_2-T_3) + V(-\Delta H)k_3{c_{A3}}^2 + Q_3
\end{aligned} \tag{3.47}$$

where

$$k_i = k_0 exp(-\frac{E}{RT_i}), \;\; i = 1,2,3 \tag{3.48}$$

The following typical values were given to the process parameters:

$F = 54l/h$, $V = 9l$, $E = 76480J/mol$, $k_0 = 1.25\text{x}10^{14}l/mol.h$,

$-\Delta H = 500\text{x}10^3 J/mol$, $\rho * c_p = 30000 J/l.K$

The steady state values of the process variables were:

$T_{0s} = 298.13K$, $T_{1s} = T_{2s} = T_{3s} = 298.13K$,

$c_{A0s} = 2.1641 mol/l$, $c_{A1s} = 1.1216 mol/l$, $c_{A2s} = 0.7071 mol/l$, $c_{A3s} = 0.5 mol/l$

These conditions correspond to a stable steady state as was verified by the simulations. The states of the system are chosen to be the concentrations and temperatures in each reactor:

$$x_1 = T_1, \quad x_2 = c_{A1}, \quad x_3 = T_2, \quad x_4 = c_{A2}, \quad x_5 = T_3, \quad x_6 = c_{A3}$$

all assumed measurable.

Figure 3.4 provides the digraph corresponding to this process, which can be used to calculate and interpret the relative orders and the nature of the resulting control laws, for the three different cases examined. More precisely :

- Case 1: As shown in Figure 3.4 and can be easily seen from the system dynamic equations, the disturbances and the manipulated input Q_3 affect the output through different paths. A change at T_0 has to go through 3 more states before affecting the output (e.g., $T_0 \longrightarrow T_1 \longrightarrow T_2 \longrightarrow T_3 \longrightarrow y = c_{A3}$). The path is shorter for c_{A0}, for which only 2 states are affected before the output ($c_{A0} \longrightarrow c_{A1} \longrightarrow c_{A2} \longrightarrow y = c_{A3}$). On the other hand, Q_3 causes the change of only 1 state (T_3) before the output. Moreover, since both the manipulated and the controlled variable are in the third tank, the effect of any of the disturbances is actually transfered to the states c_{A2} and T_2. Therefore, no measurement of the disturbances is required in the contro law, provided that the states are measured.

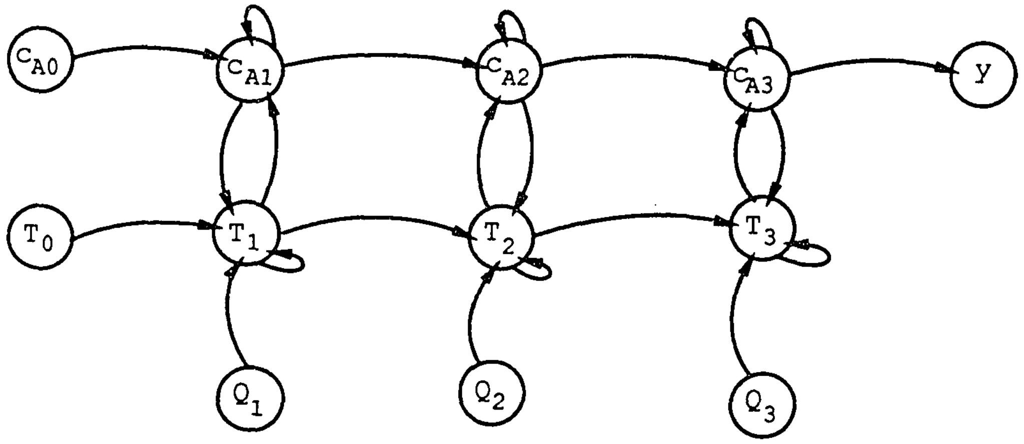

Figure 3.4: The digraph of the reactor cascade

- Case 2: Figure 3.4 shows clearly that both inputs (the disturbance T_0 and the manipulated input Q_1) immediately affect T_1 and then two more states before they affect the output. Physically, the effects of T_0 and Q_1 are very similar and a change ΔT_0 can be eliminated by a change $\Delta Q_1 = -F\rho c_p \Delta T_0$.

- Case 3: Figure 3.4 shows that the shortest path that a change at the disturbance c_{A0} must follow in order to reach the output, involves two intermediate states (c_{A1} and c_{A2}), while the one for the manipulated input Q_1 involves three (e.g. T_1, c_{A1} and c_{A2}). Therefore, the disturbance has a more direct effect on the output than the manipulated input and it is expected that predictive action will be necessary to eliminate the effect of the disturbance.

To evaluate the controller performance we tested:

1. The ability of the closed-loop system to reject disturbances (regulatory behavior)

2. The ability of the closed-loop system to follow set point changes (servo behavior)

For the two disturbance inputs considered, step changes and random noise were applied. In the simulated noise we used a standard deviation equal to 0.5 for the concentration and 20 for the temperature. The step changes were from 2.1641 to $2.5 mol/l$ for the concentration and from 298.13 to $308K$ for the temperature, and they were applied at $TIME = 0.5h$. Since no model uncertainty or unmeasured disturbances were considered, the external PI loop of the control scheme remained inactive. The performance of the feedforward/feedback control methodology (FF/FB) was compared with that of the Globally Linearizing Control (GLC) methodology (Kravaris and Chung, 1987), where no measurements of the disturbances are used in the control law.

Case 1

Setting:

$$u = Q_3 - Q_{3s}$$

$$d_1 = T_0 - T_{0s}$$

$$d_2 = c_{A0} - c_{A0s}$$

the state equations can be put in the form of Eq.3.1, where

$$f(x) = \begin{bmatrix} \frac{F}{V}(T_{0s} - x_1) - \frac{\Delta H}{\rho c_p} k_1(x_1) x_2^2 + \frac{Q_1}{V \rho c_p} \\ \frac{F}{V}(c_{A0s} - x_2) - k_1(x_1) x_2^2 \\ \frac{F}{V}(x_1 - x_3) - \frac{\Delta H}{\rho c_p} k_2(x_3) x_4^2 + \frac{Q_2}{V \rho c_p} \\ \frac{F}{V}(x_2 - x_3) - k_2(x_3) x_3^2 \\ \frac{F}{V}(x_3 - x_5) - \frac{\Delta H}{\rho c_p} k_3(x_5) x_6^2 + \frac{Q_{3s}}{V \rho c_p} \\ \frac{F}{V}(x_4 - x_6) - k_3(x_5) x_6^2 \end{bmatrix} \tag{3.49}$$

$$g(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{V \rho c_p} \\ 0 \end{bmatrix}, w_1(x) = \begin{bmatrix} \frac{F}{V} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, w_2(x) = \begin{bmatrix} 0 \\ \frac{F}{V} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.50}$$

$$h(x) = x_6 \tag{3.51}$$

Calculation of the relative orders yields:

1. $L_g h(x) = 0$

   $L_g L_f h(x) \not\equiv 0$

2. $L_{w_1} h(x) = L_{w_1} L_f h(x) = L_{w_1} L_f^2 h(x) = 0$

   $L_{w_1} L_f^3 h(x) \not\equiv 0$

3. $L_{w_2} h(x) = L_{w_2} L_f h(x) = 0$

   $L_{w_2} L_f^2 h(x) \not\equiv 0$

Consequently, $r = 2$, $\rho_1 = 4$, $\rho_2 = 3$, and $d_1, d_2 \in \mathcal{A}$. Hence, according to Theorem 3.1, the control law :

$$u = \frac{v - \sum_{k=0}^{2} \beta_k L_f^k h(x)}{\beta_2 L_g L_f h(x)} \tag{3.52}$$

eliminates the effect of d_1, d_2 on y and induces the input/output behavior:

$$\sum_{k=1}^{2} \beta_k \frac{d^k y}{dt^k} = v$$

Choosing $\beta_0 = 100$, $\beta_1 = 20$, $\beta_2 = 1$, the poles of the closed-loop system were placed at -10,-10. The behavior of the closed-loop system is shown in Figures 3.5 and 3.6. In Figure 3.5, the output is not affected under step changes and noise in c_{A0} and T_0. In Figure 3.6, the servo behavior of the closed-loop system is shown under noise in c_{A0} and T_0. The response is identical with the one obtained when the disturbances are not present.

Case 2

Setting:

$$u = Q_1 - Q_{1s}$$

$$d = T_0 - T_{0s}$$

the new f,g and w functions are :

$$f(x) = \begin{bmatrix} \frac{F}{V}(T_{0s} - x_1) - \frac{\Delta H}{\rho c_p} k_1(x_1) x_2^2 + \frac{Q_{1s}}{V \rho c_p} \\ \frac{F}{V}(c_{A0} - x_2) - k_1(x_1) x_2^2 \\ \frac{F}{V}(x_1 - x_3) - \frac{\Delta H}{\rho c_p} k_2(x_3) x_4^2 + \frac{Q_2}{V \rho c_p} \\ \frac{F}{V}(x_2 - x_3) - k_2(x_3) x_3^2 \\ \frac{F}{V}(x_3 - x_5) - \frac{\Delta H}{\rho c_p} k_3(x_5) x_6^2 + \frac{Q_3}{V \rho c_p} \\ \frac{F}{V}(x_4 - x_6) - k_3(x_5) x_6^2 \end{bmatrix} \tag{3.53}$$

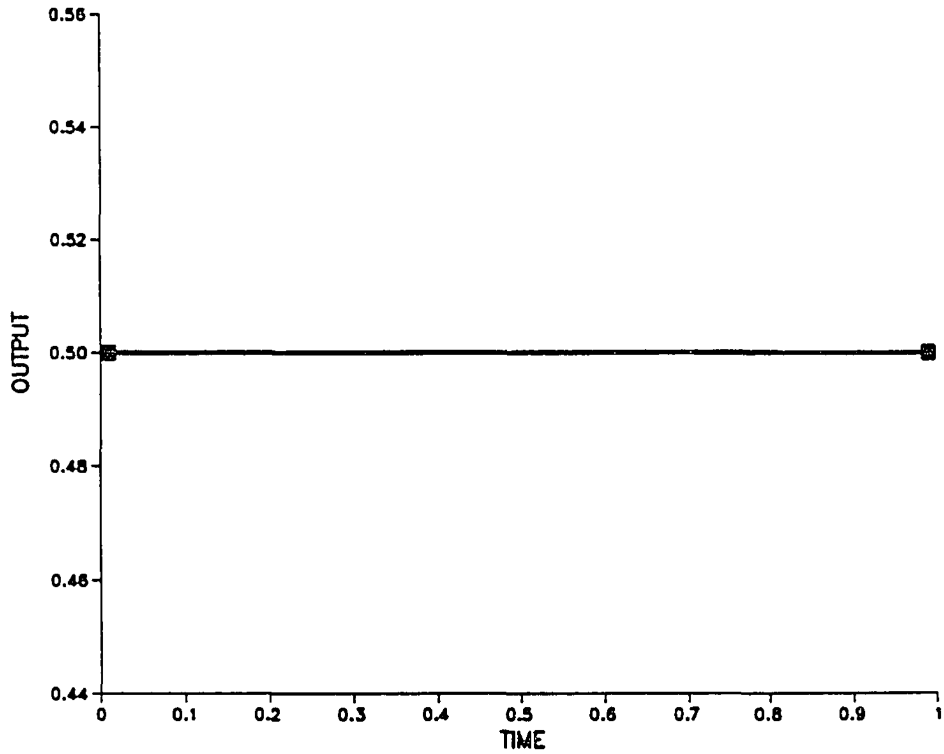

Figure 3.5: Output profile under feedforward/feedback control, for random noise and step changes in the disturbance inputs (Case 1)

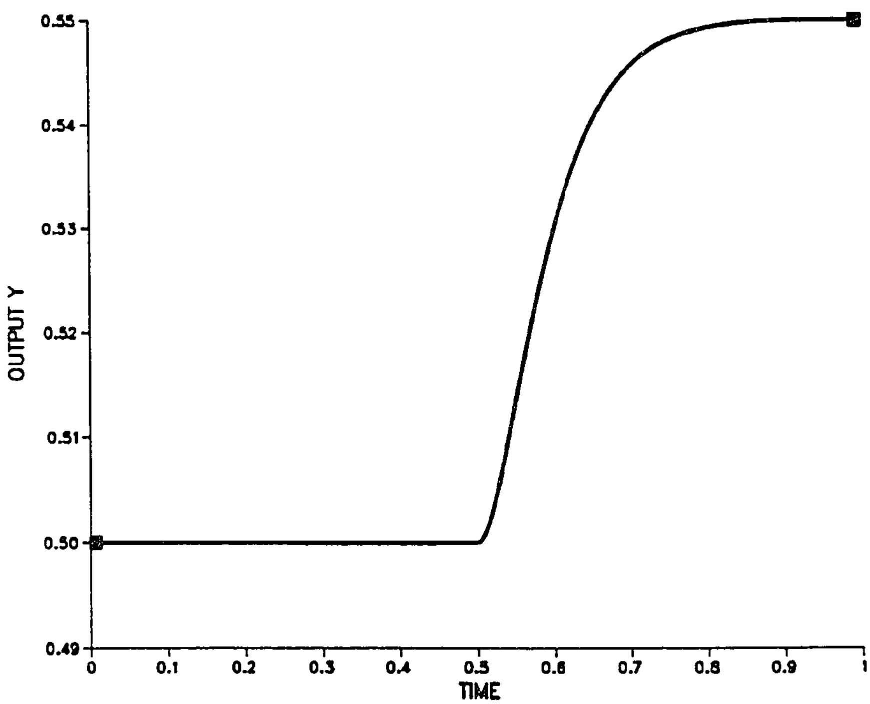

Figure 3.6: Output set-point tracking under feedforward/feedback control, for random noise in the disturbance inputs (Case 1)

$$g(x) = \begin{bmatrix} \frac{1}{V\rho c_p} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, w(x) = \begin{bmatrix} \frac{F}{V} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.54}$$

and $h(x)$ remains the same. In this case, calculation of the relative orders yields:

1. $L_g h(x) = L_g L_f h(x) = L_g L_f^2 h(x) = 0$

   $L_g L_f^3 h(x) \not\equiv 0$

2. $L_w h(x) = L_w L_f h(x) = L_w L_f^2 h(x) = 0$

   $L_w L_f^3 h(x) \not\equiv 0$

Consequently, $r = \rho = 4$, and $d \in \mathcal{B}$. According to Theorem 3.1, the control law :

$$u = \frac{v - \sum_{k=0}^{4} \beta_k L_f^k h(x) - \beta_4 L_w L_f^3 h(x) d(t)}{\beta_4 L_g L_f^3 h(x)} \tag{3.55}$$

eliminates the effect of d on the output y and induces the input/output behavior:

$$\sum_{k=1}^{4} \beta_k \frac{d^k y}{dt^k} = v$$

Choosing $\beta_0 = 30000$, $\beta_1 = 9500$, $\beta_2 = 1100$, $\beta_3 = 55$, $\beta_4 = 1$, the closed-loop poles were placed at $-10, -10, -15, -20$. Figures 3.7 and 3.8 illustrate the performance of the closed-loop system under the feedforward/feedback control law, comparing it with the performance under the GLC methodology, where pure state feedback is applied and an external PI controller is responsible for the disturbance rejection. In Figure 3.7, under the step change in the inlet temperature, the open-loop, the FF/FB and the GLC responses are compared. The PI settings in the GLC structure were chosen

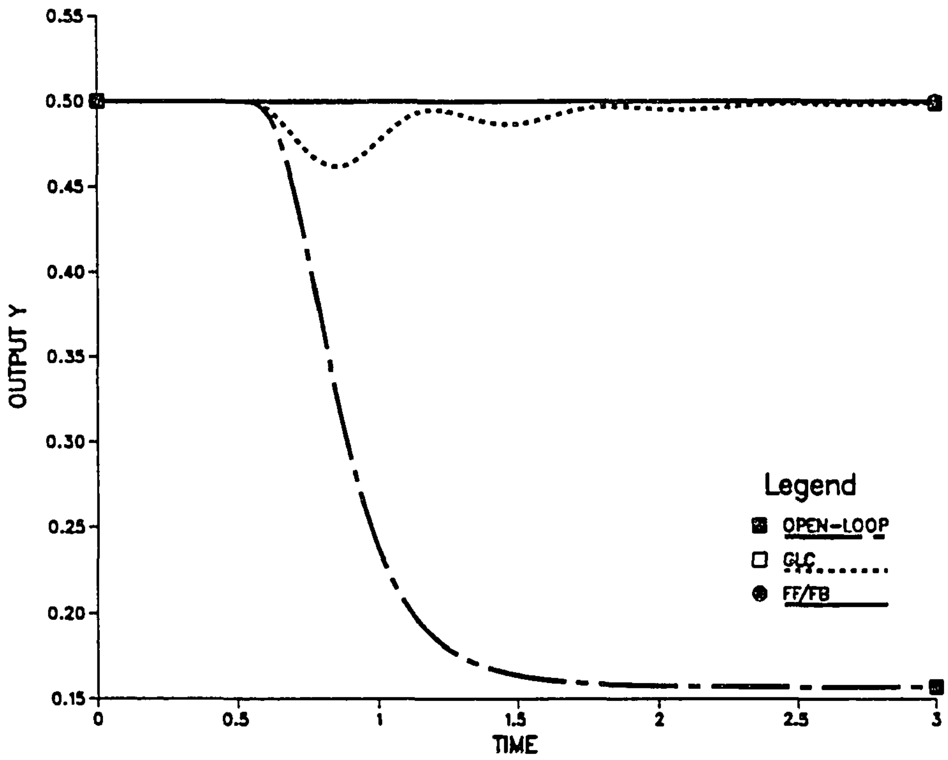

Figure 3.7: Output profiles for a step change in the disturbance input. Comparison of feedforward/feedback control, GLC and open-loop responses (Case 2)

as $K_c = 50000$ and $\tau_I = 0.5$. Feedforward compensation improves significantly the regulatory behavior of the system. In Figure 3.8, the system is forced to track a set-point change at $TIME = 0.5h$, under noise in the inlet temperature. The response in the case of the feedforward/feedback action is identical to the one under GLC in the case where no disturbance is present. Under the presence of the disturbance however, the feedforward action improves significantly the system behavior, as expected. For the PI controller in the GLC structure, we used $K_c = 5000$ and $\tau_I = 1$.

Case 3

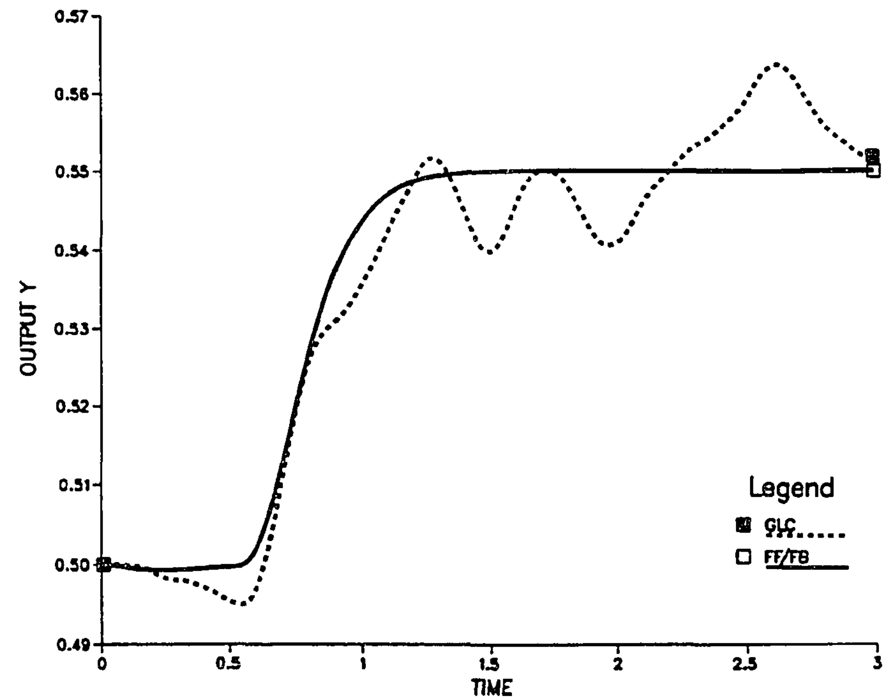

Figure 3.8: Output set-point tracking for random noise in the disturbance input. Comparison of feedforward/feedback control and GLC responses (Case 2)

Setting:

$$u = Q_1 - Q_{1s}$$

$$d = c_{A0} - c_{A0s}$$

the new f and w functions are :

$$f(x) = \begin{bmatrix} \frac{F}{V}(T_0 - x_1) - \frac{\Delta H}{\rho c_p} k_1(x_1)x_2^2 + \frac{Q_{1s}}{V \rho c_p} \\ \frac{F}{V}(c_{A0s} - x_2) - k_1(x_1)x_2^2 \\ \frac{F}{V}(x_1 - x_3) - \frac{\Delta H}{\rho c_p} k_2(x_3)x_4^2 + \frac{Q_2}{V \rho c_p} \\ \frac{F}{V}(x_2 - x_3) - k_2(x_3)x_3^2 \\ \frac{F}{V}(x_3 - x_5) - \frac{\Delta H}{\rho c_p} k_3(x_5)x_6^2 + \frac{Q_3}{V \rho c_p} \\ \frac{F}{V}(x_4 - x_6) - k_3(x_5)x_6^2 \end{bmatrix}, w(x) = \begin{bmatrix} 0 \\ \frac{F}{V} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.56}$$

where g and h remain the same as in Case 2. The calculation of relative orders goes as follows:

1. $L_g h(x) = L_g L_f h(x) = L_g L_f^2 h(x) = 0$

   $L_g L_f^3 h(x) \not\equiv 0$

2. $L_w h(x) = L_w L_f h(x) = 0$

   $L_w L_f^2 h(x) \not\equiv 0$

and yields: $r = 4$ and $\rho = 3$. Consequently, $d \in \mathcal{C}$, and a dynamic feedforward/state feedback control law has to be employed. Moreover, $r - \rho = 1$ and $L_w L_f^2 h(x) = \left(\frac{F}{V}\right)^3$. Clearly then, the condition of Eq.3.17 is satisfied and, therefore, a dynamic feedforward/static state feedback law can solve the posed synthesis problem. The required control law is obtained either directly from Eq.3.3, or alternatively from Eq.3.16, and has the form:

$$u = \frac{v - \sum_{k=0}^{4} \beta_k L_f^k h(x) - \beta_3 \left(\frac{F}{V}\right)^3 d(t) - \beta_4 L_w L_f^3 h(x) d(t) - \beta_4 \left(\frac{F}{V}\right)^3 d(t)^{(1)}}{\beta_4 L_g L_f^3 h(x)} \tag{3.57}$$

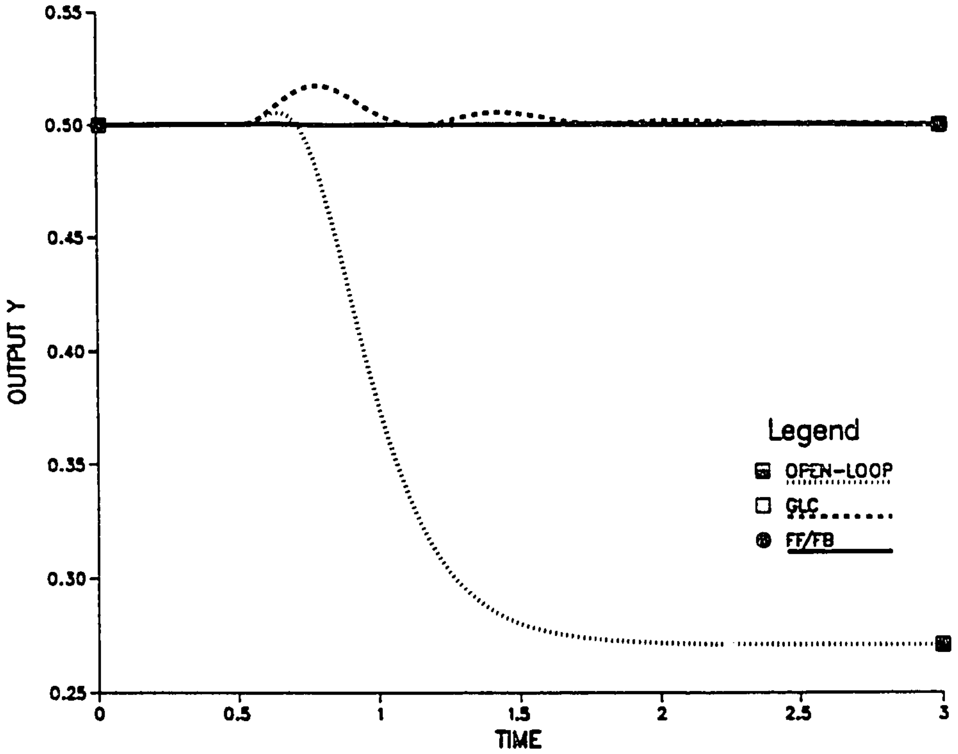

Figure 3.9: Output profiles for a step change in the disturbance input. Comparison of feedforward/feedback control, GLC and open-loop responses (Case 3)

The choice of the adjustable parameters: $\beta_0 = 30000$, $\beta_1 = 9500$, $\beta_2 = 1100$, $\beta_3 = 55$, $\beta_4 = 1$ places the closed-loop poles at $-10, -10, -15, -20$. For the implementation, the first derivative of the disturbance was approximated by a first order lead-lag, with the filter parameter equal to 0.01. Figures 3.9 and 3.10 illustrate the behavior of the system under the same conditions described in Case 2. As expected, feedforward action improves significantly the servo and regulatory behavior of the closed-loop system.

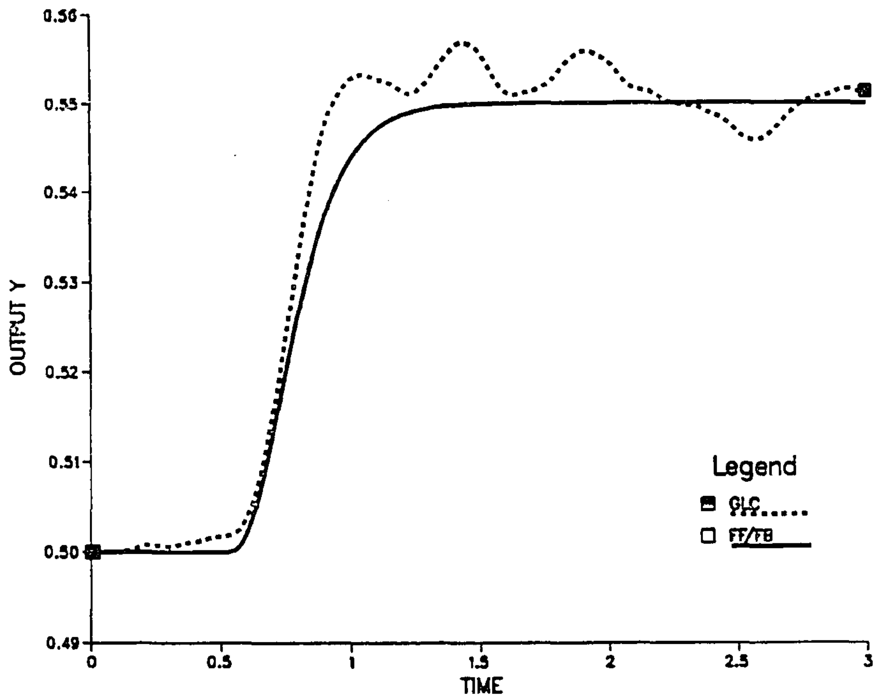

Figure 3.10: Output set-point tracking for random noise in the disturbance input. Comparison of feedforward/feedback control and GLC responses (Case 3)

Remark 3.14: In all three cases presented in the example, the corresponding figures do not show **any** effect of the disturbances on the process output. This happens because the model and measurements are assumed to be perfect, there are no active constraints on the input and there is not any time lag in the control action.

3.7 Notation

Roman Letters

A, B	=	reactant and product
E	=	activation energy, J/mol
F	=	volumetric feedrate, l/h
K_c	=	proportional gain
Q_i	=	heat input in i-th reactor, J/h
T_i	=	temperature in the i-th reactor, K
V	=	reactor volume, l
R	=	Ideal Gas constant, $J/mol.K$
c_{Ai}	=	concentration of species A in the i-th reactor, mol/l
d_κ	=	disturbance input
$d_\kappa(t)^{(k)}$	=	k-th order time derivative of the disturbance input d_κ
k_0	=	specific rate constant, $l/mol.h$
k_1, k_2, k_3	=	rate constants, $l/mol.h$
r	=	relative order of the output y with respect to the manipulated input u
s	=	the Laplace domain variable
t	=	time
u	=	manipulated input
v	=	reference input
x	=	vector of state variables
y	=	output
y_{sp}	=	output set-point

f, g, w_κ = vector fields

h, t_i = scalar fields

$-\Delta H$ = heat of reaction, J/mol

$\mathcal{A}, \mathcal{B}, \mathcal{C}$ = classes of disturbances

$\mathcal{U}$ = generalized manipulated input

Greek Letters

β_k = parameters of the feedforward/state feedback law

$\rho * c_p$ = thermal capacity, $J/l.K$

ρ_κ = relative order of the output y with respect to the disturbance d_κ

τ_I = reset time

ζ = transformed state variables

Math Symbols

$\Longrightarrow$ = implies

$>$ = greater than

$<$ = less than

$\emptyset$ = void set

$\in$ = belongs to

$\subset$ = subset

$\forall$ = for all

min = minimum element

$\mathbb{R}$ = real line

$\mathbb{R}^n$ = $n-$dimensional Euclidean space

Acronyms

BIBO	=	bounded-input bounded-output
CSTR	=	continuous stirred tank reactor
GLC	=	globally linearizing control
FF/FB	=	feedforward/feedback
SISO	=	single-input single-output

CHAPTER IV

FEEDFORWARD/FEEDBACK CONTROL OF MIMO NONLINEAR PROCESSES

4.1 Introduction

In this chapter, the general feedforward/feedback control problem for MIMO nonlinear processes will be addressed. In particular, the two-step control methodology introduced in the previous chapter for SISO nonlinear processes will be generalized to MIMO processes. In the first step, feedforward/state feedback laws will be synthesized, which: a) completely eliminate the effect of measurable disturbances on the output variables, and b) induce a well-characterized linear input/output behavior. In analogy with the SISO results, the concept of relative order will arise naturally in the control laws and will allow a transparent interpretation of their nature, consistent with intuitive considerations. Specific design objectives in the closed-loop system (e.g., degree of coupling) will be associated with appropriate choice of some adjustable parameters. The relation of the feedforward/state feedback control problem with the classical disturbance decoupling problem will also be studied. Finally, the developed feedforward/feedback control methodology will be applied to the control of temperature and number average molecular weight in a continuous polymerization reactor.

4.2 Formulation of the feedforward/feedback control problem

MIMO nonlinear processes will be considered with a state-space description of the form of Eq.2.1, i.e., :

$$\begin{aligned} \dot{x} &= f(x) + \sum_{j=1}^{m} u_j(t) g_j(x) + \sum_{\kappa=1}^{p} d_\kappa(t) w_\kappa(x) \\ y_i &= h_i(x) \ , \ i = 1, \cdots, m \end{aligned}$$

where $x \in X \subset \mathbb{R}^n$, $u(t) = [u_1(t), \cdots, u_m(t)]^T \in \mathbb{R}^m$ and $d(t) = [d_1(t), \cdots, d_p(t)]^T \in \mathbb{R}^p \ \ \forall t \in [0, \infty)$, and $y = [y_1, \cdots, y_m]^T \in \mathbb{R}^m$. In analogy with the SISO case, the general servo and regulatory control problem for such processes will be formulated as follows:

- **Step 1 (Feedforward/state feedback synthesis problem):**

  Calculate a feedforward/state feedback control law of the form:

  $$u = p(x) + q(x)v + q'(x)Q(x, d_\kappa)$$

  where $p(x)$, $q(x)$ and $q'(x)$ are matrices of appropriate dimensions, with $q(x)$ invertible on X, v is an external reference input vector and Q is a nonlinear operator that may include time derivatives, which:

  - *Completely eliminates the effect of measured disturbances on the outputs*
  - *Induces an input/output behavior between the reference inputs v and the outputs y_i that has the form:*

    $$\sum_{i=1}^{m} \sum_{k} \beta_{ik} \frac{d^k y_i}{dt^k} = v$$

    where $\beta_{ik} = (\beta_{ik}^1 \ \beta_{ik}^2 \ \cdots \ \beta_{ik}^m)^T \in \mathbb{R}^m$ are vectors of adjustable parameters

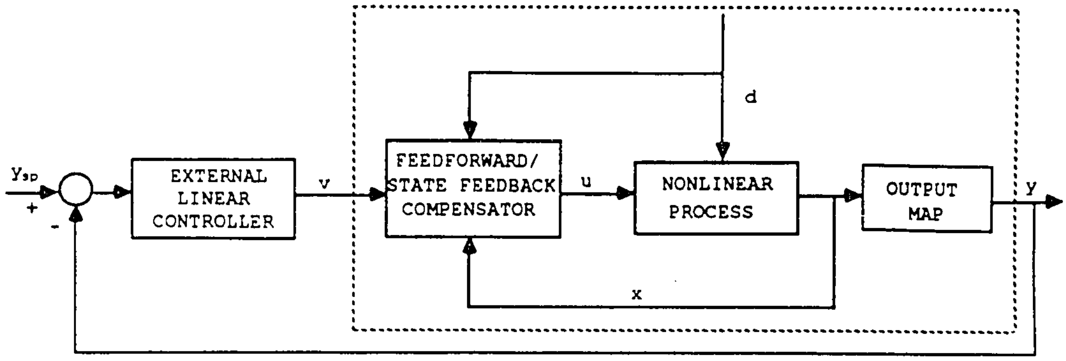

Figure 4.1: Feedforward/feedback control structure

- **Step 2 (Linear controller design problem):**

  Design a MIMO linear controller with integral action around the linear v/y loop, to achieve the desired servo and regulatory behavior, in the presence of unmeasured disturbances and/or modeling errors

The solution to the synthesis problem of Step 1 will be the main focus of the subsequent chapters, while the solution to the linear controller design problem of Step 2 will be briefly discussed in Section 4.5. The overall control configuration is shown in Figure 4.1 and it clearly depicts the resulting two-step control methodology.

The basic analysis and synthesis tools for the solution of the feedforward/state feedback synthesis problem will be the concepts of relative order introduced in Definitions 2.1 and 2.3 of Chapter II. According to Definition 2.1, r_i will denote the relative order of the output y_i with respect to the manipulated input vector u, i.e., the smallest integer for which there exists a $j \in \{1, 2, \cdots, m\}$ such that:

$$L_{g_j} L_f^{r_i - 1} h_i(x) \not\equiv 0$$

for $x \in X$. It will be assumed that each output y_i possesses a finite relative order r_i. Then, the following concept can be defined:

Definition 4.1 (Claude, 1986): *Consider the nonlinear system of Eq.2.1. The matrix:*

$$C(x) = \begin{bmatrix} L_{g_1}L_f^{r_1-1}h_1(x) & \cdots & L_{g_m}L_f^{r_1-1}h_1(x) \\ \vdots & \ddots & \vdots \\ L_{g_1}L_f^{r_m-1}h_m(x) & \cdots & L_{g_m}L_f^{r_m-1}h_m(x) \end{bmatrix} \tag{4.1}$$

is called the **characteristic matrix** *of the system.*

Remark 4.1: The characteristic matrix $C(x)$ is also referred to as the decoupling matrix (e.g., Ha and Gilbert, 1986), due to its significance to the nonlinear decoupling problem.

In analogy with the SISO treatment, it will be assumed that X does not contain any singular points, i.e., points $x \in \mathbb{R}^n$ for which $detC(x) = 0$. As long as $detC(x_0) \neq 0$, one can always redefine X in order to satisfy the above assumption. According to Definition 2.3, $\rho_{i\kappa}$ will then denote the relative order of the output y_i with respect to the disturbance input d_κ, i.e., the smallest integer for which:

$$L_{w_\kappa}L_f^{\rho_{i\kappa}-1}h_i(x) \not\equiv 0$$

for $x \in X$.

4.3 The feedforward/state feedback synthesis problem

Based on the results of Chapter III, relative orders are expected to play a fundamental role in the solution of the feedforward/state feedback synthesis problem. Referring to a nonlinear process described by Eq.2.1, the following partition of the set of disturbance inputs is proposed, into the classes $\mathcal{A}_i$, $\mathcal{B}_i$ and $\mathcal{C}_i$, associated with the output y_i:

$$\begin{aligned} d_\kappa \in \mathcal{A}_i &\iff \rho_{i\kappa} > r_i \\ d_\kappa \in \mathcal{B}_i &\iff \rho_{i\kappa} = r_i \\ d_\kappa \in \mathcal{C}_i &\iff \rho_{i\kappa} < r_i \end{aligned} \tag{4.2}$$

Note that for each output, a different, in general, partition of the set of the disturbances will be obtained. The above partition captures the relative dynamic interactions between the manipulated input vector and the individual disturbance inputs, for a particular output. More specifically,

- Disturbances that belong to class $\mathcal{A}_i$ have a less direct effect on the output y_i than the manipulated input vector
- Disturbances that belong to class $\mathcal{B}_i$ have an equally direct effect on the output y_i as the manipulated input vector
- Disturbances that belong to class $\mathcal{C}_i$ have a more direct effect on the output y_i than the manipulated input vector

Based on the intuition that has been obtained from the SISO treatment of the subject, the following properties are expected to hold concerning the nature of the regulatory control problem:

- $\mathcal{B}_i = \mathcal{C}_i = \emptyset$: pure feedback compensation will suffice to eliminate the effect of the disturbances on the output y_i
- $\mathcal{B}_i \neq \emptyset$, $\mathcal{C}_i = \emptyset$: static feedforward/state feedback compensation will be necessary to eliminate the effect of the disturbances on the output y_i
- $\mathcal{C}_i \neq \emptyset$: dynamic feedforward/state feedback compensation will be necessary to eliminate the effect of the disturbances on the output y_i

The overall control action must compensate for each class of disturbances and for each output in an appropriate way.

The above considerations arise naturally in the solution of the synthesis problem which is given in Theorem 4.1 that follows. Its proof can be found in Appendix D.

Theorem 4.1: *Consider the MIMO nonlinear process described by Eq.2.1. Let r_i and $\rho_{i\kappa}$, $i = 1, \cdots, m$, $\kappa = 1, \cdots, p$ denote the relative orders of the outputs y_i with respect to u and d_κ, respectively. Consider also the output-dependent partition of the set of disturbances defined in Eq.4.2. Then, a feedforward/state feedback law of the form:*

$$u = \left[\sum_{i=1}^{m} \beta_{ir_i} L_{g_1} L_f^{r_i - 1} h_i(x) \ \cdots \ \sum_{i=1}^{m} \beta_{ir_i} L_{g_m} L_f^{r_i - 1} h_i(x) \right]^{-1}$$
$$\Bigg\{ v - \sum_{i=1}^{m} \sum_{k=0}^{r_i} \beta_{ik} L_f^k h_i(x) - \sum_{i=1}^{m} \sum_{d_\kappa \in \mathcal{B}_i} \beta_{ir_i} d_\kappa(t) L_{w_\kappa} L_f^{r_i - 1} h_i(x)$$
$$- \sum_{i=1}^{m} \sum_{d_\kappa \in \mathcal{C}_i} \sum_{l=0}^{r_i - \rho_{i\kappa}} \sum_{k=\rho_{i\kappa}+l}^{r_i} \beta_{ik} \frac{d^l}{dt^l} \left(d_\kappa(t) L_{w_\kappa} L_f^{k-l-1} h_i(x) \right) \Bigg\} \tag{4.3}$$

- *Completely eliminates the effect of the disturbances d_κ on y_i*
- *Induces the linear input/output behavior:*

$$\sum_{i=1}^{m} \sum_{k=0}^{r_i} \beta_{ik} \frac{d^k y_i}{dt^k} = v \tag{4.4}$$

where $\beta_{ik} = [\beta_{ik}^1 \ \beta_{ik}^2 \ \cdots \ \beta_{ik}^m]^T \in \mathbb{R}^m$ are vectors of adjustable parameters with

$$det \ [\beta_{1r_1} \ \beta_{2r_2} \ \cdots \ \beta_{mr_m}] \neq 0 \tag{4.5}$$

and $v = [v_1 \ v_2 \ \cdots \ v_m]^T \in \mathbb{R}^m$ is a vector of reference inputs

Remark 4.2: Despite the apparent complexity of the control law of Eq.4.3, a rather simple structure is present. In particular, Eq.4.3 is composed of three distinct parts:

- a pure static state feedback part, which accounts for input/output linearity and eliminates, for each i, the effect of the disturbances in $\mathcal{A}_i$ on the output y_i:

$$\left[\sum_{i=1}^{m}\beta_{ir_i}L_{g_1}L_f^{r_i-1}h_i(x)\ \cdots\ \sum_{i=1}^{m}\beta_{ir_i}L_{g_m}L_f^{r_i-1}h_i(x)\right]^{-1}\left\{v-\sum_{i=1}^{m}\sum_{k=0}^{r_i}\beta_{ik}L_f^kh_i(x)\right\}$$

- a static feedforward/state feedback part which eliminates, for each i, the effect of the disturbances in $\mathcal{B}_i$ on the output y_i:

$$\left[\sum_{i=1}^{m}\beta_{ir_i}L_{g_1}L_f^{r_i-1}h_i(x)\ \cdots\ \sum_{i=1}^{m}\beta_{ir_i}L_{g_m}L_f^{r_i-1}h_i(x)\right]^{-1}\left\{-\sum_{i=1}^{m}\sum_{d_\kappa\in\mathcal{B}_i}\beta_{ir_i}L_{w_\kappa}L_f^{r_i-1}h_i(x)d_\kappa\right\}$$

- a dynamic feedforward/state feedback part which eliminates, for each i, the effect of the disturbances in $\mathcal{C}_i$ on the output y_i:

$$\left[\sum_{i=1}^{m}\beta_{ir_i}L_{g_1}L_f^{r_i-1}h_i(x)\ \cdots\ \sum_{i=1}^{m}\beta_{ir_i}L_{g_m}L_f^{r_i-1}h_i(x)\right]^{-1}$$
$$\left\{-\sum_{i=1}^{m}\sum_{d_\kappa\in\mathcal{C}_i}\sum_{l=0}^{r_i-\rho_{i\kappa}}\sum_{k=\rho_{i\kappa}+l}^{r_i}\beta_{ik}\frac{d^l}{dt^l}(L_{w_\kappa}L_f^{k-l-1}h_i(x)d_\kappa)\right\}$$

In each one of the above parts of the control law, the control action results by superimposing explicit compensation terms for each output and for each disturbance, depending on the corresponding partition of the set of disturbances. It is exactly this explicit character of the control law that results in its rather complicated form. More compact expressions can be written, by adopting a more compact vector notation.

At this point, let us summarize the basic characteristics of the proposed approach.

- Calculating the relative orders r_i and $\rho_{i\kappa}$ for every output y_i and disturbance d_κ, and

- Implementing the control law of Eq.4.3, for an appropriate choice of the adjustable parameters β_{ik}^j,

all the available process information is used, so that:

- The input/output behavior of the closed-loop system for changes in the reference input is linear

- The regulatory behavior of the closed-loop system is perfect with respect to the measurable disturbances

under the assumption of course, of a perfect model and perfect implementation of the controller. In the next section, the choice of the adjustable parameters β_{ik}^{j} will be associated with the stability characteristics and the degree of coupling in the closed-loop system. It should be mentioned that an implicit assumption in the previous development is that measurements of the system's states are available. In fact, this assumption is a key one in obtaining the perfect disturbance rejection property on the output y_i for the disturbances that belong to the class $\mathcal{A}_i$, without using measurements of these disturbances. The measurements of the states completely capture the effect of these disturbances and since the manipulated input vector has a more direct effect on the output y_i than these disturbances, it completely compensates for their effect.

Remark 4.3: The control law of Eq.4.3 simplifies greatly in the following two cases:

- $\mathcal{C}_i = \emptyset$ for every i:

$$u = \left[\sum_{i=1}^{m} \beta_{ir_i} L_{g_1} L_f^{r_i-1} h_i(x) \cdots \sum_{i=1}^{m} \beta_{ir_i} L_{g_m} L_f^{r_i-1} h_i(x) \right]^{-1}$$
$$\left\{ v - \sum_{i=1}^{m} \sum_{k=0}^{r_i} \beta_{ik} L_f^k h_i(x) - \sum_{i=1}^{m} \sum_{d_\kappa \in \mathcal{B}_i} d_\kappa(t) \beta_{ir_i} L_{w_\kappa} L_f^{r_i-1} h_i(x) \right\} \tag{4.6}$$

  which is a static feedforward/state feedback control law

- $\mathcal{B}_i = \mathcal{C}_i = \emptyset$ for every i:

$$u = \left[\sum_{i=1}^{m} \beta_{ir_i} L_{g_1} L_f^{r_i-1} h_i(x) \cdots \sum_{i=1}^{m} \beta_{ir_i} L_{g_m} L_f^{r_i-1} h_i(x) \right]^{-1} \left\{ v - \sum_{i=1}^{m} \sum_{k=0}^{r_i} \beta_{ik} L_f^k h_i(x) \right\} \tag{4.7}$$

  which is a pure state feedback control law, identical to an input/output linearizing control law in the absence of disturbances (Kravaris and Soroush, 1990)

The above results conform with the previous intuitive arguments concerning the nature of the control law, depending on the classes of disturbances present in each partition.

Remark 4.4: In the case of a SISO nonlinear process (i.e., for $m = 1$), the control law of Eq.4.3 reduces to:

$$u = \left[\beta_r L_g L_f^{r-1} h(x)\right]^{-1} \left\{ v - \sum_{k=0}^{r} \beta_k L_f^k h(x) - \sum_{d_\kappa \in B} d_\kappa(t) \beta_r L_{w_\kappa} L_f^{k-1} h(x) \right.$$

$$\left. - \sum_{d_\kappa \in C} \sum_{l=0}^{r-\rho_\kappa} \sum_{k=\rho_\kappa+l}^{r} \beta_k \frac{d^l}{dt^l} \left(d_\kappa(t) L_{w_\kappa} L_f^{k-l-1} h(x) \right) \right\}$$

which, as expected, is identical to the previous result for SISO systems (Eq.3.3).

Remark 4.5: A key assumption throughout the previous treatment was that the characteristic matrix $C(x)$ defined in Eq.4.1 was non-singular for all $x \in X$. The non-singularity of $C(x)$ is a sufficient condition for a static state feedback input/output linearizing control law to exist. It has been shown, however (Kravaris and Soroush, 1990), that input/output linearization can be achieved for a larger class of disturbance-free systems than those satisfying this assumption. Generalization of Theorem 4.1 for this class of systems is possible, but would involve several technicalities which go beyond the scope of this thesis.

Remark 4.6: The proposed methodology can be easily generalized to achieve any nonlinear input/output behavior of the form:

$$\Phi(y_1, \frac{dy_1}{dt}, \cdots, \frac{d^{r_1} y_1}{dt^{r_1}}, \cdots, y_m, \frac{dy_m}{dt}, \cdots, \frac{d^{r_m} y_m}{dt^{r_m}}) = v \tag{4.8}$$

In analogy with the SISO case, however, such a generalization does not seem particularly meaningful.

Remark 4.7: Consider the more general class of nonlinear processes with a state-space description of the form:

$$\begin{aligned}\dot{x} &= f(x) + \sum_{j=1}^{m} g_j(x)\phi_j(x, u_j(t), d_j^*(t)) + \sum_{\kappa=1}^{p} d_\kappa(t) w_\kappa(x) \\ y_i &= h_i(x)\ ,\ i = 1, \cdots, m\end{aligned} \tag{4.9}$$

where $\phi_j(x, u_j, d_j^*)$ is a scalar function solvable for u_j and d_j^* is a vector of additional measurable disturbances. The above class incorporates cases where some manipulated inputs appear in the state equations coupled with some measurable disturbances. In this case, the proposed methodology can be applied by simply letting $\mathcal{U}_j = \phi_j(x, u_j, d_j^*)$, calculating $\mathcal{U}_j$ from Eq.4.3, and then solving for the actual manipulated inputs u_j. Following this procedure, compensation for the disturbances d_j^* is also possible.

Remark 4.8: The disturbance rejection capability of the control law of Eq.4.3 with respect to the disturbances in class $\mathcal{A}_i$ can find an interesting robustness interpretation. In particular, consider a localized perturbation (model uncertainty and/or unmeasured disturbance) of arbitrary magnitude which enters the system dynamic structure in an additive way at a certain location (i.e., a certain state equation). Such a perturbation can be viewed as an unmeasurable disturbance and can be assigned an "equivalent relative order". The particular perturbation can then be included in one of the classes of disturbances defined by Eq.4.2. If it belongs to the class $\mathcal{A}_i$, it will not affect the output y_i, under the control law of Eq.4.3. The above inherent robustness feature of the control law is extremely meaningful in chemical systems, where a model uncertainty can often be identified with errors in certain system parameters, e.g., kinetic rate constants, heat transfer coefficients, etc.

Proposition 4.1 that follows provides a solution to the feedforward/state feedback synthesis problem for the special case of a linear process description.

Proposition 4.1 : *Consider a MIMO linear system of the form of Eq.2.2. Let* r_i

and $\rho_{i\kappa}$*,* $i = 1, \cdots, m$*,* $\kappa = 1, \cdots, p$ *denote the relative orders of the outputs* y_i *with respect to* u *and* d_κ*, respectively. Consider also the output-dependent partition of the set of disturbances defined in Eq.4.2. Then, a feedforward/state feedback law of the form:*

$$u = \left[\sum_{i=1}^{m} \beta_{ir_i} c_i A^{r_i-1} b_1 \; \cdots \; \sum_{i=1}^{m} \beta_{ir_i} c_i A^{r_i-1} b_m \right]^{-1}$$

$$\left\{ v - \sum_{i=1}^{m} \sum_{k=0}^{r_i} \beta_{ik} c_i A^k x - \sum_{i=1}^{m} \sum_{d_\kappa \in \mathcal{B}_i} \beta_{ir_i} d_\kappa(t) c_i A^{r_i-1} \gamma_\kappa \right.$$

$$\left. - \sum_{i=1}^{m} \sum_{d_\kappa \in \mathcal{C}_i} \sum_{l=0}^{r_i-\rho_{i\kappa}} \sum_{k=\rho_{i\kappa}+l}^{r_i} \beta_{ik} c_i A^{k-l-1} \gamma_\kappa \frac{d^l}{dt^l} (d_\kappa(t)) \right\} \tag{4.10}$$

- *Completely eliminates the effect of the disturbances* d_κ *on* y_i

- *Induces the input/output behavior:*

$$\sum_{i=1}^{m} \sum_{k=0}^{r_i} \beta_{ik} \frac{d^k y_i}{dt^k} = v$$

where $\beta_{ik} = [\beta_{ik}^1 \; \beta_{ik}^2 \; \cdots \; \beta_{ik}^m]^T \in \mathbb{R}^m$ *are vectors of adjustable parameters*

Proof: It is easily verified that, for $f(x) = Ax$, $g_j(x) = b_j$, $w_\kappa(x) = \gamma_\kappa$ and $h_i(x) = c_i x$, the following relations hold:

$$L_f^k h_i(x) = c_i A^k x$$

$$L_{g_j} L_f^k h_i(x) = c_i A^k b_j$$

$$L_{w_\kappa} L_f^k h_i(x) = c_i A^k \gamma_\kappa$$

Substituting the above relations to the control law of Eq.4.3, Eq.4.10 is easily obtained.

Motivated by the corresponding discussion and results of Chapter III, the class of MIMO nonlinear systems will now be characterized, for which the solution to the feedforward/state feedback synthesis problem is a feedforward/**static** state feedback law of the form:

$$u = p(x) + q(x)v + Q'\left(x, d(t), d(t)^{(1)}, d(t)^{(2)}, \cdots\right) \tag{4.11}$$

where $p(x), q(x)$ are matrices of appropriate dimensions, with $q(x)$ invertible on X, and Q' is a vector function which is nonsingular under nominal conditions. Such a characterization is useful both from a theoretical and practical perspective, given the result of Proposition 4.1 and the considerations regarding the implementation of the dynamic components of Eq.4.3. Theorem 4.2 that follows generalizes the result of Theorem 3.2 for MIMO nonlinear systems in the form of Eq.2.1. The proof of Theorem 4.2 is completely analogous, although notationally more complicated, to the one of Proposition 3.2, and is omitted for brevity.

Theorem 4.2 : *Consider the MIMO nonlinear system described by Eq.2.1, Let r_i and $\rho_{i\kappa}$, $i = 1, \cdots, m$, $\kappa = 1, \cdots, p$ denote the relative orders of the output y_i with respect to u and d_κ, respectively. Consider also the output-dependent partition of the set of disturbances defined in Eq.4.2, and assume that $C_i \neq \emptyset$, for some output y_i. Then, the conditions:*

$$L_{g_j}\phi_{i\ell}(x, d(t)) \equiv 0 \tag{4.12}$$

$$\ell = 0, 1, \cdots, r_i - \rho_i - 1, \ i = 1, \cdots, m, \ j = 1, \cdots, m$$

where

$$\phi_{i\ell}(x, d(t)) = \sum_{\mu=0}^{\ell} L_f^{\ell-\mu}\left(\sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} + \frac{\partial}{\partial t}\right)\left(L_f + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} + \frac{\partial}{\partial t}\right)^{\mu} L_f^{\rho_i - 1} h_i(x) \tag{4.13}$$

and

$$\rho_i = min\left\{\rho_{i1}, \cdots, \rho_{ip}\right\} \tag{4.14}$$

are necessary and sufficient in order for a feedforward/static state feedback law of the form of Eq.4.11 to:

- *Completely eliminate the effect of the disturbances d_κ on y_i, and*
- *Induce the linear input/output behavior:*

$$\sum_{i=1}^{m}\sum_{k=0}^{r_i}\beta_{ik}\frac{d^k y_i}{dt^k} = v$$

where $\beta_{ik} = [\beta_{ik}^1 \; \beta_{ik}^2 \; \cdots \; \beta_{ik}^m]^T \in \mathbb{R}^m$ are vectors of adjustable parameters.

If these conditions are satisfied, the appropriate control law takes the form:

$$\begin{aligned} u = & \left[\sum_{i=1}^{m}\beta_{ir_i}L_{g_1}L_f^{r_i-1}h_i(x) \; \cdots \; \sum_{i=1}^{m}\beta_{ir_i}L_{g_m}L_f^{r_i-1}h_i(x)\right]^{-1} \\ & \left\{v - \sum_{i=1}^{m}\sum_{k=0}^{r_i}\beta_{ik}L_f^k h_i(x) - \sum_{i=1}^{m}\sum_{d_\kappa\in\mathcal{B}_i}\beta_{ir_i}d_\kappa(t)L_{w_\kappa}L_f^{r_i-1}h_i(x)\right. \\ & \left. -\sum_{i=1}^{m}\sum_{k=\rho_i}^{r_i}\beta_{ik}\phi_{i(k-\rho_i)}\left(x, d(t), d(t)^{(1)}, \cdots, d(t)^{(k-\rho_i)}\right)\right\} \end{aligned} \quad (4.15)$$

4.4 Closed-loop design considerations

a) Design of the feedforward/feedback inner loop: Under the control law of Eq.4.3, the dynamics of the $v - y$ system is governed by:

$$(\beta_{10}y_1 + \cdots + \beta_{1r_1}\frac{d^{r_1}y_1}{dt^{r_1}}) + \cdots + (\beta_{m0}y_m + \cdots + \beta_{mr_m}\frac{d^{r_m}y_m}{dt^{r_m}}) = v \quad (4.16)$$

or, expanding the β-column notation:

$$\begin{array}{ccccccc} (\beta_{10}^1 y_1 + \cdots + \beta_{1r_1}^1\frac{d^{r_1}y_1}{dt^{r_1}}) & + & \cdots & + & (\beta_{m0}^1 y_m + \cdots + \beta_{mr_m}^1\frac{d^{r_m}y_m}{dt^{r_m}}) & = & v_1 \\ (\beta_{10}^2 y_1 + \cdots + \beta_{1r_1}^2\frac{d^{r_1}y_1}{dt^{r_1}}) & + & \cdots & + & (\beta_{m0}^2 y_m + \cdots + \beta_{mr_m}^2\frac{d^{r_m}y_m}{dt^{r_m}}) & = & v_2 \\ \vdots & & & & \vdots & & \vdots \\ (\beta_{10}^m y_1 + \cdots + \beta_{1r_1}^m\frac{d^{r_1}y_1}{dt^{r_1}}) & + & \cdots & + & (\beta_{m0}^m y_m + \cdots + \beta_{mr_m}^m\frac{d^{r_m}y_m}{dt^{r_m}}) & = & v_m \end{array} \quad (4.17)$$

or, in the Laplace domain and using a matrix fraction description:

$$y(s) = \left[(\sum_{k=0}^{r_1} \beta_{1k} s^k) \; (\sum_{k=0}^{r_2} \beta_{2k} s^k) \; \cdots \; (\sum_{k=0}^{r_m} \beta_{mk} s^k) \right]^{-1} v(s) \tag{4.18}$$

The order of the closed-loop system is $(r_1 + r_2 + \cdots + r_m) \leq n$. In analogy with the SISO case, the closed-loop system does not possess any finite zeros. On the other hand, the poles of the closed-loop system are the roots of the characteristic equation:

$$det \left[(\sum_{k=0}^{r_1} \beta_{1k} s^k) \; (\sum_{k=0}^{r_2} \beta_{2k} s^k) \; \cdots \; (\sum_{k=0}^{r_m} \beta_{mk} s^k) \right] = 0$$

Consequently, the BIBO stability characteristics in the closed-loop depend on the values of the $m(r_1 + \cdots + r_m) + m^2$ adjustable parameters β_{ik}^j.

The issue of asymptotic stability of the states in the unforced closed-loop system can be addressed following a similar procedure to the one followed for SISO systems. In particular, one can generalize the disturbance-free concept of MIMO zero dynamics (e.g., Isidori and Moog, 1988, Daoutidis and Kravaris, 1991a) to obtain a concept of zero dynamics for MIMO nonlinear systems with disturbances; then, appropriate stability conditions on the zero dynamics will guarantee the asymptotic stability of the unforced closed-loop system.

In some cases, it may be desirable to achieve input/output decoupling in the closed-loop system, i.e., to have each reference input v_j affect only the output y_i. In this case, the postulated closed-loop response is:

$$
\begin{aligned}
(\beta_{10}^1 y_1 + \cdots + \beta_{1r_1}^1 \frac{d^{r_1} y_1}{dt^{r_1}}) &= v_1 \\
(\beta_{20}^2 y_2 + \cdots + \beta_{2r_2}^2 \frac{d^{r_2} y_2}{dt^{r_2}}) &= v_2 \\
&\vdots \\
(\beta_{m0}^m y_m + \cdots + \beta_{mr_m}^m \frac{d^{r_m} y_m}{dt^{r_m}}) &= v_m
\end{aligned}
\tag{4.19}
$$

and one simply sets

$$\beta_{ik}^{j} = 0 \ , \ i \neq j$$

in the control law. Any kind of partially decoupled closed-loop response can also be achieved, by appropriate choice of the adjustable parameters β_{ik}^{j}, as long as the nonsingularity condition of Eq.4.5 is satisfied. By requesting any kind of input/output decoupling, additional structural constraints are imposed on the closed-loop system, which may cause deterioration in its performance characteristics. On the other hand, several advantages are present, such as fewer adjustable parameters ($(r_1 + \cdots + r_m + m)$, in the case of full decoupling) and the use of SISO controllers in the external loop, in which case their tuning is straightforward. Physical constraints on the manipulated input and/or physical importance of the controlled output may often dictate whether decoupling is realistic and/or desirable. In general, despite the extensive research effort in this area, there is a lack of systematic methods of fundamental rigor for assessing when decoupling is favorable, even in the case of linear systems. It should be noted that, within the proposed synthesis framework, any degree of decoupling can be achieved by simply an appropriate choice of the adjustable parameters (without any modification in the actual synthesis procedure). This fact allows a significant degree of flexibility to the designer, who can incorporate his/her own intuition and experience in the tuning procedure and test the resulting performance characteristics.

b) Design of the external linear controller: The design of a multivariable linear controller for the linear v/y system can be performed using techniques from linear control theory. Of course, if the v/y system is decoupled, the controller designer has a much simpler task of synthesizing and tuning the corresponding SISO linear controllers. In any case, the external linear controller must be designed to ensure:

- Stability of the overall closed-loop system y_{sp}/y
- Satisfactory tracking of set points and rejection of the unmeasurable disturbances
- That the magnitude of the manipulated inputs u_j will not exceed the bounds imposed by practical constraints

4.5 Feedforward/state feedback and the disturbance decoupling problem

In this section the relation between the feedforward/state feedback synthesis problem and the classical disturbance decoupling problem of the theoretical literature will be studied. Referring to a nonlinear system of the form of Eq.2.1, the disturbance decoupling problem is to find a static state feedback law $u = p(x) + q(x)v$, where $p(x), q(x)$ are matrices of appropriate dimension, with $q(x)$ invertible, such that the disturbances do not influence the output vector in the closed-loop system. In the case of a system with nonsingular characteristic matrix, the necessary and sufficient condition for solvability of this problem takes an explicit form (e.g., see Isidori et al., 1981), given in the theorem that follows:

Theorem 4.3: *Consider the nonlinear system of the form of Eq.2.1. Let r_i, $i = 1, \cdots, m$ denote the relative order of the output y_i with respect to u. Then, the disturbance decoupling problem is solvable in X, if and only if,*

$$w_\kappa(x) \in \bigcap_{i=1}^{m} \left[ker\ dh_i(x) \bigcap ker\ dL_f h_i(x) \bigcap \ldots \bigcap ker\ dL_f^{r_i-1} h_i(x)\right] \tag{4.20}$$

for every κ and every $x \in X$.

Remark 4.9: In geometric terms, the right-hand-side of the above equation is equal to the maximal controlled invariant distribution contained in $ker\ dh = \bigcap_{i=1}^{m} ker\ dh_i(x)$, where $dh_i(x)$ is the covector associated with $h_i(x)$.

Proposition 4.2 that follows provides an equivalent representation of the condition of Eq.4.20, in terms of the relative orders of the output variables. The proof can be found in Appendix D.

Proposition 4.2: *Consider the nonlinear system of the form of Eq.2.1. Let r_i and $\rho_{i\kappa}$, $i = 1, \cdots, m$, $\kappa = 1, \cdots, p$ denote the relative orders of the output y_i with respect to u and d_κ, repsectively. Then, the disturbance decoupling problem is solvable in X, if and only if,*

$$\rho_{i\kappa} > r_i \tag{4.21}$$

for every i and κ.

The condition of Eq.4.21 is equivalent to the condition $\mathcal{C}_i = \mathcal{B}_i = \emptyset$, for all i. When this condition is satisfied, the control law of Eq.4.3 takes the form of Eq.4.7, which clearly represents a solution to the disturbance decoupling problem for the system under consideration.

The condition of Eq.4.21 is rarely met in practice; this realization motivated the study of a modified disturbance decoupling problem (Moog and Glumineau, 1983) where a static feedforward/state feedback of the form $u = p(x) + q(x)v + s(x)d$ is allowed. The treatment of this problem parallels the one for the original disturbance decoupling problem. As expected, the necessary and sufficient condition for its solvability is weaker and, for the special case of nonsingular characteristic matrix, takes the following form:

Theorem 4.4: *Consider the nonlinear system of the form of Eq.2.1. Let r_i, $i = 1, \cdots, m$ denote the relative order of the output y_i with respect to u. Then, the* *modified disturbance decoupling problem is solvable in* X*, if and only if,*

$$w_\kappa(x) \in \bigcap_{i=1}^{m} \left[ker\ dh_i(x) \bigcap ker\ dL_f h_i(x) \bigcap \dots \bigcap ker\ dL_f^{r_i-1} h_i(x)\right] + span\{g_1(x), g_2(x), \cdots, g_m(x)\} \qquad (4.22)$$

for every κ *and every* $x \in X$.

Similarly with the original case, Proposition 4.3 that follows provides an equivalent formulation of the above condition in terms of the relative orders of the outputs. The proof can be found in Appendix D.

Proposition 4.3: *Consider the nonlinear system of the form of Eq.2.1. Let* r_i *and* $\rho_{i\kappa}$, $i = 1, \cdots, m$, $\kappa = 1, \cdots, p$ *denote the relative orders of the output* y_i *with respect to* u *and* d_κ*, respectively. Then, the modified disturbance decoupling problem is solvable in* X*, if and only if,*

$$\rho_{i\kappa} \geq r_i \qquad (4.23)$$

for every i *and* κ.

The condition of Eq.4.23 is equivalent to the condition: $C_i = \emptyset$, for all i. When this condition is satisfied, the control law of Eq.4.3 takes the form of Eq.4.6, which obviously represents a solution to the modified disturbance decoupling problem for the system under consideration.

Remark 4.10: Based on the above discussion, it is clear that, the conditions for the solvability of the disturbance decoupling and the modified disturbance decoupling problem take a much more transparent and easier to verify form in terms of the relative orders. Furthermore, whenever the disturbance decoupling problem or the modified disturbance decoupling problem are solvable, the control law of Eq.4.3 provides a solution to these problems, and in addition, induces a well-characterized input/output behavior. Finally, the control law of Eq.4.3 allows the elimination

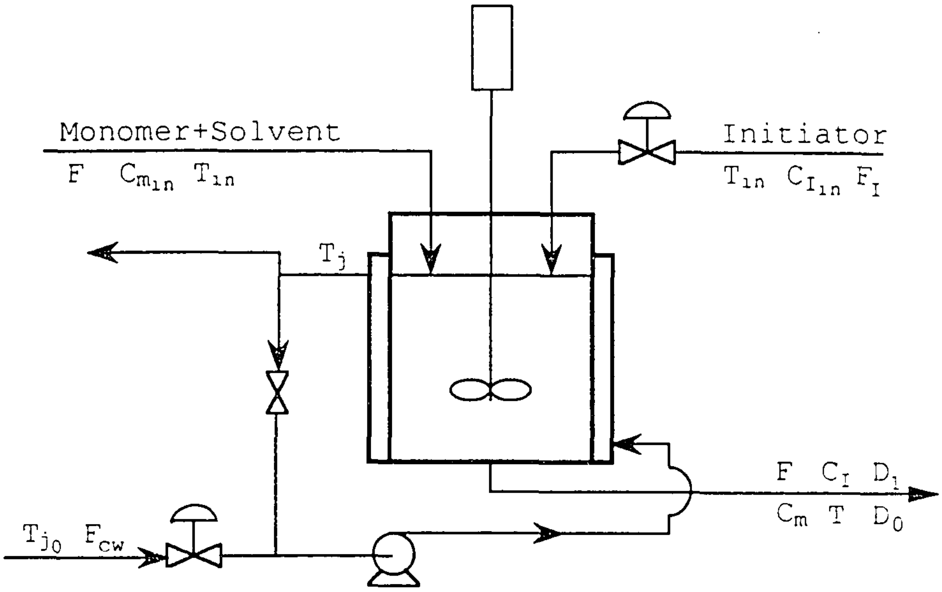

Figure 4.2: A continuous polymerization reactor

of the effect of all measurable disturbances on the outputs, even when the disturbance decoupling problem and the modified disturbance decoupling problem are not solvable.

4.6 Application of the control methodology to a continuous polymerization reactor

In this section, the developed feedforward/feedback control methodology will be applied to a polymerization reaction system. In particular, consider the CSTR shown in Figure 4.2, where free-radical polymerization of methyl methacrylate (MMA) takes place, with azo-bis-isobutyronitrile (AIBN) as initiator and toluene as solvent. The reaction is exothermic and a cooling jacket allows the heat removal. The standard mechanism of free-radical polymerization is assumed, together with the resulting rate laws (Ray, 1972, Ray et al., 1971, Congalidis et al., 1989, Schmidt and Ray, 1981, Tsoukas et al., 1982). The following assumptions are also made:

- perfect mixing in the reactor
- constant density of the reacting mixture (no volume shrinkage)
- constant heat capacity of the reacting mixture
- uniform coolant temperature in the jacket
- insulated reactor and cooling system
- constant density and heat capacity of the coolant
- no polymer in the inlet streams
- no gel effect (because of low conversion of the monomer)
- constant reactor volume (constant volumetric flowrate of the monomer stream)
- negligible flowrate of the initiator solution, in comparison to the flowrate of the monomer stream
- negligible inhibition and chain transfer to solvent reactions
- quasi-steady state and long-chain hypothesis

The dynamic behavior of the process is then described by the following mass and energy balances:

$$
\begin{aligned}
\frac{dC_m}{dt} &= -\left(Z_p exp(\frac{-E_p}{RT}) + Z_{f_m} exp(\frac{-E_{f_m}}{RT})\right) C_m P_0(C_I, T) + \frac{F(C_{m_{in}} - C_m)}{V} \\
\frac{dC_I}{dt} &= -Z_I exp(\frac{-E_I}{RT}) C_I + \frac{F_I C_{I_{in}} - F C_I}{V} \\
\frac{dT}{dt} &= Z_p exp(\frac{-E_p}{RT}) C_m \frac{(-\Delta H_P)}{\rho c_p} P_0(C_I, T) - \frac{UA}{\rho c_p V}(T - T_j) + \frac{F(T_{in} - T)}{V} \\
\frac{dD_0}{dt} &= \left(0.5 Z_{T_c} exp(\frac{-E_{T_c}}{RT}) + Z_{T_d} exp(\frac{-E_{T_d}}{RT})\right) [P_0(C_I, T)]^2 \\
&\quad + Z_{f_m} exp(\frac{-E_{f_m}}{RT}) C_m P_0(C_I, T) - \frac{F D_0}{V} \\
\frac{dD_1}{dt} &= M_m \left(Z_p exp(\frac{-E_p}{RT}) + Z_{f_m} exp(\frac{-E_{f_m}}{RT})\right) C_m P_0(C_I, T) - \frac{F D_1}{V} \\
\frac{dT_j}{dt} &= \frac{F_{cw}}{V_o}(T_{w_0} - T_j) + \frac{UA}{\rho_w c_w V_o}(T - T_j)
\end{aligned}
\tag{4.24}
$$

where

$$
P_0(C_I, T) = \left(\frac{2 f^* C_I Z_I exp(\frac{-E_I}{RT})}{Z_{T_d} exp(\frac{-E_{T_d}}{RT}) + Z_{T_c} exp(\frac{-E_{T_c}}{RT})} \right)^{0.5}
$$

Control of the temperature T and the number average molecular weight $\frac{D_1}{D_0}$ of the polymer product is considered, by manipulating the volumetric flow rate of the initiator F_I and the volumetric flow rate of the cooling water F_{cw}. The concentration of monomer in the inlet stream $C_{m_{in}}$ and the temperature of the inlet stream T_{in} are the major measurable disturbances. Thus, following the standard procedure and setting:

$$x_1 = C_m, \quad x_2 = C_I, \quad x_3 = T, \quad x_4 = D_0, \quad x_5 = D_1, \quad x_6 = T_j$$

and:

$$u_1 = F_I - F_{Is}, \quad u_2 = F_{cw} - F_{cws}$$

$$d_1 = C_{m_{in}} - C_{m_{in}s}, \quad d_2 = T_{in} - T_{ins}$$

$$y_1 = \frac{D_1}{D_0}, \quad y_2 = T$$

where the subscript s denotes steady state values, the system dynamic equations are put in the form of Eq.2.1, with $n = 6, m = 2, p = 2$ and

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \\ f_5(x) \\ f_6(x) \end{bmatrix} =$$

$$\begin{bmatrix} -\left(Z_p exp(\frac{-E_p}{Rx_3}) + Z_{f_m} exp(\frac{-E_{f_m}}{Rx_3})\right) x_1 P_0(x_2, x_3) + \frac{F(C_{m_{in}s} - x_1)}{V} \\ -Z_I exp(\frac{-E_I}{Rx_3}) x_2 + \frac{F_{Is} C_{I_{in}} - F x_2}{V} \\ Z_p exp(\frac{-E_p}{Rx_3}) x_1 \frac{(-\Delta H_P)}{\rho c_p} P_0(x_2, x_3) - \frac{UA}{\rho c_p V}(x_3 - x_6) + \frac{F(T_{ins} - x_3)}{V} \\ \left(0.5 Z_{T_c} exp(\frac{-E_{T_c}}{Rx_3}) + Z_{T_d} exp(\frac{-E_{T_d}}{Rx_3})\right) [P_0(x_2, x_3)]^2 + Z_{f_m} exp(\frac{-E_{f_m}}{Rx_3}) x_1 P_0(x_2, x_3) - \frac{F x_4}{V} \\ M_m \left(Z_p exp(\frac{-E_p}{Rx_3}) + Z_{f_m} exp(\frac{-E_{f_m}}{Rx_3})\right) x_1 P_0(x_2, x_3) - \frac{F x_5}{V} \\ \frac{F_{cws}}{V_o}(T_{w_0} - x_6) + \frac{UA}{\rho_w c_w V_o}(x_3 - x_6) \end{bmatrix}$$

$$g_1(x) = \begin{bmatrix} 0 \\ \frac{C_{I_{in}}}{V} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad g_2(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{T_{wo} - x_6}{V_o} \end{bmatrix}, \quad w_1(x) = \begin{bmatrix} \frac{F}{V} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad w_2(x) = \begin{bmatrix} 0 \\ 0 \\ \frac{F}{V} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$h_1(x) = \frac{x_5}{x_4}, \quad h_2(x) = x_3$$

For the above system, the relative orders are easily found to take the following values:

- **Output** y_1: $r_1 = 2, \ \rho_{11} = 2, \ \rho_{12} = 2$

- **Output** y_2: $r_2 = 2, \ \rho_{21} = 2, \ \rho_{22} = 1$

Consequently, the set of disturbances is partitioned as follows:

- $\mathcal{A}_1 = \emptyset, \ \mathcal{B}_1 = \{d_1, d_2\}, \ \mathcal{C}_1 = \emptyset$

- $\mathcal{A}_2 = \emptyset, \ \mathcal{B}_2 = \{d_1\}, \ \mathcal{C}_2 = \{d_2\}$

Clearly, as a result of the dynamic structure of the particular system, static feedforward/feedback compensation is required in order to eliminate the effect of d_1 and d_2 on y_1 and the effect of d_1 on y_2, while dynamic feedforward/feedback compensation will be necessary to eliminate the effect of d_2 on y_2.

The characteristic matrix $C(x)$ of this system defined by Eq.4.1 becomes:

$$C(x) = \begin{bmatrix} L_{g_1} L_f h_1(x) & L_{g_2} L_f h_1(x) \\ L_{g_1} L_f h_2(x) & L_{g_2} L_f h_2(x) \end{bmatrix} \tag{4.25}$$

where

$$L_{g_1}L_fh_1(x) = \left(\frac{\partial f_5(x)}{\partial x_2} - \frac{x_5}{x_4}\frac{\partial f_4(x)}{\partial x_2}\right)\frac{C_{I_{in}}}{Vx_4}$$

$$L_{g_2}L_fh_1(x) = 0$$

$$L_{g_1}L_fh_2(x) = \frac{\partial f_3(x)}{\partial x_2}\frac{C_{I_{in}}}{V}$$

$$L_{g_2}L_fh_2(x) = \frac{UA}{\rho c_p V}\frac{T_{w0} - x_6}{V_o}$$

It can be easily checked that $C(x)$ is generically nonsingular, which allows the straightforward application of the Theorem 4.1. The control law of Eq.4.3 takes the form:

$$u = \left[\begin{array}{cc} \beta_{12}L_{g_1}L_fh_1(x) + \beta_{22}L_{g_1}L_fh_2(x) & \beta_{12}L_{g_2}L_fh_1(x) + \beta_{22}L_{g_2}L_fh_2(x) \end{array}\right]^{-1}$$

$$\left\{ v - \sum_{i=1}^{2}\sum_{k=0}^{2}\beta_{ik}L_f^kh_i(x) \right.$$

$$- \left(\beta_{12}L_{w_1}L_fh_1(x)d_1(t) + \beta_{12}L_{w_2}L_fh_1(x)d_2(t) + \beta_{22}L_{w_1}L_fh_2(x)d_1(t)\right)$$

$$\left. - \left(\beta_{21}L_{w_2}h_2(x)d_2(t) + \beta_{22}L_{w_2}L_fh_2(x)d_2(t) + \beta_{22}\frac{d}{dt}\left(L_{w_2}h_2(x)d_2(t)\right)\right)\right\} \quad (4.26)$$

where:

$$L_fh_1(x) = \left(f_5(x) - \frac{x_5}{x_4}f_4(x)\right)\frac{1}{x_4}$$

$$L_fh_2(x) = f_3(x)$$

$$L_f^2h_1(x) = \frac{f_1(x)}{x_4}\left(\frac{\partial f_5(x)}{\partial x_1} - \frac{x_5}{x_4}\frac{\partial f_4(x)}{\partial x_1}\right) + \frac{f_2(x)}{x_4}\left(\frac{\partial f_5(x)}{\partial x_2} - \frac{x_5}{x_4}\frac{\partial f_4(x)}{\partial x_2}\right)$$
$$+\frac{f_3(x)}{x_4}\left(\frac{\partial f_5(x)}{\partial x_3} - \frac{x_5}{x_4}\frac{\partial f_4(x)}{\partial x_3}\right) + \frac{f_4(x)}{x_4^2}\left(\frac{x_5F}{V} - f_5(x) + \frac{2x_5f_4(x)}{x_4}\right)$$
$$+\frac{f_5(x)}{x_4}\left(\frac{-F}{V} - \frac{f_4(x)}{x_4}\right)$$

$$L_f^2h_2(x) = \frac{\partial f_3(x)}{\partial x_1}f_1(x) + \frac{\partial f_3(x)}{\partial x_2}f_2(x) + \frac{\partial f_3(x)}{\partial x_3}f_3(x) + \frac{\partial f_3(x)}{\partial x_6}f_6(x)$$

$$L_{w_2}h_2(x) = \frac{F}{V}$$

$$L_{w_1}L_fh_1(x) = \left(\frac{\partial f_5(x)}{\partial x_1} - \frac{x_5}{x_4}\frac{\partial f_4(x)}{\partial x_1}\right)\frac{F}{Vx_4}$$

$$L_{w_2}L_fh_1(x) = \left(\frac{\partial f_5(x)}{\partial x_3} - \frac{x_5}{x_4}\frac{\partial f_4(x)}{\partial x_3}\right)\frac{F}{Vx_4}$$

$$L_{w_1}L_fh_2(x) = \frac{\partial f_3(x)}{\partial x_1}\frac{F}{V}$$

$$L_{w_2}L_fh_2(x) = \frac{\partial f_3(x)}{\partial x_3}\frac{F}{V}$$

In order to obtain full input/output decoupling of the form:

$$\begin{cases} y_1 + \beta_{11}^1\dfrac{dy_1}{dt} + \beta_{12}^1\dfrac{d^2y_1}{dt^2} & = & v_1 \\ y_2 + \beta_{21}^2\dfrac{dy_2}{dt} + \beta_{22}^2\dfrac{d^2y_2}{dt^2} & = & v_2 \end{cases}$$

we simply set:

$$\beta_{20}^1 = \beta_{21}^1 = \beta_{22}^1 = 0,\ \beta_{20}^2 = 1$$

$$\beta_{10}^2 = \beta_{11}^2 = \beta_{12}^2 = 0,\ \beta_{10}^1 = 1$$

in Eq.4.26. The kinetic and physical parameters and the operating steady state conditions for the particular process are given in Tables 1 and 2. Integration of the system dynamic equations was performed (after appropriate dedimensionalization) by using the subroutine LSODA from the ODEBACK Library, on the Apollo network of The University of Michigan. The values for the adjustable parameters in the feedforward/feedback (FF/FB) control law of Eq.4.26 were chosen as:

$$\beta_{11}^1 = \beta_{21}^2 = 0.44h,\ \beta_{12}^1 = \beta_{22}^2 = 0.016h^2$$

to place the closed-loop poles at -2.5 and -25.0 for the two decoupled v_1/y_1 and v_2/y_2 systems. The location of the closed-loop poles was chosen so that the constraints on the magnitude of the manipulated input variables ($F_I \geq 0$ and $F_{cw} \geq 0$) be satisfied. The external linear controllers in the FF/FB control structure were chosen as two PI controllers with settings $K_c = 15$ and $\tau_I = 0.4h$. A number of simulation runs

i_j	Z_{i_j}	E_{i_j}	Reference
T_c	$3.8223 \times 10^{10} kmol/m^3.h$	$2.9442 \times 10^3 kJ/kmol$	(Schmidt and Ray, 1981)
T_d	$3.1457 \times 10^{11} kmol/m^3.h$	$2.9442 \times 10^3 kJ/kmol$	(Schmidt and Ray, 1981)
I	$3.7920 \times 10^{18} h^{-1}$	$1.2877 \times 10^5 kJ/kmol$	(Tobolsky and Baysal, 1953)
P	$1.7700 \times 10^9 kmol/m^3.h$	$1.8283 \times 10^4 kJ/kmol$	(Mahabadi and O'Driscoll, 1977)
f_m	$1.0067 \times 10^{15} kmol/m^3.h$	$7.4478 \times 10^4 kJ/kmol$	(Stickler and Meyhoff, 1978)
$f^* = 0.58$			(Tobolsky and Baysal, 1953)

Table 4.1: Kinetic parameters

F	=	1.00	m^3	F_{I_s}	=	0.01679	m^3/h
V	=	0.1	m^3	R	=	8.314	$kJ/kmol.K$
ρ	=	866	kg/m^3	M_m	=	100.12	$kg/kmol$
$C_{I_{in}}$	=	6.0	$kmol/m^3$	$C_{m_{in_s}}$	=	8.0	$kmol/m^3$
C_p	=	2.0	$kJ/kg.K$	$-\Delta H_P$	=	57,800	$kJ/kmol$
A	=	2.0	m^2	U	=	720	$kJ/m^2.h.K$
ρ_w	=	1,000	kg/m^3	c_w	=	4.2	$kJ/kg.K$
V_o	=	0.02	m^3	T_{in_s}	=	350	K
y_{1sp}	=	25,000	$kg/kmol$	y_{2sp}	=	335	K
F_{cw_s}	=	3.26363	m^3/h	T_{w_o}	=	293.2	K

Table 4.2: Process parameters and steady state values

verified the stability of the open-loop system around the operating steady state and the internal stability of the closed-loop system.

The performance of the proposed feedforward/feedback control methodology was tested in terms of rejection of step changes at the two measurable disturbances. The process was initially assumed to be at steady state. At time $t = 1h$ a step change at the inlet monomer concentration $C_{m_{in}}$ was applied, from 6 to 5 $kmol/m^3$. The process was allowed to reach a new steady state, and at time $t = 6h$ a step change at the inlet temperature T_{in} was applied, from 350 to 345 K.

Figures 4.3 through 4.7 illustrate the profiles of the two controlled outputs and the two manipulated inputs, under the assumption of perfect model and perfect measurements. The figures provide a comparison of the output and input responses under

a) The MIMO FF/FB control structure

b) The MIMO GLC structure (Kravaris and Soroush, 1990)

c) Two linear SISO PI loops (coolant flowrate/temperature, initiator flowrate/number average molecular weight)

The time derivative of the disturbance d_2 in Eq.4.26 was approximated by a lead-lag element with transfer function $\frac{s}{0.001s+1}$. In the implementation of the GLC structure, the same values of β^j_{ik} were used as in the FF/FB structure, while the external linear controllers were chosen as PI controllers with the same settings as in the FF/FB structure. Finally, in the linear control approach, the two SISO PI controllers were tuned through a trial-and-error procedure which resulted in the values $K_c = -1 \times 10^{-7} m^3/h$, $\tau_I = 0.075h$ and $K_c = -0.1 m^3/h.K$, $\tau_I = 0.075h$, respectively for "best" closed-loop performance. Due to the severe nonlinearity of

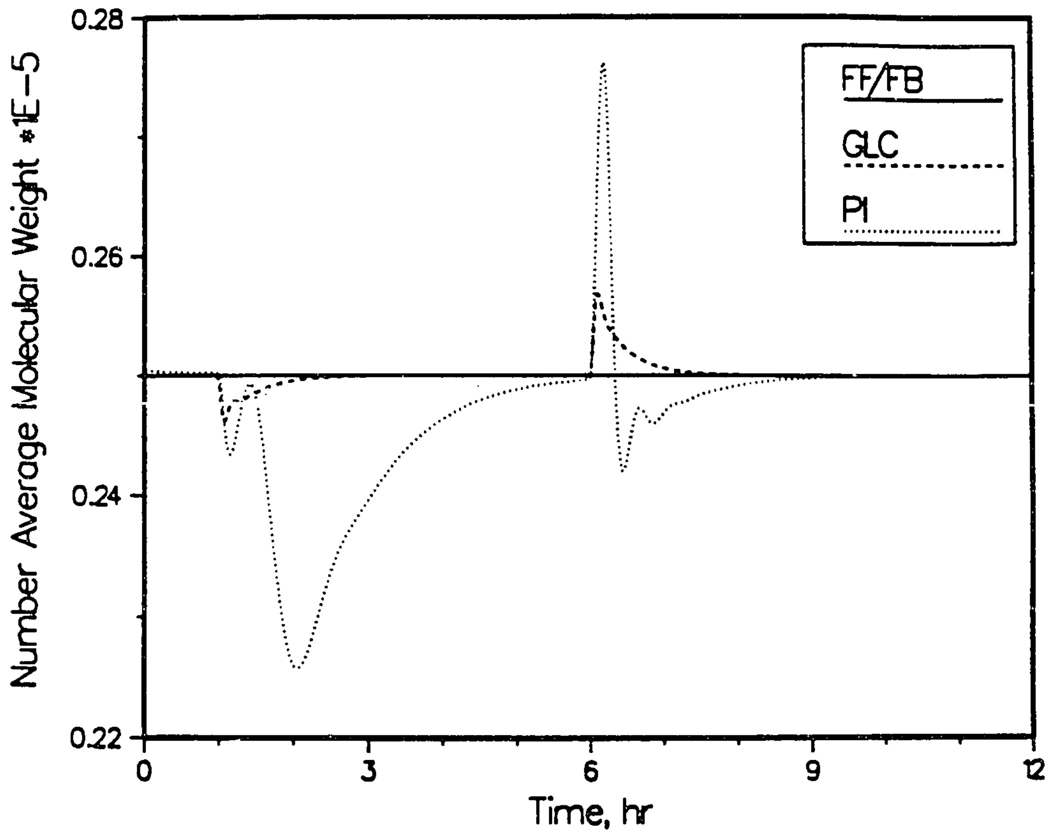

Figure 4.3: Number average molecular weight profiles under feedforward/feedback control, GLC and SISO PI control (perfect model)

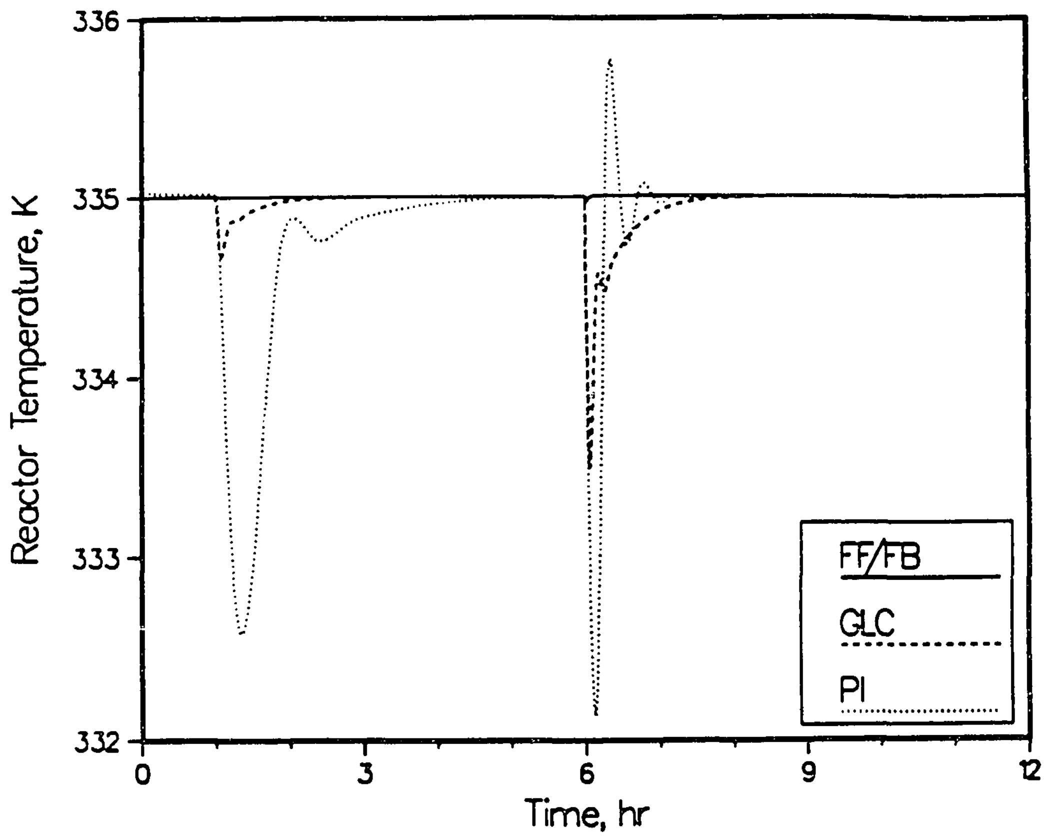

Figure 4.4: Reactor temperature profiles under feedforward/feedback control, GLC and SISO PI control (perfect model)

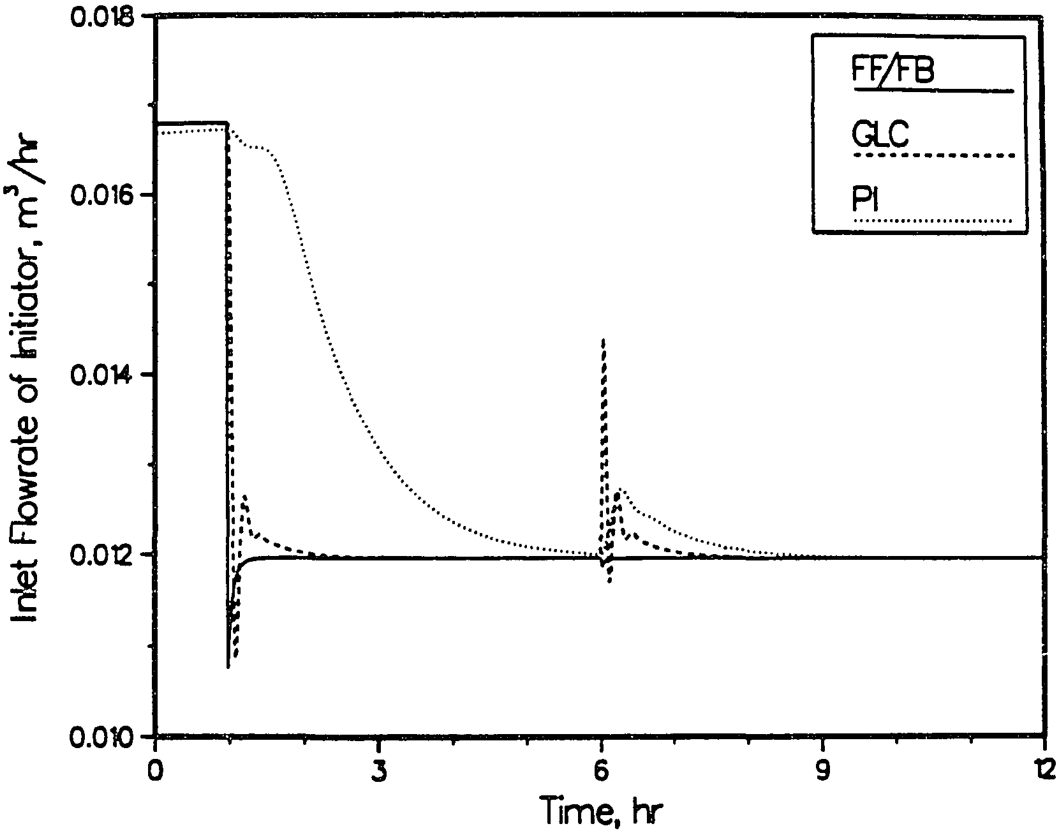

Figure 4.5: Initiator flow rate profiles corresponding to Figures 4.3 and 4.4

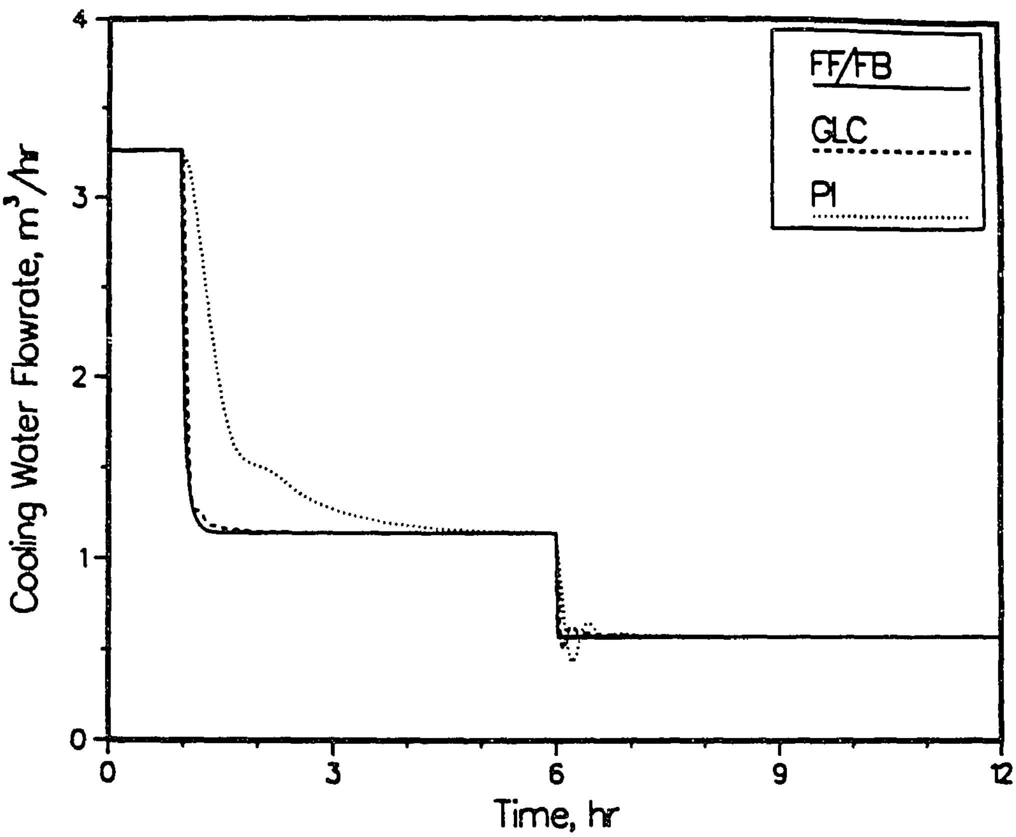

Figure 4.6: Coolant flow rate profiles corresponding to Figures 4.3 and 4.4

the process, the response characteristics were found to be very sensitive to the values of the PI controllers' settings.

Clearly, as the theory predicts, the FF/FB control law results in perfect regulation of the outputs, i.e., an obvious improvement of the closed-loop behavior compared with the one under the GLC structure (where no measurements of the disturbances are used in the control law), or the linear PI controllers.

In another set of simulation runs, assuming the same disturbance changes as previously, the robustness characteristics of the FF/FB method were tested in the face of modeling error and measurement noise. In particular we compared the closed-loop behavior of the process under:

a) Perfect model and perfect disturbance measurements

b) 20% error in the frequency factor Z_I and the heat of reaction ΔH_P

c) Sinusoidal noise in the measurements of the disturbances d_1 and d_2 of amplitudes 0.05 $kmol/m^3$ and 0.5 K, respectively, and period of oscillation of 10 min

Figures 4.7 and 4.8 depict the excellent performance of the FF/FB structure in rejecting the applied step changes in the disturbances for the case when the above model uncertainties exist. As shown in the two figures, even in the presence of the modeling errors, the output profiles are very close to the ones obtained when a perfect model is available. Figures 4.9 and 4.10 depict the performance of the FF/FB structure in rejecting the applied step changes when the disturbance measurements are corrupted with the above noise. Clearly, although the output regulation is not perfect in the presence of measurement noise, the proposed method performs very satisfactorily.

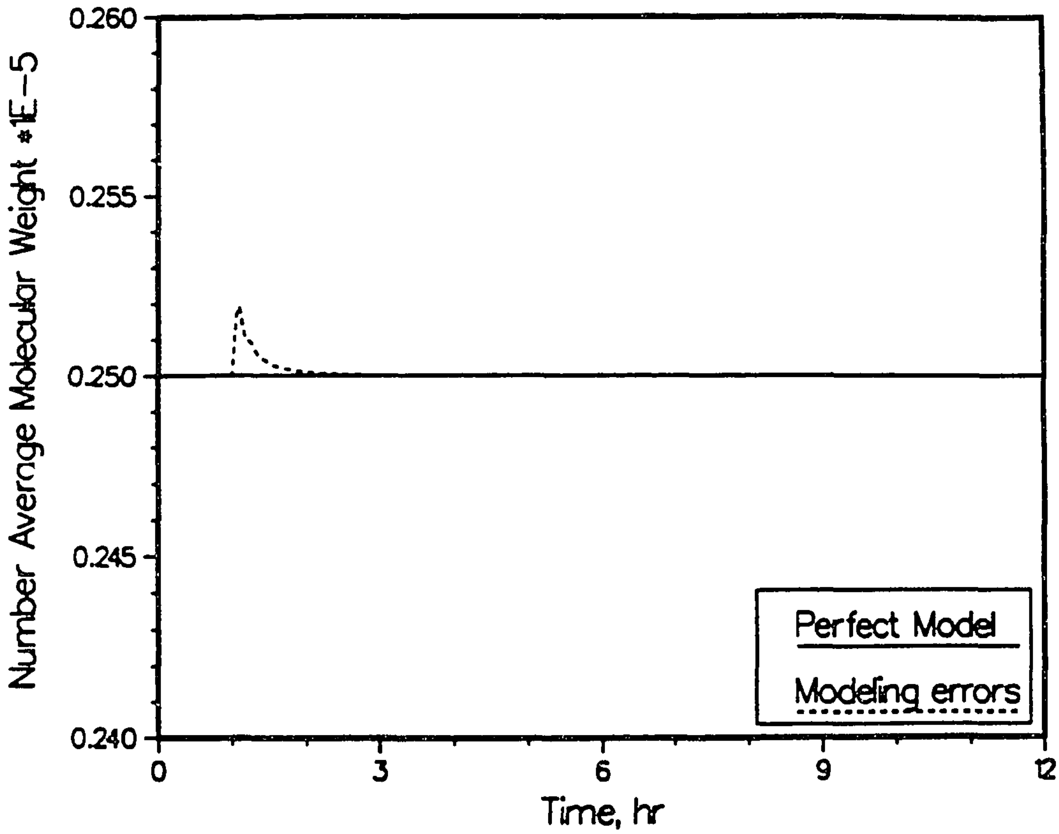

Figure 4.7: Number average molecular weight profile under feedforward/feedback control (effect of modeling error)

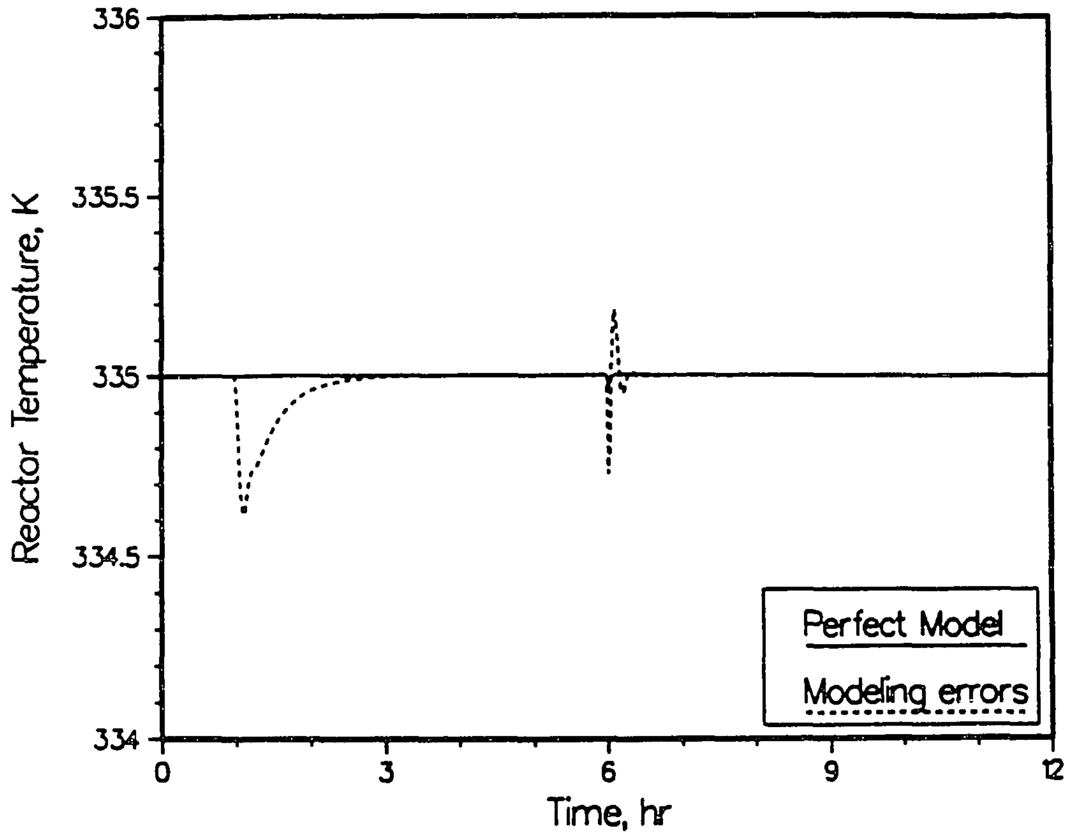

Figure 4.8: Reactor temperature profile under feedforward/feedback control (effect of modeling error)

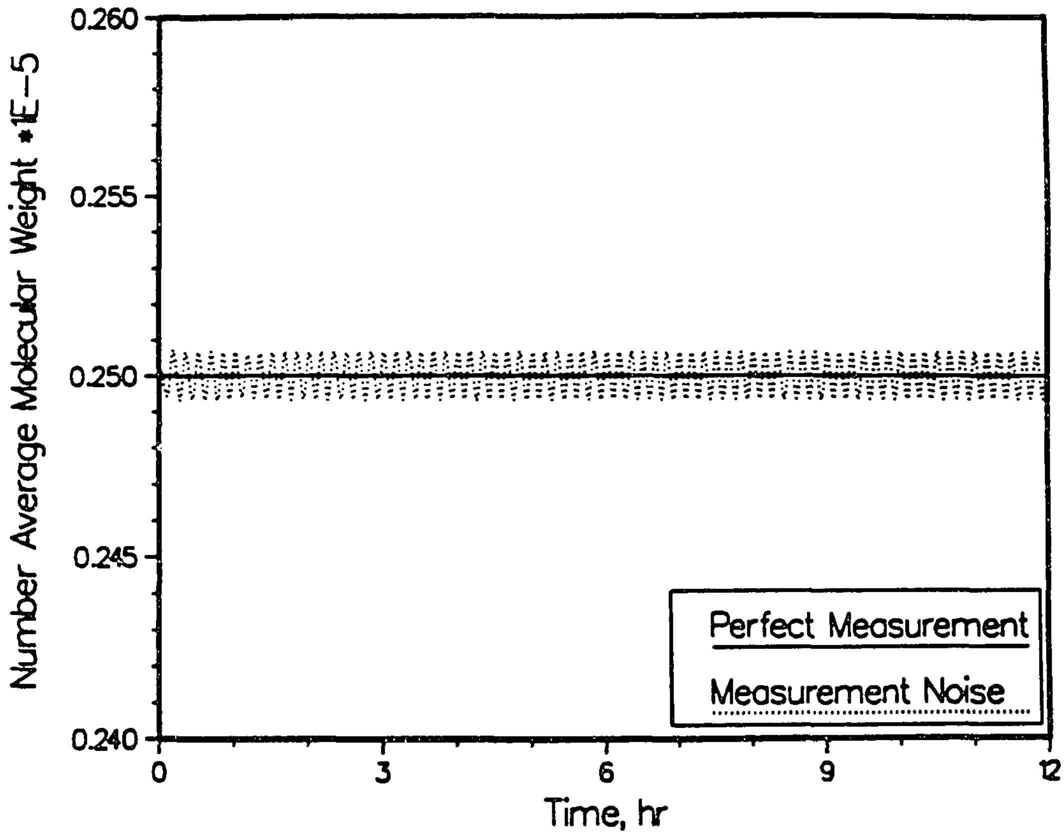

Figure 4.9: Number average molecular weight profile under feedforward/feedback control (effect of measurement noise)

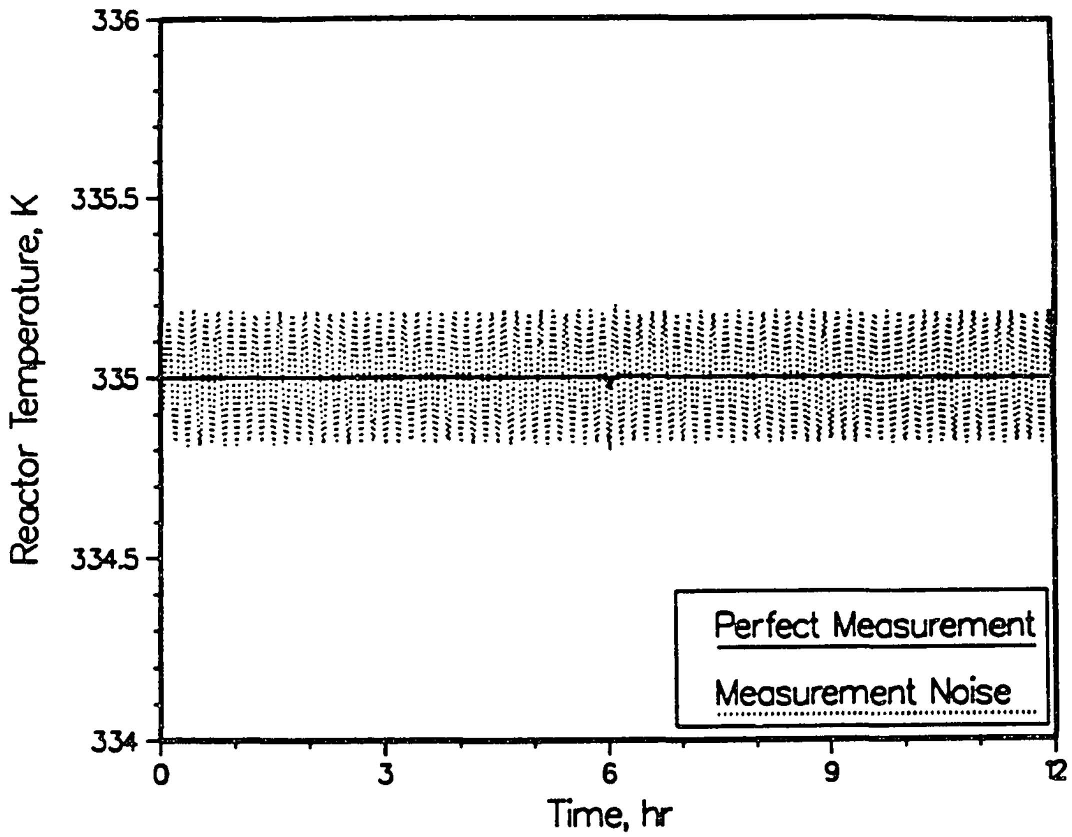

Figure 4.10: Reactor temperature profile under feedforward/feedback control (effect of measurement noise)

4.7 Notation

Roman Letters

A	=	heat tranfer area, m^2
$C(x)$	=	characteristic matrix
C_m	=	molar concentration of the monomer, $kmol/m^3$
$C_{m_{in}}$	=	molar concentration of monomer in the monomer inlet stream, $kmol/m^3$
C_I	=	molar concentration of the initiator, $kmol/m^3$
$C_{I_{in}}$	=	molar concentration of the initiator in the initiator inlet stream, $kmol/m^3$
D_0	=	molar concentration of the dead polymer chains, $kmol/m^3$
D_1	=	mass concentration of the dead polymer chains, kg/m^3
E_{f_m}, E_P, E_I	=	activation energies for chain transfer to monomer, propagation, and initiation reactions, respectively, $kJ/kmol$
E_{T_c}, E_{T_d}	=	activation energies for termination by coupling and disproportionation reactions, respectively, $kJ/kmol$
F	=	volumetric flow rate of the inlet monomer stream, m^3/h
F_I	=	volumetric flow rate of the inlet initiator stream, m^3/h
F_{cw}	=	volumetric flow rate of the inlet cooling water, m^3/h
K_c	=	proportional gain
M_m	=	molecular weight of the monomer, $kg/kmol$
P_0	=	molar concentration of the live polymer chains, $kmol/m^3$
R	=	Ideal Gas constant, $kJ/kmol.K$
T	=	reactor temperature, K
T_j	=	jacket temperature, K

T_{in} = temperature of the inlet streams in the reactor, K

T_{w0} = temperature of the inlet coolant stream, K

U = overall heat transfer coefficient, $kJ/m^2.h.K$

V = reactor volume, m^3

V_o = overall effective volume of the cooling subsystem, m^3

Z_{f_m}, Z_P = frequency factors for chain transfer to monomer, and propagation reactions, respectively, $kmol/m^3.h$

Z_{T_c}, Z_{T_d} = frequency factors for termination by coupling and disproportionation reactions, respectively, $kmol/m^3.h$

Z_I = frequency factor for initiation reaction, h^{-1}

c_p = heat capacity of the reacting mixture, $kJ/kg.K$

c_w = heat capacity of water, $kJ/kg.K$

d = vector of disturbance inputs

f^* = initiator efficiency

f, g_j, w_κ = vector fields

h_i = output scalar field

r_i = relative order of the output y_i with respect to u

t = time

u = vector of manipulated inputs

v = external reference input vector

x = vector of state variables

y = vector of outputs

y_{sp} = vector of output set-points

$-\Delta H_P$ = heat of propagation reaction, $kJ/kmol$

$\mathcal{A}_i, \mathcal{B}_i, \mathcal{C}_i$ = partition of disturbances for the output y_i

Greek Letters

β^j_{ik} = parameters of the feedforward/state feedback law

τ_I = reset time

ρ = density of the reacting mixture, kg/m^3

ρ_w = density of water, kg/m^3

$\rho_{i\kappa}$ = relative order of the output y_i with respect to the disturbance d_κ

Math Symbols

$\Longrightarrow$	=	implies
$\Longleftrightarrow$	=	is equivalent to
$\emptyset$	=	void set
$\in$	=	belongs to
$\subset$	=	subset
$\forall$	=	for all
$\cap$	=	intersection
$<.,.>$	=	inner product
T	=	transpose
min	=	minimum element
det	=	determinant of a matrix
ker	=	kernel of an operator
dh_i	=	gradient of a scalar field h_i
$\mathbb{R}$	=	real line
$\mathbb{R}^n$	=	$n-$dimensional Euclidean space

Acronyms

BIBO	=	bounded-input bounded-output
CSTR	=	continuous stirred tank reactor
FF/FB	=	feedforward/feedback
GLC	=	globally linearizing control
SISO	=	single-input single-output
MIMO	=	multiple-input multiple-output

CHAPTER V

STRUCTURAL EVALUATION OF CONTROL CONFIGURATIONS FOR MIMO NONLINEAR PROCESSES

5.1 Introduction

The first step in the synthesis of a control system for a given process is the synthesis of the control configuration. Although this step precedes the controller synthesis itself, it affects significantly the final performance of the control system. The problem of synthesis of control configurations has been investigated from various points of view in recent years (see e.g., Stephanopoulos, 1983), and, mainly for methodological purposes, can be viewed as consisting of the following two sub-problems:

1. Generation of all feasible control configurations

2. Evaluation and selection of a control configuration

The first sub-problem includes the specification of the control objectives, the identification of the available manipulated inputs and the assessment of feasibility of the resulting control configurations. Research in this area is extensive regarding linear time invariant processes, for which the system theoretic properties of state controllability, output controllability and output functional controllability have been used as feasibility criteria. On the other hand, research regarding nonlinear processes is still at the stage of understanding the corresponding system theoretic properties. In analogy with linear results, right invertibility, a concept closely related to output functional controllability, is the criterion that determines the feasibility of control configurations for most practical purposes. The first attempts to study this issue for general MIMO nonlinear systems have been within the framework of algorithmic procedures for the construction of inverses (Hirschorn, 1979b, 1981a, Singh, 1982a, 1982b, 1982c). In a differential algebraic framework (Fliess, 1985, 1986), the notion of differential output rank has generalized the notion of rank of a transfer matrix in a nonlinear setting and has led to necessary and sufficient rank conditions for invertibility, analogous to the ones for linear systems. Finally, conditions for right invertibility for a particular class of nonlinear systems have also been derived in terms of the "structure at infinity" (Nijmeijer, 1986). The implications of the above theoretical results, however, in the synthesis of control configurations have not been investigated yet.

Given a number of alternative feasible control configurations, the second subproblem consists of the evaluation of the alternative control configurations and the final selection of the one to be employed. In this direction, the majority of research effort for processes described by linear models concerns a) dynamic resilience and b) decentralized control studies. Dynamic resilience studies have mainly focused on identifying factors that pose limitations on the system invertibility (Morari, 1983) and consequently on the achievable control quality. Such factors include dead time (Holt and Morari, 1985a), right-half-plane zeros (Holt and Morari, 1985b), model uncertainty (Skogestad and Morari, 1987), etc. In decentralized control studies, a variety of static and dynamic interaction measures have been proposed for identifying favorable pairings of manipulated inputs and controlled outputs (for a review see

Jensen et al., 1986). By far the most popular analysis tool for this purpose is the relative gain array (RGA) (Bristol, 1966) and its generalizations that take into account dynamic considerations (e.g., Tung and Edgar, 1981 and Gagnepain and Seborg, 1982) or disturbance inputs (e.g., Stanley et al., 1985). All the above approaches assume a transfer function description of the process, often obtained from experimental data and therefore, are based on linear control considerations. On the other hand, in nonlinear process control theory, there are essentially no results related to the problem of evaluation of control configurations except for some results concerning the calculation of nonlinear gains (e.g., Mijares et al., 1985, Manousiouthakis and Nikolaou, 1989). One possible direction is to study the effect of nonlinearities within a linear analysis (and consequently linear controller synthesis) framework. An alternative, much more meaningful direction is to develop analytical tools and methodologies which arise from the nonlinear description of a process itself.

In this chapter, a structural perspective will be introduced in the problem of evaluation of control configurations for MIMO nonlinear processes. Structural methods have already been introduced in the generation and assessment of feasibility of control configurations for linear processes (Morari and Stephanopoulos, 1980, Govind and Powers, 1982, Johnston and Barton, 1985, Johnston et al., 1985, Russel and Perkins, 1987, Georgiou and Floudas, 1989). They are essentially based on graph-theoretic concepts and the notion of structural controllability (Lin, 1974, Shields and Pearson, 1976, Glover and Silverman, 1976). The major advantage of these methods is the genericity of the results and the minimum amount of process information that they require, which allows them to be efficiently used at the early stages of the design procedure. There has not been any attempt, however, to systematically introduce structural considerations in the evaluation and selection of control configurations either for linear or nonlinear processes. On the other hand, intuitive guidelines for the selection and pairing of manipulated inputs do make implicit use of structural considerations, through the notions of "direct effect" and "physical closeness" (see e.g., the modern process control textbooks by Stephanopoulos, 1984 and Seborg et al., 1989). The idea is that chosing a manipulated input which is "physically close" to a controlled variable (or has a "direct effect" on it), we have good chances of obtaining favorable static and dynamic characteristics for the particular input/output pair, i.e., small time delays, small time constants as well as significant static gain. Clearly though, as the size and complexity of the process increase, such intuitive considerations become obscure and sometimes misleading, especially in a MIMO context. Furthermore, there is no theoretical justification on the use of such intuitive notions as evaluation criteria. The results of Chapter II with regard to the concept of relative order established that relative order quantifies the above intuitive notions of "direct effect" and "physical closeness". Also, the controller synthesis results of Chapters III and IV showed that relative order arises naturally in the synthesis of nonlinear control laws, capturing important structural characteristics of a process. Motivated by the above, the purpose of this chapter is:

1. To identify and quantify limitations that the structure of a process poses on the control quality

2. To develop guidelines for the structural evaluation of alternative control configurations based on control quality characteristics and structural coupling considerations

The above guidelines will allow a systematic hierarchization of alternative control configurations at the early stages of the design procedure, based on a minimum amount of process information. Quantitative, static and dynamic, process information can be used at later stages of the design procedure to complement the results of the structural analysis.

Standing assumptions throughout this chapter will be the following:

1. The control of a single processing unit is considered
2. Operational, environmental, economical, safety and production requirements have resulted in a set of control objectives (controlled outputs)
3. The major disturbances have been identified (from physical considerations and possibly steady state gain information)
4. The physical phenomena with non-negligible dynamics have been identified

The term "alternative control configurations" will then imply alternative sets of manipulated inputs, while the term "multi-loop configuration" will be used to denote the specification of input/output pairs for a given set of manipulated inputs. In general, disturbance inputs that can be manipulated may also be considered as manipulated input candidates. Each control configuration will then correspond to a state-space model of the form of Eq.2.1.

In Section 5.2 that follows, the fundamental limitations that the structure of a process poses on the control quality will be studied, as they are expressed by relative orders; this will naturally lead to guidelines for the structural evaluation of control configurations on the basis of the overall servo and regulatory characteristics. Then, a matrix of relative orders will be introduced, which will allow quantifying structural coupling among input and output variables; the analysis will naturally lead to guidelines for evaluating alternative multi-loop configurations, based on structural coupling considerations. Finally, chemical engineering examples will illustrate the application of the proposed generic evaluation framework.

5.2 Structural limitations in the control quality and overall evaluation of control configurations

At a first level of evaluation of alternative control configurations (i.e., alternative sets of manipulated inputs), one would like to identify inherent limitations in the control quality imposed by the structure of the process itself. Since the whole treatment is based on structural considerations, issues like non-minimum-phase behavior, open-loop instability or constraints on the manipulated inputs are beyond consideration at this point, since their assessment requires more quantitative information. Instead, we are concerned with the general tracking and regulatory characteristics of the control configurations and the way that they are affected by structural constraints. The above issues will be investigated in the light of results on nonlinear inversion and nonlinear feedforward/state feedback control. The analysis will focus on systems with non-singular characteristic matrix, which guarantees the feasibility of the corresponding control configurations (Daoutidis and Kravaris, 1991a).

5.2.1 Relative orders in an explicit inversion control framework

In the case of a general MIMO nonlinear system, the issue of invertibility is extremely involved. Hirschorn, 1979b suggested an **algorithm** for the construction of a left inverse, that recursively generates a sequence of operators $S_1, S_2, ..., S_k$, by differentiating the output map. The sequence terminates when the output map for S_k can be solved for the manipulated input vector, in terms of derivatives of the output. Under certain conditions, invertibility of this map implies invertibility of the original system. In Theorem 5.1 that follows, an **explicit formula** for the system's inverse is derived for systems with non-singular characteristic matrix (the proof can be found in Appendix E):

Theorem 5.1: *Consider a MIMO nonlinear system of the form:*

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y_i &= h_i(x) \ , \ i = 1, \cdots, m \end{aligned} \tag{5.1}$$

where $g(x)$ is a $(n \times m)$ matrix with columns the vector fields $g_1(x), \cdots, g_m(x)$. Assume that $detC(x) \neq 0$ for $x \in X$, where $C(x)$ is the characteristic matrix defined in Eq.4.1. Then, the dynamic system :

$$\begin{aligned} \dot{\xi} &= f(\xi) + g(\xi)C(\xi)^{-1}\left(\begin{bmatrix} \dfrac{d^{r_1}y_1}{dt^{r_1}} \\ \vdots \\ \dfrac{d^{r_m}y_m}{dt^{r_m}} \end{bmatrix} - \begin{bmatrix} L_f^{r_1}h_1(\xi) \\ \vdots \\ L_f^{r_m}h_m(\xi) \end{bmatrix} \right) \\ u &= C(\xi)^{-1}\left(\begin{bmatrix} \dfrac{d^{r_1}y_1}{dt^{r_1}} \\ \vdots \\ \dfrac{d^{r_m}y_m}{dt^{r_m}} \end{bmatrix} - \begin{bmatrix} L_f^{r_1}h_1(\xi) \\ \vdots \\ L_f^{r_m}h_m(\xi) \end{bmatrix} \right) \end{aligned} \tag{5.2}$$

is a realization of the inverse of the original system.

Remark 5.1: In the case of a SISO nonlinear system ($m = 1$) with relative order r, the inverse given by Eq.5.2 reduces to:

$$\begin{aligned} \dot{\xi} &= f(\xi) + g(\xi)\frac{\dfrac{d^r y}{dt^r} - L_f^r h(\xi)}{L_g L_f^{r-1} h(\xi)} \\ u &= \frac{\dfrac{d^r y}{dt^r} - L_f^r h(\xi)}{L_g L_f^{r-1} h(\xi)} \end{aligned}$$

which, as expected, is exactly the formula for the inverse of a SISO nonlinear system originally derived by Hirschorn, 1979a.

It is important to note that the order of the output derivatives required in Eq.5.2 is determined by the relative orders $r_1, r_2, \cdots, r_m$, which therefore represent a measure of "improperness" of the inverse system. Therefore, in any explicit inversion-based control structure like IMC, Inferential Control, etc. (Economou et al., 1986, Parrish and Brosilow, 1988), the relative orders $r_1, r_2, \cdots, r_m$ will determine the order of the filter required in order to make the control action finite and consequently the order of the closed-loop response. In the above sense, the relative orders r_i play a fundamental role in "shaping" the closed-loop response.

5.2.2 Relative orders in a feedforward/state feedback control framework

The considerations of the previous subsection become even more transparent within the feedforward/state feedback controller synthesis framework of Chapter IV. Referring to MIMO nonlinear systems of the form of Eq.2.1 with non-singular characteristic matrix, the control law of Eq.4.3 induces the closed-loop response of Eq.4.4 which is of order exactly equal to $(r_1 + r_2 + \cdots + r_m)$. This should not be surprising since such an input/output linearizing control law can also be interpreted as an implicit and finite approximation of an inverse-based controller. Furthermore, considering the relative orders of the outputs y_i with respect to the external input vector v, it is clear that they are exactly equal to r_i. This implies that the order of the closed-loop response for the individual outputs y_i is exactly equal to r_i. It also implies, in loose terms, that the relative orders r_i are preserved in closed-loop and the outputs can not be made more responsive than they were in open-loop. Similar characteristics have been attributed to dead time within the framework of linear control (Holt and Morari, 1985a), which is consistent with the connection of the relative order with apparent dead time established in Chapter II. In the above feedforward/feedback framework, the role of the relative orders $\rho_{i\kappa}$ is also significant. In particular, the extent that the condition $r_i \leq \rho_{i\kappa}$ is satisfied determines the extent that measurements of the disturbances and derivatives of the disturbances are required for complete disturbance rejection on the output y_i; moreover, the difference $(r_i - \rho_{i\kappa})$ represents the order of finite approximation required for the derivatives of the disturbances in the control law.

5.2.3 Overall evaluation of control configurations

The fundamental structural limitations in the control quality expressed by the concept of relative order lead naturally to a framework for the structural evaluation of alternative control configurations, on the basis of their overall servo and regulatory characteristics. In particular, the following criteria arise as the basis of such an evaluation:

1. Low order response characteristics for the individual outputs (min r_i)
2. Low order overall response characteristics (min $(r_1 + \cdots + r_m)$)
3. More direct effect of the manipulated inputs than the disturbance inputs on the controlled outputs ($r_i < \rho_{i\kappa}$)

The intuitive basis of the above criteria lies exactly on the notions of "direct effect" and "physical closeness" (see e.g., the reactor cascade example), for which they provide a quantitative expression. Obviously, the most favorable control configuration would be one for which $r_i = 1$ and $\rho_{i\kappa} > 1$ for all outputs y_i and disturbances d_κ. When such a configuration does not exist, one must carefully hierarchize the alternative control configurations depending on the nature and the specific control needs of the process under consideration. A ranking of the outputs according to their importance may then be helpful in order to identify the most favorable control configurations. The above procedure will also allow identifying disturbances for which feedforward compensation may be required.

Remark 5.2: It is clear from the above discussion that the relative orders r_i (instead of the individual relative orders r_{ij}) capture the overall control quality characteristics. This is a consequence of the fact that we have used multivariable control considerations as the basis of the discussion. In the next section, multi-loop configurations will also be discussed and the individual relative orders r_{ij} will naturally arise.

5.3 Structural coupling and evaluation of multi-loop configurations

At a second level of evaluation, one would like to identify control configurations with favorable input/output coupling characteristics. This is especially important when one is faced with the possibility of employing a multi-loop control configuration (i.e., a partially or completely decentralized control configuration). Obviously, in this case, there is a tradeoff between the simplicity in the controller synthesis and the performance deterioration due to neglected interactions. Steady state gain and time constant considerations, encoded in appropriate interaction measures have been traditionally used in the linear control literature to identify favorable input/output pairs and evaluate the resulting configurations.

The graph-theoretic representation of a process introduced in Chapter II lends itself naturally to a notion of **structural coupling** (or structural interaction), i.e., coupling in the sense of structural interdependencies among the process variables. In the light of Theorem 2.1, relative order arises then as a natural measure of structural coupling between input and output variables. Based on the above, in what follows, we will introduce a matrix of relative orders and use it to systematically formulate intuitive guidelines for the synthesis and evaluation of multi-loop configurations based on structural coupling considerations.

Definition 5.1 (Daoutidis and Kravaris, 1991b): *For a nonlinear process with a model of the form of Eq.2.1, we define the* **relative order matrix***:*

$$M_r = \begin{bmatrix} r_{11} & \cdots & r_{1m} \\ \vdots & & \vdots \\ r_{m1} & \cdots & r_{mm} \end{bmatrix} \tag{5.3}$$

with elements the individual relative orders r_{ij} between the manipulated input and output variables.

Clearly, the relative order matrix of Eq.5.3 captures the overall pattern of structural coupling among manipulated input and output variables in the process under consideration. Before proceeding any further, the well-known notion of a structural matrix and its generic rank will now be reviewed (e.g., Shields and Pearson, 1976, Glover and Silverman, 1976):

Definition 5.2: *A* **structural matrix** *is a matrix having fixed zeros in certain locations and arbitrary entries in the remaining locations. For a given matrix, its* **equivalent structural matrix** *is the one which has zeros and arbitrary entries in exactly the same locations as the zeros and the non-zero entries of the original matrix.*

Definition 5.3: *The* **generic rank** *of a structural matrix is the maximal rank that the matrix achieves as a function of its arbitrary nonzero elements.*

Theorem 5.2 that follows will facilitate the synthesis and evaluation of multi-loop configurations based on structural coupling considerations (the proof is given in Appendix E):

Theorem 5.2: *Consider a nonlinear system in the form of Eq.2.1 and its characteristic matrix $C(x)$. Then, the generic rank of the structural matrix which is equivalent* *to $C(x)$ will be equal to m, if and only if the outputs can be rearranged so that the minimum relative order in each row of the relative order matrix appears in the major diagonal position, i.e., M_r takes the form:*

$$M_r = \begin{bmatrix} r_1 & r_{12} & \cdots & r_{1m} \\ r_{21} & r_2 & \cdots & r_{2m} \\ \vdots & \vdots & & \vdots \\ r_{m1} & r_{m2} & \cdots & r_m \end{bmatrix} \tag{5.4}$$

Remark 5.3: If the matrix $C(x)$ itself is nonsingular (i.e., has full numerical rank), its equivalent structural matrix will also have full generic rank, and the output rearrangement will therefore be possible. The converse, however, is not necessarily true.

Remark 5.4: The output rearrangement contained in Theorem 5.2 is similar to the output rearrangement suggested by Holt and Morari, 1985a in studying the effect of dead time in dynamic resilience and by Jerome and Ray, 1986 in the context of dead time compensation for MIMO linear systems. This is consistent with the connection between apparent dead time and relative order established earlier.

Given a process model with a characteristic matrix whose equivalent structural matrix has full generic rank, the result of Theorem 5.2 is important in two ways:

- The suggested output rearrangement indicates the input/output pairings u_i/y_i with the dominant structural coupling
- After the output rearrangement, the off-diagonal relative orders allow the evaluation of structural coupling between a specific input/output pair and the remaining input and output variables

In particular, off-diagonal relative orders in a row indicate the coupling between a specific output and the other inputs, and they will necessarily (due to the rearrangement) be larger or equal to the diagonal relative order. On the other hand, off-diagonal relative orders in a column indicate the coupling between a specific input and the other outputs, and there is no guarantee that they will be larger or equal to the diagonal relative order. The differences between off-diagonal and diagonal relative orders a) in a column of the relative order matrix: $(r_{li} - r_i)$, and b) in a row of the relative order matrix: $(r_{ij} - r_i)$, provide then a measure of the overall structural coupling in the system, for the particular input/output assignment. The larger these differences are, the weaker the structural coupling is in the system, and the more favorable the employment of a multi-loop configuration is from a structural point of view. In the above spirit, it is also possible to identify groups of inputs and outputs such that structural coupling among members of different groups is weak, providing thus favorable candidates for partially decentralized control structures.

Remark 5.5: In the special case of an input/output decoupled system, M_r becomes:

$$M_r = \begin{bmatrix} r_1 & \infty & \cdots & \infty \\ \infty & r_2 & \cdots & \infty \\ \vdots & \vdots & & \vdots \\ \infty & \infty & \cdots & r_m \end{bmatrix}$$

The linear analog of this case would be a diagonal transfer function matrix.

5.4 Concluding remarks

Relative order has been established as a fundamental structural concept, which quantifies the notions of "direct effect" and "physical closeness", expresses fundamental structural limitations in the control quality and allows the evaluation of structural coupling among input and output variables in a process. The above properties allowed us to develop general guidelines for the structural evaluation of alternative control configurations. In summary, for a particular process and after the alternative control configurations are identified,

- The relative orders r_{ij} and $\rho_{i\kappa}$ for all i, j, κ are calculated
- The relative order matrix M_r is formed

Then, after checking the nonsingularity of the characteristic matrix $C(x)$ (or its equivalent structural matrix), we proceed with an evaluation of the overall servo and regulatory characteristics of the alternative configurations and the evaluation of structural coupling. Clearly, the above evaluation framework is a generic one; it allows quantifying structural differences of control configurations, if there are any, and allows **a hierarchization of alternative control configurations**, often based on the specific control needs of the process under consideration. At the early stages of the design procedure, with a minimum amount of information availabe, this is clearly the best we can hope for. In later stages of the design procedure, when more quantitative information becomes available, additional analytical tools have to be employed in order to check the modeling assumptions and make sure that the structurally favorable control configurations are statically and dynamically well-defined and well-behaved.

5.5 Illustrative examples

In this section, the structural evaluation guidelines developed previously will be applied to three typical chemical engineering processes. In the first two examples and without loss of generality, the analysis will be based on detailed state-space models in order to better illustrate the procedure. In the third example, the analysis will be based on purely structural information.

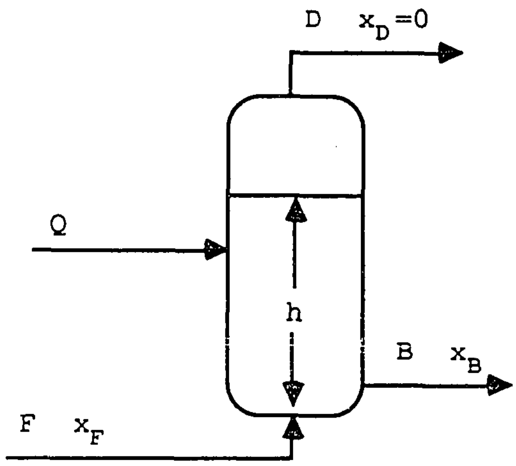

Figure 5.1: A single effect evaporator

5.5.1 A single-effect evaporator

In this example, the single-effect evaporator shown in Figure 5.1 is considered. A solution stream at solute molar concentration x_F enters the evaporator at a molar flow rate F. Heat provided by steam is used to vaporize the water, producing a vapor stream D and a liquid effluent B at solute concentration x_B. For the purpose of this example, the following simplifying assumptions are made:

1. The liquid is perfectly mixed
2. The solute concentration in the vapor stream is negligible compared with that of the liquid stream ($x_D = 0$)
3. The vapor holdup is insignificant
4. The feed and bottom stream have a constant molar density c
5. The vapor and liquid are in thermal equilibrium at all times

6. All the heat input to the evaporator is used for vaporization

7. The heat capacities of the steam chests, tube walls etc., are negligible

Under the above assumptions, the following equations describe the dynamic behavior of the process:

- Total material balance

$$Ac\frac{dh}{dt} = F - B - D \tag{5.5}$$

- Solute balance

$$Ac\frac{d(hx_B)}{dt} = Fx_F - Bx_B \tag{5.6}$$

where

A	=	cross sectional area
F, B, D	=	molar flow rates
c	=	molar density of feed and bottom streams
h	=	liquid level in the evaporator
x_F, x_B	=	solute concentration at the feed and bottom stream respectively (in mole fractions)

Assumption 3 implies that the flowrate D is equal to the rate of evaporation and together with assumption 6 imply that:

$$D = \frac{Q}{\Delta H_v} \tag{5.7}$$

where ΔH_v is the latent heat of vaporization and Q is the heat input to the evaporator. The above equation can then be substituted to the total material balance. Clearly, the variables to be controlled are the liquid level in the evaporator, h, and the concentration of the effluent stream, x_B. Available manipulated variables are the flowrate B and the heat input Q, while x_F is the major disturbance. Thus, setting:

$$x_1 = h - h_s, \; x_2 = x_B - x_{Bs}$$

and also:

$$u_1 = B - B_s, \; u_2 = Q - Q_s$$

$$d_1 = x_F - x_{Fs}$$

$$y_1 = x_1, \; y_2 = x_2$$

where the subscript s denotes a nominal steady-state value, the dynamic equations assume the following state-space form:

$$\begin{aligned}
\dot{x} &= \begin{bmatrix} 0 \\ -\dfrac{B_s x_2}{Ac(x_1 + h_s)} \end{bmatrix} \\
&+ \begin{bmatrix} -\dfrac{1}{Ac} \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} -\dfrac{\Delta H_v}{Ac} \\ \dfrac{\Delta H_v (x_2 + x_{Bs})}{Ac(x_1 + h_s)} \end{bmatrix} u_2 \qquad (5.8) \\
&+ \begin{bmatrix} 0 \\ \dfrac{F}{Ac(x_1 + h_s)} \end{bmatrix} d_1 \\
y_1 &= x_1 \\
y_2 &= x_2
\end{aligned}$$

The vector fields $f(x), g_1(x), g_2(x), w_1(x)$ and the scalar fields $h_1(x), h_2(x)$ can be easily identified from the above equations. A straightforward calculation of the relative orders r_{ij} and the relative order matrix M_r yields:

$$M_r = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

while the characteristic matrix of the above system is found to be equal to:

$$C(x) = \begin{bmatrix} L_{g_1}h_1(x) & L_{g_2}h_1(x) \\ 0 & L_{g_2}h_2(x) \end{bmatrix} = \begin{bmatrix} -(\frac{1}{Ac}) & -(\frac{\Delta H_v}{Ac}) \\ 0 & \frac{\Delta H_v(x_2 + x_{Bs})}{Ac(x_1 + h_s)} \end{bmatrix}$$

and is nonsingular, which guarantees the feasibility of the control configuration and allows the application of Theorem 5.2.

Clearly,

$$r_1 = 1, \; r_2 = 1$$

and the overall structural characteristics of the configuration are the best possible. Moreover, the relative orders of the two outputs with respect to the disturbance input take the values:

$$\rho_{11} = \infty, \; \rho_{21} = 1$$

which indicate that the output y_1 is not affected by the disturbance d_1, while y_2 is affected in a direct way and moreover $\rho_{21} = r_2$. This implies that feedforward compensation will be required for the disturbance d_1 in order to completely eliminate its effect on y_2.

Proceeding with the evaluation of structural coupling for the given control configuration, note that the relative order matrix is in a form such that all the r_i are in the major diagonal. This automatically suggests an input/output pairing of the form:

$$(u_1/y_1), \;\; (u_2/y_2)$$

i.e.,

$$(B/h), \;\; (Q/x_B)$$

as the most favorable input/output pairing from a structural point of view, while the off-diagonal relative orders in the relative order matrix indicate a significant one-way

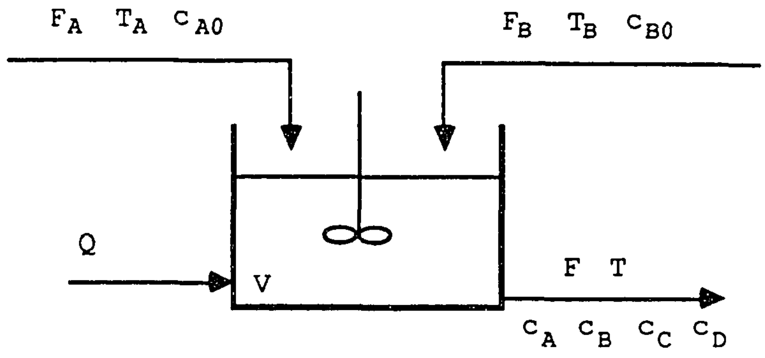

Figure 5.2: A continuous stirred tank reactor

structural coupling. The above conclusion clearly agrees with intuitive considerations based on criteria of the "direct effect" or "physical closeness" type.

5.5.2 A continuous stirred tank reactor

Consider the CSTR shown in Figure 5.2. Two solution streams consisting of species A and B, at volumetric flowrates F_A and F_B, temperatures T_A and T_B and concentrations c_{A0} and c_{B0}, respectively, enter the reactor, where the elementary reaction $A + B \longrightarrow C + D$ takes place. The effluent stream leaves the reactor at flowrate F, concentrations c_A, c_B, c_C, c_D and temperature T. Heat may be added to or removed from the system at a rate Q, using an appropriate heating/cooling system. Assuming constant density ρ and constant heat capacity C_p for the liquid streams and neglecting heat of solution effects, the material and energy balances that describe the dynamic behavior of the process take the following form:

$$
\begin{aligned}
\frac{dV}{dt} &= F_A + F_B - F \\
\frac{dc_A}{dt} &= \frac{F_A}{V}(c_{A0} - c_A) - c_A\frac{F_B}{V} - kc_Ac_Be^{(-\frac{E}{RT})} \\
\frac{dc_B}{dt} &= \frac{F_B}{V}(c_{B0} - c_B) - c_B\frac{F_A}{V} - kc_Ac_Be^{(-\frac{E}{RT})} \\
\frac{dc_C}{dt} &= -c_C\frac{F_A + F_B}{V} + kc_Ac_Be^{(-\frac{E}{RT})} \\
\frac{dT}{dt} &= \frac{F_A}{V}(T_A - T) + \frac{F_B}{V}(T_B - T) + \frac{(-\Delta H)}{\rho C_p}kc_Ac_Be^{(-\frac{E}{RT})} + \frac{1}{V\rho C_p}Q
\end{aligned}
\tag{5.9}
$$

where

C_p	=	heat capacity
E	=	activation energy
F_A, F_B, F	=	volumetric flow rates
Q	=	heat input to the tank
T_A, T_B, T	=	temperatures
V	=	volume
$-\Delta H$	=	heat of reaction
c_i	=	molar concentrations of species i
ρ	=	density

For the above process, we wish to control the volume of the liquid in the tank, V, the concentrations of the effluent stream, c_A, c_C, and the temperature of the effluent stream, T. Available manipulated variables are the flowrates F_A, F_B, F and the heat input Q. Thus, setting:

$$x_1 = V - V_s,\ x_2 = c_A - c_{As},\ x_3 = c_B - c_{Bs},\ x_4 = c_C - c_{Cs},\ x_5 = T - T_s$$

and also:

$$u_1 = F_A - F_{As},\ u_2 = F_B - F_{Bs},\ u_3 = F - F_s,\ u_4 = Q - Q_s$$

$$y_1 = x_1,\ y_2 = x_2,\ y_3 = x_4,\ y_4 = x_5$$

where the subscript s denotes a nominal steady-state value, the dynamic equations can be put in the standard state-space form of Eq.2.1. Then, the calculation of the relative orders and the relative order matrix is straightforward and yields:

$$M_r = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \infty \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 1 \end{bmatrix}$$

The characteristic matrix is given by:

$$C(x) = \begin{bmatrix} L_{g_1}h_1(x) & L_{g_2}h_1(x) & L_{g_3}h_1(x) & 0 \\ L_{g_1}h_2(x) & L_{g_2}h_2(x) & 0 & 0 \\ L_{g_1}h_3(x) & L_{g_2}h_3(x) & 0 & 0 \\ L_{g_1}h_4(x) & L_{g_2}h_4(x) & 0 & L_{g_4}h_4(x) \end{bmatrix}$$

and its equivalent structural matrix has full generic rank.

The overall structural characteristics of the control configuration are clearly the best possible, since all r_i are equal to 1.

Following Theorem 5.2, we interchange the first and the third row of M_r, obtaining the following form of the relative order matrix:

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & \infty \\ 1 & 1 & 2 & 1 \end{bmatrix}$$

with the relative orders r_i in the major diagonal. Further rearrangement of the first and second row is possible, without affecting the form of the relative order matrix.

Consequently, the input/output pairs with the dominant structural coupling are :

$$(u_1/y_2),\ (u_2/y_3),\ (u_3/y_1),\ (u_4/y_4)$$

i.e.,

$$(F_A/c_A),\ (F_B/c_C),\ (F/V),\ (Q/T)$$

or:

$$(u_1/y_3),\ (u_2/y_2),\ (u_3/y_1),\ (u_4/y_4)$$

i.e.,

$$(F_A/c_C),\ (F_B/c_A),\ (F/V),\ (Q/T)$$

On the other hand, the off-diagonal relative orders indicate a significant overall structural coupling, induced mainly by F_A, F_B.

Note that as in the previous example, the results conform with intuitive considerations about the process.

5.5.3 A heat exchanger network

Consider the network of heat exchangers shown in Figure 5.3 (Georgiou and Floudas, 1989). The energy balances that describe the dynamic behavior of the process have the following structural form:

$$\begin{aligned}
\frac{dT_1}{dt} &= \phi_1(T_1, T_2, T_{10}, F_1) \\
\frac{dT_2}{dt} &= \phi_2(T_1, T_2, T_{20}, F_2) \\
\frac{dT_3}{dt} &= \phi_3(T_3, T_4, T_{30}, F_3) \\
\frac{dT_4}{dt} &= \phi_4(T_3, T_4, T_{40}, F_4) \\
\frac{dT_5}{dt} &= \phi_5(T_5, T_6, T_{50}, F_5) \\
\frac{dT_6}{dt} &= \phi_6(T_5, T_6, T_{60}, F_6)
\end{aligned} \tag{5.10}$$

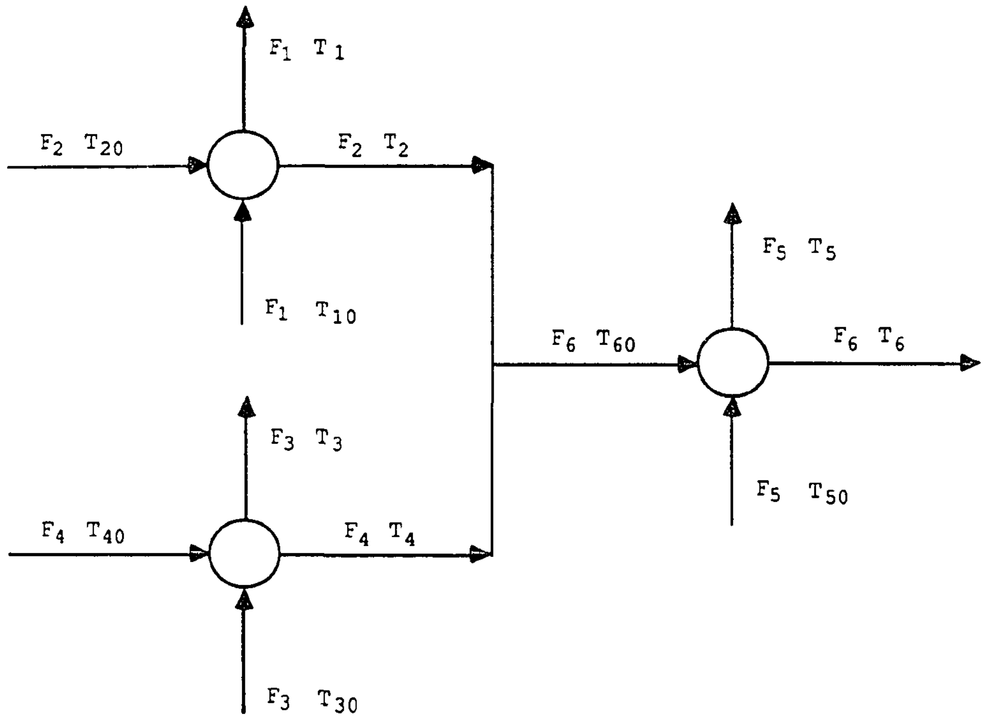

Figure 5.3: A heat exchanger network

where

$$F_i = \text{flow rate of stream } i$$

$$T_i = \text{exit temperature of stream } i$$

$$T_{i0} = \text{entrance temperature of stream } i$$

and $\phi_i(.)$ denotes a functional dependence.

Assuming steady state conditions at the mixing junction, the following algebraic equations also hold:

$$F_6 = \phi_7(F_2, F_4) \tag{5.11}$$

$$T_{60} = \phi_8(T_2, T_4, F_2, F_4) \tag{5.12}$$

Consequently, the last equation in Eq.5.10 can be more appropriately represented as:

$$\frac{dT_6}{dt} = \phi_9(T_2, T_4, T_5, T_6, F_2, F_4) \tag{5.13}$$

The control objective, determined by the operational needs of the plant under consideration, is to keep the temperatures T_1 and T_6 at some desired values. The major disturbances are considered to be the temperatures T_{30}, T_{50}. For notational consistency, let:

$$d_1 = T_{30},\ d_2 = T_{50}$$

$$y_1 = T_1,\ y_2 = T_6$$

Available manipulated inputs are the flowrates F_1, F_2 and F_4. Therefore, three alternative control configurations are possible, corresponding to the pairs of manipulated inputs (F_1, F_2), (F_2, F_4), and (F_1, F_4).

The structural dynamic model of the above process corresponds to the digraph representation shown in Figure 5.4 (where only the input nodes that correspond to the possible manipulated inputs and the disturbances are shown, for simplicity). For the three alternative control configurations under consideration, the calculation of the

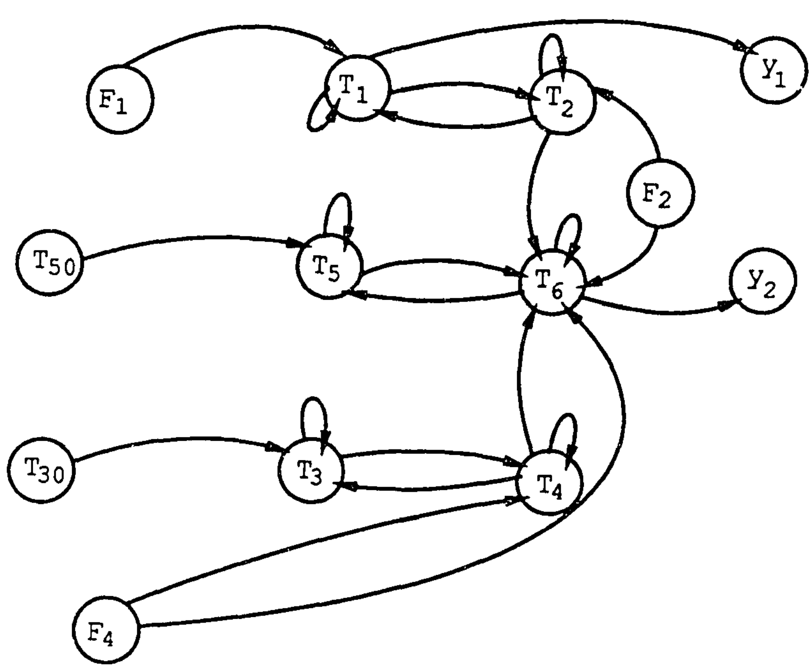

Figure 5.4: The digraph of the heat exchanger network

various relative orders can be based on the result of Theorem 2.1 and can be readily performed from the digraph representation of the process. More specifically:

Configuration 1: $u_1 = F_1$, $u_2 = F_2$

$$M_r = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

and the characteristic matrix has the form:

$$C(x) = \begin{bmatrix} L_{g_1}h_1(x) & 0 \\ 0 & L_{g_2}h_2(x) \end{bmatrix}$$

which guarantees full generic rank of its equivalent structural matrix.

Configuration 2: $u_1 = F_2$, $u_2 = F_4$

$$M_r = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} 2 & \infty \\ 1 & 1 \end{bmatrix}$$

and the characteristic matrix has the form:

$$C(x) = \begin{bmatrix} L_{g_1} L_f h_1(x) & 0 \\ L_{g_1} h_2(x) & L_{g_2} h_2(x) \end{bmatrix}$$

which also guarantees full generic rank of its equivalent structural matrix.

Configuration 3: $u_1 = F_1$, $u_2 = F_4$

$$M_r = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} 1 & \infty \\ 3 & 1 \end{bmatrix}$$

and the characteristic matrix has the form:

$$C(x) = \begin{bmatrix} L_{g_1} h_1(x) & 0 \\ 0 & L_{g_2} h_2(x) \end{bmatrix}$$

which also guarantees full generic rank of its equivalent structural matrix.

Also, the relative orders with respect to the disturbance inputs are given by:

$$\rho_{11} = \infty, \ \rho_{12} = \infty$$

$$\rho_{21} = 3, \ \rho_{22} = 2$$

Clearly, the relative orders with respect to the manipulated input vectors take the following values:

Configuration 1: $r_1 = 1$, $r_2 = 1$

Configuration 2: $r_1 = 2$, $r_2 = 1$

Configuration 3: $r_1 = 1$, $r_2 = 1$

Since $r_i < \rho_{i\kappa}$ for all i, κ, all three configurations have very favorable regulatory characteristics from a structural point of view. Configurations 1 and 3 have the best possible overall structural characteristics, since $r_1 = r_2 = 1$ for both, while configuration 2 has less favorable structural characteristics since $r_1 = 2$.

We can now proceed evaluating the structural coupling in the three configurations. The relative order matrices do not require any rearrangement and they immediately indicate the most favorable input/output pairings for each configuration. A close inspection of the off-diagonal elements indicates that configuration 2 has an unfavorable structural coupling, since the off-diagonal relative order in the first column of M_r is smaller than the diagonal. Comparing the structural coupling in configurations 2 and 3, it is clear that configuration 3 is the most favorable one, since it is characterized by the weakest structural coupling. In the case of a multi-loop configuration, the most structurally favorable input/output pairing would then be:

$$(F_1/T_1), \quad (F_4/T_6)$$

5.6 Notation

Roman Letters

$C(x)$ = characteristic matrix

M_r = relative order matrix

r_i = relative order of the output y_i with respect to the manipulated input vector u

r_{ij} = relative order of the output y_i with respect to the manipulated input u_j

t = time

u_j = manipulated input

x = vector of state variables

y_i = output to be controlled

Greek Letters

$\rho_{i\kappa}$ = relative order of the output y_i with respect to the disturbance d_κ

Acronyms

CSTR = continuous stirred tank reactor

MIMO = multiple-input multiple-output

CHAPTER VI

CONCLUSIONS

In this thesis, a unified methodological framework was developed for the synthesis of feedforward/feedback control systems for multivariable nonlinear processes. First, an original formulation of the concept of relative order was introduced, in order to study the effect of disturbance inputs on process outputs. A number of attractive properties of relative order were rigorously established: its generic calculation requires only structural information for the process, it provides a measure of sluggishness of the respone, it quantifies the intuitive notions of "direct effect" and "physical closeness" and it represents a structural analog of apparent dead time. Then, a general feedforward/feedback control problem was formulated for multivariable nonlinear processes. The key step in the solution of this problem was the synthesis of explicit feedforward/state feedback control laws that completely eliminate the effect of measurable disturbances on the process outputs and induce a well characterized linear input/output behavior. A general feedforward/feedback control structure was developed, which incorporates a linear multivariable controller with integral action to account for model uncertainty and unmeasured disturbances. Closed-loop stability, performance and degree of coupling were associated with appropriate choice of a number of adjustable parameters. The proposed methodology was applied to composition control in a cascade of chemical reactors in series and number average molecular weight and temperature control in a continuous polymerization reactor. Simulation studies verified the theoretical results and illustrated the superiority of the proposed method over existing linear and nonlinear techniques. Motivated by the fundamental properties of relative orders and the controller synthesis results, the problem of selection of control configurations was also addressed. General guidelines were developed for the evaluation and hierarchization of alternative control configurations at the preliminary stages of the design procedure, on the basis of their structural characteristics.

The results of the thesis illustrated the power of differential geometric methods in addressing typical control problems for nonlinear systems. The state-space approach was advocated throughout the thesis (as opposed to the input/output approach), because of the explicitness of the results and the transparent insight obtained from an analysis point of view. Appropriate combination of state observers and state feedback controllers appears to provide effective ways for dealing with the issue of unavailable state measurements (Daoutidis and Kravaris, 1991c, Daoutidis et al., 1990). The methodological framework introduced in the thesis can be generalized to more general forms of nonlinear systems in a straightforward fashion. Future research in this direction must also address the development of adaptive control schemes that deal effectively with parametric uncertainty and unmeasured disturbances. Other challenging problems include the development of ISE-optimal compensators for MIMO nonlinear systems with time delays or unstable inverses, as well as the development of design methods that take systematically into account modeling errors and constraints in the process variables. Regarding the problem of selection of control configurations, future research must focus on static and dynamic limitations in the control quality posed by different control configurations, as well as issues of feasibility of control configurations. Coupled with the theoretical investigations, issues related to the implementation of the control methods are also of obvious importance. These include the study of discretization and sampling effects, the development of software packages within a symbolic computation environment, and finally the experimental application of the methods.

APPENDICES

APPENDIX A

DIFFERENTIAL GEOMETRIC NOTATION

Let $f(x)$ denote a smooth vector field on $\mathbb{R}^n$ and $h(x)$ a smooth scalar field on $\mathbb{R}^n$. Then, the Lie derivative of $h(x)$ with respect to $f(x)$ is defined as:

$$L_f h(x) = \sum_{l=1}^{n} f_l(x) \frac{\partial h(x)}{\partial x_l} \tag{A.1}$$

where $f_l(x)$ denotes the l-th row of $f(x)$.

Higher order Lie derivatives can be defined inductively, as follows:

$$\begin{aligned} L_f^0 h(x) &= h(x) \\ L_f^k h(x) &= L_f L_f^{k-1} h(x), \quad k = 1, 2, \cdots \end{aligned} \tag{A.2}$$

Let $g(x)$ denote a different smooth vector field on $\mathbb{R}^n$. Then, mixed Lie derivatives of the form:

$$L_g L_f^k h(x)$$

can also be defined in an obvious way.

The Lie derivative operator is a linear first-order partial differential operator defined by:

$$L_f = \sum_{l=1}^{n} f_l(x) \frac{\partial}{\partial x_l} \tag{A.3}$$

For h_1, h_2, smooth scalar fields and f, g, smooth vector fields, the following properties of the Lie derivative operator hold:

$$
\begin{aligned}
L_f(h_1 + h_2) &= L_f h_1 + L_f h_2 \\
L_{f+g} &= L_f + L_g \\
L_{h_1 f} &= h_1 L_f \\
L_g L_f &\neq L_f L_g
\end{aligned}
\tag{A.4}
$$

APPENDIX B

PROOFS OF CHAPTER II

B.1 Proof of Theorem 2.1

Only the part of the theorem concerning the relative order r_{ij} will be proved. The same arguments will hold for ρ_{ik}. The procedure follows closely the one by Kasinski and Levine, 1984.

For the purpose of the proof, define ν_{ij} as the smallest integer such that there exist integers $k_1, k_2, \cdots, k_{\nu_{ij}} \in \{1, \cdots, n\}$ for which:

$$g_{jk_{\nu_{ij}}}(x)\frac{\partial f_{k_{\nu_{ij}-1}}(x)}{\partial x_{k_{\nu_{ij}}}} \cdots \frac{\partial f_{k_1}(x)}{\partial x_{k_2}}\frac{\partial h_i(x)}{\partial x_{k_1}} \not\equiv 0 \tag{B.1}$$

The proof of the theorem will then go through the following steps:

Step 1: It will be shown that $\nu_{ij} = \ell_{ij} - 1$.

Step 2: It will be shown that $\nu_{ij} \leq r_{ij}$.

Step 3: It will be shown that generically $\nu_{ij} = r_{ij}$.

Step 1: $\nu_{ij} = \ell_{ij} - 1$

From the definition of ν_{ij}, we have that :

$$g_{jk_{\nu_{ij}}}(x), \frac{\partial f_{k_{\nu_{ij}-1}}(x)}{\partial x_{k_{\nu_{ij}}}}, \cdots, \frac{\partial f_{k_1}(x)}{\partial x_{k_2}}, \frac{\partial h_i(x)}{\partial x_{k_1}} \not\equiv 0 \tag{B.2}$$

Consequently, according to the definition of the graph, the sequence $(u_j, x_{k_{\nu_{ij}}}, \cdots, x_{k_1}, y_i)$ corresponds to a directed path connecting u_j and y_i, of length $(\nu_{ij} + 1)$. By its definition, ℓ_{ij} is the length of the shortest path connecting u_j and y_i. Therefore, $\ell_{ij} \leq (\nu_{ij} + 1)$.

Suppose now that ℓ_{ij} is strictly less than $(\nu_{ij} + 1)$. Then, by the definition of the graph and ℓ_{ij}, there exist integers $k_1, k_2, \cdots, k_{\ell_{ij}-1} \in \{1, \cdots, n\}$, such that:

$$g_{jk_{\ell_{ij}-1}}(x) \frac{\partial f_{k_{\ell_{ij}-2}}(x)}{\partial x_{k_{\ell_{ij}-1}}} \cdots \frac{\partial f_{k_1}(x)}{\partial x_{k_2}} \frac{\partial h_i(x)}{\partial x_{k_1}} \not\equiv 0 \tag{B.3}$$

with $\ell_{ij} - 1 < \nu_{ij}$. But this leads to contradiction, since ν_{ij} is by its definition the smallest integer for which such a sequence of integeres exists. Consequently, the strict inequality does not hold and $\ell_{ij} = (\nu_{ij} + 1)$.

Step 2: $\nu_{ij} \leq r_{ij}$

In order to proceed with the proof, some auxiliary notation is needed. In particular, we define the subsets Γ_i^j and $\tilde{\Gamma}_i^j$ of $\{1, \cdots, n\}$, with $j \geq 1$, by induction, as follows:

$$\begin{aligned}
\Gamma_i^1 &= \{k_1 \in \{1, \cdots, n\} : \frac{\partial h_i(x)}{\partial x_{k_1}} \not\equiv 0\} = \tilde{\Gamma}_i^1 \\
\Gamma_i^j &= \Big\{k_j \in \{1, \cdots, n\} : \exists\, k_{j-1} \in \tilde{\Gamma}_i^{j-1}, \cdots, k_1 \in \tilde{\Gamma}_i^1 : \\
&\qquad \frac{\partial f_{k_{j-1}}(x)}{\partial x_{k_j}} \cdots \frac{\partial f_{k_1}(x)}{\partial x_{k_2}} \frac{\partial h_i(x)}{\partial x_{k_1}} \not\equiv 0\Big\} - \tilde{\Gamma}_i^{j-1} \\
\tilde{\Gamma}_i^j &= \Gamma_i^j \cup \tilde{\Gamma}_i^{j-1}
\end{aligned} \tag{B.4}$$

Also, the following analytic functions are defined:

$$\pi_i(k_1, \cdots, k_j) = \frac{\partial f_{k_{j-1}}(x)}{\partial x_{k_j}} \cdots \frac{\partial f_{k_1}(x)}{\partial x_{k_2}} \frac{\partial h_i(x)}{\partial x_{k_1}} \tag{B.5}$$

The dependence of π_i on x is suppressed mainly for notational convenience. Taking into account the previous definitions of the sets Γ_i^j and $\tilde{\Gamma}_i^j$, it can be deduced that $\pi_i(k_1, \cdots, k_j)$ is a function of the state variables x_{k_j}, with $k_j \in \tilde{\Gamma}_i^j$. Finally, the following lemma will be needed (for its proof he reader is referred to Kasinski and

Levine, 1984):

Lemma:

$$L_f^l h_i = \sum_{k_1 \in \Gamma_i^1, \cdots, k_l \in \Gamma_i^l} f_{k_l} \pi_i(k_1, \cdots, k_l) + \Phi_l(\tilde{\Gamma}_i^{l-1}) \tag{B.6}$$

where $\Phi_l(\tilde{\Gamma}_i^{l-1})$ is a linear combination with analytic coefficients of all the terms of the form $\pi_i(k_1, \cdots, k_{l-1})$ and $\dfrac{\partial^{l-s-1}\pi_i(k_1, \cdots, k_s)}{\partial x_{k_{l-1}} \cdots \partial x_{k_{s+1}}}$, for every $s < l-1$ and every $k_1, \cdots, k_{l-1} \in \tilde{\Gamma}_i^{l-1}$.

In the above lemma, the exact dependence on x is also suppressed. While $\pi_i(k_1, \cdots, k_l)$ is a function of x_{k_l}, with $k_l \in \tilde{\Gamma}_i^l$, it can also be deduced that f_{k_l} is a function of $x_{k_{l+1}}$, with $k_{l+1} \in \tilde{\Gamma}_i^{l+1}$.

The relative order r_{ij} is defined as the smallest integer for which $L_{g_j} L_f^{r_{ij}-1} h_i(x) \not\equiv 0$. Applying the above lemma to the case of the scalar field $L_f^{r_{ij}-1} h_i(x)$, the following expression is obtained:

$$L_f^{r_{ij}-1} h_i(x) = \sum_{k_1 \in \Gamma_i^1, \cdots, k_{r_{ij}-1} \in \Gamma_i^{r_{ij}-1}} f_{k_{r_{ij}-1}} \pi_i(k_1, \cdots, k_{r_{ij}-1}) + \Phi_{r_{ij}-1}(\tilde{\Gamma}_i^{r_{ij}-2}) \tag{B.7}$$

and

$$L_{g_j} L_f^{r_{ij}-1} h_i(x) = \sum_{k_{r_{ij}} \in \tilde{\Gamma}_i^{r_{ij}}} g_{jk_{r_{ij}}} \frac{\partial L_f^{r_{ij}-1} h_i(x)}{\partial x_{k_{r_{ij}}}} = \tag{B.8}$$

$$\sum_{k_1 \in \Gamma_i^1, \cdots, k_{r_{ij}} \in \Gamma_i^{r_{ij}}} g_{jk_{r_{ij}}} \pi_i(k_1, \cdots, k_{r_{ij}}) + \sum_{k_1, \cdots, k_{r_{ij}} \in \tilde{\Gamma}_i^{r_{ij}-1}} g_{jk_{r_{ij}}} \pi_i(k_1, \cdots, k_{r_{ij}-1}) \frac{\partial f_{k_{r_{ij}-1}}}{\partial x_{k_{r_{ij}}}} \tag{B.9}$$

$$+ \sum_{k_1, \cdots, k_{r_{ij}} \in \tilde{\Gamma}_i^{r_{ij}-1}} g_{jk_{r_{ij}}} \frac{\partial \pi_i(k_1, \cdots, k_{r_{ij}-1})}{\partial x_{k_{r_{ij}}}} f_{k_{r_{ij}-1}} + \sum_{k_{r_{ij}} \in \tilde{\Gamma}_i^{r_{ij}-1}} g_{jk_{r_{ij}}} \frac{\partial \Phi_{r_{ij}-1}}{\partial x_{k_{r_{ij}}}} \tag{B.10}$$

In order for the above expression not to be identically equal to zero, at least one term should be nonzero. If the first term is nonzero, then at least one product $g_{jk_{r_{ij}}} \pi_i(k_1, \cdots, k_{r_{ij}})$ should be nonzero. Since, by definition, ν_{ij} is the smallest integer such that there exist integers for which this is true, we must have $\nu_{ij} \leq r_{ij}$. If the first term is equal to zero, but the second is nonzero, there must be a product $g_{jk_{r_{ij}}}\pi_i(k_1,\cdots,k_{r_{ij}})$, with $k_1,\cdots,k_{r_{ij}} \in \tilde{\Gamma}_i^{r_{ij}-1}$ which is nonzero. In this case, we should also have $\nu_{ij} \leq r_{ij}$. Similar arguments can be used for the other terms, proving that $\nu_{ij} \leq r_{ij}$.

Step 3: Generically, $\nu_{ij} = r_{ij}$

Given the result of Step 2, suppose that ν_{ij} is strictly less than r_{ij}. Then, the following system of equations will hold:

$$\begin{aligned} L_{g_j}L_f^{\nu_{ij}-1}h_i(x) &= 0 \\ &\vdots \\ L_{g_j}L_f^{r_{ij}-2}h_i(x) &= 0 \end{aligned} \tag{B.11}$$

This is a system of $(r_{ij} - \nu_{ij})$ non-trivial partial differential equations in f, g_j, h_i and their partial derivatives. The set of solutions of the above system will be a closed subset with empty interior of the space of analytic vector-valued functions on $\mathbb{R}^n$. Consequently, $\nu_{ij} = r_{ij}$ generically, i.e., for almost all functions f, g_j, h_i.

B.2 Proof of Theorem 2.2

Under the assumptions of the theorem, in a neighborhood of x_0 and for sufficiently small times, the output y_i of the system assumes a unique Volterra series expansion of the form (Fliess, 1980):

$$y_i(t) = k_i^0(t) + \int_0^t k_i^1(t,\tau_1)u_j(\tau_1)d\tau_1 + \int_0^t\int_0^{\tau_2} k_i^2(t,\tau_2,\tau_1)u_j(\tau_2)u_j(\tau_1)d\tau_2 d\tau_1 + \cdots \tag{B.12}$$

where $k_i^j(t, \tau_j, \cdots, \tau_1)$ are the Volterra kernels associated with the output y_i, which assume a Taylor series expansion of the form:

$$k_i^0(t) = \sum_{j_1=0}^{\infty} L_f^{j_1} h_i(x_0) \frac{t^{j_1}}{j_1!} \tag{B.13}$$

$$k_i^1(t, \tau_1) = \sum_{j_2=0}^{\infty} \sum_{j_1=0}^{\infty} L_f^{j_2} L_{g_j} L_f^{j_1} h_i(x_0) \frac{(t-\tau_1)^{j_1} \tau_1^{j_2}}{j_1! j_2!} \tag{B.14}$$

$$k_i^2(t, \tau_1, \tau_2) = \sum_{j_3=0}^{\infty} \sum_{j_2=0}^{\infty} \sum_{j_1=0}^{\infty} L_f^{j_3} L_{g_j} L_f^{j_2} L_{g_j} L_f^{j_1} h_i(x_0) \frac{(t-\tau_1)^{j_1} (\tau_2 - \tau_1)^{j_2} \tau_2^{j_3}}{j_1! j_2! j_3!} \tag{B.15}$$

$$\vdots$$

The first term of the expansion, $k_i^0(t)$, which corresponds to the part of the response that depends only on the initial conditions, will vanish at the given initial condition x_0, since the output is in deviation variable form. Then, we obtain in a straightforward way the following form for the response under a unit-step change at the input:

$$\begin{aligned} y_i(t) \;=\; & [L_{g_j} h_i(x_0)]t + [L_{g_j} L_f h_i(x_0) + L_f L_{g_j} h_i(x_0) + L_{g_j}^2 h_i(x_0)] \frac{t^2}{2} \\ & + [L_{g_j} L_f^2 h_i(x_0) + L_f L_{g_j} L_f h_i(x_0) + L_f^2 L_{g_j} h_i(x_0) + 2L_{g_j}^2 L_f h_i(x_0) \\ & + L_{g_j} L_f L_{g_j} h_i(x_0) + 2 L_f L_{g_j}^2 h_i(x_0)] \frac{t^3}{6} + h.o.t. \end{aligned} \tag{B.16}$$

One can then easily verify that:

- if $r_{ij} = 1$, $y_i(t) \cong L_{g_j} h_i(x_0) t$ as $t \longrightarrow 0$
- if $r_{ij} = 2$, $y_i(t) \cong L_{g_j} L_f h_i(x_0) \frac{t^2}{2}$ as $t \longrightarrow 0$
- if $r_{ij} = 3$, $y_i(t) \cong L_{g_j} L_f^2 h_i(x_0) \frac{t^3}{6}$ as $t \longrightarrow 0$

and, by induction, $y_i(t) \cong L_{g_j} L_f^{r_{ij}-1} h_i(x_0) \frac{t^{r_{ij}}}{r_{ij}!}$ as $t \longrightarrow 0$.

B.3 Proof of Corollary 2.1

A simple proof of Corollary 2.1, independent of the result of Theorem 2.2, goes as follows:

Consider the transfer function between u and y, $G(s) = c(sI - A)^{-1}b$ and its expansion in terms of the Markov parameters (see e.g., Kailath, 1980):

$$G(s) = \frac{cb}{s} + \frac{cAb}{s^2} + \frac{cA^2b}{s^3} + \cdots \tag{B.17}$$

Then, calculating the response of the output under a unit-step change at the input, the following relation is obtained:

$$y(t) = (cb)t + (cAb)\frac{t^2}{2} + (cA^2b)\frac{t^3}{6} + h.o.t. \tag{B.18}$$

and the result of Corollary 2.1 follows immediately.

APPENDIX C

PROOFS OF CHAPTER III

C.1 Proof of Theorem 3.1

Let ρ_* be the minimal relative order ρ_i of the output with respect to the disturbances in class $\mathcal{C}$. Also, define the following subclasses of class $\mathcal{C}$:

$$\begin{aligned} \mathcal{C}^{(1)} &= \{d_\kappa \in \mathcal{C} : \rho_\kappa = \rho_*\} \\ \mathcal{C}^{(2)} &= \{d_\kappa \in \mathcal{C} : \rho_\kappa = \rho_* + 1\} \\ &\vdots \\ \mathcal{C}^{(r-\rho_*)} &= \{d_\kappa \in \mathcal{C} : \rho_\kappa = r - 1\} \end{aligned} \tag{C.1}$$

Then, a direct calculation of the derivatives of the output y up to order r yields the following expressions:

$$
\begin{aligned}
y &= h(x) \\
\frac{dy}{dt} &= L_f h(x) \\
&\vdots \\
\frac{d^{\rho_\bullet - 1} y}{dt^{\rho_\bullet - 1}} &= L_f^{\rho_\bullet - 1} h(x) \\
\frac{d^{\rho_\bullet} y}{dt^{\rho_\bullet}} &= L_f^{\rho_\bullet} h(x) + \sum_{d_\kappa \in \mathcal{C}^{(1)}} d_\kappa(t) L_{w_\kappa} L_f^{\rho_\bullet - 1} h(x) \\
\frac{d^{\rho_\bullet + 1} y}{dt^{\rho_\bullet + 1}} &= L_f^{\rho_\bullet + 1} h(x) + \sum_{d_\kappa \in \mathcal{C}^{(2)}} d_\kappa(t) L_{w_\kappa} L_f^{\rho_\bullet} h(x) \\
&\quad + \sum_{d_\kappa \in \mathcal{C}^{(1)}} \left[d_\kappa(t) L_{w_\kappa} L_f^{\rho_\bullet} h(x) + \frac{d}{dt} \left(d_\kappa(t) L_{w_\kappa} L_f^{\rho_\bullet - 1} h(x) \right) \right] \\
&\vdots \\
\frac{d^{r-1} y}{dt^{r-1}} &= L_f^{r-1} h(x) + \sum_{d_\kappa \in \mathcal{C}^{(r-\rho_\bullet)}} d_\kappa(t) L_{w_\kappa} L_f^{r-2} h(x) \\
&\quad + \sum_{d_\kappa \in \mathcal{C}^{(r-\rho_\bullet - 1)}} \left[d_\kappa(t) L_{w_\kappa} L_f^{r-2} h(x) + \frac{d}{dt} \left(d_\kappa(t) L_{w_\kappa} L_f^{r-3} h(x) \right) \right] + \cdots \\
&\quad + \sum_{d_\kappa \in \mathcal{C}^{(1)}} \left[d_\kappa(t) L_{w_\kappa} L_f^{r-2} h(x) + \frac{d}{dt} \left(d_\kappa(t) L_{w_\kappa} L_f^{r-3} h(x) \right) + \cdots \right. \\
&\quad \left. + \frac{d^{r-\rho_\bullet - 1}}{dt^{r-\rho_\bullet - 1}} \left(d_\kappa(t) L_{w_\kappa} L_f^{\rho_\bullet - 1} h(x) \right) \right] \\
\frac{d^r y}{dt^r} &= L_f^r h(x) + u(t) L_g L_f^{r-1} h(x) + \sum_{d_\kappa \in \mathcal{B}} d_\kappa(t) L_{w_\kappa} L_f^{r-1} h(x) \\
&\quad + \sum_{d_\kappa \in \mathcal{C}^{(r-\rho_\bullet)}} \left[d_\kappa(t) L_{w_\kappa} L_f^{r-1} h(x) + \frac{d}{dt} \left(d_\kappa(t) L_{w_\kappa} L_f^{r-2} h(x) \right) \right] + \cdots \\
&\quad + \sum_{d_\kappa \in \mathcal{C}^{(1)}} \left[d_\kappa(t) L_{w_\kappa} L_f^{r-1} h(x) + \frac{d}{dt} \left(d_\kappa(t) L_{w_\kappa} L_f^{r-2} h(x) \right) + \cdots \right. \\
&\quad \left. + \frac{d^{r-\rho_\bullet}}{dt^{r-\rho_\bullet}} \left(d_\kappa(t) L_{w_\kappa} L_f^{\rho_\bullet - 1} h(x) \right) \right]
\end{aligned} \tag{C.2}
$$

Using the above expressions to form the sum $\sum_{k=0}^{r} \beta_k \frac{d^k y}{dt^k}$ and substituting u from Eq.3.3, it is easily found, after some algebraic manipulations, that:

$$\sum_{k=0}^{r} \beta_k \frac{d^k y}{dt^k} = v$$

which completes the proof.

C.2 Proof of Proposition 3.2

The operation of calculating derivatives of the output y is equivalent to the recursive application of the operator $\left(L_f + d(t)L_w + u(t)L_g + \frac{\partial}{\partial t}\right)$ to $h(x)$. More specifically,

$$
\begin{aligned}
y &= h(x) \\
\frac{dy}{dt} &= \left(L_f + d(t)L_w + u(t)L_g + \frac{\partial}{\partial t}\right) h(x) \\
&\vdots \\
\frac{d^{r-1}y}{dt^{r-1}} &= \left(L_f + d(t)L_w + u(t)L_g + \frac{\partial}{\partial t}\right)^{r-1} h(x) \\
\frac{d^{r}y}{dt^{r}} &= \left(L_f + d(t)L_w + u(t)L_g + \frac{\partial}{\partial t}\right)^{r} h(x)
\end{aligned}
\tag{C.3}
$$

Given the definition of the relative orders r and ρ, the above expressions take the form:

$$
\begin{aligned}
y &= h(x) \\
\frac{dy}{dt} &= L_f h(x) \\
&\vdots \\
\frac{d^{\rho-1}y}{dt^{\rho-1}} &= L_f^{\rho-1} h(x) \\
\frac{d^{\rho}y}{dt^{\rho}} &= \left(L_f + d(t)L_w + \frac{\partial}{\partial t}\right) L_f^{\rho-1} h(x) \\
\frac{d^{\rho+1}y}{dt^{\rho+1}} &= \left(L_f + d(t)L_w + u(t)L_g + \frac{\partial}{\partial t}\right)\left(L_f + d(t)L_w + \frac{\partial}{\partial t}\right) L_f^{\rho-1} h(x) \\
&\vdots \\
\frac{d^{r-1}y}{dt^{r-1}} &= \left(L_f + d(t)L_w + u(t)L_g + \frac{\partial}{\partial t}\right)^{r-\rho-1}\left(L_f + d(t)L_w + \frac{\partial}{\partial t}\right) L_f^{\rho-1} h(x) \\
\frac{d^{r}y}{dt^{r}} &= \left(L_f + d(t)L_w + u(t)L_g + \frac{\partial}{\partial t}\right)^{r-\rho}\left(L_f + d(t)L_w + \frac{\partial}{\partial t}\right) L_f^{\rho-1} h(x)
\end{aligned}
\tag{C.4}
$$

According to the procedure of the proof of Theorem 3.1, it is clear the disturbance-free input/output behavior of Eq.3.4 can be induced by a feedforward/**static** state feedback law if and only if the manipulated input u does not appear explicitly earlier that in the r-th order derivative of the output. Imposing this condition on the expressions of the derivatives of y of order $\rho+1$ through $r-1$, the following equations are easily obtained:

$$L_g\left(\left(\left(L_f + d(t)L_w + \frac{\partial}{\partial t}\right)^{\ell} - L_f^{\ell}\right) L_f^{\rho-1}h(x) \equiv 0, \;\; \ell = 1,\cdots,(r-\rho-1)\right) \tag{C.5}$$

It is also straightforward to show by induction that the following operator identity holds:

$$\left(L_f + d(t)L_w + \frac{\partial}{\partial t}\right)^{\ell} - L_f^{\ell} = \sum_{\mu=0}^{\ell-1} L_f^{\ell-\mu-1}\left(d(t)L_w + \frac{\partial}{\partial t}\right)\left(L_f + d(t)L_w + \frac{\partial}{\partial t}\right)^{\mu} \tag{C.6}$$

Combined with the definitions of the functions $\phi_\ell(x,d)$ (Eq.3.15) and Eq.C.5, Eq.C.6 directly leads to the conditions of Eq.3.14 for $\ell = 0,1,\cdots,r-\rho-2$. Furthermore, whenever these conditions are satisfied, the expressions for the derivatives of the output y take the form:

$$\begin{array}{rcl}
y & = & h(x) \\
\dfrac{dy}{dt} & = & L_f h(x) \\
 & \vdots & \\
\dfrac{d^{\rho-1}y}{dt^{\rho-1}} & = & L_f^{\rho-1}h(x) \\
\dfrac{d^{\rho}y}{dt^{\rho}} & = & L_f^{\rho}h(x) + \phi_0\left(x, d(t)\right) \\
\dfrac{d^{\rho+1}y}{dt^{\rho+1}} & = & L_f^{\rho+1}h(x) + \phi_1\left(x, d(t), d(t)^{(1)}\right) \\
 & \vdots & \\
\dfrac{d^{r-1}y}{dt^{r-1}} & = & L_f^{r-1}h(x) + \phi_{r-\rho-1}\left(x, d(t), d(t)^{(1)}, \cdots, d(t)^{(r-\rho-1)}\right) \\
\dfrac{d^{r}y}{dt^{r}} & = & L_f^{r}h(x) + \phi_{r-\rho}\left(x, d(t), d(t)^{(1)}, \cdots, d(t)^{(r-\rho)}\right) \\
 & & +u(t)L_g\left(L_f^{r-1}h(x) + \phi_{r-\rho-1}\left(x, d(t), d(t)^{(1)}, \cdots, d(t)^{(r-\rho-1)}\right)\right)
\end{array} \tag{C.7}$$

Then, it can be easily shown that a control law of the form:

$$u = \left[\beta_r L_g\left(L_f^{r-1}h(x) + \phi_{r-\rho-1}\left(x, d(t), d(t)^{(1)}, \cdots, d(t)^{(r-\rho-1)}\right)\right)\right]^{-1}$$

$$\left\{v - \sum_{k=0}^{r}\beta_k L_f^k h(x) - \sum_{k=\rho}^{r}\beta_k\phi_{k-\rho}\left(x, d(t), d(t)^{(1)}, \cdots, d(t)^{(k-\rho)}\right)\right\} \tag{C.8}$$

induces the input/output behavior of Eq.3.4, for all values of the disturbance $d(t)$. The control law of Eq.C.8 will be in the form of Eq.3.11 (and therefore, well defined for all $x \in X$), if and only if:

$$L_g\phi_{r-\rho-1}\left(x, d(t), d(t)^{(1)}, \cdots, d(t)^{(r-\rho-1)}\right) \equiv 0 \tag{C.9}$$

which establishes the condition of Eq.3.14 for $\ell = r - \rho - 1$. Under this additional condition, Eq.C.8 reduces to the control law of Eq.3.16, which completes the proof.

APPENDIX D

PROOFS OF CHAPTER IV

D.1 Proof of Theorem 4.1

Let $\rho_i = \min(\rho_{i1}, \cdots, \rho_{ip})$ and assume without loss of generality that $\rho_i < r_i$ for every i. Also, define the following subclasses of the classes $\mathcal{C}_i$:

$$\begin{aligned}
\mathcal{C}_i^{(1)} &= \{d_\kappa \in \mathcal{C}_i : \rho_{i\kappa} = \rho_i\} \\
\mathcal{C}_i^{(2)} &= \{d_\kappa \in \mathcal{C}_i : \rho_{i\kappa} = \rho_i + 1\} \\
\vdots \quad & \quad \vdots \\
\mathcal{C}_i^{(r_i-\rho_i)} &= \{d_\kappa \in \mathcal{C}_i : \rho_{i\kappa} = r_i - 1\}
\end{aligned} \tag{D.1}$$

Then, a direct calculation of the derivatives of each output y_i up to order r_i yields the following expressions:

$$
\begin{aligned}
y_i &= h_i(x) \\
\frac{dy_i}{dt} &= L_f h_i(x) \\
&\vdots \\
\frac{d^{\rho_i-1}y_i}{dt^{\rho_i-1}} &= L_f^{\rho_i-1}h_i(x) \\
\frac{d^{\rho_i}y_i}{dt^{\rho_i}} &= L_f^{\rho_i}h_i(x) + \sum_{d_\kappa\in C_i^{(1)}} d_\kappa(t)L_{w_\kappa}L_f^{\rho_i-1}h_i(x) \\
\frac{d^{\rho_i+1}y_i}{dt^{\rho_i+1}} &= L_f^{\rho_i+1}h_i(x) + \sum_{d_\kappa\in C_i^{(2)}} d_\kappa(t)L_{w_\kappa}L_f^{\rho_i}h_i(x) \\
&\quad + \sum_{d_\kappa\in C_i^{(1)}}\left[d_\kappa(t)L_{w_\kappa}L_f^{\rho_i}h_i(x) + \frac{d}{dt}\left(d_\kappa(t)L_{w_\kappa}L_f^{\rho_i-1}h_i(x)\right)\right] \\
&\vdots \\
\frac{d^{r_i-1}y_i}{dt^{r_i-1}} &= L_f^{r_i-1}h_i(x) + \sum_{d_\kappa\in C_i^{(r_i-\rho_i)}} d_\kappa(t)L_{w_\kappa}L_f^{r_i-2}h_i(x) \\
&\quad + \sum_{d_\kappa\in C_i^{(r_i-\rho_i-1)}}\left[d_\kappa(t)L_{w_\kappa}L_f^{r_i-2}h_i(x) + \frac{d}{dt}\left(d_\kappa(t)L_{w_\kappa}L_f^{r_i-3}h_i(x)\right)\right] + \cdots \\
&\quad + \sum_{d_\kappa\in C_i^{(1)}}\left[d_\kappa(t)L_{w_\kappa}L_f^{r_i-2}h_i(x) + \frac{d}{dt}\left(d_\kappa(t)L_{w_\kappa}L_f^{r_i-3}h_i(x)\right) + \cdots\right. \\
&\quad \left. + \frac{d^{r_i-\rho_i-1}}{dt^{r_i-\rho_i-1}}\left(d_\kappa(t)L_{w_\kappa}L_f^{\rho_i-1}h_i(x)\right)\right] \\
\frac{d^{r_i}y_i}{dt^{r_i}} &= L_f^{r_i}h_i(x) + \sum_{j=1}^{m}u_j(t)L_{g_j}L_f^{r_i-1}h_i(x) + \sum_{d_\kappa\in B_i} d_\kappa(t)L_{w_\kappa}L_f^{r_i-1}h_i(x) \\
&\quad + \sum_{d_\kappa\in C_i^{(r_i-\rho_i)}}\left[d_\kappa(t)L_{w_\kappa}L_f^{r_i-1}h_i(x) + \frac{d}{dt}\left(d_\kappa(t)L_{w_\kappa}L_f^{r_i-2}h_i(x)\right)\right] + \cdots \\
&\quad + \sum_{d_\kappa\in C_i^{(1)}}\left[d_\kappa(t)L_{w_\kappa}L_f^{r_i-1}h_i(x) + \frac{d}{dt}\left(d_\kappa(t)L_{w_\kappa}L_f^{r_i-2}h_i(x)\right) + \cdots\right. \\
&\quad \left. + \frac{d^{r_i-\rho_i}}{dt^{r_i-\rho_i}}\left(d_\kappa(t)L_{w_\kappa}L_f^{\rho_i-1}h_i(x)\right)\right]
\end{aligned}
\tag{D.2}
$$

Substituting the above expressions and Eq.4.3 to the left-hand-side of Eq.4.4, it is straightforward to show that the right-hand-side of Eq.4.4 becomes equal to v, which completes the proof.

D.2 Proof of Proposition 4.2

The condition of Eq.4.20 is equivalent to:

$$\begin{aligned} < dh_i(x), w_\kappa(x) > &= 0 \\ < dL_f h_i(x), w_\kappa(x) > &= 0 \\ &\vdots \\ < dL_f^{r_i-1} h_i(x), w_\kappa(x) > &= 0 \end{aligned} \tag{D.3}$$

for every i, κ and $x \in X$. Using the Lie derivative notation, the above relations are equivalent to:

$$\begin{aligned} L_{w_\kappa} h_i(x) &= 0 \\ L_{w_\kappa} L_f h_i(x) &= 0 \\ &\vdots \\ L_{w_\kappa} L_f^{r_i-1} h_i(x) &= 0 \end{aligned} \tag{D.4}$$

for every i, κ and $x \in X$, which directly leads to the condition of Eq.4.21.

D.3 Proof of Proposition 4.3

The condition of Eq.4.22 is equivalent to the existence, for each κ, of scalar functions $\alpha_{\kappa j}(x)$, $j = 1, \cdots, m$ and a vector function $\phi_\kappa(x)$ such that :

$$w_\kappa(x) = \sum_{j=1}^{m} \alpha_{\kappa j}(x) g_j(x) + \phi_\kappa(x) \tag{D.5}$$

where

$$\begin{aligned} L_{\phi_\kappa} h_i(x) &= 0 \\ L_{\phi_\kappa} L_f h_i(x) &= 0 \\ &\vdots \\ L_{\phi_\kappa} L_f^{r_i-1} h_i(x) &= 0 \end{aligned} \tag{D.6}$$

for every i and every $x \in X$.

Then, the following relation can be easily shown to hold for all κ:

$$L_{w_\kappa} L_f^k h_i(x) = \sum_{j=1}^{m} \alpha_{\kappa j}(x) L_{g_j} L_f^k h_i(x) + L_{\phi_\kappa} L_f^k h_i(x) \;\; ; \; k = 0, 1, \cdots \qquad \text{(D.7)}$$

which, given Eq.D.6, becomes:

$$L_{w_\kappa} L_f^k h_i(x) = \sum_{j=1}^{m} \alpha_{\kappa j}(x) L_{g_j} L_f^k h_i(x) \;\; ; \; k = 0, 1, \cdots, r_i - 1 \qquad \text{(D.8)}$$

The condition of Eq.4.23 follows directly then, given the definitions of relative orders.

APPENDIX E

PROOFS OF CHAPTER V

E.1 Proof of Theorem 5.1

A constructive proof of Theorem 5.1 through Hirschorn's inversion algorithm is possible, but will not be given here because of the rather complicated procedure and the technicalities involved. Instead, Theorem 5.1 will be proved by simply verifying that the system of Eq.5.2 indeed acts as a inverse to the original system.

In particular, calculating expressions for the derivatives of the outputs y_i of the system of Eq.5.2, we get:

$$\begin{aligned} \frac{d^{r_1} y_1}{dt^{r_1}} &= L_f^{r_1} h_1(x) + L_g L_f^{r_1 - 1} h_1(x) u \\ &\vdots \\ \frac{d^{r_m} y_m}{dt^{r_m}} &= L_f^{r_m} h_m(x) + L_g L_f^{r_m - 1} h_m(x) u \end{aligned} \tag{E.1}$$

Since the characteristic matrix is nonsingular, the above set of equations can be solved for u to obtain:

$$u = C(x)^{-1} \left(\begin{bmatrix} \dfrac{d^{r_1} y_1}{dt^{r_1}} \\ \vdots \\ \dfrac{d^{r_m} y_m}{dt^{r_m}} \end{bmatrix} - \begin{bmatrix} L_f^{r_1} h_1(x) \\ \vdots \\ L_f^{r_m} h_m(x) \end{bmatrix} \right) \tag{E.2}$$

subject to the dynamics:

$$\dot{x} \;=\; f(x) + g(x)C(x)^{-1}\left(\begin{bmatrix} \frac{d^{r_1}y_1}{dt^{r_1}} \\ \vdots \\ \frac{d^{r_m}y_m}{dt^{r_m}} \end{bmatrix} - \begin{bmatrix} L_f^{r_1}h_1(x) \\ \vdots \\ L_f^{r_m}h_m(x) \end{bmatrix}\right) \tag{E.3}$$

But the u calculated above is exactly equal to the output of the dynamic system described by Eq.5.2 (just substitute x for ξ). Therefore, by definition of the inverse, Eq.5.2 is a realization of the inverse of the original system.

E.2 Proof of Theorem 5.2

First, we prove the "only if part" of the theorem. Suppose that given that the structural matrix equivalent to $C(x)$ has generic rank equal to m, the output rearrangement is not possible. This implies that there is at least one input u_{j^*} for which one of the following two is true:

1. There is no output y_i with the minimum relative order at the j^*-th column of the relative order matrix M_r, i.e., there is no output y_i such that $r_i = r_{ij^*}$.

2. There are two or more outputs, e.g. y_{i_1} and y_{i_2}, whose minimum relative order appears at the j^*-th column of the relative order matrix M_r and nowhere else, i.e., $r_{i_1} = r_{i_1j^*}$, $r_{i_2} = r_{i_2j^*}$ and $r_{i_1j} > r_{i_1j^*}$, $r_{i_2j} > r_{i_2j^*}$ for $j \neq j^*$.

In the first case, we would have:

$$L_{g_{j^*}}L_f^{r_i-1}h_i(x) = 0 \tag{E.4}$$

for every i, and therefore the j^*-th column of the characteristic matrix (and its structural equivalent) would be zero.

In the second case, we would have:

$$L_{g_{j^*}}L_f^{r_{i_1}-1}h_{i_1}(x) \not\equiv 0 \;,\; L_{g_{j^*}}L_f^{r_{i_2}-1}h_{i_2}(x) \not\equiv 0 \tag{E.5}$$

and

$$L_{g_j} L_f^{r_{i_1}-1} h_{i_1}(x) = 0 \ , \ L_{g_j} L_f^{r_{i_2}-1} h_{i_2}(x) = 0 \tag{E.6}$$

for every $j \neq j^*$. But then, the corresponding to the outputs y_{i_1} and y_{i_2} rows of the characteristic matrix would have only one non-zero element, at the same position (the j^*-th).

In both cases, a rank defficiency would result, contrary to our assumption. Therefore, by contradiction, the suggested output rearrangement is always possible.

Now, we prove the "if part" of the theorem. Suppose that the suggested output rearrangement is possible, but the structural matrix equivalent to $C(x)$ has rank defficiency. This implies either one of the following two for this matrix:

1. At least one row or column has zeros in all positions.

2. There are k ($k \geq 2$) columns or rows that cause the rank defficiency in a non-trivial way.

In the first case, we would have the case where all relative orders in a row or column are equal to infinity. In the second case, in order for rank defficiency to exist, we must have at least $m - (k - 1)$ zeros at the same positions in all k columns or rows. This leaves $(k - 1)$ or less nonzero elements at the same $(k - 1)$ positions of all k rows or columns. However, because of the rearrangement, there should k nonzero elements in the diagonal positions of these k rows or columns, i.e., in k distinct positions. In both cases, the contradiction is clear, and the theorem is proved.

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