Tax Evasion, Tax Corruption & Stochastic Growth

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Abstract

This paper presents a continuous time stochastic growth model to study the effects of tax evasion and tax corruption on the level and volatility of private investment and public spending. Our results suggest that there do exist several regimes of mean growth and growth volatility, depending upon the consumer's degree of risk aversion, the tax income yield, the risk-adjusted return of the agent's portfolio, the productivity of public spending. We find that public spending is described asymptotically by an incomplete upper Gamma distribution, while private capital is described by a power law distribution. Depending upon the values of the parameters of these distributions, growth can be characterized by extreme values (high volatility) when the return to taxation lies under a certain threshold and/or when the risk-adjusted return of investing the proceeds of illegal activities evolves above a given threshold. We provide an empirical illustration of the model.

Key words : Stochastic growth; tax evasion; tax corruption

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1 Introduction

This paper tries to shed some light on the impact of tax evasion and tax corruption on private investment and government spending, two key determinants of the growth rate and volatility of per-capita GDP. In poor countries, in which the public sector is an essential contributor to the economic growth, stagnation and severe swings in economic growth are related to the deficient tax collection systems which do not allow to provide the minimum amount of public goods and services necessary for productive activities like infrastructure, education, or investment (see Friedman et al., 2000). Following the recommendation of the multilateral and bilateral donors, as well as of the international organizations, governments are willing to reduce corruption and tax evasion to avoid loss of tax revenues. But many developing countries still appear to be stuck in a vicious circle of both tax corruption and tax evasion, a phenomenon to which the theoretical and empirical literature have paid a great attention (see, among others, Mauro, 2004). Fighting corruption may be difficult because of rent-seeking activities and building a technology that detects tax evaders is expensive. As far as we question the implication for growth and its volatility, a commonly accepted answer consists in saying that countries in such a situation are likely to achieve a bad macroeconomic equilibrium, namely the coexistence of a low growth rate and a high volatility of per-capita GDP. Indeed, according to the literature, corruption is an important factor contributing to growth volatility (see Denizer et al., 2000).

In this paper, we maintain that in such situations in which a government is unable to reduce the level of corruption and tax evasion, an alternative solution is either to allow the resources of the evaded tax to be invested in equities (by allowing a private equity market to flourish, private capital becomes a substitute for public spending in the production function), or to raise the productivity of public spending to attenuate the negative externalities of tax evasion on productive public expenditure. To develop these ideas, we use a standard portfolio argument by adopting an open economy stochastic growth model, in line with previous models like Turnovsky (1993), Grinols and Turnovsky (1993), Turnovsky (1999). Public goods and private investment are both productive inputs in the production function.

An important difference with the previous literature is that the risk that interact with growth is not exogenous, but stems from endogenous sources : tax corruption and tax evasion. Specifically, the uncertainty in the model comes from the fact that hiding income from the tax administration and offering briberies to inspectors are risky activities. Cheating is an uncertain activity because there is a probability of being detected and a probability to be confronted to a corrupted inspector. People may decide to shelter themselves from a tax payment, but at the cost of paying bribes to civil servants. Our results suggest that there do exist several regimes of mean growth and growth volatility, depending upon the consumer's degree of risk aversion, the tax income yield, the risk-adjusted return of the agent's portfolio, the productivity of public spending. Importantly, the model considers tax evasion, private capital and public

spending as endogenous variables and creates a loop between them.

We build upon the idea that tax evasion and tax corruption are non-separable when tax collection is performed by corruptible inspectors (see Hindricks et al., 1999, Sanyal et al., 2000). However, our model differs from previous models on the same topic in several respects. Lin and Yang (2001) also consider a stochastic growth model of tax evasion, but with no specific role for corruption and no role for public spending as an input in the production function. Chen (2003) also considers a model of tax evasion with productive public capital. Unlike the author, we do not consider any optimizing behavior from the government side. Further, in our model tax evasion generates uncertainty and thus a randomness on production. Dzhumashev (2007) uses a framework like ours, but his model applies to a closed economy. In our case, opening the economy allows introducing wealth effects in the model. Further, by considering a general CRRA utility function, we show that the impact on capital accumulation of tax corruption and tax evasion depends upon a trade-off between the risk aversion and the saving behavior. Finally, Cerquetti and Coppier (2011) address the issue of the effects of tax evasion and tax corruption on economic growth and they apply a game-theoretical approach to a Ramsey model. The authors focus on the strategic behaviors of consumers and bureaucrats and this issue is put of the scope of this paper.

We obtain closed-form solutions for the steady state invariant distributions. We find that public spending is described asymptotically by an incomplete upper Gamma distribution, while private capital is described by a power law distribution. These distributions have parameters described by the return to taxation and the portfolio performance of the evasion and corruption activity. The interesting point is that growth can be characterized by extreme values (high volatility) when the return to taxation lies under a certain threshold and/or when the risk-adjusted return of investing the proceeds of illegal activities evolves above a given threshold.

The remainder of the paper is organized as follows. Section 2 sets out the model. In Section 3, we study the optimal choice of the domestic agent. Section 4 presents the steady state distributions, while Section 5 contains the results of a comparative dynamics analysis. Section 6 summarizes the main results from the formal model and Section 7 presents an empirical illustration on the Southern African countries. Finally Section 8 concludes.

2　The model

This section presents a continuous time stochastic growth model. We describe a representative agent' choice and present the dynamics of saving and public spending in a stochastic environment. The source of uncertainty is not the technology but tax evasion and tax corruption. Tax People whi are frauding can be caught, but they may face corruptible bureaucrats to whom they propose bribes. Tax corruption thus occurs through bribery to avoid paying the penalty for tax evasion. Tax evasion and tax corruption are risky activities. We assume

that perceived and objective probabilities of getting caught and of paying bribes are the same and that the detection technology used by the bureaucrats is costless.

2.1 Tax fraud and tax corruption as a source of random income

2.1.1 Production

We consider an open economy called the domestic country and the rest of the world referred as the foreign country. In each country we consider a society populated with a continuum of individuals with measure 1. An individual lives forever (we have infinitely lived representative agents). She supplies her labor force inelastically to the productive sector. for purpose of simplicity we noramlize the labor supply to 1. In addition to consumers and firms, politicians live in both the domestic and foreign economies. They provide a productive input in public spending financed out of tax revenue.

Private firms in the domestic economy produce a consumption good with the following production technology:

$$c(t) = y(t) = A(t)k(t), \quad A(t) = \xi \left[g(t) \right]^{1/\xi},$$
$$(k(t), g(t)) \in [0, +\infty) \times [0, +\infty), \tag{1}$$

where $c(t)$, $y(t)$ are per-capita consumption and output, $k(t)$ and $A(t)$ are respectively the (private) capital-labor ratio and productivity. The latter is assumed to depend upon public goods and services (roads, public health, education, etc) provided by the bureaucrats or politicians and we assume decreasing returns of the technology for public goods ($\xi > 1$). The price of the consumption good is normalized to 1. $g(t)$ is per-capita public spending. Similarly, the production technology in the foreign country is given by

$$c^*(t) = y^*(t) = A^*(t)k^*(t), \quad A^*(t) = \xi^* \left[g^*(t) \right]^{1/\xi^*},$$
$$(k^*(t), g^*(t)) \in [0, +\infty) \times [0, +\infty), \tag{2}$$

For simplicity, we assume that private capital does not depreciate. g is a pure public good (government goods and services are neither rival nor excludable).

2.1.2 Tax evasion and tax corruption

Our modelling of tax evasion relies upon Allingham and Sandmo (1972) and Yitzhaki (1974). Taxes are used to finance public goods and services. We do not make explicit the production function of the public good since this is not important here.

An agent chooses to hide a fraction $e(t)$ of her income from the government and we assume that $0 < e(t) < 1$. Yet, politicians try to detect tax evasion. The probability of being detected is p ($0 < p < 1$). A consumer who is detected is asked to pay the legal tax $\tau e(t)y(t)$ plus a penalty defined as a fraction s of the

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undeclared income, $s\tau e(t)y(t)$. τ is the legal tax rate $(0 < \tau < 1)$ and we have a similar definition for the legal tax rate in the foreign country $(0 < \tau^* < 1)$. To avoid paying the penalty, the detected evader can pay a bribe to inspectors. The latter are corruptible with a probability p_1 $(0 < p_1 < 1)$. Denoting θ the penalty rate when there are no bribes, we assume that the detected evader can pay back less than θ and we denote b the penalty rate when politicians are corrupted $(b < \theta)$.

The penalty rate is thus a random variable

$$\theta_1 = \left\{ \begin{array}{ccc} \theta, & w.p. & 1 - p_1 \\ b, & w.p. & p_1 \end{array} \right|, \tag{3}$$

and the expected value of the penalty rate is $E[\theta_1] = \overline{\theta} = p_1 b + (1 - p_1)\theta$. Therefore, the random return of a unit of evaded tax is

$$x_1 = \left\{ \begin{array}{ccc} 1 - \overline{\theta}, & w.p. & p \\ 1, & w.p. & 1 - p \end{array} \right|. \tag{4}$$

The expected return on a unit of evaded tax is thus $E[x_1] = \overline{x}_1 = 1 - \overline{\theta}p$. Tax evasion is worth if $\overline{x}_1 > 0$, which implies $\overline{\theta}p < 1$. x_1 is a binomial process or a Poisson process if $p \to 0$. Assuming that the domestic economy is composed of an infinite number of consumer who behave in a similar way, both processes tends to a Normal law. Therefore x_1 converges to a Normal law with mean $\overline{x}_1$ and a variance $\sigma_1^2 = p(1 - p)\overline{\theta}^2$.

The dynamics of the random gain induced by tax evasion is described by the following stochastic differential equation (SDE):

$$dx_1(t) = \overline{x}_1 \tau e(t)y(t)dt + \sigma_1 \tau e(t)y(t)dz_1(t), \tag{5}$$

where $z_1(t)$ is a Brownian motion process.

Similarly, in the foreign country we have

$$dx_1^*(t) = \overline{x}_1^* \tau^* e^*(t)y^*(t)dt + \sigma_1^* \tau^* e^*(t)y^*(t)dz_1^*(t), \tag{6}$$

where $z_1^*(t)$ is a Brownian motion process

2.1.3 Portfolio diversification and the dynamics of wealth

A household spends a fraction of his income in consumption and uses the remaining income to buy equities whose values represent a share of the physical capital of the domestic country and of the foreign country. We assume that the population size is the same in both countries. We define

$$k(t) = k_d(t) + k_f(t) \text{ and } k^*(t) = k_d^*(t) + k_f^*(t), \tag{7}$$

where
  $k_d(t)$ is the domestic per-capita capital owned by the domestic agent,
  $k_f(t)$ is the domestic per-capita capital owned by the foreign agent,

$k_d^*(t)$ is the foreign per-capita capital owned by the domestic agent,
$k_f^*(t)$ is the foreign per-capita capital owned by the foreign agent.

Denoting $w(t)$ the average wealth of the domestic agent (per-capita wealth or saving), $n_d(t)$ and $n_d^*(t)$ the shares of domestic and foreign capital in the the domestic agent' total wealth, we have

$$n_d(t) = \frac{k_d(t)}{w(t)}, \quad n_d^*(t) = \frac{k_d^*(t)}{w(t)}, \quad w(t) = k_d(t) + k_d^*(t). \tag{8}$$

We have similar relationships for the foreign consumer:

$$n_f^*(t) = \frac{k_f^*(t)}{w^*(t)}, \quad n_f(t) = \frac{k_f(t)}{w^*(t)}, \quad w^*(t) = k_f(t) + k_f^*(t). \tag{9}$$

where $w^*(t)$ is per-capita wealth in the foreign country. We assume perfect capital mobility without restrictions on asset trade. We further assume that there is a demand for portfolio diversification. This implies that $n_d(t)$, $n_d^*(t), n_f^*(t)$ and $n_f(t)$ are strictly positive and less than 1.

Wealth (or saving) is a random variable because the expected return on tax evasion is a random variable. Each unit of income hidden yields $\overline{x}_1$ on average with more or less σ_1. Assuming that per-capita consumption evolves at a deterministic rate $c(t)dt$, we have

$$\begin{aligned} dw(t) &= \{[1 - \tau + \overline{x}_1\tau e(t)]A(t)k_d(t) + [1 - \tau^* + \overline{x}_1^*\tau^* e^*(t)]A^*(t)k_d^*(t) \\ &\quad - c(t)\}dt + \sigma_1\tau e(t)A(t)k_d(t)dz_1(t) + \sigma_1^*\tau^* e^*(t)k_d^*(t)dz_1^*(t), \end{aligned} \tag{10}$$

from which we deduce the rate of accumulation of assets by the domestic agent:

$$\frac{dw(t)}{w(t)} = \psi(t)dt + \omega_1(t)dz_1(t) + \omega_1^*(t)dz_1^*(t), \tag{11}$$

where

$$\begin{aligned} \psi(t) &= R(t)n_d(t) + R^*(t)(1 - n_d(t)) - \frac{c(t)}{w(t)}, \tag{12} \\ R(t) &= (1 - \tau + \overline{x}_1\tau e(t))A(t), \\ R^*(t) &= (1 - \tau^* + \overline{x}_1^*\tau^* e^*(t))A^*(t). \end{aligned}$$

and

$$\begin{aligned} \omega_1(t) &= \sigma_1\tau e(t)A(t), \\ \omega_1^*(t) &= \sigma_1^*\tau^* e^*(t)A^*(t), \end{aligned}$$

$R(t)$ and $R^*(t)$ are the gross rates of returns of one unit of capital invested respectively in the domestic and in the foreign countries. They depend upon the tax rates, the expected returns of a unit of evaded tax and the proportions of revenues hidden. $\omega_1(t)$ and $\omega_1^*(t)$ are the risk of one unit of capital invested in the home and foreign countries. Therefore $R(t)n_d(t) + R^*(t)(1 - n_d(t))$ is the

gross rate of return of the domestic agent's portfolio. For the foreign agent, we have similar relationships.

We assume that the following inequalities hold simultaneously $R(t) > R^*(t)$ and $\omega_1(t) > \omega_1^*(t)$ or $R(t) < R^*(t)$ and $\omega_1(t) < \omega_1^*(t)$ (this is a standard assumption in any portfolio model with risky assets). This implies that we either have $\overline{x}_1 > \overline{x}_1^*$ and $\sigma_1 > \sigma_1^*$, or $\overline{x}_1 < \overline{x}_1^*$ and $\sigma_1 < \sigma_1^*$. In the first case, the expected return from tax evasion and tax corruption is higher but more risky in the home country than in the foreign country. This happens for instance when the probability of being caught in the domestic country is lower in comparison with the same probability in the foreign country ($p < p^*$), but if, upon catching an evader, the government imposes a higher penalty ($\overline{\theta} > \overline{\theta}^*$) because the politicians are less corruptible ($p_1 < p_1^*$). However, this same situation could also arise with a higher probability of being detected ($p > p^*$) because controls are more frequent), but if, upon being detected, the penalty rate is lower because politicians are more corruptible ($\overline{\theta} < \overline{\theta}^*$ and $p_1 > p_1^*$).

2.2 The utility function

The consumer's preferences are represented by an isoelastic utility function. We assume that she obtains utility from private consumption. The objective function is

$$U = E_0 \int_0^\infty (1/\gamma) \left(c(t)\right)^\gamma e^{-\beta t} dt. \tag{13}$$

We assume that $-\infty < \gamma < 0, \beta > 0$. β is the time preference rate. $1 - \gamma$ is the Arrow-Pratt coefficient of relative risk aversion. E_0 is the expectation at time $t = 0$. Unlike other models developed in the stochastic growth literature, we assume that public spending do not enhance the marginal utility of consumption, but only the productivity of private capital. This is a major difference with, for example, Turnovsky (1999). The reason is that the situation we consider applies to poor countries, which have no social insurance systems, where the quality of institutions and governance are too weak to allow the provision of sound public service to people (for instance, public order and safety or the provision of medical services), and where political leader are not always accountable for their actions. Therefore, we do not address issues such as finding the optimal size of the public sector, or studying the provision of public spending that maximizes the welfare.

2.3 The dynamics of public spending

Public goods and services are financed out of tax income. The random return to income taxation is

$$\mu(t) = \begin{cases} \mu_1(t) = \tau(1 + se(t))A(t)k(t), & w.p. & p \\ \mu_2(t) = \tau(1 - e(t))A(t)k(t), & w.p. & 1 - p \end{cases} \tag{14}$$

Tax revenue is a random variable and so is per-capita public spending. Assuming a zero fiscal balance, the stochastic process describing the dynamics of

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public spending is therefore

$$dg(t) = \lambda_1(t)g(t)^{1/\xi}dt + \lambda_2(t)g(t)^{2/\xi}dZ_g(t), \tag{15}$$

where $Z_g(t)$ is a Brownian motion process and

$$\begin{aligned}\lambda_1(t) &= & p\mu_1(t) + (1-p)\mu_2(t) \\ &= & \xi k(t)\left\{p\tau(1+se(t)) + (1-p)\tau(1-e(t))\right\},\end{aligned} \tag{16}$$

and

$$\begin{aligned}\lambda_2(t) &= & p(1-p)\xi^2 k(t)^2 \left\{\tau^2(1+se(t))^2 + \tau^2(1-e(t))^2\right\} \tag{17}\\ & & -2\tau^2(1+se(t))(1-e(t)). \tag{18}\end{aligned}$$

(15) is a nonlinear SDE with drift and diffusion components which both depend on tax evasion behavior and tax corruption.

3 The optimal choice of the domestic agent

An agent faces the following intertemporal utility maximization problem. She maximizes (13) subject to the constraint (11) with $w(0) = w_0$.

Proposition 1 *The optimal choice of a consumer in the domestic country is given by the following unique interior solution (see the proof in Appendix 1)*

$$\begin{cases} (1-\gamma)\frac{\tilde{c}(t)}{w(t)} &= \beta - \frac{\gamma}{2}\left[1-\gamma\right]\left[(\omega_1(t))^2 + (\omega_1^*(t))^2\right]\tilde{n}_d^2 \\ &+ \frac{\gamma}{2}\left[1-\gamma\right](\omega_1^*(t))^2 \\ &- \gamma R^*(t), \\ \\ \tilde{n}_d(t) &= \dfrac{(1-\tau)A(t) - R^*(t)}{[1-\gamma](\omega_1^*(t))^2} + 1, \\ \\ \tilde{e}(t) &= \dfrac{A(t)\,\bar{x}_1\tau}{[1-\gamma][\sigma_1\tau A(t)]^2\,\tilde{n}_d(t)}. \end{cases} \tag{19}$$

The first equation is obtained from the equality between the marginal utility of consumption and the marginal utility of wealth, which leads:

$$\tilde{c}(t) = \left\{V'(w(t))\right\}^{\frac{1}{\gamma-1}} \tag{20}$$

where V is the value function.

The second equation is an arbitrage equation obtained from the first-order condition of the objective function obtained using the Jacobi-Hamilton-Bellman equation with respect to $n_d(t)$. This yields

$$R(t) - AP(w)\omega_1(t)^2 n_d(t) = R^*(t) - AP(w)\omega_1^*(t)^2 n_d^*(t), \tag{21}$$

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where $AP(w)$ is the absolute value of the Arrow-Pratt relative risk aversion coefficient assumed to be constant:

$$AP(w) = \frac{wV''(w)}{V'(w)}. \tag{22}$$

Equation (21) says that the risk-adjusted gross returns of one unit of capital invested in the domestic and foreign countries are equalized. The risk can be decomposed into several components. Its depends upon the share of capital invested out of total wealth in the domestic and foreign countries, upon the uncertainty from tax evasion and corruption and upon the agent's behavior towards risk. The risk premium is therefore a function of the degree of relative risk aversion and of the difference in the uncertainty of fraud and corrupting bureaucrats in both countries:

$$R(t) - R^*(t) = AP(w) \left[\omega_1(t)^2 n_d(t) - \omega_1^*(t) n_d^*(t)^2 \right] \tag{23}$$

The third equation is obtained by equalizing to zero the derivative of the objective function with respect to $e(t)$. This yields

$$\tilde{e}(t) = \left(\frac{1}{-AP(w)} \right) \left(\frac{\overline{x}_1}{\sigma_1^2} \right) \left(\frac{1}{\tau} \right) \left(\frac{1}{y^d(t)/w(t)} \right). \tag{24}$$

The optimal decision of tax fraud varies positively with the risk-adjusted return of fraud and with the degree of risk aversion, negatively with the tax rate and the domestic revenue as share of the agent's wealth. A system in which the tax rate is high is an incentive to cheat. Conversely, the motivation for a tax fraud diminishes as domestic production represents a high proportion of an individual's total wealth.

The optimization problem facing the foreign agent is similar, but is not studied here since our focus is on the domestic country.

The household's optimal solution does not lead to a closed-form solution, but all the variables are determined simultaneously. The solution is well defined if the three variables lie in the unit interval $(0,1)$. This requires some assumptions to guarantee that this holds.

Firstly, using the expression of $n_d(t)$ in Appendix 1, the condition $0 < \widetilde{n}_d(t) < 1$ is equivalent to imposing a lower and upper bound on $R^*(t)$:

$$(1-\tau)A(t) < R^*(t) < (1-\tau)A(t) + (1-\gamma)\omega_1^*(t)^2. \tag{25}$$

One can interpret $n_d(t)$ as the domestic financial market depth, and also as the degree of financial openness. A value near 1 indicates that there are strong restrictions on the international mobility of capital assets, while a value near 0 reflect a situation of perfect mobility and perfect substitution between domestic and foreign assets. We exclude the situation of preferred habitat where saving would be entirely invested in either domestic or foreign capital. The above inequality indicates that this is the case if investing abroad yields a minimum gross rate of return on each unit of foreign capital owned (the lower bound). To

avoid a situation of a complete depletion of domestic capital (capital outflows that would lead to $\widetilde{n}_d(t) = 0$), the foreign gross rate of return must be bounded above (the upper bound). From (19), it is seen that whenever $\widetilde{n}_d(t) > 0$, this inequality also holds for $\tilde{e}(t)$ and $\frac{\widetilde{c}(t)}{w(t)}$.

From (19), $\tilde{e}(t) < 1$ if the expected return on one unit of evaded tax, adjusted by the risk of tax evasion and tax corruption, is bounded above (the gain from cheating is limited). Formally, $\tilde{e}(t) < 1$ implies the following inequality:

$$\frac{\overline{x}_1}{\sigma_1^2} < \tau A(t)(1 - \gamma)\widetilde{n}_d(t). \tag{26}$$

Finally, the agent does not spend her whole wealth in consumption spending if the marginal utility of wealth is bounded above. Indeed, in Appendix 1 we show that

$$\frac{\widetilde{c}(t)}{w(t)} = [V'(\omega)]^{1/\gamma - 1}. \tag{27}$$

Therefore

$$\frac{\widetilde{c}(t)}{w(t)} < 1, \text{ if } V'(\omega) < \omega(t)^{\gamma - 1}. \tag{28}$$

4   Steady state distributions

4.1   Definition of the equilibrium

For a given sequence of $\left\{ A^*(t), e^*(t), \widetilde{n}_f^*(t), \frac{\widetilde{c}^*(t)}{w^*(t)}, y^*(t), k^*(t) \right\}_0^\infty$ and initial values $\tilde{e}(0)$, $\frac{\widetilde{c}(0)}{w(0)}$, $\widetilde{n}_d(0)$, $g(0)$, $y(0)$, $k(0)$,the equilibrium is a sequence

$$\left\{ A(t), e(t), \widetilde{n}_d(t), \frac{\widetilde{c}(t)}{w(t)}, y(t), k(t) \right\}_0^\infty,$$

where each variable is defined by a distribution, that satisfies the following conditions:

$\quad$ $i)$ these variables satisfy the agent's optimal choice,

$\quad$ $ii)$ domestic capital growths at the same rate as saving,

$\quad$ $iii)$ the government's budget constraint is described by the SDE (15),

$\quad$ $iv)$ the economy's capital and financial account is balanced.

$\quad$ $v)$ the constraints (25), (26) and (28) apply.

Condition (i) implies that the equilibrium path must satisfy the system (19). As shown in Appendix 1, the convexity of the maximization problem implies the unicity of the optimal solution.

Condition (ii) implies that the dynamics of capital obeys the following SDE:

$$dk(t) = \frac{dw(t)}{w(t)}k(t) = \psi(t)k(t)dt + \Omega_1(t)k(t)dZ_k(t), \tag{29}$$

where we have substituted a new diffusion component $\Omega_1(t)dZ_k(t)$ for the two local martingale terms $\omega_1(t)dz_1(t)$ and $\omega_1^*(t)dz_1^*(t)$ in Equation (11). The solution of this SDE can be written as

$$d\ln k(t) = \left\{ \psi(t) - \frac{1}{2}\Omega_1^2(t) \right\} dt + \Omega_1(t)dZ_k(t), \tag{30}$$

which implies

$$k(t) = k(0) \exp\left\{ \int_0^t (\psi(s) - \frac{1}{2}(\Omega_1(s))^2)ds \right\}. \tag{31}$$

Condition (iii) implies that the dynamics of $A(t)$ can be found by applying the Ito lemma. We have

$$A(t) = \xi \left[g(t)\right]^{1/\xi} \text{ and } dg(t) = \widetilde{\lambda}_1(t)dt + \widetilde{\lambda}_2(t)dz_g(t), \tag{32}$$

where $\widetilde{\lambda}_1(t) = \lambda_1(t) \left[g(t)\right]^{1/\xi}$ and $\widetilde{\lambda}_2(t) = \lambda_2(t) \left[g(t)\right]^{2/\xi}$ with $\lambda_1(t)$ and $\lambda_2(t)$ defined by (16) and (17). Applying the Ito lemma, we have

$$\begin{aligned} dA(t) &= \left\{ \widetilde{\lambda}_1(t)\frac{\partial A(t)}{\partial g(t)} + \frac{1}{2}(\widetilde{\lambda}_2(t))^2\frac{\partial^2 A(t)}{\partial g(t)^2} \right\} dt + \tag{33} \\ &\quad \widetilde{\lambda}_2(t)\frac{\partial A(t)}{\partial g(t)}dz_g(t), \end{aligned}$$

or

$$dA(t) = \alpha(t)A(t)dt + \beta(t)A(t)dz_g(t), \tag{34}$$

where

$$\alpha(t) = \widetilde{\lambda}_1(t)(\xi/g(t)) + (\frac{1-\xi}{\xi})(\xi/g(t)^2)\widetilde{\lambda}_2(t)^2, \tag{35}$$

$$\beta(t) = (\xi/g(t))\widetilde{\lambda}_2(t). \tag{36}$$

Equation (34) implies

$$A(t) = A(0) \exp\left\{ \int_0^t (\alpha(s) - \frac{1}{2}(\beta(s))^2)ds \right\}. \tag{37}$$

(31) and (37) are not closed-form solutions of (??) and (34) because $k(t)$ and $A(t)$ also appear in $\psi(t)$, $\Omega_1(t)$, $\alpha(t)$ and $\beta(t)$ and in (19). $k(t)$ and $A(t)$ are the two important state variables in the model, since they determine the dynamics of all the other variables. The equilibrium is described by a random sequence of the variables or by a distribution. Indeed, as is seen from our equations, the dynamics is the results of a deterministic drift component and of a diffusion component where the variance of the variables is used to define their distribution. The stochastic nature of the model entirely comesfrom the uncertain income caused by tax evasion and tax corruption.

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4.2 Sufficient conditions for the existence of a long-run stochastic steady state

We focus on the dynamics of the variables of our model in the neighborhood of the long-run stochastic steady state. Such a state is characterized, in systems of SDE, by a steady stable distribution. We study the conditions for the existence of such a distribution for per-capita GDP. Since the latter depends upon $k(t)$ and $A(t)$, we check the validity of some conditions for these variables to have a limit stable distribution. We shall prove the following two propositions:

Proposition 2 $k(t)$ *has two bounds* 0 *and* ∞. *The zero bound is inaccessible if* $\psi^s/(\Omega_1^s)^2$ *is above a threshold value (here* $1/2$*) and the infinite bound is inaccessible if* $\psi^s/(\Omega_1^s)^2$ *is below a threshold value (*$1/2$*).*

Proposition 3 $A(t)$ *has two bounds* 0 *and* ∞. *The zero bound is inaccessible if* $\alpha^s/(\beta^s)^2$ *is above a threshold value (here* $1/2$*) and the infinite bound is inaccessible if* $\alpha^s/(\beta^s)^2$ *is below a threshold value (*$1/2$*).*

Before proving these propositions, we briefly explain what they mean. An exponent s on a variable indicates that the variable is considered in the neighborhood of the random steady state. It is noteworthy that production is possible with private capital and public spending. But since the latter is financed out of tax income, if private capital is nil, then the production becomes nil and there are neither tax revenues, nor spending. The quantity $\psi^s/(\Omega_1^s)^2$ can be interpreted as a Sharpe ratio indicating the performance of the agent's portfolio that consists of domestic and foreign equities (private domestic and foreign capital). The first proposition says that, when the performance of the portfolio is high enough, there is an incentive for cheating and thus the economy never converges to a situation in which the household decides to consume her whole wealth without investing in private capital. On the other side, when income is hidden and invested into productive capital, this increases the amount of tax income available for the financing of public goods and services increases, which raises production and in turn the return to cheating. To avoid a situation in which the agent would decide to reduce her consumption to zero while investing all her wealth in private capital, the performance of the portfolio must be bounded above. It is important to notice that, for given value of p, p_1, b, θ, s, we have either $\psi^s/(\Omega_1^s)^2 < 1/2$ or $\psi^s/(\Omega_1^s)^2 > 1/2$. Therefore, if we would consider per-capita capital alone, we could not avoid either a depletion or an explosion of the economy. We therefore need other conditions on public spending, given in the second proposition. The ratio $\alpha^s/(\beta^s)^2$ can be viewed as a proxy of the risk-adjusted random income to taxation weighted by the marginal productivity of public spending (see Equation (19)). For the economy not to extinct ($A(t) = 0$), we need a minimum tax income yield ($\alpha^s > 2(\beta^s)^2$). But tax income should be bounded above to avoid an infinite accumulation of public spending that would yield an infinite per-capita output ($\alpha^s < 2(\beta^s)^2$). Again, if we would consider per-capita spending alone, we could not avoid either a depletion or an explosion of the economy. This yields four cases (see Table 1)

If both A and k equal zero or infinity, then there is no random steady state in the economy. Therefore, the set of parameters p, p_1, b, θ, s must be defined in such a way that we either have the conditions $\psi^s/(\Omega_1^s)^2 < 1/2$ and $\alpha^s/(\beta^s)^2 > 1/2$, or $\psi^s/(\Omega_1^s)^2 > 1/2$ and $\alpha^s/(\beta^s)^2 < 1/2$. Since, in the neighborhood of the random steady state, both variables are attracted towards opposite directions, the conditions imply that there is an equilibrium value for per-capita output. The two polar cases figure out two types of economies. On the one hand, if tax collection systems are efficient and government manage to fight corruption, tax evasion becomes unattractive and there is a low level of private capital due to low concealed income. But in turn, the economy will be financed by a high amount of public spending. On the other side, cheating and offering bribes to bureaucrats may be easy, thereby implying high opportunities to invest the earned income in private equities. In this case per-capita income will be financed by private capital at the expense of public spending. In our model, public and private capital are thus substitutable. One implication is that tax evasion and tax corruption are not necessarily harmful for growth, provided that there exist equity markets in which the proceeds of concealed income can be invested. If people have a low propensity to consume their wealth, but instead a high propensity to save, tax evasion can be viewed as similar to tax exemption. This is exactly the way the so-called "tax havens" function. We now prove the propositions.

Proof. We use the following lemma that apply to SDE (see Karlin and Taylor (1981)).

Lemma. Consider a diffusion process $X(t) = a(t)dt + b(t)Z_X(t)$ where $Z_X(t)$ is a Brownian motion process. Assume that this process has two bounds r_1 and r_2. Sufficient conditions: the two bounds are inaccessible, if

$\forall x_0 \in [r_1, r_2], S(r_1) = -\infty, S(r_2) = +\infty$, where

$$S(x) = \int_{x(0)}^{x} s(u)du, \quad s(x) = \exp\left\{-2\int_{x(0)}^{x} \frac{a(u)}{b^2(u)}du\right\}. \tag{38}$$

s(x) and S(x) are called respectively the scale density function and the scale function. In our case, we have

$$s(k) = [k/k(0)]^{-2\psi^s/(\Omega_1^s)^2},$$

$$S(k) = \frac{k(0)^{2\psi^s/(\Omega_1^s)^2}(\Omega_1^s)^2}{-2\psi^s + (\Omega_1^s)^2}k^{\frac{-2\psi^s+(\Omega_1^s)^2}{(\Omega_1^s)^2}} - \frac{\Omega_1^s}{-2\psi^s + (\Omega_1^s)^2}k(0),$$

and

$$\lim_{k\to 0} S(k) = -\infty, \; if \; 2\psi^s - (\Omega_1^s)^2 > 0,$$

$$\lim_{k\to\infty} S(k) = +\infty, \; if \; 2\psi^s - (\Omega_1^s)^2 < 0.$$

The proof is similar for $A(t)$. ∎

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4.3 Steady state distribution for $g(t)$

We prove and comment the following proposition:

Proposition 4 *A closed-form expression of the invariant steady-state distribution for public spending is given by the following upper incomplete Gamma distribution:*

$$P(g) = K_1^s K_3^s \frac{\Gamma(\alpha, K_2^s g^{\frac{\xi-3}{\xi}})}{\Gamma(\alpha)}, \quad \xi \in (1,3), \quad \alpha = \frac{4-\xi}{3-\xi}, \tag{39}$$

where K_1^s, K_2^s and K_3^s are constants:

$$K_1^s = \frac{1}{(\lambda_2^s)^2} \exp\left\{ K_2^s g(0)^{(\xi-3)/\xi} \right\}, K_2^s = \frac{2\lambda_1^s}{(\lambda_2^s)^2} \times \frac{\xi}{3-\xi}, \tag{40}$$

$$K_3^s = \left(\frac{1}{K_2^s} \right)^{(-4+\xi)/(\xi-3)} \frac{\xi}{\xi-3}. \tag{41}$$

λ_1^s and λ_2^s are (16) and (17) defined in the neighborhood of the random steady state and

$$\Gamma(\alpha) = \int_0^\infty g^{\alpha-1} \exp(-g) dg \quad and \quad \Gamma(\alpha, y) = \int_y^\infty g^{\alpha-1} \exp(-g) dg, \quad \alpha > 0, x > 0.$$

The proof of this proposition is in Appendix 2. Let us comment some of its implications. In our stochastic growth model, the growth rate of per-capita GDP is influenced by the volatility affecting per-capita GDP, or similarly by the volatility affecting at least one of its components, namely capital or public spending. The main characteristics of the invariant distribution of public spending g depends upon the properties of the distribution of an "auxiliary" variable $z = g^{\frac{\xi-3}{\xi}}$. The distribution of g is an upper incomplete Gamma distribution defined by using both the upper incomplete Gamma function and the Gamma function.

Notice that the upper incomplete Gamma function (the numerator of (39)) can be rewritten as follows:

$$\Gamma(\alpha, K_2^s g^{\frac{\xi-3}{\xi}}) = \Gamma(\alpha) - \gamma(\alpha, K_2^s z), \quad z = g^{\frac{\xi-3}{\xi}}, \tag{42}$$

where $\gamma(\alpha, K_2^s z)$ is the lower incomplete gamma function defined by

$$\gamma(\alpha, K_2^s z) = \int_0^{K_2^s z} g^{\alpha-1} \exp(-g) dg. \tag{43}$$

Therefore, we have

$$P(g) = K_1^s K_3^s \left(1 - \widetilde{P}(\alpha, K_2^s z) \right), \quad \widetilde{P}(\alpha, K_2^s z) = \frac{\gamma(\alpha, K_2^s z)}{\Gamma(\alpha)}. \tag{44}$$

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$\widetilde{P}(\alpha, K_2^s z)$ is the cumulative distribution function for gamma random variable z with shape parameter α and rate parameter K_2^s (or with a scale parameter $1/K_2^s$ which is the reciprocal of the rate parameter). The distribution of z can be approximated by a Normal distribution, if $\alpha > 10$, which implies the following condition on the productivity of public spending:. $\xi > 0.89$. Since, we have assumed that $\xi > 1$, this condition is always true. Therefore the limit distribution of g can be considered as being the cumulative distribution of a normal law. The limit invariant distribution is thus symmetric. As a consequence, under the assumption of decreasing productivity of public spending the "shocks" affecting public spending and per-capita output in the steady states are Gaussian. Since $\alpha > 1$, g has a unimodal distribution and the maximum is such that

$$z = [(\alpha - 1)/K_2^s] \quad \text{or equivalently} \quad g^{\max} = \{[(\alpha - 1)/K_2^s]\}^{\xi/(\xi-3)}. \tag{45}$$

Since we have a Normal distribution, the scale parameter can be interpreted as the variance of the distribution. By definition, the Kurtosis of z equals $(6/\alpha)$. The distribution thus displays heavy tails if $(6/\alpha) > 3$ (or, equivalently, if $\xi < 2$) and "normal" tails if $\xi > 2$.

As we noticed above, since g depends upon tax income, which in turn varies randomly according to the intensity of tax evasion and tax corruption, $A = \xi g^{1/\xi}$ can be interpreted as a public spending externality of tax evasion and corruption. The above results imply that, for small values of public spending productivity ("small" means lower than 2), public spending externalities can trigger drastic changes in the asymptotic behavior of per-capita public spending and thus on per-capita output. In other words, tax evasion and tax corruption can make the economy become very unstable in terms of the variability of public spending and per-capita output. The occurrence of "extreme events" in spending is linked to the fourth-order central moment of z and depends upon both ξ and the variables of the tax and corruption system. This is easily seen by noting that the fourth-order central moment is $\mu_4 = [3\alpha(2 + \alpha)]/(K_2^s)^4$. The likelihood of extreme events increases when μ_4 is big, or, equivalently when K_2^s is small. Given the definition of K_2^s, this implies a low return to income taxation.(low ratio $\lambda_1^s/(\lambda_2^s)^2$). This happens when p or s (the probability of being caught and the penalty rate) are small. In other words, tax evasion can make the economy become very unstable in terms of the variability of public spending and thus of per-capita output. Thus the model predicts that, over a long period, we should observe a higher volatility of public spending and of per-capita output growths in those countries in which the tax collection system is highly deficient, tax corruption is widespread and the productivity of public spending is low. However, this instability can be reduced if, public goods and services are highly productive ($\xi > 2$).

4.4   Steady state distribution for $k(t)$

Proposition 5 *The density function of $k(t)$ is a power law density function with a scaling parameter $\gamma = -2(1 - \psi^s/(\Omega_1^s)^2)$.:*

$$p(k) = \frac{2d_0}{(\Omega_1^s)^2} k^{-\gamma}. \tag{46}$$

where d_0 is a normalizing constant.

This density is obtained easily by computing the speed density function as for public spending (see Appendix 2). We assume that $\gamma > 0$ which implies that $\psi^s/(\Omega_1^s)^2 > 1$. To avoid that $p(k)$ diverges when $k \to 0$, we need to impose a lower bound to k. This bound exist if $k = 0$ is inaccessible (in this case, as shown above, we need $\psi^s/(\Omega_1^s)^2 > 1/2$). It is straightforward that the normalizing constant is defined by $C = (\gamma - 1)k_{\min}^{\gamma - 1}$ and this yields $d_0 = 0.5(\gamma - 1)(\Omega_1^s)^2 k_{\min}^{\gamma - 1}$. We require at least that the first moment exists, in which case $\gamma > 2$ or $\psi^s/(\Omega_1^s)^2 > 2$. The variance is finite if $2 < \gamma < 3$ or $\psi^s/(\Omega_1^s)^2 < 5/2$ and infinite if $\gamma > 3$ (thus implying heavy tails). Therefore, if $p, p_1 b, s$ are such that the performance of the agent's portfolio consisting of domestic and foreign equities is high enough ("high" means above $5/2$) then changes in per-capita capital can give rise to extreme values (or high volatility in domestic capital accumulation).

5   Impact of tax evasion and tax corruption on private capital and public spending

We first discuss the effects of changes in p_1, p, b, s on $\frac{dw(t)}{w(t)}$ (or equivalently on $\frac{dk(t)}{k(t)}$ given our definition of the equilibrium) This amounts to examining the impact of changes in $\overline{x}_1$ and σ_1 on the growth rate of saving. For purpose of illustration, we consider a situation in which the domestic agent has an incentive to cheat because she lives in a country where the tax administration is inefficient in collecting taxes and fighting bribery. We discuss the consequences of a lower probability of being caught ($\Delta p_1 < 0$), or a lower expected penalty if caught (that happens if $\Delta \overline{\theta} < 0, \Delta p_1 < 0, \Delta b < 0$). These changes imply higher expected return to corruption and tax evasion ($\Delta \overline{x}_1 > 0$) and a lower uncertainty of fraud activities. An analytical study of a comparative analysis is difficult because we do not have closed-form solutions. We shall instead use heuristic arguments, indicating which equations are affected when changes happen.

5.1   Consumption

A decrease in the probability of being caught, or lower penalty rate or higher probability of facing a corrupted bureaucrat, have the following consequences on the household's consumption decisions. Firstly, this raises the risk-adjusted

return of the unreported income ($\bar{x}_1$ increases and σ_1 decreases). The hidden income is used to buy foreign equities (or equivalently to hold a fraction of the the foreign country's physical capital). The gains from this investment are consumed (wealth effects on consumption). The wealth effect is captured by the term $-\gamma R_t^*$ in the consumption equation of (19) (remember that $\gamma < 0$). This wealth effect reduces saving (and therefore affect the growth rate of private capital negatively) and its magnitude depends upon the curvature of the utility function. The higher the domestic agent's risk aversion, the stronger the negative impact on the growth rate of saving. Further, the financing of public spending declines as tax evasion raises. This in turn reduces the domestic gross return of a unit of concealed income, $R(t)$, and therefore leads to a lower share of the domestic capital held by the household in her total wealth. A decrease in $n_d(t)$ reduces the consumption ratio as shown by the first term in the consumption equation ($\frac{\tilde{c}(t)}{w(t)}$ in (19) is positively related to $n_d(t)$). This in turn increases the growth rate of saving and therefore has a positive impact on the growth rate of private. Thirdly, a decrease in p_1, p, b, s reduces the uncertainty of tax evasion (σ_1 decreases) and the risk of domestic equities (ω_1 decreases). For the agent, this is an incentive to reduces the ratio of consumption out of her total wealth. This effect is captured by the term $\frac{\gamma}{2}\left[1 - \gamma\right](\omega_1(t))^2$ in the consumption equation in (19). The impact on the growth rate of per-capita private capital is therefore positive.

The total effects are thus ambiguous. It is natural to ask what the net effect will be in general in the developing economies. The important point here is that growth should be affected negatively in case of strong wealth effects. In the poorest countries wealth ownership is low. Therefore an agent has a lot to lose if detected when she hides income. As a consequence, this agent would tend to show a high risk aversion. Conversely, increased wealth levels tend to diminish the marginal utility of income, thereby generating a reduce aversion to cheating. Both these arguments should lead to observe a more negative impact on growth of corruption and tax evasion, through the consumption channel, in the poorest countries.

5.2   Public spending

In our model tax evasion and tax corruption is equivalent to diverting public resources that are productive. A decrease in p_1, p, b, s results in a higher $\bar{x}_1$ inducing, all things being equal, an increase in $e(t)$. The latter in turn implies a decrease in public spending (provided that s in Equation (16) is low enough such that the term $\mu_2(t)$ dominates the term $\mu_1(t)$). The magnitude of wasted public resources associated with tax evasion depends upon the taxation rate τ. The negative public spending externalities increases with the amount of lost tax income. The effect on per-capita output is negative (because y is a function of A) with a magnitude which depends on the values of ξ, p, s and τ.

Further, since there is a loop between tax evasion and public spending, a lower $A(t)$ reduces $e(t)$ but increases $n_d(t)$ (19) and thereby affects growth pos-

itively. Therefore, when the agent internalized the negative externalities of tax evasion and corruption on public spending, this makes per-capita output increase. If this second round effect dominates, we have a situation in which public spending is the main driver of per-capita output and the share of private equity diminishes. Conversely, if the negative externalities dominate, then production will be driven by private capital with a lower share of public spending.

Therefore, tax evasion and tax corruption, in addition to impacting production also influence the composition of the growth rate in terms of private and public investment. On the one side, a higher noncompliance rate and a higher tax corruption do not help the economy to capitalize on the public spending externalities. On the other hand, cheating yields individual benefits to the tax payers if there exists an equity market in which the proceeds of the concealed income can be invested.

6   How do tax evasion and tax corruption affect the economies? Summary of our results

In the model, the decision to cheat and corrupt a bureaucrat is the result of a rational choice. This decision generates negative public spending externalities in the production activity, since the amount of evaded income yields lower tax revenues that are used to finance public goods and services and which in turn determine the productivity of capital. But tax evasion and corruption are also a source of volatility of per-capita GDP, capital, spending and consumption. To summarize, we have an infinitely-lived representative household who must allocate her wealth between consumption, a domestic equity and a foreign equity. This allocation depends upon the relative risk-adjusted returns of the equities. The returns are random due to the uncertainty of being caught for non compliance with the tax declaration system, the uncertainty of the punishment (since bureaucrats may be corrupted). The agent internalizes the potential spending externalities on production caused by her behavior. Indeed, though she does not obtain utility from public expenditure, the consumer-producer knows that tax evasion and tax corruption impact the amount of per-capita spending in the economy and thus the amount of income she will receive from production. This knowledge could encourage evasion if the return on the equities generated by tax evasion is higher enough so that the positive impact on production of a higher share of private capital exceeds the negative impact of public spending externality. This is likely to happen if the agent faces a favorable gamble, for instance with a low probability of being caught and convicted and if the likelihood of paying a bribe when detected is high. A key parameter is also the degree of risk aversion because the agent may rather decide to consume the extra-income from cheating. In this case, she would reduce her share of domestic and foreign capital out of wealth because, according to her preferences, consuming an unexpected income (random income) is better then taking part in a gamble.

Tables 2 and 3 display our main findings.

Assume that we are in a poor country in which consumers have preferences characterized by a strong risk aversion and thus by a high curvature of the utility function (high γ). Further consider that the poor country also lacks developed equity markets, so that the diversion effects of public resources on the growth rate of output exceeds the positive impact of the internalization of the negative externalities of tax evasion on public spending. Finally, imagine that the productivity of public goods and services is low, that the tax administration faces difficulties in collecting taxes and that consumers escape tax payments by paying bribes to the bureaucrats. According to the tables, this country will experience a very uncomfortable situation. Indeed, not only will tax evasion and tax corruption reduce the mean growth, but per-capita output will also be highly volatile. This implies situations in which tax evasion deepens recessions. There are several ways in which a government can smooth the cyclical fluctuations of the economy. It can raise the productivity of public spending in order to reduce the degree of the public spending externality in presence of tax evasion. Another possibility is to reduce the incentive for cheating by employing an efficient technology to detect tax evasion or to fight corruption. The government may also want to limit the negative effects of tax evasion on the mean growth, by allowing them to invest their ill-gotten benefits in equity markets. However, if agents have a high risk aversion, the wealth effects on consumption will be important, thereby implying a decrease in their holding of private capital.

Now imagine a country in which a government faces tax noncompliance, but in which taxpayers want to buy domestic and foreign equities (we assume that they have a low risk aversion). Assume that, in this country, the productivity of public spending is low, that people have incentives to pay bribes to government tax collectors, that income tax evasion is widespread. Finally, let us imagine that the government is unable to implement an effective fight against corruption and tax evasion. Again, this country will experiment volatile fluctuations of the output, in addition to possible negative effects on the mean growth rate due to the diversion of public resources. To reduce the size of the output fluctuations, the government can increase the productivity of public spending. In this case, since bureaucrats cannot manage to fight tax evasion, such a policy will only reduce the volatility of public spending; but private capital will still be volatile. However, the situation is better than the initial situation in which both components of the growth rate of per-capita output were volatile. To dampen the negative effects associated with the diversion of public spending resources, the government can make the investment in equity markets an attractive activity to taxpayers by, firstly reducing the tax rate, and, secondly, by improving the productivity of public spending (these measures increase n_d). In this case, private equity markets act a substitute for anti-corruption policies and policies to fight against tax evasion.

7 An empirical illustration

We now provide an empirical support to the predictions of our model. We consider a sample of Southern African countries over the years from 2001 to 2011. We show that they are good candidates for the type of formal model studied in the paper. Countries include: Angola, Botswana, Lesotho, Madagascar, Malawi, Mauritius, Mozambique, Namibia, South Africa, Swaziland and Zambia.

7.1 Data

We collect annual data for the following series:

Size of shadow economy. This variable is measured as share of official GDP and considered here as a proxy for tax evasion. Data are taken from Schneider et al. (2010) and obtained using a MIMIC model. Observations are available for the years 2001-2007. For the years 2008-2001, we take the average of the series over the years 2001-2007.

Private investment as share of GDP. This series is taken from the African Development Bank database. We also compute the volatility of this series by taking the squared value of the difference between an observation and the mean of the series.

Control of corruption. We take this variable from the World Bank's world-wide governance indicators, 2012. The indicator is based on Kaufmann et al. (2010)'s paper. This series measures perceptions of the extent to which public power is exercised for private gain (rent-seeking behavior by elites, all forms of corruption). An increase means a lower corruption level.

Government effectiveness. This variable is also taken from the World Bank governance indicators and considered as a proxy for the productivity of public spending. It captures the perceptions of the quality of public services, the quality of policy formulation and implementation.

Government total spending as share of GDP. This variable is taken from the IMF database (International Financial Statistics) and we compute the volatility by considering the squared of the distance between a given observation and the mean of the series.

Growth rate of per-capita GDP. This series is taken from the World Bank Indicators.

S&P Global equity annual change. We consider this variable as a proxy for the degree of development of domestic equity markets. Data are available for Botswana, Mauritius, Namibia and South Africa. For the other countries of the sample we take the mean of these four countries.

7.2 Results

Though the formal model is expressed in continuous time, we consider a discretization of time and give the empirical illustration in a discrete time context. The objective is not to test the analytical structure per-se, but to test some of the predictions of the model using the economic data. We consider several panel

regressions with fixed effects to account for unobserved heterogeneity. We use a GLS estimator and in some regressions a two-step GLS approach. Tables 4 till 11 contain our results.

In Tables (4) and (5) the mean private investment and government expenditure (both as share of GDP) are regressed on the share of shadow economy and the control of corruption variables. The mean of the growth rate of per-capita GDP and government effectiveness are used as control variables in respectively the first and second regression. As far as the effect of tax evasion is concerned, the regressions show that it is positively correlated with private investment, but negatively correlated with public spending. Turning to the control of corruption, we see that private investment falls as the control of corruption increases and that the converse conclusion applies for government spending. Finally, as expected, growth enhances private investment while an improvement in government effectiveness raises government expenditure. These results support a prediction of the formal model according to which tax evasion and corruption may exert an asymmetric effect on private capital and public spending. In the theoretical model, we explained this by the negative externalities of the behavior of tax evasion on tax income and by the positive returns to cheating on private saving. An interesting point in the regressions is that the positive correlation between tax evasion and corruption (which means here a lower control of corruption) is increased when the variable capturing the development of private equity markets is added to the explanatory variables. Indeed, in Table (6) the coefficients of the shadow economy and control of corruption are found to have a stronger, respectively positive and negative effect on the mean private investment. This seems consistent with the idea that the effects of tax evasion and corruption on private capital depends upon the absorption of the proceeds of cheating by equity markets.

Conversely, when government expenditure (instrumented by its determinants in Table 5) enters the regression, the magnitude of the effects of both variables diminishes (see Table 7). The negative sign of the coefficient of government spending indicates that the private sector does not internalize the negative externalities of the illegal activities on public spending (otherwise both variables would move in the same direction).

If we consider the volatility equations (Tables 8 and 9), it appears that tax evasion increases the volatility of public spending, but does not affect at all the volatility of private investment whose coefficient is not statistically significant. The effect of the control of corruption on government spending is quite strong in comparison with its impact on private investment. Indeed, though having a similar level of significance (with a Student t-ratio of nearly 2.8), the point estimate is 25th as high in Table 9 as in Table 8. If we refer to the different cases of Table 2 in the formal model, this would suggest that the following possible situation for the Southern African economies: a situation of highly volatile public spending and normally volatile private investment, corresponding to a low incentive for cheating and a low productivity of public spending. The important role played by this last variable in determining the regime of volatility for both private investment and public spending seems to be confirmed by the

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regression in Table 10 : when government effectiveness is taken into account, the coefficient of the control of corruption inflates. Moreover, in this regression, we also add the volatility of private investment (instruments by its determinants in Table 8) to show that both volatilities (private investment and public spending) are negatively correlated.

Finally, we consider a growth equation with the mean and volatility of private and public spending (instrumented by their determinants). We add the volatility of exogenous variables (neither linked to tax evasion, nor to the control of corruption). This variable is computed by considering the residuals of the equation of the volatility of private investment and by taking the in-sample forecasts of an AR(1) process applied to the residuals. The mean private investment and government expenditure appear to have a positive effect on the growth rate of per-capita GDP (with a stronger and more significant effect of private investment), but their volatility negatively affect growth (again with a stronger magnitude of the volatility of private investment).

INSERT TABLES 4 TILL 11 ABOUT HERE

8 Discussion and Conclusion

In this paper, we proposed a theoretical model of the effects of tax evasion and tax corruption on private capital and public spending. These variables are considered as productive inputs in the production function. The model highlights several channels through which the mean and volatility of these variables are affected. We first stress the role of equity markets, showing that the evasion outcome for the private sector is not necessarily viewed as a burden, but as an opportunism and optimal response of individual agents to a governance failure from the tax administration. Tax evasion and tax corruption create a random environment - because illegal activities are risky - and the consumer take a portfolio decision (by choosing the share of private capital to hold) in conjunction with the evasion rate and her consumption ratio. Equity markets performs here the same role as a policy of tax exemption. In societies in which the share of private investment in percentage of GDP is growing, in which tax cheaters usually choose to shelter the proceeds of their illegal activities from the official financial institutions, and in which the productivity of public spending is often low, tax evasion and tax corruption may contribute to the development of private capital if people find an opportunity to invest the proceeds of their illegal activities in equity markets. We are not claiming that these activities are beneficial in a broader sense for growth, but simply that, conditional of the performance of the taxation system, tax evasion does not necessarily deepen growth or exacerbates growth volatility in an environment in which private investment is the result of a portfolio decision and of a rational choice leading the agents to take their decisions by comparing the returns to cheating and the risk of being caught and/or facing a corrupted inspector.

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A second important result is that the returns to tax evasion and tax corruption in private equity markets, the average tax income and the productivity of public spending jointly impact the volatility of the economy, through their influence on the volatility of private capital and public expenditure. We evidence several regimes of volatility for these variables. It is noteworthy that, when there is a high incentive for cheating (because the tax collection system is deficient), the negative externalities on public spending can be attenuated if its productivity is high enough. This implies that there may be a trade-off between tax governance and policies enhancing the efficiency of public goods and services on the economic growth.

Thirdly, we raise the fact that the threshold values of the parameters which determine the different configurations of the mean and volatility of the productive inputs are found endogenously by examining the invariant distributions which prevail in the random steady state. Such distributions depends upon the specification of the production function. In an AK model in which per-capita output is a linear function of per-capita private capital and a power function of per-capita government spending with decreasing returns, we show that the invariant distribution are respectively described by a power law and an upper incomplete gamma function.

We conclude by raising that our theoretical arguments seem to found support in Southern African countries. It is encouraging to find that the results of the empirical exercise yield conclusions in favor of the propositions that a) tax evasion and corruption positively impact the mean growth, b) this correlation is reinforced when the degree of development of equity markets is added to the list of regressors, c) private investment and public expenditure are substitutes once they are instrumented by tax evasion and tax corruption, d) we are able to identify a regime for the volatility of both variables.

Appendix 1. The optimal choice of the domestic and foreign agents

>We assume that

$$-\infty < \gamma < 0, \beta > 0. \tag{47}$$

We define

$$V(w(t)) = \max_{\{c(t),e(t),n_d(t)\}} E_0 \int_0^\infty (1/\gamma) \left(c(t)\right)^\gamma e^{-\beta t} dt \tag{48}$$

subject to the constraint

$$\frac{dw(t)}{w(t)} = \psi(t)dt + \omega_1(t)dz_1(t) + \omega_1^*(t)dz_1^*(t), \quad w(0) = w_0. \tag{49}$$

The optimal program is defined as

$$
\begin{aligned}
\beta V(w(t)) &= \max_{\{c(t),e(t),n_d(t)\}} \left[\frac{1}{\gamma}(c(t))^\gamma + V'(w(t))w(t)\Psi'(t) + \frac{1}{2}V''(w(t))w(t)^2\sigma_\omega^2 \right] \\
&= \max_{\{c(t),e(t),n_d(t)\}} F(c(t), e(t), n_d(t)),
\end{aligned}
\tag{50}
$$

where

$$F(c(t), e(t), n_d(t)) = \frac{1}{\gamma}(c(t))^\gamma + V'(w(t))w(t)\Psi(t) \tag{51}$$

$$+ \frac{1}{2}V''(w(t))w(t)^2\sigma_\omega^2(t).$$

V is the value function. F is of class C^3 on R^3. We define

$$\Psi(t) = [1 - \tau + \bar{x}_1\tau e(t)] A(t)n_d(t) +$$
$$[1 - \tau^* + \bar{x}_1^*\tau^*e^*(t)] A^*(t)(1 - n_d(t)) - \frac{c(t)}{w(t)}, \tag{52}$$
$$\sigma_\omega^2(t) = [\sigma_1\tau e(t)A(t)n_d(t)]^2 + [\sigma_1^*\tau^*e(t)^*A(t)^*(1 - n_d(t))]^2.$$

Using the fact that

$$\Psi(t)w = [1 - \tau + \bar{x}_1\tau e(t)] A(t)n_d(t)w + [1 - \tau^* + \bar{x}_1^*\tau^*e^*(t)] A^*(t)(1 - n_d(t))w - c(t),$$

we obtain

$$\begin{array}{rcl}
\frac{\partial F(c,e,n_d)}{\partial c} & = & c^{\gamma-1} - V'(w), \\
\frac{\partial F(c,e,n_d)}{\partial e} & = & V'(w)A(t)\,n_d\,w\bar{x}_1\tau + V''(w)w^2\,[\sigma_1\tau A(t)n_d(t)]^2\,e(t) \\
\frac{\partial F(c,e,n_d)}{\partial n_d} & = & V'(w)\{[1 - \tau + \bar{x}_1\tau e(t)] A(t)w - [1 - \tau^* + \bar{x}_1^*\tau^*e^*(t)] A^*(t)w\}, \\
& + & V''(w)w^2\left\{[\sigma_1\tau e(t)A(t)]^2\,n_d(t) - [\sigma_1^*\tau^*e(t)^*A(t)^*]^2\,(1 - n_d(t))\right\}.
\end{array}$$

Hence, we deduce the function F presents an extremum in $(\tilde{c}, \tilde{e}, \tilde{n}_d)$ given by:

$$\begin{cases}
\tilde{c}(t) & = & \{V'(w)\}^{\frac{1}{\gamma-1}} \\
\tilde{e}(t) & = & \frac{-V'(w)A(t)\,\tilde{n}_d(t)\,w\bar{x}_1\tau}{V''(w)w^2[\sigma_1\tau A(t)\tilde{n}_d(t)]^2} \\
\tilde{n}_d(t) & = & \frac{-V'(w)\{[1 - \tau + \bar{x}_1\tau\tilde{e}(t)]A(t) - [1 - \tau^* + \bar{x}_1^*\tau^*\tilde{e}^*(t)]A^*(t)\}}{V''(w)w\left\{[\sigma_1\tau\tilde{e}(t)A(t)]^2 + [\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*]^2\right\}} \\
& + & \frac{[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*]^2}{\left\{[\sigma_1\tau\tilde{e}(t)A(t)]^2 + [\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*]^2\right\}}.
\end{cases} \tag{53}$$

Multiplying the second equation in (53) by $\tilde{n}_d(t)$, we get

$$\begin{array}{rcl}
\tilde{n}_d(t)(\tilde{e}(t)) & = & \frac{-V'(w)A(t)\,\tilde{n}_d^2(t)\,w\bar{x}_1\tau}{V''(w)w^2[\sigma_1\tau A(t)]^2\tilde{n}_d^2(t)} \\
& = & \frac{-V'(w)A(t)\,w\bar{x}_1\tau}{V''(w)w^2[\sigma_1\tau A(t)]^2}.
\end{array}$$

$$\tilde{n}_d(t)(\tilde{e}(t)) = \frac{-V'(w)A(t)\,w\bar{x}_1\tau}{V''(w)w^2\,[\sigma_1\tau A(t)]^2} \tag{54}$$

Analogously the Bellman equation for the foreigner is given by:

$$\begin{array}{rcl}
\beta V(w^*) & = & \max_{\{c^*, e^*, n_f\}} \left[\frac{1}{\gamma}(c^*)^\gamma + V'(w^*)w^*\Psi^* + \frac{1}{2}V''(w^*)w^{*2}\sigma_\omega^{*2}\right] \\
& = & \max_{\{c^*, e^*, n_f^*\}} F^*(c^*(t), e^*(t), n_f^*(t))
\end{array}$$

$$\tag{55}$$

where

$$F^*(c^*(t), e^*(t), n_f^*(t)) = \frac{1}{\gamma}(c^*(t))^\gamma + V'(w^*)w^*\Psi^* + \frac{1}{2}V''(w^*)(w^*)^2(\sigma^*)^2_\omega(t).$$

$$\begin{aligned}
\Psi^*(t) &= [1 - \tau + \bar{x}_1\tau e(t)]\,A(t)n_f t \\
&+ [1 - \tau^* + \bar{x}_1^*\tau^* e^*(t)]\,A^*(t)n_f^*(t) - \frac{c^*(t)}{w^*(t)}, \\
(\sigma^*)^2_\omega(t) &= [\sigma_1\tau e(t)A(t)n_f]^2 + [\sigma_1^*\tau^* e(t)^* A(t)^*(1 - n_f)]^2.
\end{aligned}$$

Using the fact that

$$\begin{aligned}
\Psi^*(t)w^*(t) &= [1 - \tau + \bar{x}_1\tau e(t)]\,A(t)n_f(t)w^*(t) \\
&+ [1 - \tau^* + \bar{x}_1^*\tau^* e^*(t)]\,A^*(t)n_f^*(t)w^*(t) - c^*(t)
\end{aligned}$$

we obtain

$$\begin{aligned}
\frac{\partial F^*(c^*, e^*, n_f)}{\partial c^*} &= c^{*\gamma-1} - V'(w^*), \\
\frac{\partial F^*(c^*, e^*, n_f)}{\partial e^*} &= V'(w^*)A^*(t)\,n_f^*\,w^*\bar{x}_1^*\tau^* \\
&+ V''(w^*)w^{*2}\left[\sigma_1^*\tau^* A^*(t)n_f^*(t)\right]^2 e^*(t) \\
\frac{\partial F^*(c^*, e^*, n_f)}{\partial n_f} &= V'(w^*)w^*[1 - \tau + \bar{x}_1\tau e(t)]\,A(t) \\
&- V'(w^*)w^*[1 - \tau^* + \bar{x}_1^*\tau^* e^*(t)]\,A^*(t), \\
&+ V''(w^*)w^{*2}[\sigma_1\tau e(t)A(t)]^2\,n_f(t) \\
&- V''(w^*)w^{*2}[\sigma_1^*\tau^* e(t)^* A(t)^*]^2(1 - n_f(t)).
\end{aligned}$$

Hence, we deduce the function F^* presents an extremum in $(\tilde{c}^*, \tilde{e}^*, \tilde{n}_f)$ given by:

$$\begin{cases}
\tilde{c}^*(t) &= \{V'(w^*)\}^{\frac{1}{\gamma-1}} \\
\tilde{e}^*(t) &= \dfrac{-V'(w^*)A^*(t)\,w^*\bar{x}_1^*\tau^*}{V''(w^*)w^{*2}\left[\sigma_1^*\tau^* A^*(t)\right]^2(1 - \tilde{n}_f)(t)} \\
\tilde{n}_f(t) &= \dfrac{-V'(w)\{[1-\tau+\bar{x}_1\tau\tilde{e}(t)]A(t)-[1-\tau^*+\bar{x}_1^*\tau^*\tilde{e}^*(t)]A^*(t)\}}{V''(w)w\left\{[\sigma_1\tau\tilde{e}(t)A(t)]^2+\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2\right\}} \\
&+ \dfrac{\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2}{\left\{[\sigma_1\tau\tilde{e}(t)A(t)]^2+\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2\right\}}.
\end{cases} \tag{56}$$

Multiplying the second equation in (53) by $(1 - \tilde{n}_f(t))$, we get

$$(1 - \tilde{n}_f(t))(\tilde{e}(t) = \frac{-V'(w^*)A^*(t)\,w^*\bar{x}_1^*\tau^*}{V''(w^*)w^{*2}\left[\sigma_1^*\tau^* A^*(t)\right]^2}. \tag{57}$$

Lemma 6 *Assume that the restrictions (47) hold. Then F and F^* present respectively a strict local maximum in $(\tilde{c}, \tilde{e}, \tilde{n}_d)$ and $(\tilde{c}^*, \tilde{e}^*, \tilde{n}_f)$ defined by (53) and (56).*

Proof. We will prove that F presents a strict local maximum in $(\tilde{c}, \tilde{e}, \tilde{n}_d)$ defined by (53). One proceeds exactly in the same way to prove that F^* presents a strict local maximum in $(\tilde{c}^*, \tilde{e}^*, \tilde{n}_f)$ defined by (56).

Denote by $(H_{ij})_{1\leq i,j\leq 3}$, the components of the Hessian Matrix of F. Then,

$$
\begin{aligned}
H_{11} = \frac{\partial^2 F(\tilde{c},\tilde{e},\tilde{n}_d)}{\partial c^2} &= (\gamma - 1)\tilde{c}(t)^{\gamma-2}, \\
H_{12} = \frac{\partial^2 F(\tilde{c},\tilde{e},\tilde{n}_d)}{\partial e \partial c} &= 0, \\
H_{13} = \frac{\partial^2 F(\tilde{c},\tilde{e},\tilde{n}_d)}{\partial n_d \partial c} &= 0, \\
H_{22} = \frac{\partial^2 F(\tilde{c},\tilde{e},\tilde{n}_d)}{\partial e^2} &= V''(w)w^2 \left[\sigma_1 \tau A(t)\tilde{n}_d(t)\right]^2 \\
H_{21} = \frac{\partial^2 F(\tilde{c},\tilde{e},\tilde{n}_d)}{\partial c \partial e} &= 0 \\
H_{23} = \frac{\partial^2 F(\tilde{c},\tilde{e},\tilde{n}_d)}{\partial n_d \partial e} &= V'(w)A(t)\, w\bar{x}_1\tau + 2V''(w)w^2 \left[\sigma_1 \tau A(t)\right]^2 \tilde{n}_d(t)\tilde{e}(t) \\
&= -V'(w)A(t)\, w\bar{x}_1\tau \\
H_{33} = \frac{\partial^2 F(\tilde{c},\tilde{e},\tilde{n}_d)}{\partial n_d^2} &= V''(w)w^2 \left\{ [\sigma_1\tau\tilde{e}(t)A(t)]^2 + [\sigma_1^*\tau^*\tilde{e}(t)^*A(t)^*]^2 \right\} \\
H_{31} = \frac{\partial^2 F(\tilde{c},\tilde{e},\tilde{n}_d)}{\partial c \partial n_d} &= 0 \\
H_{32} = \frac{\partial^2 F(\tilde{c},\tilde{e},\tilde{n}_d)}{\partial e \partial n_d} &= V'(w)\bar{x}_1\tau A(t)w + 2V''(w)w^2 \left[\sigma_1\tau A(t)\right]^2 \tilde{n}_d(t)\tilde{e}(t).
\end{aligned}
$$

With a isoelastic utility function, the value function is of the following form:

$$V(w) = \mu w^\gamma \tag{58}$$

where

$$\mu = \frac{1}{\gamma}\left(\frac{c}{w}\right)^{\gamma-1} \tag{59}$$

Thus,

$$
\begin{aligned}
V'(w) &= \mu\gamma w^{\gamma-1}, \\
V''(w) &= \mu\gamma[\gamma-1]w^{\gamma-2}
\end{aligned}
\tag{60}
$$

Since $\gamma < 0$, we have $\mu < 0$. Consequently, $V'(w) > 0$ and $V''(w) < 0$. Thus $H_{22} < 0$. On the other hand g being a non negative function, we have $\tilde{c}(t) > 0$ for all $t \in \mathbb{R}_+$ and then $H_{11} < 0$ since $\gamma - 1 < 0$ and $H_{22}H_{11} > 0$. Now using (57), we have

$$
\begin{aligned}
H_{22}H_{33} &= \left(V''(w)w^2\right)^2 [\sigma_1\tau A(t)]^4 (\tilde{n}_d(t)\tilde{e}(t))^2 \\
&+ \left(V''(w)w^2\right)^2 [\sigma_1\tau A(t)\tilde{n}_d(t)]^2 [\sigma_1^*\tau^*\tilde{e}(t)^*A(t)^*]^2 \\
&= \left(-V'(w)A(t)\, w\bar{x}_1\tau\right)^2 + \left(V''(w)w^2\right)^2 [\sigma_1\tau A(t)\tilde{n}_d(t)]^2 [\sigma_1^*\tau^*\tilde{e}(t)^*A(t)^*]^2 \\
&= \left(V'(w)A(t)\, w\bar{x}_1\tau\right)^2 + \left(V''(w)w^2\right)^2 [\sigma_1\tau A(t)\tilde{n}_d(t)]^2 [\sigma_1^*\tau^*\tilde{e}(t)^*A(t)^*]^2.
\end{aligned}
$$

Consequently,

$$
\begin{aligned}
H_{22}H_{33} - H_{23}^2 &= \left(V'(w)A(t)\, w\bar{x}_1\tau\right)^2 + \left(V''(w)w^2\right)^2 [\sigma_1\tau A(t)\tilde{n}_d(t)]^2 [\sigma_1^*\tau^*\tilde{e}(t)^*A(t)^*]^2 \\
&\quad - \left(V'(w)A(t)\, w\bar{x}_1\tau\right)^2 \\
&= \left(V''(w)w^2\right)^2 [\sigma_1\tau A(t)\tilde{n}_d(t)]^2 [\sigma_1^*\tau^*\tilde{e}(t)^*A(t)^*]^2.
\end{aligned}
$$

This means that $H_{22}H_{33} - H_{23}^2 > 0$ and $H_{11}(H_{22}H_{33} - H_{23}^2) < 0$.

Thus, the principal minors associate to the Hessian matrix of F satisfies

$$H_{11} < 0; \quad H_{22}H_{11} > 0; \quad H_{11}(H_{22}H_{33} - H_{23}^2) < 0.$$

■

Using (53) and (60), we have

$$
\begin{cases}
\tilde{e}(t) &= \dfrac{A(t)\,\bar{x}_1\tau}{[1-\gamma][\sigma_1\tau A(t)]^2\tilde{n}_d(t)} \\[2mm]
\tilde{n}_d(t) &= \dfrac{\{[1-\tau+\bar{x}_1\tau\tilde{e}(t)]A(t)-[1-\tau^*+\bar{x}_1^*\tau^*\tilde{e}^*(t)]A^*(t)\}}{[1-\gamma(1+\eta)]\left\{[\sigma_1\tau\tilde{e}(t)A(t)]^2+\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2\right\}} \\[2mm]
&+ \dfrac{\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2}{\left\{[\sigma_1\tau\tilde{e}(t)A(t)]^2+\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2\right\}}.
\end{cases}
\tag{61}
$$

and

$$
\tilde{n}_d(t)\tilde{e}(t) = \frac{A(t)\,\bar{x}_1\tau}{[1-\gamma]\left[\sigma_1\tau A(t)\right]^2}
\tag{62}
$$

Thus,

$$
\begin{aligned}
[1-\gamma]\left\{[\sigma_1\tau\tilde{e}(t)A(t)]^2+\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2\right\}\tilde{n}_d &= [1-\tau+\bar{x}_1\tau\tilde{e}(t)]\,A(t) \\
&- [1-\tau^*+\bar{x}_1^*\tau^*\tilde{e}^*(t)]\,A^*(t) \\
&+ [1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2
\end{aligned}
$$

That is,

$$
\begin{aligned}
[1-\gamma]\left[\sigma_1\tau A(t)\right]^2\tilde{e}^2(t)\tilde{n}_d &= -[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2\tilde{n}_d \\
&+ (1-\tau)A(t)+(\bar{x}_1\tau A(t))\tilde{e}(t) \\
&- \left[1-\tau^*+\bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t) \\
&+ [1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2
\end{aligned}
$$

Multiplying this latter identity by $\tilde{n}_d$,

$$
\begin{aligned}
[1-\gamma]\left[\sigma_1\tau A(t)\right]^2\tilde{e}^2(t)\tilde{n}_d^2 &= -[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2\tilde{n}_d^2 \\
&+ (1-\tau)A(t)\tilde{n}_d+(\bar{x}_1\tau A(t))\tilde{n}_d\tilde{e}(t) \\
&- \left[1-\tau^*+\bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t)\tilde{n}_d \\
&+ [1-\gamma(1+\eta)]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2\tilde{n}_d
\end{aligned}
$$

and using (62),

$$
\begin{aligned}
&[1-\gamma]\left[\sigma_1\tau A(t)\right]^2\left(\tfrac{A(t)\,\bar{x}_1\tau}{[1-\gamma][\sigma_1\tau A(t)]^2}\right)^2 = \\
&-[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2\tilde{n}_d^2+ \\
&(1-\tau)A(t)\tilde{n}_d+(\bar{x}_1\tau A(t))\left(\tfrac{A(t)\,\bar{x}_1\tau}{[1-\gamma][\sigma_1\tau A(t)]^2}\right)- \\
&\left[1-\tau^*+\bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t)\tilde{n}_d+ \\
&[1-\gamma(1+\eta)]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2\tilde{n}_d
\end{aligned}
$$

which gives

$$
\begin{aligned}
&\tfrac{(A(t)\,\bar{x}_1\tau)^2}{[1-\gamma][\sigma_1\tau A(t)]^2} = \\
&-[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2\tilde{n}_d^2+ \\
&(1-\tau)A(t)\tilde{n}_d+\tfrac{(A(t)\,\bar{x}_1\tau)^2}{[1-\gamma][\sigma_1\tau A(t)]^2}- \\
&\left[1-\tau^*+\bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t)\tilde{n}_d+ \\
&[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2\tilde{n}_d.
\end{aligned}
$$

or
$$\begin{aligned}
0 \;=\; & -[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2 \tilde{n}_d^2 + \\
& (1-\tau)A(t)\tilde{n}_d \\
& - \;\left[1-\tau^* + \bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t)\tilde{n}_d \\
& + \;[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2 \tilde{n}_d.
\end{aligned}$$

Hence, assuming that $\tilde{n}_d \neq 0$, we get

$$\begin{aligned}
[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2 \tilde{n}_d \;=\; & (1-\tau)A(t) \\
& - \;\left[1-\tau^* + \bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t) \\
& + \;[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2.
\end{aligned}$$

and

$$\tilde{n}_d \;=\; \frac{(1-\tau)A(t)-[1-\tau^*+\bar{x}_1^*\tau^*\tilde{e}^*(t)]A^*(t)}{[1-\gamma(1+\eta)]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2} + 1$$

Set $\Delta = \left[\sigma_1\tau\tilde{e}(t)A(t)\right]^2 + \left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2$. Then

$$\begin{aligned}
[1-\gamma]\Delta\tilde{n}_d \;=\; & \left[1-\tau + \bar{x}_1\tau\tilde{e}(t)\right]A(t) - \left[1-\tau^* + \bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t) \\
& + \;[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2
\end{aligned} \tag{63}$$

From the Bellman equation (50), we have

$$\beta V(w) = \left[\frac{1}{\gamma}(\tilde{c}g^\eta)^\gamma + V'(w)w\tilde{\Psi} + \frac{1}{2}V''(w)w^2\tilde{\sigma}_\omega^2\right]$$

where

$$\tilde{\Psi}(t) = \left[1-\tau + \bar{x}_1\tau\tilde{e}(t)\right]A(t)\tilde{n}_d(t) + \left[1-\tau^* + \bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t)(1-\tilde{n}_d(t)) - \frac{\tilde{c}(t)}{w(t)},$$
$$\tilde{\sigma}_\omega^2(t) = \left[\sigma_1\tau\tilde{e}(t)A(t)\tilde{n}_d\right]^2 + \left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*(1-\tilde{n}_d)\right]^2.$$

Note that

$$\begin{aligned}
\tilde{\sigma}_\omega^2(t) \;=\; & \left[\sigma_1\tau\tilde{e}(t)A(t)\tilde{n}_d\right]^2 + \left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*(1-\tilde{n}_d)\right]^2 \\
\;=\; & \Delta\tilde{n}_d^2 + \left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2(1-2\tilde{n}_d).
\end{aligned}$$

From (63),

$$\begin{aligned}
\left[1-\tau + \bar{x}_1\tau\tilde{e}(t)\right]A(t) - \left[1-\tau^* + \bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t) \;=\; & [1-\gamma]\Delta\tilde{n}_d \\
& - \;[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2
\end{aligned}$$

$$\begin{aligned}
\tilde{\Psi}(t) \;=\; & \left\{\left[1-\tau + \bar{x}_1\tau\tilde{e}(t)\right]A(t) - \left[1-\tau^* + \bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t)\right\}\tilde{n}_d(t) \\
& + \;\left[1-\tau^* + \bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t) - \frac{\tilde{c}(t)}{w(t)} \\
\;=\; & [1-\gamma]\Delta\tilde{n}_d^2 - [1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2 \tilde{n}_d \\
& + \;\left[1-\tau^* + \bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t) - \frac{\tilde{c}(t)}{w(t)}
\end{aligned}$$

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Thus using (58) and the fact that $c^{\gamma-1} = V'(w)$ yields

$$
\begin{aligned}
\beta\mu w^\gamma &= \tfrac{1}{\gamma}(\tilde{c})^\gamma + V'(w)w\tilde{\Psi} + \tfrac{1}{2}V''(w)w^2\tilde{\sigma}_\omega^2 \\
&= \tfrac{1}{\gamma}\tilde{c}^\gamma + \mu\gamma w^{\gamma-1}w\tilde{\Psi} \\
&+ \tfrac{1}{2}\mu\gamma[\gamma-1]w^{\gamma-2}w^2\tilde{\sigma}_\omega^2 \\
&= \tfrac{1}{\gamma}\left(\tfrac{\tilde{c}}{w}\right)\mu\gamma w^\gamma + \mu\gamma w^\gamma\tilde{\Psi} \\
&+ \tfrac{1}{2}\mu\gamma[\gamma-1]w^\gamma\tilde{\sigma}_\omega^2
\end{aligned}
$$

which after simplification gives

$$
\beta = \left(\frac{\tilde{c}}{w}\right) + \gamma\tilde{\Psi} + \frac{1}{2}\gamma[\gamma-1]\tilde{\sigma}_\omega^2.
$$

Hence replacing $\tilde{Psi}$ by its expression in the latter identity, we obtain

$$
\begin{aligned}
\tfrac{\beta}{1+\eta} &= \left(\tfrac{\tilde{c}}{w}\right) + \gamma[1-\gamma]\Delta\tilde{n}_d^2 - \gamma[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2\tilde{n}_d \\
&+ \gamma\left[1-\tau^* + \bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t) - \gamma\tfrac{\tilde{c}(t)}{w(t)} \\
&+ \tfrac{1}{2}\gamma[\gamma(1+\eta)-1]\Delta\tilde{n}_d^2 + \tfrac{1}{2}\gamma[\gamma(1+\eta)-1]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2(1-2\tilde{n}_d) \\
&= (1-\gamma)\left(\tfrac{\tilde{c}}{w}\right) + \tfrac{\gamma}{2}[1-\gamma(1+\eta)]\Delta\tilde{n}_d^2 - \tfrac{\gamma}{2}[1-\gamma(1+\eta)]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2 \\
&+ \gamma\left[1-\tau^* + \bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t).
\end{aligned}
$$

$$
\begin{aligned}
(1-\gamma)\left(\tfrac{\tilde{c}}{w}\right) &= \beta - \tfrac{\gamma}{2}[1-\gamma]\Delta\tilde{n}_d^2 \\
&+ \tfrac{\gamma}{2}[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A(t)^*\right]^2 \\
&- \gamma\left[1-\tau^* + \bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t)
\end{aligned}
$$

Finally

$$
\begin{cases}
\tilde{n}_d &= \dfrac{(1-\tau)A(t)-[1-\tau^*+\bar{x}_1^*\tau^*\tilde{e}^*(t)]A^*(t)}{[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A^*(t)\right]^2} + 1 \\[3mm]
(1-\gamma)\left(\tfrac{\tilde{c}}{w}\right) &= \beta - \tfrac{\gamma}{2}[1-\gamma]\Delta\tilde{n}_d^2 \\
&+ \tfrac{\gamma}{2}[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A^*(t)\right]^2 \\
&- \gamma\left[1-\tau^* + \bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t) \\[3mm]
\tilde{e}(t) &= \dfrac{A(t)\bar{x}_1\tau}{[1-\gamma][\sigma_1\tau A(t)]^2\tilde{n}_d(t)} \\[3mm]
.
\end{cases}
$$

Proceeding as for the domestic country, we can prove that the optimum equilibrium solution of the Bellman equation for the foreign country is

$$
\begin{cases}
\tilde{n}_f & = \dfrac{[1-\tau+\bar{x}_1\tau\tilde{e}(t)]A(t)-(1-\tau^*)A^*(t)}{[1-\gamma(1+\eta)][\sigma_1\tau\tilde{e}(t)A(t)]^2} \\[2em]
(1-\gamma)\left(\dfrac{\tilde{c}^*}{w^*}\right) & = \beta - \gamma\left[1-\tau+\bar{x}_1\tau\tilde{e}(t)\right]A(t)\tilde{n}_f \\
& \quad -\ \gamma\left[1-\tau^*+\bar{x}_1^*\tau^*\tilde{e}^*(t)\right]A^*(t)(1-\tilde{n}_f) \\
& \quad +\ \dfrac{\gamma}{2}[1-\gamma][\sigma_1\,\tau\,\tilde{e}(t)A(t)]^2\tilde{n}_f^2 \\
& \quad +\ \dfrac{\gamma}{2}[1-\gamma]\left[\sigma_1^*\tau^*\tilde{e}^*(t)A^*(t)\right]^2(1-\tilde{n}_f)^2 \\[2em]
\tilde{e}^*(t) & = \dfrac{A^*(t)\,\bar{x}_1^*\tau^*}{[1-\gamma]\left[\sigma_1^*\tau^*A^*(t)\right]^2(1-\tilde{n}_f(t))} \\[2em]
. &
\end{cases}
$$

and

$$
(1-\tilde{n}_f(t))\tilde{e}^*(t) = \frac{A^*(t)\,\bar{x}_1^*\tau^*}{[1-\gamma]\left[\sigma_1^*\tau^*A^*(t)\right]^2} \tag{64}
$$

Appendix 2. Steady state distribution for public spending

We use the following lemma that applies to SDE (see Mandel, 1968, or Karlin and Taylor, 1981).

Lemma. Let $X(t)$ be a stochastic process described by

$$
dX(t) = a(X)dt + b(X)dW(t),
$$

where $w(t)$ is a Brownian motion process. this process has a time-invariant or steady-state density function $p(x)$, if and only if the speed density $m(x)$ satisfies

$$
\int_{b_1}^{b_2} m(x)dx < \infty, \quad p(x) = c_0 m(x), \quad \int_{b_1}^{b_2} p(x)dx = 1, \quad x\epsilon X(t),
$$

where c_0 is a normalizing constant, b_1 and b_2 are two bounds and

$$
m(x) = \frac{1}{b^2(x)s(x)},
$$

where $s(x)$ is defined in (38).

We consider Equation (15) in the neighborhood of the stationary distribution:

$$
dg = \lambda_1^s g^{1/\xi}dt + \lambda_2^s g^{2/\xi}dZ_g(t),
$$

$$
\lambda_1^s = \xi k^s p\tau(1+se^s) + (1-p)\tau(1-e^s),
$$

$$
\begin{aligned}
\lambda_2^s & = p(1-p)(\xi^s)^2(k^s)^2\left\{\tau^2(1+s(e^s))^2 + \tau^2(1-(e^s))^2\right\} \tag{65} \\
& \quad -2\tau^2(1+s(e^s))(1-(e^s)), \tag{66}
\end{aligned}
$$

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where the index s indicates that we consider the value of a variable on the steady-state distribution (this could be for instance the mode of the distribution). We have

$$m(g) = \frac{1}{(\lambda_2^s)^2 g^{4/\xi}} \exp\left\{\frac{2\lambda_1^s}{(\lambda_2^s)^2} \int_{g(0)}^{g} \frac{u^{1/\xi}}{u^{4/\xi}} du\right\},$$

or

$$m(g) = \frac{1}{(\lambda_2^s)^2} g^{-4/\xi} \exp\left\{-\frac{2\lambda_1^s}{(\lambda_2^s)^2} \times \frac{\xi}{3-\xi}\left(-g(0)^{(\xi-3)/\xi}\right) + g^{(\xi-3)/\xi}\right\},$$

and finally

$$m(g) = K_1^s g^{-4/\xi} \exp\left\{-K_2^s g^{(\xi-3)/\xi}\right\},$$

where

$$K_1^s = \frac{1}{(\lambda_2^s)^2} \exp\left\{K_2^s g(0)^{(\xi-3)/\xi}\right\} \quad \text{and} \quad K_2^s = \frac{2\lambda_1^s}{(\lambda_2^s)^2} \times \frac{\xi}{3-\xi}, \quad \xi \neq 3.$$

The time-invariant probability density function $p(g)$ is

$$p(g) = c_0 m(g).$$

To obtain a closed-form of the density function, we show that the invariant distribution function can be written using the Gamma and upper incomplete Gamma functions.

Let us write the distribution function as

$$P(g) = \int_0^g p(u)du = c_0 K_1^s \int_0^g u^{-4/\xi} \exp\left\{-K_2^s u^{(\xi-3)/\xi}\right\} du, \quad K_2^s > 0.$$

The condition $K_2^s > 0$ implies that $\xi < 3$, since $\lambda_1^s > 0$. Define

$$x = K_2^s u^{(\xi-3)/\xi}.$$

This implies

$$u = \left(\frac{1}{K_2^s}\right)^{\xi/(\xi-3)} x^{\xi/(\xi-3)}, \quad du = \frac{\xi}{\xi-3}\left(\frac{1}{K_2^s}\right)^{\xi/(\xi-3)} x^{3/(\xi-3)} dx,$$

and

$$\lim_{u\to g} x = K_2^s g^{(\xi-3)/\xi} \quad \text{and} \quad \lim_{u\to 0} x = +\infty.$$

we therefore have

$$\int_0^g p(u)du = c_0 K_1^s K_3^s \int_x^{\infty} x^{-1/(\xi-3)} \exp(-x)dx,$$

where

$$K_3^s = \left(\frac{1}{K_2^s}\right)^{(-4+\xi)/(\xi-3)} \frac{\xi}{\xi-3}.$$

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By definition

$$\Gamma(\alpha, x) = \int_x^\infty y^{\alpha-1}\exp(-y)dy, \;\; y > 0, \alpha > 0.$$

Denoting $\alpha = (\xi - 4)/(\xi - 3)$ and setting $c_0 = \Gamma(\alpha)$, with

$$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1}\exp(-y)dy,$$

we have

$$P(g) = K_1^s K_3^s \frac{\Gamma(\alpha, K_2^s g^{(\xi-3)/\xi})}{\Gamma(\alpha)}, \;\; \xi \in (1, 3) \cup (4, +\infty).$$

The steady-state density function is thus

$$p(g) = K_4^s g^{-1/\xi}\exp\left\{-K_2^s g^{(\xi-3)/\xi}\right\}$$

where $K_4^s = \frac{1}{\Gamma(\alpha)}K_1^s K_3^s\left(-K_2^s\right)^{1/(3-\xi)}$.

The first and second moments of the density function are

$$E(g) = \int_0^\infty gp(g)dg = I.$$

and

$$V(g) = E\left[g^2\right] - (E(g))^2 = K_4^s\Gamma(\delta, K_2^s g^{(\xi-3)/\xi}), \quad \delta = (2\xi - 1)/\xi = J$$

By making a change of variable, we have

$$I = I(\beta) = -K_4^s\left(K_2^s\right)^{(2+\xi)/(3-\xi)}\int_0^\infty x^{(2+\xi)/(3-\xi)}\exp(-x)dx, \;\; \beta = \frac{(2+\xi)}{(3-\xi)}, \; \xi < 3,$$

and an integration by parts yields

$$I(\beta) = \beta I(\beta - 1) = ... = \beta^k I(\beta - k),$$

When $k \to \infty$, $\beta \to 1$. Moreover $I(\beta - 1) < I(\beta)$ and therefore $E(g)$ is bounded.

The variance is bounded if $E\left[g^2\right]$ is bounded which is straightforward to prove using similar arguments. Note that the first and second moments exist if $\xi < 3$.

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Table 1. Four cases of accessible bounds

	$\psi^s/(\Omega_1^s)^2 < 1/2$	$\psi^s/(\Omega_1^s)^2 > 1/2$
$\alpha^s/(\beta^s)^2 < 1/2$	$k = 0$ is possible	$k = \infty$ is possible
	$A = 0$ is possible	$A = 0$ is possible
$\alpha^s/(\beta^s)^2 > 1/2$	$k = 0$ is possible	$k = \infty$ is possible
	$A = \infty$ is possible	$A = \infty$ is possible

Table 2. Impact of tax evasion and corruption on private capital and public spending

		Low incentive for cheating
Impact on	Magnitude increases with	p, p_1, b, s are high (or increase)
Private capital	Degree of financial openness (n_d^*)	$(+)$ Wealth effects on consumption ratio
	and risk aversion (γ)	
		$(-)$Positive externality of public spending on consumption
	$p, \bar{\theta}, \tau$	$(-)$Higher risk of investing in private capital :ω_1
Public spending	τ which influences the tax income yield $\alpha^s/(\beta^s)^2$	$(+)$Output-enhancing public spending
Private capital	Equity market depth (n_d)	$(-)$Internalization : higher public spending reduces the agent's incentive to accumulate private capital

Table 2 (continued). Impact of tax evasion and corruption on private capital and public spending

		High incentive for cheating
Impact on	Magnitude increases with	p, p_1, b, s are low (or decrease)
Private capital	Degree of financial openness (n_d^*)	$(-)$ Wealth effects on consumption ratio
	and risk aversion (γ)	
		$(+)$Negative externality of public spending on consumption
	$p, \bar{\theta}, \tau$	$(+)$lower risk of investing in private capital :ω_1
Public spending	τ which influences the tax income yield $\alpha^s/(\beta^s)^2$	$(-)$Diversion of productive public spending
Private capital	Equity market depth (n_d)	$(+)$Internalization : lower public spending increases the agent's incentive to accumulate private capital

Table 3. Impact of tax evasion and corruption on the volatility of growth components

		Low incentive for cheating	High incentive for cheating
		$1 < \psi^s/(\Omega_1^s)^2 < 5/2$	$\psi^s/(\Omega_1^s)^2 > 5/2$
Low productivity	$\xi < 2$	High volatility in public spending	High volatility in public spending
of public spending		Normal volatility in private capital	High volatility in private capital
High productivity	$\xi > 2$	Normal volatility in public spending	Normal volatility in public spending
of public spending		Normal volatility in private capital	High volatility in private capital

Table 4. Regression 1 : private investment (mean)

Explanatory variable	Constant	Shadow economy	Control of corruption	Growth	$\overline{R}^2$	F-stat
Coefficient	-51.91***	1.07***	-3.61**	0.15***	0.42	7.96
t-statistics	(-7.32)	(9.00)	(-2.30)	(4.50)		(0.0)

Note : *, **, *** indicate a coefficient significant respectively at 10%; 5% and 1% level of significance.

Table 5. Regression 2 : Government spending (mean)

Explanatory variable	Constant	Shadow economy	Control of corruption	Government effectiveness	$\overline{R}^2$	F-stat
Coefficient	391.17***	-1.49***	134.57***	89.25***	0.79	31.09
t-statistics	(6.84)	(-3.37)	(5.99)	(6.89)		(0.0)

Table 6. Regression 3 : private investment (mean)

Explanatory variable	Constant	Shadow economy	Control of corruption	Growth	Financial depth	$\overline{R}^2$	F-stat
Coefficient	-84.51***	1.59***	-8.98***	0.26***	0.04***	0.72	23.49
t-statistics	(-14.83)	(16.53)	(-7.29)	(10.25)	(10.73)		(0.0)

Table 7. Regression 4 : private investment (mean)

Explanatory variable	Constant	Shadow economy	Control of corruption	Government spending	$\overline{R}^2$	F-stat
Coefficient	-5.49***	0.12***	-2.40***	-0.03***	0.95	158.43
t-statistics	(-6.23)	(7.25)	(-7.64)	(-8.83)		(0.0)

Table 8. Regression 5 : private investment (volatility)

Explanatory variable	Constant	Shadow economy	Control of corruption	Growth	$\overline{R}^2$	F-stat
Coefficient	16.44**	-0.15	5.59***	0.24***	0.58	11.86
t-statistics	(2.37)	(-1.38)	(2.85)	(6.36)		(0.0)

Table 9. Regression 6 : Government spending (volatility)

Explanatory variable	Constant	Shadow economy	Control of corruption	Government effectiveness	$\overline{R}^2$	F-stat
Coefficient	57.02	5.69**	139.80***	84.59**	0.11	5.95
t-statistics	(0.42)	(2.09)	(2.89)	(2.46)		(0.0)

Table 10. Regression 7 : Government spending (volatility)

Explanatory variable	Constant	Shadow economy	Control of corruption	Government effectiveness	Private invest. volatility	$\overline{R}^2$	F-stat
Coefficient	21.68***	15.5***	1471.55***	654.56***	-80.30	0.86	-174.30
t-statistics	(16.20)	(8.73)	(25.3)	(20.52)	(-20.80)		(0.0)

Table 11. Regression 8 : Growth of per-capita GDP

Explanatory variable	Constant	Private inv (mean)	Gov. spending (mean)	Private inv. (volatility)	Gov.spending (volatility)	Volatility (others)
Coefficient	-6.05***	5.59***	0.68***	-6.05***	-0.76***	8.40***
t-statistics	(-9.02)	(46.50)	(38.52)	(-15.16)	(-84.63)	(8.95)

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