Three Essays in Macroeconomics and International Economics

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Economics)
in The University of Michigan
2016

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This dissertation is dedicated to Ling Yu and Baoshun Zhang.

ACKNOWLEDGEMENTS

This dissertation would not have been possible without the guidance and support of my committee members: Dmitriy Stolyarov, Rüdiger Bachmann, Miles Kimball, and Joshua Hausman. I am grateful to Dmitriy for pushing me to always improve with his intellectual rigor and unreserved dedication, to Rüdiger for leading me to go beyond the edge of research, to Miles for his endless support, encouragement, and inspiration, and to Joshua for all the helpful advice. I also want to specially thank Jinhui Bai, who is not on my dissertation committee but guides me more than an adviser. This dissertation has also benefited immensely from many seminar participants. I am also very grateful to many faculty and administrators at the University of Michigan Department of Economics.

I have been extremely lucky to have many incredible colleagues to get help from: Minjoon Lee, Chenyue Hu, Michael Gelman, Christian Proebsting, Gretchen Lay, Rishi Sharma, Bartley Tablante, Katherine Lim, Ben Meiselman, Daniel Reck, Christoph Boehm, Gaurav Khanna, Ben Thompson, Prachi Jain, and Qinggong Wu. I am also so fortunate to have support from many great friends at Michigan: Ben Hopkins, Jungeun Lim, Courtney Bearns Tablante, Daphne Chang, Bingxin Chen, Yun Chen, Daniela Morar, Sreyoshi Das, Morris Hamilton, Kimberly Lin, Paolo Abarcar, Daniel Schaffa, and Chenyu Yang.

Finally, I would not have made it this far without my family have always been behind me. I want to thank my parents and sister for their patience, my parents-in-law for their trust, and especially my wife, for her love and challenge.

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CHAPTER 1

Inequality and House Prices

This paper studies the interaction between inequality and house prices using an incomplete market model with heterogeneous households. The model links cross-sectional household portfolio saving decisions to housing market outcomes, and it can account for the positive empirical relationship between growth in inequality and changes in house prices. It also illustrates a new house price formation mechanism in which an investment motive among the wealthy plays a key role. A quantitative application of the theory rationalizes the recent substantial housing boom accompanied by rising household saving rates in China. The theory in this paper shows that market frictions can have a differential impact cross-sectionally, increasing inequality. Inequality can in turn amplify frictions in the market.

1.1 Introduction

In the past several decades, many countries have seen rising trends in both inequality[1] (exemplified by the top income share series documented in Atkinson et al. (2011)) and house prices (see Knoll et al. (2014)). Each of these phenomena has drawn attention from policy makers and researchers alike. This paper argues that

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[1]Inequality has many dimensions. This paper mostly looks at the dimensions over income and wealth. Moreover, because of the lack of administrative data on wealth, cross-sectionally comparable wealth data are limited. Therefore, inequality measures in most of the empirical analyses of this paper are along the income dimension similar as in Atkinson et al. (2011). However, wealth inequality and income inequality are closely related, at least to certain degrees of aggregation (see Saez and Zucman (2014)).

rising inequality accompanied by rising house prices is not a coincidence, and identi-
fies a two-way causality mechanism where income inequality and house price run-ups
feed on each other.

Figure 1.1.1: Top income share growth and house price appreciation

(a) Cross-country   (b) China
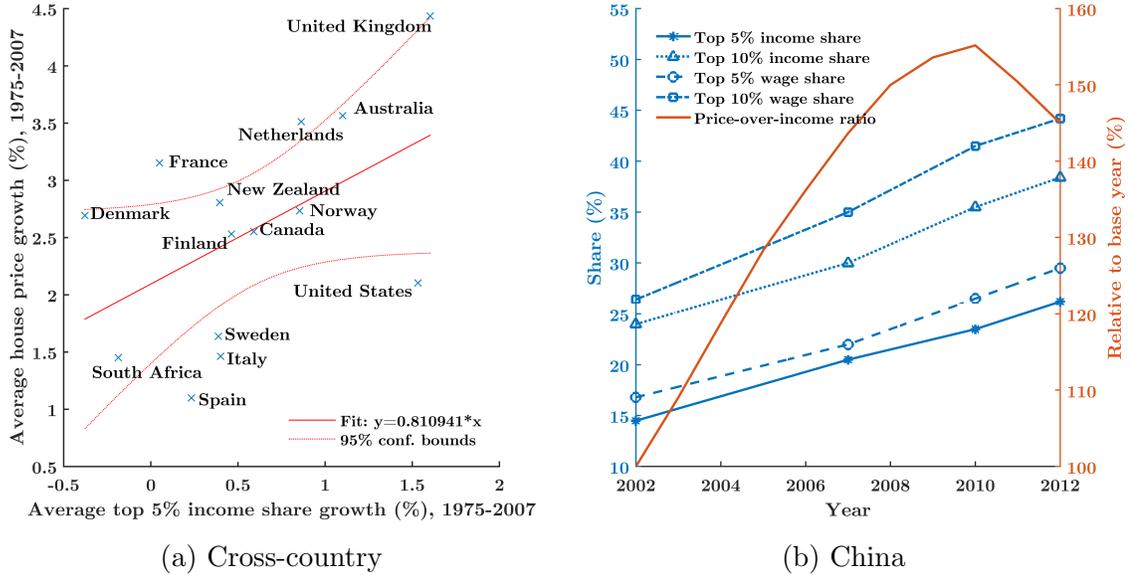
Notes:

1.For each observation in Panel (a), the average growth rates are obtained by averaging the yearly
growth rates over the entire sample periods. In Panel (a), cross-country (real) house price indexes are
taken from Federal Reserve Bank of Dallas's `International House Price Database` as described
in Mack and Martínez-García (2011); cross-country top 5% income shares are taken from Alvaredo
et al. (2015) (income excludes capital gains). Appendix 1.A contains more details about data
source, variable construction, robustness check, and other related empirical analysis (in particular,
an analysis of the cross-state data in the United States).

2. All the income inequality measures in Panel (b) are calculated by author from *Chinese Household
Income Project (CHIP) 2002, 2007* (n.d.) and *China Family Panel Studies (CFPS) 2010, 2012*
(n.d.). Both CHIP and CFPS are large-scale national household surveys, which are arguably the
best available data sources on household income and financial assets in China. Appendix 1.C.2 has
more descriptions of the two data sets. The price-over-income ratio is the data trend of housing
price over per capita income. The construction of the income inequality measures and the price-
over-income ratio is explained in more detail in Appendix 1.C.3.

Cross country correlation of income inequality change and house price growth
is strongly positive, as shown in Panel (a) of Figure 1.1.1.[2]This correlation is also
shown to be robust at the cross-state level in the United States (see Appendix 1.A

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[2]This paper uses the top 5 percent income share as the income inequality measure. Although there
is no natural division between high-income households and low-income households in an economy, the
top 5 percent are often considered as high-income households in the literature that studies the cross-
sectional heterogeneity in household consumption and saving behavior, see Kumhof et al. (2015)
for example. This division also fits the purpose of this study since the top 5 percent households

for a detailed analysis). Moreover, Panel (b) of Figure 1.1.1 provides the time series from China illustrating this pattern: house price growth far outpaced income growth (red line) during the recent period of rising income inequality.

In this paper, I first lay out a mechanism that might rationalize the positive correlation between income inequality and house price shown in Panel (a) of Figure 1.1.1. The mechanism is then embedded in a heterogeneous agent incomplete markets framework in the style of Huggett (1993) and Aiyagari (1994) augmented with endogenous rent-or-own decision and dynamic portfolio choice. The model features endogenous house prices, rents and wealth distribution. I apply the model to account for the dynamics of house prices in China in Panel (b) of Figure 1.1.1 . The quantitative exercise also provides an explanation for the rising Chinese household saving rate.

The basic intuition motivates the mechanisms in the paper is that housing is not only a consumption durable but also an investment asset. What's more, as an investment asset, housing has an access barrier that naturally favors the wealthy, especially when there is a tight borrowing constraint. Therefore, wealthy households can act faster in the housing market and tend to own more investment houses: housing demand and thus its price will depend positively on inequality.

To put the intuition into work, I first set up a simple equilibrium model with income heterogeneity and a frictional housing market. Households in the model consume goods and housing services, and can save in either liquid bonds or housing. Since a homeowner is assumed to gain no direct utility from housing ownership and is allowed to rent out any amount of housing services at no cost, household's rent-to-own decision is linked to its portfolio choice. Moreover, due to frictions in the housing ownership market[3], the adjustment in housing assets is both large and infrequent, which I refer to as lumpy. The lumpiness induces a financial rate of return premium of housing over bonds. In the end, housing is essentially a lumpy financial asset with a return premium to households.

To see how equilibrium housing price depends on inequality, it's informative to look at cross-sectional household portfolio saving decision in the presence of a lumpy housing asset. In particular, housing demand from wealthy households is responsive to changes in individual and market conditions since housing adjustment costs matter less as households get wealthier. Housing demand from the poor, however, is rigid because their housing investment is constrained (or close to constrained) by the

are likely to be wealthy enough to buy houses for investment purposes. Analyzing the investment motive in housing is the focus of this paper.

[3]There are several frictions in adjusting the housing asset, with fixed adjustment cost as the main one.

lumpiness. As a result, the aggregate demand and aggregate price of housing respond positively to a rise in inequality (a shift of resources from the irresponsive poor to the responsive wealthy increases housing demand).

The basic model illustrates the mechanism that make the equilibrium housing price positively respond to inequality. To quantitatively evaluate how much the mechanism can account for housing market dynamics in a realistic setting, I introduce heterogeneous dynastic households facing uninsurable idiosyncratic income shocks to the basic model to study the recent housing boom in China. I analyze a non-stationary equilibrium where the model predicts the evolution of the wealth distribution as well as the equilibrium house price trajectory. I first parameterize the model to match important dimensions of aggregate and cross-sectional data of the Chinese economy before 2002. I then feed in changes in both aggregate income and income inequality from 2002 to 2012 taken from the data to the calibrated model and solve for the perfect foresight transition path equilibrium. The model is shown to reproduce the observed housing price run-up mainly through an endogenous feedback loop between house prices and wealth inequality. This feedback loop causes house prices to respond strongly to changes in fundamentals. The mechanism is, again, based on the differential responsiveness in cross-sectional housing demand.

The feedback mechanism is initiated by expected income growth, which will ultimately lead to house price appreciation. Due to potential capital gains, a period of high premium in housing is thus anticipated among households. In response, the most responsive wealthy households stock up on housing wealth initially, causing a surge in house prices. Due to the skewed housing wealth distribution, the price surge causes a higher dispersion in capital income and thus higher wealth inequality follows. Higher wealth inequality, in turn, helps sustain the growth in house prices. This is because, while housing demand from wealthy households saturates, demand from households with modest wealth starts to pick up after a short delay. Even with a high premium, transaction frictions and liquidity constraints still limit less-wealthy households' response in the housing market. Accordingly, the inequality gap between wealthy housing investors and the rest continues to widen as house prices keep creeping up. This creates a self-reinforcing feedback loop between house price and wealth inequality. In the end, persistent episodes of faster-than-income housing price appreciation is rationalized by relays of housing purchases in the cross section – market frictions cause households taking turns to be active in the housing market: wealthy households are the first movers and initially rising prices force the poor to delay the housing purchase.

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Not only can the model rationalize the observed housing price run-up, it also provides a new explanation for the recent prolonged rising aggregate personal saving rate trajectory in China. First of all, all households initially save more rather than spend more when facing a temporal positive income shock. This is because, for the wealthy, the incentives to tap in the temporary higher return premium in housing market dominates the desire to spend more. For the poor, a stronger saving motive is dictated by the faster-than-income housing price appreciation. The high saving rate is then sustained by the portfolio re-balancing motive. Households in the model all desire a certain level of liquid wealth to ensure against temporary income risks. After the initial portfolio shift toward housing wealth, while waiting for the capital gain in housing asset to fully realize, households need to save up more liquid wealth since it is still the better form of buffer against idiosyncratic income shocks (housing asset is too expensive to adjust frequently).

Broadly, the mechanism behind the interaction between inequality and housing prices illustrates a new channel through which inequality and macroeconomic forces can interact. In particular, market frictions can have a differential impact cross-sectionally, increasing inequality. An increase in inequality can in turn amplify frictions in the market.

The rest of the paper is organized as follows. Section 1.2 discusses related literature. To illustrate the key mechanisms in this paper, Section 1.3 introduces a simplified version of the model to illustrate key mechanisms. It also presents facts about the Chinese economy to inform the model's assumptions and provide an evidence base for testing the theory. Section 1.4 calibrates the quantitative model to the Chinese economy to study the Chinese housing boom between 2002 and 2012, and Section 1.5 concludes. An Appendix that contains additional empirical analyses, proofs, and numerical algorithms follows.

1.2   Related Literature

The modeling approach in this paper builds on a growing strand of literature that tries to rationalize housing price dynamics in an incomplete-market setting with heterogeneous households. See, for example, Kiyotaki et al. (2011), Sommer et al. (2013), Iacoviello and Pavan (2013), and Favilukis et al. (forthcoming). The model in this paper differs from the rest of the literature in the sense that it provides a

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new channel to relate inequality to housing market outcomes.[4] This is achieved by linking rent-or-own decision to portfolio choice. Market frictions make housing choices lumpy, and the lumpiness in housing affects housing choices differently across wealth groups. Thus, housing market outcomes are closely linked to cross-sectional household portfolio saving decisions. The model highlights the investment motives of wealthy households in driving house price dynamics.

In addition, the modeling approach of this paper also takes elements from the literature initiated by Grossman and Laroque (1990) that studies optimal consumption problems under fixed adjustment costs. Of particular interest here are those papers that analyze portfolio choices in the presence of housing, see, for example, Flavin and Yamashita (2002), Fischer and Stamos (2013), and Corradin et al. (2014). In general, this strand of literature adopts a representative agent framework and studies partial equilibrium implications of lumpy housing for consumption and investment decisions with exogenously specified processes for house prices. In contrast, this paper explores the differential impact of the fixed adjustment costs on portfolio choices across wealth groups and investigates its implications for endogenous house price formation.

There is a strand of recent literature that investigated the connection between inequality and house price in the context of static spatial or assignment models. For example, Nieuwerburgh and Weill (2010) adopt a spatial model with changes in cross-sectional productivity differences to explain the increase in house price dispersion across US metropolitan areas. Määttänen and Terviö (2014) and Landvoigt et al. (2015) both use assignment models to study how the distributions of house prices within a metro area are affected by the distribution of assorted households characters. My modeling approach is very different from those static models. In addition, my model features endogenous wealth distribution and the inter-temporal feedback loop. The focus of my paper is on the dynamic interaction between aggregate house prices and inequality.

A few papers study mechanisms whereby house prices may "overreact" to changes in fundamental factors. Ortalo-Magn and Rady (2006) show that house prices can overshoot when changes in income affect the number of credit-constrained owners moving up the property ladder, while in this paper the overreaction in house prices

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[4]Favilukis et al. (forthcoming) also point out the importance of a realistic wealth distribution in determining housing price dynamics. However, their mechanism is very different from that of this paper. In their paper, since housing assets cannot be a vehicle for investment (because housing assets are non-tradable), wealth heterogeneity only serves the purpose of generating a large fraction of housing demand from financially constrained households. This aspect of their mechanism makes equilibrium house price sensitive to changes in financing constraints and interest rates.

is caused by the investment motives of the unconstrained wealthy housing investors. Kahn (2008) finds that regime-switching productivity growth trends in output can generate house prices that are substantially more volatile than output. But the model in Kahn (2008) abstracts from heterogeneity and financial frictions, both of which are central for the mechanisms in this paper. Piazzesi and Schneider (2009) provide a search model to illustrate that a small number of optimistic traders can have a big impact on house prices. This paper shows that wealthy housing investors can drive the house price dynamics.

Finally, the quantitative exercise in this paper links the two strands of literature that study house prices and household saving rates in China. Unlike this paper, which tries to theoretically rationalize the fast appreciation of house prices, most existing papers in the emerging literature that studies China's house prices focus on empirically explaining the high house price level in China. See Wei et al. (2012) and Wang and Zhang (2014) for example. Garriga et al. (2014) and Chen and Wen (2014) are two exceptions that study the growth rate of housing price theoretically. Garriga et al. (2014) use a spatial model to explore the role of structural transformation and the resulting rural-urban migration in the house price dynamics in urban areas of China. However, their model forces housing demand to be determined only by migrants moving from rural areas to cities, which does not seem to be consistent with the excessively strong investment housing demand from existing homeowners found in the data (Chen and Wen (2014)). Chen and Wen (2014) propose a growing-bubble theory to explain the fast house price growth, but their basis is a general asset bubble framework with no particular relevance to the nature of housing assets (since housing is an intrinsically valueless asset in their baseline setup). In contrast, the theory provided by this paper stresses the crucial role played by the lumpy nature of housing as an investment vehicle.

China's high and rising household saving rates have attracted a growing literature investigating the mechanisms behind them. Many factors have been proposed as the possible drivers of these rising saving rates, but no answer has been found conclusive (see, for example, Modigliani and Cao (2004), Chamon and Prasad (2010), Chamon et al. (2013), and Curtis et al. (2015), among many others). Among those factors, rising house prices is one of the usual suspects. Under a partial equilibrium setting (where house prices are exogenously given), Wang and Wen (2012) and Bussière et al. (2013) both show that the saving rates of certain household groups can increase with house prices, provided that certain conditions hold. However, to my best understanding, there is no existing theory that can endogenously generate fast-growing house prices

and rising saving rates at the same time, as this paper does. This paper provides new intuition in understanding the seemingly puzzling persistent high household saving rates in contemporary China.

1.3 The Basic Model

This section introduces basic elements of the model in a simplified setting. The simple model produces the result that house prices positively depend on exogenous inequality measures.[5] The mechanisms behind the results are analytically illustrated and intuitions for further quantitative analysis are discussed.

1.3.1 Environment

Key elements of the model setting include income heterogeneity, a frictional housing ownership market, and separable rent-to-own decisions. Under this setup, housing becomes a lumpy financial asset that affects heterogeneous households differently. Below, we describe the model environment in details.

Endowments and generational structure In this economy, time is discrete and extends from $t = 0, \ldots, \infty$. There are no aggregate or household-specific uncertainty.

Aggregate endowment Y is constant every period. Two types of young households are born at the beginning of each period, referred to, respectively, as high earners (with constant population share μ in each generation) and low earners $(1 - \mu)$. Households live 2 periods and receive income only when they are young. The share of endowment received by high earners is π. Each generation has a constant population mass of one half. Note that the relative endowment between the high earner and low earner is $\frac{\pi(1-\mu)}{\mu(1-\pi)}$, which increases with both high income share π and low population share $1 - \mu$. Note that π and $1 - \mu$ are the two parameters that govern the exogenous endowment inequality of this economy.

There is also a fixed amount of housing stock H in the economy, which produces flow services each period. Newborn households are not endowed with any housing assets (i.e., the older generation owns all the housing stock).

Markets and timing Households can trade the ownership of housing assets. Other than housing, households can also invest in a one-period fully enforceable bond b,

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[5]The distribution for non-capital income is exogenously specified in the model, while capital income and wealth distributions are endogenously determined.

which pays a fixed gross interest rate of $1 + r$. I assume a small open economy so that bonds are supplied by an outside intermediary with infinite supply elasticity. Households enter a period with last period's bond holdings b and housing assets h (newborn households have no assets). At the beginning of the period, a household receives its income, i.e., the output share for the young, and the returns from financial assets for the old (the return from housing assets is the rental income earned).

Asset markets open first in order to allow the older generation to sell off assets for consumption. Young households can buy or borrow bonds from the outside intermediary at the fixed price $1/(1+r)$ with a collateralized constraint for borrowing. In particular, households cannot borrow more than a fraction λ_h of their housing assets. The collateralized borrowing constraint is to guarantee debt payment (default is not allowed in the model). Households can also trade housing assets, but there is a minimum holding size requirement in the housing asset market.[6] To be specific, a household can only hold housing stock at a quantity larger than or equal to $\underline{h}$ with unit price P.[7]

After asset markets close, households consume goods and housing services (as owners and/or renters). Unlike housing asset markets, the housing rental market is frictionless in that households can buy any amount of services s at the market unit price R. The model's frictionless rental market may be a bit extreme, but it captures the idea that a person can rent a very small shared living space that is not, however, available for sale.[8]

Household preferences and the nature of housing   Households have the same individual preferences. In particular, they all receive utility from consuming output c and housing services s. I assume the following separable flow utility function:

$$U(c, s) = (1 - \varphi) \ln c + \varphi \ln s,$$

---

[6]This minimum size friction here serves the purpose of generating lumpiness in the housing market in a tractable way. Although the minimum size friction is frequently adopted in the quantitative housing literature (see, for example, Iacoviello and Pavan (2013)), it is not essential for the mechanisms in this paper as long as there are other frictions in place that make housing lumpy. See Section 1.3.5 for more discussion.

[7]The specification of a (quality-adjusted) per-unit price of housing is natural here since the total stock is fixed and the individual housing choice is continuous when it is above the minimum threshold size $\underline{h}$. The general focus of this paper is the aggregate housing price, not the cross-sectional price dispersion.

[8]In the quantitative housing literature, when the rental decision is explicitly modeled, it is common to assume that rental units come in smaller sizes than houses, see Iacoviello and Pavan (2013) and Sommer et al. (2013) for example.

where $\varphi \in (0, 1)$ reflects the utility weight of housing services in total consumption.[9] The discount factor is constant at β across all the households.

Note that housing ownership does not enter into household's utility function. Moreover, this paper assumes two separate markets for housing services and housing stock. Considering a household that owns h units of housing stock, the housing consumption of this household is not necessarily h, since household members are allowed to either rent out part of their housing stock to become landlords ($s < h$) or to purchase more rental services and be net renters ($s > h$). This means that although housing services and housing stock are necessarily the same in aggregate ($S = H$), individual housing ownership and consumption can be different ($s \neq h$) without any friction.[10] This structure separates individual decisions about housing consumption and housing investment. It stresses the financial aspects of housing, allowing housing purchase to be an investment decision only.[11]

Therefore, housing in this economy is essentially a financial asset that delivers a dividend at the rate of $\frac{R}{P}$.

1.3.2 Household's problem and optimal decision rules

In each period, a household chooses housing and non-housing expenditures. Let the total expenditure be x ($x = c + Rs$). Then the indirect utility function resulting from optimal expenditure allocation is

$$u(x) = \ln x + (1 - \varphi) \ln(1 - \varphi) + \varphi \ln \varphi - \varphi \ln R.$$

Old households only make the above consumption choices. The young households' problem also involves a consumption-saving decision and a portfolio choice between h and b for their saving. Using the indirect utility function, this problem can be written as

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[9]The log-specification over consumption and housing services provides tractability. A more general utility function will be considered in the quantitative model in section 1.3.

[10]Although in reality becoming a landlord bears with it various costs, such as management costs and differential tax treatments, this paper abstracts from those costs, in order to keep the exposition clean.

[11]Sommer et al. (2013) allow households to invest in rental properties. However, housing investment decisions in their model can not be easily separated from housing consumption decisions, since becoming a landlord is assumed to shift one's utility function.

$$\max_{\{x,h,b\}} \ln x + \beta \ln x' \qquad (1.3.1)$$

$$\text{s.t. } x + b + Ph = y$$

$$x' = (1+r)b + (P+R)h$$

$$b \geq -\lambda_h Ph \qquad (1.3.2)$$

$$h \in \{0, [\underline{h}, +\infty]\}. \qquad (1.3.3)$$

Optimal decision rules   Before characterizing the equilibrium, we first discuss the optimal decision rules for households for given aggregate situations. In particular, we consider the "interior" case where aggregate prices satisfy $\frac{R}{P} > r$ (housing has a higher financial rate of return than bonds).[12]

Figure 1.3.1: Policy functions for housing and bonds
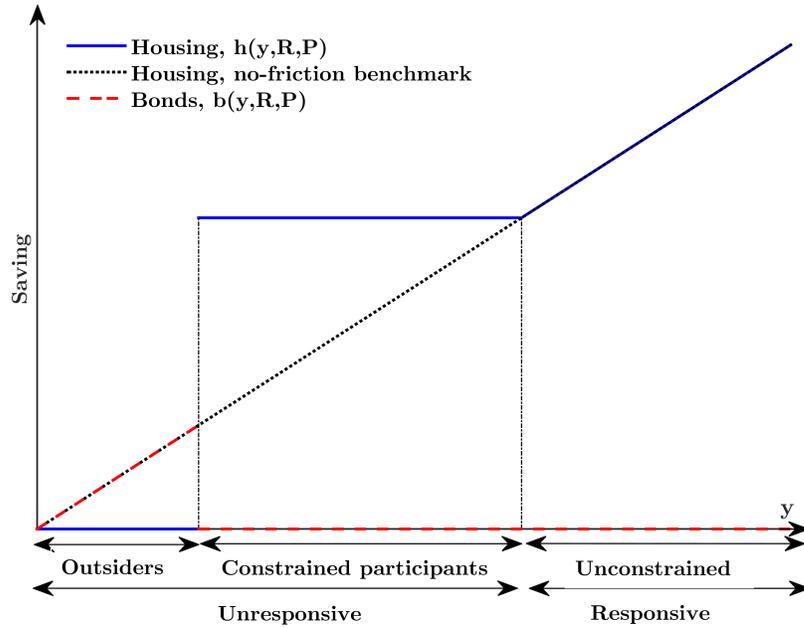
Notes: This figure illustrates the equilibrium policy functions for both housing asset and bonds. The policy functions correspond to one set of parameter values that satisfies the conditions for equilibria in which $\frac{R}{P} > r$. Moreover, for the ease of exposition, borrowing is not allowed ($\lambda_h = 0$).

Note that, when $\frac{R}{P} > r$, optimal portfolio choice depends on income level y, due to

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[12]The "corner" case where aggregate prices satisfy $\frac{R}{P} = r$ is trivial and is explained in Appendix 1.B.

the minimum size friction in housing ownership and the borrowing constraint. More-over, whenever the optimal saving portfolio consists of non-zero housing, constraint (1.3.2) will bind; i.e., it is in the household's interest to fully utilize the collateral-ized borrowing opportunity. Figure 1.3.1 plots optimal rules for saving in housing and bonds as functions of lifetime income y (the analytical derivation is in Appendix 1.B).

If there is either no friction in the housing market ($\underline{h} = 0$) or no limit in bor-rowing ($\lambda_h = 1$), denoted as the no-friction benchmark, households will simply save a constant share of total resources. And all of the savings will go to housing (the dotted line). However, with housing market frictions and borrowing limits, certain households' saving decisions are distorted. In particular, households with low levels of resources cannot enter the housing market and save only in bonds (shown as "out-siders" in Figure 1.3.1); although households with modest resources participate in the housing market, they are constrained by the frictions and have to save more relative to the no-friction benchmark (as "constrained participants"). Only those households with high enough resources are not affected by the frictions (as "unconstrained" par-ticipants). As a result, households with low and modest resources are in large zones of inaction: their housing demand is "unresponsive" to changes in income.[13] In contrast, affluent households have "responsive" housing demand.

1.3.3 Definition of equilibrium

The stationary equilibrium associated with parameters μ, π, Y, r, λ_h, H and $\underline{h}$ consists of prices (R and P), decision rules ($b(y, R, P)$, $h(y, R, P)$, and $x(y, R, P)$), and a stationary distribution $\Gamma(y, b, h)$ such that,

1. Given R and P, $b(y, R, P)$, $h(y, R, P)$ and $x(y, R, P)$ solve the young household's problem defined in (1.3.1).

2. The stationary distribution $\Gamma(y, b, h)$ is induced by $b(y, R, P)$, $h(y, R, P)$, and $x(y, R, P)$.

3. Markets clear:

   (a) $\iint_{\Gamma(y,b,h)} h(y, b, h) = H$,

   (b) $\iint_{\Gamma(y,b,h)} s(y, b, h) = H$.

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[13]Note that housing demand is very responsive (it jumps) at one critical point where switching occurs between "outsiders" and "constrained participants".

Note that, in the model, the distribution of y is exogenously given, and both rent R and house price P are endogenously determined by market clearing conditions. Although there are separate market clearing conditions for housing rental and ownership, R and P are jointly determined in the equilibrium since housing demand is a function of its financial rate of return $\frac{R}{P}$. Moreover, since housing assets are risk-free in a stationary environment, due to the minimum holding size friction, the equilibrium return from housing has to be at least as high as the return from the risk-free bond (i.e., $\frac{R}{P} \geq r$).

1.3.4 The analytical result

With the policy functions identified in Figure 1.3.1, equilibrium prices can be derived using market clearing conditions (3a) and (3b). Similarly, I only consider the "interior" equilibrium in which the return in housing is higher than bonds ($\frac{R}{P} > r$) and all young households own some housing asset.[14]

In equilibrium, the old households do not own any housing asset, the low-earner young households own the minimum-size house, and the high-earner young households save a constant share of their income in housing, thus, asset market clearing condition (3a) can be simplified to[15]

$$\frac{\mu}{2} \times \frac{\pi Y}{\mu} \times \frac{\beta}{(1+\beta)\lambda_h} + \frac{(1-\mu)}{2} \times P\underline{h} = PH,$$

which implies the equilibrium housing price

$$P = \frac{\beta \pi Y}{(1+\beta)\lambda_h(2H - (1-\mu)\underline{h})}.$$

Note that equilibrium housing price P increases with both inequality measures π and $1 - \mu$. In equilibrium, the low earners participate in the housing market but are constrained while high earners are unconstrained.[16] So it is easy to understand why an increase in inequality will cause house prices to go up: an increase in either π or $1 - \mu$ generates more housing demand from the top earners (who are "responsive") but does not suppress demand from the bottom earners (who are "unresponsive").

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[14] The model also has "corner" equilibria in which low-earner young households could not afford the minimum-size house, when there is no restriction on parameter values. See Appendix 1.B for the full characterization.

[15] See Appendix 1.B for the detailed derivation.

[16] There are only two income levels among all households in the basic model. When allowing for more household types, the constrained participants would be households in the middle income range.

1.3.5 Discussion

In this section, I discuss several key intuitions derived from the simple model that are generalizable to more realistic settings.

First, housing purchases are lumpy due to frictions in the market. In the simple model, the lumpiness is built in by the minimum purchase size friction. In reality, there are many other reasons for housing purchases to be lumpy. Non-convex adjustment cost, considered later in the quantitative model, is one obvious reason.[17] Second, housing has to provide a higher equilibrium rate of return than bonds to compensate the frictions in trading. Thus, housing is a preferred asset than bonds when a household can "comfortably" afford it (housing is lumpy). Third, the responsiveness in housing demand differ across wealth groups due to the lumpiness. Wealthy households are more responsive than the poor in the housing market. This is especially the case if credit conditions are tight.

As a result, house prices will respond to inequality when housing is both lumpy and pays a premium. Since wealthy households are more responsive in the housing market, an increase in inequality will increase the housing price. Rental price, however, does not respond to changes in inequality. This is because all households spend a constant share of their income on rental consumption and the rental price is roughly proportional to aggregate output.

In sum, the simple model in this section explains the basic mechanisms behind the positive association between inequality and house prices. Those mechanisms will be further developed to make quantitative analysis.

1.3.6 The case of China

The mechanism that the above model describes depends on housing market frictions and housing being the preferred asset (differential asset returns). In this section, I provide independent evidence that this is the case in China. In particular, I look at Chinese household finance in urban China[18] and document that there are just two major asset classes, and the return on housing exceeds that on other financial assets. I also discuss that Chinese housing markets are subject to substantial frictions.

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[17]See Iacoviello and Pavan (2007) for a discussion of various realistic frictions that make housing investment lumpy within the context of the US. The lumpiness is even more relevant for economies with underdeveloped financial markets. Section 1.4.1 contains more details about the formulation, calibration, and evaluation of the frictions that make housing lumpy in the quantitative model.

[18]There is a big urban-rural divide in China. This paper does not consider rural China.

1.3.6.1 Household finance in China

The most prominent feature of household finance in China might be that households have an asset portfolio dominated by housing and fixed bank deposits.

Figure 1.3.2: Household asset position and interest rate

	China	US
Housing net	79.4%	34.5%
Housing gross	81.8%	44.6%
Housing debt	-2.4%	-10.1%
Fixed deposits	5.3%	0.9%
Other financial	5.5%	40.1%
Other	9.7%	24.6%
Total	100%	100%

(a) Asset composition

(b) Interest rate
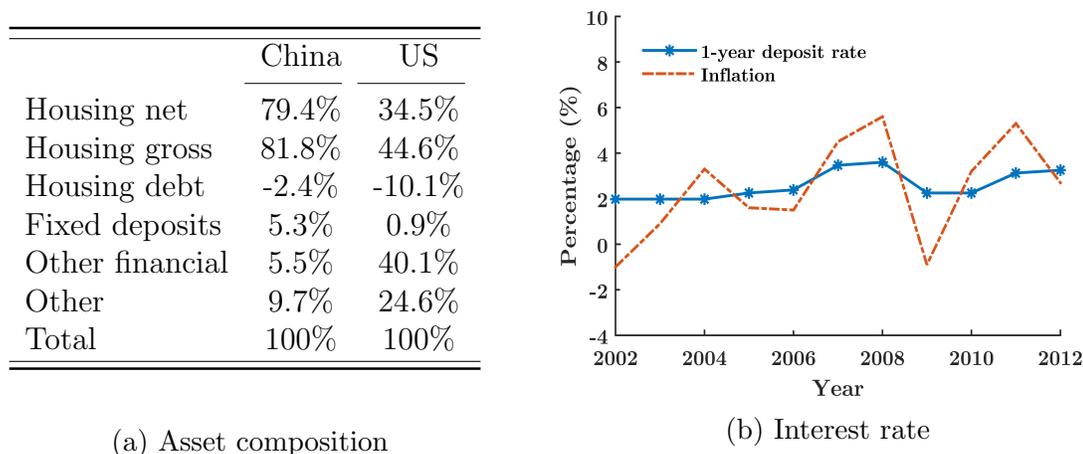
Notes: The statistics in Panel (a) are calculated by the author using the CFPS 2012 and the Survey of Consumer Finances (SCF) 2013. See Appendix 1.C for the detailed calculation procedure. In Panel (b), the 1-year deposit rate is from the People's Bank of China and the inflation rate is from the NBS of China.

Panel (a) in Figure 1.3.2 compares the components of total assets in 2012 between China and the United States. The share of housing (net of mortgages) in total assets in China is more than two times as big as that in the United States. Moreover, the share of financial assets in China is only about one fourth of the US share. Half of the financial assets in Chinese household portfolios are in terms of fixed bank deposits, which achieve low returns (sometimes even negative in real terms, as shown in Panel (b) in Figure 1.3.2). In contrast, the share of fixed deposits is negligible for US households.

Chinese households have good reason to save in housing and bank deposits: in fact, they have no good alternatives. First of all, the Chinese government implements strict capital controls so that saving in capital markets outside of China is off the table for most households. Second, domestic stock markets were underdeveloped and offered meager returns in the last two decades. As shown in Table 1.3.1, at least during the 2003–2013 period, stock market index returns are dominated by the housing price index according to a simple mean and variance comparison.

Table 1.3.1: Returns of Stock Market Index vs. Housing Price Index (2003–2013)

	Mean		Std. Dev.	
	Stock	Housing	Stock	Housing
2003-2013	0.073	0.157	0.515	0.154
2003-2008	0.090	0.204	0.662	0.105
2009-2013	0.053	0.109	0.339	0.191

Notes: The numbers in this table are directly extracted from Table 3 and Table 4 in Fang et al. (2015). The housing price index is for first-tier cities only. See Fang et al. (2015) for more details and discussion.

To sum up, the basic setup of the model (i.e., households choosing portfolios of housing and fixed return bonds) is well suited for the case study of China.

Another important feature of household finance in China is that Chinese households save a lot. Moreover, the average urban household saving rate in China rose significantly during most of the first decade of the 21st century (see Chamon and Prasad (2010), Yang et al. (2013), and Curtis et al. (2015)). The saving rate dynamics will also be one of the focus in the later quantitative analysis.

1.3.6.2 Tight credit conditions in the housing market

In China, housing became a commodity that individuals can purchase only after the early 1990s (before that local governments or work units allocated housing).[19] Commercial mortgages could rarely be seen until late 1990s. After mortgages are available, the minimum down payment requirement for first home purchases in China is strictly regulated by the People's Bank of China. It has changed over time but was never lower than two levels: 30% or 40%.[20] Mortgages to finance purchases of second homes can be subject to even higher down payment requirements.

Moreover, due to the lack of a official credit record system (no official credit record system exists until 2006), loan approval in China is very strict. The most important factor considered by the bank for mortgage approval is usually the applicant's income. Although there is no clear-cut loan to income criterion universally set for all the banks, guidelines from bank regulators usually require that a borrower's ratio of monthly mortgage payment to income should be lower than 50%.[21]

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[19]A more detailed discussion of the housing market privatization process in China can be found in Appendix 1.C.1.

[20]Down payment requirement in the US.

[21]For a comprehensive discussion of residential mortgages in China, see Fang et al. (2015).

Tight credit conditions also make China a good case to apply the theory (as explained in the basic model analysis, tight credit conditions work in favor of the mechanism).

Summary   The simple model in this section qualitatively explains how inequality and housing market outcomes can interact due to heterogeneity in cross-sectional portfolio allocation. It is of more interest to quantitatively evaluate how much the proposed theory can account for facts about portfolio allocation in a cross section as well as the dynamics of house prices and wealth inequality. For the quantitative analysis, this paper chooses the Chinese economy as a case study. This is because features of the Chinese economy map well into the model's key assumptions, as made evident by institutional backgrounds and stylized facts presented above.[22]

To make a quantitative test of the theory, in particular, to evaluate the model's ability to match the housing price and the saving rate trajectories given the cross-sectional household characteristics, a quantitative model equipped with more realistic features will be presented in the next section. The facts documented in this section will be used to inform the specification of the quantitative model in Section 1.4.1. Some of the facts presented in this section will also be used for the calibration and evaluation of the quantitative model in Section 1.4.2 and Section 1.4.3.

1.4   A Quantitative Analysis

The basic model identified a mechanism that make the equilibrium housing price positively respond to inequality (holding aggregate resources constant). In this section, I embed this mechanism into a richer model with heterogeneous dynastic households facing uninsurable idiosyncratic income shocks. I will demonstrate that the full model can quantitatively account for the 2002-2012 housing price run-up in China with realistic exogenous shocks, and it can also match the dynamics of other important aggregate variables such as saving rate and wealth gini.

1.4.1   Model setup

Compared to the basic model in Section 1.2, I make two main extensions in the quantitative model: households become infinite-lived and they face uninsurable id-

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[22]Although some institutional features are unique to China, the theory still generally applies to other economies, as long as sufficient frictions exist in the housing market. Factors such as institutional details that can affect the lumpiness of housing, financial market developments, and the inequality level will determine to what extent the mechanism will be effective.

iosyncratic earnings risks, which are shared by standard incomplete-market settings such as in Huggett (1993) and Aiyagari (1994). The main addition is a frictional housing market. Relative to standard incomplete markets models with heterogeneous households, the model in this paper has a frictional housing market equilibrium. The setup of the model will allow equilibrium house prices and rents to depend on the wealth distribution as well as its endogenous evolution.

In discussing the model environment, I focus on the extensions and give only brief consideration to the parts that are similar to those of the basic model. Moreover, I try to keep variable definitions and notations consistent with those in Section 1.2 whenever possible.

1.4.1.1   Household preferences and endowments

The setup is still an endowment economy, with no aggregate uncertainty and populated by two types of earners differing in income processes. Superscript $i \in \{B, T\}$ will be used to denote bottom and top earners, respectively. In addition to the basic model, there are preference shocks change household patience parameter periodically.[23]

Each type i earner is ex-ante identical and infinitely lived, with lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta_t^i u\left(c_t, s_t\right),$$

where β_t^i describes the cumulative discount factor between period 0 and period t, and I allow it to differ between bottom and top earners and leave it to the calibration to pick the respective values. In particular, $\beta_t^B = \tilde{\beta}^B \beta_{t-1}^B$, where $\tilde{\beta}^B$ is an idiosyncratic shock following a three-state, first-order Markov process. But $\beta_t^T = \tilde{\beta}^T \beta_{t-1}^T$, where $\tilde{\beta}^T$ is a standard constant one period discount factor. The instantaneous utility function $u\left(c_t, s_t\right)$ is modeled as a non-separable form (the basic model is a special case where $\sigma = 1$), following Kiyotaki et al. (2011) and others:

$$u\left(c_t, s_t\right) = \frac{\left(\left(\frac{c_t}{1-\varphi}\right)^{1-\varphi}\left(\frac{s_t}{\varphi}\right)^{\varphi}\right)^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}},$$

where σ is the risk aversion parameter and $\varphi \in (0, 1)$ again reflects the utility weight

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[23]This follows Krusell and Smith (1998) and allows the model to obtain rich aggregate wealth dynamics while respecting computational constraints.

of housing services in the consumption aggregate of c and s. Note that although the Cobb-Douglas preference has implications for the degree of substitutability between housing and non-housing consumption, it still produces the feature that the housing expenditure share is constant.[24]

Households in the economy also face idiosyncratic endowment shocks denoted as ε (the same for bottom and top earners), which are the same as unemployment shocks. In particular, ε follows a first-order Markov process with two states $\{0, 1\}$; $\varepsilon = 1$ means that the corresponding household is employed, receives a normal endowment, according to her type ($\bar{y}^B$ for a bottom earner and $\bar{y}^T$ for a top earner). If $\varepsilon = 0$, the household is unemployed, receiving a safety-net endowment that equals a fraction ω of the endowment at normal times. Note that the sum of the bottom earners' income always equals $(1 - \mu)Y$ and the sum of the top earners' income always equals μY.[25]

1.4.1.2   The household decision problem

Households in this model accumulate wealth to insure against idiosyncratic endowment shocks. They also make a portfolio choice between liquid wealth (bonds) and illiquid wealth (housing). In each period, households decision making not only depend on individual states but also the aggregate distribution. In particular, the state variables relevant to a type i earner's decision making include the individual state vector k_i, which includes (ε, $\tilde{\beta}^i$, b, h); the aggregate state variables Y, π, and Λ, where Λ denotes the measure of households over k^i and i; and the expectation of future individual states $k^{i\prime}$ as well as aggregate states Y', π' and Λ'.

Denote $V^i(\varepsilon, \tilde{\beta}^i, b, h; \Lambda, Y, \pi, H)$ as the current period value function for a type i earner, and the dynamic programming problem is the following:

$$V^i(\varepsilon, \tilde{\beta}^i, b, h; \Lambda, Y, \pi, H) = \max_{x, b', h'} u\left(x; R\right) + \tilde{\beta}^i E[V^{i\prime}(\varepsilon', \tilde{\beta}^{i\prime}, b', h'; \Lambda', Y', \pi', H') | \varepsilon, \tilde{\beta}^i]$$

---

[24]This feature is consistent with the empirical finding that, over time and across cities, the expenditure share on housing is constant (see Iacoviello and Pavan (2013)).

[25]Note that the endowment process is kept simple in the model, which does not take certain cross-section and life-cycle aspects of the income distribution into account. There are two main reasons for adopting this modeling approach. The first simply has to do with data availability. A panel data set with a close tracking of individual income would be needed to identify the persistence parameter in the income process. The second reason is that a more elaborate endowment process will affect not only inequality in earnings in the cross-section but also the individual earnings uncertainty. However, for the transition exercise in Section 1.4, I want to isolate the effect of changes in cross-section inequality from changes in individual earnings uncertainty. The current modeling approach for the endowment process better fits that purpose.

s.t.

$$x + b' + Ph' + \Phi(h', h) = \varepsilon\bar{y}^i + (1 - \varepsilon)\omega\bar{y}^i + (1 + r)b + R'h + (1 - \delta)P'h$$
$$b' \geq \bar{b}(\lambda_h, h', \lambda_y, \varepsilon)$$
$$h' \in \{0, [\underline{h}, +\infty)\},$$

for $i \in \{B, T\}$.

$\bar{b}(\lambda_h, h', \lambda_y, \varepsilon)$ denotes the borrowing constraint. In addition to the loan to value ratio criterion in the basic model, there is an additional loan to income criterion. In particular, not only are households unable to borrow more than a fraction λ_h of their housing stock, they cannot borrow more than a fraction λ_y of their expected earnings either; i.e.,

$$\bar{b}(\lambda_h, h', \lambda_y, \varepsilon) = \max\{-\lambda_h Ph', -\lambda_y \mathbb{Y}(\varepsilon, \bar{y}^i)\},$$

where $\mathbb{Y}(\varepsilon, \bar{y}^i) = E[\sum_{k=0}^{N} \frac{y_k}{(1+r)^k}|\varepsilon, \bar{y}^i]$ approximates the expected present discounted value of lifetime endowment. The endowment at period k is denoted as y_k, and N is the approximate length of a life-cycle. This constraint is consistent with the usual lending criteria in the mortgage market that take into account minimum down payments, ratios of debt payments to income, and current and expected future employment conditions.

$\Phi(h', h)$ denotes housing adjustment costs. Unlike in the basic model, a homeowner incurs a cost $\Phi(h', h)$ whenever she adjusts her housing stock:

$$\Phi(h', h) = f_1 P|h' - h| + f_2 \mathbb{1}_{\{h' \neq h\}} P\underline{h}. \tag{1.4.1}$$

Note that there are both linear and fixed components in the adjustment cost function. The linear component $f_1 P|h' - h|$ captures common practices in the housing market that require, for instance, commissions paid to realtors to be equal to a fraction of the value of the house being sold. The fixed component $f_2 \mathbb{1}_{\{h' \neq h\}} P\underline{h}$, which is in terms of a percentage value of the minimum size house, captures other costs associated with housing transactions, such as registration fees and search costs.[26]

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[26] By paying the cost, housing stock can be adjusted to any future level $h' \in \{0, [\underline{h}, +\infty)\}$. The assumption that the fixed cost component applies to adjustments that are minor relative to the existing housing stock essentially prevents homeowners from making small improvements to their houses.

1.4.1.3 Definition of equilibrium

Denote C as the aggregate non-housing consumption and denote B as the aggregate bonds savings. Given a starting distribution of households Λ_0 and a sequence of aggregate state variables $\{Y_t, \pi_t, H_t\}$, a competitive equilibrium is then defined as: sequences of individual value and policy functions $\{V_t^i(\varepsilon, \tilde{\beta}^i, b, h; \Lambda_t, Y_t, \pi_t, H_t), b_t^{i'}(\varepsilon, \tilde{\beta}^i, b, h; \Lambda_t, Y_t, \pi_t, H_t), s_t^{i'}(\varepsilon, \tilde{\beta}^i, b, h; \Lambda_t, Y_t, \pi_t, H_t), \text{ and } h_t^{i'}(\varepsilon, \tilde{\beta}^i, b, h; \Lambda_t, Y_t, \pi_t, H_t)\}$; pricing functions $P(\Lambda_t, Y_t, \pi_t, H_t)$ and $R(\Lambda_t, Y_t, \pi_t, H_t)\}$, such that, at each period t,

1. Given V_{t+1}^i, $\{V_t^i, b_t^{i'}, h_t^{i'}\}_{i\in\{B,T\}}$ solve household's problem.

2. Markets clear at all times.

   (i) Housing market: $\int_{\Lambda_t} \sum_i h_t^{i'}(\varepsilon, \tilde{\beta}, b, h; \Lambda_t, Y_t, \pi_t, H_t) = H_{t+1}$

   (ii) Rental market: $\int_{\Lambda_t} \sum_i s_t^{i'}(\varepsilon, \tilde{\beta}, b, h; \Lambda_t, Y_t, \pi_t, H_t) = H_{t+1}$

   (iii) Bond market: $\int_{\Lambda_t} \sum_i b_t^{i'}(\varepsilon, \tilde{\beta}, b, h; \Lambda_t, Y_t, \pi_t, H_t) = B_{t+1}$

   (iv) Goods market: $\int_{\Lambda_t} \sum_i \Phi(h_t^i, h_{t+1}^i) + \delta P_t H_t + C_t = Y_t + r_t B_t$

3. Λ_{t+1} is generated by $\{b_t^{i'}, h_t^{i'}\}_{i\in\{B,T\}}$ and exogenous processes ε and $\tilde{\beta}^i$.

Note that the formula $\int_{\Lambda_t} \sum_i \Phi(h_t^i, h_{t+1}^i)$ summarizes the total transaction costs incurred by homeowners for adjusting the housing stock, which depends on both the total transaction volume and the transaction frequency (see Equation [1.4.1]). Therefore, according to the goods market clearing condition (2iv), prices P_t and R_t depend on both the transaction volume and the transaction frequency of housing stock. This observation has implications for the computation of the model.

Moreover, the analysis will focus on transition path resulting from temporary shocks. Accordingly, I assume that exogenous state is consistent with the economy eventually reaching a stationary equilibrium where aggregate state variables are all time-invariant, i.e., we only consider cases when $\{Y_t, \pi_t, H_t\} = (\bar{Y}, \bar{\pi}, \bar{H})$ after a finite number of periods.

1.4.1.4 Computation strategy

Since the considered economy will always reach stationarity after in finite periods, the solution starts with solving the stationary equilibrium associated with $(\bar{Y}, \bar{\pi}, \bar{H})$. With stationarity, the equilibrium conditions in the above section imply an economy with a stationary distribution Λ^* in which households behave optimally and markets always clear with constant house price and rental price P^* and R^*. Note that, to

solve their individual problems, households in the stationary economy only need to know market prices and returns for both bonds and housing. Since P^* and R^* are sufficient statistics to determine the return on housing, the problem is reduced to finding the pair of prices P^* and R^* that can clear all the markets when households all behave optimally. Details on how to solve the stationary equilibrium can be found in Appendix 1.D.1.

The next step is to find how the economy will evolve before entering stationarity (on the transition path). Note that solving the household optimization problem along the transition path requires adding time to the state variables listed in the steady-state problem described earlier in the paper because both current-period states and future states affect households' optimal decisions. The computation is still done by a fixed point iteration procedure. Unlike in the steady-state equilibrium, instead of iterating over a set of stationary aggregate variables, the algorithm now iterates over the sequences of prices P_t, bond savings B_t, transaction volumes Ω_t, and transaction frequencies F_t along which the optimal decisions of households clear all the three markets. Given a sequence of those aggregate variables, the dynamic programming problem can be solved recursively, moving backward in time from time period T. Details of the computational procedure can be found in Appendix 1.D.2.

1.4.1.5 Household decision rules

As discussed in the basic model in Section 1.3.5, the cross-sectional difference in housing demand responsiveness is the key to generating an interaction between housing market outcomes and inequality. Therefore, I show next to what extent the quantitative model features this differential responsiveness in housing demand.

I first discuss the housing decision for non-homeowners, for whom bond is the only financial asset to begin with. Their decision rule is similar to that in Figure 1.3.1: there is a threshold amount (depending on the exact household type) of liquid assets such that, if assets exceed the threshold, non-homeowners become first-time home-owners. Also, the larger the initial liquid assets are, the less likely a household is to borrow for the purpose of financing a housing purchase.

As for homeowners, Figure 1.4.1 from the baseline calibration in the next section plots their housing adjustment decisions as a function of initial housing and liquid wealth, across 4 different types, employed or unemployed, top earners or bottom earn-ers (holding time preference parameter constant at its middle value). A homeowner can stay put, upgrade, downsize, or completely sell off her housing stock (thus exiting the housing market).

Figure 1.4.1: Housing adjustment functions

(a) Employed bottom earners

(b) Employed top earners

(c) Unemployed bottom earners

(d) Unemployed top earners
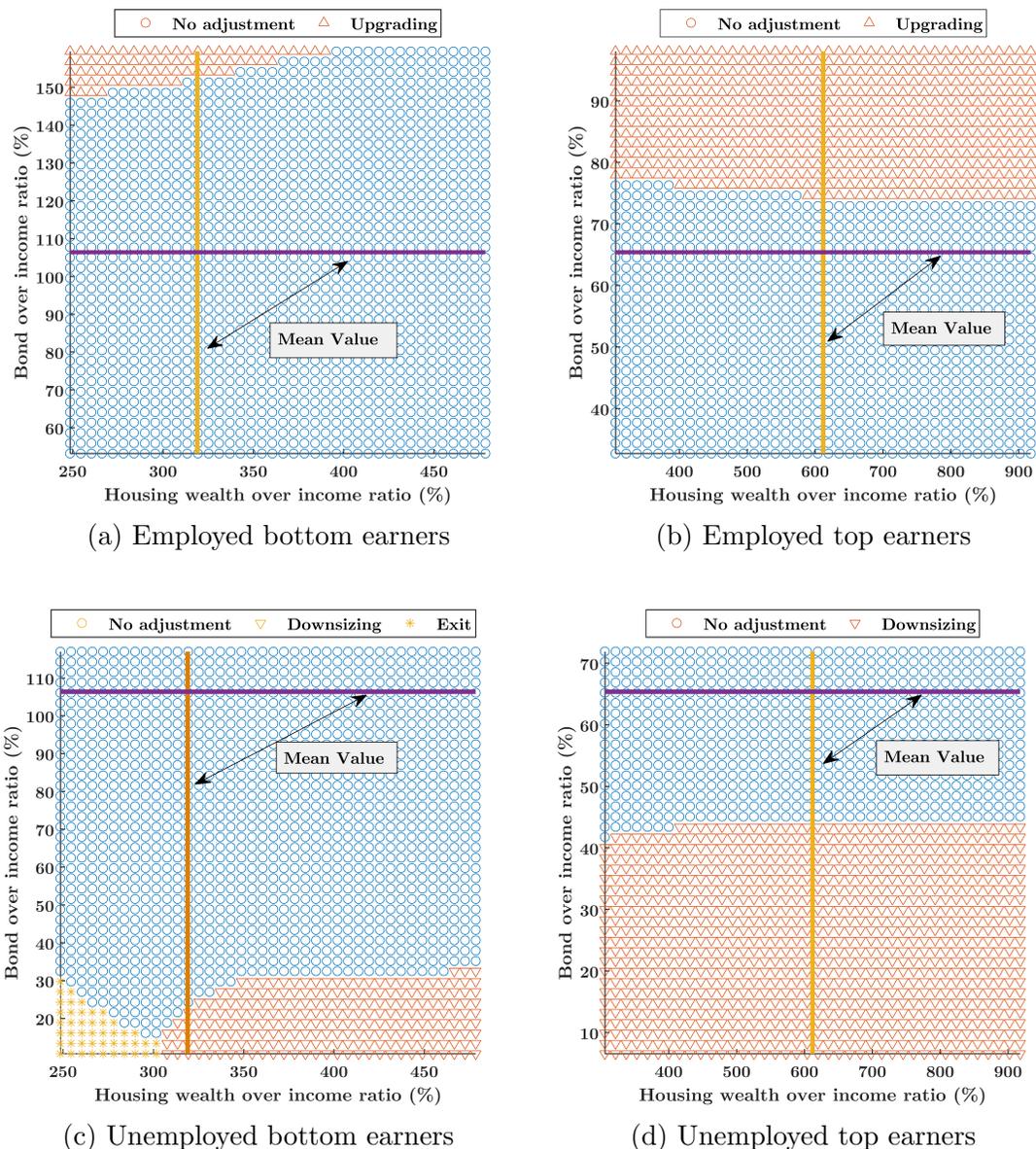
Notes: The figure illustrates the equilibrium optimal housing adjustment policy as functions of initial housing wealth and bond wealth. The plots in Panel (a) and Panel (c) correspond to a medium patient bottom earner ($\tilde{\beta} = \tilde{\beta}_m$).

One key feature of the decision rules is that bottom earners have much larger inaction zones – i.e., unresponsive housing demand; while top earners adjust their housing stock much more frequently – i.e., responsive housing demand. The unresponsive housing demand from the bottom earners is also reflected in the feature that when the amount of liquid assets is small, the housing tenure decision depends

on the initial level of housing wealth non-monotonically. Consider, for instance, an unemployed homeowner with liquid assets equal to about thirty percent of annual income (i.e., a bottom earner). If the initial house value is really small, the homeowner pays the adjustment cost and, because of his low liquid assets, completely quits the housing market. If the initial house value is in the middle range, the homeowner does not change house size since, given the modest size, quitting the housing market is too big an adjustment, while downsizing a bit is not economical given the sizable fixed adjustment costs. If the initial house size is large, there is enough room to optimally downsize the housing stock instead of completely selling it off.

Moreover, across all types of households, larger liquid assets increase the chance of upgrading one's housing stock, and transaction costs create a region of inaction where the housing stock is constant. If liquid wealth falls below a certain level, the household either downsizes or exits the housing market. Moreover, when a household does adjust her housing stock, the size of the adjustment also depends on her liquid wealth level.

1.4.2 Calibration with data before 2002

Parameters of the model are calibrated using data from the Chinese economy before 2002, and the calibrated model are then used to study the dynamics of the economy between 2002 and 2012.

The calibration involves mainly two stages. The first stage is to set the parameters that can either be directly observed or be estimated from the data. Important parameters in this category includes, individual income process, employment process, and housing market frictions.

The second stage is to calibrate the unobservable parameters in the model: preference parameters mostly. The model is first solved with the assumption that the the Chinese economy is at a stationary equilibrium in the year of 2002.[27] The preference parameters are then chosen such that the cross-sectional properties on household assets of household finances in the model best matches that of the data in the year 2002.

1.4.2.1 Calibration results

Table 1.4.1 summarizes the parameters used in the baseline model (the estimated and the calibrated parts). Detailed descriptions of the calibration procedure follow

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[27]Alternative calibration without the stationarity assumption will be considered in future sensitivity analysis.

in the main text.

Table 1.4.1: Summary of Parameters

Description	Parameter	Value	Source/Target
Conventional values			
Relative risk aversion	σ	1.00	Standard value
Total endowment	Y	1.00	Normalization
Total housing stock	H	1.00	Normalization
Estimated from the data			
Risk free interest rate	r	0.02	People's Bank of China
Top 10% wage share	π	0.26	Top 10% wage share[a]
Transition matrix of employment process	$\Pi_{\varepsilon,\varepsilon'}$	-	Giles et al. (2005)
Mean endowment for bottom earner	y^B	0.90	Derived from $\Pi_{\varepsilon,\varepsilon'}$
Mean endowment for top earner	y^T	2.86	Derived from $\Pi_{\varepsilon,\varepsilon'}$
Unemployment replacement rate	ω	0.07	China Labor Statistical Yearbook
Minimum size housing	$\underline{h}$	0.57	First quartile value[a]
Borrowing constraint against housing	λ_h	0.60	Down payment requirement
Linear transaction cost	f_1	0.03	Association of Realtors
Calibrated in the model			
Depreciation rate	δ	0.03	Wu et al. (2012)
Utility share of housing service	φ	0.24	Bond share in total: 0.23[a]
Discount factor for top earners	$\tilde{\beta}^T$	0.97	Top 10% asset-over-income: 10.5[a]
Medium discount factor for bot earners	$\tilde{\beta}_m^B$	0.96	Aggregate asset-over-income: 5.4[a]
Discount factor variation for bot earners[b]	$\Delta\tilde{\beta}^B$	0.01	Wealth gini: 0.48[a]
Transition matrix of discount factor	$\Pi_{\tilde{\beta},\tilde{\beta}'}$	-	See main text
Borrowing constraint against income	λ_y	0.10	Aggregate loan-to-value: 0.05[a]
Fixed transaction cost	f_2	0.12	Housing-transaction-over-GDP: 0.06

[a]: Calculated using CHIP 2002. See Appendix 1.C for detailed procedure.
[b]: The discount factor grids are symmetric; i.e., $\Delta\tilde{\beta}^B = \tilde{\beta}_h^B - \tilde{\beta}_m^B = \tilde{\beta}_m^B - \tilde{\beta}_l^B$.

Endowments Both output and the housing stock are normalized to be 1. I set π to 26%, according to the top 10% wage income share in CHIP 2002. Wage income is the data counterpart for the exogenous endowment in the model. The reason to choose the top 10% as the top earners is because the income share of the top 5% in China is too low compared to the other countries in Panel (a) in Figure 1.1.1 (the calculated top 5 percent income share in China at 2002, while it is 30% at the same time in the United States). The normal endowment levels for bottom earners and top earners, $\bar{y}^B$ and $\bar{y}^T$ respectively, are then set to meet the conditions that $(1-u)\bar{y}^B + u\bar{y}^B = (1-\mu)Y$ and $(1-u)\bar{y}^T + u\bar{y}^T = \mu Y$, where u denotes the aggregate unemployment rate and $\mu = 0.1$.

The employment process $\Pi_{\varepsilon,\varepsilon'}$ is chosen so that the average duration of an unemployment spell is two years and the unemployment rate u is 10%, which is roughly in line with the findings about China's unemployment rate in Giles et al. (2005). The transition matrix of the employment status is as follows (rows indicate the current

state, and columns indicate next period's state; both the first row and the first column correspond to $\varepsilon = 1$ – i.e., employment):

$$\begin{bmatrix} 17/18 & 1/18 \\ 1/2 & 1/2 \end{bmatrix}.$$

The unemployment insurance replacement rate ω is set to be 7%. This level of unemployment insurance is in line with the data. According to the China Labor Statistical Yearbook 1999 to 2005, the overall replacement rate from unemployment insurance is about 14% of the worker's wage, and the maximum duration of benefits for unemployment insurance recipients is 2 years (the same as the unemployment spell in the model). However, the China Labor Statistical Yearbook also reports that about half of all unemployed workers were eligible for unemployment benefits from the unemployment insurance system from 1999 to 2005. Thus, I set ω to be 7% (half of the 14%), the same for both the top and the bottom earners.

Preferences The baseline risk aversion parameter is set at $\gamma = 1$. The utility share of housing service φ is set at 0.24 to match the housing wealth and bonds wealth split among all households in CHIP 2002. I calibrate $\tilde{\beta}^T$ to be 0.9657 according to the net asset-over-income ratio (10.5) of the population's top 10% wealthiest. The shock to the patience level of bottom earners, $\tilde{\beta}^B$, takes on values from a symmetric grid, ($\tilde{\beta}_l^B = 0.9427, \tilde{\beta}_m^B = 0.9527, \tilde{\beta}_h^B = 0.9627$), with 80% of the bottom earners adopting the middle value and 10% each adopting the extreme points in the invariant distribution. The expected duration of the extreme discount factors is 50 years. As in Krusell and Smith (1998), this is meant to capture, albeit in a somewhat crude way, a dynastic element in the evolution of preferences. Transitions can only occur to adjacent values, where the transition probability from either extreme value to the middle grid is 1/50 and from the middle grid to either extreme value is 1/400. This Markov chain for $\tilde{\beta}^B$ has been chosen to match the aggregate wealth-over-income ratio (5.4) and the Gini coefficient (0.48) of the wealth distribution. The transition matrix of $\tilde{\beta}$ is as follows (rows indicate the current state, and columns indicate next period's state; both the first row and the first column correspond to $\tilde{\beta} = \tilde{\beta}_l$; both the second row and the second column correspond to $\tilde{\beta} = \tilde{\beta}_m$):

$$\begin{bmatrix} 49/50 & 1/50 & 0 \\ 1/400 & 199/200 & 1/400 \\ 0 & 1/50 & 49/50 \end{bmatrix}.$$

Market arrangements The risk-free interest rate r is set as the one year bank deposit rate in 2002 taken from the People's Bank of China. The minimum-size house ($\underline{h}$) is set according to the housing asset value at first quartile among all the homeowners in CHIP 2002. This value is roughly 2 times the average annual wage income among all the homeowners in CHIP 2002. The maximum loan-to-value ratio restriction is set at $\lambda_h = 0.60$, corresponding to a constant down payment requirement of 40%.[28] The maximum loan-to-future-income ratio restriction λ_y is chosen to be 0.10.[29] The two borrowing requirements in the model together determine the overall tightness of credit conditions in the model, generating an aggregate loan-to-value ratio of 5% consistent with CHIP 2002.

Note that the mortgage option setup in the model allows households to draw on a home equity line of credit (subject to a loan to income criterion). Thus, loans are essentially payment-option mortgages with a required interest rate payment and a pre-approved home equity line of credit. In principle, this means that a borrower can choose to only cover mortgage interest payments but not to pay down principal every period. However, mortgage loans in China are all installment loans: equal installments of the loan must be paid each period before maturity, although early retirement is allowed. It turns out that this is not a problem for the model. Households usually do borrow when they make new housing purchases, but they pay back their debts gradually afterward. This is because unemployment shocks and sizable adjustment costs in housing assets induce households to desire a certain amount of liquid wealth buffer. Moreover, the tight loan to income criterion further dampens the effect of a home equity line of credit since the borrowing constraints become much tighter when households are unemployed.

The depreciation rate of the housing asset δ is set to 0.03, to approximate both the maintenance and depreciation cost of holding housing wealth.[30] I choose $f_1 = 0.03$ to roughly approximate the amount of commissions for realtors when transacting houses,

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[28]The minimum down payment requirement for first home purchases in China is strictly regulated by the People's Bank of China. It has changed over time but within a narrow band between two levels: 30% or 40%. Mortgages to finance purchases of second homes can be subject to even higher down payment requirements.

[29]In China, the most important factor considered by the bank for mortgage approval is usually the applicant's income (no official credit record system exists until 2006). However, there is no clear-cut loan to income criterion universally set for all the banks. Guidelines from bank regulators usually require that a borrower's ratio of monthly mortgage payment to income should be lower than 50%. For a comprehensive discussion of residential mortgages in China, see Fang et al. (2015).

[30]There is no direct source to calculate the depreciation rate of housing in China. Previous literature routinely assumes a depreciation rate of 2.5%–3%. See Wu et al. (2012) and Chivakul et al. (2015), for example.

and I calibrated $f_2 = 0.12$ to target the housing transaction volume-over-GDP ratio in 2002 calculated according to data from National Bureau of Statistics of China. Overall, the adjustment cost function is used to approximate various realistic costs associated with transacting houses in a simplified way.[31] The setup serves the purpose of generating lumpiness in housing that has a differential impact across wealth groups. This feature is essential for the mechanism in the paper.

1.4.2.2   Calibration validity check

In order to check the performance of the calibration, I analyze the match of several important untargeted data moments, the distribution of wealth and its composition in particular.

Wealth distribution   Table 1.4.2 examines the wealth distribution in 2002 produced by the model. Compared to the data, the model matches almost all the moments (only gini coefficient was explicitly targeted in the calibration).

Table 1.4.2: Comparison of distribution of wealth

	% of wealth held by top					Fraction with	Gini
	1%	5%	10%	20%	30%	wealth< 0	coefficient
Model (baseline)	10%	24%	36%	55%	66%	1%	0.48
Data (CHIP 2002)	8%	23%	35%	53%	64%	2%	0.48

Notes: The wealth distribution in the data is calculated from CHIP 2002. Appendix 1.C contains more details about the calculation procedure.

Note that, when compared to the US data, wealth concentration is at a much lower level in China.[32] Although this might be a reflection of the true inequality level difference between the two economies, it might also due, at least in part, to the underrepresentation of the top richest households in CHIP 2002.[33] However, those at the top who are missing from the survey data are unlikely to hold mainly housing in

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[31]There are more than 20 different fees and taxes associated with housing transactions in China. Moreover, fees and taxes differ substantially according to the type of transaction and ownership rights (houses in China have different types of ownership rights due to institutional reasons). Details of those fees and taxes can be found at `http://esf.fang.com/zt/201403/esfjysfmx.html` (in Chinese, provided by a *NYSE* listed private realtor company *SouFunHoldings*).

[32]According to the SCF 2001, the net worth shares of the top 1 and 5 percent wealthy households in the US are 32% and 57%, respectively.

[33]Xie and Jin (forthcoming) find that, after supplementing CFPS 2012 with data from the China Rich List reports, wealth concentration in China is comparable to that of the United States, at least according to measures of top wealth shares.

their portfolios since they tend to have access to assets with better returns.[34] Thus, overlooking the richest households here is unlikely to affect the results in this paper.

Figure 1.4.2: Comparison of cross-sectional asset composition
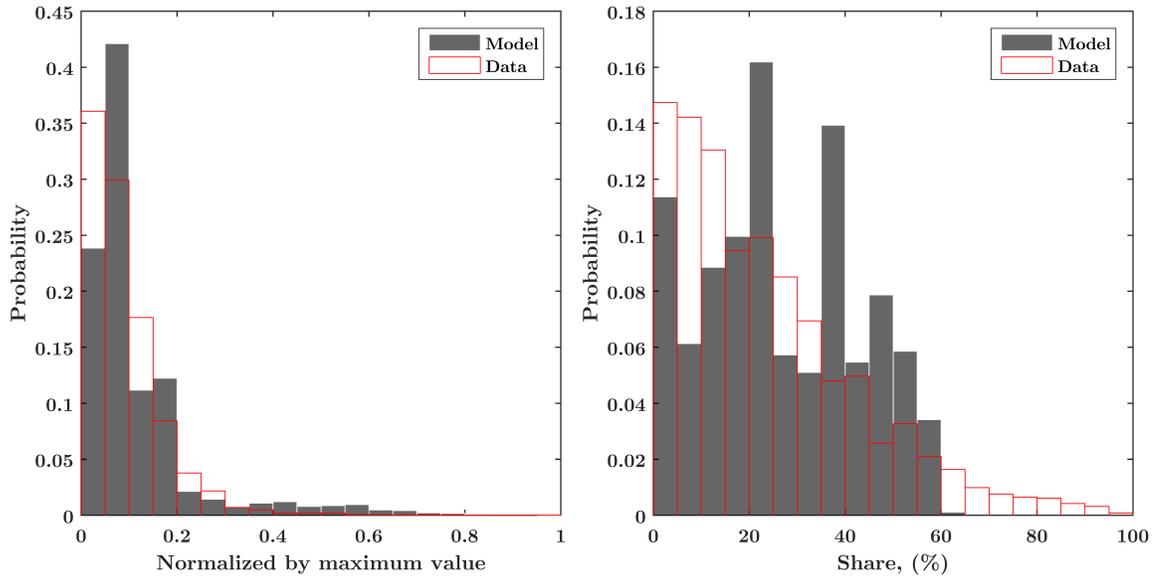
(a) Housing-wealth-over-income ratio   (b) Bond wealth share among homeowners

Notes: Panel (a) plots the distributions of net housing-wealth-over-income ratio both in the model and in the data. Each distribution is normalized by dividing its maximum value, so that the range is between 0 and 1. Panel (b) plots the distributions of bond wealth share in total wealth among homeowners. To be consistent with the model, bond wealth in the data is defined as fixed bank deposits, and total wealth is defined as bank deposits plus net housing wealth. The two data distributions in both panels are obtained from CHIP 2002.

Cross-sectional asset composition   Since the difference in cross-sectional portfolio choices is central to the paper's mechanism, it is important to examine how well the model matches with the real portfolio composition in the data. Panel (a) in Figure 1.4.2 shows that the model does a good job of capturing the fact that most households own similar housing wealth, a manifestation of the minimum size friction. Panel (b) in Figure 1.4.2 shows that liquidity conditions vary a lot among homeowners, both in the data and in the model. Households in the model ideally desire to hold a balanced composition between liquid wealth and housing wealth. Although housing assets earn a higher return, liquid wealth is more suitable for the purpose

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[34]The rate of return to capital has stayed comfortably over 20% throughout 1993 to 2005, according to Bai et al. (2006). Although these high corporate returns are not available to normal households, the wealthiest do have access to them.

of smoothing temporary income shocks. The model can produce a wide range of liquidity conditions because, due to the adjustment costs, households opt for depleting liquid wealth when making new housing purchases, and gradually accumulating it back to the ideal level afterward.

1.4.3 The transition after 2002

In this section, I conduct the following transition experiment: starting from the calibrated economy in 2002 (as calibrated in Section 1.4.2) as Λ_0, I set aggregate income Y_t and top income share π_t to be the same as the data between 2002 and 2012 and to stay constant after 2012.[35] I then solve the transition equilibrium. The main goal of this transition exercise is to quantitatively evaluate how much the proposed mechanism in this paper can account for the dynamics of house prices in China, along with the dynamics of aggregate saving as well as wealth inequality (in terms of wealth gini).

1.4.3.1 Results

In this section, I study the economy's transition from 2002 until 2012. I particularly focus on the outcomes of three aggregate variables: house price, wealth inequality and aggregate saving rate.

In the three panels I present below in Figure 1.4.3, the starting point is the initial 2002 steady state; the solid line denotes the transition path outcomes of the baseline model, the dotted line with circles denotes the data, and the dashed line denotes the transition path outcomes of a counterfactual model where income growth is taken from the data but the inequality stays at its 2002 level. This simulation separates the contributions of the two aggregate shocks, informing the role played by inequality shocks. Note that I only present the transitional dynamics until 2012, since the economy pretty much starts monotonically converging to the new steady state by then and the transition to new steady state afterward is very gradual.[36]

The main object of interest is the evolution of house prices. In particular, the central question of the paper is whether the large-scale housing price appreciation can be accounted for by the relatively parsimonious model setup within a rational expectation equilibrium framework. Panel (a) in Figure 1.4.3 shows the model's prediction for house price dynamics along the transition path (solid line), presented

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[35]Alternative experiments will be considered in future sensitivity analysis.

[36]The steady-state aggregate variables are close to their levels in the year 2020 although the full transition will take about 20 more periods forward.

as the aggregate price-over-income ratio relative to the initial steady state. Note that aggregate income in the model is assumed to grow at 8% annually between 2002 and 2012, and at 0% afterward in the baseline setting.

The results show that the amplification mechanism of the baseline model is sufficient for rationalizing a large run-up of house prices. In particular, the growth rate of house prices in the baseline model is significantly higher than the growth rate of income during early periods of the transition and then gradually drops below income growth and even becomes negative toward the end of the time period examined. The existing data show a similar price-over-income trend in the first phase of the model outcome although the data trend begins less abruptly and is sustained over a longer period than that in the model.[37] As will be discussed in the next section, this difference might be reconcilable by incorporating more realistic features such as continued income growth or urbanization.

Panel (b) and Panel (c) in Figure 1.4.3 show that the hump-shaped house price-over-income dynamics is accompanied by similar patterns in the evolutions of wealth inequality and the aggregate saving rate in the baseline model.[38] Again, the outcomes of the baseline model are a bit more dramatic than the data counterpart, for similar reasons as argued above. Overall, the results in Figure 1.4.3 show that the baseline model can roughly account for both the trend dynamics and the changes in magnitude for all the three aggregate variables in consideration.

The role played by the inequality shocks can be seen by comparing the outcomes of the baseline model and those of the counterfactual model. In all the three panels in Figure 1.4.3, the results of the counterfactual model are qualitatively similar to those of the baseline model, but are much mitigated in the absolute levels. This suggests that the strength of the mechanism at work in this model crucially depends on the evolution of the inequality measure in the economy, which is largely controlled by the top income share π in the model. Thus, the inequality shocks fed into the baseline model are not only empirically relevant but also important for amplifying the extent of the housing market responses and matching them with the observed magnitude of house price run-ups.

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[37]I focus on the data trend here since the raw data is volatile, especially due to the interruption of the 2008 economic crisis, while the model outcomes are smooth since I only consider a one-time period of anticipated temporary shocks.

[38]In the baseline model, wealth inequality only falls a bit after the early surge and remains at a much higher level than the initial steady state afterward. This is not the case for the saving rate and the price-over-income ratio. The is because the baseline model assumes that the increase in top income share π_t is permanent. In the absence of permanent π_t shocks, wealth inequality in the counterfactual model also returns to its original steady-state level eventually.

Figure 1.4.3: Transitional dynamics
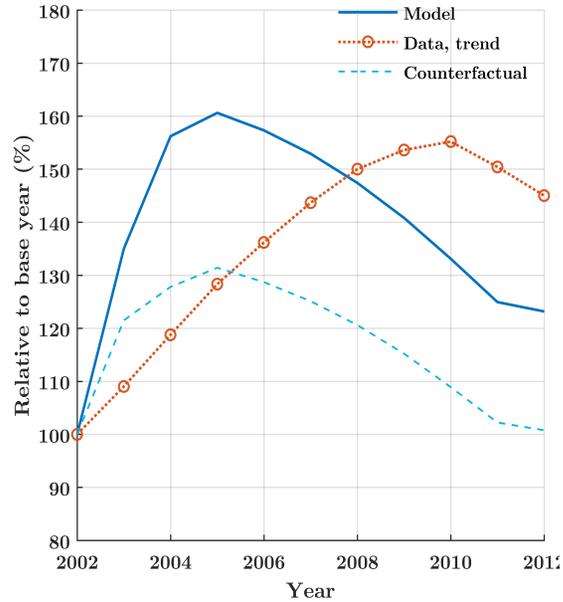
(a) Aggregate housing-price-over-income ratio
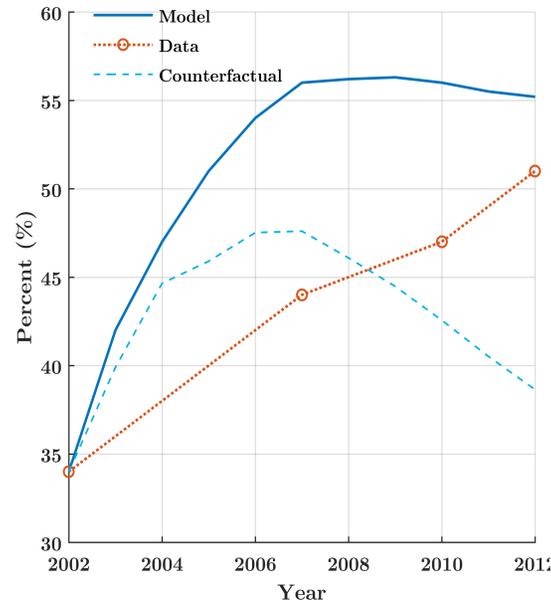
(b) Top 10% wealth share
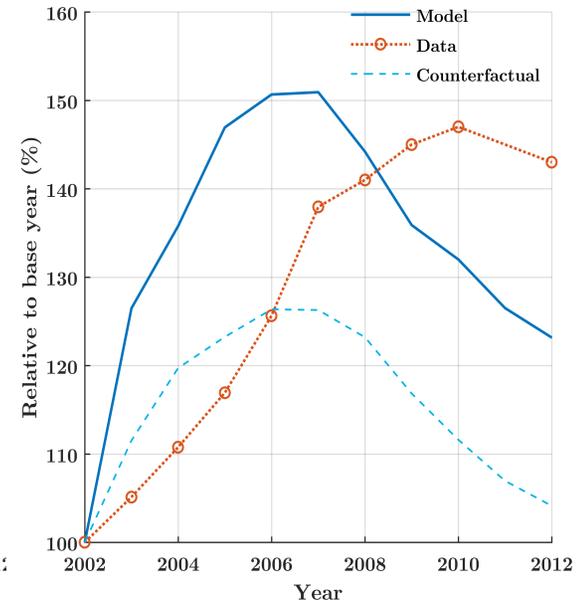
(c) Aggregate saving rate

Notes: The price-over-income data trend is taken from Panel (b) of Figure 1.1.1. The data in Panel (b) is calculated by author from the same sources as those in Panel (b) of Figure 1.1.1. See Appendix 1.C for more details. The data before 2009 for Panel (c) is taken from Yang et al. (2013), where the saving rates are calculated from Urban Household Surveys conducted by National Bureau of Statistics of China. For the years 2010 and 2012, the saving rates are calculated by author from the same sources as those in Panel (b) of Figure 1.1.1. Note that the model does not attempt to explain the level of the saving rate in the data (due to its fairly simplified assumptions); so only the relative changes in the saving rates are compared here.

1.4.3.2   Mechanisms

In the following, I investigate the mechanism behind the baseline model outcome. As discussed in earlier parts of the paper, one key feature of the model is the differential responsiveness in housing demand across wealth groups. The differential responsiveness turns out to be crucial for the transitional analysis, too. In particular, it implies an endogenous feedback loop between house prices and wealth inequality. It is exactly this feedback loop that causes house prices to overreact to temporary shocks. The general intuition about the mechanism is the following.

The feedback mechanism is initiated by expected income growth, which will ultimately lead to house price appreciation. Due to potential capital gains, a period of high premium in housing is thus anticipated among households. In response, the most responsive wealthy households stock up on housing wealth initially, causing a surge in house prices. Due to the skewed housing wealth distribution, the price surge causes a higher dispersion in capital income and thus higher wealth inequality follows. Higher wealth inequality, in turn, helps sustain the growth in house prices. This is because, while housing demand from wealthy households saturates, demand from households with modest wealth starts to pick up after a short delay. Even with a high premium, transaction frictions and liquidity constraints still limit less-wealthy households' response in the housing market. Accordingly, the inequality gap between wealthy housing investors and the rest continues to widen as house prices keep creeping up. This creates a self-reinforcing feedback loop between house price and wealth inequality.

The loop ends eventually, when most of the capital gains (due to income growth) in the housing market are realized. The return premium sinks close to its steady-state level, wealthy households begin to draw down their over-sized housing stock, and at about the same time, households that initially had meager resources are able to enter the housing market after continued saving. Housing wealth becomes more dispersed as house prices adjust back to trend.

Figure 1.4.4 plots the transitional dynamics for the housing premium and the buyer distribution in the baseline model, to help confirm the above intuition. According to the top graph in Figure 1.4.4, the housing premium stays high only for early periods when there are high capital gains in housing. Although the high premium dwarfs the transaction costs in housing, early buyers who are able to enjoy the whole high return periods are still mostly households from the top wealth tertile. This is because tight borrowing limits and a high entry barrier prevents households with low wealth from responding quickly. Therefore, most of the initial housing demand comes from a

portfolio shift from bonds to housing among the responsive wealthy. Households with relatively plentiful resources can tap into some of the high returns in the housing market during later periods of the high return phase, with modest saving efforts. Households with initial low resources (mostly non-homeowners) are forced to save more and wait longer to enter the housing market.

Figure 1.4.4: Housing premium and buyer distribution during transition
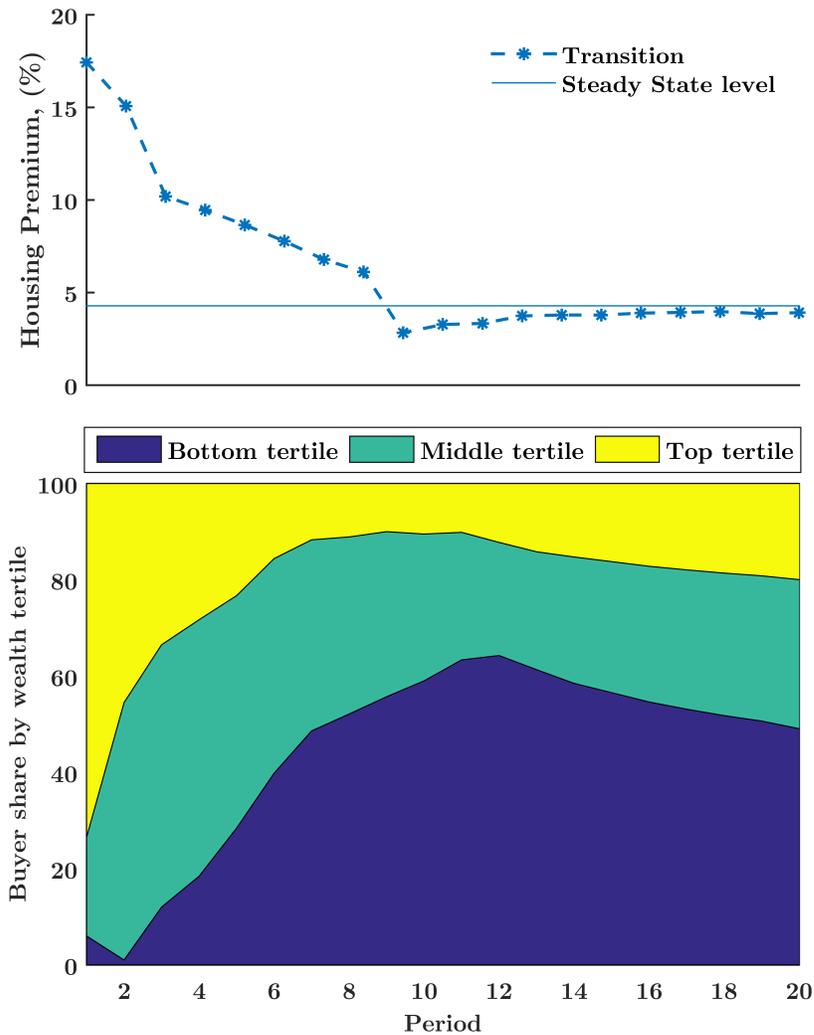
Notes: The housing premium in period t is calculated as $\frac{R_t}{P_t} + \frac{P_t}{P_{t-1}} - \delta - r$. Each tertile buyer share in period t is obtained by accruing all the households who purchased housing assets (either by upgrading or making first-time purchases) in period t according to wealth tertile in period t.

The transition dynamics of house prices and the buyer distribution make it clear that aggregate saving can increase due to a strong saving motive for housing purchases

from households with few resources. However, the initial sharp rise in aggregate saving is also partly due to the portfolio re-balancing behavior of the wealthy. As explained earlier, households in the model desire a certain level of liquid wealth. After the initial portfolio shift toward housing wealth, while waiting for the capital gain in housing asset to fully realize, the wealthy need to save up more liquid wealth since it is still the better form of buffer against idiosyncratic income shocks.

In sum, the model rationalizes a house price run-up relative to income growth and its adjustment back to trend in a perfect foresight equilibrium with one-time shocks. It also explains increased wealth concentration and a rising aggregate saving rate. The mechanism links portfolio choices of the wealthy and the poor with house price dynamics and expectations, stressing the role of responsive investment behavior among wealthy households.

1.4.3.3 Discussion

In this section, I discuss some of the missing elements in the baseline transition exercise, additional sensitivity analyses, and potential extensions.

Other demand factors The baseline transition exercise only considers one-time anticipated income shocks (both growth and inequality), abstracting from changes in many other demand factors. Although this choice makes our results clean for interpretation, other demand factors might play important roles in the housing boom. We briefly discuss the implications of other demand factors here.

In particular, this paper does not consider changes in credit conditions.[39] This choice is consistent with the empirical evidence. Although the amount of mortgage borrowing did increase over the years in China, the regulation has not changed much, in the sense that both the mortgage interest rates and down payment requirements vary within a relatively small range (see Fang et al. (2015) for more details). One might worry that borrowing from the shadow banking sector might mean that some lenders actions may work independently from official regulations. However, according to the two sources of household survey data analyzed in this paper (CHIP 2002 and

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[39]Recent literature explaining the early 2000s US housing booms focuses on credit expansions (see, for example, Kiyotaki et al. (2011), Sommer et al. (2013), and Favilukis et al. (forthcoming)). Housing market outcomes are sensitive to credit conditions in those models because house prices are mainly driven by consumption demand among the majority poor agents. Due to the very little cross-sectional variation in wealth across low income agents, small changes in credit availability can cause large groups of identical, low income, low wealth households to move between renting and homeownership. However, the large swings in homeownership often produced by those models contradict the data.

CFPS 2012), the loan-to-value ratio at the household level remained low in the two survey waves, about 5% in both 2002 and 2012, which suggests that shadow financing is not a significant concern. If we allowed a controlled relaxation in credit conditions during the transition exercise, the model would be able to produce larger housing booms.

The large-scale urbanization process, rural-urban migration in particular, is a more relevant exogenous demand factor. According to the two latest National Population Censuses, the urban population in China increased at an average annual rate of about 4%. Chen and Song (2014) find that urbanization accounts for 80.4% of the total urban population growth. Moreover, among the urbanized population, rural-urban migration accounts for more than half. This means that including urbanization factors would introduce a large number of poor households who need to save up a long time for housing purchases to the model. Accordingly, the mechanism in the paper would get strengthened. In particular, this would likely cause the housing boom to last longer.

This paper does not consider demographic factors either. In particular, life-cycle components are not explicitly modeled. The intergenerational link is strong in China in the sense that parents are usually able and willing to provide financial support to their children's home purchases.[40] This means that the "actual" housing demand in China might present less of a life-cycle pattern than a standard life-cycle housing demand model would imply. This is partly supported by the empirical observation that the saving behavior does not vary significantly along the life-cycle dimension.[41] Thus life-cycle does not seem to be a factor of the first order for housing market outcomes in the short and medium run.[42]

Aggregate uncertainty   The paper does not model aggregate uncertainty. Due to the modeling approach of this paper (which treats housing as a lumpy financial asset), adding aggregate uncertainty can be expected to strength the mechanism in the model. Since housing becomes a risky asset in the presence of aggregate uncertainty, it can be expected that housing would be compensated with a risk premium in addition to the illiquidity premium. The lumpiness of housing would thus increase. This

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[40]This is partly because of the Chinese tradition that children support parents in their old age (and sometimes even live in the same house).

[41]According to Chamon and Prasad (2010), the age profile of savings among Chinese households has an unusual pattern in recent years, with younger and older households having relatively high savings rates.

[42]Curtis et al. (2015) show that long-run demographic change in China can significantly affect the aggregate household saving rate.

might amplify the differential responsiveness in housing demand. Taking aggregate uncertainties into account would also help the model in matching the volatility in house prices in the data.

Supply side factors Finally, housing supply is fixed in the model.[43] Allowing for housing production is unlikely to affect the qualitative result of this paper either, especially since land is an important factor in housing construction. As discussed earlier, according to Wu et al. (2015), house price growth in China is driven by rising land values, not by construction costs. However, incorporating housing production can be an important future avenue to explore. The cross-sectional variation in housing supply elasticities could be an important factor in accounting for the regional variations in house price growth.

In sum, the qualitative results from the baseline transition experiment are expected to hold even considering various additional elements.

1.5 Conclusion

This paper develops a heterogeneous agent equilibrium model to study the interaction of inequality, income growth, and house price dynamics. Important features of the housing market environment are explicitly captured: housing delivers returns both in terms of rental dividends and capital gains; market frictions make housing adjustment lumpy. House prices and rents (and therefore the financial rate of return on housing) are jointly determined in equilibrium. The modeling approach of the paper allows housing purchases to be purely financial decisions. Thus, housing market outcomes are closely linked to cross-sectional household portfolio saving decisions: prices and inequality endogenously interact.

The model can account for the positive empirical relationship between inequality and house prices. More importantly, a transitional equilibrium analysis of the Chinese economy can rationalize its observed house price growth accompanied by a rise in the private savings rate. Key mechanisms behind the results are based on the differential responsiveness in housing demand across wealth groups, which is due to the differential impact of the lumpy housing.

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[43]Combined with the fixed population mass, the model essentially assumes exogenous changes in housing demand are met by the supply side. This turns out not to be a bad assumption. According to Wu et al. (2015), in the aggregate, the growth in households demanding housing units driven by fundamentals roughly matches with new housing construction in urban China, although with considerable regional variation.

This work contributes to the housing literature by illustrating a new price formation mechanism, which highlights the role played by the investment motive among the wealthy.[44] Another important contribution is that the paper provides a tractable quantitative model that are suitable to study housing market outcomes in China. Moreover, the theory in this paper shows that market frictions can have differential impact cross-sectionally, increasing inequality. Inequality can in turn amplify market frictions. This adds to our understanding of a broader topic: how inequality and macroeconomic forces can interact.

At its current stage, this research only focuses on explaining the mechanisms behind certain housing market outcomes. Important welfare and policy implications are the next step. For example, the interaction between inequality and return premium in the housing market directly speaks to the efficient provision of housing services in a steady state. If housing supply is explicitly modeled, increased inequality might boost housing production and ultimately improve welfare. Moreover, in the short run, low-income households can be priced out of the housing market due to the feedback loop between house prices and wealth inequality. This highlights the potentially large welfare cost to low-income households during fast economic growth.

Possible extensions of this work may have applications beyond explaining housing market outcomes. For example, one could develop a related model to explain the cross-sectional pattern in debt-to-income ratios and its implication on aggregate default risk. In an environment in which heterogeneous households face investment opportunities with different risk-return perspectives and different investment costs, wealthy households can afford to load on riskier and costlier assets with a low debt-to-income ratio, earning high returns. While the wealthy households bid up the prices for high-return assets, the poor households are either "trapped" in risk-free assets with meager returns or forced to borrow a lot (relative to income) and bear higher default risks. The mechanism can potentially explain the cross-sectional debt-to-income ratio patterns in the United States and shed light on the development of financial crises.

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[44]This mechanism might be an important force behind the recent housing boom in the United States: evidence from the SCF shows that the housing assets owned by wealthy households increased much more dramatically than the rest of the housing stock during the housing price run-up in the early 2000s.

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1.A Appendix: Additional stylized facts about inequality, house prices and rents

Panel (a) of Figure 1.1.1 in Section 1.1 presents the raw correlation between top income share growth and the appreciation of house prices across countries. In this section, I explain more details of the data source and variable construction. I also conduct some robustness checks on these correlations by controlling for some factors, such as income and population growth and housing supply elasticity. Moreover, I do the same analysis with data across states in the United States. Lastly, I also show that rents do not co-move with top income shares. But due to limitations on data availability, the analysis with rental data is limited only to the US cross-state level.

US cross-state housing prices, rents and inequality   The theory in this paper suggests that housing price positively correlates with inequality, and it consists with the cross country data. It would be more convincing to see whether the data shows the same consistency at a more disaggregate level since housing market conditions might vary considerably more across countries. Therefore I do the same analysis as that of Panel (a) of Figure 1.1.1 using data across states in the United States, and the result is shown in Panel (a) of Figure 1.A.1. The same positive correlation is found at a high significant level.

The theory in this paper suggests that rents depend mostly on average resources instead of the inequality level. It is important to verify this result to justify the model setup in this paper. However, at the cross-country level we lack a long enough series of rents data, especially if we want them to be comparable. Thus, the analysis using rental data is limited to only the US cross-state level. Panel (a) of Figure 1.A.1 plots the correlation between growth in the top 5 percent income share and rental prices across states in the United States.

As can be seen in Figure 1.A.1, rents do not co-move with top income shares.[45] This observation is consistent with the theory posited in this paper.

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[45]There is a hint of a positive correlation between those two in the raw data, but it's not significant. Moreover, after we add the controls as in Panel (c) of Figure 1.A.2, the hint of correlation is completely gone.

Figure 1.A.1: US cross-state analysis of inequality, housing prices, and rents

(a) Housing prices and inequality   (b) Rents and inequality
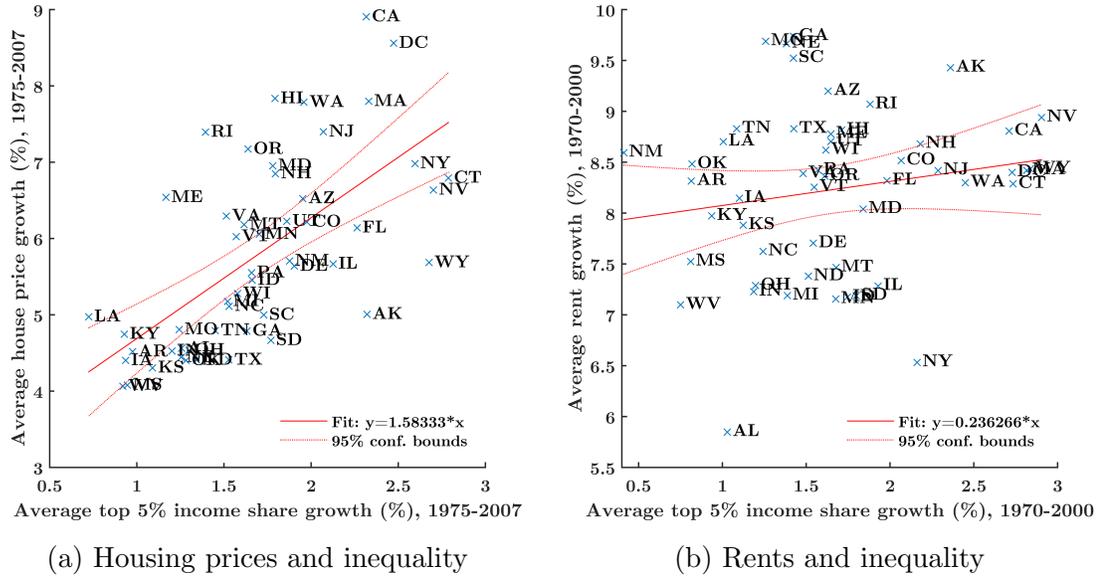
Notes: In Panel (a), US cross-state (nominal) house prices are from `FHFA All-Transactions Indexes`; US cross-state top 5% income shares are from `Frank-Sommeiller-Price Series` (income excludes capital gains) as described in Frank et al. (2015). For each observation in Panel (a), the average growth rates are obtained by averaging the yearly growth rates over the entire sample periods. In Panel (b), the inequality measure is the same as that of Panel (a). US cross-state (nominal) rents are taken from `Historical Census of Housing Tables` (decennial), provided by the US Census Bureau, Housing and Household Economic Statistics Division. Gross rent is the monthly amount of rent plus the estimated average monthly cost of utilities and fuels. Monthly rents were computed for specified renter-occupied units paying cash rent. This category excludes one-family houses on ten or more acres.

The choice of house price indexes   Every house price index is representative in its own way and has different strengths and weaknesses. Even within the same geographic area, house price indexes from different sources can differ significantly, depending on construction methodology, sample composition, and data aggregation. Therefore, sometimes it is crucial to choose the appropriate index according to specific purposes.

The focus of this paper is on the relationship between house prices and inequality. One of the paper's central empirical and theoretical arguments is that the investment motives of wealthy households play important roles in driving house price dynamics. Since wealthy households tend to purchase more expensive houses than the rest of the population, using a value-weighted index, in which expensive houses have a greater influence on estimated price changes, will naturally bias the results of the analysis. This is especially true if more expensive houses have different price dynamics than

less expensive ones.[46] Therefore, for this study, it is important to choose a house price index that weights price trends equally for all residential properties. To this end, I choose the all-transactions index produced by the Federal Housing Finance Agency (FHFA) for the US cross-state analysis.

The FHFA indexes are available from 1976 to the present for all US states; this makes them great for cross-section analysis. Each state-level index is constructed using a weighted, repeat sales method that compares transaction prices of the same property over time. This method is preferred because it avoids composition biases from quality changes in the stock of houses in transaction.

For the cross-country analysis, I try to use house price indexes that are consistent with the US FHFA index methodology. These types of indexes are identified and compiled for many countries in the Federal Reserve Bank of Dallas's `International House Price Database` as described in Mack and Martínez-García (2011). For the purpose of this study, I use the house price indexes expressed in real terms to control for potential correlation between changes in inequality and the overall price level in one country.

Robustness checks with other data sources   The weakness of the FHFA index is that its underlining sample composites only houses purchased with conforming mortgages, and this might understate the sensitivity of house prices to alternative credit linking to investment purposes, especially those cash transactions. Moreover, in specific regions and during certain times, housing purchases with sub-prime, jumbo, and other non-conforming loans can contribute significantly to changes in house prices as shown by some researchers. To address these issues, I repeat the same analysis using an alternative index: the `Zillow Home Value Index (ZHVI)`, which is also available across all US states, although extending back only to 1996. ZHVI also uses repeat-sales methods to address composition biases, making it conceptually similar to the FHFA index, with the additional advantage of a broader sample coverage.[47] The results of the analyses are similar when using either the FHFA index or the ZHVI for the same time periods. Similarly, for the cross-country analysis, the robustness of the

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[46]In an ongoing project, from various data sources, my coauthor and I document that the price dynamics of houses from different price ranges does differ. In particular, more expensive houses tend to appreciate faster than the rest of the houses.

[47]Details of the ZHVI construction methodology can be found at `http://www.zillow.com/research/zhvi-methodology-6032/`. In general, ZHVI is in nature similar to the S&P/Case-Shiller indexes, but with subtle differences in sample collection (see `http://cdn1.blog-media.zillowstatic.com/3/ZHVI-InfoSheet-04ed2b.pdf` for a detailed comparison of the two indexes). I choose ZHVI because it has better geographic coverage.

result is verified by using an alternative set of comparable house price indexes from the `BIS Residential Property Price database`.

The choice of inequality measures The inequality measure in the empirical part of this paper focuses on households at the upper end of the income distribution. According to the theory of the paper, the more relevant dimension to look at should be household wealth. This mismatch happens because there is insufficient comparable wealth data at the cross-section over time (see Saez and Zucman (2014) for an account of the series of wealth distribution only within the United States using capitalized income tax data). Cross-sectional data on income distribution has been better documented; thanks to the recent effort by Alvaredo et al. (2015) and Frank et al. (2015). Thus, the measurement for inequality in this paper is consistently the top income (excluding capital incomes) shares.

Control for average income growth and housing supply elasticity Some important factors that affect the dynamics of house prices might also affect changes in inequality. Two obvious ones are average income and housing supply elasticity. Figure 1.A.2 conducts the same correlation analysis as in Panel (a) of Figure 1.1.1 in Section 1.1 and Figure 1.A.1 in this section with additional elements to control for those factors. In particular, I add cross-sectional series on personal disposable income per capita to control for changes in average income and population growth. Consistent with the cross-country house prices data, cross-country disposable income per capita series are expressed in real terms. Moreover, I also try to control for cross-sectional differences in housing supply elasticity, although in very crude ways. For both cross-country and cross-state analysis, the supply elasticity measures I have are only for a cross section and thus could not account for changes in supply elasticity itself over time. This is only going to be problematic if changes in the supply elasticity correlate with changes in inequality, which is not considered evidently the case in the literature.

As can be seen in Figure 1.A.2, the positive association between top income share growth and appreciation of house prices remains significant after those controls. It even gets stronger at the cross-country level. Moreover, the no correlation result still holds for the case of rental prices. This evidence provides us with more confidence in the causal relationship between inequality and house prices.

Figure 1.A.2: The same analysis with controls
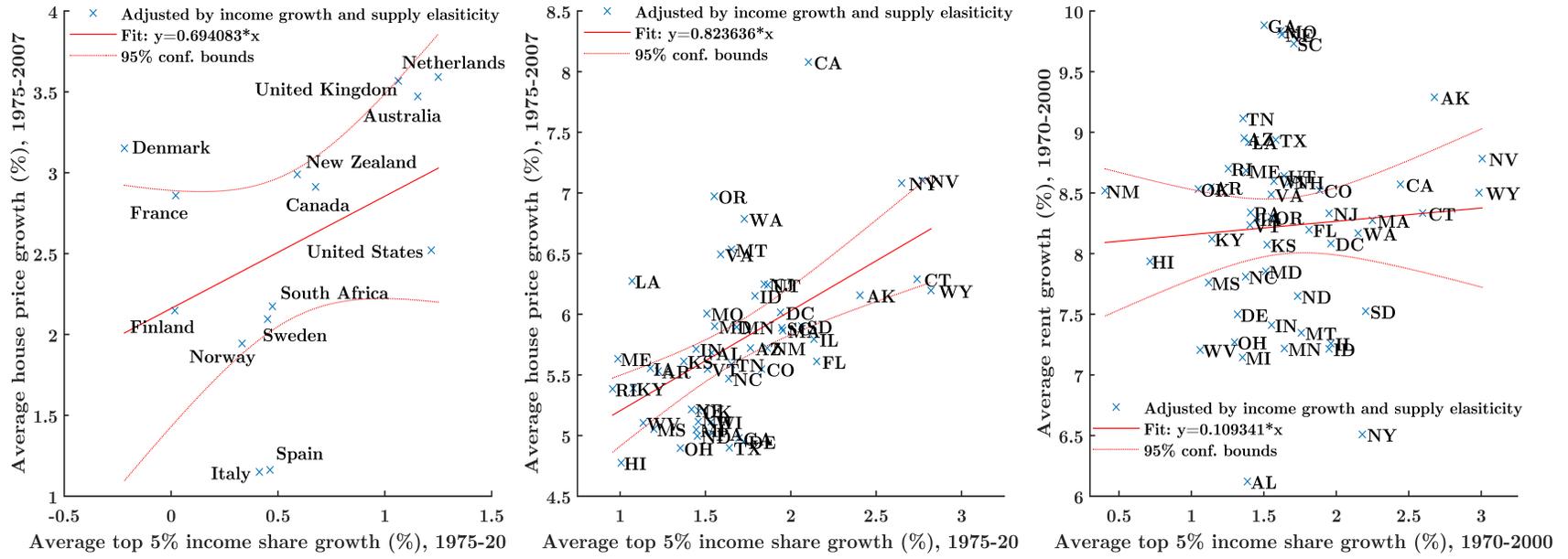
(a) Cross-country housing price & inequality (b) Cross-state housing price & inequality (c) Cross-state rents and inequality

Notes: House prices, rents, and inequality measures are exactly the same as those in Panel (a) of Figure 1.1.1 in Section 1.1 and Figure 1.A.1. Income is measured by personal disposable income per capita, with the same panel structure as the price and inequality data. The average growth rates of income are obtained by averaging the yearly growth rates over the entire sample periods. Supply elasticity measures are available only as a cross-section. In Panel (a), cross-country personal disposable income per capita (real) series are taken from Federal Reserve Bank of Dallas's `International House Price Database` as described in Mack and Martínez-García (2011); cross-country housing supply elasticities are cross-sectional estimates from Caldera and Johansson (2013). In Panel (b) and (c), US cross-state (nominal) personal disposable income per capita series are from `BEA Regional Economic Accounts`; US cross-state housing supply elasticities are approximated by the cross-sectional Wharton Land-Use Regulatory Index (WRLURI) from Gyourko et al. (2008).

1.B   Appendix: Equilibrium characterization of the basic model

Before characterizing the equilibrium, we first recast the definition of equilibrium in more detail in the following.

Definition of equilibrium   The steady-state stationary equilibrium associated with μ, π, Y, r, λ_h, H and $\underline{h}$ is defined as: a set of prices R and P, a set of policy functions $b(y, R, P)$, $h(y, R, P)$ and $x(y, R, P)$ for young households, as well as a stationary distribution $\lambda(y, b, h)$, such that

1. Given R and P, $b(y, R, P)$, $h(y, R, P)$ and $x(y, R, P)$ solve the young household's problem defined in (1.3.1).

2. The stationary distribution $\lambda(y, b, h)$ is induced by μ, φ, Y, r, H and $\underline{h}$ and $b(y, R, P), h(y, R, P), x(y, R, P)$.

3. Markets clear.

    (a) $\iint_{\lambda(y,b,h)} h(y, b, h) = H$

    (b) $\iint_{\lambda(y,b,h)} s(y, b, h) = H$

1.B.1   Equilibrium with $\frac{R}{P} = r$

Although housing has a minimum size friction than bonds, equilibrium with $\frac{R}{P} = r$ does exit, providing the condition that the risk free interest rate is sufficiently high and the saving motive is sufficiently strong. For equilibrium with $\frac{R}{P} = r$, there is no strict portfolio choice problem. Regardless of the income level y, the solution to problem (1.3.1) is simply

$$x = \frac{1}{1 + \beta} y, \ b + Ph = \frac{\beta}{1 + \beta} y,$$

subject to constraints (1.3.2) and (1.3.3).

Combining the above optimal consumption decision with the rental market clearing condition (3b), we can recover R; i.e.,

$$\frac{1}{2} \frac{1}{1 + \beta} \varphi Y + \frac{1}{2} \frac{(1 + r)\beta}{1 + \beta} \varphi Y = RH \Rightarrow R = (1 + \frac{\beta r}{1 + \beta}) \frac{\varphi Y}{2H}.$$

Note that in the above derivation we used the condition that $Rs = \varphi x$. R increases with r, due to a pure wealth effect from the older generation. But the increase is less than one for one, since there is still a substitution effect from the younger generation.

House price simply follows from the no arbitrage condition between bonds and housing:

$$P = (\frac{1}{r} + \frac{\beta}{1+\beta})\frac{\varphi Y}{2H}.$$

P decreases with r due to the fact that equilibrium R grows less than one for one with the growth of r. This makes housing less attractive than bonds. Note that in this case income inequality does not affect either P or R. This is the result of the log utility specification: the marginal propensity to consume does not change along the wealth dimension.

The equilibrium existence condition further requires that housing asset market clearing condition (3a), constraint (1.3.2), and (1.3.3) are satisfied. Those conditions imply:

$$\lambda_h P\underline{h} \leq \frac{\beta}{1+\beta}\frac{\pi Y}{\mu} \text{ and } \lambda_h PH \leq \begin{cases} \frac{\beta}{1+\beta}\pi\frac{Y}{2} & \text{if } \lambda_h P\underline{h} > \frac{\beta}{1+\beta}\frac{(1-\pi)Y}{(1-\mu)} \\ \frac{\beta}{1+\beta}\frac{Y}{2} & \text{otherwise} \end{cases}.$$

Plugging the above conditions into the equilibrium pricing functions, we have the following two cases.

Case One: both bottom and top earners can afford the minimum house when the parameters satisfy the following condition:

$$K > \varphi\lambda_h \text{ and } r \geq \max\left\{\frac{1+\beta}{\beta}\frac{\varphi\lambda_h}{K-\varphi\lambda_h}, \frac{1+\beta}{\beta}\frac{\varphi\lambda_h}{1-\varphi\lambda_h}\right\},$$

where $K \equiv 2\frac{(1-\pi)H}{(1-\mu)\underline{h}}$, which measures the strength of inequality and market friction. This is the case where both inequality and market friction are small.

Case Two: only top earners can afford the minimum house when the set of parameters either satisfies the following condition:

$$K < \varphi\lambda_h \text{ and } r \geq \frac{1+\beta}{\beta}\frac{\varphi\lambda_h}{\pi-\varphi\lambda_h},$$

or satisfies the following condition:

$$K > \varphi\lambda_h \text{ and } \frac{1+\beta}{\beta}\frac{\varphi\lambda_h}{K-\varphi l} \geq r \geq \frac{1+\beta}{\beta}\frac{\varphi\lambda_h}{\pi-\varphi\lambda_h}.$$

Note that in one situation bottom earners could not afford the minimum house because of the high levels of inequality and market friction; in the other situation, the reason is due to the relatively low interest rate level.

1.B.2 Equilibrium with $\frac{R}{P} > r$

For equilibrium with $\frac{R}{P} > r$, due to the minimum size friction in housing ownership and the borrowing constraint, optimal portfolio choice depends on income level y. Moreover, whenever the optimal portfolio consists of non-zero housing, constraint (1.3.2) will bind; i.e., it is in the household's interest to fully utilize the collateralized borrowing opportunity. In the following, we first derive the policy functions for the situation where $\frac{R}{P} > r$.

Policy functions First, note that constraint (1.3.3) necessarily binds for households with sufficiently low income. In particular, let's denote

$$\underline{y} \equiv \lambda_h P \underline{h}$$

as the income level at which the minimum house is affordable only with zero consumption. Then for households with an income level lower than $\underline{y}$, they are forced to save in terms of bonds, and the solution to problem (1.3.1) for $0 < y \leq \underline{y}$ is simply

$$x = \frac{1}{1+\beta} y, \ h = 0, \ b = \frac{\beta}{1+\beta} y.$$

Second, for households with high enough income, constraint (1.3.3) will not bind. Due to the assumed log preference, the cutoff income level can be conveniently identified as

$$\bar{y} \equiv \frac{1+\beta}{\beta} \underline{y}.$$

Thus, the solution to problem (1.3.1) for $y \geq \bar{y}$ is simply

$$x = \frac{1}{1+\beta} y, \ h = \frac{\beta y}{1+\beta} \frac{1}{\lambda_h P}, \ b = (1 - \lambda_h) P h.$$

For households with an income level in the middle range $\underline{y} \leq y \leq \bar{y}$, they are unable to optimally choose more than $\underline{h}$ units of housing assets. Moreover, due to the log preference, their optimal decision must be either choosing exactly $\underline{h}$ units of housing or opting for zero housing.

If a household decides to choose $\underline{h}$ units of housing, its current consumption will suffer from its over-saving, she but the household can enjoy a higher return from its savings. In the end, the household's lifetime utility level will be

$$\ln(y - \underline{y}) + \beta \ln\left(\underline{y}(1+\rho)\right),$$

where

$$\rho \equiv r + \frac{1}{\lambda_h}(\frac{R}{P} - r)$$

denotes the net financial rate of return from holding the housing asset.

If a household chooses zero housing, the optimal consumption-saving decision would be the same as for those households with an income lower than $\underline{y}$, and the associate utility level would be

$$\ln(\frac{1}{1+\beta}y) + \beta \ln\left(\frac{\beta(1+r)}{1+\beta}y\right).$$

Denote $d(y)$ as the difference between the two utility levels, or, in simplified form,

$$d(y) = \ln\left((1+\beta)\frac{y - \underline{y}}{y}\right) - \beta \ln\left(\frac{\beta(1+r)y}{(1+\beta)\underline{y}(1+\rho)}\right).$$

It can be shown that $d(y)$ monotonically increases with y when $y \in [\underline{y}, \bar{y}]$. Moreover, $d(\underline{y}) = -\infty$ and $d(\bar{y}) > 0$. This ensures that the solution to the equation $d(y^*) = 0$ is unique, and $d(y) < 0$ if and only if $y < y^*$. Therefore, it's optimal to purchase $\underline{h}$ units of housing for households with income levels between y^* and $\bar{y}$, while households save only in bonds when they have an income level below y^*; i.e.,

$$h(y, R, P) = \begin{cases} \frac{\beta}{1+\beta}\frac{y}{\lambda_h P} & \text{if } y \geq \bar{y} \\ \underline{h} & \text{if } y^* < y < \bar{y} \\ 0 & \text{otherwise} \end{cases}, \quad b(y, R, P) = \begin{cases} -\frac{\beta}{1+\beta}\frac{(1-\lambda_h)y}{\lambda_h} & \text{if } y \geq \bar{y} \\ -(1-\lambda_h)P\underline{h} & \text{if } y^* < y < \bar{y} \\ \frac{\beta}{1+\beta}y & \text{otherwise} \end{cases}.$$

Characterizing the equilibrium With the policy functions identified above, equilibrium prices can be derived using market clearing conditions (3a) and (3b). In the following, we discuss three different equilibrium outcomes case by case.

Case One: When $\frac{(1-\pi)Y}{(1-\mu)} \geq \bar{y}$, asset market clearing condition (3a) implies

$$\frac{\beta Y}{2(1+\beta)\lambda_h} = PH \Rightarrow P = \frac{\beta Y}{2(1+\beta)\lambda_h H}.$$

Rental market clearing condition (3b) implies

$$\frac{\varphi Y}{2(1+\beta)} + \frac{\beta\varphi Y(1+\rho)}{2(1+\beta)} = RH \Rightarrow \frac{R}{P} = \frac{\varphi}{1-\varphi}\left(\lambda_h(\frac{1}{\beta} + 1) + (\lambda_h - 1)r\right).$$

Equilibrium rental price can then be derived as

$$R = \frac{\varphi \beta Y}{2(1-\varphi)(1+\beta)H}\left(\left(\frac{1}{\beta}+1\right)+\frac{(\lambda_h-1)}{\lambda_h}r\right)$$

Note that neither prices nor returns depend on any inequality measure in this case. However, for this type of equilibrium to exist, we need a relatively equal income distribution, small market friction $\underline{h}$, and a low interest rate. In particular, those conditions imply the following:

$$K \geq 1 \text{ and } r < \frac{\varphi \lambda_h(1+\beta)}{\beta(1-\lambda_h\varphi)}.$$

Case Two: When $\frac{\pi Y}{\mu} > \bar{y} > \frac{(1-\pi)Y}{(1-\mu)} \geq y^*$, asset market clearing condition (3a) implies

$$\frac{\beta \pi Y}{2(1+\beta)\lambda_h} + \frac{(1-\mu)}{2}P\underline{h} = PH \Rightarrow P = \frac{\beta \pi Y}{(1+\beta)\lambda_h(2H-(1-\mu)\underline{h})}.$$

Rental market clearing condition (3b) implies

$$\pi Y + \frac{\beta \pi Y}{(1+\beta)}\rho + (1-\pi)Y + (1-\mu)P\underline{h}\rho = \frac{2RH}{\varphi}$$

$$\Rightarrow \frac{R}{P} = \frac{\varphi}{1-m\varphi}\left(\lambda_h\left(\frac{1}{\beta}+1\right)n + (\lambda_h-1)rm\right).$$

Equilibrium rental price can then be derived as

$$R = \frac{\varphi \beta Y}{(1-m\varphi)(1+\beta)(2H-(1-\mu)\underline{h})}\left(\left(\frac{1}{\beta}+1\right)n + \frac{(\lambda_h-1)}{\lambda_h}rm\right),$$

where $m \equiv (1 + (1-\mu)\frac{1-\lambda_h}{\lambda_h}\frac{\underline{h}}{2H})$ and $n \equiv (1-(1-\mu)\frac{\underline{h}}{2H})\frac{1}{\pi}$.

Note that in this case an increase in either the top earner's income share π or the bottom earner's population share $1-\chi$ will cause house prices to increase. This is a case in which income inequality is modest compared to the amount of the housing market friction. The equilibrium outcome is that the bottom earners participate in the housing market but are liquidity constrained; i.e., $\bar{y} > \frac{(1-\pi)Y}{(1-\chi)} \geq y^*$. The equilibrium condition requires the following:

$$\frac{\beta \pi}{1-(1-\mu)\underline{h}/H/2} \leq K(1+\beta) \leq \frac{\pi(1+\beta)}{1-(1-\mu)\underline{h}/H/2}, \ r < \frac{\varphi \lambda_h(1+\beta)n}{\beta(1-\lambda_h\varphi m)} \text{ and } \varphi m < 1.$$

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Case Three: When $\frac{\pi Y}{\mu} > \bar{y} > y^* > \frac{(1-\pi)Y}{(1-\mu)}$, asset market clearing condition (3a) implies

$$\frac{\beta \pi Y}{2(1+\beta)\lambda_h} = PH \Rightarrow P = \frac{\beta \pi Y}{2(1+\beta)\lambda_h H}.$$

Rental market clearing condition (3b) implies

$$\varphi Y + \beta \varphi \left(\pi Y(1+\rho) + (1-\pi)Y(1+r)\right) = 2(1+\beta)RH$$

$$\Rightarrow \frac{R}{P} = \frac{\varphi}{1-\varphi}\left(\lambda_h(\frac{1}{\beta} + \frac{1+r}{\pi}) - r\right).$$

Equilibrium rental price can then be derived as

$$R = \frac{\varphi \beta \pi Y}{2(1-\varphi)(1+\beta)H}\left((\frac{1}{\beta} + \frac{1+r}{\pi}) - \frac{r}{\lambda_h}\right).$$

Note that both prices and returns only depend on the inequality measure π. However, for this type of equilibrium to exist, we need a relatively unequal distribution and a big market friction $\underline{h}$. In particular, the equilibrium conditions require the following:

$$K \leq \frac{\beta \phi y^*}{(1+\beta)\underline{y}} \text{ and } r < \frac{\varphi l(1+\beta)}{\beta(\pi - l\varphi)}.$$

1.C Appendix: Institutional background and data for China

1.C.1 Backgrounds

Before 1978 China had a centrally planned socialist economy in which the ownership of land (and housing) was nationalized and public rental housing was the predominate form of urban housing provision. Little has been changed about China's housing welfare system during early 1980s, after China's major reform in 1978. It was not until 1988 that the Chinese constitution was amended to allow for land transactions, which set the legal stage for the privatization of housing in China.

In the mid-1990s, the Chinese central government identified the final goal of housing reform as the creation of a new urban housing system that suited socialist market economics: "commodity housing" was allowed for market transaction. Moreover, employees in the state sector were encouraged to purchase full or partial property rights of their current apartment units at subsidized prices. However, the housing reform of this period did not manage to completely shift the system away from state-provided housing. Commodity housing was still relatively rare during this period.

An important milestone in housing policy occurred in 1998, when the central government announced that welfare housing distribution in urban China would be abandoned at the end of 1998 and completely replaced by market provision. Market provision of housing surged after 1998: the share of private housing units among all completed housing units almost doubled in 4 years, increasing from about 30% in 1998 to over 50% in 2002 (Wu et al. (2014)). Moreover, during the same period, the People's Bank of China lowered the mortgage interest rate five times to encourage home purchases (Fang et al. (2015)).

According to Walder and He (2014), in 2002, 78% of all households in urban China owned their homes with partial or full property rights and only 16% continued to live in rent-subsidized work unit housing: the housing privatization program was roughly complete. For more discussion about the housing market development in China after the early 2000s, readers are referred to Fang et al. (2015) and Wu et al. (2015).

1.C.2 Household income and wealth data in China

The availability of household-level data from China is limited. One common data source being used in the literature to study questions related to cross-sectional income in China is the annual Urban Household Survey (UHS) conducted by the National Bureau of Statistics (for example, see Piketty and Qian (2009); Chamon and Prasad (2010); and Yang et al. (2013) for example). However, UHS does not adequately

account for household wealth.

One of the few options for detailed information on both household income and household wealth during 1990s and the first decade of the 21st century is the Chinese Household Income Project (CHIP). Due to increasing interest in household finance and the wealth distribution in China, more efforts have been made in data collection, and several sources are now available after the end of the first decade of the 21st century. The China Family Panel Studies (CFPS), which this paper adopts, has arguably the best quality data for the purpose of this study.[48]

In the following section, I describe the two data sources in more detail. In the interest of saving space, this appendix does not present detailed calculation procedures for all of the distributional statistics used in the main text of this paper; however, these calculations are available upon request.

The Chinese Household Income Project (CHIP)   The Chinese Household Income Project, carried out by a team of international economists in collaboration with China's National Bureau of Statistics, has conducted a series of national surveys covering the years 1988, 1995, 2002, and 2007. Each wave of CHIP contains both a rural survey and an urban survey, and the implementation is largely consistent across waves, with the 2007 wave as an exception. In this study, we only use the urban survey in each wave.

The urban surveys collect detailed information on household wealth and its components, including financial assets, market value of private housing, production assets, and value of durable consumer goods. In particular, the housing value in CHIP is estimated by asking households to assess the market value of their owned housing. This is similar to the procedure used by the Survey of Consumer Finance.

One major drawback of CHIP is that it does not contain sample weights. For more details about CHIP, such as its sample size, sampling procedure, and geographical coverage, readers are referred to Shi and Zhao (2007).

The China Family Panel Studies (CFPS)   The China Family Panel Studies (CFPS) are a nationally representative, annual longitudinal survey of Chinese communities, families, and individuals. The survey was launched in 2010 by the Institute

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[48]Another major household survey that aims to capture Chinese household wealth and that has recently become available is the China Household Finance Survey (CHFS), which is conducted by the Southwestern University of Finance and Economics, China. However, the CHFS data suggest a wealth-over-income ratio that is too high (as high as about 20, according to some initial analysis of CHFS 2012). Xie and Jin (forthcoming) make more comparisons between the CHFS and the CFPS.

of Social Science Survey of Peking University, China. The data on a follow-up survey conducted in 2012 are also available. Like CHIP, during each wave CFPS conducts two separate surveys for rural and urban areas. In this study, we only use the urban survey in each wave.

The CFPS dataset contains comprehensive measurements of assets, including housing assets, financial assets (which are further broken down into subcategories, such as fixed bank deposits, stocks, and mutual funds, etc.), business assets, detailed items of durable goods, and liabilities from housing and other sources. As in CHIP, in CFPS information on housing values is obtained by directly asking homeowners how much their houses are worth. Some missing housing values are imputed according to the size of the house and unit value for similar housing types. Dropping those imputed values do not affect any of the data moments used in this paper.

Detailed sampling weights are available for each wave of CFPS as well as for cross-sectional analysis between 2010 and 2012 waves. A detailed account of the advantage of CFPS in capturing Chinese household wealth can be found in Xie and Jin (forthcoming).

1.C.3 Income inequality and house prices in China

Panel (b) in Figure 1.1.1 show four series of income inequality measures in urban China: income and wage shares of the top 5 and top 10 percent. Note that, unlike the income-tax based cross-sectional income data used in panel (a) of Figure 1.1.1, administrative income statistics for China are not available. Existing data on income distribution in China are largely based on household surveys (see Piketty and Qian (2009)). Income inequality measures based on household surveys are often less comparable to those based on administrative sources. Moreover, the income concentration in China at 2002 (calculated by author) is much lower than most countries in Panel (a) in Figure 1.1.1 (the top 5 percent income share is 14% in China at 2002, while it is 30% at the same time in the United States); thus, I show four different income inequality series for China. Section 1.3.5 discusses the wealth inequality comparison between China and the United States.

Panel (b) in Figure 1.1.1 also presents the residential land prices series in urban China, together with a series of per capita income. Note that I only present here a land price index because reliable house prices data is publicly unavailable for China.[49]

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[49] According to China's official house price indices, the average housing price (urban) appreciation is only mild compared to aggregate income growth. However, these indices are mistrusted and widely criticized due to the lack of quality adjustments in their construction. Constant quality price

To simplify the discussion, I will refer to this land price index as an approximation of house prices in the rest of the paper.[50] All the income inequality measures grew significantly during the considered time periods. In particular, the income share of the top 5 percent grew from 14% to 26% between 2002 and 2012: in other words, it has almost doubled. During the same time, house prices grew significantly faster than the already fast income growth.

The source for the years 2002 and 2003 is the Ministry of Land and Resources of China. The trend is constructed by HP filtering the raw data with frequency parameter $\lambda = 6.25$.

In Panel (b), the residential land price indexes are Wharton/NUS/Tsinghua Chinese Residential Land Price Indexes taken from Deng et al. (2014). Note that I only present here a land price index because reliable house prices data is publicly unavailable for China. See footnote 49 for more details. Real per capita income is the Average Wage of Employed Persons in Urban Units taken from National Bureau of Statistics (NBS) of China (deflated by the Urban Household Consumer Price Index).

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series developed by academic researchers mostly appreciate at a faster pace than aggregate income, although with regional variations. See Wu et al. (2014), Deng et al. (2014), Wu et al. (2015) and Fang et al. (2015) for more details. Still, a well trusted aggregate house price index is not publicly available for China. The residential land price index presented here is relatively more developed and taken from Deng et al. (2014).

[50]In general, land prices can be very different from house prices. However, house prices and land prices go hand in hand in China during the time periods considered in this paper. According to Fang et al. (2015) and Wu et al. (2015), house prices growth roughly matches with the growth in land prices in China over the past decade. Moreover, Wu et al. (2015) show that house prices growth in China is driven by rising land values, not by construction costs.

1.D Appendix: Computation

1.D.1 Solving for the stationary equilibrium

The stationary equilibrium is formally defined by the conditions presented in Section 1.3.3, with the assumption that $\{Y_t, \pi_t, H_t\}$ always stay constant as $(\bar{Y}, \bar{\pi}, \bar{H})$. The additional assumption implies an economy with a stationary distribution Λ^* over $(\varepsilon, \tilde{\beta}^i, b, h, i)$ in which households behave optimally and markets always clear with prices $\{P^*, R^*\}$. We approximate the distribution Λ^* by an economy with a finite number of households, each characterized by its individual type $(\varepsilon, \tilde{\beta}^i, b, h, i)$.

Note that, to solve their individual problems, households in this economy only need to know market prices and returns for both bonds and housing. Since P^* and R^* are sufficient statistics to determine the return on housing, the problem is reduced to finding the pair of prices P^* and R^* that can clear all the markets when households all behave optimally. However, directly searching for the equilibrium price pair $\{P^*, R^*\}$ can be very time consuming because it involves repeatedly re-solving individual optimization problems over all the combinations of $\{P, R\}$ and simulating data to check for market clearing conditions. This is especially the case since in the price searching process it is hard to find an updating rule that can make P and R monotonically converge due to their intercorrelation.

A more efficient approach is to use the goods market clearing condition (2iv) to identify the correlation between P and R, along with other aggregate state variables: aggregate savings in bonds A, housing transaction volume (Ω), and the housing transaction frequency (F), and ultimately to eliminate one of the two prices. Here we choose to eliminate the rental price R. By treating R as a residue from condition (2iv), and searching for a stationary combination of the other aggregate variables $\{P^*, A^*, \Omega^*, F^*\}$, it is easy to find a monotonically converging price updating rule when utilizing the downward sloping demand property. The algorithm outlined in the following describes this computation procedure in detail:

Let $\{P^k, A^k, \Omega^k, F^k\}$ represent the kth guess of the set of aggregate state variables.

Step 1, Guess a set of aggregate variables $\{P^k, A^k, \Omega^k, F^k\}$.

Step 2, Given the set of aggregate variables, solve for the households' optimal decision rules $\{b_i^k, h_i^k\}_{i \in \{B,T\}}$. This step of the algorithm requires solving the households' value functions V_i^k. To find V_i^k, I first approximate the household's continuation value function with a set of interpolation grids for (b, h). I then use a value func-

tion iteration procedure to solve the household's parametric dynamic programming problem as defined in Section 1.3.3. Note that, by imposing $\{P^k, A^k, \Omega^k, F^k\}$ in the household optimization problem, we can derive R^k according to the goods market clearing condition (2iv).

Step 3, Using the above obtained optimal decisions, we simulate the economy using M households and Z periods. M needs to be large enough and Z needs to be long enough such that the simulated economy becomes stationary within Z periods. In practice, M is chosen to be $100,000$ and Z is set at $1,000$.[51]

Step 4, Calculate relevant aggregate statistics in the ending period of the simulation when the economy is stable. In particular, we collect total demand for housing assets and total demand for housing rentals in order to check equilibrium conditions.

Step 5, Check whether housing asset market clearing condition (2iii) and housing rental market clearing condition (2ii) are satisfied.[52] If not, update the guess for the set of aggregate variables $\{P^k, A^k, \Omega^k, F^k\}$ and go back to *Step 1*. Note that the updating rule can be straightforwardly set according to the downward sloping demand property since we only have one price in the set of aggregate variables.

1.D.2 Solving for the transition path equilibrium

This appendix describes the computation method to solve for the perfect foresight transition path equilibrium between two steady states, as described in Section 1.4.1.

As solving for the steady-state equilibrium, the computation is done by a fixed point iteration procedure. The main difference is that, instead of iterating over a set of stationary aggregate variables, we now need to iterate over a set of sequences of aggregate state variables.

Moreover, there is a setup stage when we need to get the following objects ready: a sample distribution Λ_0 over $(\varepsilon, \tilde{\beta}^i, b, h, i)$ drawn from the initial steady state, an initial guess of the length for the transition period T, and the value functions from the new steady-state equilibrium from period T onward.

After the setup stage, the algorithm begins by setting the relevant aggregate variables in periods $t = 0$ and $t = T$ equal to their initial and final steady-state values.

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[51]Note that we divide those M households into bottom earner group and top earner group, according to their population share. When M is set at $100,000$, the economy usually stabilizes after about 100 periods, starting from reasonable initial conditions. We set $Z = 1,000$ for extra caution.

[52]Note that the good market clearing condition (2iv) is automatically satisfied by the derivation of R^k.

Next, a guess is made for the remaining sequences of aggregate state variables along the transition path. The transition path is found using the following algorithm:

Let $\{P_t^k, A_t^k, \Omega_t^k, F_t^k\}_{t=1}^{t=T-1}$ represent the kth guess of the set of sequences of aggregate state variables along the transition path.

Step 1, Guess a set of aggregate variables $\{P_t^k, A_t^k, \Omega_t^k, F_t^k\}_{t=1}^{t=T-1}$.

Step 2, Given the set of aggregate variables, solve for the households' optimal decision rules at each time period $t \leq T - 1$. Note that the decision rules can be solved backward, starting from period $T-1$, taking the sequence of aggregate state variables $\{P_t^k, A_t^k, \Omega_t^k, F_t^k\}_{t=1}^{t=T}$ and ending period value functions as given. Similarly as in solving the steady-state equilibrium, in each period, we first need to derive R_t^k according to goods market clearing condition before we can solve for the households' optimal decision rule.

Step 3, Using the above obtained sequences of optimal decisions, we simulate the initial distribution Λ_0 along the transition path from period $t = 1$ to period $t = T-1$.

Step 4, Calculate relevant aggregate statistics in each period, based on the simulation results.

Step 5, Check whether market clearing conditions in each period are satisfied. If not, update the guess for the set of aggregate variables $\{P_t^k, A_t^k, \Omega_t^k, F_t^k\}_{t=1}^{t=T-1}$ and go back to *Step 1*.

Finally, we update the length of the transition period T if necessary. We only need to do this if the converged aggregate state variables in period $t = T - 1$ do not match with those in period T, the new steady-state values. In practice, any $T \geq 50$ will result in the same computed equilibrium, within a given level of the convergence precision.

CHAPTER 2

The Welfare and Distributional Effects of Fiscal Uncertainty: a Quantitative Evaluation

Coauthored with Rüdiger Bachmann, Jinhui Bai, and Minjoon Lee

This study explores the welfare and distributional effects of fiscal uncertainty using a neoclassical stochastic growth model with incomplete markets. In our model, households face uninsurable idiosyncratic risks in their labor income and discount factor processes, and we allow aggregate uncertainty to arise from both productivity and government purchases shocks. We calibrate our model to key features of the U.S. economy, before eliminating government purchases shocks. We then evaluate the distributional consequences of the elimination of fiscal uncertainty and find that, in our baseline case, welfare gains decline with private wealth holdings.

2.1   Introduction

One consequence of the financial crisis has been the perception of high policy uncertainty in both the U.S. and in Europe. In one study, Baker et al. (2015) analyze Internet news and find a (causal) relationship between high policy uncertainty and subdued aggregate economic activity. In another study, Fernández-Villaverde et al. (2011) find large contractionary effects of fiscal uncertainty on economic activity accompanied by inflationary pressure, especially when the nominal interest rate is at the zero lower bound. These findings are reinforced by a 2012 Economic Policy Survey among business economists (Economic Policy Survey (2012)): "the vast majority of a panel of 236 business economists 'feels that uncertainty about fiscal policy is holding

back the pace of economic recovery'." Finally, Azzimonti (2014) shows that, in recent decades, political polarization in the U.S. has increased, which may lead to heightened fiscal uncertainty.[1]

Most of the existing research on fiscal uncertainty has focused on the aggregate effects of short-run fluctuations of uncertainty on various macroeconomic variables (see below for a more detailed discussion of the literature). However, this literature has not explored the welfare and distributional consequences of fiscal uncertainty.[2] In this paper, we provide such an analysis and address the following question: how large are the welfare costs of fluctuations in government purchases for different wealth households?

To do so, we follow the approach of Krusell and Smith (1998) and use an incomplete market model where heterogeneous households face uninsurable idiosyncratic risks in their labor income and discount factor processes. We then calibrate this model with U.S. data, in particular data on U.S. wealth inequality. Our model has aggregate uncertainty arising from both productivity and government purchases shocks. We thus specify government purchases shocks as the only fundamental source of fiscal uncertainty. In line with the data, we further assume that government purchases shocks are independent of aggregate productivity and employment conditions. Government purchases enter the utility function of the households as separable goods.[3]

Because the government partially funds its expenditures through taxation, purchases fluctuations generate uncertain household tax rates. To capture the distributional effects of fiscal shocks through taxation, we model key features of the progressive U.S. income tax system. In a progressive tax system, aggregate government purchases fluctuations may lead to changes in the distribution of household-specific tax rates and thus to idiosyncratic after-tax income uncertainty. We also employ an empirical tax revenue response rule, which includes government debt and is estimated from U.S. data.

To eliminate fiscal uncertainty, we follow Krusell and Smith (1999) and Krusell et al. (2009), start from a stochastic steady state of the economy with both productivity and government purchases shocks, and remove the fiscal shocks at a given point in time by replacing them with their conditional expectations, while retaining the aggregate productivity process. We then compute the transition path towards the

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[1]McCarty et al. (2006) makes a similar point in the political science literature.

[2]We follow the widespread use in the recent literature and treat "fiscal uncertainty" and "fiscal volatility" as synonymous in this paper.

[3]We consider other utility specifications with complementary and substitutable private-public good relationships, respectively, in extensions to the baseline calibration.

new stochastic steady state in full general equilibrium. Based on the quantitative solution for this transition path, we then compare the welfare of various household groups in the transition-path equilibrium to their welfare level with both aggregate shocks in place.

Our results show that the magnitude of aggregate welfare costs from fiscal shocks is comparable to that of the welfare costs of business cycle fluctuations reported in Lucas (1987, 2003), even though in our model aggregate (spending) fluctuations lead to idiosyncratic after-tax income uncertainty. The welfare gains of eliminating fiscal uncertainty are decreasing in household wealth. However, alternative implementations of the progressive U.S. federal income tax system and the empirical tax revenue response rule may reverse this pattern. We also find that in two counterfactual tax systems, a linear income tax system and a lump sum tax system, the welfare gains of eliminating fiscal uncertainty increase with household wealth. An investigation of the reasons behind these distributional results uncovers the mechanisms through which fiscal uncertainty influences economic welfare. We conclude that the details of the (progressive) tax system determine which wealth group experiences the tax uncertainty burden caused by government purchases shocks, and thus benefits the most from the elimination of these shocks.

Since uncertain tax rates pre-multiply income levels, they generate – loosely speaking – multiplicative after-tax income risk.[4] Just as with the additive endowment risk in early incomplete market models, this after-tax income risk leads households to self-insure through precautionary saving. Wealth-rich households can thus achieve a higher degree of self-insurance relative to wealth-poor households. Indeed, we find that wealth-rich households show less consumption volatility both before and after the elimination of fiscal uncertainty. They also experience a smaller reduction in consumption fluctuations due to the policy change. Consequently, from a precautionary saving perspective, wealth-poor households should gain more when fiscal uncertainty is eliminated.

However, due to the multiplicative nature of the after-tax income risk, the tax-rate uncertainty induced by government purchases fluctuations also creates a rate-of-return risk to savings, which in turn impacts the quality of capital and bonds as saving vehicles.[5] From this perspective, eliminating fiscal uncertainty facilitates the intertemporal transfer of resources, and thus benefits the wealth-rich, who have more

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[4]This is cleanest in a linear tax system. However, even in a progressive tax system, the fluctuating average tax rates work like multiplicative after-tax income risk.

[5]Angeletos and Calvet (2006), in a seminal contribution on risk in incomplete markets, discuss this tension between labor endowment risk and rate-of-return uncertainty.

resources to save. Which of the two effects dominates, depends – as we will show – on the details of the implementation of the progressive tax system in the model. If the tax rate uncertainty induced by government purchases fluctuations is concentrated on households in the middle of the wealth distribution, the precautionary saving effect dominates, and the welfare gains are decreasing in household wealth. Under an implementation where the wealth-rich households are significantly exposed to the tax-rate volatility caused by fiscal uncertainty, the quality-of-saving-vehicle channel starts to matter more, and the distributional welfare gain pattern might be reversed.

Finally, the distributional effects of eliminating fiscal uncertainty can depend on general equilibrium price changes.[6] The precautionary saving and quality-of-saving-vehicle channels lead to endogenous responses of the aggregate capital stock along the transition path, changing both the pre-tax capital rate-of-return and real wages. In our baseline specification, the aggregate capital stock declines after the elimination of fiscal uncertainty. This favors the wealth-rich through higher pre-tax rates of return, but disadvantages the wealth-poor through lower pre-tax real wages.

In addition to our baseline, we consider three counterfactual fiscal regimes: a balanced budget with a progressive tax system, a linear tax system, and a lump-sum tax system, with the latter two allowing for government debt. In another variation, we show that when private and public consumption are complements, the overall welfare gains from eliminating government purchases fluctuations are higher, because a higher government purchases level leads to a higher marginal utility of private consumption when taxes are high (because government purchases are high). That is, taxes are high when households would receive substantial utility from additional private consumption, making government purchases fluctuations all the more unpleasant. Finally, motivated by recent policy discussions of the possible permanence of heightened fiscal uncertainty, we examine the welfare consequences if we double the historical government purchases volatility level. Our results suggest that the welfare effects of fiscal uncertainty are symmetric between zero and twice the pre-crisis volatility of government purchases.

In addition to its substantive contributions, our study makes a technical contribution to the literature. Specifically, we merge the algorithm for computing the *deterministic* transition path in heterogeneous-agent economies from Huggett (1997) and Krusell and Smith (1999), and the algorithm for computing a *stochastic* recursive equilibrium in Krusell and Smith (1998), to show that an approximation of the wealth

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[6]There is also a direct utility effect because households are risk averse with respect to government purchases fluctuations.

distribution and its law of motion by a finite number of moments can also be applied to a stochastic transition path analysis. Recall that after fiscal uncertainty is eliminated, our economy is still subject to aggregate productivity shocks. This solution method may prove useful for other quantitative studies of stochastic transition-path equilibria.

Related Literature

Besides the general link to the literature on incomplete markets and wealth inequality (see Heathcote et al. (2009) for an overview), our study is most closely related to three strands of literature.

First, our paper contributes to research on the welfare costs of aggregate fluctuations (see Lucas (2003) for a comprehensive discussion). As in Krusell and Smith (1999), Mukoyama and Sahin (2006) and Krusell et al. (2009), we quantify the welfare and distributional consequences of eliminating macroeconomic fluctuations. However, while these studies focus on TFP fluctuations, we examine the welfare consequences of eliminating fluctuations in government purchases. Our study complements theirs by examining fluctuations due to fiscal policy, arguably a more plausible candidate shock to be (fully) eliminated by a policy maker – they are, after all, the result of a policy decision.

Second, our paper relates to the recent literature about the effects of economic uncertainty on aggregate economic activity. Most of the research in this stream of literature has focused on the amplification and propagation mechanisms for persistent, but temporary uncertainty shocks, which are typically modeled and measured as changes to the conditional variance of traditional economic shocks. These uncertainty shocks include second-moment shocks to aggregate productivity, and policy and financial variables, which are often propagated through physical production factor adjustment costs, sticky prices, or financial frictions (see e.g., Arellano et al. (2012), Bachmann and Bayer (2013, 2014), Baker et al. (2015), Basu and Bundick (2012), Bloom (2009), Bloom et al. (2012), Born and Pfeifer (2014), Croce et al. (2012), Fernández-Villaverde et al. (2011), Gilchrist et al. (2014), Kelly et al. (2014), Mumtaz and Surico (2015), Mumtaz and Zanetti (2013), Nodari (2014), Pastor and Veronesi (2012, 2013), and Stokey (2015)). Other studies investigate the effects of uncertainty in the time-varying parameters of monetary or fiscal feedback rules (Bi et al. (2013), Davig and Leeper (2011), and Richter and Throckmorton (2015)). Our study complements this literature by focusing on the welfare and distributional effects of a permanent elimination of fiscal fluctuations.

Finally, our work contributes to the growing body of literature on macroeconomic

policy in environments with heterogeneous agent (Auclert (2015), Bachmann and Bai (2013), Bhandari et al. (2013), Boehm (2015), Brinca et al. (2015), Ferriere and Navarro (2014), Gornemann et al. (2012), Gomes et al. (2013), Heathcote (2005), Kaplan and Violante (2014), Li (2013) and McKay and Reis (2013)). There is also a budding empirical literature on the distributional consequences of policy actions: see Coibion et al. (2012) for the case of monetary policy, and Giorgi and Gambetti (2012) for the case of fiscal policy.

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 discusses its calibration. Section 4 describes our solution method. Section 5 presents the baseline findings on the welfare and distributional effects of eliminating government purchases fluctuations, while Section 6 investigates these welfare and distributional effects in alternative model specifications. We close in Section 7 with final comments and relegate the details of the quantitative procedure to various appendices.

2.2 Model

Following Aiyagari (1994) and Huggett (1993), we model an incomplete market setting where a continuum of infinitely-lived heterogeneous households face uninsurable idiosyncratic risks in their labor efficiency processes. We also include aggregate productivity shocks as well as shocks to a household's discount factor, as in Krusell and Smith (1998). We then add aggregate uncertainty from government purchases shocks. In our model exposition, we focus our discussion on the fiscal elements.

2.2.1 The private sector

Our households are *ex-ante* identical, with preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta_t u\left(c_t, G_t\right), \tag{2.2.1}$$

where β_t denotes the cumulative discount factor between period 0 and period t. In particular, $\beta_t = \tilde{\beta}\beta_{t-1}$, where $\tilde{\beta}$ is an idiosyncratic shock following a three-state, first-order Markov process. Furthermore, c_t denotes private consumption, and G_t the public good provided by the government (government purchases).

The strictly concave flow utility function has constant relative risk aversion (CRRA)

with respect to a constant-elasticity-of-substitution (CES) aggregate of c and G,

$$u\left(c_t, G_t\right) = \frac{\left(\theta c_t^{1-\rho} + (1-\theta) G_t^{1-\rho}\right)^{\frac{1-\gamma}{1-\rho}} - 1}{1 - \gamma}, \qquad (2.2.2)$$

where γ is the risk aversion parameter and $1/\rho$ is the elasticity of substitution between c and G. We discuss the details of the G_t-process in the next subsection.

Our households also face idiosyncratic employment shocks. We denote the employment process by ε, which follows a first-order Markov process with two states $\{0, 1\}$. $\varepsilon = 1$ denotes that the household is employed, providing a fixed amount of labor $\tilde{l}$ to the market, and is paid the market wage, w. $\varepsilon = 0$ represents the unemployed state of a household who receives an unemployment insurance payment that equals a fraction ω of the current wage income of an employed household.

We represent the aggregate production technology as a Cobb-Douglas function:

$$Y_t = z_t F(K_t, L_t) = z_t K_t^\alpha L_t^{1-\alpha}, \qquad (2.2.3)$$

where K_t is aggregate capital, L_t is aggregate labor efficiency input, and z_t is the aggregate productivity level. z_t follows a two-state (z_g, z_b) first-order Markov process, where z_g and z_b denote aggregate productivity in good and bad times, respectively. Note that, because of the law of large numbers, L_t equals $(1 - u_t)\tilde{l}$, where u_t is the unemployment rate. We also allow the unemployment rate to take one of two values: u_g in good times and u_b in bad times. In this way, u_t and z_t move perfectly together.

We now specify the standard aggregate resource constraint:

$$C_t + K_{t+1} + G_t = Y_t + (1-\delta)K_t, \qquad (2.2.4)$$

where C_t represents aggregate consumption, and δ the depreciation rate.

The markets in our model are perfectly competitive. Labor and capital services are traded on spot markets each period, at factor prices $r(K_t, L_t, z_t) = \alpha z_t K_t^{\alpha-1} L_t^{1-\alpha} - \delta$ and $w(K_t, L_t, z_t) = (1-\alpha)z_t K_t^\alpha L_t^{-\alpha}$. In addition, we assume that the households can trade one-period government bonds on the asset market in each period t. For computational tractability, we follow Heathcote (2005) and assume that government bonds pay the same rate-of-return as physical capital in all future states in $t + 1$. Because of the assumed perfect substitutability between capital and bonds, each household has access to effectively only one asset in self-insuring against stochastic shocks. We use a to denote a household's total asset holdings, i.e., the sum of physical

capital and government bonds.

2.2.2 Fiscal uncertainty and the government budget

Our model has three government spending components: government purchases, G_t, aggregate unemployment insurance payments, Tr_t, and aggregate debt repayments, $(1 + r_t)B_t$. Government purchases are the only fundamental source of fiscal uncertainty. They follow an AR(1) process in logarithms:

$$\log(G_{t+1}) = (1 - \rho_g)\overline{\log(G)} + \rho_g \log(G_t) + (1 - \rho_g^2)^{\frac{1}{2}}\sigma_g \epsilon_{g,t+1}, \qquad (2.2.5)$$

where ρ_g is a persistence parameter, $\overline{\log(G)}$ is the unconditional mean of $\log(G_t)$, $\epsilon_{g,t+1}$ is an innovation term which is normally distributed with mean zero and variance one, and σ_g is the unconditional standard deviation of $\log(G_t)$. Note that the government purchases process is independent of the process for aggregate productivity. As is well known and as we show below, government purchases are roughly acyclical in U.S. quarterly data.

The aggregate unemployment insurance payment, $Tr_t = u_t \omega w_t \tilde{l}$, depends on both the unemployment rate, u_t, and the size of the unemployment insurance payment for each household, $\omega w_t \tilde{l}$.

We assume that government spending at time t is financed through a combination of aggregate tax revenue, T_t, and new government debt, B_{t+1}. As in Bohn (1998) and Davig and Leeper (2011), we model the aggregate tax revenue net of transfers (as a fraction of GDP) as an (increasing) function of the debt-to-GDP ratio, making the debt-to-GDP ratio stationary. We can thus specify the following fiscal rule for determining tax revenue:

$$\frac{T_t - Tr_t}{Y_t} = \rho_{T,0} + \rho_{T,Y}log(\frac{Y_t}{\overline{Y}}) + \rho_{T,B}\frac{B_t}{Y_t} + \rho_{T,G}\frac{G_t}{Y_t}, \qquad (2.2.6)$$

where $(\rho_{T,0}, \rho_{T,Y}, \rho_{T,B}, \rho_{T,G})$ is a vector of positive coefficients and $\overline{Y}$ is a constant number equal to the unconditional mean of GDP in the ergodic distribution.[7] Furthermore, $\rho_{T,Y}$ captures the automatic stabilizer role of the U.S. tax system when $\rho_{T,Y} > 0$, and $\rho_{T,B}$ and $\rho_{T,G}$ reflect the capability of the endogenous revenue adjustment system in maintaining long-run fiscal sustainability. Note that our fiscal rule implies that the government purchases level (relative to GDP) and the GDP gap are

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[7]$\overline{Y}$ serves as a normalization to make the coefficients of the fiscal rule scale-free. Also, while $\rho_{T,B} > 0$ is necessary for the debt-to-GDP ratio to be stationary, this condition is not imposed. Instead, all coefficients in equation (2.2.6) are estimated from the data, and this estimated $\rho_{T,B}$ just turns out to be positive.

the main non-debt determinants of the primary surplus.

Given the total tax revenue in (2.2.6), we can use the government budget constraint to determine the dynamics of aggregate government debt B_{t+1}:

$$B_{t+1} = (1 + r_t)B_t + (G_t + Tr_t - T_t). \qquad (2.2.7)$$

2.2.3 The progressive tax system

Because the distribution of the tax burden across households is important for quantifying the distributional effects of fiscal policies, we model the tax system to approximate the current U.S. tax regime as realistically as possible while maintaining a certain tractability. Specifically, the government uses a flat-rate consumption tax and a progressive income tax to raise the aggregate tax revenue T_t. The consumption tax is given by:

$$\tau^c(c_t) = \tau_c c_t. \qquad (2.2.8)$$

This specification allows the model to capture sources of tax revenue other than income taxes, which in turn provides a total income tax burden that is in line with the data.

Following Castañeda et al. (2003), we specify the progressive income tax function as:

$$\tau^y(y_t) = \begin{cases} \tau_1 \left[y_t - \left(y_t^{-\tau_2} + s \right)^{-\frac{1}{\tau_2}} \right] + \tau_0 y_t & \text{if } y_t > 0 \\ 0 & \text{if } y_t \leq 0, \end{cases} \qquad (2.2.9)$$

where $(\tau_0, \tau_1, \tau_2, s)$ is a vector of tax coefficients and y_t is taxable household income; or $y_t = r_t a_t + w_t \varepsilon_t \tilde{l}$.[8] The first term in the above equation is based on Gouveia and Strauss' (1994) characterization of the effective federal income tax burden of U.S. households. The federal income tax accounts for about 40% of federal government revenue and is the main driver of progressivity in the U.S. tax system (Piketty and Saez, 2007). The linear term, $\tau_0 y_t$, is used to capture any remaining tax revenue, including state income taxes, property taxes and excise taxes.

With these tax specifications, a household's budget constraint can be written as:

$$(1 + \tau_c)c_t + a_{t+1} = a_t + y_t - \tau^y(y_t) + (1 - \varepsilon_t)\omega w(K_t, L_t, z_t)\tilde{l}. \qquad (2.2.10)$$

Note that equation (2.2.6) specifies a fiscal rule to calculate the aggregate govern-

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[8]Unlike in Castañeda et al. (2003), where households cannot borrow and thus cannot have negative income, y_t can be negative in our model in rare cases, so that we have to specify the tax function also for the case of $y_t < 0$.

ment tax revenue. Equations (2.2.8) and (2.2.9), on the other hand, model the concrete tax instruments with which the government collects the aggregate tax revenue. These two sets of equations are compatible only if we treat one of the parameters in equation (2.2.9) as an endogenous tax instrument, to be determined in equilibrium, rather than a fixed tax parameter. Following Conesa and Krüger (2006) and Conesa et al. (2009), we choose, in the baseline specification, s for this endogenous parameter, s_t, and denote the resulting tax function by $\tau^y(y_t; s_t)$. Adjusting s means that the overall progressivity of the tax system becomes the main tax instrument to raise the tax revenue required by the fiscal rule, while the top marginal (average) tax rates are approximately constant at $\tau_0 + \tau_1$.[9] This adjustment allows us to satisfy the empirical fiscal rule that describes aggregate U.S. tax adjustments well, and, more importantly, ensures the stationarity of the debt-to-GDP ratio. Consequently, we take the empirical fiscal rule as given and endogenously adjust one aspect of the tax system to make the two sets of equations compatible as in Davig and Leeper (2011) and Fernández-Villaverde et al. (2011).

Given our tax function specification, we can now specify total tax revenue as follows:

$$T_t = \tau_c C_t + \int_0^1 \left[\tau_0 y_{i,t} + \tau_1 \left(y_{i,t} - \left(y_{i,t}^{-\tau_2} + s_t \right)^{-\frac{1}{\tau_2}} \right) \right] * \mathbb{1}(y_{i,t} > 0) di. \qquad (2.2.11)$$

Equation (2.2.11) defines an implicit function of s_t. Recall that T_t is governed by G_t, Y_t, B_t, and Tr_t through the fiscal rule specified in equation (2.2.6). This means that, for a given inherited level of bond holdings, B_t, s_t fluctuates in response to changes in both G_t and the income distribution. As a result, in our baseline model the *aggregate* uncertainty in G_t translates into *idiosyncratic* tax rate uncertainty.

2.2.4   The household's decision problem and the competitive equilibrium

In this subsection, we discuss the household's dynamic decision problem, which is determined by both the idiosyncratic state vector $(a, \varepsilon, \tilde{\beta})$ and the aggregate state vector (Γ, B, z, G), where Γ denotes the measure of households over $(a, \varepsilon, \tilde{\beta})$. We

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[9]Both the derivative of equation (2.2.9) and equation (2.2.9) divided by y_t converge to $\tau_0 + \tau_1$ for large y_t. In Section 6, we examine two alternative specifications, where we let τ_0 and τ_1, respectively, be the tax instruments that adjust endogenously.

begin by letting H_Γ denote the equilibrium transition function for Γ:[10]

$$\Gamma' = H_\Gamma(\Gamma, B, z, G, z'). \tag{2.2.12}$$

We next let H_B denote the (exogenous) transition function for B, as described in equation (2.2.7):

$$B' = H_B(\Gamma, B, z, G). \tag{2.2.13}$$

Finally, we let S denote the equilibrium function for the endogenous tax parameter s, which is implicitly determined in equation (2.2.11):

$$s = S(\Gamma, B, z, G). \tag{2.2.14}$$

The dynamic programming problem faced by a household can now be written as follows:

$$V(a, \varepsilon, \tilde{\beta}, \Gamma, B, z, G; H_\Gamma, S) = \max_{c, a'} \{ u(c, G) + \tilde{\beta} E[V(a', \varepsilon', \tilde{\beta}', \Gamma', B', z', G'; H_\Gamma, S) | \varepsilon, \tilde{\beta}, z, G] \}$$

subject to:

$$(1 + \tau_c)c + a' = a + y - \tau^y(y; s) + (1 - \varepsilon)\omega w(K, L, z)\tilde{l}$$
$$y = r(K, L, z)a + w(K, L, z)\varepsilon\tilde{l},$$
$$a' \geq \underline{a},$$
$$\Gamma' = H_\Gamma(\Gamma, B, z, G, z'),$$
$$B' = H_B(\Gamma, B, z, G),$$
$$s = S(\Gamma, B, z, G),$$

where ε and $\tilde{\beta}$ follow the processes specified in Section 2.2.1, G follows the process specified in equation (2.2.5), and $\underline{a}$ is an exogenously set borrowing constraint. Finally, we can summarize the optimal saving decision for households in the following policy function:

$$a' = h(a, \varepsilon, \tilde{\beta}, \Gamma, B, z, G; H_\Gamma, S). \tag{2.2.15}$$

Our recursive competitive equilibrium is then defined as: the law of motion H_Γ,[11]

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[10]Note that z', but not G', is an argument of H_Γ. This is because, in our setting, which reflects the setting in Krusell and Smith (1998), the future z affects the employment transition process, while the G-process is independent of other processes. Note that we also leave time subscripts and switch into recursive notation now.

[11]Note that since H_B is exogenously determined by equation (2.2.7), it is not an equilibrium

individual value and policy functions $\{V, h\}$, pricing functions $\{r, w\}$, and the S-function for the endogenous parameter s, such that:

1. $\{V, h\}$ solve the household's problem.

2. $\{r, w\}$ are competitively determined.

3. S satisfies equation (2.2.11) with the fiscal rule (2.2.6) replacing T_t.

4. H_Γ is generated by h.

The economy without a fluctuating G_t is identical, except for the deterministic G_t-process.

2.3 Calibration

In this section, we discuss our model calibration beginning with basic parameters. The frequency of our model economy is quarterly. We parameterize the model to match important aggregate and cross-sectional statistics of the U.S. economy (Table 2.3.1).

2.3.1 Basic parameters

We set the relative risk aversion parameter $\gamma = 1$, and the elasticity of substitution between private consumption and the public good $1/\rho = 1$. To calibrate the weight of private consumption in the utility function, θ, we assume that the Lindahl-Samuelson condition holds for our economy in the long-run. This means that there is efficient provision of public goods, i.e., there are equalized marginal utilities from private and public goods. Mathematically, this is represented as $\int_0^1 \frac{(1-\theta)/G_t}{\theta/c_{it}} di = 1$, on average over many time periods. With this procedure, θ is calibrated to 0.722.

We take other parameter values directly from Krusell and Smith (1998): the depreciation rate is $\delta = 0.025$, the capital elasticity of output in the production function is $\alpha = 0.36$, and labor supply is normalized to $\tilde{l} = 0.3271$. We allow our aggregate productivity process, z_t, to take on two values, $z_g = 1.01$ and $z_b = 0.99$, with unemployment rates of $u_g = 0.04$ and $u_b = 0.1$, respectively. The transition matrix for z_t is as follows:

$$\begin{bmatrix} 0.875 & 0.125 \\ 0.125 & 0.875 \end{bmatrix},$$

_____

object.

Table 2.3.1: Summary of parameters

Parameter	Value	Description	Source / Target
Taken from the literature			
$1/\rho$	1.00	Substitutability between c and G	Standard value
γ	1.00	Relative risk aversion	Standard value
α	0.36	Capital share	Standard value
δ	0.025	Depreciation rate	Standard value
$\tilde{l}$	0.3271	Hours of labor supply of employed	Normalization
(z_l, z_h)	(0.99, 1.01)	Support of aggr. productivity	Krusell and Smith (1998)
$\Pi_{z,z'}$	See text	Transit matrix of aggr. productivity	Krusell and Smith (1998)
(u_g, u_b)	(4%, 10%)	Possible unemployment rates	Krusell and Smith (1998)
$\Pi_{\varepsilon\varepsilon'|zz'}$	See text	Transit matrix of emp. process	Krusell and Smith (1998)
ω	0.10	Replacement rate	Krusell and Smith (1998)
(τ_1, τ_2)	(0.258, 0.768)	Progressive tax function	Gouveia and Strauss (1994)
Estimated from the data			
τ_0	5.25%	Income tax parameter	
τ_c	8.14%	Consumption tax rate	
$\rho_{T,B}$	0.0173	Debt coefficient of fiscal rule	
$\rho_{T,Y}$	0.2820	Output coefficient of fiscal rule	
$\rho_{T,G}$	0.4835	Government purchases coefficient	
$(G_l/G_m, G_h/G_m)$	(0.951, 1.049)	Size of the G-shock	
$\Pi_{G,G'}$	See text	Transit matrix of the G-process	
Calibrated in the model			
θ	0.7221	Weight on private consumption	Lindahl-Samuelson condition
G_m	0.2319	Middle grid of the G-process	Mean G/Y (20.86%)
$\underline{a}$	-4.15	Borrowing constraint	Share with negative wealth (11%)
$\rho_{T,0}$	0.1007	Intercept of tax revenue rule	Mean B/Y (30%)
$\tilde{\beta}_m$	0.9919	Medium value of discount factor	Mean K/Y (2.5)
$\tilde{\beta}_h - \tilde{\beta}_m, \tilde{\beta}_m - \tilde{\beta}_l$	0.0046	Size of discount factor variation	Gini coeff. (0.79)
$\Pi_{\tilde{\beta},\tilde{\beta}'}$	See text	Transit matrix of discount factor	Share of top 1% wealthy (30%)

where rows represent the current state and columns represent the next period's state. The first row and column correspond to z_g. The transition matrix for the employment status, ε, is a function of both the current aggregate state (z) and the future aggregate state (z'). There are thus four possible cases, (z_g, z_g), (z_g, z_b), (z_b, z_g), and (z_b, z_b), corresponding to the following employment status transition matrices:[12]

$$\begin{bmatrix} 0.33 & 0.67 \\ 0.03 & 0.97 \end{bmatrix}, \begin{bmatrix} 0.75 & 0.25 \\ 0.07 & 0.93 \end{bmatrix}, \begin{bmatrix} 0.25 & 0.75 \\ 0.02 & 0.98 \end{bmatrix}, \begin{bmatrix} 0.60 & 0.40 \\ 0.04 & 0.96 \end{bmatrix},$$

where the first row and column correspond to $\varepsilon = 0$ (unemployed).

We calibrate the borrowing constraint and the idiosyncratic time preference process to match key features of the overall wealth distribution in the U.S. The borrowing constraint is set to $\underline{a} = -4.15$ to match the fraction of U.S. households with negative wealth holdings, 11%.[13]

$\tilde{\beta}$ takes on values from a symmetric grid, ($\tilde{\beta}_l = 0.9873, \tilde{\beta}_m = 0.9919, \tilde{\beta}_h = 0.9965$). In the invariant distribution, 96.5% of the population is in the middle state, and 1.75%

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[12]The numbers are rounded to the second decimal point.

[13]We check that the total resources available to a household, taking into account unemployment insurance benefits and the borrowing limit, are never negative under this calibration.

is distributed across each of the extreme points. The expected duration of the extreme discount factors is set at 50 years, to capture a dynastic element in the evolution of time preferences (Krusell and Smith (1998)). In addition, transitions occur only across adjacent values, where the transition probability from either extreme value to the middle grid is 1/200, and the transition probability from the middle grid to either extreme value is 7/77200. This Markov chain for $\tilde{\beta}$ allows our model to generate a long-run U.S. capital-output ratio of 2.5, and a Gini coefficient for the U.S. wealth distribution of 0.79, as reported by Krusell and Smith (1998). It also allows our model to match the wealth share of the top 1% (Krusell and Smith (1998)). An accurate calibration of this moment is important because, as we will show, the welfare effects of fiscal uncertainty for top wealth holders, characterized by high levels of buffer-stock savings and high capital income, can be quantitatively rather different from those for other households.

2.3.2 Fiscal parameters

Regarding the fiscal parameters, we set the unemployment insurance replacement rate, ω, to 10% of the current market wage income, in line with the data. From Stone and Chen (2014) we know that the overall replacement rate from unemployment insurance is about 46% of a worker's wage, and its average pre-2008 benefits duration is 15 weeks. This translates to about 53% of a worker's quarterly wage. In our case, since we spread the unemployment benefits through the agent's whole unemployment period and the average duration of unemployment in the model is about 2 quarters, this translates to about 27% of the quarterly wage level. Moreover, from Auray et al. (2014) we know about 60% out of all the unemployed workers were eligible for unemployment benefits from 1989 to 2012, and that about 75% of those eligible for benefits actually collected them. Thus, we set our unemployment insurance payment to be 10% of the market wage.[14]

To estimate the parameters related to fiscal policy and individual tax rates, we use U.S. quarterly data from 1960I to 2007IV. We restrict the data window up to 2007IV because, arguably, fiscal policy was special during and after the Great Recession and for calibration purposes we want to focus on "normal" times. We provide the details

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[14]Our calibration also matches the aggregate data on unemployment insurance well: 0.0049 for the average unemployment insurance to output ratio (0.0041 in the data), and 0.0021 for its standard deviation, after removing a linear trend (0.0019 in the data). In both the model and the data, the unemployment insurance-to-output ratio is countercyclical. Also note that in Krusell and Smith (1998), the unemployment insurance is treated as a fixed amount, ψ, calibrated to be about 10% of the long-run quarterly wage.

of our fiscal parameter estimation in Appendix 2.A. Here we briefly outline the general procedure.

For the government purchases process, we use the Rouwenhorst method (Rouwenhorst (1995)) to construct a three-state first-order Markov chain approximation to the AR(1) process of the linearly detrended $\log(G)$ series.[15] The middle grid point of the G-process, G_m, is calibrated using the average G/Y-ratio in the data; see Appendix A.1 for the details.

To determine the parameters of our fiscal tax revenue rule, we first estimate the federal revenue rule as in Bohn (1998) and Davig and Leeper (2011), and the state and local rule without debt. We then take the weighted average of the federal rule and the state and local rule to get the general government tax revenue function, the empirical counterpart of our model. We describe the details of this procedure in Appendix A.2.

For the consumption tax parameters and the linear part of the income tax function, we follow standard procedures and calculate the time series of the corresponding tax rates from the quarterly NIPA data (see, e.g., Fernández-Villaverde et al. (2011) and Mendoza et al. (1994)). We then take the time-series average values to obtain the following tax rates: $\tau_c = 8.14\%$ and $\tau_0 = 5.25\%$; see Appendix A.3 for the details.

To model the progressivity of the U.S. tax system, we set the parameters of the progressive part of the income tax function to the values estimated by Gouveia and Strauss (1994) for U.S. data from 1989 (see Castañeda et al. (2003) and Conesa and Krüger (2006)), the last year in their sample. Recall that the progressive part of the income tax has the following form:

$$\tau_1 \left[y - \left(y^{-\tau_2} + s \right)^{-\frac{1}{\tau_2}} \right]. \tag{2.3.1}$$

Note that equation (2.3.1) is linearly homogeneous in y, if s is readjusted appropriately. Consequently, doubling one's income would lead to a doubling of the tax revenue collected from this income, if s adjusts to the new scale. Since the units of s are an innocuous choice, we can use the estimated numbers from Gouveia and Strauss (1994), $(\tau_1, \tau_2) = (0.258, 0.768)$.[16] As mentioned earlier, s is an equilibrium object

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[15]Kopecky and Suen (2010) show that the Rouwenhorst method has an exact fit in terms of five important statistical properties: unconditional mean, unconditional variance, correlation, conditional mean and conditional variance. The last two properties are important for our elimination of fiscal uncertainty, where both the conditional mean and variance matter for the transition-path equilibrium.

[16]Note that the estimation in Gouveia and Strauss (1994) is carried out on annual federal income tax data, whereas our model frequency is quarterly. Given the nonlinear nature of the tax function

determined by the fiscal rule.

2.3.3 The wealth distribution and business cycle moments

In this section, we examine the wealth distribution and the business cycle moments, focusing on the fiscal variables, generated by our calibrated model. For our model to be a suitable laboratory for the experiment of eliminating fiscal uncertainty, and for producing reliable quantitative answers to our welfare and distributional questions, it should broadly match these aspects of the data.

Table 2.3.2 compares the long-run wealth distribution generated by our model with both the data and the model results in Krusell and Smith (1998). From Table 2.3.2, we see that our wealth distribution is a good match for the U.S. wealth distribution, especially for those in the top 1 percent.

Table 2.3.2: Wealth distribution

	% of wealth held by top					Fraction with	Gini
	1%	5%	10%	20%	30%	wealth< 0	coefficient
Model	31%	59%	72%	81%	87%	11%	0.79
K&S	24%	54%	72%	87%	91%	11%	0.81
Data	30%	51%	64%	79%	88%	11%	0.79

Notes: The wealth distribution in the data is taken from Krusell and Smith (1998). Household wealth in our model is the sum of physical capital and government bonds.

Table 2.3.3 provides the results of a comparison between the key business cycle moments generated by the model and those from the data. This comparison includes output, tax revenue, and government purchases volatility and persistence. We calculate the same moments for the output ratios of tax revenue, government purchases and federal government debt. Finally, we examine the co-movements of these series with output and government purchases.

From Table 2.3.3, we see that our baseline model is successful in matching most of the business cycle moments, with the exception of output volatility (which is about 70% larger in the model). Even without fiscal uncertainty, as in Krusell and Smith (1998), the model produces higher output fluctuations than found in the data, while the introduction of fiscal uncertainty does not contribute substantially to the volatility of output. To check whether our welfare results are affected by this feature of the model, we conduct a robustness check where we recalibrate the aggregate productivity

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(equation 2.3.1), this may raise a time aggregation issue. We therefore estimate the implied tax function from simulated annual income and annual tax payment data (aggregated from simulated quarterly observations). The results from this estimation are very close to those from the annual data.

Table 2.3.3: Business cycle moments

A: Data (1960 I - 2007 IV)						
	Y	T-Tr	G	(T-Tr/Y)	(G/Y)	(B/Y)
Standard deviation	0.0149	0.0543	0.0134	0.0123	0.0083	0.0772
Autocorrelation	0.8616	0.8134	0.7823	0.9045	0.9573	0.9945
Corr(Y,X)	1	0.7242	0.0992	0.4791	-0.3826	-0.0472
Corr(G,X)	0.0992	0.0352	1	0.0345	0.4806	-0.0281
B: Model simulation						
	Y	T-Tr	G	(T-Tr/Y)	(G/Y)	(B/Y)
Standard deviation	0.0235	0.0415	0.0123	0.0063	0.0086	0.0403
	(0.0018)	(0.0041)	(0.0025)	(0.0009)	(0.0012)	(0.0151)
Autocorrelation	0.5841	0.5870	0.6978	0.8185	0.8254	0.9754
	(0.0561)	(0.0558)	(0.0582)	(0.0572)	(0.0546)	(0.0305)
Corr(Y,X)	1	0.9892	-0.0013	0.6939	-0.6433	-0.1791
	(0)	(0.0043)	(0.1294)	(0.1044)	(0.0940)	(0.1537)
Corr(G,X)	-0.0013	0.1316	1	0.2488	0.3804	-0.0086
	(0.1294)	(0.1296)	(0)	(0.1116)	(0.1079)	(0.0571)

Notes: In Panel A, $Y, T - Tr$ and G are HP-filtered (with a smoothing parameter of 1600) real log series of output, tax revenue net of transfers and government purchases, respectively. $(T - Tr)/Y$, G/Y and B/Y are linearly detrended output ratios of tax revenue net of transfers, government purchases and federal government debt, respectively. The data sources are documented in Appendix A.2. In Panel B, all variables are defined and filtered the same way as those in Panel A. The reported numbers are the average values from 1,000 independent simulations of the same length as the data (192 quarters). We show the standard deviations across these simulations in parentheses.

process so that the model matches the output volatility in the data, leaving our baseline results unchanged.

2.4   Computation

2.4.1   Stochastic steady state

To compute the model's equilibrium with two aggregate shocks, we use the approximate aggregation technique proposed by Krusell and Smith (1998).[17] This technique assumes that households act as if only a limited set of moments of the wealth distribution matters for predicting the future of the economy, and that the aggregate result of their actions is consistent with their perceptions of how the economy evolves. However, in contrast to Krusell and Smith (1998), we find that higher moments of the wealth distribution are necessary in our model with progressive taxation. That is,

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[17]The solution method for the stochastic steady state of the model with only aggregate productivity shocks is the same, except that $G_t = G_m, \forall t$.

the accurate description of our economy's evolution requires a combination of average physical capital and the Gini coefficient of the wealth distribution.

Furthermore, the optimization problem in our model requires households to know the endogenous tax parameter, s. We therefore approximate the function S, as defined in equation (2.2.14), with a parameterized function of the same moments that represent the wealth distribution.[18] We can now state the following functional forms for H_Γ and S:[19]

$$log(K') = a_0(z, G) + a_1(z, G)log(K) + a_2(z, G)B + a_3(z, G)(log(K))^2 + a_4(z, G)B^2$$
$$+ a_5(z, G)B^3 + a_6(z, G)log(K)B + a_7(z, G)Gini(a), \qquad (2.4.1)$$

$$Gini(a') = \tilde{a}_0(z, G) + \tilde{a}_1(z, G)log(K) + \tilde{a}_2(z, G)B + \tilde{a}_3(z, G)(log(K))^2 + \tilde{a}_4(z, G)B^2$$
$$+ \tilde{a}_5(z, G)B^3 + \tilde{a}_6(z, G)log(K)B + \tilde{a}_7(z, G)Gini(a), \qquad (2.4.2)$$

$$log(s) = b_0(z, G) + b_1(z, G)log(K) + b_2(z, G)B + b_3(z, G)(log(K))^2 + b_4(z, G)B^2$$
$$+ b_5(z, G)B^3 + b_6(z, G)log(K)B + b_7(z, G)Gini(a), \qquad (2.4.3)$$

where K denotes the average physical capital, and $Gini(a)$ denotes the Gini coefficient of the wealth distribution. We compute the equilibrium using a fixed-point iteration procedure from the parameters in equations (2.4.1)-(2.4.3) onto themselves; see Appendix 2.B.1 for the details of the computational algorithm and Appendix 2.B.2 for the estimated equilibrium laws of motions.

A check of the one-step-ahead forecast accuracy yields $R2$s above 0.999997 for H_Γ (equations (2.4.1) and (2.4.2)), and above 0.9992 for S (equation (2.4.3)). However, as den Haan (2010) points out, high $R2$-statistics are not necessarily indicative of multi-step-ahead forecast accuracy. Hence, we also examine the 10-year ahead forecast errors of our model. This check shows that our forecast errors are small and unbiased; see Appendix 2.B.2 for the details.

2.4.2   Transition-path equilibrium

To study the welfare effects of eliminating fiscal uncertainty, we start with the ergodic distribution of the two-shock equilibrium. From time $t = 1$, we let G_t follow

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[18]This is in the same spirit as the bond price treatment in Krusell and Smith (1997).

[19]These specific functional forms perform best among a large set of (relatively parsimonious) functional forms tested.

its deterministic conditional mean along the transition path until it converges to G_m. While we do not take a stance on how this stabilization is brought about (Lucas (1987) and Krusell et al. (2009)), we do note that, in contrast to stabilizing aggregate productivity shocks, the G_t-process is arguably under more direct government control.

As stated, during the transition periods G_t follows a time-dependent deterministic conditional-mean process until it converges to G_m, i.e.,

$$G_t = [\mathbb{1}(G_1 = G_l), \mathbb{1}(G_1 = G_m), \mathbb{1}(G_1 = G_h)] \, \Pi_{GG'}^{t-1} \begin{bmatrix} G_l \\ G_m \\ G_h \end{bmatrix}, \qquad (2.4.4)$$

where $\Pi_{GG'}$ is the transition probability matrix of the G-process in the two-shock economy discussed in Appendix 2.A. Note that, depending on G_1, the G_t-paths will have different dynamics. For example, if $G_1 = G_m$, G_t will stay at G_m for all $t \geq 1$, and the economy will immediately transition to its long-run G level. However, if the economy starts the transition away from G_m, G_t converges to G_m over time through the deterministic process described in (2.4.4). In this case, the counterfactual economy will go through transitional dynamics to eventually reach the productivity-shock-only stochastic steady state.

Recall the assumption that the government purchases process is independent from other stochastic processes, which implies that none of the other exogenous stochastic processes changes during or after the elimination of the fiscal shocks. Therefore, our counterfactual economy features aggregate productivity shocks both during and after the transition. This creates a new technical challenge in addition to those present in previous transition path analyses of heterogeneous-agent economies (e.g., Huggett (1997) and Krusell and Smith (1999)). While these studies model a deterministic aggregate economy along the transition path, our stochastic setting with aggregate uncertainty produces an exponentially higher number of possible aggregate paths as the length of the transition period increases. This feature precludes computation of the equilibrium for all possible realizations of aggregate productivity shocks.

To address this challenge, we extend the approximate aggregation technique for our two-shock equilibrium to the transition-path setting: that is, we postulate that time-dependent prediction functions govern the evolution of the economy on the transition path, through the following set of laws of motions:

$$\Gamma_{t+1} = H_{\Gamma,t}^{trans}(\Gamma_t, B_t, z_t), \qquad (2.4.5)$$

$$s_t = S_t^{trans}(\Gamma_t, B_t, z_t), \qquad (2.4.6)$$

where t denotes an arbitrary period along the transition path. At the end of the transition path, the laws of motions converge to those in our one-shock equilibrium. Consequently, solving for the transition-path equilibrium is equivalent to finding the appropriate approximations for (2.4.5) and (2.4.6), such that the realized evolution of the economy is consistent with the postulated evolution; see Appendix 2.B.3 for the details of the algorithm. We find that the same functional forms we use for the stochastic steady state economy yield accurate predictions also for the transition-path equilibrium. That is, for every period on the transition path, we achieve a similar forecast accuracy as in the stochastic steady state two-shock economy; see Appendix 2.B.4 for the details.

2.5 Results

Following Lucas (1987), we measure the welfare costs of fiscal uncertainty as the proportional change in a household's life-time consumption (Consumption Equivalent Variation or λ), such that:

$$E_1[\sum_{t=1}^{\infty} \beta_t u((1+\lambda)c_t, G_t)] = E_1[\sum_{t=1}^{\infty} \beta_t u(\tilde{c}_t, \tilde{G}_t)], \qquad (2.5.1)$$

where c_t is consumption in the baseline economy with G_t-fluctuations, while $\tilde{c}_t$ is consumption in the counterfactual economy with a deterministic $\tilde{G}_t$-process.

2.5.1 Baseline results

To obtain our baseline results, we first calculate welfare gains conditional on wealth, employment status and time preference for every sample economy in the transition-path computation,[20] using the value functions from our two-shock and transition-path equilibria.[21] We then average these across the sample economies, including all possible values of G_1, the government purchases level when fiscal uncertainty is eliminated. The results, presented in Table 2.5.1, can thus be interpreted as the *ex-ante* expected welfare gains from eliminating fiscal uncertainty.

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[20]To start the transition-path simulation, we draw a large set (16,000) of independent joint distributions over $(a, \varepsilon, \tilde{\beta})$ from the simulation of the two-shock equilibrium; see Appendix 2.B.3 for the details.

[21]The right side of (2.5.1) is the value function from the transition-path equilibrium. Given the log-log utility assumption in the baseline calibration, the left side of (2.5.1) can be expressed using the value function from the two-shock equilibrium and λ; see Appendix 2.B.5 for the details of the derivation.

Table 2.5.1: Expected welfare gains λ (%)

	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	0.0298	0.0303	0.0307	0.0307	0.0304	0.0301	0.0290	0.0240	0.0242
$\varepsilon = 1$	0.0298	0.0302	0.0306	0.0307	0.0304	0.0301	0.0290	0.0240	0.0242
$\varepsilon = 0$	0.0299	0.0304	0.0309	0.0308	0.0305	0.0302	0.0290	0.0240	0.0242
$\tilde{\beta} = \tilde{\beta}_l$	0.0290	0.0293	0.0293	0.0291	0.0289	0.0287	0.0275	0.0240	0.0226
$\tilde{\beta} = \tilde{\beta}_m$	0.0299	0.0312	0.0310	0.0307	0.0304	0.0301	0.0291	0.0241	0.0236
$\tilde{\beta} = \tilde{\beta}_h$	0.0262	0.0316	0.0313	0.0309	0.0304	0.0300	0.0272	0.0232	0.0266

Notes: The wealth groups are presented in ascending order from left to right. The welfare number for a particular combination of ε (or $\tilde{\beta}$) and a wealth group is calculated as follows: we first draw a large set (16,000) of independent joint distributions over $(a, \varepsilon, \tilde{\beta})$ from the simulation of the two-shock equilibrium. These distributions are used to start the computation of the transition-path equilibria. For each sample economy, we then find all the individuals that fall into a particular wealth×employment status or wealth×preference category, and calculate their welfare gain according to equation (2.5.1). We then take the average over the individuals in a particular category to find the welfare numbers for a given sample economy. To arrive at the numbers in this table, we finally take the average across all the 16,000 samples.

The results in Table 2.5.1 show that the aggregate welfare gain, i.e., the average welfare change across the whole population, is about 0.03%, comparable in size to the results in Lucas (1987) and Krusell et al. (2009). We further find that the welfare gains decrease with wealth, employment status does not affect the welfare changes, and time preferences do not present a clear welfare gain pattern. In the next subsection, we examine the mechanisms affecting the welfare gains along the wealth dimension.

2.5.2 The mechanisms

Our analyses show that the decreasing-with-wealth welfare gain pattern is the result of three interacting channels: a direct utility change channel, a saving channel, and a general equilibrium price channel. In the direct utility change channel, the utility gains resulting from household risk aversion with respect to government purchases fluctuations are isolated. In the saving channel, two types of fiscal risk arising from tax rate fluctuations interact: an after-tax-wage risk and an after-tax-rate-of-return risk. These risks have a different effect on the precautionary saving behavior of households and the quality of both capital and bonds as saving vehicles. Finally, in the general equilibrium price channel, factor price changes along the transition path are reflected.

In the following sub-sections, we discuss each channel in turn. Given the separa-

bility of private and public goods in our baseline calibration, we can exactly separate the direct utility channel from the other two. By contrast, we cannot exactly separate the saving channel from the general equilibrium price channel.

2.5.2.1 The direct utility change channel

Since a household's utility over G is strictly concave, eliminating fluctuations in G leads to a direct increase in expected lifetime utility. To isolate this direct utility gain, we compute a λ_c such that:

$$E_1[\sum_{t=1}^{\infty} \beta_t u((1+\lambda_c)c_t, G_t)] = E_1[\sum_{t=1}^{\infty} \beta_t u(\tilde{c}_t, G_t)], \quad (2.5.2)$$

where c_t, $\tilde{c}_t$, and G_t are defined in the same way as before. That is, we calculate the amount of private consumption compensation required to achieve the same lifetime utility under the uncertain G-process as from consumption in the transition-path equilibrium. Since the stochastic G-process enters both sides of equation (2.5.2), λ_c isolates the joint effect of the saving and general equilibrium price channels. Thus, the difference between λ and λ_c measures the direct utility channel. Furthermore, with a separable flow utility function, λ_c can be computed using the following simpler equation:

$$E_1[\sum_{t=1}^{\infty} \beta_t log((1+\lambda_c)c_t)] = E_1[\sum_{t=1}^{\infty} \beta_t log(\tilde{c}_t)]. \quad (2.5.3)$$

The results, presented in Table 2.5.2, show positive, albeit smaller consumption-related welfare when fiscal uncertainty is eliminated. Thus, we conclude that the direct utility channel is quantitatively important for the overall level of welfare changes, but, distributionally, there is a more pronounced decline in welfare gains as private wealth increases.

2.5.2.2 The saving channel

Fluctuations in government purchases lead to more volatile individual tax rates, resulting in greater after-tax labor and capital income risk. Consequently, the effect of eliminating this uncertainty depends on a household's (heterogeneous) degree of self-insurance against income risks. As in other Bewley-type incomplete market economies, our households engage in precautionary saving. Wealthier households can better insure themselves against after-tax income risk. As a result, wealth-poor

Table 2.5.2: Expected welfare gains from private consumption, λ_c (%)

	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	0.0087	0.0095	0.0097	0.0096	0.0093	0.0090	0.0078	0.0028	0.0030
$\varepsilon = 1$	0.0087	0.0095	0.0097	0.0096	0.0093	0.0090	0.0078	0.0028	0.0030
$\varepsilon = 0$	0.0088	0.0096	0.0098	0.0097	0.0094	0.0091	0.0078	0.0028	0.0030
$\tilde{\beta} = \tilde{\beta}_l$	0.0086	0.0089	0.0088	0.0086	0.0084	0.0082	0.0070	0.0036	0.0021
$\tilde{\beta} = \tilde{\beta}_m$	0.0088	0.0101	0.0099	0.0096	0.0093	0.0090	0.0080	0.0030	0.0025
$\tilde{\beta} = \tilde{\beta}_h$	0.0044	0.0098	0.0095	0.0091	0.0086	0.0082	0.0054	0.0014	0.0048

Notes: The welfare numbers in this table are calculated as those in Table 2.5.1, using (2.5.3) instead of (2.5.1).

households should be more likely to benefit from the elimination of uncertainty.

However, the tax-rate uncertainty induced by the G-shocks also creates a rate-of-return risk on after-tax capital income, affecting the quality of both capital and bonds as saving vehicles. This rate-of-return risk makes households' intertemporal transfer of resources riskier. Since wealth-rich households have more exposure to this rate-of-return risk, they should be more likely to benefit from the elimination of fiscal uncertainty.

In short, the saving channel effects can favor the wealth-poor or the wealth-rich, depending on whether the precautionary saving or the quality-of-assets effect dominates. The strength of the quality-of-assets effect depends on how much individual tax-rate uncertainty fluctuations in government purchases generate for the wealth-rich households.

We illustrate the distributional effects from precautionary saving through changes in the volatility of consumption for different wealth levels. Although the quality of assets as saving vehicles also influences consumption volatility, we nevertheless believe this volatility appropriately captures the overall ability of households to self-insure through precautionary saving. Table 2.5.3, Panel A, presents the volatilities of $\tilde{\beta}(\varepsilon)$- and wealth-specific consumption time series for our two-shock economy (measured as the variance of log consumption).[22] These results show that household consumption

[22]The wealth percentiles in Table 2.5.3 are constructed differently than those in Table 2.5.1 and 2.5.2. Recall that, in Table 2.5.1 and 2.5.2, we calculate wealth percentiles for each sample economy. By contrast, for Table 2.5.3, we calculate wealth percentiles from the long-run ergodic distribution of the stochastic two-shock economy. This is more appropriate for analyzing consumption changes for a certain wealth percentile over time, as the associated wealth level remains constant. To compute the type-specific consumption volatilities, we proceed in several steps: First, for each combination of ε, $\tilde{\beta}$ and wealth, we collect the consumption data of a household with that particular characteristic. We do so for every period of the simulation used in the stochastic steady state equilibrium computation. We then construct a set of ε-, $\tilde{\beta}$- and wealth-specific consumption time

Table 2.5.3: Consumption volatility ($\times 100$)

Percentile	1%	5%	25%	50%	75%	95%	99%
	A: The two-shock economy						
All	0.120	0.110	0.102	0.095	0.087	0.040	0.014
$\varepsilon = 0$	0.222	0.125	0.109	0.100	0.091	0.041	0.014
$\varepsilon = 1$	0.113	0.109	0.102	0.095	0.087	0.040	0.014
$\tilde{\beta} = \tilde{\beta}_l$	0.112	0.096	0.088	0.084	0.078	0.037	0.012
$\tilde{\beta} = \tilde{\beta}_m$	0.121	0.110	0.103	0.096	0.087	0.040	0.014
$\tilde{\beta} = \tilde{\beta}_h$	0.123	0.115	0.105	0.099	0.092	0.045	0.016
	B: Difference between the two-shock and the one-shock economy						
All	0.029	0.029	0.027	0.025	0.023	0.011	0.004
$\varepsilon = 0$	0.022	0.026	0.027	0.025	0.023	0.011	0.004
$\varepsilon = 1$	0.030	0.029	0.027	0.025	0.023	0.011	0.004
$\tilde{\beta} = \tilde{\beta}_l$	0.027	0.026	0.024	0.023	0.021	0.010	0.004
$\tilde{\beta} = \tilde{\beta}_m$	0.029	0.029	0.027	0.025	0.023	0.011	0.004
$\tilde{\beta} = \tilde{\beta}_h$	0.031	0.030	0.028	0.026	0.024	0.012	0.005

Notes: Consumption volatility is measured as the variance of log consumption and calculated as follows: for each combination of ε, $\tilde{\beta}$, and wealth (see footnote 22 for how the wealth percentiles are calculated), we collect the consumption data of a household with that particular characteristic. We do so for every period of the simulation used in the stochastic steady state equilibrium computation and then construct a set of ε-, $\tilde{\beta}$-, and wealth-specific consumption time series. Next, we calculate the volatility of these disaggregate consumption time series and take the weighted (by the unconditional probability of each state) average over ε ($\tilde{\beta}$) to get the numbers conditional on $\tilde{\beta}$ (ε). To compute the consumption volatility for the one-shock economy, we use the stochastic steady state equilibrium simulation with only aggregate productivity shocks.

volatility is decreasing along the wealth dimension, which is consistent with the notion that wealthier households have less to gain from the elimination of fiscal uncertainty, as they are already better insured in the two-shock economy. Furthermore, the results in Panel B show that the difference in consumption volatilities between the two-shock and one-shock economies is substantially lower for our wealth-rich households.[23]

We next analyze the quality-of-assets effect by examining the change in saving behavior, across wealth groups, after the elimination of fiscal uncertainty. Reduced needs for saving (self-insurance) would indicate that the precautionary saving effect

series. Next, we calculate the volatility of these disaggregate consumption time series and take the weighted (by the unconditional probability of each state) average over ε ($\tilde{\beta}$) to obtain the volatilities conditional on $\tilde{\beta}$ (ε).

[23]For the unemployed households, the relationship is not monotone in wealth. This is because wealth-poor unemployed households have negligible income, so they are not exposed to much tax-rate uncertainty until they are re-employed. They thus experience a relatively smaller volatility reduction between the two and one-shock economies.

Figure 2.5.1: Policy function comparison - saving
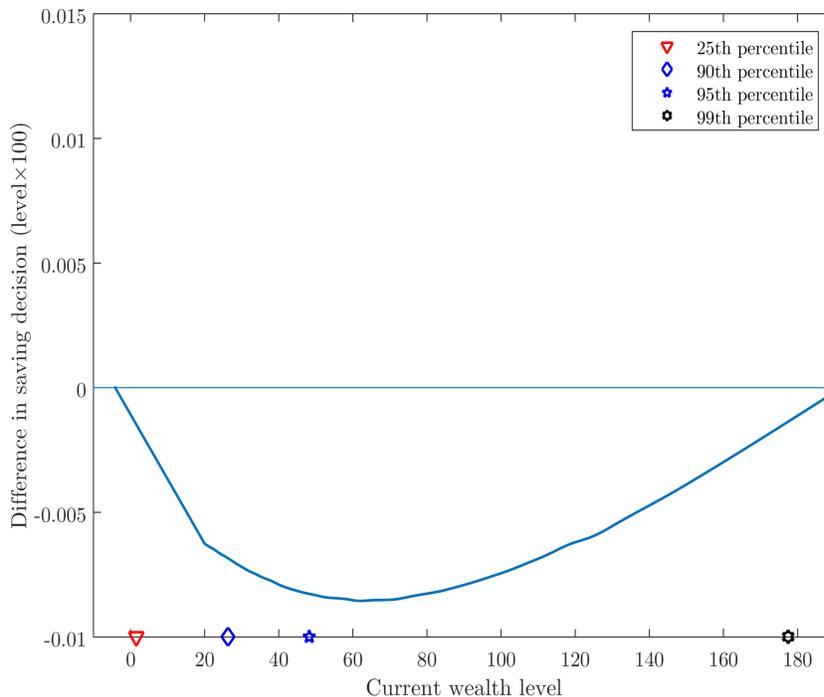
Notes: This figure shows the difference between the first-period policy function for saving from the transition equilibrium (with $G_1 = G_m$) and that from the two-shock equilibrium (with $G_1 = G_m$), evaluated at the long-run averages of (K, B), $z = z_g$, $\varepsilon = 1$, and $\tilde{\beta} = \tilde{\beta}_m$.

dominates, increased demand for saving would indicate a stronger quality-of-assets effect. The results in Figure 2.5.1 show a reduction in saving in the first period of the transition-path equilibrium compared to the two-shock equilibrium across all wealth classes.[24] Figure 2.5.1 also shows that the quality-of-assets effect is eventually increasing in wealth, but never dominates the precautionary saving effect. We will show in Section 2.6.1 that this dominance is specific to the baseline tax system as it depends on which tax instrument is used to raise the tax revenue required by the fiscal rule. Recall, that we use the parameter s in the progressive tax function for the cyclical tax revenue adjustment, and that τ_0 and τ_1 are constants. Since the marginal tax rate faced by the wealth-rich is close to $\tau_0 + \tau_1$, it is largely invariant to fluctuations in aggregate productivity or government purchases. Consequently, the wealth-rich face little tax-rate uncertainty in the baseline two-shock economy.

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[24]The policy function difference for saving is evaluated at $G_1 = G_m$, the long-run averages of $(K; B)$, $z = z_g$, $\varepsilon = 1$ and $\tilde{\beta} = \tilde{\beta}_m$. However, similar patterns hold for other combinations of state variables.

2.5.2.3 General equilibrium price channel

Figure 2.5.2: Expected aggregate capital path comparison
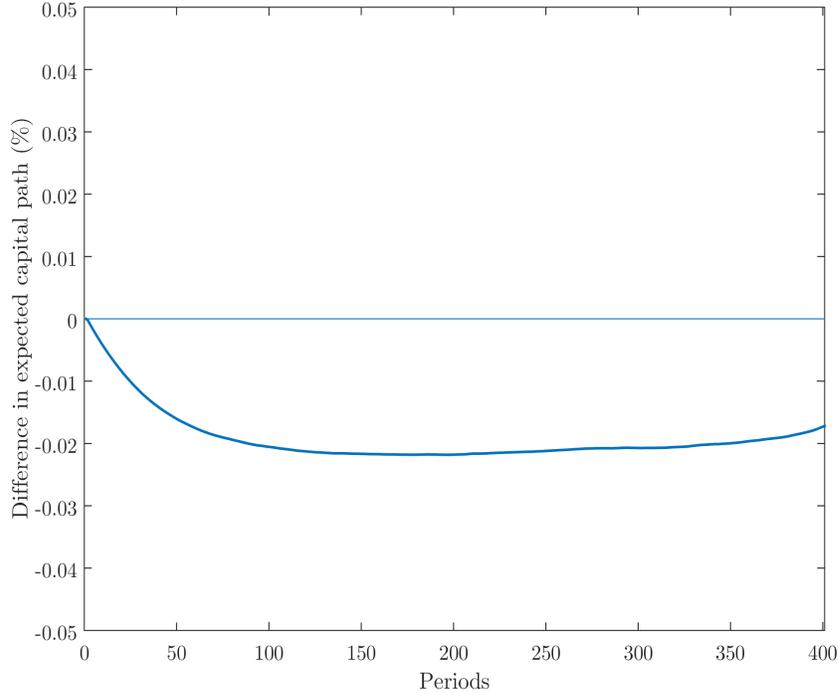
Notes: This figure shows the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium. We use the same 16,000 sample economies and the same sequences of z-shocks (for both the transition and the two-shock aggregate capital paths) as in the transition-path computation and then take the average. The G-shock sequences in the two-shock simulations are constructed in such a way that the cross-sectional joint distribution of (z, G)-shocks in each period is close to the invariant joint distribution.

We next examine the general equilibrium channel. In our model with a representative neoclassical firm, factor price changes follow aggregate capital stock changes. If the elimination of fiscal uncertainty lowers the aggregate capital stock, then pre-tax capital returns, all else being equal, will increase relative to wages. Because wealth-rich (wealth-poor) households have higher (lower) capital income shares, the wealth-rich (wealth-poor) households will benefit (lose) from this relative factor price change. As a result, changes in the aggregate capital stock will have distributional effects.

To examine the direction of the general equilibrium price channel for our baseline scenario, we compute the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium. The results

in Figure 2.5.2 show that the expected aggregate capital path in the transition-path equilibrium is slightly lower than it is in the two-shock equilibrium. We conclude that the general equilibrium factor price changes favor wealth-rich households. Recall that the overall distributional effect from the elimination of fiscal uncertainty favors the wealth-poor households. Overall, we see then that the saving channel dominates the general equilibrium price channel.

Table 2.5.4: Expected welfare gains from private consumption, λ_c (%), conditional on G_1

$G_1 = G_l$

	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
					Wealth Group				
All	0.0140	0.0155	0.0156	0.0153	0.0149	0.0145	0.0128	0.0059	0.0031
$\varepsilon = 1$	0.0140	0.0154	0.0156	0.0153	0.0149	0.0144	0.0128	0.0059	0.0031
$\varepsilon = 0$	0.0142	0.0156	0.0158	0.0155	0.0151	0.0146	0.0129	0.0059	0.0031
$\tilde{\beta} = \tilde{\beta}_l$	0.0144	0.0149	0.0148	0.0145	0.0141	0.0138	0.0118	0.0064	0.0025
$\tilde{\beta} = \tilde{\beta}_m$	0.0141	0.0160	0.0158	0.0153	0.0149	0.0145	0.0129	0.0059	0.0025
$\tilde{\beta} = \tilde{\beta}_h$	0.0091	0.0167	0.0165	0.0160	0.0154	0.0149	0.0112	0.0051	0.0051

$G_1 = G_m$

	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
					Wealth Group				
All	0.0087	0.0095	0.0097	0.0096	0.0094	0.0091	0.0078	0.0027	0.0028
$\varepsilon = 1$	0.0087	0.0095	0.0097	0.0096	0.0093	0.0091	0.0078	0.0027	0.0028
$\varepsilon = 0$	0.0088	0.0097	0.0099	0.0097	0.0094	0.0091	0.0079	0.0027	0.0028
$\tilde{\beta} = \tilde{\beta}_l$	0.0086	0.0089	0.0089	0.0087	0.0085	0.0082	0.0071	0.0036	0.0021
$\tilde{\beta} = \tilde{\beta}_m$	0.0088	0.0101	0.0099	0.0097	0.0094	0.0091	0.0080	0.0029	0.0024
$\tilde{\beta} = \tilde{\beta}_h$	0.0043	0.0097	0.0094	0.0090	0.0086	0.0081	0.0053	0.0013	0.0046

$G_1 = G_h$

	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
					Wealth Group				
All	0.0034	0.0034	0.0037	0.0038	0.0037	0.0036	0.0028	-0.0001	0.0031
$\varepsilon = 1$	0.0033	0.0034	0.0037	0.0038	0.0037	0.0036	0.0028	-0.0001	0.0031
$\varepsilon = 0$	0.0034	0.0035	0.0039	0.0039	0.0037	0.0036	0.0028	-0.0001	0.0031
$\tilde{\beta} = \tilde{\beta}_l$	0.0027	0.0028	0.0027	0.0027	0.0026	0.0025	0.0022	0.0007	0.0019
$\tilde{\beta} = \tilde{\beta}_m$	0.0034	0.0040	0.0040	0.0038	0.0037	0.0036	0.0030	0.0001	0.0026
$\tilde{\beta} = \tilde{\beta}_h$	0.0001	0.0029	0.0026	0.0023	0.0020	0.0017	-0.0003	-0.0018	0.0050

Notes: The welfare numbers in this table are calculated as in Table 2.5.2, but separately for $G_1 = G_l, G_m, G_h$, using 8,000 simulations for $G_1 = G_m$ and 4,000 simulations each for $G_1 = G_l, G_h$.

2.5.3   Distributional analysis conditional on G_1

Finally we examine the welfare gains from eliminating fiscal uncertainty *conditional* on G_1, the level of government purchases at the time the policy change is insti-

tuted. The results in Table 2.5.4 for λ_c reveal similar overall decreasing-with-wealth welfare gain patterns. However, when $G_1 = G_h$, the welfare gains are essentially flat implying that the general equilibrium price channel must be stronger in this case. This interpretation is consistent with the results in Figure 2.5.3, regarding the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium, conditional on G_1. For $G_1 = G_h$, we find the steepest decline in aggregate capital after the elimination of fiscal uncertainty, but an ultimately flat welfare gain pattern. By contrast, for $G_1 = G_l$, the aggregate capital path is flatter in the transition to a new stochastic steady state, but the welfare gains show the steepest decline.

Figure 2.5.3: Expected aggregate capital path comparison, conditional on G_1
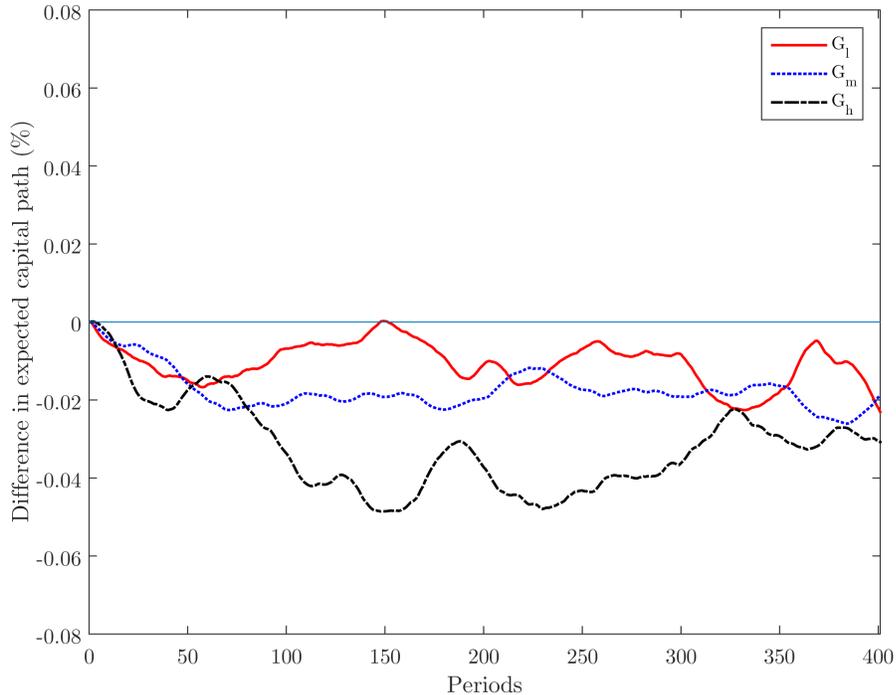
Notes: This figure shows the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium conditional on G_1. We use the same 16,000 sample economies and the same sequences of z-shocks (for both the transition and the two-shock aggregate capital paths) as in the transition-path computation and then average by G_1: 8,000 simulations for $G_1 = G_m$, and 4,000 simulations each for $G_1 = G_l, G_h$. Note that, due to our conditioning on G_1 and the subsequent smaller sample sizes, the expected aggregate capital paths in Figure 2.5.3 are more volatile compared to those in Figure 2.5.2.

2.6 Alternative specifications and additional experiments

In this section, we examine the welfare and distributional consequences of eliminating fiscal uncertainty under the following alternative model specifications: different flow utility functions, different adjustments to the progressive tax function, counterfactual fiscal regimes, and a model without TFP uncertainty. In addition, we examine our results when we double fiscal uncertainty, as well as when the elimination of fiscal uncertainty is accompanied by a sudden change in the level of government purchases. In each case, we re-calibrate parameter values when necessary to preserve target moment-data consistency. We summarize the welfare change results in terms of λ_c in Table 2.6.1. Table 2.C.1 in Appendix 2.C reports the corresponding λ-measures.

2.6.1 Alternative specifications

Non-separable utility. Recall that our baseline specification assumes a separable flow utility function in private and public consumption ($\rho = 1$), which implies that government purchases uncertainty affects the household decisions only indirectly, through equilibrium tax rate changes. By contrast, if public and private consumption are non-separable, then this uncertainty has a direct effect on the consumption-saving decision, since the government purchases level affects the marginal utility of private consumption.[25] We thus examine two alternative specifications where public consumption is an Edgeworth substitute (complement), $\rho = 0.5$ ($\rho = 1.5$), to private consumption.[26]

The results in rows 2 and 3 of Table 2.6.1 show that when G and c are complements (substitutes), the welfare gains from the elimination of fiscal uncertainty are larger (smaller) than in the baseline scenario. This is because the positive conditional comovement between G and taxes in the estimated fiscal rule makes uncertainty in G more costly when c and G are complements.[27] Since households face higher tax rates (lower disposable income) when G and the marginal utility of private consumption are high, the utility gain from fiscal uncertainty elimination in the case of complements is larger than in the separable case. An analogous argument applies when G and c are substitutes.

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[25]See Fiorito and Kollintzas (2004) for an overview of utility specifications for public consumption.

[26]To calculate λ_c with a non-separable utility function, we calculate the left side of (2.5.2) as a discounted sum of flow utilities under various values of λ_c, using the equilibrium policy function. We then find a value of λ_c that satisfies the equation numerically, using a bisection search.

[27]In the estimated fiscal rule, the tax-output ratio responds to the government-purchases-output ratio with a coefficient of $\rho_{T,G} = 0.484$; see Appendix A.2. for the details.

Table 2.6.1: Expected welfare gains from private consumption, λ_c (%), under different model specifications

	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
					Wealth Group				
Baseline	0.0087	0.0095	0.0097	0.0096	0.0093	0.0090	0.0078	0.0028	0.0030
Non-separable Utility Function									
Substitute ($\rho = 0.5$)	-0.0002	-0.0017	0.0008	0.0018	-0.0013	0.0027	-0.0030	-0.0078	-0.0011
Complement ($\rho = 1.5$)	0.0385	0.0267	0.0292	0.0346	0.0293	0.0459	0.0473	0.0372	0.0355
Different Tax Function Adjustment									
Adjusting τ_0	0.0084	0.0083	0.0086	0.0087	0.0084	0.0082	0.0077	0.0091	0.0135
Adjusting τ_1	0.0082	0.0081	0.0085	0.0085	0.0082	0.0079	0.0076	0.0101	0.0159
Counterfactual Fiscal Regimes									
Balanced Budget	0.0076	0.0097	0.0093	0.0087	0.0083	0.0079	0.0068	-0.0002	-0.0015
Linear Tax	0.0072	0.0067	0.0068	0.0070	0.0070	0.0070	0.0071	0.0103	0.0163
Lump-sum Tax	0.0073	0.0070	0.0070	0.0070	0.0069	0.0068	0.0071	0.0129	0.0204
Additional Experiments									
Constant TFP	0.0084	0.0091	0.0093	0.0092	0.0090	0.0087	0.0077	0.0034	0.0040
Double Volatility of G	-0.0071	-0.0081	-0.0083	-0.0081	-0.0077	-0.0073	-0.0061	-0.0011	-0.0012
Sudden change in G	0.0114	0.0148	0.0143	0.0135	0.0127	0.0119	0.0094	0.0001	-0.0015

Tax function adjustments. Recall that, in our baseline specification, the tax function parameter s is determined endogenously to satisfy the government's fiscal rule (equation (2.2.6)), while the linear (τ_0) and top marginal (τ_1) tax rates in the progressive tax function are fixed. In this case, a fluctuating s does not generate substantial tax-rate uncertainty for the very rich households as their marginal tax rate is close to the upper bound $\tau_0 + \tau_1 = const$. By contrast, it is the tax rates for the middle of the income distribution that respond the most to changes in s.

We therefore consider the following two alternative adjustments in the tax function: adjusting τ_0, the linear part in the income tax function, and adjusting τ_1, the parameter that governs the taxes for the top income brackets.[28] In both cases, s is fixed at its long-run average from the baseline stochastic two-shock economy. The simulated distributions of both τ_0 and τ_1 are symmetrically centered around their values in the baseline scenario (i.e., the ones estimated from the data).

The results in rows 4 and 5 of Table 2.6.1 show that other tax function adjustments yield similar overall welfare gains as the baseline case. However, unlike in the baseline scenario, the welfare gain for the top 5% of households is larger than the average welfare gain.

Since our wealth-rich are still better insured against fiscal uncertainty, the precautionary saving aspect does not drive our different distributional results: when we compute the analogue of Table 6 for our two alternative cases, the wealth-rich still

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[28]In the first case, all the households face the same tax rate changes in terms of absolute magnitude. However, in relative terms, the tax rate change is decreasing with income level. In the second case, all the households face the same percentage change in their tax rates. However, in terms of absolute magnitude, the tax rate change is increasing with income level.

experience the lowest reduction in consumption volatility. In addition, the general equilibrium channel in our two alternatives favors the wealth-poor households: the aggregate capital stock increases after the elimination of fiscal uncertainty, which leads to higher wages and a lower pre-tax capital rate-of-return. However, when τ_0 or τ_1 are adjusted to satisfy the fiscal rule, the wealth-rich face a higher tax rate uncertainty when government purchases fluctuate. They thus enjoy larger gains from a reduction in rate-of-return risk. We therefore conclude that the quality of capital (bonds) as a saving vehicle increases after the elimination of fiscal uncertainty, meaning that the wealth-rich benefit more.

Figure 2.6.1: Policy function comparisons - saving, adjusting τ_1
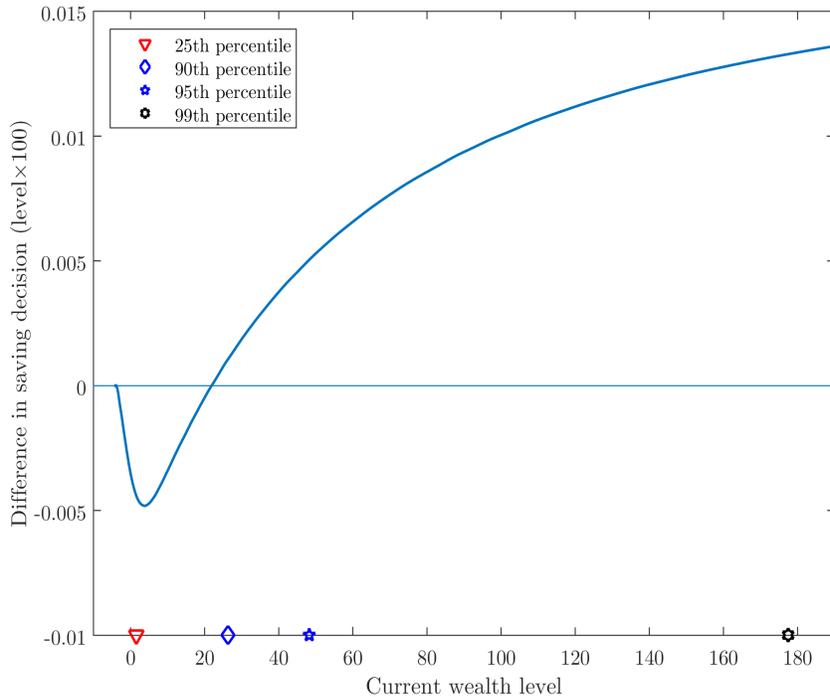
Notes: This figure shows the difference between the first-period policy function for saving from the transition equilibrium (with $G_1 = G_m$) and that from the two-shock equilibrium (with $G_1 = G_m$), evaluated at the long-run averages of (K, B), $z = z_g$, $\varepsilon = 1$, and $\tilde{\beta} = \tilde{\beta}_m$.

We illustrate this in Figure 2.6.1. From Figure 2.6.1, we see that there is a reduction in saving in the first period of the transition-path equilibrium compared to the two-shock equilibrium until approximately the 90th wealth percentile.[29] However,

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[29]Figure 2.6.1 is the analog of Figure 2.5.1 for the τ_1-adjustment case. The τ_0-adjustment case looks very similar to the τ_1-adjustment case.

this saving reductions for less wealthy households is weaker than in the baseline case. In addition, the wealth-rich now increase their saving after the elimination of fiscal uncertainty. Thus, in this case, the quality-of-assets mechanism dominates the precautionary saving mechanism for wealth-rich households.

The different welfare gain patterns in the baseline scenario vis-à-vis the τ_1/τ_0-adjustments have important policy implications: the distributional effects of eliminating fiscal uncertainty depend on which wealth group experiences the tax uncertainty burden that is caused by government purchases shocks.

Counterfactual fiscal regimes.   We next use three counterfactual fiscal regimes and examine their respective welfare results. This analysis will shed additional light on the mechanisms behind the welfare effects of the elimination of fiscal uncertainty. In our first regime, a balanced budget scenario, we dispense with the fiscal rule (equation (2.2.6)) and assume that government spending is financed exclusively through tax revenue. In our next two regimes, a linear (lump-sum) tax scenario, we keep the fiscal rule but change the progressive tax system to a linear (lump-sum) tax, $\tau_1 = 0$ ($\tau_0 = 0$ and $\tau_2 = -1$). The linear tax rate (the lump-sum tax amount) are then endogenously determined to satisfy the fiscal rule. Note that the lump-sum tax is imposed only on employed households to avoid negative after-tax incomes.[30]

We present the results of this set of analyses in rows 6 to 8 of Table 2.6.1. Overall, the welfare gains are similar to those obtained in our baseline analysis. Distributionally, the negative slope of the welfare gains along the wealth dimension is steeper in the balanced-budget regime. The results in Table 2.6.2 show the changes in consumption volatility between the two and one-shock economy for the balanced budget case (Panel B) compared to those in the baseline case (Panel A). In particular the balanced budget volatility changes for the wealth-rich are quite small, reflecting the negative correlation between the aggregate tax rate ($(T - Tr)/Y$) and the rate-of-return to capital (Table 2.6.3). By contrast, our baseline case has a positive correlation between the aggregate tax rate and the rate-of-return to capital, which is higher in the one-shock case.[31] In short, since in the balanced-budget regime the wealth-rich tend

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[30]We note that this treatment is slightly different from all the other cases, where tax payments are zero if and only if $y \leq 0$. With lump sum taxes, this could potentially introduce a discontinuity in the budget constraint of the unemployed at $y = 0$ (for the employed, it must hold that $y - T \geq 0$ at the borrowing limit, and thus, *a fortiori*, for all employed households).

[31]The reason for the different sign of these correlations lies in the different cyclicality of the average tax rate (net of transfers) in the two models (the rate-of-return to capital is always procyclical, independent of the fiscal regime). In the baseline model, just as in the data, the average tax rate is procyclical because the estimated fiscal rule captures automatic stabilizers: $\rho_{T,Y} > 0$. In the balanced budget regime, by contrast, the cyclicality of the average tax rate is essentially determined

Table 2.6.2: Consumption volatility differences between the two-shock and the one-shock economy

Percentile	1%	5%	25%	50%	75%	95%	99%
	A: Baseline						
All	0.029	0.029	0.027	0.025	0.023	0.011	0.004
$\varepsilon = 0$	0.022	0.026	0.027	0.025	0.023	0.011	0.004
$\varepsilon = 1$	0.030	0.029	0.027	0.025	0.023	0.011	0.004
$\tilde{\beta} = \tilde{\beta}_l$	0.027	0.026	0.024	0.023	0.021	0.010	0.004
$\tilde{\beta} = \tilde{\beta}_m$	0.029	0.029	0.027	0.025	0.023	0.011	0.004
$\tilde{\beta} = \tilde{\beta}_h$	0.031	0.030	0.028	0.026	0.024	0.012	0.005
	B: Balanced budget						
All	0.016	0.014	0.012	0.011	0.009	0.004	0.001
$\varepsilon = 0$	0.013	0.013	0.011	0.010	0.009	0.004	0.001
$\varepsilon = 1$	0.016	0.014	0.012	0.011	0.009	0.004	0.001
$\tilde{\beta} = \tilde{\beta}_l$	0.017	0.015	0.012	0.011	0.009	0.004	0.001
$\tilde{\beta} = \tilde{\beta}_m$	0.016	0.014	0.012	0.011	0.009	0.004	0.001
$\tilde{\beta} = \tilde{\beta}_h$	0.013	0.012	0.011	0.010	0.009	0.004	0.001

Notes: Panel A is Panel B in Table 2.5.3. Panel B is calculated here the same way as Panel A (see notes to Table 2.5.3), but from the balanced budget simulations.

to have higher tax rates when the rate-of-return to capital is low, they enjoy a smaller consumption volatility reduction.

Table 2.6.3: Correlation between the aggregate tax rate and the capital rate-of-return

	A: Baseline		B: Balanced Budget	
	two-shock	one-shock	two-shock	one-shock
$\text{Corr}(r,(T-Tr)/Y)$	0.8307	0.8971	-0.5760	-0.7895
	(0.0666)	(0.0495)	(0.1506)	(0.0942)

Notes: r and $(T-Tr)/Y$ are the rate-of-return to capital and the ratio of tax revenue net of transfers to output from model simulations, both linearly detrended. The correlations are the average values from 1,000 independent simulations of the same length as the data (192 quarters). We show the standard deviations across these simulations in parentheses.

Turning next to the linear tax case, we find that the welfare gains are increasing in wealth, just as in the case of τ_1- or τ_0-adjustment, as seen in row 7 of Table 2.6.1. Indeed, the mechanism is the same: in a linear tax regime, using the tax rate for cyclical tax adjustment increases the after-tax rate-of-return uncertainty for the wealth-rich.

by its denominator, aggregate output, because aggregate taxes are given by the government purchases level, which is assumed to be unrelated to the business cycle. This underscores the empirical importance of having a fiscal rule and including debt in our analysis.

Consequently, the elimination of this uncertainty provides them with a better saving vehicle. Indeed, when we compare the saving policy function between the two-shock and the transition-path equilibrium, we find an almost identical pattern as that in the τ_1-adjustment case (Figure 2.6.1).

Finally, our results for the lump-sum tax case (row 8 of Table 2.6.1) show that the welfare gains are again increasing in wealth. We also find again that the policy function comparison between the transition-path and the two-shock equilibrium looks similar to that in Figure 2.6.1. In other words, the rich save more when fiscal uncertainty is eliminated. However, since for the lump-sum tax case there is no change in the after-tax capital return uncertainty after the elimination of fiscal uncertainty, a different mechanism than for the linear tax (τ_1-adjustment) case must be at work. Indeed, the aggregate capital stock decreases after the elimination of uncertainty in the lump-sum tax case, that is, the general equilibrium factor price effect favors the wealth-rich. Overall, since there is no distortion from high tax rates, the higher return makes capital more attractive as a saving vehicle after the elimination of uncertainty, leading to benefits for the wealth-rich. The only difference is that the quality-of-asset change is an increase in the average expected return instead of a decrease in the return volatility.

No TFP uncertainty. Recall that our baseline scenario adopts the same TFP and unemployment processes as used in Krusell and Smith (1998). However, these choices produce an output volatility in the model that is 70% larger than that in the data (Section 2.3.3). To examine whether this difference affects our welfare results, we match the output volatility in the data by keeping TFP constant (at $z = 1$), but allowing unemployment rate fluctuations. The results in row 9 in Table 2.6.1 are similar to those from the baseline model, suggesting that the excess output volatility in our model does not influence our welfare results.

2.6.2   Additional experiments

Transition to a higher level of fiscal uncertainty. As mentioned, one topic that has received vigorous debate is how permanently heightened fiscal policy uncertainty might impact aggregate economic activity and welfare. To address this question, we let the economy transition to a level of fiscal uncertainty which is twice that in our baseline economy. Appendix 2.C provides the details of the computational implementation of this experiment.

The penultimate row in Table 2.6.1 shows the welfare changes from this magnified

uncertainty experiment. As in the baseline experiment, higher fiscal uncertainty leads to a welfare loss for every wealth group, with wealth-rich households experiencing a smaller loss. Overall, the numbers suggest that, at least for the range between zero and twice the pre-crisis level of fiscal volatility, the welfare effects of fiscal uncertainty are roughly symmetric.

Sudden change in the level of government purchases. In a final experiment, we examine the consequences of a concomitant sudden change in the government purchases level by letting government purchases move to and stay at their unconditional mean value, G_m, immediately after the elimination of fiscal uncertainty. We view this and the baseline scenario, where government purchases gradually converge to their long-run level, as two extreme ways of how fiscal uncertainty can be eliminated.

From the results in the last row of Table 2.6.1, we see that the unconditional welfare gains with a sudden change in the level of government purchases are similar but not identical to those in the baseline case, suggesting that the effect of a sudden change in G is not symmetric between $G_1 = G_l$ and $G_1 = G_h$ (the $G_1 = G_m$-case is the same as in the baseline scenario).

By contrast, the results in Table 2.6.4 show that the welfare changes conditional on $G_1 = G_l$ and $G_1 = G_h$ are one order of magnitude larger than those in the baseline case (Table 2.5.4). For the $G_1 = G_l$-case, the welfare changes are increasing in the wealth level, while the opposite pattern holds for the case of $G_1 = G_h$. However, these patterns are not driven by the elimination of fiscal uncertainty *per se*. The sudden change in the level of government purchases (and hence taxation) leads to a faster aggregate capital stock adjustment and a larger effect on welfare. For instance in the $G_1 = G_l$-case, the sudden increase in government purchases leads to a faster decrease in aggregate capital, output, and average welfare. However, since lower aggregate capital levels (higher pre-tax rates of return) favor the wealth-rich capital income earners, the welfare change pattern increases with wealth. Following a similar intuition, the distributional effect for the $G_1 = G_h$-case is reversed.

2.7 Conclusion

The recent recession and the economy's slow recovery have sparked a debate over the economic effects of fiscal uncertainty. Commentators have argued that uncertainty over future taxation and spending policies negatively affects contemporary economic outcomes. In this study, we quantify the welfare costs of fiscal uncertainty and their

Table 2.6.4: Expected welfare gains from private consumption, λ_c (%), conditional on G_1, sudden change in G

$G_1 = G_l$	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	-0.3864	-0.5347	-0.5032	-0.4653	-0.4372	-0.4089	-0.3128	0.0591	0.3914
$\varepsilon = 1$	-0.3862	-0.5362	-0.5041	-0.4655	-0.4374	-0.4092	-0.3134	0.0590	0.3914
$\varepsilon = 0$	-0.3877	-0.5263	-0.4954	-0.4628	-0.4345	-0.4060	-0.3056	0.0602	0.3913
$\tilde{\beta} = \tilde{\beta}_l$	-0.5197	-0.5606	-0.5458	-0.5158	-0.4839	-0.4556	-0.3308	-0.0158	0.2921
$\tilde{\beta} = \tilde{\beta}_m$	-0.3908	-0.5110	-0.4927	-0.4648	-0.4372	-0.4091	-0.3194	0.0443	0.3696
$\tilde{\beta} = \tilde{\beta}_h$	-0.0065	-0.4392	-0.4149	-0.3848	-0.3572	-0.3274	-0.1639	0.1501	0.4899
$G_1 = G_h$	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	0.4146	0.5748	0.5407	0.5000	0.4692	0.4383	0.3347	-0.0642	-0.4033
$\varepsilon = 1$	0.4144	0.5764	0.5416	0.5002	0.4694	0.4385	0.3352	-0.0641	-0.4033
$\varepsilon = 0$	0.4164	0.5665	0.5333	0.4977	0.4666	0.4356	0.3279	-0.0646	-0.4036
$\tilde{\beta} = \tilde{\beta}_l$	0.5554	0.5995	0.5835	0.5501	0.5154	0.4847	0.3511	0.0154	-0.3067
$\tilde{\beta} = \tilde{\beta}_m$	0.4194	0.5508	0.5304	0.4995	0.4692	0.4385	0.3417	-0.0486	-0.3818
$\tilde{\beta} = \tilde{\beta}_h$	0.0102	0.4786	0.4509	0.4173	0.3868	0.3541	0.1764	-0.1602	-0.4991

distribution in a neoclassical stochastic growth environment with incomplete markets. In our model, aggregate uncertainty arises from both productivity and government purchases shocks. Government spending is financed by a progressive tax system, modeled after important features of the U.S. tax system. We calibrate the model to U.S. data and evaluate the welfare and distributional consequences of eliminating government purchases shocks.

Our baseline results show that the welfare gains of eliminating fiscal uncertainty are decreasing in wealth. However, we also find that the effects of eliminating fiscal uncertainty depend on the fiscal regime and which group's after-tax rate-of-return to capital is affected the most by fiscal uncertainty. We also find that the welfare gains of eliminating fiscal uncertainty are higher when public and private consumption are complements.

While our study provides insight into the impact of eliminating fiscal uncertainty, it should be viewed as a first step towards a comprehensive analysis of the welfare and distributional implications of fiscal uncertainty. Future research could explore how our results change if nominal frictions that cause relative price distortions are added to the model. It could also examine wait-and-see effects. There is also no role in our model for a direct influence of government purchases on the unemployment process and thus cyclical idiosyncratic risk. Including this feature in a future quantitative analysis would require the development of a statistical model of how government purchases influence idiosyncratic unemployment processes, but such a model is elusive in the literature. Instead, since government purchases appear to be independent of the

cycle in U.S. post-war data, we have also used this assumption in the model. Furthermore, we have chosen to place exogenous uncertainty fundamentally within the level of government purchases, while the uncertainty of individual tax rates is derived from our model. Among fiscal data, we view the official aggregate data on government purchases as cleanest and least subject to construction choices, but recognize that the data on tax rates collected in Mertens (2013) could provide an alternative route. Finally, we model government purchases as a symmetric autoregressive process. However, future research could examine fiscal uncertainty in an economy facing the risk of very large government purchases as a very rare and dramatic event.

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2.A Appendix: Estimation of the fiscal parameters

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For the calibration, we use quarterly data from 1960I to 2007IV.

2.A.1 Government purchases process

We first construct a real government purchases (G) series by deflating the "Government consumption expenditures and gross investment" series (from NIPA table 3.9.5, line 1) with the GDP deflator (from NIPA table 1.1.9, line 1). We then estimate an AR(1) process for the linearly detrended real $\log(G)$ series. We use the Rouwenhorst method (see Rouwenhorst (1995)) to approximate this zero-mean AR(1) process with a three-state Markov Chain. This gives us a transition probability matrix, and a grid in the form $(-m,0,m)$, where m represents the percentage deviation from the middle grid point. The middle grid point of the G-process, G_m, is then calibrated to match the time series average of nominal G over nominal GDP from U.S. national accounting data (nominal GDP is from NIPA table 1.1.5, line 1), 20.86%. The grid for G is given by (G_l, G_m, G_h), where $G_h = (1+m)G_m$ and $G_l = (1-m)G_m$, and the discretized G-process on $[0.2205, 0.2319, 0.2433]$ has the following transition matrix:

$$\begin{bmatrix} 0.9607 & 0.0389 & 0.0004 \\ 0.0195 & 0.9611 & 0.0195 \\ 0.0004 & 0.0389 & 0.9607 \end{bmatrix}.$$

2.A.2 Fiscal rule

2.A.2.1 Methodology and estimation results

We first estimate the fiscal rule separately at two levels of government: the federal government level and the state/local level, allowing for debt only at the federal level.[32] We then construct a composite rule, using the share of federal government purchases in total government purchases.

---

[32]When we estimate one equation, using the sum of federal and the state-local level data, the estimation result implies a non-stationary government debt process.

The empirical specification for the federal fiscal rule is based on Bohn (1998) and Davig and Leeper (2011) and takes the following form:

$$\frac{T_t^F - Tr_t^F}{Y_t} = \rho_{T,0}^F + \rho_{T,B}^F \frac{B_t}{Y_t} + \rho_{T,Y}^F log(\frac{Y_t}{\bar{Y}_t}) + \rho_{T,G}^F \frac{G_t^F}{Y_t}, \qquad (2.A.1)$$

where:

Y_t: Nominal GDP (Line 1 of NIPA table 1.1.5).

$T_t^F - Tr_t^F$: Federal government current receipts (Line 1 of NIPA table 3.2) minus federal government transfer expenditure (Line 25 of NIPA table 3.2).

B_t: Market value of privately held gross federal debt at the beginning of a quarter: data are from the Federal Reserve Bank of Dallas (`http://www.dallasfed.org/research/econdata/govdebt.cfm`).

$\bar{Y}_t$: Nominal CBO potential GDP: data are from the CBO website (`http://www.cbo.gov/publication/42912`).

G_t^F: Nominal federal government consumption expenditures and gross investment (Line 23 of NIPA table 1.1.5).

At the state and local level, we drop the debt-to-GDP ratio term, yielding the following equation for the state and local level:

$$\frac{T_t^{SL} - Tr_t^{SL}}{Y_t} = \rho_{T,0}^{SL} + \rho_{T,Y}^{SL} log(\frac{Y_t}{\bar{Y}_t}) + \rho_{T,G}^{SL} \frac{G_t^{SL}}{Y_t}, \qquad (2.A.2)$$

where:

$T_t^{SL} - Tr_t^{SL}$: State and local government receipts (Line 1 of NIPA table 3.3) minus state and local government transfer expenditure (Line 24 of NIPA table 3.3).

G_t^{SL}: Nominal state and local government consumption expenditures and gross investment (Line 26 of NIPA table 1.1.5).

We then linearly detrend all ratio variables, except for $log(Y_t/\bar{Y}_t)$, before estimating equations (2.A.1) and (2.A.2).

Table 2.A.1 summarizes the estimation results.

Table 2.A.1: Estimated coefficients of the fiscal rule

	constant	B_t/Y_t	$log(Y_t/\bar{Y}_t)$	G_t/Y_t
Federal	-0.009	0.017	0.321	0.146
	(0.001)	(0.004)	(0.032)	(0.096)
State and local	0.001	–	-0.039	0.771
	(0.001)	–	(0.015)	(0.063)

2.A.2.2 The composite fiscal rule

The composite fiscal rule used in our model is given by:

$$
\begin{aligned}
\frac{T_t - Tr_t}{Y_t} &= \rho_{T,0} + \rho_{T,B}\frac{B_t}{Y_t} + \rho_{T,Y}log(\frac{Y_t}{\bar{Y}_t}) + \rho_{T,G}\frac{G_t}{Y_t} \\
&= \rho_{T,0} + \rho_{T,B}^F\frac{B_t}{Y_t} + (\rho_{T,Y}^F + \rho_{T,Y}^{SL})log(\frac{Y_t}{\bar{Y}_t}) + (\gamma^F\rho_{T,G}^F + (1-\gamma^F)\rho_{T,G}^{SL})\frac{G_t}{Y_t},
\end{aligned}
$$
(2.A.3)

where γ^F is calibrated as the average share of federal government purchases within total government purchases: 0.46. This yields the following fiscal rule parameters:

$$\rho_{T,B} = 0.017, \quad \rho_{T,Y} = 0.282, \quad \rho_{T,G} = 0.484.$$

We use $\rho_{T,0}$ to match the average debt-to-GDP ratio in the data: 30%.

2.A.3 Consumption and income tax parameters

For the consumption tax function and the linear part of the income tax function, we use the average tax rate calculated from the data.

To be specific, the average tax rate on consumption is defined as:

$$\tau_c = \frac{TPI - PRT}{PCE - (TPI - PRT)},$$
(2.A.4)

where the numerator is taxes on production and imports (TPI, NIPA table 3.1, line 4) minus state and local property taxes (PRT, NIPA table 3.3, line 8). The denominator is personal consumption expenditures (PCE, NIPA table 1.1.5, line 2) net of the numerator. We calculate the average $\tau_{c,t}$ over our sample period as our τ_c parameter: 8.14%.

For income taxes, we use the state level tax revenue to approximate the linear part:

$$\tau_0 = \frac{PIT + CT + PRT}{\text{Taxable Income}}, \tag{2.A.5}$$

where PIT (NIPA table 3.3, line 4) is state income tax, CT (NIPA table 3.3, line 10) is state tax on corporate income, and PRT (NIPA table 3.3, line 8) is state property taxes. Note that we exclude the social insurance contribution in the numerator since we do not have social security expenditures in the model. The denominator is GDP minus consumption of fixed capital (NIPA table 1.7.5, line 6), since our model has a depreciation allowance for capital income. Averaging $\tau_{0,t}$ from 1960I to 2007IV yields $\tau_0 = 5.25\%$.

2.B   Appendix: Computational algorithm

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2.B.1   Computational algorithm for the two-shock stochastic steady state economy

Step 0: We first select a set of summary statistics for the wealth distribution, $\{K, Gini(a)\}$, and fix the functional form of the equilibrium rules in equations (2.4.1)-(2.4.3). We then set the interpolation grids for $(a, K, Gini(a), B)$ to be used in the approximation of the household's continuation value function and policy function. We use an initial guess of coefficients $\{a_0^0, \ldots, a_7^0\}$, $\{\tilde{a}_0^0, \ldots, \tilde{a}_7^0\}$, $\{b_0^0, \ldots, b_7^0\}$ to obtain initial conjectures for $\{H_\Gamma^0, S^0\}$, and set up a convergence criterion $\varepsilon = 10^{-4}$.

Step 1: At the nth iteration, imposing $\{H_\Gamma^n, S^n\}$ in the household optimization problem, we use a value function iteration to solve the household's parametric dynamic programming problem as defined in Section 2.2.4. From this process, we obtain the continuation value function $V^n(a, \varepsilon, \tilde{\beta}, K, Gini(a), B, z, G; H_\Gamma^n, S^n)$.

Step 2: We next simulate the economy using N_H households and T periods. In each period t of the simulation, we first calculate the equilibrium $s_t^{eq,n}$ using equation (2.2.11) and $\{H_\Gamma^n\}$. Then we solve the household's optimization problem for the current $(K_t^n, Gini_t^n(a), B_t^n, z_t^n, G_t^n, s_t^{eq,n})$ using $V^n(a, \varepsilon, \tilde{\beta}, K, Gini(a), B, z, G; H_\Gamma^n, S^n)$

as the continuation value function and $\{H_\Gamma^n, H_B\}$. This is a one-shot optimization problem. The aggregate states in the next period follow from our aggregation of the optimal household decisions. From this step, we collect the time series $\{K_t^n, Gini_t^n(a), B_t^n, z_t^n, G_t^n, s_t^{eq,n}\}_{t=1}^T$.

Step 3: With these time series, we obtain separate OLS estimates of $\{\hat{a}_0^n, \ldots, \hat{a}_7^n\}$, $\{\hat{\tilde{a}}_0^n, \ldots, \hat{\tilde{a}}_7^n\}$, $\{\hat{b}_0^n, \ldots, \hat{b}_7^n\}$, for each z and G combination, which, with a slight abuse of notation, we summarize as $(\hat{H}_\Gamma^n, \hat{S}^n)$.

Step 4: If $|H_\Gamma^n - \hat{H}_\Gamma^n| < \epsilon$ and $|S^n - \hat{S}^n| < \epsilon$, we stop. Otherwise, we set

$$H_\Gamma^{n+1} = \alpha_H \times \hat{H}_\Gamma^n + (1 - \alpha_H) \times H_\Gamma^n$$
$$S^{n+1} = \alpha_S \times \hat{S}^n + (1 - \alpha_S) \times S^n$$

with $\alpha_H, \alpha_S \in (0, 1]$, and go to Step 1.

Step 5: Finally, we check whether the $R2$s (the multiple-step-ahead forecast errors) of the final OLS regressions are sufficiently high (small) for the equilibrium rules to be well approximated. If they are not, we change the functional forms in Step 0 and repeat the algorithm.

In Step 1, we iterate on the value function until it converges at a set of collocation points, chosen to be the grid points of $(a, K, Gini(a), B)$ defined in Step 0. In each step of the value function iteration, we use multi-dimensional cubic splines on this interpolation grid to approximate the continuation value function. For each collocation point of the state variables $(a, K, Gini(a), B)$ as well as the exogenous aggregate state variables (z, G), we use $\{H_\Gamma^n, S^n, H_B\}$ to infer the values of $(K', Gini'(a), B', s)$. Given the aggregate variables $(K', Gini'(a), B', s)$, we maximize the Bellman equation numerically along the a'-dimension using Brent's method, as described in Press et al. (2007). The same method is used in the numerical optimization part of Step 2.

In Step 2, we use $N_H = 90,000$ households and run 12 parallel simulations of length $T = 18,000$ each.[33] Following Krusell and Smith (1998), we also enforce that at each t these 90,000 households are distributed according to the stationary distribution of the Markov chains governing ε and β. We thus avoid introducing artificial aggregate uncertainty due to small deviations from the law of large numbers. To eliminate sampling error, we use the same series of aggregate shocks for all iterations and all model simulations.

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[33] Although each simulation has 19,000 periods, we discard the initial 1,000 observations in the estimation.

The algorithm is implemented in a mixture of C/C++ and MATLAB, which are then connected through MATLAB's CMEX interface.[34]

2.B.2 Results for the stochastic steady state

2.B.2.1 Estimated laws of motions for the two-shock equilibrium

H_Γ for aggregate capital in *good* times (state z_g), with low (G_l), medium (G_m), and high (G_h) government purchases levels are, respectively (in the above order):

$$log(K') = 0.1422 + 0.9125log(K) - 0.0019B + 0.0124(log(K))^2 + 0.0000B^2 \quad (G_l)$$
$$+ 0.0000B^3 + 0.0007log(K)B + 0.0007Gini(a), \quad R2 = 0.999999,$$
$$log(K') = 0.1374 + 0.9151log(K) - 0.0017B + 0.0120(log(K))^2 + 0.0000B^2 \quad (G_m)$$
$$- 0.0000B^3 + 0.0006log(K)B + 0.0007Gini(a), \quad R2 = 0.999999,$$
$$log(K') = 0.1223 + 0.9265log(K) - 0.0014B + 0.0098(log(K))^2 + 0.0000B^2 \quad (G_h)$$
$$- 0.0000B^3 + 0.0005log(K)B + 0.0006Gini(a), \quad R2 = 0.999999.$$

H_Γ for aggregate capital in *bad* times (state z_b), with low, medium, and high government purchases levels are, respectively:

$$log(K') = 0.1116 + 0.9299log(K) - 0.0016B + 0.0094(log(K))^2 + 0.0000B^2 \quad (G_l)$$
$$+ 0.0000B^3 + 0.0006log(K)B + 0.0010Gini(a), \quad R2 = 0.999999,$$
$$log(K') = 0.1084 + 0.9313log(K) - 0.0015B + 0.0093(log(K))^2 + 0.0000B^2 \quad (G_m)$$
$$+ 0.0000B^3 + 0.0005log(K)B + 0.0009Gini(a), \quad R2 = 0.999999,$$
$$log(K') = 0.0983 + 0.9385log(K) - 0.0012B + 0.0079(log(K))^2 + 0.0000B^2 \quad (G_h)$$
$$- 0.0000B^3 + 0.0004log(K)B + 0.0008Gini(a), \quad R2 = 0.999999.$$

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[34]On a 12-core 2.67 GHz Intel Xeon X5650 Linux workstation, the typical run time for the value function iteration lies around several hours (it gets shorter as the initial guess gets more accurate), while that for one simulation loop is about 40 minutes. Starting from a guess close to the equilibrium, it takes about 40 iterations to converge.

H_Γ for the Gini coefficient of the wealth distribution in *good* times (state z_g), with low, medium, and high government purchases levels are, respectively:

$$Gini(a') = -0.0300 + 0.0087log(K) + 0.0024B + 0.0017(log(K))^2 - 0.0000B^2 \quad (G_l)$$
$$- 0.0000B^3 - 0.0008log(K)B + 0.9975Gini(a), \quad R2 = 0.999999,$$

$$Gini(a') = -0.0455 + 0.0213log(K) + 0.0026B - 0.0009(log(K))^2 - 0.0000B^2 \quad (G_m)$$
$$+ 0.0000B^3 - 0.0009log(K)B + 0.9975Gini(a), \quad R2 = 0.999999,$$

$$Gini(a') = -0.0431 + 0.0194log(K) + 0.0025B - 0.0005(log(K))^2 - 0.0000B^2 \quad (G_h)$$
$$+ 0.0000B^3 - 0.0008log(K)B + 0.9976Gini(a), \quad R2 = 0.999999.$$

H_Γ for the Gini coefficient of the wealth distribution in *bad* times (state z_b), with low, medium, and high government purchases levels are, respectively:

$$Gini(a') = -0.0747 + 0.0472log(K) + 0.0025B - 0.0066(log(K))^2 - 0.0000B^2 \quad (G_l)$$
$$- 0.0000B^3 - 0.0009log(K)B + 0.9992Gini(a), \quad R2 = 0.999998,$$

$$Gini(a') = -0.0853 + 0.0559log(K) + 0.0027B - 0.0083(log(K))^2 - 0.0000B^2 \quad (G_m)$$
$$+ 0.0000B^3 - 0.0010log(K)B + 0.9992Gini(a), \quad R2 = 0.999998,$$

$$Gini(a') = -0.0804 + 0.0519log(K) + 0.0027B - 0.0075(log(K))^2 - 0.0000B^2 \quad (G_h)$$
$$+ 0.0000B^3 - 0.0010log(K)B + 0.9992Gini(a), \quad R2 = 0.999998.$$

S in *good* times (state z_g), with low, medium, and high government purchases levels are, respectively:

$$log(s) = 8.3647 - 8.3407log(K) - 0.5945B + 2.0612(log(K))^2 + 0.0095B^2 \quad (G_l)$$
$$+ 0.0012B^3 + 0.3437log(K)B - 0.0907Gini(a), \quad R2 = 0.99992,$$

$$log(s) = 11.4913 - 10.8071log(K) - 0.8477B + 2.5649(log(K))^2 + 0.0109B^2 \quad (G_m)$$
$$+ 0.0021B^3 + 0.4499log(K)B - 0.0830Gini(a), \quad R2 = 0.99983,$$

$$log(s) = 16.1515 - 14.4841log(K) - 1.2486B + 3.3065(log(K))^2 + 0.0117B^2 \quad (G_h)$$
$$+ 0.0036B^3 + 0.6183log(K)B - 0.0747Gini(a), \quad R2 = 0.99957.$$

S in *bad* times (state z_b), with low, medium, and high government purchases levels are, respectively:

$$log(s) = 7.7967 - 7.9011log(K) - 0.5212B + 1.9637(log(K))^2 + 0.0076B^2 \quad (G_l)$$
$$+ 0.0018B^3 + 0.3255log(K)B - 0.1058Gini(a), \quad R2 = 0.99991,$$
$$log(s) = 11.0908 - 10.4747log(K) - 0.8318B + 2.4831(log(K))^2 + 0.0087B^2 \quad (G_m)$$
$$+ 0.0032B^3 + 0.4553log(K)B - 0.0978Gini(a), \quad R2 = 0.99970,$$
$$log(s) = 15.8242 - 14.2123log(K) - 1.2243B + 3.2384(log(K))^2 + 0.0074B^2 \quad (G_h)$$
$$+ 0.0052B^3 + 0.6220log(K)B - 0.0935Gini(a), \quad R2 = 0.99924.$$

2.B.2.2 Multi-step ahead forecast errors

To compute the multi-step ahead forecast errors, we compare the aggregate paths from the equilibrium simulation with those generated by the approximate equilibrium rules. To be specific, starting from the first period, we pick a set of aggregate states 80 periods apart along the final simulation of the stochastic steady state calculation, $\{K_{(i-1)\times 80+1}, B_{(i-1)\times 80+1}, Gini_{(i-1)\times 80+1}(a)\}_{i\in\{1,2,\dots\}}$. We then simulate each set of aggregate states for 40 periods with the equilibrium laws of motions H_Γ and H_B (with the same aggregate shock sequences as those in the equilibrium simulation), and record the aggregate capital level in the last period of each simulation $\{K^H_{(i-1)\times 80+1+40}\}_{i\in\{1,2,\dots\}}$. We then calculate the ith 10-year (40-period) ahead forecast error for aggregate capital as $u_i^{40} = K_{(i-1)\times 80+1+40} - K^H_{(i-1)\times 80+1+40}$, where K_t is the aggregate capital level at period t from the equilibrium simulation. Note that the forecast errors generated this way are independent of each other because 40 periods are discarded between each simulation. With the large number of simulations in the stochastic steady state calculation (12 parallel simulations each for 18,000 periods), we generate about 2,700 such forecast errors. The mean of these 10-year-ahead forecast errors is 0.0002%, and the root mean squared error (RMSE) of the 10-year-ahead forecast is about 0.08% of the long-run average capital level, suggesting little bias and high overall forecast accuracy in our laws of motions.

2.B.2.3 Estimated laws of motions for the one-shock equilibrium

H_Γ for aggregate capital in *good* times (state z_g) and *bad* times (state z_b) are, respectively:

$$log(K') = 0.1329 + 0.9189log(K) - 0.0016B + 0.0112(log(K))^2 + 0.0000B^2 \quad (z_g)$$
$$- 0.0000B^3 + 0.0006log(K)B + 0.0007Gini(a), \quad R2 = 0.999999,$$
$$log(K') = 0.1045 + 0.9346log(K) - 0.0014B + 0.0086(log(K))^2 + 0.0000B^2 \quad (z_b)$$
$$+ 0.0000B^3 + 0.0005log(K)B + 0.0009Gini(a), \quad R2 = 0.999999.$$

H_Γ for Gini coefficient of wealth distribution in *good* times (state z_g) and *bad* times (state z_b) are, respectively:

$$Gini(a') = -0.0509 + 0.0258log(K) + 0.0026B - 0.0018(log(K))^2 - 0.0000B^2 \quad (z_g)$$
$$+ 0.0000B^3 - 0.0009log(K)B + 0.9976Gini(a), \quad R2 = 0.999999,$$
$$Gini(a') = -0.0997 + 0.0678log(K) + 0.0030B - 0.0108(log(K))^2 - 0.0000B^2 \quad (z_b)$$
$$- 0.0000B^3 - 0.0011log(K)B + 0.9992Gini(a), \quad R2 = 0.999997.$$

S in *good* times (state z_g) and *bad* times (state z_b) are, respectively:

$$log(s) = 10.0396 - 9.6184log(K) - 0.7878B + 2.3216(log(K))^2 + 0.0117B^2 \quad (z_g)$$
$$+ 0.0017B^3 + 0.4253log(K)B - 0.0841Gini(a), \quad R2 = 0.99991,$$
$$log(s) = 9.3062 - 9.0204log(K) - 0.7350B + 2.1870(log(K))^2 + 0.0096B^2 \quad (z_b)$$
$$+ 0.0025B^3 + 0.4157log(K)B - 0.1008Gini(a), \quad R2 = 0.99987.$$

2.B.3 Computational algorithm for the transition-path equilibrium

Step 0: Set up:

We choose the starting G level (three possibilities) and guess a length for the transition period T_{trans}.

We next assume specific functional forms for $\{H_{\Gamma,t}^{trans}, S_t^{trans}\}_{t=1}^{T_{trans}}$.

We then select the interpolation grid for $(a, K, Gini(a), B)$ used in the spline approximation of the household's continuation value function.

This calculation also requires the following inputs:

1. H_Γ^{1s} and S^{1s}: laws of motions from the one-shock equilibrium.

2. $V_{1s}(a, \varepsilon, \tilde{\beta}, K, Gini(a), B, z; H_\Gamma^{1s}, S^{1s})$: the value function for households from the one-shock equilibrium.

3. N_{trans} independent joint distributions over $(a, \varepsilon, \tilde{\beta})$ (each with N_H households) drawn from the two-shock equilibrium simulation to start the transition-path equilibrium simulations. To get a balanced sample for each combination of z and t, exactly half of these distributions are collected during good times $(z = z_g)$.

4. N_{trans} different aggregate productivity shock paths $\{\{z_t^i\}_{t=1}^{T_{trans}}\}_{i=1}^{N_{trans}}$, where z_1^i matches with the productivity level in the ith collected joint distribution and the $z_{t>1}^i$ are randomly drawn following its Markov process.

Step 1: We start from an initial coefficient guess $\{\{a_{0,t}^0, \ldots, a_{7,t}^0\}, \{\tilde{a}_{0,t}^0, \ldots, \tilde{a}_{7,t}^0\}, \{b_0^{0,t}, \ldots, b_{7,t}^0\}\}_{t=1}^{T_{trans}}$ to get our initial conjectures $\{H_{\Gamma,t}^{trans,0}, S_t^{trans,0}\}_{t=1}^{T_{trans}}$. Set up a convergence criterion ε.

Step 2: In the nth iteration, we compute the household's value function at each period by backward induction, with imposed laws of motions $\{H_{\Gamma,t}^{trans,n}, S_t^{trans,n}\}_{t=1}^{T_{trans}}$. To be specific, $\forall t \in \{1, \ldots, T_{trans}\}$, given the continuation value $V_{t+1}^{trans,n}(a, \varepsilon, \tilde{\beta}, K, Gini(a), B, z; H_{\Gamma,t+1}^{trans,n}, S_{t+1}^{trans,n})$, we calculate $V_t^{trans,n}(a, \varepsilon, \tilde{\beta}, K, Gini(a), B, z; H_{\Gamma,t}^{trans,n}, S_t^{trans,n})$. Note that $V_{T_{trans}+1}^{trans,n}(\cdot) = V_{1s}(\cdot)$. We store the value functions at each point in time of the transition path.

Step 3: In this step, we simulate the N_{trans} economies with the corresponding productivity shock paths. For the simulation of the ith economy, in each period t, we first calculate the equilibrium $s_t^{eq,n,i}$ using equation (2.2.11) and $H_{\Gamma,t}^{trans,n}$. Then we solve the household's optimization problem for $(K_t^{n,i}, Gini_t^{n,i}(a), B_t^{n,i}, z_t^{n,i}, s_t^{eq,n,i})$ using $V_{t+1}^{trans,n}(a, \varepsilon, \tilde{\beta}, K, Gini(a), B, z; H_{\Gamma,t+1}^{trans,n}, S_{t+1}^{trans,n})$ and $\{H_{\Gamma,t}^{trans,n}, S_t^{trans,n}, H_B\}$. The aggregate states in the next period follow from aggregating the optimal household decisions. We finally collect the following panel data $\{K_t^{n,i}, Gini_t^{n,i}(a), B_t^{n,i}, z_t^{n,i}, s_t^{eq,n,i}\}_{t=1}^{T}\}_{i=1}^{N_{trans}}$.

Step 4: $\forall t \in \{1, \ldots, T_{trans}\}$, we run OLS for each point in time along the transition path to get estimates of $\{\hat{a}_{0,t}^n, \ldots, \hat{a}_{7,t}^n\}, \{\hat{\tilde{a}}_{0,t}^n, \ldots, \hat{\tilde{a}}_{7,t}^n\}, \{\hat{b}_{0,t}^n, \ldots, \hat{b}_{7,t}^n\}$, which, with a slight abuse of notation, we summarize as $\{\hat{H}_{\Gamma,t}^{trans,n}, \hat{S}_t^{trans,n}\}_{t=1}^{T_{trans}}$.

Step 5: If $\max_t |H_{\Gamma,t}^{trans,n} - \hat{H}_{\Gamma,t}^{trans,n}| < \varepsilon$ and $\max_t |S_t^{trans,n} - \hat{S}_t^{trans,n}| < \varepsilon$, we stop. Otherwise, $\forall t \in \{1, \ldots, T\}$, we set:

$$H_{\Gamma,t}^{trans,n+1} = \alpha_H \times \hat{H}_{\Gamma,t}^{trans,n} + (1 - \alpha_H) \times H_{\Gamma,t}^{trans,n}$$
$$S_t^{trans,n+1} = \alpha_S \times \hat{S}_t^{trans,n} + (1 - \alpha_S) \times S_t^{trans,n}$$

with $\alpha_H, \alpha_S \in (0,1]$, and go to Step 2.

Step 6: We check the convergence of the last period's laws of motion to those from the one-shock equilibrium. To be specific, starting from the aggregate states observed in the last period's simulations, $\{z_T^i, K_T^i, B_T^i, Gini_T^i(a)\}_{i=1}^{N_{trans}}$, we calculate the differences in the predicted values of aggregate capital, the wealth Gini coefficient and s, between when we use the converged last period's laws of motions of the transition-path equilibrium, $\{H_{\Gamma,T_{trans}}^{trans}, S_{T_{trans}}^{trans}\}$, and when we use the laws of motions from the one-shock economy, $\{H_{\Gamma,1s}, S_{1s}\}$. If the differences are comparable in size to those of the one-step prediction errors of the laws of motions from the one-shock stochastic steady state equilibrium, we go to Step 7. Otherwise, we go back to Step 0 and increase T_{trans}.

Step 7: We check whether the $R2$s (the multiple-step-ahead forecast errors) of the final OLS regressions are sufficiently high (small) for the equilibrium rules to be well approximated. If they are not, we change the functional forms in Step 0 and repeat the algorithm.[35]

The numerical methods for interpolation and optimization used to solve the household's maximization problem in Step 2 and Step 3 are the same as those in the computation of the stochastic steady state. The only difference is that the procedure does not involve a value function iteration since we use backward induction to solve for the value function for each period, starting from the value function from the stochastic steady state without fiscal uncertainty.

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[35]We choose $\varepsilon = 10^{-4}, N_H = 90,000$, and $T_{trans} = 400$. N_{trans} is set to be 8,000 when we start from G_m and 4,000 for the other two cases, to keep the ratios between the numbers of observations across different cases the same as those implied by the ergodic distribution of the G-process. On a 32-core 2.4 GHz Intel Xeon E5-4640 Linux workstation, the typical run time for the value function calculation takes about an hour, and for one simulation loop, it takes about six hours. Starting from a guess based on the weighted average between the two-shock and the one-shock laws of motion, it takes about 40 to 50 iterations to converge.

2.B.4 Estimated laws of motions for transition-path equilibrium

Here we present the estimated laws of motions for selected periods and every combination of z_t and G_1.

Table 2.B.1: Transition-path equilibrium: starting from $G_1 = G_l$, $z_t = z_g$

t	$a_{0,t}$	$a_{1,t}$	$a_{2,t}$	$a_{3,t}$	$a_{4,t}$	$a_{5,t}$	$a_{6,t}$	$a_{7,t}$	G values	$R2$
1	0.1390	0.9150	-0.0017	0.0118	0.0000	-0.0000	0.0006	0.0007	0.2205	0.999999
2	0.1423	0.9122	-0.0018	0.0124	0.0000	-0.0000	0.0007	0.0007	0.2210	0.999999
3	0.1446	0.9104	-0.0018	0.0128	0.0000	-0.0000	0.0007	0.0007	0.2214	0.999999
...	...	...	...	...	...	...	...			
388	0.1278	0.9231	-0.0015	0.0103	0.0000	-0.0000	0.0005	0.0006	0.2319	0.999999
399	0.1303	0.9210	-0.0016	0.0108	0.0000	-0.0000	0.0006	0.0006	0.2319	0.999999
400	0.1321	0.9196	-0.0016	0.0110	0.0000	-0.0000	0.0006	0.0006	0.2319	0.999999
	$\tilde{a}_{0,t}$	$\tilde{a}_{1,t}$	$\tilde{a}_{2,t}$	$\tilde{a}_{3,t}$	$\tilde{a}_{4,t}$	$\tilde{a}_{5,t}$	$\tilde{a}_{6,t}$	$\tilde{a}_{7,t}$	G values	$R2$
1	-0.0394	0.0162	0.0027	0.0002	-0.0000	0.0000	-0.0009	0.9975	0.2205	0.999999
2	-0.0340	0.0117	0.0026	0.0011	-0.0000	-0.0000	-0.0009	0.9975	0.2210	0.999999
3	-0.0451	0.0208	0.0028	-0.0008	-0.0000	0.0000	-0.0009	0.9975	0.2214	0.999999
...	...	...	...	...	...	...	...			
388	-0.0524	0.0271	0.0025	-0.0021	-0.0000	0.0000	-0.0008	0.9975	0.2319	0.999999
399	-0.0475	0.0231	0.0025	-0.0013	-0.0000	0.0000	-0.0008	0.9975	0.2319	0.999999
400	-0.0548	0.0291	0.0026	-0.0025	-0.0000	0.0000	-0.0009	0.9976	0.2319	0.999999
	$b_{0,t}$	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$	$b_{5,t}$	$b_{6,t}$	$b_{7,t}$	G values	$R2$
1	10.1857	-9.8044	-0.6990	2.3558	0.0105	0.0012	0.3858	-0.0880	0.2205	0.99985
2	10.7885	-10.3002	-0.7003	2.4586	0.0099	0.0014	0.3867	-0.0892	0.2210	0.99985
3	10.8166	-10.3177	-0.7052	2.4617	0.0100	0.0014	0.3888	-0.0890	0.2214	0.99986
...	...	...	...	...	...	...	...			
388	9.2488	-8.9958	-0.7253	2.1997	0.0126	0.0013	0.3996	-0.0880	0.2319	0.99993
399	9.4318	-9.1361	-0.7483	2.2264	0.0134	0.0011	0.4088	-0.0873	0.2319	0.99993
400	9.4819	-9.1765	-0.7500	2.2345	0.0134	0.0011	0.4095	-0.0871	0.2319	0.99993

Table 2.B.2: Transition-path equilibrium: starting from $G_1 = G_l$, $z_t = z_b$

t	$a_{0,t}$	$a_{1,t}$	$a_{2,t}$	$a_{3,t}$	$a_{4,t}$	$a_{5,t}$	$a_{6,t}$	$a_{7,t}$	G values	$R2$
1	0.1138	0.9280	-0.0018	0.0098	0.0000	0.0000	0.0006	0.0010	0.2205	0.999999
2	0.1213	0.9219	-0.0020	0.0110	0.0000	0.0000	0.0008	0.0010	0.2210	0.999999
3	0.1098	0.9312	-0.0016	0.0092	0.0000	0.0000	0.0006	0.0010	0.2214	0.999999
...	...	...	...	...	...	...	...			
388	0.1000	0.9382	-0.0013	0.0079	0.0000	0.0000	0.0004	0.0009	0.2319	0.999999
399	0.0977	0.9402	-0.0013	0.0074	0.0000	-0.0000	0.0005	0.0009	0.2319	0.999999
400	0.0987	0.9393	-0.0013	0.0076	0.0000	-0.0000	0.0005	0.0009	0.2319	0.999999
	$\tilde{a}_{0,t}$	$\tilde{a}_{1,t}$	$\tilde{a}_{2,t}$	$\tilde{a}_{3,t}$	$\tilde{a}_{4,t}$	$\tilde{a}_{5,t}$	$\tilde{a}_{6,t}$	$\tilde{a}_{7,t}$	G values	$R2$
1	-0.1048	0.0716	0.0035	-0.0115	-0.0000	-0.0000	-0.0013	0.9991	0.2205	0.999997
2	-0.1013	0.0685	0.0037	-0.0108	-0.0000	-0.0000	-0.0014	0.9991	0.2210	0.999998
3	-0.0858	0.0556	0.0037	-0.0081	-0.0000	-0.0000	-0.0014	0.9991	0.2214	0.999997
...	...	...	...	...	...	...	...			
388	-0.1108	0.0768	0.0032	-0.0126	-0.0000	0.0000	-0.0012	0.9992	0.2319	0.999998
399	-0.1003	0.0681	0.0034	-0.0108	-0.0000	-0.0000	-0.0012	0.9991	0.2319	0.999997
400	-0.0972	0.0655	0.0034	-0.0103	-0.0000	-0.0000	-0.0013	0.9991	0.2319	0.999997
	$b_{0,t}$	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$	$b_{5,t}$	$b_{6,t}$	$b_{7,t}$	G values	$R2$
1.0000	9.6792	-9.4256	-0.6023	2.2721	0.0084	0.0018	0.3583	-0.1046	0.2205	0.99988
2.0000	8.7794	-8.6901	-0.5785	2.1223	0.0087	0.0017	0.3485	-0.1027	0.2210	0.99989
3.0000	10.3272	-9.9497	-0.6354	2.3792	0.0092	0.0018	0.3717	-0.1024	0.2214	0.99986
...	...	...	...	...	...	...	...			
388	8.5682	-8.4447	-0.6632	2.0752	0.0123	0.0016	0.3855	-0.1017	0.2319	0.99993
399	8.7976	-8.6271	-0.6809	2.1114	0.0120	0.0017	0.3930	-0.1025	0.2319	0.99993
400	8.7089	-8.5561	-0.6756	2.0973	0.0120	0.0016	0.3908	-0.1025	0.2319	0.99993

Table 2.B.3: Transition-path equilibrium: starting from $G_1 = G_m$, $z_t = z_g$

t	$a_{0,t}$	$a_{1,t}$	$a_{2,t}$	$a_{3,t}$	$a_{4,t}$	$a_{5,t}$	$a_{6,t}$	$a_{7,t}$	G values	$R2$
1	0.1345	0.9176	-0.0016	0.0115	0.0000	-0.0000	0.0006	0.0007	0.2319	0.999999
2	0.1348	0.9173	-0.0016	0.0115	0.0000	0.0000	0.0006	0.0007	0.2319	0.999999
3	0.1351	0.9170	-0.0016	0.0116	0.0000	0.0000	0.0006	0.0007	0.2319	0.999999
...	...	...	...	...	...	...	...			
388	0.1311	0.9203	-0.0015	0.0109	0.0000	0.0000	0.0006	0.0007	0.2319	0.999999
399	0.1326	0.9190	-0.0015	0.0112	0.0000	0.0000	0.0005	0.0008	0.2319	0.999999
400	0.1327	0.9189	-0.0015	0.0112	0.0000	0.0000	0.0005	0.0008	0.2319	0.999999
	$\tilde{a}_{0,t}$	$\tilde{a}_{1,t}$	$\tilde{a}_{2,t}$	$\tilde{a}_{3,t}$	$\tilde{a}_{4,t}$	$\tilde{a}_{5,t}$	$\tilde{a}_{6,t}$	$\tilde{a}_{7,t}$	G values	$R2$
1	-0.0470	0.0224	0.0028	-0.0011	-0.0000	-0.0000	-0.0009	0.9975	0.2319	0.999999
2	-0.0506	0.0253	0.0029	-0.0017	-0.0000	-0.0000	-0.0010	0.9975	0.2319	0.999999
3	-0.0523	0.0267	0.0029	-0.0020	-0.0000	-0.0000	-0.0010	0.9975	0.2319	0.999999
...	...	...	...	...	...	...	...			
388	-0.0520	0.0265	0.0029	-0.0019	-0.0000	-0.0000	-0.0010	0.9975	0.2319	0.999999
399	-0.0497	0.0247	0.0027	-0.0016	-0.0000	-0.0000	-0.0009	0.9975	0.2319	0.999999
400	-0.0537	0.0279	0.0028	-0.0022	-0.0000	-0.0000	-0.0010	0.9975	0.2319	0.999999
	$b_{0,t}$	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$	$b_{5,t}$	$b_{6,t}$	$b_{7,t}$	G values	$R2$
1	11.6816	-10.9556	-0.8705	2.5939	0.0122	0.0018	0.4589	-0.0846	0.2319	0.99984
2	11.6892	-10.9601	-0.8751	2.5946	0.0120	0.0019	0.4607	-0.0855	0.2319	0.99983
3	11.8277	-11.0691	-0.8836	2.6160	0.0123	0.0019	0.4641	-0.0857	0.2319	0.99983
...	...	...	...	...	...	...	...			
388	10.4730	-9.9769	-0.7954	2.3957	0.0125	0.0015	0.4283	-0.0848	0.2319	0.99992
399	10.3018	-9.8403	-0.7797	2.3687	0.0122	0.0015	0.4220	-0.0855	0.2319	0.99993
400	9.9651	-9.5653	-0.7746	2.3124	0.0121	0.0015	0.4199	-0.0853	0.2319	0.99993

Table 2.B.4: Transition-path equilibrium: starting from $G_1 = G_m$, $z_t = z_b$

t	$a_{0,t}$	$a_{1,t}$	$a_{2,t}$	$a_{3,t}$	$a_{4,t}$	$a_{5,t}$	$a_{6,t}$	$a_{7,t}$	G values	$R2$
1	0.1072	0.9322	-0.0015	0.0091	0.0000	-0.0000	0.0005	0.0010	0.2319	0.999999
2	0.1071	0.9324	-0.0015	0.0091	0.0000	-0.0000	0.0005	0.0010	0.2319	0.999999
3	0.1077	0.9319	-0.0015	0.0092	0.0000	-0.0000	0.0005	0.0010	0.2319	0.999999
...	...	...	...	...	...	...	...			
388	0.1003	0.9379	-0.0013	0.0079	0.0000	0.0000	0.0004	0.0010	0.2319	0.999999
399	0.1018	0.9367	-0.0014	0.0082	0.0000	0.0000	0.0005	0.0010	0.2319	0.999999
400	0.1001	0.9381	-0.0014	0.0079	0.0000	0.0000	0.0005	0.0010	0.2319	0.999999
	$\tilde{a}_{0,t}$	$\tilde{a}_{1,t}$	$\tilde{a}_{2,t}$	$\tilde{a}_{3,t}$	$\tilde{a}_{4,t}$	$\tilde{a}_{5,t}$	$\tilde{a}_{6,t}$	$\tilde{a}_{7,t}$	G values	$R2$
1	-0.0976	0.0660	0.0030	-0.0104	-0.0000	-0.0000	-0.0011	0.9991	0.2319	0.999998
2	-0.0829	0.0538	0.0028	-0.0079	-0.0000	-0.0000	-0.0010	0.9991	0.2319	0.999998
3	-0.0868	0.0570	0.0029	-0.0085	-0.0000	-0.0000	-0.0011	0.9991	0.2319	0.999998
...	...	...	...	...	...	...	...			
388	-0.0944	0.0633	0.0031	-0.0098	-0.0000	0.0000	-0.0011	0.9992	0.2319	0.999998
399	-0.0977	0.0662	0.0028	-0.0105	-0.0000	-0.0000	-0.0010	0.9991	0.2319	0.999998
400	-0.0957	0.0645	0.0028	-0.0101	-0.0000	0.0000	-0.0010	0.9992	0.2319	0.999998
	$b_{0,t}$	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$	$b_{5,t}$	$b_{6,t}$	$b_{7,t}$	G values	$R2$
1	11.1724	-10.5413	-0.8306	2.4971	0.0106	0.0026	0.4544	-0.1022	0.2319	0.99983
2	10.4475	-9.9508	-0.8006	2.3768	0.0101	0.0026	0.4424	-0.1020	0.2319	0.99986
3	10.9516	-10.3594	-0.8225	2.4595	0.0103	0.0026	0.4514	-0.1017	0.2319	0.99985
...	...	...	...	...	...	...	...			
388	9.2879	-9.0151	-0.7167	2.1881	0.0112	0.0021	0.4077	-0.1022	0.2319	0.99993
399	9.7104	-9.3685	-0.7219	2.2617	0.0113	0.0020	0.4100	-0.1005	0.2319	0.99992
400	9.6848	-9.3502	-0.7156	2.2586	0.0113	0.0020	0.4074	-0.1008	0.2319	0.99992

Table 2.B.5: Transition-path equilibrium: starting from $G_1 = G_h$, $z_t = z_g$

t	$a_{0,t}$	$a_{1,t}$	$a_{2,t}$	$a_{3,t}$	$a_{4,t}$	$a_{5,t}$	$a_{6,t}$	$a_{7,t}$	G values	$R2$
1	0.1279	0.9219	-0.0015	0.0107	0.0000	-0.0000	0.0006	0.0007	0.2433	0.999999
2	0.1296	0.9206	-0.0016	0.0110	0.0000	-0.0000	0.0006	0.0007	0.2429	0.999999
3	0.1272	0.9226	-0.0015	0.0106	0.0000	-0.0000	0.0006	0.0007	0.2424	0.999999
...	...	...	...	...	...	...	...			
388	0.1408	0.9124	-0.0019	0.0125	0.0000	-0.0000	0.0007	0.0008	0.2319	0.999999
399	0.1454	0.9086	-0.0019	0.0133	0.0000	0.0000	0.0007	0.0008	0.2319	0.999999
400	0.1372	0.9153	-0.0017	0.0119	0.0000	0.0000	0.0006	0.0008	0.2319	0.999999
	$\tilde{a}_{0,t}$	$\tilde{a}_{1,t}$	$\tilde{a}_{2,t}$	$\tilde{a}_{3,t}$	$\tilde{a}_{4,t}$	$\tilde{a}_{5,t}$	$\tilde{a}_{6,t}$	$\tilde{a}_{7,t}$	G values	$R2$
1	-0.0416	0.0179	0.0026	-0.0002	-0.0000	0.0000	-0.0009	0.9975	0.2433	0.999999
2	-0.0341	0.0119	0.0024	0.0011	-0.0000	-0.0000	-0.0008	0.9975	0.2429	0.999999
3	-0.0472	0.0227	0.0025	-0.0012	-0.0000	0.0000	-0.0008	0.9975	0.2424	0.999999
...	...	...	...	...	...	...	...			
388	-0.0451	0.0209	0.0026	-0.0008	-0.0000	-0.0000	-0.0009	0.9975	0.2319	0.999999
399	-0.0376	0.0148	0.0024	0.0005	-0.0000	-0.0000	-0.0008	0.9974	0.2319	0.999999
400	-0.0393	0.0161	0.0025	0.0002	-0.0000	-0.0000	-0.0008	0.9974	0.2319	0.999999
	$b_{0,t}$	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$	$b_{5,t}$	$b_{6,t}$	$b_{7,t}$	G values	$R2$
1	17.8423	-15.8786	-1.2955	3.5942	0.0142	0.0032	0.6363	-0.0752	0.2433	0.99960
2	17.5167	-15.6076	-1.2901	3.5373	0.0147	0.0030	0.6337	-0.0768	0.2429	0.99962
3	16.0065	-14.3765	-1.2447	3.2855	0.0146	0.0029	0.6151	-0.0760	0.2424	0.99966
...	...	...	...	...	...	...	...			
388	10.0928	-9.6530	-0.8079	2.3267	0.0132	0.0014	0.4329	-0.0827	0.2319	0.99993
399	10.5458	-10.0150	-0.8425	2.3989	0.0144	0.0012	0.4466	-0.0823	0.2319	0.99993
400	10.7527	-10.1863	-0.8392	2.4344	0.0134	0.0014	0.4457	-0.0824	0.2319	0.99993

Table 2.B.6: Transition-path equilibrium: starting from $G_1 = G_h$, $z_t = z_b$

t	$a_{0,t}$	$a_{1,t}$	$a_{2,t}$	$a_{3,t}$	$a_{4,t}$	$a_{5,t}$	$a_{6,t}$	$a_{7,t}$	G values	$R2$
1	0.1020	0.9355	-0.0014	0.0085	0.0000	-0.0000	0.0005	0.0009	0.2433	0.999999
2	0.1025	0.9351	-0.0014	0.0086	0.0000	-0.0000	0.0005	0.0009	0.2429	0.999999
3	0.1020	0.9355	-0.0014	0.0085	0.0000	-0.0000	0.0005	0.0009	0.2424	0.999999
...	...	...	...	...	...	...	...			
388	0.1131	0.9274	-0.0016	0.0101	0.0000	-0.0000	0.0006	0.0010	0.2319	0.999999
399	0.1134	0.9271	-0.0017	0.0101	0.0000	-0.0000	0.0006	0.0011	0.2319	0.999999
400	0.1193	0.9223	-0.0019	0.0111	0.0000	-0.0000	0.0007	0.0011	0.2319	0.999999
	$\tilde{a}_{0,t}$	$\tilde{a}_{1,t}$	$\tilde{a}_{2,t}$	$\tilde{a}_{3,t}$	$\tilde{a}_{4,t}$	$\tilde{a}_{5,t}$	$\tilde{a}_{6,t}$	$\tilde{a}_{7,t}$	G values	$R2$
1	-0.0671	0.0408	0.0024	-0.0052	-0.0000	-0.0000	-0.0009	0.9991	0.2433	0.999998
2	-0.0875	0.0576	0.0029	-0.0087	-0.0000	-0.0000	-0.0010	0.9991	0.2429	0.999998
3	-0.0912	0.0605	0.0032	-0.0092	-0.0000	-0.0000	-0.0012	0.9992	0.2424	0.999998
...	...	...	...	...	...	...	...			
388	-0.0876	0.0576	0.0028	-0.0086	-0.0000	-0.0000	-0.0010	0.9992	0.2319	0.999998
399	-0.0987	0.0669	0.0027	-0.0106	-0.0000	-0.0000	-0.0009	0.9991	0.2319	0.999998
400	-0.0983	0.0666	0.0025	-0.0105	-0.0000	-0.0000	-0.0009	0.9991	0.2319	0.999998
	$b_{0,t}$	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$	$b_{5,t}$	$b_{6,t}$	$b_{7,t}$	G values	$R2$
1	14.7092	-13.2959	-1.2257	3.0504	0.0144	0.0037	0.6194	-0.0944	0.2433	0.99961
2	16.4578	-14.7219	-1.2851	3.3401	0.0133	0.0040	0.6442	-0.0937	0.2429	0.99952
3	19.4670	-17.1930	-1.3600	3.8468	0.0131	0.0041	0.6750	-0.0948	0.2424	0.99945
...	...	...	...	...	...	...	...			
388	10.2379	-9.7773	-0.7763	2.3407	0.0123	0.0019	0.4317	-0.1007	0.2319	0.99992
399	9.7252	-9.3655	-0.7442	2.2579	0.0108	0.0023	0.4191	-0.0994	0.2319	0.99992
400	9.7874	-9.4211	-0.7399	2.2703	0.0116	0.0020	0.4171	-0.0994	0.2319	0.99993

2.B.5 Calculating the welfare gain

This appendix shows how to use the value functions from the in the transition-path equilibrium and the two-shock equilibrium to conduct the welfare cost calculation implicitly defined in equation (2.5.1), which we restate here for convenience:

$$E_1[\sum_{t=1}^{\infty} \beta_t u((1+\lambda)c_t, G_t)] = E_1[\sum_{t=1}^{\infty} \beta_t u(\tilde{c}_t, \tilde{G}_t)]. \tag{2.5.1}$$

We denote the value functions from the transition-path equilibrium and the two-shock equilibrium by $\tilde{V}$ and V, respectively, where

$$\tilde{V} = E_1[\sum_{t=1}^{\infty} \beta_t u(\tilde{c}_t, \tilde{G}_t)] \text{ and } V = E_1[\sum_{t=1}^{\infty} \beta_t u(c_t, G_t)].$$

Note that the right side of (2.5.1) is exactly $\tilde{V}$. Under the assumption of a log separable utility function, the left side of (2.5.1) can be expressed as:

$$E_1[\sum_{t=1}^{\infty} \beta_t u((1+\lambda)c_t, G_t)] = E_1[\sum_{t=1}^{\infty} \beta_t u(c_t, G_t)] + E_1[\sum_{t=1}^{\infty} \beta_t \theta \log(1+\lambda)]$$

$$= \theta \log(1+\lambda) E_1[\sum_{t=1}^{\infty} \beta_t] + V.$$

This allows us to rewrite (2.5.1) as:

$$\theta \log(1+\lambda) E_1[\sum_{t=1}^{\infty} \beta_t] + V = \tilde{V}.$$

Thus, λ can be calculated as follows:

$$\lambda = \exp\left(\frac{\tilde{V} - V}{\theta E_1[\sum_{t=1}^{\infty} \beta_t]}\right) - 1.$$

Note that, since β_1 is known at time 1, the value of the denominator in the parentheses is a function of β_1. The calculation of λ is straightforward using the transition matrix governing the Markov process for $\tilde{\beta}$.

Under a non-separable utility function, we solve for λ numerically. That is, we calculate the left side of (2.5.1) as a discounted sum of flow utilities under various values of λ and the equilibrium policy function and then find a value of λ that satisfies the equation, using a bisection search.

2.C   Appendix: Alternative specifications and additional experiments

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2.C.1 λ under different model specifications

Table 2.C.1 reports the λ-measure of welfare gains under different model specifications.

Table 2.C.1: Expected welfare gains, λ (%), under different model specifications

	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
Baseline	0.0298	0.0303	0.0307	0.0307	0.0304	0.0301	0.0290	0.0240	0.0242
Non-separable Utility Function									
Substitute ($\rho = 0.5$)	0.0062	0.0046	0.0071	0.0082	0.0052	0.0090	0.0034	-0.0015	0.0053
Complement ($\rho = 1.5$)	0.1233	0.1101	0.1142	0.1196	0.1136	0.1309	0.1324	0.1233	0.1218
Different Tax Function Adjustment									
Adjusting τ_0	0.0295	0.0291	0.0296	0.0298	0.0295	0.0293	0.0288	0.0303	0.0347
Adjusting τ_1	0.0293	0.0289	0.0295	0.0296	0.0293	0.0290	0.0287	0.0313	0.0371
Counterfactual Fiscal Regimes									
Balanced Budget	0.0288	0.0306	0.0303	0.0299	0.0294	0.0291	0.0280	0.0210	0.0198
Linear Tax	0.0284	0.0277	0.0279	0.0282	0.0282	0.0282	0.0283	0.0315	0.0375
Lump-sum Tax	0.0280	0.0276	0.0276	0.0277	0.0276	0.0275	0.0279	0.0336	0.0412
Additional Experiments									
Constant TFP	0.0295	0.0299	0.0303	0.0303	0.0301	0.0298	0.0288	0.0246	0.0252
Double Volatility of G	-0.0282	-0.0290	-0.0293	-0.0293	-0.0288	-0.0284	-0.0272	-0.0223	-0.0225
Sudden change in G	0.0343	0.0371	0.0370	0.0363	0.0355	0.0347	0.0324	0.0237	0.0227

2.C.2   Doubling fiscal uncertainty

To implement the experiment of doubling fiscal uncertainty, we start by solving a new two-shock equilibrium with a government purchases variance twice that in the baseline model, while keeping the other parameter values the same as in the baseline calibration. We use the Rouwenhorst method to discretize the counterfactual AR(1) process and to obtain a new grid for G (G_l^{new}, G_m^{new}, G_h^{new}) =($(1 - m_{new})G_m$, G_m, $(1+m_{new})G_m$), with the same transition probability matrix as in the baseline model. We then start from the ergodic distribution of the original two-shock equilibrium and let the economy gradually transit to the new two-shock equilibrium as follows: at $t = 1$, after a particular G-state is realized from the old grid, (G_l, G_m, G_h), the households in the economy learn that, from the next period on ($t \geq 2$), the G-states

will evolve according to a different process, with the same transition probabilities, but with new period-t grid values that are conditional on the period-1 state, G_1, as $(G_l^{new}, G_m^{new}, G_h^{new}) + G_{t,G_1}^{adj}$. G_{t,G_1}^{adj} is an adjustment term that makes the period-t conditional mean as of $t = 1$ equal to those of the original process.[36] Mechanically, G_{t,G_1}^{adj} is zero for all t when $G_1 = G_m$, and positive (negative) and decreasing (increasing) to zero when $G_1 = G_l$ ($G_1 = G_h$). We plot G_{t,G_1}^{adj} in Figure 2.C.1.

Figure 2.C.1: Government purchases adjustment (G_{t,G_1}^{adj})
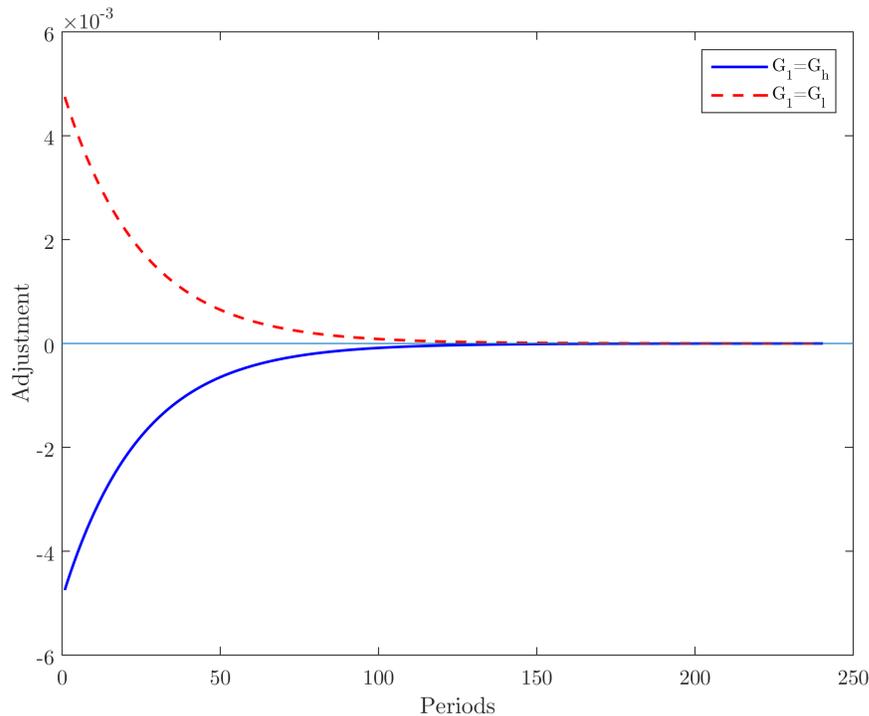
We follow similar steps as in the baseline case to solve the model. Note that, unlike in the baseline case, we now have an uncertain government purchases level along the transition path in addition to the aggregate productivity shocks. However, we do not condition on G in the transition-equilibrium laws of motions. Instead, we incorporate G as a continuous variable in $H_{\Gamma_{t+1}}$ and S_{t+1}, and pool the regressions for the laws of motion for $t = 1, 2, \ldots, 30$. Note that we do condition the laws of motion on the period-1 value of government purchases (G_1). We find that the following (relatively

---

[36]Note that this adjustment term isolates the effect of a change in fiscal uncertainty without a sudden level adjustment.

parsimonious) functional forms perform well:

$$
\begin{aligned}
log(K'_{t+1}) =& a_{0,t}(z_t, G_1) + a_{1,t}(z_t, G_1)log(K_t) + a_{2,t}(z_t, G_1)B_t + a_{3,t}(z_t, G_1)Gini(a_t) \\
& + a_{4,t}(z_t, G_1)B_t^5 + a_{5,t}(z_t, G_1)G_t^2 + a_{6,t}(z_t, G_1)B_t^5 G_t^2, \\
Gini(a'_{t+1}) =& \tilde{a}_{0,t}(z_t, G_1) + \tilde{a}_{1,t}(z_t, G_1)log(K_t) + \tilde{a}_{2,t}(z_t, G_1)B_t + \tilde{a}_{3,t}(z_t, G_1)Gini(a_t) \\
& + \tilde{a}_{4,t}(z_t, G_1)B_t^5 + \tilde{a}_{5,t}(z_t, G_1)G_t^2 + \tilde{a}_{6,t}(z_t, G_1)B_t^5 G_t^2, \\
log(s_t) =& b_{0,t}(z_t, G_1) + b_{1,t}(z_t, G_1)log(K_t) + b_{2,t}(z_t, G_1)B_t + b_{3,t}(z_t, G_1)Gini(a_t) \\
& + b_{4,t}(z_t, G_1)B_t^2 + b_{5,t}(z_t, G_1)G_t^2 + b_{6,t}(z_t, G_1)B_t^5 G_t^2.
\end{aligned}
$$

CHAPTER 3

Optimal Trade Costs after Sovereign Defaults

Coauthored with Chenyue Hu

This paper offers new theoretical and empirical insights into the effect of sovereign defaults on trade. We contend that sovereign debt renegotiation is associated not with trade sanction but with trade benefit between debtor countries and creditor countries. We find empirical support for the argument from the changes in trade share after debt renegotiations as well as the Aid-for-trade statistics. We build a two-country DSGE model with incomplete financial market to explain why trade sanction is not observed. We reason that creditors lower trade costs with debtors in hopes of collecting the remaining debt during debt renegotiations. The adjustment in turn affects debtors' default decisions. The model departs from the existing literature on sovereign defaults by building on the strategic interaction between debtors and creditors. We solve the model numerically to determine the optimal trade costs given different combinations of debt and income levels.

3.1 Introduction

The danger of default exists with every financial loan, and sovereign debt is no exception. Holders of sovereign debt face additional uncertainty stemming from the lack of supernational legal entities. The recent debt crises in Europe and Latin America have demonstrated the need to study both creditors' and debtors' incentives and decisions in the initiation, negotiation and settlement process of sovereign debt contracts. This paper aims to contribute to the discussion by focusing on a novel mechanism that has been overlooked in previous work.

Globalization since the second half the twentieth century has featured both trade liberalization and financial mobility across borders. The two channels should not be studied in isolation, as both are important sources of individual countries' economic development as well as world risk sharing. As Tomz and Wright (2013) point out, theoretical models are missing while empirical evidence is ambiguous over how trade and sovereign default interact. Our paper addresses this gap in the literature by providing new empirical and theoretical results that bring together the trade and borrowing channels to explain sovereign default settlement.

Trade, in previous literature on sovereign default, has played a trivial if any role. For instance, Bulow and Rogoff (1989) argue default may lead to a decline in international trade, which is interpreted as a constant output loss in their model. Their approach is followed in the majority of sovereign default papers including Aguiar and Gopinath (2006), Yue (2010) , Bai and Zhang (2010), to name just a few. Tomz and Wright (2013) summarize three reasons why trade could suffer after default happens: (1) creditors' trade restrictions as a means of punishment (a.k.a. trade sanction), (2) the collapse of trade credit, and (3) creditors' asset seizures. None of these reasons can be captured by direct output loss, let alone the strategic behaviors that arise from these features. Instead, our paper will focus on how trade costs may change before and after sovereign default.

Rose (2005) explains empirically the cost of trade after sovereign default. Using government-to-government debt default information from the Paris Club, he finds that debt renegotiation has significantly negative effects on contemporaneous and lagged trade volume in a gravity regression. We find his results inspiring and intriguing but not fully explored. Trade volume will naturally fall with the deterioration of economic terms, which may not be fully picked up by their gravity variables. It is the *relative* share instead of the *absolute* value of trade that measures the existence and severity of punishment in the bilateral borrowing relationship. We replicate Rose's analysis on an expanded dataset that includes fifteen additional years. Similar to Rose (2005) we find trade volume falls, but we also find that trade share increases significantly (by around 5%) after debt renegotiation happens. This is a surprising result that runs contrary to the traditional trade sanction arguments.

The finding is robust to the inclusion of exchange rates in the regressions, which implies that the positive trade effect is not driven by currency depreciation that often accompanies sovereign defaults. At the same time, the conclusion still holds when we control for bilateral trade elasticity, eliminating the compositional effect of consumption goods as the reason for the change in trade share. Novy (2013) argues that

trade share can be used to infer time-varying bilateral trade costs directly from the model's gravity equation without imposing arbitrary trade cost functions. Based on all the argument above, we hypothesize trade costs change as a creditor's reaction to debt renegotiation. As there lacks comprehensive and consistent data on direct measurement of trade costs, we resort to OECD's data on aid for trade and find there is noticeable increase in trade-related assistance from creditors when debt renegotiation happens. This is complementary evidence for lower trade costs after defaults.

Our findings lead us to rethink the creditors' incentives: why would creditors be willing to lower their trade costs with defaulters? In practice, before a default reaches its final resolution, there is a renegotiation stage where the creditor and the debtor could agree on debt settlement based on the current income of the debtor and the size of the debt. Our hypothesis is that in the renegotiation stage, it is sometimes optimal for the creditor to lower trade costs so that the debtor is more likely to service the debt.

We build a dynamic stochastic general equilibrium model to develop our hypothesis. Our model differs from a standard sovereign default model in the following ways. First, it is a two-country model instead of a small open economy. Additionally, because we are interested in whether the model's prediction of trade share can match our empirical finding, we will study a creditor-debtor two-country model integrated with a world market. Second, our model includes a trade component. The consumption bundle in a country consists of domestic goods, financial partner's (whether it be creditor or debtor) goods, and goods from the world market, with an elasticity of substitution among them. Third, creditors are risk averse. Creditors in most sovereign default models are risk-neutral and perfectly competitive for tractability reasons. Bond prices are directly linked to the world interest rate once the probability of default is computed. The assumption will be relaxed in our model as we assume a concave utility function. We are able to get a market-clearing bond price under the constant relative risk aversion (CRRA) assumption.

In our story, the amount creditors hope to collect from debtors induces the adjustment of bilateral trade costs. At the same time, the change in trade benefit affects debtors' probability to service the debt. At the end of the day, default probability, bond prices and optimal trade costs can all be endogenously derived as the solution to the general equilibrium model. Trade and debt channels are more correlated and interactive in our model than in any previous work, and can explain our novel empirical finding.

Our contribution is three-fold. First, we identify an interesting but overlooked

phenomenon through our empirical analysis, which calls the widely-accepted trade sanction argument into question. Second, we propose a new mechanism which links bilateral trade and bilateral borrowing. Third, we develop computation techniques that allow us to numerically solve a sovereign default model with more realistic features, such as risk-averse creditors.

In our new approach, we have maintained several important features from previous work. Eaton and Gersovitz (1981) proposed financial autarky as a means to support debtors' incentive to repay the debt. In our model, defaulters are also denied access to new loans. Our paper is also related to the recent work of Gu (2015) but with a different focus. She introduces vertical integration into the bilateral trade relationship between a creditor and a debtor while examining the dynamics of terms of trade and trade volumes, while our work aims to provide an answer to the optimal trade cost a creditor imposes on a debtor after debt renegotiation takes place. In terms of empirical analysis, our paper is in line with Martinez and Sandleris (2011) who find debtors' bilateral trade with creditor countries does not fall more than trade with other countries. On the computation side, we follow Hatchondo et al. (2010)'s recommendation and use cubic spline interpolation rather than discrete state space technique to approximate the value functions to reduce computational burden.

The remainder of the paper proceeds as follows: Section 2 presents the empirical findings. Section 3 describes the model as well as the properties of the recursive equilibrium. Section 4 elaborates on the algorithm, parameterizations and numerical results. Section 5 concludes.

3.2   Empirical Analysis

In this section, we present our findings about the effect of sovereign defaults on trade. We are interested in the dynamics of trade share after a debt renegotiation because it is a more accurate measure of trade sanction/benefit between a creditor and a debtor than trade volume: if there were trade sanction, creditors would disproportionably depress their trade with debtors, which would lead to lower creditor-debtor trade share after debt renegotiation.

Following Rose (2005), we track sovereign default episodes since 1956 from the Paris Club. It is an informal group of financial officials from 19 of some of the world's biggest economies, which provides financial services such as war funding, debt restructuring, debt relief and debt cancellation to indebted countries and their creditors. We

Figure 3.2.1: Debtors' Trade Share in Creditors' and Non-creditors' Imports
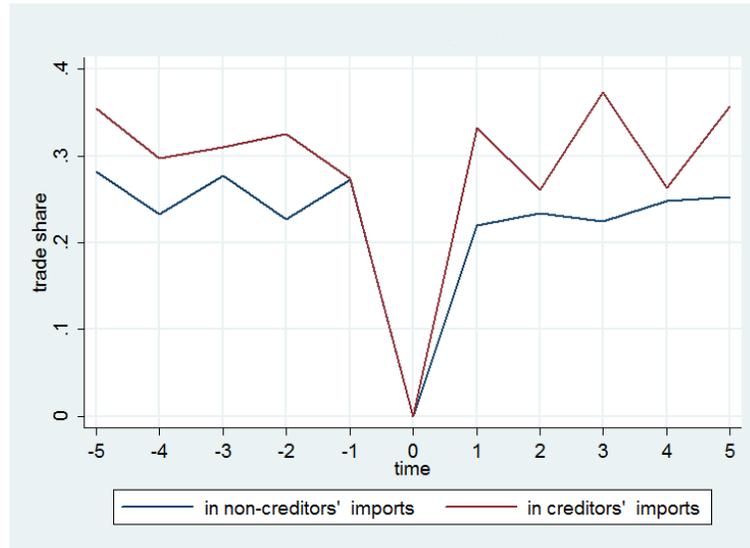
recognize that there are diverse forms of international lending besides the debt exchanges between governments[1] , yet the Paris Club has remained a central player in the resolution of developing and emerging countries' debt problems. We can track the date, the list of creditors, the amount of debt and the terms of treatment of sovereign debt renegotiations from the association. Another reason that we only consider government-to-government bilateral agreements is that private lending does not have so direct impact as public lending on the flow of international trade. After all, governments are the major players to design trade policies and sign trade treaties.

Before we move to regression analysis, it is intuitive to first show the trends of trade share graphically around sovereign default periods. In Figure 3.2.1, we plot debtors' share in creditors' and non-creditors' imports averaged over all the default episodes. Trade share reaches its trough in the default year (denoted as zero on the x-axis) when debtors' economies experience hardest hit. However, it is noticeable that debtors' trade with creditors is able to recover sooner and better than that with non-creditors: while the trade share in non-creditors' imports is lower than the level before defaults, the trade share in creditor's imports bounces back and even higher than the level before defaults.

I herein use a panel regression to quantify the effect of sovereign defaults on trade. The first step we take is to replicate Rose (2005)'s results with extended data till 2012.

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[1]Besides government to government bilateral debt under the Paris Club umbrella, debtor countries also issue commercial bank debt under the London Club, or issue bond debt. For detailed elaboration and comparison of different forms of sovereign debts, see Das et al. (2012).

The original gravity model of Rose is

$$ln(X_{ijt}) = \beta_0 + \beta_1 ln(Y_i Y_j)_t + \beta_2 ln(Y_i Y_j / Pop_i Pop_j)_t + \beta_3 ln D_{ij} + \beta_4 Lang_{ij}$$

$$+\beta_5 Cont_{ij} + \beta_6 FTA_{ijt} + \beta_7 Landl_{ij} + \beta_8 Island_{ij} + \beta_9 ln(Area_i Area_j)$$

$$+\beta_{10} ComCol_{ij} + \beta_{11} CurCol_{ijt} + \beta_{12} Colony_{ij} + \beta_{13} ComNat_{ij} + \beta_{14} CU_{ijt}$$

$$+\beta_{15,0} IMF_{ijt} + \sum_k \beta_{15,k} IMF_{ijt-k} + \phi RENEG_{ijt} + \sum_m \phi_m RENEG_{ijt-m} + \epsilon_{ijt}$$

X_{ijt} is the trade flow between country i and j at time t. Y denotes real GDP and Pop denotes population, so that Y/Pop is income per capita. Dij represents the distance between i and j and $Area$ represents a country's land mass. Binary variables include $Lang$ (common language), $Cont$ (common border), FTA (regional trade agreement), $ComCol$ (common colonizer after 1945), $CurCol$ (colonies at time t), $ComNat$ (part of the same nation at time t) and CU (same currency). $Landl$ and $Island$ are the numbers of landlocked and island countries in the country pair, which take the value of 0,1, or 2. $RENEG_{ijt}$ is a binary variable which is unity if i and j renegotiated international debt at time t and zero otherwise, meanwhile IMF_{ijt} is one/two if one/both of i or/and j began an IMF program at t and zero otherwise. Lagged $RENEG$ and lagged IMF are also listed as explanatory variables, considering the change of trade flow is a gradual and persistent process.

Our first goal is to extend Rose's data by 15 years to reflect the recent trend of sovereign defaults. In collecting the data, we do our best to choose similar, if not the same data sources as Rose, in order to make the results consistent and comparable. We get the trade data from the 'Direction of Trade Statistics (DOTS)' dataset by the International Monetary Fund (IMF). The values are in current US dollars, so we deflate them by the US CPI (82-84=100) from BLS. GDP and population data are taken from World Bank's 'World Development Indicator'. In the case of missing values, we turn to Penn World Table. Values of other common variables for a gravity model including distance, contiguity, language and colonization are available in the CEPII dataset[2]. The information about regional trade agreements is updated with the records from the World Trade Organization. Lastly, we get the list for the IMF programs from Axel Dreher. See Table 5 in Appendix for detailed categories.

The results on trade volumes we get are similar to Rose's, in both sign and mag-

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[2]It is a square gravity dataset for all pairs of countries, downloadable at http://econ.sciences-po.fr/thierry-mayer

nitude of the estimated coefficients. Table 1 lists the estimates in fixed-effect and random-effect model with contemporaneous and fifteen lags of the dummy variable for debt renegotiation. In all the cases (i.e. bilateral trade, trade from debtor to creditor (denoted as country1to2), and trade from creditor to debtor (country2to1)), the linearly-combined coefficients of contemporaneous and lagged debt renegotiation $—\sum_{t=1}^{15} RENEG$ — are all negative, whether we employ a fixed-effect or random-effect model. This result indicates that bilateral trade volumes between a creditor and a debtor decrease after a sovereign default.

Table 3.2.1: Linearly Combined Contemporaneous and Lagged Effects of Debt Renegotiation on Trade Volumes

	Coefficient	Std. Err.	t	[95 percent Conf.	Interval]
bilateral FE	-1.098	0.118	-9.300	-1.329	-0.867
bilateral RE	-1.608	0.119	-13.490	-1.842	-1.375
trade 1to2 FE	-1.416	0.150	-9.460	-1.710	-1.123
trade 1to2 RE	-2.177	0.151	-14.410	-2.473	-1.881
trade 2to1 FE	-1.426	0.144	-9.930	-1.708	-1.145
trade 2to1 RE	-1.891	0.144	-13.090	-2.174	-1.608

After replicating Rose's original results, we go a step further to analyze trade share. We believe it is the relative but not the absolute change in trade that reflects the existence and severity of trade sanction after defaults take place. To this end, we create two variables $Impw1to2$ and $Impw2to1$ to measure 1) of all a debtor's imports, how much comes from its creditor, and 2) of all a creditor's imports, how much comes from its debtor. Then we replace trade volumes with these two measures as the dependent variable in the regression.

We deal with two caveats when we analyze trade share. First, we add exchange rates as an independent variable as currency depreciation may bias the results. For instance, the collapse of South America during the 1970's debt crisis affected the currency values of nearly all the countries in the whole region. The covariance between exchange rates across Latino countries was different from that between Latino countries and developed countries, which mattered for the change of trade share. To correct this bias, we collect data of exchange rates starting from 1948 from the International Financial Statistics (IFS). The original data are in the units of currency per US dollars, which can be converted to obtain the relative exchange rates between two arbitrary currencies under the no-arbitrage condition in the foreign exchange market. Second, we need to control for bilateral trade elasticity for fear that the change in

trade share is simply driven by the composition effect given different trade baskets among countries. Elasticity is estimated with the equation

$$log\ trade_{ij,t} = \alpha_0 + \alpha_1 log\ GDP_{i,t} + \alpha_2 log\ GDP_{j,t} + \sum_{t=0}^{m} log\ trade_{ij,t-m}$$

Table 3.2.2: Linearly Combined Effects of Debt Renegotiation on Trade Share

	Coefficient	Std. Err.	t	$p > |t|$	[95 percent Conf.	Interval]
share 2in1	0.0590	0.0048	12.2000	0.0000	0.0495	0.0684
share 1in2	0.0628	0.0047	13.4600	0.0000	0.0537	0.0720

Table 2 lists the regression results in the fixed-effect model. We present the coefficient and the standard error of $\sum_{t=1}^{15} RENEG$, the linear combination of coefficients on paris,paris1-15. It is a good measure of the effects of sovereign defaults on trade. From the table, debt renegotiation has significantly positive effect on trade share. The coefficients on default variables are not only significant but also remarkable in magnitude. For instance, a sovereign default is associated with a 5% increase of debtors' share in creditors' trade. The increase is impressive, given the number of trade partners available nowadays in the integrated world market. We believe the increase in trade share indicates that sovereign defaults do not lead to trade sanction, but are instead associated with trade benefit.

Trade share has been used by the economists at the World Bank to uncover trade costs. The approach was developed by Head and Ries (2001) and extended by Novy (2013), who derived a micro-founded measure of bilateral trade costs that indirectly infers trade frictions from observable trade data. The measure turns out to be consistent with a broad range of leading trade theories including Ricardian and heterogeneous firms models. The bilateral comprehensive trade costs are calculated as follows

$$\tau_{ij} = \left(\frac{X_{ii}X_{jj}}{X_{ij}X_{ji}}\right)^{\frac{1}{2(\sigma-1)}} - 1$$

where τ_{ij} represents the geometric average of trade costs between countries i and j relative to domestic trade costs within each country. Its value means that trading goods between i and j involves, on average for all tradable goods, additional costs amounting to approximately τ_{ij} of the value of goods — as compared to when the two countries trade these goods within their borders. It covers tariffs, transportation costs, and other unobservable trade barriers. X_{ij}, X_{ji} denote trade between i and j,

X_{ii}, X_{jj} denote domestic expenditure.

With a little math, it is easy to see an increase in trade share is *equivalent* to a decrease in trade cost. Thus, we hypothesize that trade costs between a creditor and a debtor are lower after default happens.

While our argument will be much stronger if we can support our hypothesis with a consistent and continuous data set of visible and invisible trade costs, such data set is rare.[3] Alternatively, we turn to case studies with OECD's aid for trade dataset to see whether the efforts to boost bilateral trade are strengthened when debt renegotiation happens. For the purpose of the paper, we restrict our attention to the categories of aid which are directly related to trade policy adjustment (See Table 7 for details). Figure 3.A.1 plots the change in aid (only for trade policy purposes) around the default period for the following three cases: Honduras in 2004, Congo in 2008 and Burundi in 2009. In the years of sovereign defaults, creditors double or triple their expense in trade-related aid to help defaulters out. They are generous with trade benefit instead of strict with harsh trade punishment. The case studies serve as indirect evidence for our hypothesis that creditors lower their trade costs with debtors.

In a nutshell, sovereign renegotiation is associated with increased bilateral trade share between debtors and creditors. This empirical result, in line with Martinez and Sandleris (2011), contradicts the prediction of the trade sanction theory of sovereign debt. Based on Novy (2013)'s trade costs theory and aid-for-trade data, we believe bilateral trade costs decrease after debt renegotiation.

3.3   Model

In the previous section, we challenge the conventional trade sanction theory of sovereign default. Our explanation for this interesting observation is that a creditor is willing to compromise in the trade channel in order to minimize its loss from the financial side. In another word, when the creditor finds the debtor on the brink of defaulting, it is willing to lower trade costs to boost the debtor's exports such that the debtor is more probable to service the debt. The reduced trade costs will in turn determine a debtor's willingness to repay. The strategic interaction between the two parties are solved in the Markov perfect equilibrium of our model.

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[3]Bilateral tariff and non-tariff data from the World Bank's WITS is discontinuous and available only for the past decade. Trade costs in our paper are broader in definition, so it is hard to find direct comprehensive evidence.

3.3.1 Model Environment

There are three countries: a creditor, a debtor and the rest of the world (ROW). Although commonly used for sovereign default problems, a model with a small open economy is not able to capture the strategic interaction between a creditor and a debtor. Meanwhile, a standard two-country model is not helpful in studying the trend of trade share after sovereign defaults. To this end, we will build a creditor-debtor two-country model integrated with a world market (or ROW).

There are two countries in the model, $i = c, d$ (c denotes the creditor and d denotes the debtor). Each country is an endowment economy with a country-specific good. For simplicity, we assume the income of the creditor $\bar{A}$ in each period is constant and large enough for the country to always be the lender. On the other hand, the income of the debtor follows an AR(1) process

$$y_t = \rho_y y_{t-1} + (1 - \rho_y)\bar{y} + \epsilon_t$$

where $\epsilon_t \sim N(0, \sigma_i^2)$ and $\bar{y}$ is the long run mean of y_t.

Other than the two countries, there is a world goods market that both countries can interact with. Specifically, our world market consists of two parts — world financial market and world final goods market. In the world financial market, there is a risk free asset called world bond with rate r. The world goods market supplies one kind of consumption good c_w. To simplify the price and quantity of the world goods, we assume $c_{iw} = c_{wi}$ which indicates the world goods can always be traded one-for-one for domestic goods.

Country i's objective is to maximize the expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{i,t}^{1-\gamma}}{1-\gamma}$$

where utility takes the form of constant relative risk aversion (CRRA) from consumption

$$C_{i,t} = [\theta_{ii} c_{ii,t}^\rho + \theta_{ij} c_{ij,t}^\rho + (1 - \theta_{ii} - \theta_{ij}) c_{iw,t}^\rho]^{\frac{1}{\rho}}$$

The consumption composite of country i consists of domestic goods (c_{ii}), foreign goods (c_{ij}) and world goods (c_{iw}) with elasticity of substitution $\frac{1}{1-\rho}$. For simplicity, we assume preference is symmetric across countries $\theta_{ii} = \theta_{ii} = \theta_h$ and $\theta_{ij} = \theta_{ji} = \theta_f$.

The market clearing condition of goods i states that

$$c_{ii} + c_{ji} + c_{wi} = y_i$$

Let p_{ii} represent the price of goods i in its source country. There is a trade cost $\tau_{ij} > 1$ imposed by country i on goods coming from country j, reflecting trade restrictions like tariff. Thus, the effective price of imports from country j to country i is $p_{ij} = \tau_{ij} p_{jj}$. As we are mainly interested in the impact of the creditor's trade policies on the debtor's default decisions, we assume $\tau_{dc} \equiv 1$. An implication of this assumption is that there is no trade retaliation or reciprocity from the debtor's side. On the other hand, the creditor has some flexibility in adjusting trade costs $\tau_{cd} \in [\underline{\tau}, \overline{\tau}]$. Tariff also becomes part of the creditor's income for the model to yield a non-corner solution to τ_{cd}. As we will show later, τ_{cd} is a crucial policy instrument that affects not only bilateral trade but also bilateral debt. The trade costs between a country and the rest of the world are set equal to zero for simplicity: $\tau_{iw} = \tau_{wi} = 1$, $i = \{c, d\}$.

The debtor issues one-period risky bonds to the creditor. The bond market features limited enforcement in that the debtor can default on its debt. Two default states (S_0, S_1) coexist in the debt problem:

State 0 (S_0): The debtor repays the bilateral debt previously and retains financial ties with the creditor.

State 1 (S_1): The debtor defaults previously and is stuck in financial autarky.

In S_0, the debtor chooses from two default options ($D \in \{D_0, D_1\}$). It either services the debt (D_0) and stays in S_0, or defaults (D_1) and downgrades to financial autarky in the next period. In S_1, it no longer issues debt and consumes its endowment.

The timeline of the model is summarized in Figure 2. At the beginning of period t, the debtor can issue risky bond b to the creditor if it is in S_0. The creditor lends money, chooses risk-free asset b_c from the world financial market and sets trade cost τ. When the one-period bond matures at $t+1$, the debtor observes the realization of its current endowment and chooses either to repay the debt so as to stay in S_0, or to default and move to S_1. Meanwhile the creditor sets τ' based on state variables b, b_c and y. If the debtor defaults previously, it is in financial autarky (S_1). Following Aguiar and Gopinath (2006), there is an exogenous probability λ for the debtor in S_1 to regain access to borrowing.

Figure 3.3.1: Timeline

in S0: debtor borrows b
and creditor sets bc and τ

debtor repays and borrows b',
creditor sets τ' and bc'

t   t+1   t+2

in S1: debtor borrows
nothing and consumes
its income

debtor defaults and
downgrades to S1, creditor
sets τ' and bc'

debtor returns to S1 with
probability λ
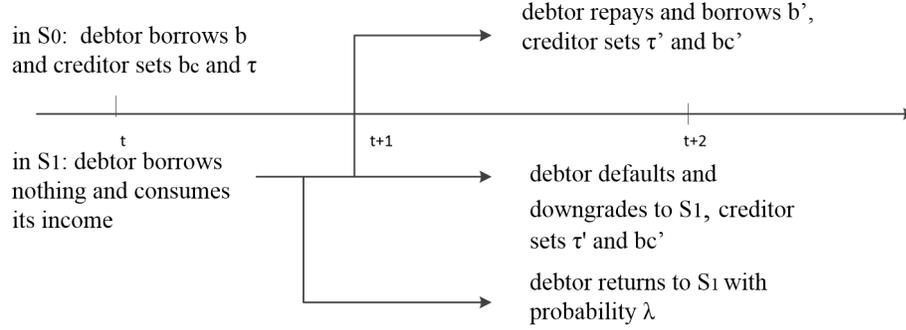
3.3.2 Recursive Equilibrium

State space in the model consists of default state $s \in S = \{S_0, S_1\}$ and a set of fundamental macroeconomic variables including the debtor's income, bilateral bond and the creditor's wealth w. Denote the set as $x = (y, b, w) \in X$. Agents' value functions and decision rules will depend on $S \times X$. In this section, we solve for the creditor's and the debtor's problem and define the equilibrium of the model.

3.3.2.1 Debtor's Problem

In S_0, the debtor enters a period with b and observes the endowment realization y. If it chooses not to default, it issues a new bond b' at the price $q(y, b', w)$ (denominated in the debtor's goods price). If it chooses to default, its debt b is written off but it moves to financial autarky at the beginning of next period. Denote the value function of a debtor who has not previously defaulted by $V_d(S_0, y, b, w)$.

$$V_d(S_0, y, b, w) = \max\{W_0(y, b, w), W_1(y, b, w)\}$$

where $W_0(y, b, w)$ is the welfare by choosing D_0 and $W_1(y, b, w)$ is the welfare by choosing D_1. A debtor makes its default decision upon the comparison of the two welfare levels

$$D_s = arg \max_s W_s(y, b, w), \quad s = \{0, 1\}$$

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More specifically, $W_0(y, b, w)$ can be expressed as

$$W_0(y, b, w) = \max_{C_d \geq 0, b'} U(C_d) + \beta E[V_d(S_0, y', b', w')|y]$$

subject to

$$C_d + q(y, b', w)b' \leq y + b$$

Notice that everything is denominated in debtor country's domestic good. Thus, the debtor's total expenditure on consumption is

$$C_d = \frac{c_{dd}p_{dd} + c_{dc}p_{cc} + c_{dw}p_{dw}}{p_{dd}}$$

From now on, we normalize p_{dd} to be one and define $\frac{p_{cc}}{p_{dd}} \equiv p$. Thus, $C_d = c_{dd} + c_{dc}p + c_{dw}$. We also discipline the level of bonds with the financial constraint following Aiyagari (1994)

$$b' \geq -\frac{\bar{y}}{r}$$

where r is calibrated to the world interest rate. As long as the debtor does not borrow $b' > 0$, it saves the money in the world financial market at rate $q_f = \frac{1}{1+r}$.

Similarly, $W_1(y, b, b_c)$ the welfare of choosing D_1 follows

$$W_1(y, b, w) = \max_{C_d \geq 0} U(C_d) + \beta E[V_d(S_1, y', 0, w')|y]$$

subject to

$$C_d \leq y$$

A country in S_1 is in financial autarky, but it has exogenous probability λ of returning to S_0 in the next period. Hence, its value function becomes

$$V_d(S_1, y, 0, w) = \max_{C_d \geq 0} U(C_d) + \beta(\lambda E[V_d(S_0, y', 0, w')|y] + (1 - \lambda)E[V_d(S_1, y', 0, w)|y]$$

subject to

$$C_d \leq y$$

3.3.2.2 Creditor's Problem

In this paper, we assume the creditor does not face any income uncertainty for simplicity. Its endowment level $(y_c = \bar{A})$ is so high that it will always be the lender in the bilateral debt relationship.

When the creditor deals with the debtor who hasn't defaulted in the last period,

its value function is

$$V_c(S_0, y, b, w) = \max_{C_c, b'_c \geq 0, \tau_0} U(C_c) + \beta E[\pi_{00}(y, b', w)V_c(S_0, y', b', w')|y$$

$$+\pi_{01}(y, b', w)V_c(S_1, y', b', w')|y]$$

subject to

$$C_c - \frac{q(a, b', w)b'}{p} + q_f b'_c \leq y_c - \frac{b}{p} + b_c$$

where

$$C_c = \frac{c_{cc}p + \tau_{cd}c_{cd} + c_{cw}p}{p}$$

$\pi_{mn}(y, b', w')$ represents the debtor's probability of going to state S_n from state S_m conditional on y. There is a cutoff income value y^* of the debtor that it is going to default if and only if the realization of y is below the value. Thus, we have

$$\pi_{00}(y, b', w) = Pr(y' > y^*|y) = \int_{y^*}^{\bar{y}} f(y'|y)dy' = 1 - \pi_{01}(y, b', w)$$

If the debtor is in the default state, the creditor's value function $V_c(S_1, y, b, w)$ is

$$V_c(S_1, y, b, w) = \max_{C_c, b'_c \geq 0, \tau_1} U(C_c) + \beta E[\lambda V_c(S_0, y', 0, w')|y + (1 - \lambda)V_c(S_1, y', 0, w')|y]$$

subject to the budget constraint

$$C_c + q_f b'_c \leq y_c + b_c$$

Financial wealth is the aggregate holding of the two bonds. Since there is possibility of default, we need to multiply risky asset by the debtor's repayment decision $D \in \{1, 0\}$ where $D = 1$ represents the repayment case and $D = 0$ represents the default case.

$$w = D(-b) + b_c$$

3.3.2.3   Bond Price

The creditor can choose between two assets: a risky asset and a risk free asset. The former is the bilateral bond at price q. The latter is the bond purchased from the world financial market at $q_f = \frac{1}{1+r}$. If the debtor is in the default state, the creditor's saving which is the difference between its income and consumption is used solely to purchase risk-free asset b_c. If the debtor has good credit history, the creditor's saving

is divided between b and b_c. In this case, the bilateral bond price can be determined by the creditor's Euler equation

$$q \frac{\partial V_c}{\partial C_c} = \beta E \frac{\partial V_c'}{\partial C_c'}$$

The right hand side is the expected marginal utility from tomorrow's consumption, which incorporates the default probability of the debtor. As is pointed out by Lizarazo (2013), bond price is higher in the case where creditors are risk-averse due to the fact that there is covariance between creditor' consumption level and debtors' default decision.

3.3.2.4 Goods Price

p denotes the creditor's goods price p_{cc} relative to the debtor's goods price p_{dd}. Based on the creditor's budget constraint,

$$\frac{c_{cc}p + \tau_{cd}c_{cd} + c_{cw}p}{p} - \frac{q(a, b', w)b'}{p} + q_f b_c' = y_c - \frac{b}{p} + b_c$$

we find p is determined jointly by debt b, wealth w and trade cost τ. In the model, the creditor chooses optimal wealth and trade cost to maximize its utility. In this process, it is considering the gain from both the lending channel and the trade channel. This explains why τ may deviate from its optimal value when the two countries do not borrow and lend to each other. The debtor anticipates the lower trade cost and strategically makes its borrowing the default decision. This mechanism can be used to explain why both debt level and default probability are higher than expected.

3.3.2.5 Markov Perfect Equilibrium

We now proceed to define the equilibrium of the model.

A *Markov Perfect Equilibrium* consists of the debtor's value function $V_d(S, X)$, the creditor's value function $V_c(S, X)$, bond holdings b', b_c', consumption choices C_c, C_d, default decisions D, trade costs τ, bond pricing schedule $q(y, b', w)$, and relative goods price p, such that

1. Given the bond price q, goods price p, trade costs τ_{cd}, the creditor's wealth w and consumption C_c, the debtor chooses optimal C_d, D and b' to maximize its

expected lifetime utility.

2. Given the debtor's default decisions D, bond holding b' and consumption C_d, the creditor chooses optimal τ_{cd}, b_c and C_c to maximize its expected lifetime utility.

3. Bond market clears at q and goods market clears at p.

3.4 Computation

In this two-country model, the creditor and the debtor decide interactively their policy rules. The numerical solution to the model is found over the space of three state variables, b the bilateral bond, w the creditor's wealth and y the debtor's income.

The solution to the problem is found through a combination of policy function iteration and value function iteration. We first divide all the three state variables into grids and compute the initial value function at each grid based on different default states. Second, we derive interactively the optimal choice of bond holding of both countries and the creditor's optimal trade cost τ. In this process, we approximate the value function by cubic spline interpolation, which is significantly more efficient and accurate than the discrete state space technique which is commonly used for the computation of sovereign default problems, as is pointed out by Hatchondo et al. (2010). After we find optimal policy functions, we solve for the debtor's default decision and update its value function. We continue the iterating process until the difference between value functions in consecutive iterations is smaller than my precision criterion. The algorithm is described in detail below.

3.4.1 Algorithm

Step 1. Discretize b, w, y and compute the corresponding consumption of the debtor at all the grid nodes. In different default states S_0, S_1, calculate the utility from consumption V_0^0, V_1^0. The initial value guess is the higher of the two $V^0 = \max\{V_0^0, V_1^0\}$.

Step 2. In default state S_1, solve for the creditor's optimal choice of tariff τ_1 and bond holding b_c. With τ_1, calculate the price level that clears the goods market and the resulting debtor's value function V_1^1.

Step 3. In repayment state S_0, guess an initial value of tariff τ_0^0 and calculate the corresponding price level.

Step 4. Given the creditor's choice, solve the debtor's problem to get the optimal borrowing in the next period b', with which to update the best responding bond holding b'_c and τ_0^1 by maximizing the value function of the creditor.

Step 5. Continue the iterating process until τ_0 converges, at which time compute the debtor's interpolated value function V_0^1.

Step 6. Compare the debtor's value function V_0^1, V_1^1, and find the maximum $V^1 = \max\{V_0^1, V_1^1\}$.

Step 7. Repeat Step 2 - Step 6, until value function converges, $|V^{i+1} - V^i| \le \epsilon_v$.

3.4.2 Calibration

Parameters in the model are chosen in our best effort to match either stylized facts or classical literature on the topic. The coefficient of relative risk aversion σ is set to 2. Discount factor β is set to be relatively low as in Aguiar and Gopinath (2006) to speed up convergence of solution and to get a reasonable prediction of default occurrence. We set the elasticity of substitution between goods ρ to be 2 and the weight of domestic/partner's goods in consumption is $\theta_h = \theta_f = .3$ in the benchmark case. These two parameters are important in reflecting the relative significance of bilateral trade. We will do a numerical exercise by looking at value functions and default decisions when varying the values of ρ. Also following Aguiar and Gopinath (2006), we assume income in the debtor country follows an AR(1) process with coefficient of autocorrelation $\rho_y = .9$ and standard deviation 3.4%. The advantage of choosing the parameter values in a classic paper is that we can directly compare our results, and highlight the contribution our model — which is the trade channel — to the existing literature. To this end, we also temporarily set $b_c = 0$ and focus on bilateral lending. To start with, we assume the endowment of the creditor is twice that of the debtor $\bar{A} = log2$. The relative economy size also comes into play in affecting the creditor's willingness to adjust trade costs and forgive debt.

All the parameter values are summarized in Table 3.

Table 3.4.1: Parametrization

Parameter	Description	Value
β	quarterly discount factor	0.80
σ	coefficient of relative risk aversion	2
r	international risk-free rate	0.01
λ	probability autarky ends	0.1
	Income process	
ρ_y	coefficient of autocorrelation in endowment of debtor	0.9
σ_y	standard deviation of endowment shocks of debtor	0.034
$\bar{y}$	average endowment level of debtor	.00058
$\bar{A}$	constant endowment level of creditor	$log2$
	In the benchmark case	
θ	weight of home/partner's goods in consumption	0.3
ρ	elasticity of substitution between goods	0.75

Table 3.4.2: Comparison across Models

Variable	Description	AG's result	Our result
$std(c)$	consumption volatility	4.37	4.03
$std(tb/y)$	trade-balance volatility	.17	2.81
$corr(y,c)$	correlation between income and consumption	.99	.79
$corr(y,tb)$	correlation between income and trade-balance	-.33	-.10
$avg(b/y)$	average debt ratio	.27	.34
$d\%$	default probability	.02%	.48%

3.4.3 Results

3.4.3.1 Comparison with Previous Work

We first compare the performance of our model with that of Aguiar and Gopinath (2006) (AG for short hereafter) in capturing the features of sovereign defaults. We use 150 simulation samples with 500 periods and report statistics in the Table 4. Among all the statistics, consumption volatility and average debt ratio are similar across models. Trade-balance volatility is much greater in my model, as the price adjusts based on the two countries' endowment as well as creditor's trade costs. Correlation between income and consumption turns out to be smaller in our model, partly due to the additional uncertainty from price changes. We also manage to simulate counter-cyclical trade balance, but the value is greater than that in AG since creditors adjust trade costs to boost debtors' exports. Lastly, both debt level and default probability are much higher in our model. The additional trade benefits encourage debtors to take on more debt than what they can afford.

3.4.3.2 Trade Costs

In this part, we evaluate the adjustment of trade costs in our model. The following two graphs represent the trend in τ in the two default states S_0 and S_1 given different combinations of endowment y and debt b.

It is easy to spot the monotonic relationship between τ_1 and y. When there is no outstanding debt in S_1, a debtor's price of exports negatively comoves with its endowment. As the elasticity of substitution between goods is below unity in the baseline case, price adjusts in the same direction as tariff revenue. Thus it is in the creditor's interest to set a high trade cost when the debtor's endowment is low. And the optimal tariff in the default state is independent of initial debt b as the tariff does not affect repayment probability.

In S_0 with outstanding debt (which corresponds to the debt renegotiation stage in data), the optimal tariff not only covaries with the debtor's endowment but also the debtor's amount of outstanding debt. For a relatively low level of debt, when we control for b, we find τ_0 decreases in the debtor's endowment y. This fact can be explained by the same reasoning as in the S_1 state: trade policies do not matter for the debtor's default decision because it is always in the debtor's interest to service the debt. Hence, the creditor chooses trade costs simply to maximize its revenue. We also find in this region that controlling for the level of y, τ_0 first decreases and then increases in initial debt. This is largely due to the curvature of the interior solution to the goods market clearing condition. We find interesting jumps in optimal tariffs above a certain debt level. It is within this region that the debtor is on the brink of defaulting and has non-smooth choices of b'. The shape of the surface can be explained by the following reasons. When debt is high, the debtor has higher probability to default. To avoid the financial loss of sovereign defaults, the creditor is willing to sacrifice in the trade channel by choosing a lower value of τ. Hence, the solution to the optimal τ_0 plummets in the region. It is worth-noting that the creditor and the debtor are best responding to each others choices. In expectation of lower τ in S_0, the debtor is also willing to take more debt than in an ordinary setting.

Next, let us compare side by side τ_0 and τ_1 by fixing the initial debt level to a high level and a low level.

In the case where initial debt is equal to zero, τ_0 and τ_1 are very close in value. τ_0 is slightly greater since the debtor is going to borrow from the creditor in the current period, the loss in wealth caused by lending is partially compensated by the increased

Figure 3.4.1: Optimal Trade Costs in S_0 and S_1
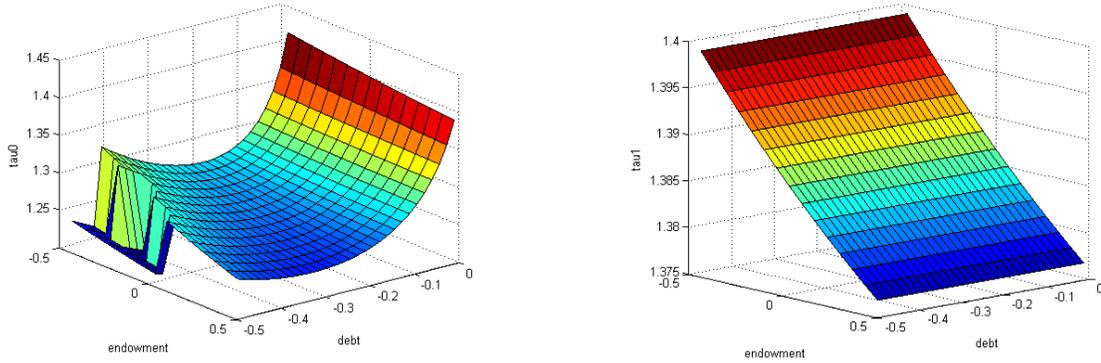
tariff revenue. Once the level of debt goes up, τ_0 is going to be significantly lower than τ_1. This is consistent with the main empirically finding of the paper: when the debtor is on the brink of defaulting, the creditor has the incentive to lower trade costs in order to increase repayment probability.

Figure 3.4.2: τ under different endowment
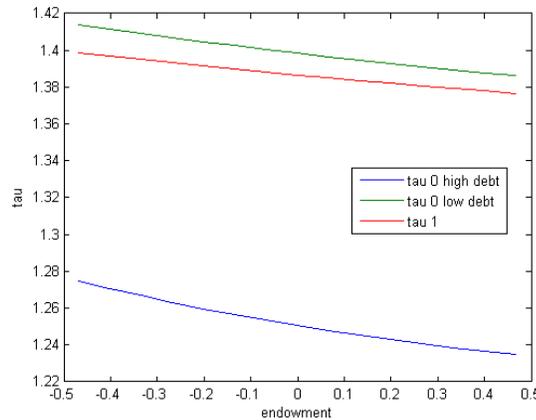
3.5 Conclusion

This paper identifies the increase in bilateral trade share between a creditor and a debtor when sovereign default happens. The finding runs contrary to the traditional trade sanction theory. We build a model which incorporates the trade channel in a sovereign debt problem to account for the phenomenon. The model builds on the strategic interaction between the creditor and the debtor. By solving the model

numerically, we are able to capture counter-cyclical trade balance and high default probability that are closer to data than other models.

We consider extending our model in the following ways so that it reflects reality better. First, we can build a production-economy model instead of endowment-economy model. Many debtors are in need of developed countries' support for capital goods and investment. By introducing two sectors (consumption goods and capital goods) into the model, the two countries will be more dependent on each other. Second, we consider introducing a partial default state into the model to reflect the renegotiation stage in sovereign defaults better. The equilibrium will feature financial haircut, grace period and dynamics in trade simultaneously. But the extension does come at the cost of a higher level of computation complexity. Lastly, we can relax the assumption of constant creditors' income, and study the creditors' incentives in different economic conditions. To sum up, there is much interesting interaction between the trade channel and the borrowing channel. We hope future research will explore the mechanism in depth so that we can have a better understanding of sovereign defaults.

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3.A   Appendix: Additional tables and charts

Table 3.A.1: IMF Programs

IMF SBA	IMF Standby Arrangement agreed
IMF EFF	IMF Extended Fund Facility Arrangement agreed
IMF SAF	IMF Structural Adjustment Facility Arrangement agreed
IMF PRGF	IMF Poverty Reduction and Growth Facility Arrangement agreed
IMF SBA 5	IMF Standby Arrangement in effect for at least 5 months in a particular year
IMF EFF 5	IMF Extended Fund Facility Arrangement in effect for at least 5 months in a particular year
IMF SAF 5	IMF Structural Adjustment Facility Arrangement in effect for at least 5 months in a particular year
IMF PRGF 5	IMF Poverty Reduction and Growth Facility Arrangement in effect for at least 5 months in a particular year

Figure 3.A.1: Aid for Trade during Sovereign Defaults
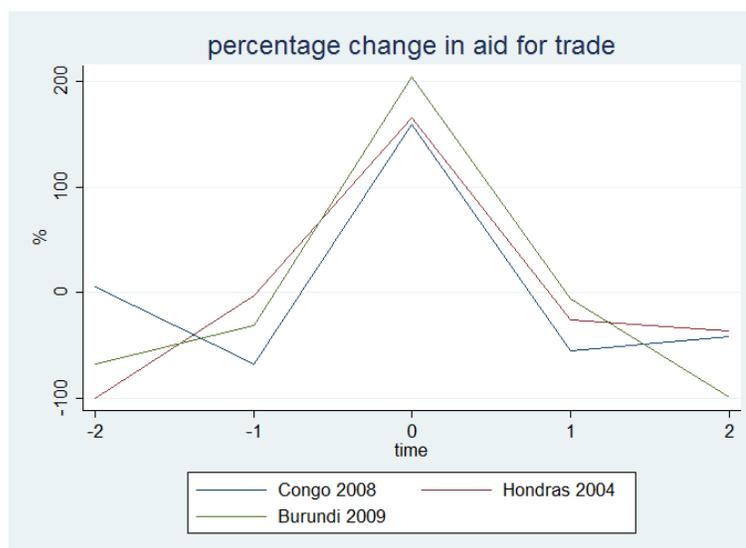
Table 3.A.2: Effect of Debt Renegotiations on Trade Volumes

	bilateral FE	bilateral RE	trade 1to2 FE	trade 1to2 RE	trade 2to1 FE	trade 2to1 RE
paris	-0.0115	-0.0114	0.0259	0.0124	-0.0496	-0.0459
	(-0.30)	(-0.32)	-(0.54)	(-0.25)	(-1.08)	(-0.99)
parisl1	-0.0438	-0.0515	-0.0432	-0.0639	-0.102*	-0.106*
	(-1.14)	(-1.32)	(-0.89)	(-1.30)	(-2.19)	(-2.25)
parisl2	-0.0358	-0.044	-0.0194	-0.0392	-0.0832	-0.0882
	(-0.92)	(-1.11)	(-0.39)	(-0.79)	(-1.76)	(-1.85)
parisl3	-0.057	-0.0746	-0.0732	-0.103*	-0.0705	-0.0861
	(-1.45)	(-1.87)	(-1.48)	(-2.05)	(-1.48)	(-1.79)
parisl4	-0.0376	-0.059	-0.0183	-0.0516	-0.0724	-0.0937
	(-0.95)	(-1.47)	(-0.37)	(-1.02)	(-1.51)	(-1.93)
parisl5	-0.0568	-0.0662	-0.0507	-0.0695	-0.115*	-0.125*
	(-1.43)	(-1.64)	(-1.01)	(-1.36)	(-2.38)	(-2.56)
parisl6	0.0226	-0.00173	0.0153	-0.0228	-0.0317	-0.0539
	(-0.56)	(-0.04)	(-0.3)	(-0.44)	(-0.65)	(-1.10)
parisl7	-0.0389	-0.07	-0.0862	-0.134**	-0.053	-0.0812
	(-0.96)	(-1.71)	(-1.69)	(-2.58)	(-1.08)	(-1.64)
parisl8	-0.0588	-0.0972*	-0.0904	-0.148**	-0.0588	-0.094
	(-1.44)	(-2.35)	(-1.76)	(-2.85)	(-1.19)	(-1.88)
parisl9	-0.0794	-0.112**	-0.116*	-0.167**	-0.043	-0.0737
	(-1.89)	(-2.63)	(-2.19)	(-3.11)	(-0.84)	(-1.43)
parisl10	-0.109*	-0.144***	-0.156**	-0.209***	-0.0897	-0.122*
	(-2.55)	(-3.32)	(-2.89)	(-3.82)	(-1.73)	(-2.33)
parisl11	-0.136**	-0.178***	-0.132*	-0.192***	-0.168**	-0.206***
	(-3.10)	(-4.00)	(-2.39)	(-3.43)	(-3.17)	(-3.84)
parisl12	-0.0615	-0.108*	-0.0911	-0.158**	-0.0728	-0.115*
	(-1.33)	(-2.31)	(-1.56)	(-2.67)	(-1.30)	(-2.04)
parisl13	-0.0779	-0.130**	-0.0963	-0.166**	-0.120*	-0.169**
	(-1.63)	(-2.68)	(-1.60)	(-2.70)	(-2.07)	(-2.87)
parisl14	-0.125*	-0.195***	-0.212***	-0.303***	-0.118*	-0.184**
	(-2.53)	(-3.88)	(-3.40)	(-4.80)	(-1.97)	(-3.03)
parisl15	-0.196***	-0.277***	-0.277***	-0.378***	-0.175**	-0.251***
	(-3.83)	(-5.34)	(-4.31)	(-5.80)	(-2.83)	(-4.01)
imf	-0.128***	-0.155***	-0.201***	-0.233***	-0.144***	-0.170***
	(-28.48)	(-34.03)	(-33.79)	(-38.87)	(-25.29)	(-29.80)
imfl1	-0.0248***	-0.0401***	-0.0366***	-0.0552***	-0.0290***	-0.0432***
	(-5.08)	(-8.11)	(-5.71)	(-8.50)	(-4.73)	(-6.98)
imfl2	-0.00372	-0.0132*	-0.000829	-0.0122	0.00468	-0.00298
	(-0.72)	(-2.53)	(-0.12)	(-1.79)	-0.73	(-0.46)
imfl3	-0.000458	-0.0109*	0.0129	0.000624	0.00785	-0.000803
	(-0.09)	(-2.00)	-1.85	-0.09	-1.18	(-0.12)
imfl4	0.0281***	0.0134*	0.0425***	0.0247***	0.0239***	0.0116
	(-5.1)	(-2.39)	(-5.95)	(-3.41)	(-3.48)	(-1.68)
imfl5	0.104***	0.0720***	0.147***	0.109***	0.125***	0.0981***
	(-19.68)	(-13.48)	(-21.66)	(-15.77)	(-19.09)	(-14.9)

custrict	0.421***	0.398***	0.389***	0.387***	0.296***	0.325***
	(-10.54)	(-10.33)	(-7.43)	(-7.68)	(-5.88)	(-6.7)
ldist	0.255***	-1.334***	0.0988	-1.556***	0.515***	-1.438***
	(-4.04)	(-80.74)	(-1.21)	(-70.94)	(-6.58)	(-66.55)
lrgdp	0.332***	0.543***	0.216***	0.528***	0.487***	0.676***
	(-54.99)	(-119.28)	(-25.7)	(-84.79)	(-61.38)	(-113.06)
lrgdppc	0.155***	0.00498	0.357***	0.102***	0.0924***	-0.0336***
	(-18.36)	(-0.79)	(-30.63)	(-11.97)	(-8.41)	(-4.10)
comlang	-0.0284	0.243***	0.0850**	0.419***	-0.271***	0.107***
	(-1.27)	(-12.56)	(-2.81)	(-16.05)	(-9.51)	(-4.33)
border	0.0191	0.976***	0.0234	0.983***	-0.158	1.177***
	(-0.1)	(-11.27)	(-0.09)	(-8.63)	(-0.62)	(-10.44)
regional	0.295***	0.275***	0.291***	0.264***	0.321***	0.312***
	(-10.39)	(-9.58)	(-8.1)	(-7.28)	(-9.27)	(-8.92)
landl	1.158***	-0.559***	1.202***	-0.623***	0.689***	-0.658***
	(-26.48)	(-27.48)	(-20.6)	(-22.97)	(-12.35)	(-24.75)
island	0.422***	0.197***	0.436***	0.219***	0.456***	0.254***
	(-20.25)	(-11.84)	(-15.76)	(-9.9)	(-16.89)	(-11.74)
lareap	0.496***	0.172***	1.419***	0.259***	0.531***	0.149***
	(-5.67)	(-36.3)	(-12.35)	(-40.79)	(-4.74)	(-23.9)
comcol	0.574***	0.197***	0.551***	0.222***	-0.216*	0.195***
	(-6.99)	(-5.03)	(-4.88)	(-4.22)	(-2.08)	(-3.8)
curcol	0.348***	0.539***	0.374***	0.617***	0.644***	0.797***
	(-4.05)	(-6.39)	(-3.43)	(-5.76)	(-6.13)	(-7.75)
colony	0.314**	1.408***	0.327**	1.534***	0.261*	1.368***
	(-3.23)	(-18.2)	(-2.66)	(-15.43)	(-2.21)	(-14.21)
comctry	-0.701***	-1.077***	-0.670***	-1.132***	-0.776***	-1.146***
	(-12.23)	(-19.33)	(-9.24)	(-16.06)	(-11.11)	(-16.93)
cons	-18.87***	-6.088***	-37.82***	-7.988***	-28.41***	-11.06***
	(-8.99)	(-33.77)	(-13.72)	(-33.16)	(-10.59)	(-47.02)

t statistics in parentheses. * significant at 10%, * significant at 1%, * significant at 1%
Country 1 denotes a debtor; country 2 denotes a creditor.

Table 3.A.3: Trade Policy, Regulations and Trade-Related Adjustment

CRS Code	Description	Clarifications / Additional notes on coverage
33110	Trade policy and administrative management	Trade policy and planning; support to ministries and departments responsible for trade policy; trade-related legislation and regulatory reforms; policy analysis and implementation of multilateral trade agreements e.g. technical barriers to trade and sanitary and phytosanitary measures (TBT/SPS) except at regional level (see 33130); mainstreaming trade in national development strategies (e.g. poverty reduction strategy papers); wholesale/retail trade; unspecified trade and trade promotion activities.
33120	Trade facilitation	Simplification and harmonisation of international import and export procedures (e.g. customs valuation, licensing procedures, transport formalities, payments, insurance); support to customs departments; tariff reforms.
33130	Regional trade agreements (RTAs)	Support to regional trade arrangements [e.g. Southern African Development Community (SADC), Association of Southeast Asian Nations (ASEAN), Free Trade Area of the Americas (FTAA), African Caribbean Pacific/European Union (ACP/EU)], including work on technical barriers to trade and sanitary and phytosanitary measures (TBT/SPS) at regional level; elaboration of rules of origin and introduction of special and differential treatment in RTAs.
33140	Multilateral trade negotiations	Support developing countries effective participation in multilateral trade negotiations, including training of negotiators, assessing impacts of negotiations; accession to the World Trade Organisation (WTO) and other multilateral trade-related organisations.
33150	Trade-related adjustment	Contributions to the government budget to assist the implementation of recipients own trade reforms and adjustments to trade policy measures by other countries; assistance to manage shortfalls in the balance of payments due to changes in the world trading environment.

Source: OECD Aid for Trade