

# ESSAYS ON INDUSTRIAL ORGANIZATION

by

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# DEDICATION

To My Family.

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# CHAPTER I

## Introduction

In this dissertation, I use empirical structural models to analyze how firms innovate, position products and form relationships. The first chapter asks two questions: 1) whether, from a welfare point of view, oligopolistic competition leads to too few or too many products in a market, 2) how competition affects the composition of product offerings. The chapter addresses these two questions in the context of the U.S. smartphone market. My co-author and I find that there are too few products and a reduction in competition further decreases the number of products and reduces product variety. These results suggest that merger policies should be stricter when we take into account the effects of a merger on product choice in addition to those on pricing.

The second chapter studies the effects of vertical integration on innovation in the chipset and smartphone industry. I formulate and estimate a dynamic structural model of the upstream chipset maker Qualcomm and downstream smartphone handset makers. The two sides make dynamic investment decisions and negotiate chipset prices via Nash bargaining. Using the estimates, I simulate market outcomes should Qualcomm merge with a downstream handset maker. I find that the vertical merger would significantly increase innovation rates and social welfare, driven primarily by the investment coordination of the two merged firms. I

also explore the roles of upstream product availability, downstream product substitution and consumer price sensitivity.

The third chapter examines two-sided matching markets with transferrable utilities. Agents in two-sided matching games vary in characteristics that are unobservable in typical data on matching markets. My co-authors and I investigate the identification of the distribution of unobserved characteristics using data on who matches with whom. In full generality, we consider many-to-many matching and matching with trades. The distribution of match-specific unobservables cannot be fully recovered without information on unmatched agents, but the distribution of a combination of unobservables, which we call unobserved complementarities, can be identified. Using data on unmatched agents restores identification. We estimate the contribution of observables and unobservable complementarities to match production in venture capital investments in biotechnology and medical firms.

## CHAPTER II

# Competition, Product Proliferation and Welfare: A Study of the U.S. Smartphone Market

with Ying Fan

### 2.1 Introduction

In many markets such as the printer market, the CPU market and the smartphone market, firms typically offer multiple products across a wide spectrum of quality. In these markets, product proliferation is an outcome of firms' oligopolistic competition in product space. Does such competition result in too few or too many products from a welfare point of view? How does a change in competition affect the number and composition of product offerings? In this paper, we study these two questions in the context of the U.S. smartphone industry.

For the first question, in theory, it is possible that oligopolistic competition results in either excessive or insufficient product proliferation. On the one hand, a profit-maximizing firm will have a product in the market as long as the profit gains are greater than the costs, but some of the profit gains may come from business stealing. Because firms do not take into account this negative externality, there may be too many products. On the other hand, unlike a social

planner, firms do not internalize consumer surplus. If consumer surplus increases when a product is added to the market, there may also be too few products. Therefore, whether competition leads to too few or too many products in the market is an empirical question.

For the second question, the effect of a merger on product offerings is also theoretically ambiguous. When two firms merge, the merged firm internalizes the business stealing effect and thus may reduce its number of products. This is a direct effect. However, there may also exist a countervailing indirect effect: a merger is likely to soften price competition. As a result, the profit gains from adding a product may be larger, leading to an increase in the number of products.

Combining these two research questions, this paper sheds light on how to adjust the leniency of competition policies when product offerings are endogenous. If competition leads to too many products and a merger reduces product offerings, then merger policies should be more lenient. Conversely, if a merger reduces product offerings when there are already too few products in the market, then merger policies should be stricter.

To study our research questions, we focus on the U.S. smartphone market for several reasons. First, the smartphone industry has been one of the fastest growing industries in the world, with billions of dollars at stake. Worldwide smartphone sales grew from 122 million units in 2007 to 1.4 billion units in 2015 (Gartner (2007) and Gartner (2015)), with about 400 billion dollars in global revenue in 2015 (GfK (2016)). Second, product proliferation is a prominent feature of this industry. For example, in the U.S. market during our sample period, Samsung, on average, simultaneously offered 11 smartphones with substantial quality and price variation.

In order to address our research questions and quantify welfare, we develop a structural

model of consumer demand and firms' product and price decisions. We describe the demand side using a random coefficient discrete choice model, where product quality is a linear function of a set of key product characteristics, and consumers have heterogeneous tastes for quality. We describe the supply side using a static three-stage structural model. In the first stage, smartphone firms choose products from a set of potential products, with each product associated with a fixed cost in each period. In the second stage, these firms set their wholesale prices for carriers based on the product portfolio of each firm as well as realized demand and marginal cost shocks. In the third stage, carriers set their retail prices.

Our data come from the Investment Technology Group (ITG) Market Research. This data set provides information on all smartphone products in the U.S. market between January 2009 and March 2013. For every month during this period, we observe both the price and the quantity of each smartphone sold through each of the four national carriers in the U.S. (i.e., AT&T, T-Mobile, Sprint, and Verizon). In addition, we observe key specifications of each product, such as battery talk time and camera resolution.

Using these data, we estimate our model of smartphone demand and marginal cost following an estimation procedure similar to that in Berry, Levinsohn and Pakes (1995a). The estimation results are intuitive: on average, consumers prefer smartphones with longer battery talk time, higher camera resolution, a more advanced chipset, a larger screen and a lighter weight. We use these results to calculate a product quality index, a linear combination of product characteristics weighted by the respective estimated demand coefficients. We then use our quality index to propose a measure of product variety such that adding a product identical to existing products in terms of the observed key characteristics (hence the quality index) has no impact on our variety measure. Therefore, this measure allows us to distinguish "meaningful" product differentiation from obfuscation. Our results show that product



variety within the U.S. smartphone market increases over time during our sample.

On the supply side, we find that marginal cost increases in quality and decreases over time. We also obtain bounds on fixed costs. Specifically, we assume that the observed product portfolio of a firm is profit maximizing in a Nash equilibrium. Consequently, removing or adding a product should not increase the firm's profit. Based on these conditions, for any product in the market in a month, we obtain an upper bound of its fixed cost in that month; and for any product not in the data in a given month, we obtain a lower bound.

Based on the estimated demand, marginal cost and fixed cost bounds, we conduct counterfactual simulations to address our research questions. To answer the question of whether there are too few or too many products in the market, we conduct two sets of counterfactual simulations for March 2013, the last month in our sample period. In one set of counterfactual simulations, we remove products while in the other set, we add products. To separate product variety from product innovation, we remove (add) only products below the quality frontier of each firm.<sup>1</sup> Our results show that removing a product decreases total surplus, even considering the maximum saving in the fixed cost. These results are robust no matter which product or which two products we remove. In the second set of simulations, we add a product that fills a gap in the quality spectrum. We find that consumer surplus, carrier surplus, and smartphone firms' total variable profit all increase. The change in total welfare is the sum of these increases minus the fixed cost of the added product. We find that the former is about 2.3 times the lower bound of the latter. Therefore, as long as the fixed cost is not more than 2.3 times of its lower bound, total surplus should increase. To put this ratio in perspective, note that the average upper bound is about 1.2 times of the average lower bound we obtain in our estimation. Overall, these counterfactual simulation results suggest

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<sup>1</sup>The next chapter focuses on innovation in the smartphone industry and its upstream chipset industry.

that there are too few products under oligopolistic competition.

Turning to the second research question of how a change in competition affects product offerings, we simulate the effect of a hypothetical merger between Samsung and LG on product offerings, prices, and welfare in March 2013. We also repeat the simulation for a Samsung-Motorola merger and an LG-Motorola merger. Again, to separate product variety from innovation, we allow firms to adjust only those products below their quality frontier. However, different from addressing the first research question, for which we only need to compute the new pricing equilibrium given certain product offerings in the market, we now need to compute the post-merger equilibrium in both product choice and pricing. Computing the product-choice equilibrium is challenging because, in theory, a firm can drop any subset of its current products or add any number of new products after a merger, leading to a large action space. To keep the problem tractable, we restrict the set of potential products for each firm to those offered by this firm in either February or March 2013, plus two additional products that vary in quality. Even with this restriction, a firm's action space can still be prohibitively large. For example, the merged Samsung-LG entity has 31 potential products, implying a choice set of  $2^{31} \approx 2.4 \times 10^9$  product portfolios. Therefore, to further deal with this computational challenge, we use a heuristic algorithm to find a firm's best-response product portfolio given the portfolios of its competitors, and embed this optimization algorithm in a best-response iteration to solve for the post-merger product-choice equilibrium. Results from Monte Carlo simulations show that our algorithm performs well at least for optimal product portfolio problems with a small number of potential products.<sup>2</sup>

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<sup>2</sup>In the Monte Carlo simulations, we study product-choice problems where the number of potential products is small enough for us to enumerate all possible product portfolios and determine the optimal one. We find that the failure rate for the heuristic algorithm (i.e., the percentage of simulations where the heuristic algorithm fails to find the true optimal product portfolio) is always lower than 0.3%, regardless of the starting point for the heuristic algorithm.

Using this algorithm, we find that after the Samsung-LG merger, the number of products in the market decreases. In particular, the merged firm drops 3 products (out of a per-merger combined total of 26 products) while competing firms altogether add one product. This reduction in the overall number of products also decreases product variety. Due to the decrease in product offerings and the accompanying increase in the prices, we find that consumers are worse off and total welfare also decreases after the merger. These findings hold for the other two mergers (Samsung-Motorola and LG-Motorola) as well.

In summary, we find that there are too few products in the market. We also find that a reduction in competition as a result of a merger further decreases product variety. These findings are robust to several variations to both the demand side and the supply side of the model. The combination of these findings suggests that merger policies may have to be stricter when we take into account the effect of a merger on product offerings in addition to its effect on prices.

By studying the welfare implications of product proliferation and how competition affects them, this paper contributes to the literature of endogenous product choice.<sup>3</sup> Two papers in the literature, Fan (2013a) and Berry et al. (forthcoming), are most closely related to this paper. Fan (2013a) also studies the effect of a merger considering firms' endogenous product choices. However, whereas Fan (2013a) keeps the number of products fixed, our model allows firms to adjust both the number and composition of products after a merger.<sup>4</sup>

Interestingly, despite the differences in focus and industries, the two papers make similar

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<sup>3</sup>Examples in this literature include Seim (2006a), Draganska, Mazzeo and Seim (2009), Watson (2009), Chu (2010), Crawford and Yurukoglu (2012), Sweeting (2013), Eizenberg (2014a), Nosko (2014), Crawford et al. (2015a), Orhun, Venkataraman and Chintagunta (2015) and Wollmann (2015). See Crawford (2012) for a survey of this literature. Examples in the theoretical literature on this topic include Johnson and Myatt (2003) and Shen, Yang and Ye (2016).

<sup>4</sup>Wollmann (2015) studies the importance of accounting for product entry in predicting price changes due to different policies and thus also allows firms to adjust the number of products.

policy recommendations: merger policies should be tougher when we take into account firms' post-merger adjustments in their product portfolios, whether such adjustments only concern the characteristics of a fixed set of products or also involve changes in the number of products. In another study, Berry et al. (forthcoming) examine the optimal level of product variety in a local radio market.<sup>5</sup> Our study differs from their work by considering product variety in a multi-product oligopoly setting instead of a single-product oligopoly setting. This difference in market structure may explain why they find too much product variety in the local radio market, but we find too few products in the U.S. smartphone industry. Compared to a single-product firm, a multiple-product firm has an additional reason for not adding a product: to avoid cannibalization. As a result, in a market with multiple-product firms, it is more likely that there are too few products.

This paper is also related to the stream of research that studies the smartphone industry. For example, Sinkinson (2014) studies the motivations behind the exclusive contract between Apple and AT&T for the early iPhones. In another study, Zhu, Liu and Chintagunta (2015) quantify the welfare effects of this exclusive contract. Luo (2015) examines the operation system network effect. Finally, my next chapter studies the effect of vertical integration on innovation in the smartphone industry and its upstream chipset industry. We complement these papers by studying the welfare implications of product choices and the effects of competition with endogenous product choice.

The rest of the paper is organized as follows. We describe the data in Section 2.2. We develop the model of the smartphone market in Section 2.3 and present the estimation results in Section 2.4. Section 2.5 first describes counterfactual simulations and then discusses the

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<sup>5</sup>Thomas (2011) studies a similar question from the firm perspective and finds that decentralized decision making by multinational firms leads to too many products in the sense that a firm's profit would increase with fewer products.

results. We discuss the robustness of the results in Section 2.6. Finally, we conclude in Section 2.7.

## 2.2 Data

Our data come from the Investment Technology Group (ITG) Market Research. This data set covers all smartphones sold in the U.S. market between January 2009 and March 2013. For every carrier in the U.S. and every month during our sample period, we observe the price and sales for each smartphone sold through that carrier in that month. We also observe key specifications of each product such as battery talk time and camera resolution. The price information provided by the ITG for the four major national carriers (AT&T, Verizon, Sprint, and T-Mobile) is the so-called subsidized price or the average price for a smartphone device that a carrier charges a consumer who uses this carrier’s network service.<sup>6</sup> Note that the subsidized price for a smartphone is not the true cost of buying the smartphone because the consumer also needs to pay for the service plan. As will be explained later, we include carrier/year-specific fixed effects in the model to capture the average service cost for a consumer. Furthermore, since non- major or fringe carriers often provide only prepaid service plans and serve only one regional market, we drop these observations from our analyses.<sup>7</sup>

In the end, our sample consists of 3256 observations, each of which is a smartphone/carrier/month combination. Table 2.1 presents the summary statistics on the quantity, price and product characteristics. From Table 2.1, we can see that the average monthly sales of a product are

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<sup>6</sup>The average is taken over transactions in a month. Note that the carrier fee structure is relatively stable during our sample period. In April 2013 (right after our sample period), however, T-Mobile launched an “Uncarrier” campaign, which abandoned service contracts and subsidies for devices. Other carriers followed suit.

<sup>7</sup>The total U.S. market share of these fringe carriers in terms of smartphones sold is about 10%.

around 77,000 while the standard deviation of the monthly sales is about twice the mean. We also find a sizable variation in price across observations: the price is 122 dollars on average, with a standard deviation of 85. For each product, we observe product characteristics such as battery talk time, camera resolution, screen size measured by the diagonal of the screen, and weight. We also observe the generation of the chipset used by each product. For example, there are five Apple smartphones in our data (i.e., iPhone 3G, iPhone 3Gs, iPhone 4, iPhone 4s and iPhone 5), each of which uses a chipset of a different generation. The standard deviations of these product characteristics are about 17% to 47% of their corresponding means, indicating a wide variety of products across our sample.

Table 2.1: Summary Statistics

Variable	Mean	Std. Dev.	Min	Max
Quantity (1000)	77.54	146.04	0.04	1419
Price (\$)	122.16	85.24	0 <sup>a</sup>	406.9
Battery Talk Time (hours)	7.08	2.93	3	22
Camera Resolution (megapixel)	4.65	2.18	0 <sup>b</sup>	13
Chipset Generation 2	0.23	0.42	0	1
Chipset Generation 3	0.25	0.43	0	1
Chipset Generation 4	0.14	0.34	0	1
Chipset Generation 5	0.09	0.29	0	1
Screen Size (inch)	3.44	0.73	2.20	5.54
Weight (gram)	135.31	22.72	89.5	193
Observations (smartphone/carrier/months)	3256			

<sup>a</sup>Four observations in our sample have a 0 price

<sup>b</sup>One product in our sample (BlackBerry 8830) does not have a camera

There are 18 smartphone firms and 260 smartphones in the sample. Table 2.2 lists the top six firms according to their average monthly smartphone sales: Apple, Samsung, BlackBerry, HTC, Motorola and LG. From Table 2.2, we see that Apple is the undisputed leader in the industry, with an average monthly sales of about 2 million units, followed by Samsung with an average monthly sales of 0.76 million. The table also shows that all of these six firms offer

Table 2.2: List of Top Six Smartphone Firms

Firm	Headquarters	Avg. Monthly Sales <sup>a</sup> (million units)	Avg. Number of Products <sup>a</sup>
Apple	U.S.	1.99	2.10
Samsung	Korea	0.76	11.08
BlackBerry	Canada	0.61	8.33
HTC	Taiwan	0.60	10.35
Motorola	U.S.	0.46	7.90
LG	Korea	0.33	6.76

<sup>a</sup>Averaged across months.

multiple products simultaneously. For example, on average, Samsung offers 11 products in a given month, followed by HTC with an average of 10 products in a given month.

Table 2.3 shows that the multiple products offered by a firm have different qualities and prices. In this table, we report two within-(firm/month) dispersion measures for price and product characteristics. To calculate within-(firm/month) price dispersion, for example, we first compute the standard deviation of price across all observations of a given firm in a given month. We set the standard deviation to 0 for firm/months with a single observation. We then take the average of these standard deviations across all 557 firm/months in the sample, and report this average in Column 1 of Table 2.3. Similarly, we compute the difference between the highest and the lowest price among all observations in the same firm/month and take the average across firm/months to obtain the average range within a firm/month, as shown in Column 2. We find that the average within-(firm/month) standard deviation in price is 42.42 dollars, which is about 1/2 of the overall standard deviation of price across all observations (see Table 2.1), implying that within-(firm/month) variation is an important component of total price variation. The within-(firm/month) variation of product characteristics is also significant. For example, Column 2 for chipset generation shows

that smartphone firms on average simultaneously offer products whose chipsets are one generation apart. Overall, Table 2.3 provides evidence for product proliferation in smartphone industry.

Table 2.3: Summary Statistics on Quality and Price Dispersion within a Firm/Month

	Average Std. Dev.	Average Range
Price (\$)	42.42	122.50
Battery Talk Time (hours)	1.04	3.10
Camera Resolution (megapixel)	0.81	2.16
Chipset Generation	0.36	0.93
Screen Size (inch)	0.21	0.61
Weight (gram)	11.12	32.23

## 2.3 Model

### 2.3.1 Demand

In this section, we develop our model. We begin with the demand side, which we describe using a random-coefficient discrete choice model. Since our data are aggregated at the smartphone/carrier/month level, we assume that a consumer’s choice is a smartphone/carrier combination, indexed by  $j$ . Furthermore, we assume that the utility that consumer  $i$  gets from purchasing  $j$  in period  $t$  is:

$$u_{ijt} = \beta_i q_j - \alpha p_{jt} + \lambda_{m(j)} + \kappa_{c(j)t} + \xi_{jt} + \varepsilon_{ijt}, \quad (2.3.1)$$

where  $q_j$  is a quality index which depends on the observable product characteristics  $\mathbf{x}_j$  as  $q_j = \mathbf{x}_j \boldsymbol{\theta}$ , where  $\boldsymbol{\theta}$  are parameters to be estimated. The random coefficient  $\beta_i$  captures



consumers' heterogeneous tastes for quality and is assumed to follow a normal distribution with mean  $\beta$  and variance  $\sigma^2$ . Since we cannot separately identify  $\beta$ ,  $\sigma$  and  $\theta$  as they enter the utility function as  $\beta\theta$  and  $\sigma\theta$ , we normalize the first dimension of  $\theta$  to be 1. Finally, we denote the price of  $j$  in period  $t$  by  $p_{jt}$ .

To capture consumers' average taste for a brand, we include a brand fixed effect,  $\lambda_{m(j)}$ , where  $m(j)$  represents the smartphone firm (i.e., the brand) of  $j$ . To capture the average quality and cost of carrier  $c$ 's network service in period  $t$  as well as a general time trend in consumers' tastes for smartphones, we include a carrier/year fixed effect.<sup>8</sup> Finally, to capture seasonality in demand, we include a quarter fixed effect. For simplicity of notation, we denote both the carrier/year fixed effect and the quarter fixed effect by one term  $\kappa_{c(j)t}$ , where  $c(j)$  represents the carrier of choice  $j$ . The term  $\xi_{jt}$  represents a demand shock, and the error term  $\varepsilon_{ijt}$  captures consumer  $i$ 's idiosyncratic taste, which is assumed to be i.i.d. and to follow a type-I extreme value distribution. We normalize the mean utility of the outside option to be 0. Thus, the utility of the outside option is  $u_{i0t} = \varepsilon_{i0t}$ .

Under the type-I extreme value distributional assumption of  $\varepsilon_{ijt}$ , we can express the market share of choice  $j$  in period  $t$  as:

$$s_{jt}(\mathbf{q}_t, \mathbf{p}_t, \boldsymbol{\xi}_t) = \int \frac{\exp(\beta_i q_j - \alpha p_{jt} + \lambda_{m(j)} + \kappa_{c(j)t} + \xi_{jt})}{1 + \sum_{j' \in \mathcal{J}_t} \exp(\beta_i q_{j'} - \alpha p_{j't} + \lambda_{m(j')} + \kappa_{c(j')t} + \xi_{j't})} dF(\beta_i), \quad (2.3.2)$$

where  $\mathcal{J}_t$  denotes the set of all products in period  $t$ ,  $\mathbf{q}_t = (q_j, j \in \mathcal{J}_t)$  is a vector of the quality indices of all products in the market, and  $\mathbf{p}_t$  and  $\boldsymbol{\xi}_t$  are analogously defined. Finally,  $F(\beta_i)$

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<sup>8</sup>By using fixed effects to capture service plan features and prices, we implicitly assume that they are exogenous. We do so for two reasons. First, we do not have data on carriers' service plans. It is also difficult to compare service plans provided by different carriers as they differ in many dimensions. Second, a carrier typically does not redesign its service plans when a new smartphone is introduced to the market. Thus, it is plausible to assume that carriers' service plans are exogenous to smartphone firms' product and price decisions

is the distribution function of the random coefficient  $\beta_i$ .

Following Berry, Levinsohn and Pakes (1995b), we define the mean utility of  $j$  in period  $t$  as

$$\delta_{jt} = \beta q_j - \alpha p_{jt} + \lambda_{m(j)} + \kappa_{c(j)t} + \xi_{jt}, \quad (2.3.3)$$

and invert it out based on equation (2.3.2).

### 2.3.2 Supply

The supply side of the model is described by a static three-stage game. In the first stage, firms choose their products. Next, after observing the demand and the marginal cost shocks, firms choose the wholesale prices charged to the carriers. Finally, carriers chooses the subsidized retail prices. We describe these three stages in reverse order.

#### Decisions on Prices

At the third stage, carriers observe the set of products available on each carrier (denoted by  $\mathcal{J}_{ct}$ ), the wholesale prices ( $w_{jt}$ ) and the demand shocks ( $\xi_{jt}$ ). They choose the retail prices ( $p_{jt}$ ) to maximize their respective profit. Suppose that the profit that carrier  $c$  obtains through its service is  $b_{ct}$  per consumer. Carrier  $c$ 's profit for each unit of a product sold is therefore  $p_{jt} + b_{ct} - w_{jt}$ . We do not observe  $b_{ct}$  or  $w_{jt}$ . But we can invert out  $\tilde{w}_{jt} = w_{jt} - b_{c(j)t}$  from the first-order condition on the price  $p_{jt}$ . Specifically, carrier  $c$ 's profit-maximizing problem is

$$\max_{p_{jt}, j \in \mathcal{J}_{ct}} \sum_{j \in \mathcal{J}_{ct}} N s_{jt}(\mathbf{q}_t, \mathbf{p}_t, \boldsymbol{\xi}_t) (p_{jt} - \tilde{w}_{jt}), \quad (2.3.4)$$

where  $N$  is the market size. The first-order condition allows us to invert out  $\tilde{w}_{jt}$  as:

$$\tilde{w}_{jt} = p_{jt} + [\Delta_{ct}^{-1} \mathbf{s}_{ct}]_{jt}, \quad (2.3.5)$$

where  $\Delta_{ct}$  represents a  $|\mathcal{J}_{ct}| \times |\mathcal{J}_{ct}|$  matrix whose  $(j, j')$  element is  $\frac{\partial s_{j't}}{\partial p_{jt}}$ , and  $\mathbf{s}_{ct} = (s_{jt}, j \in \mathcal{J}_{ct})$ . We denote the equilibrium of this stage by  $p_{jt}^*(\tilde{\mathbf{w}}_t, \mathbf{q}_t, \boldsymbol{\xi}_t)$ , where  $\tilde{\mathbf{w}}_t = (\tilde{w}_{jt}, j \in \mathcal{J}_t)$  and  $(\mathbf{q}_t, \boldsymbol{\xi}_t)$  have been analogously defined in Section 2.3.

At the second stage, smartphone firms choose the wholesale prices that they charge carriers after observing the demand shocks and the marginal cost shocks. We assume that marginal cost depends on the quality of a product ( $q_j$ ), time fixed effects ( $\gamma_t$ ), and a  $jt$ -specific shock ( $\eta_{jt}$ ).<sup>9</sup> Specifically, we assume that the marginal cost is  $mc_{jt} = \gamma_t + \gamma_1 \exp(q_j) + \eta_{jt}$ . Let  $\tilde{m}c_{jt} = mc_{jt} - b_{c(j)t}$ , and  $\tilde{\gamma}_{ct} = \gamma_t - b_{ct}$ . With these notations, we can re-write the marginal cost as

$$\tilde{m}c_{jt} = \tilde{\gamma}_{c(j)t} + \gamma_1 \exp(q_j) + \eta_{jt}. \quad (2.3.6)$$

A smartphone firm  $m$ 's profit-maximizing problem is therefore

$$\max_{\tilde{w}_{jt}, j \in \mathcal{J}_{mt}} \sum_{j \in \mathcal{J}_{mt}} (\tilde{w}_{jt} - \tilde{m}c_{jt}) N s_{jt}(\mathbf{q}_t, \mathbf{p}_t^*(\tilde{\mathbf{w}}_t, \mathbf{q}_t, \boldsymbol{\xi}_t), \boldsymbol{\xi}_t), \quad (2.3.7)$$

where  $\mathcal{J}_{mt}$  represents the choices offered by firm  $m$  in period  $t$ . The first-order condition is

$$s_{jt} + \sum_{j' \in \mathcal{J}_{mt}} (\tilde{w}_{j't} - \tilde{m}c_{j't}) \left( \sum_{j'' \in \mathcal{J}_t} \frac{\partial s_{j't}}{\partial p_{j''t}} \frac{\partial p_{j''t}^*}{\partial \tilde{w}_{jt}} \right) = 0, \quad (2.3.8)$$

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<sup>9</sup>Note that  $j$  indexes a smartphone/carrier combination. Therefore, the marginal cost shock is at the smartphone/carrier/time level. Marginal cost may vary across carriers because different radio technologies are used for products sold by different carriers. Moreover, carriers sometimes require firms to preload different softwares on a smartphone, which may come with different costs.

or equivalently,

$$\tilde{w}_{jt} + [\Delta_{mt}^{-1} \mathbf{s}_{mt}]_{jt} = \tilde{\gamma}_{c(j)t} + \gamma_1 \exp(q_j) + \eta_{jt}, \quad (2.3.9)$$

where  $\mathbf{s}_{mt} = (s_{jt}, j \in \mathcal{J}_{mt})$ , and  $\Delta_{mt}$  represents a  $|\mathcal{J}_{mt}| \times |\mathcal{J}_{mt}|$  matrix whose  $(j, j')$  element is

$\left( \sum_{j'' \in \mathcal{J}_t} \frac{\partial s_{j't}}{\partial p_{j''t}} \frac{\partial p_{j''t}^*}{\partial \tilde{w}_{jt}} \right)$ . Combining equations (2.3.5) and (2.3.9) yields

$$p_{jt} + [\Delta_{ct}^{-1} \mathbf{s}_{ct}]_{jt} + [\Delta_{mt}^{-1} \mathbf{s}_{mt}]_{jt} = \tilde{\gamma}_{c(j)t} + \gamma_1 \exp(q_j) + \eta_{jt}, \quad (2.3.10)$$

which we bring to the data for estimation.

As can be seen from equation (2.3.10), this pricing model is a simple linear pricing model, which implies double marginalization. In Section 2.6, we consider several alternative pricing models for robustness analyses.

## 2.4 Estimation

Table 2.4 reports the estimation results on demand and marginal cost. Our demand estimation results indicate that consumers on average favor products with longer battery talk time, higher camera resolution, a more advanced chipset, a larger screen and a lighter weight.

For example, we find that a one-hour increase in battery talk time is equivalent to a price decrease of 6.5 dollars for an average consumer. Similarly, a one-megapixel increase in camera resolution is equivalent to a price decrease of 10.9 dollars, while an increase in the screen size by 0.1 inches is equivalent to a price decrease of 11.7 dollars. Finally, we find that each generation upgrade is equivalent to a price drop between 30 to 78 dollars. The estimated standard deviation of consumers' taste for quality is about 40% of the average

taste, suggesting that consumers are heterogenous in their willingness-to-pay for quality. In our estimation, we include Apple, BlackBerry and Samsung dummies and group all other brands as a baseline brand in the utility function. Our estimates show that there is a large premium for Apple (417 dollars), followed by BlackBerry, and then Samsung.<sup>10</sup> Our estimation results also suggest that there is an advantage to be a flagship product, which is probably related to firms' differential advertising spending on flagship versus non-flagship products.

Table 2.5 reports the price semi-elasticities for top-five products on AT&T in March 2013: Motorola's Atrix HD, Samsung's Galaxy S III and Apple's iPhone 4, iPhone 4s and iPhone 5. The table shows that a \$10 increase in the price of a product leads to about 6% decrease in its demand.<sup>11</sup> Unsurprisingly, the own price semi-elasticities are larger than the cross semi-elasticities.

We construct the quality index for each product based on the estimated coefficients of the product characteristics. Table 2.6 reports the elasticities of quality based on the estimated quality index, again for the top-five AT&T products in March 2013. Across all five products, we see that a 1% increase in the quality index corresponds to about a 5% to 8% increase in sales.

To see the evolution of smartphone quality over time, we divide the brand fixed effects by the mean taste for quality and then add it to the quality index. In Figure 2.4.1, we plot the maximum and median of this index across all products in each month. We also plot the maximum of this index for Apple and Samsung, respectively. Figure 2.4.1 shows that

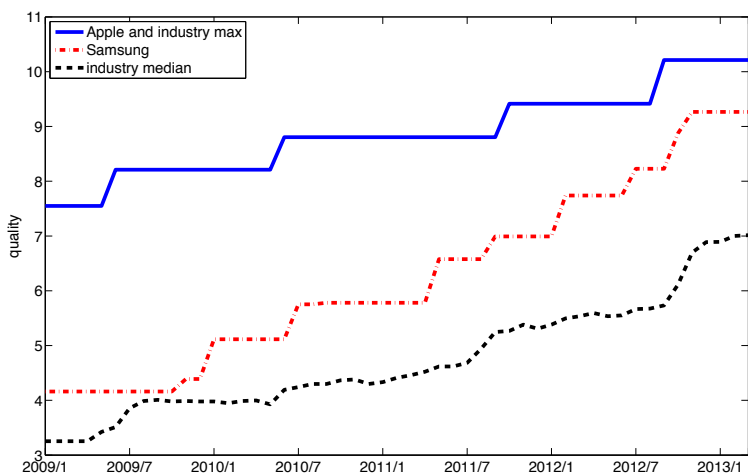
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<sup>10</sup>Note that even though the estimated BlackBerry-dummy coefficient is larger than that of Samsung, considering the product characteristics, the average quality of Samsung products in a month is generally higher than that of BlackBerry products, especially later in our sample.

<sup>11</sup>Given that we have data on only the subsidized retail price, which is not the actual price for a consumer to buy a smartphone, we do not compute the price elasticity.

the Apple quality frontier line perfectly coincides with the industry quality frontier line and that this line experiences a discrete jump whenever a new iPhone product is introduced, confirming the perception that iPhone products drive the quality frontier. Figure 2.4.1 also shows that the median quality index stays at a relatively constant distance from the frontier and that Samsung has narrowed the quality gap between its smartphone products and Apple’s iPhones.

Figure 2.4.1: Smartphone Quality over Time



We use the same quality index used in Figure 2.4.1 to construct a measure of product variety and show its evolution over time. Specifically, we measure product variety in a market with  $n$  products as  $\left[ \sum_{k=2}^n (q^{(k)} - q^{(k-1)})^{1/2} \right]^2$ , where  $q^{(1)} < \dots < q^{(n)}$  are the qualities of the  $n$  products sorted in an ascending order. Note that this measure resembles the CES utility function, and has three desirable properties. First, given the quality range (i.e.,  $q^{(n)} - q^{(1)}$ ), this measure is maximized when products are equidistant. The maximum is  $(n - 1) (q^{(n)} - q^{(1)})$ . Second, this maximum is increasing in the number of products  $n$  and the quality range  $(q^{(n)} - q^{(1)})$ . Third, adding a product identical to one of the existing

products in terms of the key observable characteristics (and hence also in terms of the quality index) has no impact on the product variety measure. In other words, if firms take a strategy of obfuscation, i.e., add products that differ from existing products only in trivial features such as names or colors, our product variety measure will recognize such a strategy and will not count such products in measuring variety.

Given the first property of the product variety measure, we can give the following “as if” interpretation to the measure: a value of  $x$  for the product variety measure is as if there are  $x/(q^{(n)} - q^{(1)}) + 1$  equidistant products. In Figure 2.4.2, we plot the number of smartphones, our measure of product variety, and the “as if” number of equidistant products every month during our sample. Figure 2.4.2(a) shows that the number of smartphones available in the market increases over time, from 33 in January 2009 to 70 in March 2013. This increase is accompanied by an increase in both the product variety measure (see Figure 2.4.2(b)) and the “as if” number of equidistant products (see Figure 2.4.2(c)), indicating that the increase in the number of smartphones is not completely driven by obfuscation.

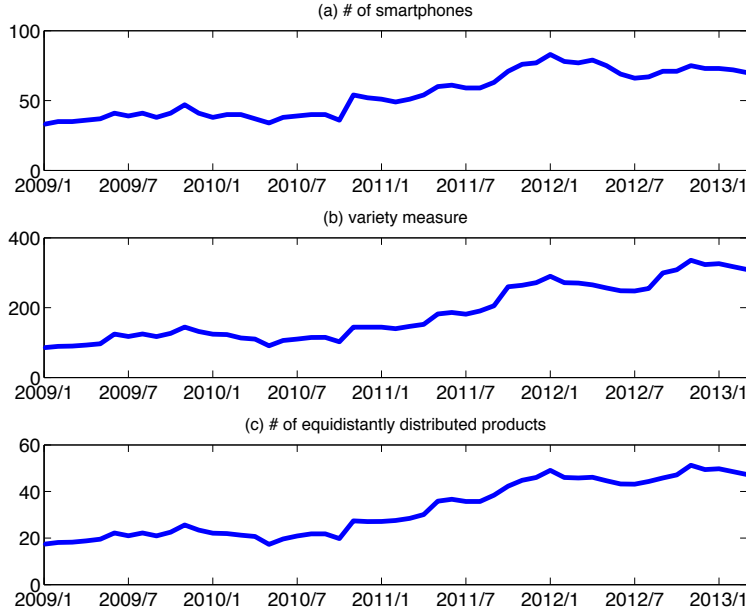
On the supply side, we find that marginal cost increases in product quality. Though not reported in Table 2.4, the estimated carrier/year fixed effects indicate that marginal cost is decreasing over time.<sup>12</sup> Based on the estimates of the demand and marginal cost functions, we obtain an upper bound of the fixed cost for each non-flagship smartphone/month combination in the data. The average upper bound, averaged across all such smartphone/month combinations, is 6.16 million dollars.<sup>13</sup> Figure 2.4.3(a) plots these upper bounds. The hori-

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<sup>12</sup>It should be noted that the estimated marginal cost is, in fact, a smartphone firm’s marginal cost minus the carrier’s per-consumer service profit. The estimated time trend, therefore, accounts for changes in the marginal cost of smartphones as well as changes in the service profit of a carrier.

<sup>13</sup>As mentioned, 14 non-flagship smartphones are sold through several carriers. In reporting this statistic, we divide the upper bound for each smartphone/month by the corresponding number of carriers. When we do not do so, this average upper bound becomes 6.58.

Figure 2.4.2: Product Variety over Time

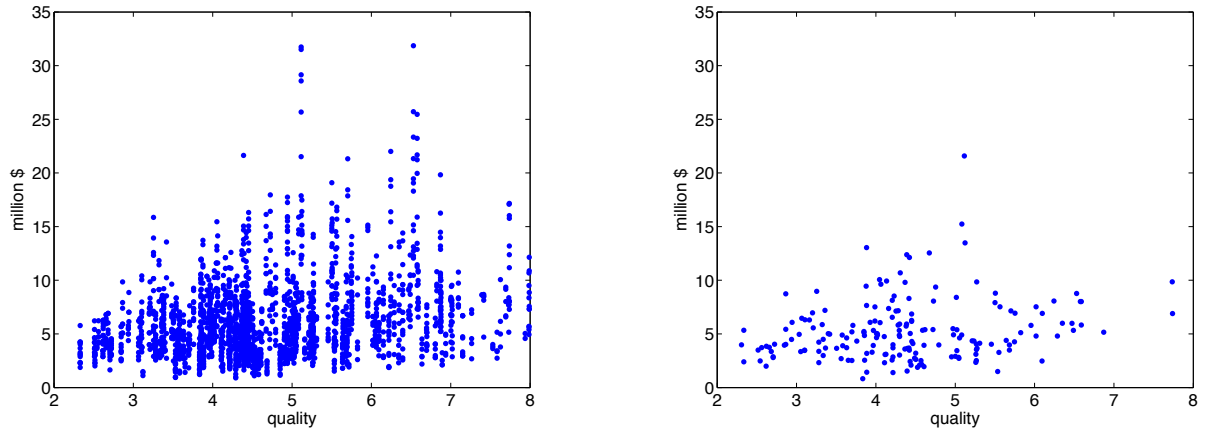


zontal axis represents the quality of a product, the same quality index in Figure 2.4.1. The vertical axis represents the upper bound of the fixed cost. Figure 2.4.3(a) suggests that the upper bound of the fixed cost is positively correlated with product quality. In Figure 2.4.3(b), we plot the lower bounds for discontinued non-flagship products.<sup>14</sup> The average lower bound is 5.27 million dollars.

<sup>14</sup>There are 7 smartphone/month combinations where a product is discontinued from multiple carriers. For these smartphone/months, we report the lower bound of the fixed cost of having this smartphone provided through each carrier separately.



Figure 2.4.3: Bounds of Fixed Costs (Million \$)



## 2.5 Counterfactual Simulations

In this section, we conduct counterfactual simulations to address the two research questions of interest. In all counterfactual simulations, we keep the set of flagship products as fixed and only allow the number and the composition of non-flagship products to be adjusted.<sup>15</sup> Therefore, for simplicity of exposition, a product in this section refers to a non-flagship product whenever it is not explicitly specified.

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<sup>15</sup>In a robustness analysis where we allow firms to also adjust old flagship products, we obtain the same results, i.e., we find that firms do not adjust these products.

### 2.5.1 Are there too few or too many products?

There are two reasons why product offerings in an oligopoly market are inefficient. First, given the competitors' products, a firm chooses to offer a product as long as the marginal profit from doing so is positive. However, the firm does not consider the potential negative externality from stealing market share when making its decisions. As a result, there might be too many products in the market from a welfare point of view. Second, consumer surplus is not part of a firm's objective function. If consumer surplus increases when a product is added to the market, there might also be too few products. Because of these two potentially countervailing forces, whether there are too few or too many products is an empirical question.

To address this question, we first conduct counterfactual simulations where we remove a product.<sup>16</sup> Specifically, for March 2013, the last month of our data, we remove the lowest-quality product in the month, solve for the new pricing equilibrium, and then compute the corresponding consumer surplus and producer surplus. We repeat this counterfactual simulation removing the median (highest)-quality product, and report the results in Table 2.7. Each column of the table corresponds to a simulation where a different product is removed. In the first three rows of the table, we report changes in consumer surplus, carrier surplus (i.e., the sum of carriers' profits) and the sum of smartphone firms' variable profits. All three measures are expectations over the demand and the marginal cost shocks. In the last row, we report the upper bound of the removed product's fixed cost, which is the maximum possible saving in fixed costs.

The results across all three columns of Table 2.7 show that consumers are worse off when a

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<sup>16</sup>For any product removed, we remove it from all carriers.

product is removed. This is partially due to changes in prices after the product is removed, but mainly because of the direct effect of removing the product. Specifically, when we hold the prices of the remaining products fixed, we find that changes in consumer surplus are (-0.94, -2.19, -11.57) million dollars across the three columns, which account for most of the total change in consumer surplus. To put these changes in consumer surplus in perspective, note that the average monthly sales of a product is 77540 units and the average subsidized price is 122 dollars (see Table 2.1). Considering an average service plan price of 60 dollars per month over 24 months, consumers pay a total of  $(60 \times 24 + 122) \times 77540$  dollars, which is about 10 times the consumer surplus from removing the highest-quality product.

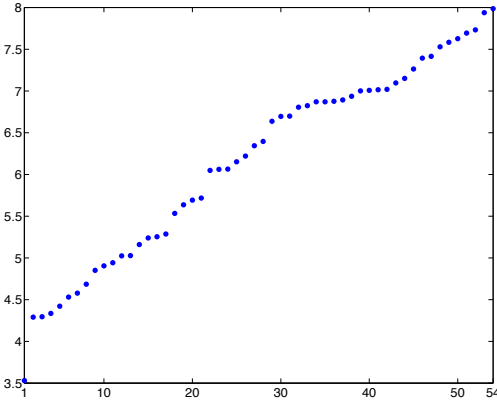
Carriers' profits also drop. As for smartphone firms, the comparison of the third row and the last row shows that if the fixed cost is at its upper bound, the total smartphone producer surplus increases after a product is removed. This result confirms the intuition that because firms do not internalize the business stealing effect, there may be excessive product proliferation, especially if the fixed cost is high. However, this effect is dominated by the effect of product offerings on consumer surplus: summing over the four rows of Table 2.7, we see that removing a product leads to a decrease in total welfare, even considering the maximum possible saving in the fixed cost.

Comparing results across the three columns, we can see that the changes in all welfare measures become larger as we move from removing the lowest to the highest-quality product. The main conclusion, however, remains the same: total welfare decreases even considering the maximum possible saving in the fixed cost. In fact, when we repeat the above exercise for each of the 54 products in March 2013, we find that our results hold in all 54 simulations. Specifically,  $\Delta(\text{consumer surplus})$ ,  $\Delta(\text{carrier surplus})$  and  $\Delta(\text{smartphone producer variable profits})$  are always negative; the sum of them plus the upper bound of the removed product's

fixed cost is always negative. These results indicate that removing any product in the market leads to a decrease in total welfare, even considering the maximum possible saving in the fixed cost. Finally, because it is a theoretical possibility that removing multiple products together may increase total welfare, we have also repeated the exercise removing any two products and find that the conclusion still holds.

In summary, the above results suggest that removing any one or two of the existing products in this market is welfare-decreasing. However, does adding a product lead to an increase in welfare? To answer this question, we consider adding a product that fills a gap in the quality spectrum. Specifically, we plot the qualities of the products in March 2013 in Figure 2.5.1, find the largest gap in quality above 4 (the gap between 5.72 and 6.05) and add a product whose quality is at the midpoint of the gap (5.88).

Figure 2.5.1: Quality of Products in March 2013



We conduct four simulations where this product is added to Samsung’s, LG’s, HTC’s or Motorola’s product portfolio, respectively. After Apple, they are the four largest smartphone firms in March 2013 according to their sales in that month. In all four simulations, we choose

Sprint, the carrier with the least number of products, as the carrier. The simulation results are presented in Table 2.8, each column of which represents a different simulation.

Not surprisingly, consumers are better-off with the additional product in the market (Row 1). Carriers also earn more profits (Row 2). Smartphone firms' total variable profit increases (Row 3). For the added product, we obtain a lower bound on its fixed cost, which is reported in Row 4 of Table 2.8. The ratio of the sum of the first three rows to the last is around 2.3 for all four simulations. This implies that as long as the fixed cost is not more than 2.3 times of its estimated lower bound, the sum of the first three rows minus the fixed cost of the added product (i.e., the change in total welfare) is positive. To put the number 2.3 in perspective, note that the average upper bound and the average lower bound we report in Section 2.4 are, respectively, 6.16 and 5.17, with a ratio of 1.2.

Overall, our simulation results from removing products and adding a product suggest that there are too few products. As mentioned, there are two countervailing forces: firms do not consider the business-stealing externality, which may lead to excessive product offerings; firms do not consider consumer surplus, which may lead to insufficient product proliferation. Our results suggest that the second effect dominates the first.

### **2.5.2 How does competition affect product offerings?**

To study how competition affects product offerings, we simulate the effect of a hypothetical merger between Samsung and LG in March 2013, the second and the third largest smartphone firms in terms of sales in that month, following Apple. In the appendix of the paper version of the chapter, we show the effects of a Samsung-Motorola merger and an LG-Motorola merger, where Motorola is the fourth largest smartphone firm in March 2013. In these

merger simulations, we compute the post-merger equilibrium in both product offerings and pricing. In contrast, in Section 2.5.1, we only need to compute the new pricing equilibrium for given product offerings in the market.

Computing the post-merger product-choice equilibrium can be challenging because the product-choice action space for a firm can be very large. A firm can choose to drop any set of products or add any number of products after the merger. To keep the problem tractable, we restrict the set of potential products for each firm in the merger simulations to be the firm's products in the data in either March or February 2013, plus two additional potential products that fill gaps in the quality spectrum.<sup>17</sup> As shown in the plot of the qualities of products in March 2013 (Figure 2.5.1), the quality spectrum exhibits gaps between 5.72 and 6.05 and between 6.40 and 6.64. We find the respective midpoints of these gaps (5.88 and 6.52) and allow each firm to add a product whose quality is either 5.88 or 6.52. These two products can be sold through any of the four carriers in the sample. Products in February or March 2013 are sold through their respective carriers observed in the data. In sum, with this set of potential products, our simulation allows a firm to drop any subset of its existing products, add back any subset of its discontinued products, add one or two additional products, or use a combination of the above three types of adjustments.

Even with this restricted set of potential products, the choice set for a firm can still be too large. This is because a smartphone firm chooses a product portfolio, which is a subset (of any size) of the potential products. In other words, the choice set of a firm is the

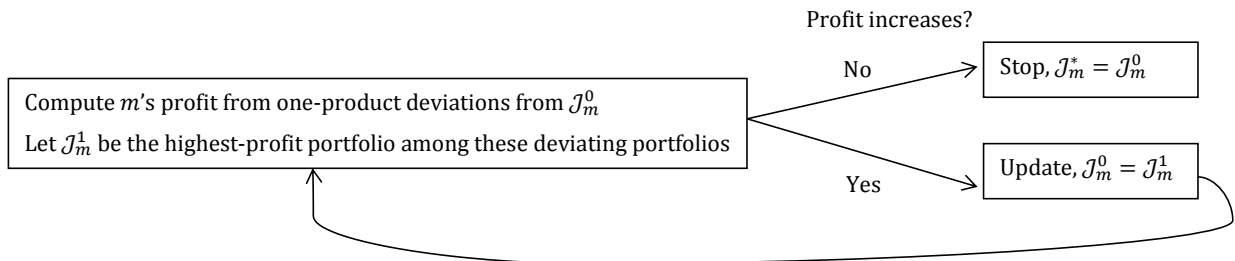
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<sup>17</sup>Since we do not have an estimate of the brand effect for the merged Samsung-LG entity, in the merger simulation, we assign the Samsung brand effect to products originally offered by Samsung before the merger, and the LG brand effect to those originally offered by LG. To be consistent, we allow four additional potential products for the merged firm Samsung-LG, two of which carry the Samsung brand effect and two of which carry the LG brand effect. In the appendix of the paper version of the chapter, we repeat the merger simulation by assuming that the post-merger Samsung-LG brand effect is the average of the pre-merger Samsung and LG brand effects. The results are robust to this alternative assumption.

power set of its potential products. For example, the merged Samsung-LG entity has 31 potential products, and thus a choice set of  $2^{31}$  ( $\approx 2.4 \times 10^9$ ) product portfolios. Moreover, to compute the profit of each product portfolio, we need to compute the corresponding pricing equilibrium, making the computational burden prohibitively high. To address this issue, we use a heuristic algorithm to compute a firm's optimal product portfolio given its competitors' product portfolios. This algorithm is then embedded in a best-response iteration to solve for the post-merge product-choice equilibrium.

We use firm  $m$  as an example to describe the heuristic algorithm for a firm's optimal product portfolio problem, and depict the algorithm in Figure 2.5.2.

Figure 2.5.2: Algorithm for Computing the Best-Response Product Portfolio



Let  $\bar{\mathcal{J}}_m$  represent firm  $m$ 's potential products (for example,  $\bar{\mathcal{J}}_m = \{j_1, \dots, j_n\}$ ). We start with a portfolio  $\mathcal{J}_m^0 \subseteq \bar{\mathcal{J}}_m$  (for example,  $\mathcal{J}_m^0 = \{j_1, \dots, j_{n_1}\}$  where  $n_1 \leq n$ ). We compute firm  $m$ 's profit from each of the following deviations from  $\mathcal{J}_m^0$ :  $\mathcal{J}_m^0 \setminus \{j_k\}$ ,  $k = 1, \dots, n_1$  or  $\mathcal{J}_m^0 \cup \{j_k\}$ ,  $k = n_1 + 1, \dots, n$ . Note that each deviation differs from  $\mathcal{J}_m^0$  in only one product: either a product in  $\mathcal{J}_m^0$  is removed or a potential product not in  $\mathcal{J}_m^0$  is added. Let  $\mathcal{J}_m^1$  be the highest-profit deviating product portfolio. If firm  $m$ 's profit corresponding to  $\mathcal{J}_m^1$  is smaller than that corresponding to  $\mathcal{J}_m^0$ , this procedure stops and returns  $\mathcal{J}_m^0$  as the best response. Otherwise, we compute  $m$ 's profit from any one-product deviation from  $\mathcal{J}_m^1$  by either adding a potential product to or dropping a product from  $\mathcal{J}_m^1$ . We continue this process until firm

$m$ 's profit no longer increases. This algorithm allows us to translate a problem growing exponentially in the number of potential products into one growing linearly in it.<sup>18</sup>

In this algorithm, even though we impose a one-product deviation restriction in each step of the algorithm, the optimal product portfolio found by the algorithm can be very different from the starting portfolio in both product number and composition. This is because each step of the algorithm leads to a one-product deviation and strictly increases profit prior to convergence. Therefore, as long as the algorithm does not converge after only one step, it yields a product portfolio that deviates from the starting product portfolio by more than one product. Note that product composition can also change as the algorithm drops one product in one step and adds another in a later step.

To evaluate the performance of the algorithm, we conduct Monte Carlo simulations, as discussed in the appendix of the paper version of the chapter. These simulations suggest that our algorithm works well, at least for relatively small problems where we can solve for the true optimal product portfolio without using the heuristic algorithm. Given that we impose a one-product deviation restriction in each step, we also check and confirm that no firm has a two-product profitable deviation at the equilibrium found by the heuristic algorithm in our merger simulations below.

We embed this algorithm in a best-response iteration, where firms take turns updating to their best-response product portfolio. We repeat this iteration until no firm has an incentive to deviate. In the iteration, we loop firms according to their monthly sales in March 2013, either ascending or descending. These two best-response iterations yield the same equilibrium

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<sup>18?</sup> uses a similar idea to avoid excessive computation burden in studying firm acquisition problems. Specifically, he assumes that when a firm decides on which set of firms to acquire, it makes a sequential decision of whether to acquire each firm according to a pre-specified sequence of potential acquirees. Our algorithm is less restrictive: in each step, a firm evaluates all one-product deviations simultaneously rather than being constrained to one such deviation determined by a pre-specified sequence.



in our merger simulations.

As for fixed costs, we draw the fixed cost for each potential product from a range consistent with the bounds obtained in the estimation and report the average merger effects, averaged over different sets of fixed-cost draws. Specifically, for each product in the data, we have obtained an upper bound of its fixed cost (denoted by  $\bar{F}_{jt}$ ). For such a product, we randomly draw five fixed-cost values from the range  $[0.5\bar{F}_{jt}, \bar{F}_{jt}]$ . Similarly, for each potential product not in the data, we have obtained a lower bound of its fixed cost  $\underline{F}_{jt}$ . We draw five fixed-cost values from  $[\underline{F}_{jt}, 5\underline{F}_{jt}]$ . In the appendix of the paper version of the chapter, we consider two alternative ranges for the fixed costs. In one alternative, we fix the length of the range to be  $(\bar{F} - \underline{F})$ , where  $\bar{F} = 6.16$  and  $\underline{F} = 5.27$  are the average upper and lower bounds reported in Section 2.4. In the other alternative, we define the range according to the quality of a product. Our merger simulation results are robust to these two alternative fixed-cost ranges.

Table 2.9 presents the baseline merger simulation results. These results show an average decrease of 2.50 products after the merger, mainly driven by the merged firm dropping products: the average change for the merged firm is -3.40 while that for the non-merging firms is 0.80. We also find that the merged firm drops products across the quality spectrum except the very top. Specifically, we find that the average number of products dropped from each quality quartile (below the pre-merger 25% quality quantile, [25%, 50%), [50%, 75%), and above 75%) is 0.6, 1, 1, and 0, respectively. Overall, the product variety measure decreases by 21.41 (from 360.25). We use the following back-of-the-envelope calculation to understand the magnitude of such a change. Before the merger, the range of the quality spectrum is 6.68. The pre-merger product variety measure (360.25) is “as if” there are 54.93 equidistant products  $(360.25/6.68 + 1)$ , while the post-merger product variety measure (338.84) is “as if” there are 51.72 equidistant products. Therefore, a change of -21.41 in the

product variety measure is equivalent to a decrease of about 3.21 in the number of “as if” equidistant products.

Regarding changes in quality and price, we find little change in the sales-weighted average quality in the market after the merger, but an increase in the sales-weighted average retail price of 1.42 dollars. This is largely due to price increases for the merged firm’s products. Specifically, the results in Row (9) of Table 2.9 show that the sales-weighted average retail price of the merged firm’s products increases by about 8.87 dollars. Overall, sales for the merged firm decreases and that for the non-merging firms increases, with a net change of -87,526 units. The decrease in product offerings and the increase in prices eventually lead to a reduction in consumer surplus by around 28 million dollars. Carriers are also worse off. The total smartphone profit, however, increases by around 13.39 million dollars, among them, 1.74 million dollars are attributed to the increase in the merged firm’s profit and the remaining 11.64 million dollars are due to changes in non-merging firms’ profits with an average increase of 1.06 million dollars per non-merging firm. Despite the increase in smartphone producer surplus, overall welfare decreases by around 30.84 million dollars.

In summary, the results from this counterfactual simulation show that a reduction in competition leads to a decrease in the number of products across the quality spectrum. This decrease is accompanied by an increase in prices, leading to a decline in consumer and carrier surplus and eventually a reduction in overall welfare, despite an increase in smartphone producer surplus. Our simulations of other mergers yield similar results (see the appendix of the paper version of the chapter for the Samsung-Motorola and LG-Motorola merger). The combination of our findings in the previous section (i.e., the market contains too few products) and our findings in this section (i.e., a merger further reduces product offerings) suggests that merger policies should be stricter when we take into account the effect of a

merger on product offerings.

This conclusion is also consistent with a merger simulation where we keep the set of products fixed and allow firms to adjust only prices after the merger. In such a merger simulation, we find that the changes in consumer surplus, carrier profit, and smartphone firm profit are all smaller (in absolute value). They are -19.46, -10.83 and 8.98 million dollars, respectively. In contrast, they are -28.11, -16.58 and 13.39 million dollars when post-merger adjustments in both product offerings and prices are allowed. The decrease in total surplus is also smaller, again suggesting that the merger policy should be stricter considering firms' endogenous product choice.

## 2.6 Robustness Analyses

In this section, we conduct three robustness analyses. We change the demand side of the model in the first two robustness analyses and the supply side in the third. For each robustness analysis, we both re-estimate the model and repeat the counterfactual simulations.

On the demand side, one concern with our discrete choice model is that the assumption of independent idiosyncratic shocks may lead us to overestimate the effect of removing or adding a product on consumer surplus. To address this concern, we conduct two robustness analyses where we add more random coefficients in order to allow greater correlation among the utilities that a consumer gets from different products.

In the first robustness analysis, we add a random coefficient for the Apple dummy variable and allow this random coefficient to be correlated with the quality random coefficient. The estimation results in Table 2.10 indicate that the standard deviation of the Apple-dummy

random coefficient is 2.625 and that this random coefficient is highly correlated with the quality random coefficient (the estimated correlation is 0.991). Unfortunately, both estimates are statistically insignificant. For the parameters common to both models, both the estimates and the statistical significance levels are robust. More importantly, the results from the counterfactual simulations, which allow us to address our research questions, are also robust (see Tables 2.11). For example, we still find that removing a product reduces total surplus even considering the maximum possible saving in the fixed cost, that adding a product increases total surplus as long as the fixed cost is not much higher than its lower bound and that a merger leads to a reduction in product offerings and eventually a decrease in total welfare.

In the second robustness analysis, we add a random coefficient for each carrier dummy variable. In Table 2.12, we show the estimation results show that the standard deviations of all carrier dummy variable coefficients, except that for T-Mobile, are small and statistically insignificant. The estimates for the parameters common to the two models are robust as are counterfactual simulation results (see Table 2.13).

On the supply side, in the pricing model of the baseline specification, we assume that smartphone firms and carriers make their pricing decisions sequentially: smartphone firms make decisions on wholesale prices before carriers make decisions on retail prices. It is possible that they make the pricing decisions jointly. This is especially likely for Apple and AT&T during the time when they had an exclusive contract (i.e., AT&T was the sole seller for iPhones before February 2011). In the third robustness analysis, we allow Apple and AT&T to set their pre-February 2011 iPhones prices jointly to maximize their joint profit from iPhones.<sup>19</sup>

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<sup>19</sup>At the same time, other carriers choose their retail prices to maximize their profits and AT&T chooses its retail prices for its non-iPhone products to maximize its profit from non-iPhone products.

Specifically, we take the demand estimates from the baseline model, re-estimate the marginal cost functions and fixed cost bounds and repeat the counterfactual simulations. Our results in Table 2.14 indicate that our findings remain robust.

In the appendix of the paper version of the chapter, we present the results of additional robustness analyses. First, we consider a model where all smartphone firms and all carriers jointly set retail prices. We also provide a uniform framework following Villas-Boas and Hellerstein (2006) to discuss the differences between the baseline model and these two alternative pricing models. Finally, we show that our results are robust to two additional deviations to the simple linear pricing model.

## 2.7 Conclusion

In this chapter, my co-author and I study how oligopolistic competition impacts product offerings in the U.S. smartphone market. To this end, we develop and estimate a model for the demand and supply of smartphones. We first conduct counterfactual simulations where we add or remove products to determine whether there are too few or too many products in the market. We then use merger simulations to study the effects of competition on product offerings, prices, and overall welfare. Our findings show that there are too few products in the market and that a reduction in competition decreases product number and product variety and reduces total welfare. These results suggest that merger policies should be stricter when we take into account the effect of a merger on product choice.

We conclude by highlighting a few caveats. First, similar to many papers in the endogenous product choice literature, our paper uses a static model to describe consumer demand and

firm behavior.<sup>20</sup> On the supply side, this modeling choice is somewhat justifiable as we focus on non-flagship products which presumably do not involve a large sunk cost such as the R&D cost. However, consumers may be dynamic, which will lead to firm dynamic behavior. For example, it may be costly for consumers to switch from one carrier to another. Given such frictions, firms may consider how their decisions in the current period affect their payoffs in the future. Note that, in a reduced-form way, our carrier/year fixed effects in the utility function capture an average switching cost.<sup>21</sup> Similarly, our estimated fixed cost in a reduced-form way captures both the true fixed cost and the effect of a product on future firm profits. That said, we acknowledge that we keep the carrier/year fixed effect and the fixed cost constant in the counterfactual simulations and therefore do not discuss industry dynamics.

Second, our model does not explain the choice of a carrier by a smartphone firm. As a result, we do not discuss the effect of competition on the carrier choice for each product, which may affect the pricing equilibrium, and thus a smartphone firm's product offerings. We could expand our definition of potential products for each firm to allow firms to choose carriers.<sup>22</sup> However, given that doing so increases the computational burden substantially and that in the data, we do not observe smartphone firms moving one product from one carrier to another, we leave this for future research.

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<sup>20</sup>See, for example, Seim (2006b), Fan (2013b), Eizenberg (2014b), and Crawford, Shcherbakov and Shum (2015a).

<sup>21</sup>For instance, the fixed effect for Verizon in a year captures its opponents' market shares in the previous year, which determines the proportion of consumers who have to pay switching costs to buy a Verizon product this year. Therefore, this fixed effect somewhat captures the average switching cost for consumers to buy a Verizon product.

<sup>22</sup>For example, we could add the combination of a firm's products in March 2013 with all carriers as additional potential products for the firm.

Table 2.4: Estimation Results

	Parameter	Std. Error
Demand		
Quality coefficient		
Battery Talk Time (hours)	0.056***	0.013
Camera Resolution (megapixel)	0.093***	0.036
Chipset Generation 2	0.460***	0.113
Chipset Generation 3	0.718***	0.147
Chipset Generation 4	1.055***	0.200
Chipset Generation 5	1.674***	0.280
Screen Size (inch)	1	
Weight (gram)	-0.002*	0.001
Quality random coefficient		
Mean	0.779***	0.128
Std. Dev.	0.300***	0.079
Price (\$)	-0.007***	0.002
Apple	2.779***	0.094
BlackBerry	1.237***	0.121
Samsung	0.338***	0.069
Flagship?	0.597***	0.065
Carrier/Year and Quarter Fixed Effects		Yes
Marginal Cost (\$)		
exp(quality/10)	518.521***	2.504
Apple	-30.221***	0.115
BlackBerry	98.749***	0.433
Samsung	-20.413***	0.131
Carrier/Year Fixed Effects		Yes

\* indicates 90% level of significance. \*\*\* indicates 99% level of significance.

Table 2.5: Demand Semi-Elasticities with Respect to Price

	Atrix HD	Galaxy S III	iPhone 4	iPhone 4s	iPhone 5
Atrix HD	-6.600	0.089	0.160	0.213	0.398
Galaxy S III	0.065	-6.570	0.163	0.217	0.409
iPhone 4	0.047	0.066	-6.526	0.175	0.309
iPhone 4s	0.052	0.073	0.145	-6.476	0.337
iPhone 5	0.058	0.083	0.155	0.203	-6.289

Note: Top-five products on AT&T in in March 2013. (Row  $i$ , Column  $j$ ): percentage change in market share of product  $j$  with a \$10 change in product  $i$ 's retail price.

Table 2.6: Demand Elasticities with Respect to Quality

	Atrix HD	Galaxy S III	iPhone 4	iPhone 4s	iPhone 5
Atrix HD	7.875	-0.125	-0.148	-0.224	-0.488
Galaxy S III	-0.087	8.207	-0.152	-0.23	-0.506
iPhone 4	-0.059	-0.086	5.168	-0.173	-0.357
iPhone 4s	-0.066	-0.098	-0.129	5.906	-0.397
iPhone 5	-0.077	-0.114	-0.141	-0.21	6.762

Note: Top-five products on AT&T in in March 2013. (Row  $i$ , Column  $j$ ): percentage change in market share of product  $j$  with a 1 percentage change in product  $i$ 's quality.

Table 2.7: Welfare Changes when a Product is Removed, March 2013 (million \$)

Removed product	Lowest-quality	Median	Highest
$\Delta$ (consumer surplus)	-0.92	-2.52	-12.67
$\Delta$ (carrier surplus)	-0.83	-1.39	-9.13
$\Delta$ (smartphone producer variable profits)	-0.50	-0.90	-3.24
Upper bound of savings in fixed costs	0.94	2.19	12.14

Table 2.8: Welfare Changes when a Product is Added, March 2013 (million \$)

	HTC	LG	Motorola	Samsung
$\Delta$ (consumer surplus)	2.43	2.43	2.51	2.79
$\Delta$ (carrier surplus)	1.26	1.27	1.29	1.53
$\Delta$ (smartphone producer variable profits)	1.04	1.03	1.00	1.64
Lower bound of added fixed costs	2.10	2.11	2.13	2.62



Table 2.9: The Effect of Samsung-LG Merger, March 2013

	Variable	Pre-merger	Post-merger	Change
(1)	Number of Non-flagship Products	54	51.40	-2.60
(2)	merged firm	26	22.60	-3.40
(3)	non-merging firms	28	28.80	0.80
(4)	Variety	360.25	338.84	-21.41
(5)	Sales-weighted avg quality	8.40	8.42	0.02
(6)	merged firm	7.32	7.34	0.02
(7)	non-merging firms	6.25	6.25	0.0003
(8)	Sales-weighted avg price (\$)	110.00	111.43	1.42
(9)	merged firm	156.08	164.95	8.87
(10)	non-merging firms	91.23	91.51	0.28
(11)	Total sales	7,002,268	6,914,742	-87,526
(12)	merged firm	2,027,077	1,875,141	-151,936
(13)	non-merging firms	4,975,192	5,039,601	64,410
(14)	Consumer surplus (million \$)	1681.21	1653.10	-28.11
(15)	Carrier profit (million \$)	1266.42	1249.84	-16.58
(16)	Smartphone firm profit (million \$)	1115.45	1128.83	13.39
(17)	merged firm	270.90	272.64	1.74
(18)	non-merging firms	844.55	856.20	11.64

Note: except in Rows (1) - (3), all variables are computed based on all products, including both the flagship products and the non-flagship products.

Table 2.10: Robustness Analysis: Allowing an Apple Random Coefficient

	Parameter	Std. Error
Demand		
Quality coefficient		
Battery Talk Time (hours)	0.052***	0.016
Camera Resolution (megapixel)	0.109***	0.046
Chipset Generation 2	0.444***	0.137
Chipset Generation 3	0.743***	0.180
Chipset Generation 4	1.145***	0.261
Chipset Generation 5	1.857***	0.385
Screen Size (inch)	1	
Weight (gram)	-0.002*	0.002
Quality random coefficient		
Std. Dev., Quality	0.214**	0.104
Std. Dev., Apple FE	2.625	2.248
Correlation	0.991	1.559
Price (\$)	-0.006	0.079
Apple	0.030	2.059
BlackBerry	1.149***	0.132
Samsung	0.337***	0.069
Flagship?	0.592***	0.069
Carrier/Year and Quarter Fixed Effects		Yes
Marginal Cost (\$)		
exp(quality/10)	544.583***	2.908
Apple	-252.177***	0.150
BlackBerry	104.275***	0.510
Samsung	-20.101***	0.151
Carrier/Year Fixed Effects		Yes

\*\*\* indicates 99% level of significance.

Table 2.11: Robustness Analysis: Allowing an Apple Random Coefficient. Simulation

Removed product	Lowest-quality	Median	Highest
$\Delta(\text{consumer surplus})$	-1.13	-3.14	-7.45
$\Delta(\text{carrier surplus})$	-1.03	-2.08	-4.20
$\Delta(\text{smartphone producer variable profits})$	-0.68	-1.14	-1.89
Upper bound of savings in fixed costs	1.16	2.70	5.82

	HTC	LG	Motorola	Samsung
$\Delta(\text{consumer surplus})$	2.67	2.68	2.72	3.18
$\Delta(\text{carrier surplus})$	1.72	1.73	1.75	2.11
$\Delta(\text{smartphone producer variable profits})$	1.20	1.20	1.18	1.85
Lower bound of added fixed costs	2.34	2.35	2.36	2.98

Variable	Pre-merger	Post-merger	Change
Number of non-flagship products	54	46.60	-7.40
Variety	324.84	283.61	-41.23
Sales-weighted avg quality	6.879	6.882	0.003
Sales-weighted avg price (\$)	94.62	99.63	5.01
Total sales	7,398,499	7,190,089	-208,409
Consumer surplus (million \$)	2632.84	2563.01	-69.83
Carrier profit (million \$)	1648.47	1605.85	-42.63
Smartphone firm profit (million \$)	1776.96	1811.47	34.52

Table 2.12: Robustness Analysis: Allowing Carrier Random Coefficients

	Parameter	Std. Error
Demand		
Quality coefficient		
Battery Talk Time (hours)	0.067**	0.032
Camera Resolution (megapixel)	0.112***	0.043
Chipset Generation 2	0.456***	0.177
Chipset Generation 3	0.780***	0.229
Chipset Generation 4	1.097***	0.275
Chipset Generation5	1.786***	0.373
Screen Size (inch)	1	
Weight (gram)	-0.001	0.002
Std. Dev. of Random Coefficients		
Quality	0.349*	0.213
AT&T	0.018	23.410
Sprint	0.394	33.860
T-Mobile	4.241**	1.997
Verizon	0.394	33.860
Price (\$)	-0.008***	0.003
Apple	2.741***	0.192
BlackBerry	1.253***	0.175
Samsung	0.335***	0.076
Flagship?	0.587***	0.114
Carrier/year and quarter dummies		Yes
Marginal Cost (\$)		
Exp(quality/10)	2.816***	163.3324
Apple	0.134***	-351.291
BlackBerry	0.521***	167.6449
Samsung	0.148***	-193.061
Carrier/year dummies		Yes

\* indicates 90% level of significance. \*\* indicates 95% level of significance.

\*\*\* indicates 99% level of significance.

Table 2.13: Robustness Analysis: Allowing Carrier Random Coefficients, Simulation

Removed product	Lowest-quality	Median	Highest
$\Delta(\text{consumer surplus})$	-0.99	-2.39	-10.54
$\Delta(\text{carrier surplus})$	-1.15	-1.40	-10.38
$\Delta(\text{smartphone producer variable profits})$	-0.12	-0.66	-0.56
Upper bound of savings in fixed costs	0.96	2.05	10.42

	HTC	LG	Motorola	Samsung
$\Delta(\text{consumer surplus})$	1.96	1.92	2.03	2.44
$\Delta(\text{carrier surplus})$	0.95	0.95	0.99	1.26
$\Delta(\text{smartphone producer variable profits})$	0.8	0.81	0.77	1.34
Lower bound of added fixed costs	1.62	1.61	1.66	2.18

Variable	Pre-merger	Post-merger	Change
Number of non-flagship products	54	28.80	-25.20
Variety	379.09	232.45	-146.64
Sales-weighted avg quality	8.38	8.44	0.05
Sales-weighted avg price (\$)	94.71	102.50	7.80
Total sales	7,893,045	7,697,927	-195,118
Consumer surplus (million \$)	2230.96	2171.69	-59.26
Carrier profit (million \$)	1577.60	1559.69	-17.91
Smartphone firm profit (million \$)	1299.89	1376.39	76.51

Table 2.14: Robustness Analysis: Apple and AT&T Joint Price Setting before February 2011

	Parameter	Std. Error
Exp(quality/10)	460.828***	2.274
Apple	6.473***	0.107
BlackBerry	86.426***	0.393
Samsung	-17.546***	0.119
Carrier/year dummies	Yes	

\*\*\* indicates 99% level of significance.

Removed product	Lowest-quality	Median	Highest
$\Delta$ (consumer surplus)	-0.80	-2.59	-14.08
$\Delta$ (carrier surplus)	-0.72	-1.43	-10.22
$\Delta$ (smartphone producer variable profits)	-0.47	-1.00	-4.22
Upper bound of savings in fixed costs	0.83	2.29	13.93

	HTC	LG	Motorola	Samsung
$\Delta$ (consumer surplus)	2.41	2.41	2.49	2.61
$\Delta$ (carrier surplus)	1.26	1.27	1.29	1.44
$\Delta$ (smartphone producer variable profits)	1.11	1.11	1.08	1.66
Lower bound of added fixed costs	2.13	2.14	2.17	2.52

Variable	Pre-merger	Post-merger	Change
Number of non-flagship products	54	51.60	-2.40
Variety	360.25	339.96	-20.29
Sales-weighted avg quality	8.34	8.36	0.02
Sales-weighted avg price (\$)	128.08	130.32	2.24
Total sales	6,792,576	6,697,845	-94,731
Consumer surplus (million \$)	1632.88	1602.54	-30.34
Carrier profit (million \$)	1225.29	1208.31	-16.97
Smartphone firm profit (million \$)	1044.85	1058.08	13.23

## CHAPTER III

### Does Vertical Integration Increase Innovation?

#### 3.1 Introduction

In vertical industries, upstream and downstream innovations are often complementary. Upstream firms upgrade the core technologies essential to enhance performance, and downstream firms combine the new technology with new designs in the next generation of consumer products. There are many examples, such as traction batteries (upstream) and electric vehicles (downstream) and CPU's (upstream) and personal computers (downstream). Given the prevalence of vertical structures, this paper studies the effects of vertical integration on pricing, innovation and welfare.

There is a large theoretical literature on vertical integration.<sup>23</sup> Theory suggests that vertical integration may affect both investment and price decisions. Vertical integration may be pro-investment by aligning the investment incentives of the merged firms. Firms also set prices differently when an upstream firm is merged with a downstream firm, producing the well known efficiency trade-off between two pricing forces: on one hand, double marginalization

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<sup>23</sup>Examples of surveys include Perry (1989), Holmström and Roberts (1998), Tirole (1999) and ? and Aghion and Holden (2011). The list is far from exhaustive.

is reduced within the integrated firm, which may charge consumers lower prices; on the other hand, the integrated firm has an incentive to charge higher prices to downstream rivals (raising rivals' cost or the foreclosure effect). However, predictions of the net effects on firm profits may depend on the nature of downstream product competition and how firms set prices,<sup>24</sup> and because investment is driven by the marginal value of investment, i.e. a firm's post-investment profit less the pre-investment profit, it is unclear how the price effects of vertical integration affect investment.

Understanding the relative magnitudes and the interaction of the investment coordination effects and price effects of vertical integration is crucial for policies and regulations. For example, the potential tradeoff between one firm's innovation and industry-wide innovation was a key issue in the European antitrust case against Microsoft in 2004. Microsoft owned the popular proprietary operating systems used on computer servers and foreclosed other server software companies. Microsoft argued in its defense that the foreclosure would increase its own innovation. The European Commission, however, believed that if Microsoft were to provide downstream rivals (server software producers) with reasonable access to its upstream technology (operating system), "the positive impact on the level of innovation in the whole industry outweighed the negative impact of the dominant undertaking's incentives to innovate".<sup>25</sup> The court ruled against Microsoft. In effect, the authorities believed that preventing foreclosures would increase the innovation of other downstream firms, and the benefits would be greater than the potential increase of the integrated firm's innovation.

In this paper, I use an empirical model of pricing and dynamic investment to study the investment coordination and price effects of vertical integration in the context of the chipset

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<sup>24</sup>Predictions can be more definitive in some more special cases, such as Riordan (1998).

<sup>25</sup>Case T-201/04, Microsoft Corp. v. Commission, 2007 E.C.R. II-3825 (Ct. First Instance).



and smartphone industry. The main novelty is the specification of both dynamic upstream and downstream firms. The upstream industry consists of a dominant firm (“Qualcomm”) and a fringe, and the downstream firms are a finite number of handset makers. Qualcomm invests to increase the quality of its chipsets. Similarly, downstream handset makers invest to increase the quality of their handsets, but some handset makers are constrained by Qualcomm’s chipset quality. Handset makers also choose the proportion of handsets that use Qualcomm chipsets. A handset maker’s sunk cost of innovation depends on the amount of quality increase and the proportion of handsets using Qualcomm. The dynamic game of innovation determines the set of products in every period. Conditional on the set of products, Qualcomm and handset makers first negotiate chipset prices via Nash bargaining. Handset makers then take the chipset prices as given, set retail prices and sell to the consumers. Firms’ period profits are determined by the subgame perfect equilibrium of the overall static pricing game. When deciding whether to innovate, firms weigh the gains in the present discounted values of future profits due to innovation against the sunk costs, and the dynamic decisions form a Perfect Bayesian Equilibrium.

The model is estimated on a data set of the US smartphone market from 2009 to 2013. The estimation procedure has three steps. First, price and quantity data allow me to estimate a static random coefficient logit model of consumer demand for smartphones. I refer to a linear combination of product characteristics, where the weights are given by the estimated demand coefficients, as the quality index of the products,<sup>26</sup> and I use this index to construct the quality frontiers of Qualcomm and handset makers, detailed in Section 3.5. Next, I recover chipset prices and non-chipset marginal costs of smartphones using data on chipset markups and firms’ equilibrium pricing strategies. The first two steps do not involve estimating

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<sup>26</sup>The demand model is the same as Fan and Yang (2016)

the dynamic model. The estimates and the pricing equilibrium assumption imply firms' period profit functions. In the last step, I use the estimated period profit functions and the evolution of quality frontiers of Qualcomm and handset makers to estimate the cost function of innovation. To keep the computation tractable, I estimate a dynamic game among the upstream Qualcomm and three handset makers: Apple, Samsung and HTC, the top three handset makers by revenue. The three handset makers account for 70.2% of the total revenue during the sample period. Consistent with data, I assume that Apple only uses its own chipsets, while HTC only uses Qualcomm chipsets. Samsung can adjust the proportion of its handsets using Qualcomm chipsets. Samsung and HTC are constrained by Qualcomm chipset quality, while Apple is not (it can innovate to a quality level not yet reached by Qualcomm). I use the method of simulated moments to estimate the model. The dynamic model is solved for every trial of parameters. To ensure the existence and uniqueness of the dynamic equilibrium, I make two assumptions: 1) the dynamic game has a finite horizon, and 2) firms make investment decisions sequentially within every period.<sup>27</sup>

I examine the counterfactual should Qualcomm merge with HTC, a key handset maker that primarily uses Qualcomm chipsets. Using the model, I simulate the net effects of vertical integration and decompose the net effects into the investment coordination effects and price effects. In the main specification, I find that upstream innovation, defined as the average increase of quality per period, increases by 38%, and the innovation rate of the integrated handset maker increases by 23%. The increase primarily comes from the investment coordination of Qualcomm and HTC: the integrated firm internalizes the marginal value of HTC innovation for Qualcomm and the marginal value of Qualcomm innovation for HTC. The price effects also increase the integrated firm's innovation, but are much smaller

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<sup>27</sup>The related robustness checks are discussed in the appendix of the paper version of the chapter.

than the investment coordination effects. Moreover, Samsung is less often constrained by the upstream innovation, and Samsung innovation increases by 26%, while Apple innovation increases by 21%. Primarily due to the increase in innovation, consumer surplus increases by over 20%. In addition, while the raising rivals' cost effect increases Samsung's retail prices, the elimination of double marginalization lowers HTC prices, and the overall price effects increase the consumer surplus. The total consumer and producer surplus (variable profits) increases by over 18%. The findings thus suggest that vertical integration policies should fully take into account a vertical merger's dynamic implications and in particular the effects of investment coordination, which may be much larger than the price effects.

**Contributions and Related Literature** This paper's dynamic model of innovation is grounded in the theory of incomplete contracts. Specifically, I assume that vertically separated firms cannot contract on the amount of investment prior to the realization of investment. The assumption rules out the possibility that vertically separated firms can sign a "cooperative" contract, where one side agrees to make investment in exchange for a future transfer. The investment coordination effect studied in the counterfactual exercise quantifies the efficiency gains of allowing Qualcomm and HTC to overcome the non-contractibility of investment. Lafontaine and Slade (2007) surveys the empirical literature on vertical integration. Examples of empirical work that examines the competitive effects of vertical integration using reduced form analyses includes Waterman and Weiss (1996), Chipty (2001), Hastings (2004), Hastings and Gilbert (2005), Chen and Waterman (2007) and Hortacsu and Syverson (2007), to name a few. Static structural models have also been used to understand the effects of vertical integration in, for example, Asker (2004), Brenkers and Verboven (2006), Houde (2012) and Crawford, Lee, Whinston and Yurukoglu (2015b). The model in this paper endo-

genizes both the dynamic investment decisions as well as the pricing of intermediate goods. I contribute to the growing literature that analyzes innovation with dynamic oligopoly models (Ericson and Pakes (1995), Goettler and Gordon (2011), Borkovsky (2012), Igami (2015) and others) by modeling the complementarity of innovations between the upstream and downstream firms. The static model of product competition is built on the empirical bilateral bargaining framework developed in Horn and Wolinsky (1988). This type of models has been widely used to analyze the pricing of services and physical goods in vertical industries. Examples include Crawford and Yurukoglu (2012), Grennan (2013), Gowrisankaran, Nevo and Town (2014), Ho and Lee (2015) and Crawford, Lee, Whinston and Yurukoglu (2015b). Like many papers in this literature, I assume that firms in my model use linear price contracts. Another strand of the empirical structural literature on vertical relations studies the pricing and welfare effects of different upstream-downstream relationships (examples include Sudhir (2001), Villas-Boas (2007) and Mortimer (2008)).

Closely related to this paper, Crawford, Lee, Whinston and Yurukoglu (2015b) studies how vertical integration affects program carriage choices, prices and ultimately welfare in the US television market using a multi-stage static model. I focus on the dynamic process of innovation, where firms have rational expectations about the future evolution of the industry. An interesting feature of the model in this paper is that the states and actions of the upstream firm (Qualcomm's quality level and its investment to increase the quality) do not directly affect the current period profit of itself or downstream firms, and Qualcomm is solely motivated to innovate by the expectation that downstream firms will innovate and adopt Qualcomm chipsets in the future.

Lastly, I examine how different definitions of the disagreement point in the bargaining game may affect counterfactual predictions. Many structural empirical studies assume that prod-

ucts whose prices are negotiated would be dropped at the point of disagreement. I also consider alternative definitions, where a downstream firm switches to an alternative upstream product if the negotiation breaks down. I find that, while the main conclusions are robust, the bargaining model where handset makers drop products at the point of disagreement predicts larger price changes in the counterfactual scenario of vertical integration, even when different bargaining models are estimated on the same data set. The exercise suggests that researchers should conduct robustness checks regarding the definitions of the disagreement point, when there is no additional institutional knowledge to support the use of a particular model.

**Road Map** In the rest of the paper, I first describe the institutional setting and data in Section 3.2. Next, I detail the dynamic model of innovation in Section 3.3 and the static model of bargaining and pricing in Section 3.4. Section 3.5 discusses the estimation of the model, and Section 3.6 reports the counterfactual experiments.

## 3.2 Industry and Data

The key upstream firm in this industry, Qualcomm, produces application chipsets.<sup>28</sup> This chipset is the CPU of a smartphone, but it may also combine the functions of a GPU, modem and other components (Yang et al. (2014)). The price of a chipset is usually between \$16 to \$40 (Woyke (2014)). According to reports published by iHS, a tear-down company that tracks component prices, the chipset accounts for 10 to 20% of the material cost of a

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<sup>28</sup>Sometime a smartphone modem is also called a communication chipset. Throughout this paper a chipset always means the application chipset.

smartphone.<sup>29</sup>

Qualcomm is the most important company in the upstream chipset industry. In 2009, 53% of non-Apple smartphones sold in the US carried a Qualcomm chipset, and the figure increased to 72% in the first quarter of 2013. Although Qualcomm is able to capture the lion's share of the chipset market, there is no lack of competition. iPhone 4s and later models use Apple's own A series chipsets. Samsung uses its Exynos series chipsets on some of its handsets. There are also a large number of independent chipset makers, such as MediaTek (Taiwan), Texas Instruments (US) and NVIDIA (US). Compared with its competition, one of Qualcomm's advantages is its ability to combine many components (including the modem, GPS, GPU and others) onto a single chipset. By using Qualcomm chipsets, handset makers may save the development cost and the cost of procuring and combining other components. In addition, an integrated chipset saves energy and thus extends battery life, and the more compact design may also save space and allow handset makers to install additional components and add functionality. Much like the role of CPU for personal computers, chipsets are central to the performance of smartphones, because many of the metrics that consumers value, such as processing power (CPU), support of fast network (modem), graphic processing (GPU) and energy efficiency, are determined by the chipset. The innovations of chipsets center on improving all components within the processor without sacrificing too much energy efficiency or significantly increasing manufacturing costs (Yeap (2013)).

Qualcomm products are categorized in multiple generations released over time. Snapdragon S1, the first generation of chipsets observed in the data, was released in October 2008. Generation S2, S3 and S4 are released in April 2010, October 2010 and January 2012. S4

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<sup>29</sup>iHS publishes the marginal cost estimates of select handsets through its press releases. I have collected its published data, which are available upon request.

is the last generation observed in the data. A later generation typically features significant gains in performance (more cores and higher frequency) and energy efficiency. Following Qualcomm’s release of a new generation, its competitors release comparable products. Table 3.15 reports the origins of chipsets used in major non-Apple handset makers.

Table 3.15: Chipset Origin, % of Handset Maker Retail Revenues, 2009 to 1st Quarter 2013

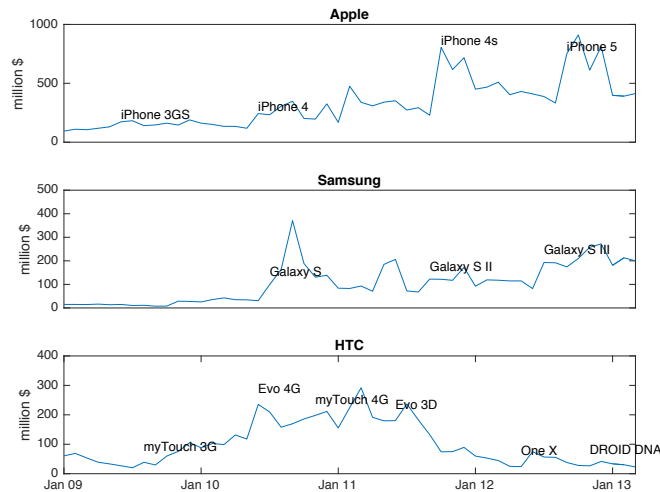
	Samsung	HTC	Motorola	LG	BlackBerry
Qualcomm	33.93	99.60	14.73	94.10	51.10
Samsung	62.52	0.00	0.00	0.00	0.00
TI	2.68	0.21	74.18	5.05	0.00
NVIDIA	0.47	0.10	8.70	0.85	0.00
Other	0.40	0.09	2.39	0.00	48.90

The downstream smartphone industry is relatively concentrated. From 2009 to 2012, the top three handset makers were Apple, Samsung and HTC as measured by retail sales revenues, and they accounted for 70% of the US market. Most smartphones in the US are sold through the four major national wireless carriers, AT&T, Verizon, T-Mobile and Sprint. Over time, handset makers adopt new chipsets in new phones and provide many additional improvements, such as larger and clearer screens, longer talk time, and higher definition cameras. Handset makers spend billions of dollars on combining new technologies and designing new phones. It is worth noting that the chipset quality nonetheless plays a central role in this process. For example, several times more power is consumed by GPU and CPU than the display when a smartphone is used to play video games (Chen et al. (2013)). A smartphone maker would not be able to use large screens for its phones without first obtaining energy-efficient chipsets.

Smartphone sales are driven by the release of new products. I plot the monthly retail sales revenues in Figure 3.2.1. The sales spikes tend to coincide with the release of flagship

products. The innovation of Qualcomm, marked by the release of new generation of chipsets, occurs 2 to 6 months before handset makers adopt the chipsets.

Figure 3.2.1: Handset Maker Monthly Sales Revenues



The data set includes the aggregate sales, retail prices and characteristics of smartphones sold through the four national carriers, from January 2009 to March 2013.<sup>30</sup> The observation is at the handset-carrier-month level. The US market accounts for about 15% of the global shipment in Q4 2011 (Gartner (2012)), but is likely more important to the high end handset makers. For example, it is reported that the US market accounted for over 40% of Apple’s revenue in 2010.<sup>31</sup> Throughout the analysis, I assume that the US market accounts for a constant proportion of the world market. While I do not observe chipset prices directly, I collect the accounting gross margin data of Qualcomm from its quarterly financial reports. I use the data as the sales-weighted average markup of Qualcomm chipsets. The average

<sup>30</sup>Smartphone quantity and price data are acquired from the ITG Market Research, and the chipset information is scrapped from technology websites and press releases.

<sup>31</sup>CSI Market.



markup data allow me to impute product-specific chipset prices, detailed in Section 3.5. Additional data and institutional details of the smartphone industry can be found in Fan and Yang (2016).

### 3.3 A Dynamic Model of Upstream and Downstream Innovation

Time is discrete  $t = 1, 2, \dots, T$ . The upstream consists of Qualcomm and a non-strategic fringe. Downstream firms are comprised of a finite number of firms  $\mathcal{N}$ . The Qualcomm state variable is the quality frontier  $q^Q$ . The state variables of a handset maker  $n$  include the proportion of  $n$ 's handsets using Qualcomm and  $n$ 's quality frontier,  $s^n = \{\eta^n, q^n\}$ ,  $\eta^n \in (0, 1)$ . The industry state consists of  $s = \{t, q^Q, \{\eta^n, q^n\}_{n \in \mathcal{N}}\}$ . The empirical game estimated later would include Apple and HTC whose  $\eta$ 's are fixed, and Apple is not constrained by Qualcomm innovation. To simplify the presentation of the model, I assume in this section that all handset makers can adjust their proportions of handsets using Qualcomm, and are constrained by Qualcomm's quality frontier.

In every period, Qualcomm chooses quality increments of its quality frontier,  $a^Q \in \{0, \Delta, 2\Delta, \dots, K_1\Delta\}$ , and the next period Qualcomm state transitions to  $q_{t+1}^Q = q_t^Q + a^Q$ . The action in data that corresponds with Qualcomm innovation is its release of new chipsets. A handset maker also chooses quality increments  $a_q^n \in \{0, \delta, 2\delta, \dots, K_2\delta\}$ . If  $n$  does not innovate ( $a_q^n = 0$ ), the proportion stays the same. If  $n$  innovates ( $a_q^n > 0$ ),  $n$  also chooses its proportion of handsets using Qualcomm from a discrete set,  $a_\eta^n \in \{\eta_1, \eta_2, \dots, \eta_{K_3}\}$ , at the new quality level  $q_{t+1}^n = q_t^n + a_q^n$ .  $n$ 's state transition can be summarized as the following:

when  $n$  takes action  $a^n = \{a_q^n, a_\eta^n\}$ , the next period state becomes

$$\begin{cases} q_{t+1}^n = q_{t+1}^n + a_q^n, \eta_{t+1}^n = a_\eta^n, & \text{if } a_q^n > 0 \\ q_{t+1}^n = q_t^n, \eta_{t+1}^n = \eta_t^n, & \text{if } a_q^n = 0 \end{cases}$$

The action in data corresponding with handset maker  $n$ 's innovation is the launch of a handset whose quality is higher than any of  $n$ 's previous handsets.

The game starts in  $t = 1$ . In every period, firms first receive period profits  $\pi_t(s_t)$ , and make dynamic decisions sequentially. Qualcomm moves first, and handset makers move in the sequence  $n_1, \dots, n_N$ :

- Qualcomm draws i.i.d cost shock  $\varepsilon_t^Q$ , takes action  $a_t^Q$  and pays a sunk cost of  $C^Q(a_t^Q, \varepsilon_t^Q)$ .
- Handset maker  $n_1$  observes Qualcomm's decision, draws i.i.d cost shock  $\varepsilon_t^{n_1}$ , takes action  $a_t^{n_1}$  and pays  $C^{n_1}(a_t^{n_1}, \varepsilon_t^{n_1})$
- ...
- Handset maker  $n_N$  observes all previous actions, draws i.i.d cost shocks  $\varepsilon_t^{n_N}$ , takes action  $a_t^{n_N}$  and pays  $C^{n_N}(a_t^{n_N}, \varepsilon_t^{n_N})$ .

The dynamic optimization problem of Qualcomm in period  $t$  solves

$$\max_{a^Q} \left( -C^Q(a^Q, \varepsilon_t^Q) + \beta E \left( V_{t+1}^Q(s_{t+1}) | s_t \right) \right),$$

where the expectation is taken over the action probabilities of firms who have not moved in

period  $t$ . The value function of Qualcomm satisfies the Bellman equation

$$V_t^Q(s_t) = \pi^Q(s_t) + \int_{\varepsilon_t^Q} \left\{ -C^Q(a^{Q^*}, \varepsilon_t^Q) + \beta E(V_{t+1}^Q(s_{t+1}) | s_t) \right\}, \quad (3.3.1)$$

where the strategy  $a^{Q^*}$  is a function of its own cost shock. Similarly, a handset maker  $n$  solves

$$\max_{a^n} \left( -C^n(a^n, \varepsilon_t^n) + \beta E(V_{t+1}^n(s_{t+1}) | a_{\mathcal{N}(n)}, s_t) \right),$$

but subject to the constraint that  $q_t^n + a_q^n \leq q_{t+1}^Q$ .  $a_{\mathcal{N}(n)}$  denotes the actions of firms that have moved before  $n$ . The Bellman equation of  $n$  is identical to (3.3.1), with the superscript  $Q$  replaced by  $n$ . Also note that  $n$ 's strategy is a function of its shocks and the actions of firms that have moved. Players in this game have private information and move sequentially, and I solve for the Perfect Bayesian Nash Equilibrium (PBE). The last period value function is specified as  $V_T = \frac{\pi(s_T)}{1 - \beta}$ .

While the assumptions of a finite horizon and sequential moves are quite strong, they provide three crucial benefits: 1) the dynamic equilibrium is unique, 2) solving the dynamic game does not involve value function iterations and suffers no convergence problem (Egesdal, Lai and Su (2015)), 3) the finite horizon assumption also helps to capture the non-stationarity in data (Igami (2015)). I explore the robustness of both assumptions in the appendix of the paper version of the chapter.

The investment cost is specified as

$$C^Q(a^Q, \varepsilon_t^Q) = \begin{cases} 0, & a^Q = 0 \\ \exp\left(\gamma_0^Q + (\gamma_1^Q + \sigma^Q \varepsilon_t^Q) a^Q\right) & a^Q > 0 \end{cases}$$

$$C^n(a^n, \varepsilon_t^n) = \begin{cases} 0, & a_q^n = 0 \\ \exp(\gamma_0^n + (\gamma_1^n + \sigma^n \varepsilon_t^q) a_q^n + \gamma_2^n a_\eta^n) & a_q^n > 0 \end{cases}$$

The cost shocks  $\varepsilon$  follow the standard normal distribution.

In this model, I assume that dynamic innovation decisions are not contractible. Therefore it is not possible that HTC enters into a contract with Qualcomm about Qualcomm innovation before Qualcomm’s innovation is realized. Such contracts would effectively achieve vertical integration. Grossman and Hart (1986) suggests that without investment coordination, two vertically separated firms would invest below the socially optimal level because neither firm fully internalizes the benefit of investment on the other. Central to the concept of “incompleteness” is the difficulty of communicating a firm’s investment decisions to others before the realization of the investment. Indeed, while the technology capability of a firm is abstracted into a scalar  $q$  in the model, coordinating innovations in the real world potentially would require the chipset maker and handset maker to agree on the joint development of many dimensions of the technology. Identifying and agreeing to the exact nature of innovation may be hard enough. The legal costs of writing down contracts that enumerate all aspects of cooperative development could be high. Enforcement may be hard, because in the case of contract violations, firms may need to disclose proprietary designs in a legal proceeding. Given these considerations, I assume that firms cannot contract on future innovation.<sup>32</sup>

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<sup>32</sup>The ex ante communication difficulty may be overcome in an infinite horizon dynamic game. There may exist a PBE where firms condition strategies on past actions and Qualcomm may be able to credibly slow down innovation and “punish” HTC, if HTC does not pay Qualcomm a transfer after a Qualcomm innovation. The additional assumptions of a finite horizon and sequential moves rule out this possibility: the game admits a unique solution, which can be obtained via backward induction. In this equilibrium, the downstream firms would never voluntarily pay Qualcomm a transfer.

## 3.4 Bargaining Model

The profit function  $\pi(s_t)$  is given by a static model of bargaining. I assume that prices are set in the following order:

1. Qualcomm and handset makers negotiate chipset prices via Nash bargaining.
2. Handset makers take the chipset prices and other components of the marginal cost as given, set retail prices and sell to consumers.

I start with the demand function.

### 3.4.1 Consumer Demand

I model the consumer demand for smartphones using a random coefficient logit model (Berry, Levinsohn and Pakes (1995a)). Index consumers by  $i$  and handsets by  $j$ . The utility of consumer  $i$  purchasing handset  $j$  in period  $t$  is

$$\begin{aligned} u_{ijt} &= \beta_{0i}q_j - \alpha p_{jt} + \theta_{n(j)} + \kappa_{c(j)t} + \xi_{jt} + \epsilon_{icjt} \\ &= \underbrace{\bar{\beta}_0 q_j - \alpha p_{jt} + \theta_{n(j)} + \kappa_{c(j)t} + \xi_{jt}}_{\mu_{jt}} + \sigma \nu_i q_j + \epsilon_{ijt} \end{aligned}$$

where  $q_j = x_j \beta$  is the linear quality index of handset characteristics,  $\beta_{0i}$  is a normally distributed scalar random coefficient that captures the heterogeneous tastes for quality:  $\beta_{0i} = \bar{\beta}_0 + \sigma \nu_i$ ,  $\nu_i \sim \mathcal{N}(0, 1)$ ,  $p_{jt}$  is the retail price of the smartphone,  $\theta_n$  is the handset maker fixed effect,  $\kappa_{ct}$  is the carrier-year fixed effect plus quarter fixed effect that captures carrier service heterogeneity and the values of time-varying outside options (this term is referred to as

carrier-time fixed effects in the rest of the paper),  $\xi_{jt}$  is the unobserved product quality, and  $\epsilon_{ijt}$  is an i.i.d type I extreme value shock. Smartphone characteristics in  $x_j$  include the screen size, camera resolution and chipset generation fixed effects. The mean portion of the consumer utility function is collected in the term  $\mu_{jt}$ , and the utility of no purchase is normalized to zero plus an i.i.d type I extreme value shock  $\epsilon_{i0t}$ . The demand for  $j$  is given by

$$D_{jt} = D_0 \int \frac{\exp(\mu_{jt} + \sigma \nu_i q_j)}{1 + \sum_{j' \in \mathcal{J}_t} \exp(\mu_{j't} + \sigma \nu_i q_{j'})} dF_{\nu_i},$$

where  $\mathcal{J}_t$  is the set of all products available in period  $t$ ,  $D_0$  is the market size and  $F_{\nu_i}$  is the CDF of  $\nu_i$ . I next discuss the pricing of smartphones and chipsets.

### 3.4.2 Prices of the Smartphones

Denote the set of handset maker  $n$ 's product as  $\mathcal{J}_{nt}$ . Given the chipset prices  $\psi_{jt}$  and other parts of the marginal cost  $\omega_{jt}$ , handset maker  $n$  sets wholesale prices  $w_{jt}, \forall j \in \mathcal{J}_{nt}$ , to maximize its profit

$$\sum_{j \in \mathcal{J}_{nt}} (w_{jt} - \psi_{jt} - \omega_{jt}) D_{jt}.$$

The non-chipset marginal cost of a smartphone is specified as a function of observed characteristics plus a shock:

$$\omega_{jt} \equiv \underbrace{\lambda_q \exp(q_{jt}) + \lambda_{n(j)} + \lambda_{Q(j)} + \zeta_{c(j)t}}_{\text{quality, handset maker FE}} + \underbrace{\varkappa_{jt}}_{\text{shock}}. \quad (3.4.1)$$

use Qualcomm?

carrier-time FE

Carrier pricing of handsets can be complex. To simplify computation, I assume that the carrier subsidy on product  $j$  is specified as

$$r_{jt} = \tilde{\lambda}_q \exp(q_{jt}) + \tilde{\lambda}_{n(j)} + \tilde{\lambda}_{Q(j)} + \tilde{\zeta}_{c(j)t} + \tilde{\varkappa}_{jt},$$

such that the retail price satisfies  $p_j = w_{jt} - r_{jt}$ . Handset maker  $n$ 's profit maximization problem can be re-written as

$$\max_{p_j, j \in \mathcal{J}_{nt}} \sum_{j \in \mathcal{J}_{nt}} (p_{jt} - \psi_{jt} - (\omega_{jt} - r_{jt})) D_{jt}.$$

Given this equivalence, I assume that handset makers choose retail prices directly. To save notation, I re-define  $\omega_{jt}$  as  $\omega_{jt} - r_{jt}$ , and correspondingly, the coefficients in the non-chipset component  $\lambda$  as  $\lambda - \tilde{\lambda}$  and the shock  $\varkappa$  as  $\varkappa - \tilde{\varkappa}$ . Equilibrium retail prices satisfy the following first order condition:

$$s_{jt} + \sum_{j' \in \mathcal{J}_{nt}} (p_{j't} - \psi_{j't} - \omega_{j't}) \frac{\partial s_{j't}}{\partial p_{jt}} = 0, \forall j' \in \mathcal{J}_{nt}.$$

In vector notation similar to Eizenberg (2014a), the vector of retail prices  $p$  satisfies

$$p - \psi - \omega = (L * \Delta)^{-1} s, \tag{3.4.2}$$

where  $L$  is a  $|\mathcal{J}_t| \times |\mathcal{J}_t|$  product origin matrix ( $L_{jj'} = 1$  if both  $j$  and  $j'$  belong to  $\mathcal{J}_{nt}$  and 0 otherwise),  $\Delta_{jj'}$  is the derivative of the demand for  $j'$  with respect to the price of  $j$ , and  $*$  represents element-by-element multiplication. If the price equilibrium is unique at this stage, the derived demand for chipsets on handset  $j$  is well defined. However, there may be

multiple Nash-Bertrand equilibria under logit demand with random coefficients and multi-product firms (Echenique and Komunjer (2007)). To select an equilibrium, I use (3.4.2) as a fixed point mapping to solve for the equilibrium iteratively. The starting point is the price of the product in data that is closest in quality. Denote  $D^* = D(p^*(\psi, \omega))$  as the derived demand for chipsets.

### 3.4.3 Nash Bargaining and Chipset Prices

The bargaining game in the first stage of the static game determines the equilibrium chipset prices between Qualcomm and handset makers. I first write down Qualcomm's profit function. Qualcomm earns profits from chipset sales:

$$\pi_t^Q(\psi) = \sum_{j \in \mathcal{J}_{Qt}} (\psi_{jt} - \underline{\psi}) D_{jt}^*$$

where  $\mathcal{J}_{Qt}$  is the set of handsets using Qualcomm chipsets and  $\underline{\psi}$  is the marginal cost for Qualcomm to manufacture a chipset.<sup>33</sup> The chipset prices are set in a bargaining equilibrium. Qualcomm negotiates with each handset maker  $n$  separately. Denote the vector of chipset prices specific to a Qualcomm- $n$  bargaining pair as  $\boldsymbol{\psi}_{nt} = (\psi_{jt}, j \in \mathcal{J}_{Qt} \cap \mathcal{J}_{nt})$ .

**Definition 1.** Chipset prices  $\boldsymbol{\psi}_{nt}$  for all products in  $\mathcal{J}_{Qt} \cap \mathcal{J}_{nt}$  are set to maximize the Nash product corresponding with the bargaining pair of Qualcomm and handset maker  $n$ , conditional on other chipset prices  $\boldsymbol{\psi}_{-nt}$ :

$$\left[ \pi_t^Q(\boldsymbol{\psi}_{nt}, \boldsymbol{\psi}_{-nt}) - \tilde{\pi}_t^Q(\boldsymbol{\psi}_{-nt}) \right]^{\tau_t} \cdot \left[ \pi_t^n(\boldsymbol{\psi}_{nt}, \boldsymbol{\psi}_{-nt}) - \tilde{\pi}_t^n(\boldsymbol{\psi}_{-nt}) \right]^{1-\tau_t},$$

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<sup>33</sup>In reality, Qualcomm does not own any chipset manufacturing facility, and it outsources the production to dedicated fabrication plants.



where  $\tilde{\pi}$  is the disagreement payoff, and  $\tau_t$  is the bargaining weight.<sup>34</sup>

Many papers using empirical bargaining games such as Crawford and Yurukoglu (2012), Grennan (2013) and Crawford et al. (2015b) employ the assumption that, at the disagreement point, products in  $\mathcal{J}_{Qt} \cap \mathcal{J}_{nt}$  are dropped, and firms earn profits from the rest of the products with the remaining products' chipset prices fixed and downstream equilibrium recalculated. In absence of Qualcomm's chipset competitors, there is reason to believe that the bargaining weight  $\tau$  should be close to 0.5: Collard-Wexler, Gowrisankaran and Lee (2014) interprets the parameter as the relative discount factor, and the discount factors are likely equal across agents.<sup>35</sup> However, as Qualcomm's competitors are able to offer substitutable chipsets, this weight is unlikely to be 0.5. Another plausible disagreement point definition may be that, while the bargaining weight is fixed at 0.5, handset makers switch to an outside alternative temporarily. I therefore consider three types of disagreement payoffs:

1.  $n$  switches to a non-Qualcomm source and does not pay chipset prices to Qualcomm, but the qualities of products in  $\mathcal{J}_{Qt} \cap \mathcal{J}_{nt}$  decrease by  $\bar{q}_t$ , their chipset prices are set to 0, and Qualcomm does not provide chipsets for these products. Chipset prices of other products are held fixed, and the downstream equilibrium is recalculated.  $\tau_t = 0.5$ .  $\bar{q}_t = \bar{q} + \sigma_q \varpi_t$ .  $\varpi_t$  is standard normal i.i.d across time, and  $(\bar{q}, \sigma_q)$  are parameters to be estimated.
2.  $n$  switches to a non-Qualcomm source and does not pay chipset prices to Qualcomm, but procures chipsets for products in  $\mathcal{J}_{Qt} \cap \mathcal{J}_{nt}$  at a price  $\bar{\psi}_t$ , and Qualcomm does not

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<sup>34</sup>Crawford and Yurukoglu (2012) shows that alternative definitions of a bargaining pair do not strongly affect their counterfactual equilibrium price predictions.

<sup>35</sup>This parameter may also reflect differences of bargaining abilities due to, for example, the access to information (Grennan and Swanson (2016)).

provide chipsets for these products. The qualities of these products do not change. Chipset prices of other products are held fixed, and the downstream equilibrium is recalculated.  $\tau_t = 0.5$ .  $\bar{\psi}_t = \bar{\psi} + \sigma_\psi \varpi_t$ .  $\varpi_t$  is standard normal i.i.d across time, and  $(\bar{\psi}, \sigma_\psi)$  are parameters to be estimated.

3. Products in  $\mathcal{J}_{Qt} \cap \mathcal{J}_{nt}$  are dropped, and firms earn profits from the rest of the products. Chipset prices of other products are held fixed, and the downstream equilibrium is recalculated. The bargaining weight is an unknown parameter  $\tau_t \in (0, 1)$  to be estimated.  $\bar{\tau}_t = \bar{\tau} + \sigma_\tau \varpi_t$ .  $\varpi_t$  is standard normal i.i.d across time, and  $(\bar{\tau}, \sigma_\tau)$  are parameters to be estimated.

In the bargaining equilibrium, the vector of all Qualcomm chipset prices  $\psi$  satisfies the following first order condition:

$$\psi = \underline{\psi} + \Theta^{-1}\Phi, \tag{3.4.3}$$

where  $\Theta$  and  $\Phi$  are defined (differently) for each bargaining model in the appendix of the paper version of the chapter.<sup>36</sup>

It should be noted that the assumption of linear contracts between handset makers and Qualcomm is not completely innocuous. This assumption introduces double marginalization, an inefficiency that vertical integration can reduce. Unfortunately, contracts and contracting processes are confidential, and it is hard to obtain detailed information on these procurement agreements. I nonetheless was able to download from SEC a copy of modem procurement contract (with numbers redacted) between Qualcomm and a client (not a smartphone maker). The contract specifies the unit price, quantity and date of delivery. If two vertical monopolies

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<sup>36</sup>There may also be multiple bargaining equilibria. To define the period profit for each firm, I use (3.4.3) as a fixed point mapping to iteratively solve for the equilibrium chipset prices, starting from  $1.2\underline{\psi}$ .

can contract on both the price and quantity and firms have complete information about downstream demand, there may exist a contract equivalent to a lump sum transfer agreement that avoids double marginalization. On the other hand, if firms in fact only negotiate prices and the quantity is given by the demand function, the agreement is a linear contract. Without knowing which is the case, I follow the many papers in the literature of empirical bargaining games and assume that firms use linear contracts. Future work will explore alternative specifications.

### 3.4.4 Period Profit

Collect the number of products, product qualities, chipset origins, the bargaining parameter ( $\bar{q}_t$ ,  $\tau_t$  or  $\bar{\psi}_t$ ) and carrier-time fixed effects in a vector  $y$ . Using the equilibrium selection rules above, Qualcomm and handset maker profits can be written as a function of  $y$ , demand shocks and marginal cost shocks,  $\pi_t^Q(y, \xi, \varkappa)$  and  $\pi_t^n(y, \xi, \varkappa)$ . Note that  $y$  does not include the state variable of Qualcomm.

In this paper, I focus on how firms adjust quality frontiers, and assume that  $y$  is a realization from the distribution  $g(Y; s_t, \theta)$ : the set of products is a stationary distribution conditional on the state variables defined in Section 3.3.<sup>37</sup> The specification of  $g(\cdot)$  is in the appendix of the paper version of the chapter. In particular, the distribution is based on handset makers' state variables alone. As  $q^n$  increases, the highest and average qualities of  $n$ 's smartphones increase; at a higher  $\eta_n$ , a larger number of  $n$ 's handsets use Qualcomm. Firms use  $\pi^Q(s_t) \equiv E_{Y, \xi, \varkappa | s_t}(\pi_t^Q(Y, \xi, \varkappa))$  and  $\pi^n(s_t) \equiv E_{Y, \xi, \varkappa | s_t}(\pi_t^n(Y, \xi, \varkappa))$  to make dynamic innovation decisions.

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<sup>37</sup>See Fan and Yang (2016) for a study on product variety.

The assumptions of the static demand and pricing and the stationarity of product distribution allow the period profits to be computed separately from the dynamic game. The integration of  $\pi_t^Q(Y, \xi, \varkappa)$  and  $\pi_t^n(Y, \xi, \varkappa)$  over the distribution of products, demand shocks and cost shocks is time-consuming but only needs to be done once, because the random variables are distributed i.i.d over time. No knowledge of the innovation costs or the dynamic equilibrium is required to compute period profits. The profits are then taken as input to the estimation and simulation of the dynamic game. In reality, smartphones may be both durable goods and network goods (Sinkinson (2014) and Luo (2014)). While the framework in this paper does not include dynamic consumers and endogenous network effects, the demand function partially captures both effects with  $\kappa_{ct}$ , and the model implicitly assumes that the two effects are exogenous.

## 3.5 Estimation

### 3.5.1 Demand and Smartphone Marginal Cost

Demand is estimated using standard BLP instruments on the full sample from January 2009 to March 2013. Each month is treated as an independent market. The estimates of the demand model are presented in the left panel of Table 3.16. The characteristics  $x_j$  used to construct the quality index include the screen size,<sup>38</sup> chipset generation, camera resolution, weight and the talking time on full battery. The screen size coefficient is normalized to be 1. The chipset generation fixed effect corresponds with each generation of Qualcomm chipsets and comparable products. Generation 1 corresponds with phones that do not use chipsets

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<sup>38</sup>The screen size is measured as the diagonal length of the phone, as is standard in this industry, and the unit is inch.

or use uncategorized old chipsets, and the coefficient is normalized to be 0. The brand fixed effects of Apple, Samsung and BlackBerry are also included. The detailed definitions of variables and interpretations of the coefficients are documented in Fan and Yang (2016). The demand estimates are reasonably intuitive, with higher generation, camera resolution, lower weights and longer battery talk time contributing positively to the index. The Apple brand fixed effect in the demand function is large, worth over \$400 to consumers.

Using the estimates, the quality index of a product is constructed as  $q_j = x_j\beta$ . I construct the quality frontier of a handset maker in period  $t$  as the highest quality in and before period  $t$ ,  $\max_{t' \leq t} q_j, j \in \mathcal{J}_{nt'}$ . Because the sales of a handset maker are driven by its flagship products, and these flagship products often have the highest quality, this construction captures the essence of handset maker innovation. To construct the frontier corresponding with each generation's Qualcomm chipsets, I use the highest quality of handsets using that generation's chipsets plus 0.25. For example, the highest quality handset that uses generation 4 is Galaxy Note, and its quality is 7.17. The generation 4 chipset quality is then 7.42. It is possible that the observed handsets on a generation's chipsets never "reach the full potential", and a handset maker could use the chipset to produce a phone whose quality may be much higher than 7.17. My construction may seem conservative, but a 0.25 increase in quality is nontrivial, because it is almost the size of the increase of the generation coefficient from generation 2 to 3 and from 3 to 4.

I now discuss how to estimate the marginal cost function (3.4.1). Given the estimated demand function and observed prices, the full marginal cost  $\omega + \psi$  can be inverted using the first order condition (3.4.2). To estimate the coefficients in (3.4.1), I need to break out the chipset prices  $\psi$ . To impute  $\psi$ , I rely on the average Qualcomm markup data in its

Table 3.16: Demand Side Estimates

		Est	Se
	Screen Size (inch)	1	-
	Chipset Generation 2	0.460	0.113
	Chipset Generation 3	0.718	0.147
	Chipset Generation 4	1.055	0.200
$\beta$	Chipset Generation 5	1.674	0.280
	Camera Resolution (megapixel)	0.093	0.036
	Weight (gram)	-0.002	0.001
	Battery Talk Time (hours)	0.056	0.013
$\sigma$	Std, Quality	0.300	0.079
$\bar{\beta}_0$	Mean, Quality	0.779	0.128
$\alpha$	Price (\$)	0.007	0.002
$\theta_n$	Apple	2.779	0.094
Carrier-time and other brand fixed effects included			

Table 3.17: Supply Side Estimates

		Model 1		Model 2		Model 3	
		Est	Se	Est	Se	Est	Se
$\lambda_q$	exp (quality/10) (\$)	425.884	2.818	425.968	2.818	425.308	2.816
$\lambda_Q$	Use Qualcomm? (\$)	-12.537	0.266	-12.745	0.266	-12.738	0.266
Carrier-time and brand fixed effects included							

quarterly financial reports<sup>39</sup> and the equilibrium first order conditions corresponding with the Nash product, (3.4.3). I set Qualcomm's marginal cost of manufacturing a chipset to be \$20. Consider Bargaining Model 1. For every  $\bar{q}$ , I can solve for a vector of chipset prices consistent with the observed retail prices using (3.4.3) in every  $t$ . If the solution is unique, then there exists a one-to-one relationship between  $\bar{q}$  and the vector of equilibrium chipset prices. Because a vector of chipset prices implies a unique sales-weighted average Qualcomm

<sup>39</sup>This figure is defined as 1-(cost of goods sold/total revenue), which does not include fixed and sunk costs such as administrative overhead and R&D. Nevo (2001) argues that this is an upper bound on the markup.

Table 3.18: Bargaining Parameters and Mean Chipset Prices

Bargaining Parameter Estimates <sup>a</sup>								
Model 1			Model 2			Model 3		
	Est	Se		Est	Se		Est	Se
$\bar{q}$	0.222	0.015	$\bar{\psi}$ (\$)	57.884	2.510	$\tau$	0.097	0.006
$\sigma_q$	0.062	0.011	$\sigma_\psi$ (\$)	10.340	1.828	$\sigma_\tau$	0.025	0.004
Mean Chipset Prices (\$) <sup>b</sup>								
Model 1			Model 2			Model 3		
36.746			36.749			36.700		

<sup>a</sup>: mean and standard deviations are calculated from inverted  $\bar{q}_t$ ,  $\bar{\psi}_t$  or  $\tau_t$  specific to each quarter on a sample of 17 quarters.

<sup>b</sup>: averaged over all imputed chipset prices of Samsung and HTC.

chipset markup, there exists a one-to-one relationship between  $\bar{q}$  and the average Qualcomm markup. I use this relationship to invert a  $\bar{q}_t$  for every period. In practice, I can always obtain a unique solution when solving for the chipset prices from multiple starting points.

Table 3.17 shows that the marginal cost estimates from three different bargaining models are consistent. The non-chipset components' costs increase with the quality of the smartphone. Using a Qualcomm chipset saves \$12 to \$13 in marginal cost for the non-chipset part of the phone. An alternative interpretation is that it costs about \$12 to acquire a non-Qualcomm chipset. The estimates of the bargaining parameters are presented in Table 3.18. The financial reports I used are quarterly, and the sample includes 17 quarters. For every quarter, depending on the bargaining model, I invert a  $\bar{q}_t$ ,  $\bar{\psi}_t$  or  $\tau_t$  by matching the (un-discounted) sales-weighted average Qualcomm markup of the quarter with the gross margin data. I assume that the random variables are normal and distributed i.i.d over quarters, and I estimate the mean and standard deviation using the 17 observations. During the inversion process, I use a minimization algorithm to match markup with data. I run the algorithm from 10 different starting points and always find a unique solution. The average Qualcomm

chipset prices are about \$37. Notably, the mean bargaining weight in Model 3 is close to 0, implying that Qualcomm bargaining power is low, consistent with the fact that Qualcomm faces competition from other chipset makers.

### 3.5.2 Sunk Cost of Innovation

The estimates of the static pricing game allow me to simulate period profits at any given point in the state space. For any parameter value of the sunk cost function, I am able to solve for the unique Perfect Bayesian Equilibrium. I therefore employ a nested fixed point simulated estimator that matches moments from the model with data. To limit the computational burden, I estimate a dynamic game of Qualcomm and the three top handset makers: Apple, Samsung and HTC. I assume that the order of moves is Qualcomm, HTC, Samsung and Apple. This order is chosen so that the two firms using Qualcomm are the first to react to Qualcomm's actions. Apple is assumed to always use non-Qualcomm chipsets ( $\eta^A = 0$ ) and not constrained by Qualcomm quality frontier; HTC innovation is constrained by Qualcomm, and always chooses  $\eta^{HTC} = 1$ : the chipsets of all HTC phones are supplied by Qualcomm and their prices are determined in the bargaining equilibrium; Samsung innovation is also constrained by Qualcomm, but can adjust the proportion of Qualcomm chipsets used on Samsung handsets. It should be noted that Samsung uses its own chipsets in some of its flagship smartphones. However, these smartphones are typically launched much later than the corresponding Qualcomm generation. For example, Samsung Note II (the 5th chipset generation) using Samsung's own Exynos chipsets was launched in Oct 2012, while the corresponding generation of Qualcomm chipsets was launched in January 2012, and the first smartphone using Qualcomm's generation 5 chipsets was launched in April 2012. The model is solved by backward induction from the last period of data, March 2013. Additional details



are provided in the appendix of the paper version of the chapter, where I also check the sensitivity of the assumptions of a finite horizon and sequential moves. In the first exercise, I solve the model from August 2013, with the carrier-time fixed effects in the period profit functions for April to August 2013 extrapolated from demand estimates in earlier periods. In the second exercise, I reverse the order of downstream firms' moves, and firms move in the sequence of Qualcomm, Apple, Samsung and HTC.

To underscore the importance of potential cost heterogeneity, I estimate a firm specific intercept  $\gamma_0$ . I also estimate a different  $\gamma_1$  specific to Apple, Samsung and Qualcomm. I restrict  $\gamma_1^{HTC} = \gamma_1^{Samsung}$ , and I also restrict  $\sigma^{Apple} = \sigma^{Samsung} = \sigma^{HTC}$  and estimate a different  $\sigma^{Qualcomm}$ , giving me a total of 10 parameters to estimate. I use the following moments:

1. mean innovation rates, defined as  $\bar{v} = (q_T - q_1) / T$ ;
2. variance of innovation rates,  $\sum_{t=1}^{T-1} (q_{t+1} - q_t - \bar{v})^2 / T$ ;
3. the mean distance from Qualcomm frontier to the maximum of HTC and Samsung's frontiers,  $\left( \sum_{t=1}^T \left( q_t^Q - \max(q_t^S, q_t^H) \right) \right) / T$ ;
4. the mean proportion of Qualcomm chipsets on Samsung products,  $\sum_{t=1}^T \eta_t / T$ .

There are exactly 10 moments for 10 parameters: the exact identification case also helps to diagnose the performance of the minimizer of the objective function, because the minimum is 0. I use the genetic algorithm to minimize the objective function.

The moments are sensitive to the parameters.  $\gamma_0^n$ 's are mainly identified by the mean innovation rates. Both the mean and variance of innovation rates react sharply to changes

in  $\gamma_1^n$ . The distance between Qualcomm and the maximum of HTC and Samsung frontiers is sensitive to  $\gamma_1^Q$  and  $\sigma^{handset}$ . The mean innovation rates of Samsung and Qualcomm and the mean proportion of Qualcomm chipsets are sensitive to  $\gamma_2^{Samsung}$ . The mean innovation rates of Samsung, HTC and Qualcomm and the mean proportion of Qualcomm chipsets are sensitive to  $\sigma^Q$ .<sup>40</sup>

Table 3.19: Estimates of Innovation Costs

Bargaining Model		1		2		3	
		Est	Se	Est	Se	Est	Se
$\gamma_0^n$	Apple	-0.28	0.22	-0.28	0.62	-0.28	0.48
	Samsung	-0.07	1.39	-0.07	1.79	-0.07	1.63
	HTC	-0.14	0.30	-0.17	0.41	-0.14	0.64
	Qualcomm	-4.78	0.68	-4.78	0.35	-4.78	0.94
$\gamma_1^n$	Apple	2.68	0.26	2.68	0.41	2.68	0.98
	HTC/Samsung	1.55	0.00	1.55	0.01	1.55	0.09
	Qualcomm	0.41	0.21	0.41	0.17	0.41	0.85
$\gamma_2^n$	Samsung	-4.00	0.93	-4.39	0.84	-4.00	0.70
$\sigma^n$	Handset Makers	1.60	0.12	1.60	0.28	1.60	0.41
	Qualcomm	0.44	0.78	0.43	0.87	0.44	0.77

The estimates based on the period profit functions of each bargaining model are reported in Table 3.19. The intercept estimates  $\gamma_0$  are small and close to 0 except for Qualcomm. The estimates show that increasing one unit of quality (0.25) is more costly for Apple than for Samsung and HTC, and using a higher proportion of Qualcomm chipsets reduces the innovation sunk costs. The estimates are consistent with the possibility that Qualcomm is more likely to work with a customer that primarily uses Qualcomm chipsets to reduce the handset maker's development costs.

To put the parameters in perspective, I simulate the dynamic model at the estimated param-

<sup>40</sup>Gentzkow and Shapiro (2014) provides a (standardized) sensitivity measure to formally quantify this type of discussion.

eter value 120 times, starting from January 2009. For the simplicity of presentation, I only discuss the results using Bargaining Model 1 (handset makers' product qualities decrease at the disagreement point); the results from the other two models are very similar. The evolution of qualities, averaged across all simulation paths, is displayed alongside the observed data in Fig. 3.5.1. While the simulation does not capture the “lumpiness” of the innovation process in data,<sup>41</sup> it does approximate the trend reasonably well. The simulated average total investment expenditures of Apple, Samsung, HTC and Qualcomm from 2009 to 2012 are 29.71, 7.45, 3.60 and 0.69 billion dollars. To examine whether these figures are sensible, I sum up the operating expenses<sup>42</sup> in Apple and HTC's financial reports<sup>4344</sup>, discounted by an annual rate of  $0.99^{12} = 0.89$ . Apple and HTC's total accounting expenditures were \$41.68 and \$6.83 billion dollars during the period. As discussed earlier, the US market may account for as much as 40% of Apple revenues, and the accounting figures match the simulation in scale.

## 3.6 Counterfactual Simulation

I investigate the effects of a Qualcomm-HTC merger. HTC is a natural choice for this counterfactual because of its high dependence on Qualcomm chipsets. Moreover, Apple, the unconstrained handset maker, and Samsung, which can flexibly adjust the proportion of its handsets using Qualcomm, are a pair of downstream rival firms that would realistically react to a vertical merger. Samsung may decrease the use of Qualcomm chipsets because of the

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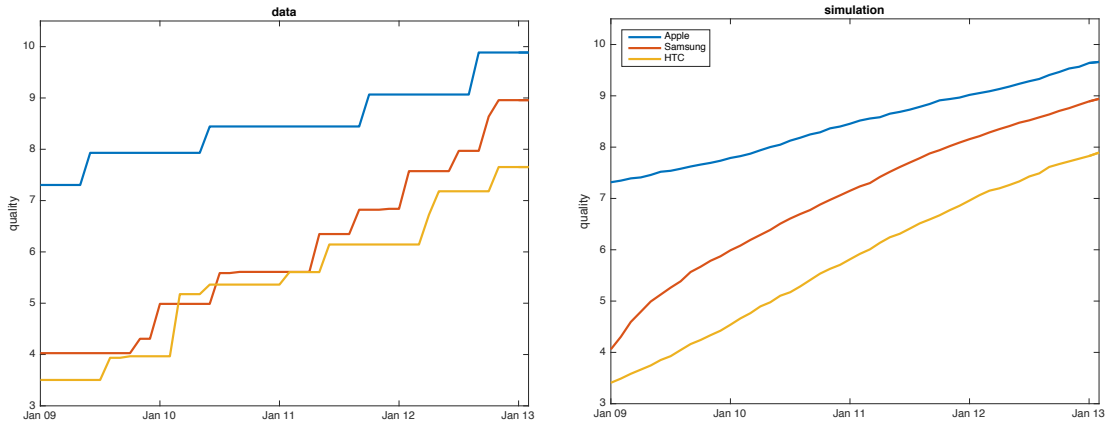
<sup>41</sup>Averaging across a large number of simulation paths would erase all lumpiness.

<sup>42</sup>This item in the financial report includes R&D, selling, general and administrative costs but does not include the manufacturing costs of the goods sold.

<sup>43</sup>Samsung and Qualcomm have major operations outside the application chipset and smartphone industry, and the expenses in their accounting reports are less relevant.

<sup>44</sup>HTC's fiscal year coincides with the calendar year, and Apple's ends in September.

Figure 3.5.1: Quality Evolution, Adjusted for Handset Maker Fixed Effects



raising rivals' cost incentive, but if the integrated firm innovates faster, it may also increase its use of Qualcomm chipsets to reduce development costs and increase innovation. In the first part of the section, I simulate the effects of vertical integration and decompose the effects into the investment coordination effects and price effects. In the second part, I analyze how the parameter estimates drive the results. Specifically, I focus on the parameters that govern the upstream product (chipset) availability from non-Qualcomm producers, the substitution of the downstream products and the price sensitivity of the consumers. I simulate the baseline and every counterfactual scenario 1200 times for the period of January 2010 to December 2011, starting from the state of January 2010 in data.

### 3.6.1 Vertical Integration

Vertical integration has two effects. First, the integrated firms invest to maximize the joint value function, internalizing the marginal effect of HTC innovation on Qualcomm and vice versa. Secondly, the integrated firm reduces double marginalization within the merged par-

ties but may raise the rival's (Samsung) costs. With the first effect, the new dynamic programming problem for Qualcomm and HTC becomes

$$\begin{aligned} & \max_{a^Q} \{ -C^Q(a^Q, \varepsilon^Q) + \beta E(V_{t+1}^{VI}(s_{t+1}) | a^Q, s_t) \} \\ & \max_{a^{HTC}} \{ -C^{HTC}(a^{HTC}, \varepsilon^{HTC}) + \beta E(V_{t+1}^{VI}(s_{t+1}) | a^Q, a_{N(HTC)}, s_t) \}, \end{aligned} \quad (3.6.1)$$

and the Bellman equation for the joint firm is

$$V_t^{VI}(s_t) = \tilde{\pi}^{VI} + E(-C^Q(a^{Q*}, \varepsilon^Q) - C^{HTC}(a^{HTC*}, \varepsilon^{HTC}) + \beta V_{t+1}^{VI}(s_{t+1}) | s_t), \quad (3.6.2)$$

where the expectation is taken over  $(\varepsilon^Q, \varepsilon^{HTC})$ , the corresponding strategies of Qualcomm and HTC, and the action probabilities of their rivals.  $\tilde{\pi}^{VI}$  is the sum of  $\tilde{\pi}^Q$  and  $\tilde{\pi}^{HTC}$ , the joint equilibrium profit under vertical integration. The first order conditions that define the new equilibrium prices in the static pricing game are outlined in the appendix of the paper version of the chapter.

I conduct three sets of simulations: baseline, investment coordination only, and full vertical integration with both investment coordination and price effects. The purpose of the second simulation is to parse out the investment coordination effect. Specifically, I simulate the outcomes where firms still price their products as if they were still separate, but Qualcomm and HTC pool their profits when making dynamic investment decisions: i.e. I replace  $\tilde{\pi}^{VI} = \tilde{\pi}^Q + \tilde{\pi}^{HTC}$  with  $\pi^Q + \pi^{HTC}$ . The difference between this simulation and the baseline simulation shows the net investment coordination effects, while the difference between this simulation and the full vertical integration simulation shows the additional price effects.

Table 3.20 reports the simulation based on static profits given by Bargaining Model 1. The first column (baseline) reports the simulation results at the estimated parameters, and the

second column reports the results where Qualcomm and HTC coordinate investment but not pricing. The fourth column reports the results when Qualcomm and HTC coordinate both investment and pricing (full VI). The third column reports the percent change from the baseline to investment coordination-only, which shows the investment coordination effect of VI, and the last column reports the percentage change from investment coordination-only to the full VI case, reflecting the price effects of VI.

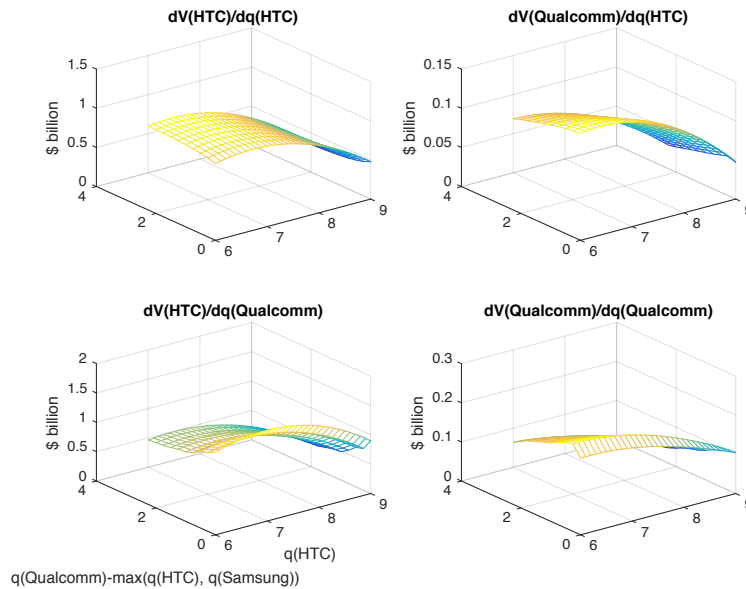
Table 3.20: Counterfactual Results, Jan 2010 to Dec 2011, Model 1

		Baseline	VI, investment coordination only	Percent Change	VI	Percent Change
Innovation Rate: $(q_T - q_1) / T$	Apple	0.06	0.07	20.69%	0.07	-0.24%
	Samsung	0.10	0.12	26.32%	0.12	1.07%
	HTC	0.10	0.12	22.59%	0.12	0.45%
	Qualcomm	0.09	0.13	38.21%	0.13	0.72%
Producer Surplus (\$ billion)	Apple	5.84	6.57	12.36%	6.55	-0.27%
	Samsung	2.86	3.43	19.86%	3.46	0.91%
	HTC+Qualcomm	0.78	1.03	31.17%	1.04	1.33%
Consumer Surplus (\$ billion)		8.91	10.59	18.89%	10.69	0.95%
CS+PS (\$ billion)		18.4	21.62	17.49%	21.75	0.59%
% Using Qualcomm	Samsung	39.52%	51.71%	30.83%	51.88%	0.32%
Investment (\$ billion)	Apple	17.66	25.34	43.45%	25.22	-0.45%
	Samsung	3.47	3.09	-10.82%	3.09	-0.09%
	HTC	1.80	2.98	65.88%	3.02	1.45%
	Qualcomm	0.37	0.91	144.08%	0.93	1.73%
Retail Price (\$)	Apple	102.76	116.68	13.54%	116.47	-0.18%
	Samsung	121.18	143.95	18.79%	145.42	1.02%
	HTC	115.36	135.38	17.36%	120.64	-10.88%
Chipset Price (\$)	Samsung	39.03	39.08	0.12%	39.71	1.61%
	HTC	38.63	38.72	0.24%	-	-

The first main observation is that vertical integration substantially increases the innovation

of all firms, in particular that of Qualcomm. Qualcomm and HTC innovate faster, because they each internalize the marginal value of one's innovation on the other. To visualize the magnitude of the effect, I examine the first order difference,  $V(q + \delta) - V(q)$ , where  $\delta = 0.25$ . To simplify notation, I denote the first order difference as  $\frac{\partial V}{\partial q}$ . I plot the baseline  $\frac{\partial V^Q}{\partial q^Q}$ ,  $\frac{\partial V^Q}{\partial q^{HTC}}$ ,  $\frac{\partial V^{HTC}}{\partial q^Q}$ , and  $\frac{\partial V^{HTC}}{\partial q^{HTC}}$  in Fig. 3.6.1, where Apple and Samsung quality levels are fixed at 4.5 and 6.5. Per unit increase of Qualcomm or HTC quality, HTC value function would increase in the range of 0.5 to 1.5 billion dollars. Per unit increase of HTC quality increases Qualcomm value function by about 0.05 to 0.15 billion dollars, and the effect of Qualcomm's own quality change is slightly larger.

Figure 3.6.1: Marginal Effects of Qualities on HTC and Qualcomm Value Functions



Samsung also innovates faster and is less constrained by Qualcomm. The average number of months that  $q^{\text{Samsung}} = q^Q$  is reduced from 3.23 months to 1.25 months. Furthermore, an examination of the second order difference shows that in equilibrium, being constrained by

Qualcomm is less harmful to Samsung. Denote this second order difference as  $\frac{\partial^2 V^{Samsung}}{\partial q^Q \partial q^{Samsung}}$ , similar to the notation above. I normalize Samsung's marginal value of innovation to 0 when Samsung is constrained. If it is profitable for Samsung to innovate, but Samsung is unable to carry it out because of the Qualcomm constraint, then Samsung's marginal value of innovation should increase sharply (from 0) when Samsung becomes unconstrained. In this case,  $\frac{\partial^2 V^{Samsung}}{\partial q^Q \partial q^{Samsung}}$  should be positive, and a larger value implies greater harm from the constraint, because Samsung likely has to delay more profitable innovations. In Fig. 3.6.2, I plot the second order difference using Samsung's baseline and VI value functions. Apple and HTC qualities are fixed at 5.5 and 6, so Samsung is likely constrained if  $q^Q - q^{Samsung}$  is close to 0.<sup>45</sup> At the baseline, the second order derivative is indeed positive and large, suggesting that Samsung is likely to miss profitable innovation opportunities when constrained. As Qualcomm innovates faster in the case of VI, the magnitude of the second order difference decreases.

Apple's faster innovation results from the long-run equilibrium changes in the competition. I plot the increase of Apple's marginal value of innovation from the baseline to the case of VI,  $\left(\frac{dV^A}{dq^A}\right)_{VI} - \left(\frac{dV^A}{dq^A}\right)_{baseline}$  in Fig. 3.6.3. The quality of Qualcomm is fixed at 0.75 above Samsung, and the quality of HTC at 6. While  $\left(\frac{dV^A}{dq^A}\right)_{VI} - \left(\frac{dV^A}{dq^A}\right)_{baseline}$  does not strongly respond to different levels of Samsung quality, it is large, increasing and concave in Apple quality.

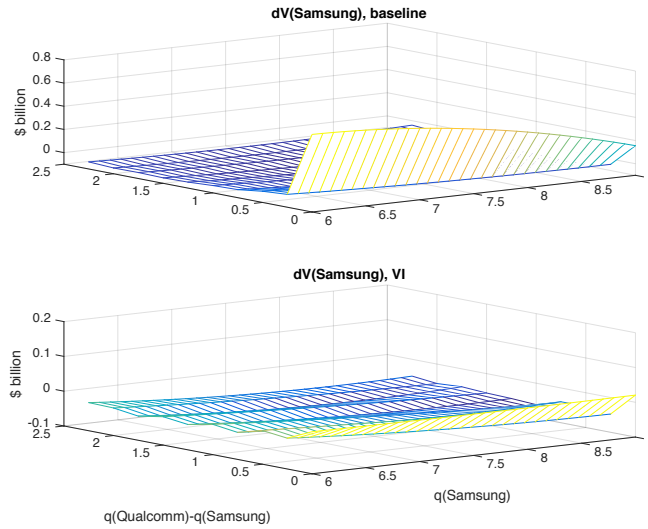
The second observation is that investment coordination effects account for almost all the changes due to VI. The price effects are only about 1% of the investment coordination effects. While the magnitude is small, the price effects decrease Apple innovation while

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<sup>45</sup>Note that Samsung can be constrained even if  $q^Q - q^{Samsung} > 0$ : Samsung can choose the size of its quality increase, and may want to increase its quality greater than  $q^Q - q^{Samsung}$ .

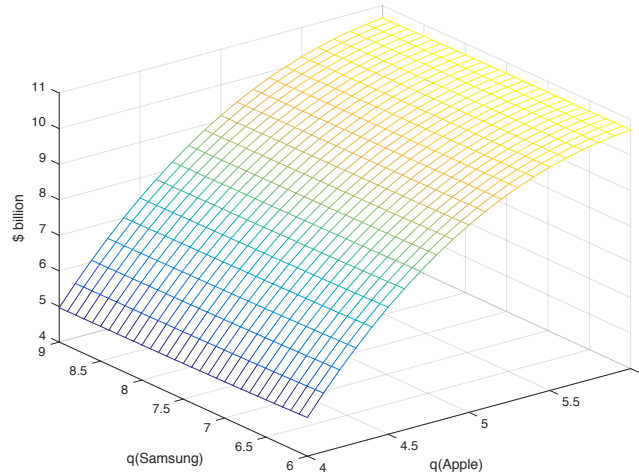


Figure 3.6.2: Marginal Effect of Qualcomm Quality on the Marginal Value of Samsung Innovation



increasing other firms' innovation. The decrease of Apple innovation is intuitive, because on average, chipset prices increase with smartphone qualities, and higher quality HTC products see a larger decrease in marginal costs with the elimination of double marginalization, which may also reduce retail prices of higher quality products more than of lower quality products. This downward pricing pressure thus reduces Apple's expected returns from innovation. On the other hand, Samsung is able to adjust the proportion of handsets using Qualcomm, and a higher proportion reduces innovation costs. Recall that Samsung faces the tradeoff between increasing its usage of Qualcomm chipsets, which leads to lower innovation costs, faster innovation and higher profits in the long run, and decreasing the usage of Qualcomm chipsets, which may lead to slower innovation but higher profits in the short run. The simulation shows that Samsung chooses to respond to the price effects by increasing the Qualcomm chipset usage to innovate faster. Both the investment coordination effects and

Figure 3.6.3: Change of Apple Marginal Value Function due to the Qualcomm-HTC Merger



the price effects reduce the overall Samsung innovation costs.

Lastly, vertical integration significantly increases consumer and producer surplus. While much of the increase comes from the investment coordination effects, the price effects also increase both surpluses. I report sales-weighted average retail prices and chipset prices (averaged over sales of handsets using a Qualcomm chipset). The elimination of double marginalization decreases HTC retail prices, while the effect of raising rivals' cost and Samsung' faster innovation (due to the price effects) increase Samsung's prices. In particular, while higher quality products may be priced higher, HTC retail prices decrease by almost 11% even when price effects increase HTC innovation.

Turning to the predictions using Bargaining Model 2 and 3 in Table 3.21 and 3.22, I find that Model 2 (replacing the Qualcomm chipset with an alternative at cost  $\bar{\psi}$  at the point of disagreement) predicts changes similar in magnitudes. However, the more "traditional" Bargaining Model 3, where handset makers drop products at the point of disagreement,

predicts much larger price effects of vertical integration, although the qualitative effects are consistent with model 1 and 2. Nonetheless, this exercise suggests that researchers should conduct robustness checks or look for external validation when defining the disagreement point: different definitions of the disagreement point may generate different predictions for a counterfactual exercise, even when the models are estimated on the same data set.

Table 3.21: Baseline and Counterfactual Results based on Bargaining Model 2

		Baseline	VI, investment coordination only	Percent Change	VI	Percent Change
Innovation Rate: $(q_T - q_1) / T$	Apple	0.06	0.07	19.63%	0.07	-0.28%
	Samsung	0.09	0.12	29.97%	0.12	0.96%
	HTC	0.10	0.12	22.97%	0.12	0.27%
	Qualcomm	0.09	0.13	41.04%	0.13	0.55%
Producer Surplus (\$ billion)	Apple	5.84	6.52	11.50%	6.50	-0.22%
	Samsung	2.69	3.40	26.21%	3.41	0.38%
	HTC+Qualcomm	0.79	1.05	32.59%	1.05	0.85%
Consumer Surplus (\$ billion)		8.74	10.53	20.49%	10.61	0.75%
CS+PS (\$ billion)		18.06	21.49	18.96%	21.58	0.40%
% Using Qualcomm	Samsung	37.82%	52.85%	39.73%	51.72%	-2.13%
Investment (\$ billion)	Apple	17.57	24.86	41.50%	24.73	-0.50%
	Samsung	3.43	2.94	-14.39%	2.95	0.49%
	HTC	1.77	2.87	62.48%	2.91	1.29%
	Qualcomm	0.37	0.94	156.00%	0.96	1.57%
Retail Price (\$)	Apple	102.76	115.88	12.76%	115.68	-0.17%
	Samsung	113.86	143.15	25.72%	143.19	0.03%
	HTC	117.70	137.48	16.81%	122.17	-11.14%
Chipset Price (\$)	Samsung	39.09	39.14	0.11%	39.69	1.41%
	HTC	39.09	39.11	0.06%	-	-

Table 3.22: Baseline and Counterfactual Results based on Model 3

		Baseline	VI, investment coordination only	Percent Change	VI	Percent Change
Innovation Rate: $(q_T - q_1) / T$	Apple	0.06	0.07	20.03%	0.07	-0.51%
	Samsung	0.09	0.12	32.85%	0.13	3.39%
	HTC	0.10	0.13	20.07%	0.13	2.58%
	Qualcomm	0.10	0.13	36.35%	0.13	2.04%
Producer Surplus (\$ billion)	Apple	5.88	6.54	11.30%	6.48	-0.95%
	Samsung	2.47	3.40	37.60%	3.54	4.08%
	HTC+Qualcomm	0.82	1.05	28.16%	1.14	8.22%
Consumer Surplus (\$ billion)		8.58	10.57	23.22%	10.80	2.14%
CS+PS (\$ billion)		18.4	21.62	17.49%	21.75	0.59%
% Using Qualcomm	Samsung	45.46%	51.40%	13.06%	55.39%	7.77%
Investment (\$ billion)	Apple	17.45	24.84	42.39%	24.59	-1.01%
	Samsung	3.76	2.71	-27.96%	2.93	8.14%
	HTC	1.82	2.82	55.14%	3.20	13.38%
	Qualcomm	0.41	0.95	133.08%	1.04	8.76%
Retail Price (\$)	Apple	103.31	116.36	12.63%	115.63	-0.63%
	Samsung	106.89	140.80	31.72%	152.57	8.36%
	HTC	119.19	136.96	14.91%	126.24	-7.83%
Chipset Price (\$)	Samsung	37.46	37.79	0.87%	44.60	18.03%
	HTC	37.49	37.69	0.53%	-	-

### 3.6.2 Comparative Statics

The previous section has shown that the effects of VI on innovation are large, positive and driven by the investment coordination effects. In this section, I explore how the parameters that govern the price effects may change the results using Bargaining Model 1. In particular, I examine the effects of three parameters: the decrease in quality at the point of disagreement  $\bar{q}$ , consumer taste dispersion parameter  $\sigma$  and the price coefficient  $\alpha$ . Increasing  $\bar{q}$  allows

Qualcomm to negotiate a higher chipset prices and reduces handset maker profits. Higher  $\sigma$  increases the proportion of consumers with a high willingness to pay for quality, thus likely increasing demand, but it also increases substitution between similar quality products and may increase the raising rivals' cost incentives. Higher price coefficients reduce the consumption of smartphones and likely would reduce the smartphone markups and chipset prices. In the counterfactuals in Table 3.23, I increase one of the parameters by 10% in each of the three simulations.

At higher  $\bar{q}$ , the price effects either reduce innovation or have a smaller positive impact on the innovation of all handset makers. The price effects reduce the innovation of Apple from -0.24% to -0.36%, which suggests that Qualcomm's higher bargaining ability has a larger negative impact on the marginal value of rival innovation. HTC innovation attributable to the price effects is proportionally smaller, suggesting that with higher upstream bargaining ability, the marginal value of downstream innovation for the integrated firm is smaller.

The innovation rates are higher when  $\sigma$  is increased by 10% at the baseline, but the gains due to the vertical integration are smaller. The price effects have a larger negative impact on Apple innovation, and larger positive impact on the innovation of other firms, consistent with the intuition that price effects are stronger with a higher degree of downstream competition.

In both scenarios, differently from Table 3.20, Samsung uses a smaller proportion of Qualcomm chipsets even when Samsung innovation increases due to the price effects, which suggests that Samsung innovates faster not because of the savings of the innovation costs, but because of reduced payments to the integrated firm.

Increasing consumers' price sensitivity reduces the innovation rates at the baseline level, but the increase in innovation due to the investment coordination effects is similar to Table 3.20

in magnitude. The price effects become proportionally smaller. As consumers become more price sensitive, Qualcomm and handset makers set prices closer to the marginal cost, limiting the effects of eliminating double marginalization and raising rivals' cost.

The analysis above helps to identify situations where the price effects may affect innovation more strongly: poor upstream substitutes, high degree of downstream market competition and low consumer price sensitivity. However, even when the price effects are larger, they are unlikely to dominate the coordination effects, and do not necessarily decrease rival downstream firms' innovation.

Table 3.23: Comparative Statics. Innovation Rate:  $(q_T - q_1) / T$

	Baseline	VI, investment coordination only	Percent Change	VI	Percent Change	
$\bar{q}$	Apple	0.06	0.07	23.21%	0.07	-0.36%
	Samsung	0.10	0.12	26.68%	0.12	0.72%
	HTC	0.10	0.12	23.25%	0.12	0.07%
	Qualcomm	0.10	0.13	36.30%	0.13	1.07%
	% of Samsung using Qualcomm	36.15%	49.56%	37.12%	48.51%	-2.13%
$\sigma$	Apple	0.08	0.09	11.98%	0.09	-1.18%
	Samsung	0.10	0.12	19.51%	0.12	1.43%
	HTC	0.11	0.13	17.86%	0.13	1.17%
	Qualcomm	0.10	0.13	27.52%	0.13	1.06%
	% of Samsung using Qualcomm	36.76%	55.57%	51.18%	55.25%	-0.57%
$\alpha$	Apple	0.05	0.05	3.70%	0.05	0.00%
	Samsung	0.07	0.11	50.12%	0.11	0.62%
	HTC	0.08	0.11	39.91%	0.11	0.47%
	Qualcomm	0.07	0.12	72.46%	0.12	0.58%
	% of Samsung using Qualcomm	37.15%	44.88%	20.81%	44.97%	0.20%

## 3.7 Discussion and Conclusion

This paper estimates a new model that combines bilateral bargaining with dynamic innovation to analyze the impact of vertical integration on innovation, pricing and welfare in the chipset and smartphone industry. Using the estimated model, I simulate the counterfactual experiment of a vertical merger, and find that vertical integration increases innovation primarily through the channel of investment coordination. The results suggest that the dynamic effect of vertical integration may be large, providing support for giving more weight to this factor in a vertical merger review.

Several simplifying assumptions underlie the model. Most importantly, I abstract away from vertical integration's effects on the cost primitives. Given the possibility of cost reduction in the case of a successful merger, the results here are a lower bound on the positive impact of vertical integration. Secondly, I model the pricing game without considering the strategic roles of carriers. In some sense the approach follows the CPU literature such as Goettler and Gordon (2011) and Nosko (2014) that abstracts from the role of downstream computer assemblers. In practice, I find that equilibrium prices may not exist for a three stage pricing game, where Qualcomm and handset makers first negotiate chipset prices, then handset makers sell to the carriers, and carriers sell to the consumers. On the other hand, Fan and Yang (2016) finds that the response of smartphone variety to a market structure change such as a horizontal merger is not sensitive to various assumptions of carrier conduct. Thirdly, I do not consider serially correlated unobserved cost variables, which may be a concern given that the data frequency is monthly. Omitting these variables would bias the estimates of the innovation costs. However, including such a cost component does not change any of the economic argument why innovation increases with a vertical merger. Fourthly, I assume that

Qualcomm and handset makers use linear contracts. I will explore alternative assumptions in the future. The paper also does not discuss patent issues in this industry. Qualcomm collects percentage fees (1 to 5%) of the wholesale smartphone prices because of its many communication technology patents. These fees form a large part of the overall Qualcomm profits, but are not directly related to Qualcomm's efforts in application chipset innovation. Lastly, vertical integration may also affect innovation through an information channel: during product development, downstream firms may need to interact with upstream firms, and vertical integration could change the integrated firm's incentive to protect or exploit the proprietary information of rival downstream firms (Allain, Chambolle and Rey (2011)). The integrated firm may imitate its downstream rivals and equilibrium innovation rates may be reduced.



## CHAPTER IV

# Unobserved Heterogeneity in Matching Games with an Application to Venture Capital

with Jeremy T. Fox and David H. Hsu

### 4.1 Introduction

Matching games model the sorting of agents to each other. Men sort to women in marriage based on characteristics such as income, schooling, personality and physical appearance, with more desirable men typically matching to more desirable women. Upstream firms sort to downstream firms based on the product qualities and capacities of each of the firms. This paper is partially motivated by such applications in industrial organization and entrepreneurial finance, where downstream firms pay upstream firms money, and thus it is reasonable to work with transferable utility matching games Koopmans and Beckmann (1957); Gale (1960); Shapley and Shubik (1971); Becker (1973). In particular, we explore an empirical application in corporate finance and management, where the upstream firms are

venture capitalists and the downstream firms are entrepreneurial biotech and medical firms. There has been recent interest in the structural estimation of (both transferable utility and non-transferable utility) matching games.<sup>46</sup> The papers we cite are unified in estimating some aspect of the preferences of agents in a matching game from data on who matches with whom as well as the observed characteristics of agents or of matches. The sorting patterns in the data combined with assumptions about equilibrium inform the researcher about the structural primitives in the market, namely some function that transforms an agent’s own characteristics and its potential partner’s characteristics into some notion of utility or output. These papers are related to, but are not special cases of, papers estimating discrete, non-cooperative (Nash) games, like the entry literature in industrial organization and the discrete outcomes peer effects literature.<sup>47</sup> Matching games typically use the cooperative solution concept of pairwise stability.<sup>48</sup>

The empirical literature cited previously structurally estimates how various structural or equilibrium objects, such as payoffs or preferences, are functions of the characteristics of agents *observed in the data*. For example, Choo and Siow (2006) study the marriage market in the United States and estimate how the equilibrium payoffs of men for women vary by the ages of the man and the woman. Sørensen (2007) studies the matching of venture

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<sup>46</sup>See, among others: Dagsvik (2000); Boyd et al. (2003); Choo and Siow (2006); Sørensen (2007); Fox (2016); Gordon and Knight (2009); Chen and Song (2013); Ho (2009); Park (2013); Yang et al. (2009); Logan et al. (2008); Levine (2009); Baccara et al. (2012); Siow (2015); Salanié and Galichon (2012); Chiappori et al. (2015); Weese (2015); Christakis et al. (2010); Echenique et al. (2013); Menzel (2015); Uetake and Watanabe (2012); Agarwal (2015); Agarwal and Diamond (2014); Akkus et al. (2015).

<sup>47</sup>See, among others: Berry (1992); Bresnahan and Reiss (1991); Mazzeo (2002); Tamer (2003); Bajari et al. (2010); Seim (2006a); Brock and Durlauf (2007); de Paula and Tang (2012).

<sup>48</sup>Transferable utility matching games (particularly those with “contracts” or “trades” that specify endogenous product attributes) are equivalent to models of hedonic equilibrium (Brown and Rosen, 1982; Ekeland et al., 2003; Heckman et al., 2010; Chiappori et al., 2010). Unlike the empirical literature on hedonic equilibrium, the estimation approaches in most matching papers do not rely on data on equilibrium prices or transfers. Compared to the current work, the hedonic papers do not allow for unobserved characteristics.

capitalists to entrepreneurs as a function of observed venture capitalist experience. Fox (2016) studies matching between automotive assemblers (downstream firms) and car parts suppliers (upstream firms) and asks how observed specialization measures in the portfolios of car parts sourced or supplied contribute to agent profit functions.

The above papers all use data on a relatively limited set of agent characteristics. In Choo and Siow (2006), personality and physical attractiveness are not measured, even though those characteristics are likely important in determining the equilibrium pattern of marriages. Similarly, in Fox (2016) each firm's product quality is not directly measured and is only indirectly inferred. In Sørensen (2007), the unobserved ability of each venture capitalist and the business prospects of each entrepreneurial firm are not measured. If matching based on observed characteristics is found to be important, it is a reasonable conjecture that matching based on *unobserved characteristics* is also important. Our empirical work on biotech and medical venture capital investments complements the earlier work by Sørensen (2007) on venture capital; we estimate distributions of functions of match-specific unobservables. Akerberg and Botticini (2002) provide empirical evidence that farmers and landlords sort on unobservables such as risk aversion and monitoring ability, without formally estimating a matching game or the distribution of these unobservables.

Our discussion of the empirical applications cited above suggests that unobserved characteristics are potentially important. As the consistency of estimation procedures for matching games depends on assumptions on the unobservables, empirical conclusions might be more robust if the estimated matching games allow richly specified distributions of unobserved agent heterogeneity. This paper investigates what data on the sorting patterns between agents can tell us about the distributions of unobserved agent characteristics relevant for sorting. In particular, we study the nonparametric identification of distributions of unob-

served agent heterogeneity in two-sided matching games. With the distribution of unobservables, the researcher can explain sorting and construct counterfactual predictions about market assignments. This paper allows for this empirically relevant heterogeneity in partner preferences using data on only observed matches (who matches with whom), not data from, say, an online dating site on rejected profiles (Hitsch, Hortacısu and Ariely, 2010) or on equilibrium transfers, such as wages in a labor market (Eeckhout and Kircher, 2011). Transfers are often confidential data in firm contracts (Fox (2016)) and are rarely observed in marriage data Becker (1973).

In the following specific sense, this paper on identification is ahead of the empirical matching literature because, when this paper was first written, no empirical papers had parametrically estimated distributions of unobserved agent characteristics, or of match characteristics without assuming independence across matches, in matching games. Thus, this paper seeks to introduce a new topic for economic investigation, rather than to simply loosen parametric restrictions in an existing empirical literature. This paper contributes to the literature on the nonparametric (allowing infinite dimensional objects) identification of transferable utility matching games (Fox, 2010; Graham, 2011). Our paper is distinguished because of its focus on identifying distributions of unobservables, rather than mostly deterministic functions of observables. Our focus in identification on using data on many markets with finite numbers of agents in each (transferable utility) market follows Fox (2010).<sup>49</sup>

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<sup>49</sup>In addition to our study of identifying distributions of unobservables, there are many modeling differences between our paper and the literature on transferable utility matching games following the approach of Choo and Siow (2006), including Salanié and Galichon (2012), Chiappori, Salanié and Weiss (2015), Graham (2011) and Fox (2016). We use data on many markets with finite numbers of players and different realizations of observables and unobservables in each market; the Choo and Siow (2006) approach has been applied to one large market with an infinity of agents. We require at least one continuous, observable characteristic per match or per agent; the Choo and Siow (2006) literature allows only a finite number of observable characteristic values. The production functions corresponding to these finite unobservables are usually recoverable without further functional form assumptions; we require a particular match or agent characteristic

We first consider a baseline model, which is stripped down to focus on the key problem of identifying distributions of heterogeneity from sorting data. In our baseline transferable utility matching game, the primitive that governs sorting is the matrix that collects the production values for each potential match in a matching market. The production level of each match is additively separable in observable and unobservable terms. The observable term is a match-specific characteristic. The unknown primitive is therefore the distribution (representing randomness across markets) of the matrix that collects the unobservable terms in the production of each match in a market. We call this distribution the distribution of match-specific unobservables. Match-specific unobservables nest many special cases, such as agent-specific unobservables.

We first show that that the distribution of match-specific unobservables is not identified in a one-to-one matching game with data on who matches with whom but without data on unmatched or single agents. We provide two main theoretical results and many extensions. Our first main theoretical result states that the distribution of a change of variables of the unobservables, the distribution of what we call **unobserved complementarities**, is identified. We precisely define unobserved complementarities below. Our identification proof works by tracing the joint (across possible matches in a market) cumulative distribution function of these unobserved complementarities using the match-specific observables. We also show that knowledge of the distribution of unobserved complementarities is sufficient for computing assignment probabilities. Our second main theoretical result says that the distribution of the primitively specified, match-specific unobservables is actually identified when unmatched agents are observed in the data.

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to enter production additively separably. Unobservables in the Choo and Siow (2006) literature are typically i.i.d. shocks for the finite observable types rather than unobserved agent characteristics or unobserved preferences on observed, ordered characteristics, such as random coefficients. This is one interpretation of the “separability” assumption of Chiappori, Salanié and Weiss.

Our main theoretical results can be intuitively understood by reference to a classic result in Becker (1973). He studies sorting in two-sided, transferable utility matching games where agents have scalar characteristics (types). He shows that high-type agents match to high-type agents if the types of agents are complements in the production of matches. Many production functions for match output exhibit complementarities. Say in Becker (1973)'s model male and female types are  $x_m$  and  $x_w$ , respectively. A production function with horizontal preferences, such as  $-(x_m - x_w)^2$ , and one with vertical preferences, such as  $2x_mx_w$ , can both have the same cross-partial derivative, here 2. Becker (1973)'s result that complementarities alone drive sorting means that data on sorting cannot tell these two production functions apart. In our more general class of matching games, we cannot identify the distribution of match-specific unobservables. However, we can identify the distribution of our notion of unobserved complementarities. These results are analogous to Becker (1973)'s results, for a more general class of matching games.

Our second main theoretical result uses data on unmatched agents. In a matching game, agents can unilaterally decide to be single or not. If all other agents are single and hence available to match, the fact that one particular agent is single can only be explained by the production of all matches involving that agent being less than the production from being single. This type of direct comparison between the production of being single and the production of being matched is analogous to the way identification proceeds in discrete Nash games, where the payoff of a player's observed (in the data) strategy must be higher than strategies not chosen, given the strategies of rivals. Thus, the availability of data on unmatched agents introduces an element of individual rationality that maps directly into the data and is therefore useful for identification of the primitive distribution of match-specific characteristics.

Many empirical researchers might be tempted to specify a parametric distribution of match-specific unobservables. Our theoretical results together suggest that estimating a matching model with a parametric distribution of match-specific unobservables will not necessarily lead to credible estimates without using data on unmatched agents, as a more general non-parametrically specified distribution is not identified. Also, we present an example of a multivariate normal distribution of match-specific characteristics whose parameters are not parametrically identified. One could instead impose a parametric distribution for unobserved complementarities, as we do in our empirical work on biotech and medical venture capital.

We examine several extensions to the baseline model that add more empirical realism. Our baseline model imposes additive separability between unobservables and observables in the production of a match. We examine an extension where additional observed characteristics enter match production and these characteristics may, for example, have random coefficients on them, reflecting the *random preferences* of agents for partner characteristics. For example, observationally identical men are often observed to marry observationally distinct women. One important hypothesis is that these men have heterogeneous preferences for the observable characteristics of women. In a model with random preferences, we identify the distribution of unobserved complementarities conditional on the characteristics of agents and matches other than the match-specific characteristics used in the baseline model. Identifying a distribution of unobservables conditional on observables follows identification work using special regressors in the multinomial choice literature (Lewbel, 2000; Matzkin, 2007; Berry and Haile, 2009).

In another extension, we identify fixed-across-markets but heterogeneous-within-a-market coefficients on the match-specific characteristics used in the baseline model. This relaxes the assumption that the match-specific characteristics enter the production of each match

in the same manner. Another extension considers models where key observables vary at the agent and not the match level and enter match production multiplicatively. We can identify the distribution of unobserved complementarities if match unobservables are equal to the product of agent unobservables.

Our results on one-to-one, two-sided matching games extend naturally to many-to-many matching (Kelso Jr and Crawford (1982); Roth and Sotomayor (1990); Sotomayor (1999)). Our application to venture capital uses the many-to-one special case. We discuss another extension to the model of matching with trades in Hatfield et al. (2013), who significantly generalize Crawford and Knoer (1981). In matching with the trades, the same agent can make so-called trades both as a buyer and a seller and can have complicated preferences over the set of trades. An individual trade generalizes a match in that a trade can list other specifications, such as the number of startup board seats given to a venture capitalist. The matching with trades model has many special cases and is the most general model we provide identification results for.

We use our theoretical results to motivate an empirical investigation into matching between biotech and medical entrepreneurs and venture capitalists. Venture capital is a key way entrepreneurial innovation is funded. We use detailed data on the observed matches between entrepreneurial startups and venture capitalist firms over a ten year period. We collect information on the geographic locations of both startups and venture capitalists, on the patent stocks of startups, and on the past experience of venture capitalists in various biotech and medical sectors. Despite these observed characteristics being as detailed as any data set on venture capital that could realistically be collected by academic researchers, we find that the distribution of unobserved complementarities suggests that unobserved characteristics play a large role in match production.



## 4.2 Baseline Identification Results

We mainly analyze a two-sided, one-to-one matching game with transferable utility (Koopmans and Beckmann (1957); Gale (1960); Shapley and Shubik (1971); Becker (1973); Roth and Sotomayor (1990, Chapter 8)). This section imposes that all agents must be matched in order to focus purely on the identification coming from agent sorting and not from the individual rationality decision to be single. We also use a simple space of explanatory variables. We change these assumptions in later sections.

### 4.2.1 Baseline Model

We use the terms “agents” and “firms” interchangeably. In a one-to-one matching game, an upstream firm  $u$  matches with a downstream firm  $d$ . In biotech and medical venture capital, upstream firms are venture capitalists and downstream firms are biotech entrepreneurs. Upstream firm  $u$  and downstream firm  $d$  can form a match  $(u, d)$ . The monetary transfer from  $d$  to  $u$  is denoted as  $t_{u,d}$ ; we will not require data on the transfers. The **production** or **total profit** from a match  $\langle u, d \rangle$  is

$$z_{u,d} + e_{u,d}, \tag{4.2.1}$$

where  $z_{u,d}$  is a scalar match-specific characteristic observed in the data and  $e_{u,d}$  is a scalar match-specific characteristic unobserved in the data, but observable to all firms in the matching game. In our empirical work on venture capital, one match-specific characteristic  $z_{u,d}$  is the distance between the headquarters of firms  $u$  and  $d$ .<sup>50</sup> The match-specific, unob-

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<sup>50</sup>Distance  $z_{u,d}$  is always positive and likely enters match production with a negative sign; we can always construct a new regressor  $\tilde{z}_{u,d} = -(z_{u,d} - E[z_{u,d}])$  that enters with a positive sign and has mean zero.

served characteristic  $e_{u,d}$  generalizes special cases such as  $e_{u,d} = e_u \cdot e_d$ , where  $e_u$  and  $e_d$  are unobserved upstream and downstream firm characteristics, respectively. We allow a match-specific coefficient on each  $z_{u,d}$  and, separately, use only agent-specific explanatory variables below.

We can more primitively model production for a match  $\langle u, d \rangle$  as the sum of the profit of  $u$  and the profit of  $d$ , where the possibly negative transfer  $t_{u,d}$  between  $d$  and  $u$  enters additively separately into both individual profits and therefore cancels in their sum.<sup>51</sup>

However, only production levels matter for the matches that form, and we will not attempt to identify upstream firm profits separately from downstream firm profits.

There are  $N$  firms on each side of the market.  $N$  can also represent the set  $\{1, \dots, N\}$ . In this section, there can be no unmatched firms. The matrix

$$\begin{pmatrix} z_{1,1} + e_{1,1} & \cdots & z_{1,N} + e_{1,N} \\ \vdots & \ddots & \vdots \\ z_{N,1} + e_{N,1} & \cdots & z_{N,N} + e_{N,N} \end{pmatrix}$$

describes the production of all matches in a market, where the rows are upstream firms and the columns are downstream firms. Let

$$E = \begin{pmatrix} e_{1,1} & \cdots & e_{1,N} \\ \vdots & \ddots & \vdots \\ e_{N,1} & \cdots & e_{N,N} \end{pmatrix}, Z = \begin{pmatrix} z_{1,1} & \cdots & z_{1,N} \\ \vdots & \ddots & \vdots \\ z_{N,1} & \cdots & z_{N,N} \end{pmatrix}$$

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<sup>51</sup>If the profit of  $u$  at some market outcome is  $\pi_{u,d}^u + t_{u,d}$  and the profit of  $d$  is  $\pi_{u,d}^d - t_{u,d}$ , then the production of the match  $\langle u, d \rangle$  is equal to  $\pi_{u,d}^u + \pi_{u,d}^d = z_{u,d} + e_{u,d}$ . We will not attempt to learn the distributions of the unobservable portions of  $\pi_{u,d}^u$  and  $\pi_{u,d}^d$  separately (Fox, 2010).

be the matrices of unobservables and observables, respectively, in a market.<sup>52</sup>

A feasible one-to-one **assignment**  $A$  is a set of matches  $A = \{\langle u_1, d_1 \rangle, \dots, \langle u_N, d_N \rangle\}$ , where for this section each firm is matched exactly once. There are  $N!$  feasible assignments. An **outcome** is a list of matches and transfers between matched agents:

$$\{\langle u_1, d_1, t_{u_1, d_1} \rangle, \dots, \langle u_N, d_N, t_{u_N, d_N} \rangle\}.$$

An outcome is **pairwise stable** if it is robust to deviations by pairs of two firms, as defined in references such as Roth and Sotomayor (1990, Chapter 8).<sup>53</sup> An assignment  $A$  is called **pairwise stable** if there exists an underlying outcome (including transfers) that is pairwise stable.

The literature cited previously proves that the existence of a pairwise stable assignment is guaranteed and that an assignment  $A$  is pairwise stable if and only if it maximizes the **sum of production**

$$s(A; E, Z) = \sum_{\langle u, d \rangle \in A} (z_{u, d} + e_{u, d}).$$

If  $z_{u, d}$  or  $e_{u, d}$  have continuous support,  $s(A; E, Z)$  has a unique maximizer with probability 1 and therefore the pairwise stable assignment is unique with probability 1. The **sum of the unobserved production** of assignment  $A$  relative to the particular assignment  $A_1 = \{\langle 1, 1 \rangle, \dots, \langle N, N \rangle\}$  is

$$\tilde{s}(A; E) = \sum_{\langle u, d \rangle \in A} e_{u, d} - \sum_{\langle u, d \rangle \in A_1} e_{u, d}. \quad (4.2.2)$$

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<sup>52</sup>Because the scalar  $z_{u, d}$  is an element of the matrix  $Z$ , we do not use upper and lower case letters (or other notation) to distinguish random variables and their realizations. Whether we refer to a random variable or its realization should be clear from context.

<sup>53</sup>We omit standard definitions here that can be easily found in the literature.

A market is defined to be the pair  $(E, Z)$ ; agents in a market can match and agents in different markets cannot. A researcher observes the assignment  $A$  and the match-specific characteristics  $Z$  for many markets. In other words, in each matching market the researcher observes who matches with whom  $A$  and the characteristics  $Z$  of the realized and potential matches. This allows the identification of  $\Pr(A | Z)$ , the probability of assignment  $A$  being the pairwise stable assignment given the market-level match characteristics  $Z$ . Researchers do not observe transfers, which are often part of confidential contracts.

$Z$  is independent of the unobservable matrix  $E$ . We assume that  $Z$  has full and product support, meaning that any  $Z \in \mathbb{R}^{N^2}$  is observed.<sup>54</sup> Each match characteristic  $z_{u,d}$  enters production additively, the sign and coefficient on each  $z_{u,d}$  in production is common across matches (normalized to be 1), each  $z_{(u,d)}$  has large support, and  $Z$  is independent of  $E$ .<sup>55</sup>

Similar large support explanatory variables have been used to prove point identification in the binary and multinomial choice literature (Manski, 1988; Ichimura and Thompson, 1998; Lewbel, 1998, 2000; Matzkin, 2007; Gautier and Kitamura, 2013; Berry and Haile, 2009; Fox and Gandhi, 2016). In this literature, failure to have large support often results in set rather than point identification of the distribution of heterogeneity.<sup>56</sup> We discuss the failure of the

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<sup>54</sup>The example of the match-characteristic distance may not vary independently over all of  $\mathbb{R}^{N^2}$  because distance is computed using the agent-specific characteristics latitude and longitude. In our empirical work, we use a second match-specific characteristic that conceptually can vary independently in  $N^2$  dimensions: the past experience of a venture capitalist with investments in the four-digit sector of a startup. Experience in a sector varies independently in  $N^2$  dimensions if all startups are in different four-digit sectors.

<sup>55</sup>We could in principle address the statistical dependence of  $E$  and  $Z$  with instrumental variables. We do not explore this. We should mention that the  $e_{u,d}$  and  $z_{u,d}$  for the *realized* matches in the pairwise stable assignment  $A$  will likely be statistically dependent because of the conditioning on the dependent variable  $A$ , part of the outcome to the game.

<sup>56</sup>Consider a binary choice model of buying a can of soda (or not) where the large support regressor is the (negative) price of the soda, which varies across the dataset. If we assume that consumers' willingnesses to pay for the can of soda are bounded by \$0 and \$10, we can point identify the distribution of the willingness to pay for soda if observable prices range between \$0 and \$10. If prices range only between \$0 and \$5, we can identify the fraction of consumers with values above \$5 by seeing the fraction who purchase at \$5. We cannot identify the fraction with values above \$6, or any value greater than \$5. If we do not restrict the support

support condition in our matching context below. In this paper, we use large support match characteristics in part to focus on reasons specific to matching games for the failure of point identification.<sup>57</sup>

## 4.2.2 Data Generating Process

The unknown primitive whose identification we first explore is the CDF  $G(E)$ , which reflects how the match unobservables vary across matching markets. We do not restrict the support of  $E$  and we do not assume independence across the  $e_{u,d}$ 's within matching markets. Hence, we allow for many special cases, such as the case  $e_{u,d} = e_u \cdot e_d$  mentioned earlier.

The probability of assignment  $A$  occurring given the match characteristics  $Z$  is

$$\Pr(A | Z; G) = \int_E 1[A \text{ pairwise stable assignment} | Z, E] dG(E), \quad (4.2.3)$$

where  $1[A \text{ pairwise stable assignment} | Z, E]$  is equal to 1 when  $A$  is a pairwise stable assignment for the market  $(E, Z)$ .

The distribution  $G$  is said to be **identified** whenever, for  $G^1 \neq G^2$ ,  $\Pr(A | Z; G^1) \neq \Pr(A | Z; G^2)$  for some pair  $(A, Z)$ .  $G^1$  and  $G^2$  give a different probability for at least one assignment  $A$  given  $Z$ . If  $G$  has continuous and full support so that all probabilities  $\Pr(A | Z; G)$  are nonzero (for every  $(A, Z)$ ,  $s(A; E, Z)$  will be maximized by a range of  $E$ )

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of the willingness to pay, we need prices to vary across all of  $\mathbb{R}$  (including negative prices if consumers may have negative willingnesses to pay) for point identification of the distribution of the willingness to pay for soda.

<sup>57</sup>Our use of large support and the use of large support in most of the literature on binary and multinomial choice does not constitute identification at infinity as used in certain proofs to study Nash games by, for example, Tamer (2003). Identification at infinity in a Nash game uses only extreme values of regressors for all but one player to, in effect, turn a multi-player game into a single-player decision problem. We use large explanatory variable values only to identify the tails of distributions of heterogeneity.

and continuous in the elements of  $Z$ , the existence of one such pair  $(A, Z)$  implies that a set of  $Z$  with positive measure satisfies  $\Pr(A | Z; G^1) \neq \Pr(A | Z; G^2)$ .

All of our positive identification results will be constructive, in that we can trace a distribution such as  $G(E)$  using variation in an object such as  $Z$ . Also, our identification arguments can be used to prove the consistency of a nonparametric mixtures estimator for a distribution  $G$  of heterogeneous unobservables  $E$ , as Fox, il Kim and Yang (2015) show for a particular, computationally simple mixtures estimator.<sup>58</sup> Other mixtures estimators can be used, including maximum simulated likelihood, the EM algorithm, NPMLE, and MCMC.<sup>59</sup> In the empirical work to biotech venture capital, we use the simulated method of moments in a parametric model, because of the large numbers of firms in our matching markets (McFadden, 1989; Pakes and Pollard, 1989).

### 4.2.3 Non-Identification of the Distribution of Match-Specific Characteristics

As maximizing  $s(A; E, Z)$  determines the assignment seen in the data, the ordering of  $s(A; E, Z)$  across assignments  $A$  as a function of  $E$  and  $Z$  is a key input to identification. We can add a constant to the production of all matches involving the same upstream firm and the ordering of the production  $s(A; E, Z)$  of all assignments will remain the same. This non-identification result is unsurprising: the differential production of matches and hence

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<sup>58</sup>The proof of consistency in Fox, il Kim and Yang (2015) for one estimator requires the heterogeneous unobservable (such as  $E$ ) to have compact support, which is not required here for identification. A second estimator in Fox, il Kim and Yang (2015) allows the support of  $E$  to be  $\mathbb{R}^{\dim(E)}$ .

<sup>59</sup>For large markets, these estimators all have computational problems arising from the combinatorics underlying the set of matching game assignments. Fox (2016) uses a maximum score estimator to avoid these computational problems, but does not estimate a distribution of unobservables. Our identification arguments do not address computational issues. Likewise, random variables such as  $E$  are of large dimension and nonparametrically estimating a CDF such as  $G(E)$  will result in a data curse of dimensionality.

assignments governs the identity of the pairwise stable assignment in any market.

We will show another non-identification result. Consider the two realizations of matrices of unobservables

$$E_1 = \begin{pmatrix} e_{1,1} & e_{1,2} & \cdots & e_{1,N} \\ e_{2,1} & e_{2,2} & \cdots & e_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ e_{N,1} & e_{N,2} & \cdots & e_{N,N} \end{pmatrix}, E_2 = \begin{pmatrix} e_{1,1} & e_{1,2} + 1 & \cdots & e_{1,N} \\ e_{2,1} - 1 & e_{2,2} + 1 - 1 & \cdots & e_{2,N} - 1 \\ \vdots & \vdots & \ddots & \vdots \\ e_{N,1} & e_{N,2} + 1 & \cdots & e_{N,N} \end{pmatrix}.$$

It is easy to verify that  $s(A; E_1, Z) = s(A; E_2, Z)$  for all  $A, Z$ , which means that the pairwise stable assignment  $A$  is the same for  $E_1$  and  $E_2$ , for any  $Z$ . Therefore it is not possible to separately identify the relative frequencies of  $E_1$  and  $E_2$  in the data generating process; the support of the random matrix  $E$  is too flexible.

We summarize the two counterexamples in the following non-identification proposition.

**Proposition 2.** *The distribution  $G(E)$  of market-level unobserved match characteristics is **not** identified in a matching game where all agents must be matched.*

Consider a simple case focusing on two upstream firms and two downstream firms. If we see the matches  $\langle u_1, d_1 \rangle$  and  $\langle u_2, d_2 \rangle$  in the data, we cannot know whether this assignment forms because  $\langle u_1, d_1 \rangle$  has high production,  $\langle u_2, d_2 \rangle$  has high production,  $\langle u_1, d_2 \rangle$  has low production, or  $\langle u_2, d_1 \rangle$  has low production. The non-identification result implies that parametric estimation of  $G(E)$  under these assumptions may not be well founded, in that the generalization removing the parametric restrictions is not identified.

#### 4.2.4 Unobserved Assignment Production

The pairwise stable assignment  $A$  maximizes the function  $s(A; E, Z) = \sum_{\langle u,d \rangle \in A} (z_{u,d} + e_{u,d})$ . This looks like a single agent, the social planner, maximizing a utility function. Rough intuition from the multinomial choice literature, cited earlier, suggests that the distribution  $H(\tilde{S})$  of

$$\tilde{S} = (\tilde{s}(A_2; E), \dots, \tilde{s}(A_{N!}; E)) = \left( \sum_{\langle u,d \rangle \in A_2} e_{u,d} - \sum_{\langle u,d \rangle \in A_1} e_{u,d}, \dots, \sum_{\langle u,d \rangle \in A_{N!}} e_{u,d} - \sum_{\langle u,d \rangle \in A_1} e_{u,d} \right)$$

might be identified, where the long vector  $\tilde{S}$  collects the unobserved production of  $N! - 1$  assignments relative to the reference assignment  $A_1 = \{\langle 1, 1 \rangle, \dots, \langle N, N \rangle\}$ . Directly citing the multinomial choice literature requires a vector of  $N! - 1$  assignment-specific observables with support  $\mathbb{R}^{N!-1}$ , where a hypothetical assignment-specific observable would enter only  $s(A; E, Z)$  for a particular  $A$ . Assignment-specific observables do not exist in our matching game. However, the distribution of  $H(\tilde{S})$  is identified using only the variation in match-specific characteristics  $Z$  assumed earlier.

**Lemma 3.** *The distribution  $H(\tilde{S})$  of unobserved production for all assignments is identified.*

The proof, in the appendix of the paper version of the chapter, shows that large and product support on  $Z$  allows us to trace  $H(\tilde{S})$ . The identification argument is therefore constructive. Failure of large and product support results in partial identification of  $H(\tilde{S})$ .

**Example 4.** For a running example, consider the case  $N = 3$ . The matrix of match



characteristics is

$$E = \begin{pmatrix} e_{1,1} & e_{1,2} & e_{1,3} \\ e_{2,1} & e_{2,2} & e_{2,3} \\ e_{3,1} & e_{3,2} & e_{3,3} \end{pmatrix}.$$

There are six possible assignments,

$$\begin{aligned} A_1 &= \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle\} \\ A_2 &= \{\langle 1, 2 \rangle, \langle 2, 1 \rangle, \langle 3, 3 \rangle\} \\ A_3 &= \{\langle 1, 3 \rangle, \langle 2, 2 \rangle, \langle 3, 1 \rangle\} \\ A_4 &= \{\langle 1, 2 \rangle, \langle 2, 3 \rangle, \langle 3, 1 \rangle\} \\ A_5 &= \{\langle 1, 1 \rangle, \langle 2, 3 \rangle, \langle 3, 2 \rangle\} \\ A_6 &= \{\langle 1, 3 \rangle, \langle 2, 1 \rangle, \langle 3, 2 \rangle\} \end{aligned} \tag{4.2.4}$$

and

$$\tilde{S} = \begin{pmatrix} \tilde{s}(A_2; E) \\ \tilde{s}(A_3; E) \\ \tilde{s}(A_4; E) \\ \tilde{s}(A_5; E) \\ \tilde{s}(A_6; E) \end{pmatrix} = \begin{pmatrix} e_{1,2} + e_{2,1} + e_{3,3} - (e_{1,1} + e_{2,2} + e_{3,3}) \\ e_{1,3} + e_{2,2} + e_{3,1} - (e_{1,1} + e_{2,2} + e_{3,3}) \\ e_{1,2} + e_{2,3} + e_{3,1} - (e_{1,1} + e_{2,2} + e_{3,3}) \\ e_{1,1} + e_{2,3} + e_{3,2} - (e_{1,1} + e_{2,2} + e_{3,3}) \\ e_{1,3} + e_{2,1} + e_{3,2} - (e_{1,1} + e_{2,2} + e_{3,3}) \end{pmatrix}. \tag{4.2.5}$$

Lemma 3 states that the distribution  $H(\tilde{S})$  is identified using variation in

$$Z = \begin{pmatrix} z_{1,1} & z_{1,2} & z_{1,3} \\ z_{2,1} & z_{2,2} & z_{2,3} \\ z_{3,1} & z_{3,2} & z_{3,3} \end{pmatrix}.$$

### 4.2.5 Unobserved Complementarities

The random vector  $\tilde{S}$  has  $N! - 1$  elements. Estimating a joint distribution of  $N! - 1$  elements is not practical in typical datasets. We now introduce the concept of unobserved complementarities as an intuitive, lower-dimensional random variable whose distribution is point identified if and only if  $H(\tilde{S})$  is point identified.

As described in the introduction, Becker (1973) shows that complementarities govern sorting when there is one characteristic (schooling) per agent. Likewise, references such as Fox (2010) and Graham (2011) prove that complementarities in *observed* agent or match characteristics are identified using data on matches. Likewise, while it is not possible to identify the distribution of unobserved match characteristics, we will show that the distribution of *unobserved* complementarities can be identified.

**Definition.** The **unobserved complementarity** between matches  $\langle u_1, d_1 \rangle$  and  $\langle u_2, d_2 \rangle$  is

$$c_{u_1, d_1, u_2, d_2} = e_{u_1, d_1} + e_{u_2, d_2} - (e_{u_1, d_2} + e_{u_2, d_1}). \quad (4.2.6)$$

The unobserved complementarities capture the change in the unobserved production (unobserved profits) when two matched pairs  $\langle u_1, d_1 \rangle$  and  $\langle u_2, d_2 \rangle$  exchange partners and the matches  $\langle u_1, d_2 \rangle$  and  $\langle u_2, d_1 \rangle$  arise.

Fixing a realization of the unobserved match characteristics  $E$ , one can calculate the market-level array (of four dimensions) comprising all unobserved complementarities

$$C = (c_{u_1, d_1, u_2, d_2} \mid u_1, u_2, d_1, d_2 \in N). \quad (4.2.7)$$

We only consider values  $C$  formed from valid values of  $E$ .

There are  $N^4$  values  $c_{u_1, d_1, u_2, d_2}$  in  $C$  given any realization  $E$ . However, all unobserved complementarities can be formed from a smaller set of other unobserved complementarities by addition and subtraction. Let

$$b_{u,d} = c_{1,1,u,d} = e_{1,1} + e_{u,d} - (e_{1,d} + e_{u,1}) \quad (4.2.8)$$

be an unobserved complementarity fixing the identities of the upstream firm  $u_1$  and the downstream firm  $d_1$  to both be 1. Let the matrix  $B$  be

$$B = \begin{pmatrix} b_{2,2} & \cdots & b_{2,N} \\ \vdots & \ddots & \vdots \\ b_{N,2} & \cdots & b_{N,N} \end{pmatrix},$$

which contains all unique values of  $b_{u,d}$  for a market.  $B$  is a matrix of  $(N - 1)^2$  elements. The following lemma shows we can restrict attention to  $B$  instead of  $C$  and hence focus on identifying the joint distribution  $F(B)$  of the heterogeneous matrix  $B$ .

**Lemma 5.**

1. *Every element of  $C$  is a linear combination of elements of  $B$ . The specific linear combination does not depend on the realizations of  $C$  or  $B$ .*
2. *For any CDF  $F(B)$ , there exists  $G(E)$  generating  $F(B)$  by the appropriate change of variables in (4.2.8).*
3. *If  $E$  is a exchangeable random matrix in upstream agent indices and also exchangeable in downstream agent indices, then so is  $B$ .*

By the first part of the lemma, we can focus on identifying the distribution of the  $(N - 1)^2$  elements in  $B$  instead of all  $N^4$  elements in  $C$ . By the second statement in the lemma, we can restrict attention to identifying  $F(B)$  without restrictions on the support of  $B$  or the dependence between the elements of  $B$ , as any  $F(B)$  is compatible with some distribution  $G(E)$  of the primitive matrix of match-specific unobservables  $E$ . Further, the third statement in the lemma shows that in the typical empirical context where the distribution of primitive unobservables is exchangeable in agent indices, the distribution of unobserved complementarities is also exchangeable in agent indices. The proof in the appendix of the paper version of the chapter has a formal definition of exchangeability in agent indices. We now present examples of some of the claims in the lemma.

**Example. 4** ( $N = 3$ ) There are  $3! = 6$  assignments. There are 12 unobserved complementarities  $c_{u_1, d_1, u_2, d_2}$  in  $C$ . There are 4 unobserved complementarities in  $B$ :

$$B = \begin{pmatrix} b_{2,2} & b_{2,3} \\ b_{3,2} & b_{3,3} \end{pmatrix} = \begin{pmatrix} e_{1,1} + e_{2,2} - (e_{1,2} + e_{2,1}) & e_{1,1} + e_{2,3} - (e_{1,3} + e_{2,1}) \\ e_{1,1} + e_{3,2} - (e_{1,2} + e_{3,1}) & e_{1,1} + e_{3,3} - (e_{1,3} + e_{3,1}) \end{pmatrix}. \quad (4.2.9)$$

The first part of Lemma 5 claims that the 12 elements in  $C$  can be constructed from the 4 elements in  $B$ . For one example,

$$c_{2,2,3,3} = e_{2,2} + e_{3,3} - (e_{2,3} + e_{3,2}) = b_{2,2} - b_{2,3} - b_{3,2} + b_{3,3}.$$

**Example 6.** Let the distribution  $G(E)$  be exchangeable in agent indices for upstream and downstream firms separately. Also let  $G(E)$  be multivariate normal with zero means. Under

exchangeability, zero means and the multivariate normality of  $E$ , the variance matrix of the distribution  $G$  is parameterized by four unique parameters as

$$\begin{aligned}\text{Cov}(e_{u_1,d_1}, e_{u_2,d_2}) &= \psi_1, \text{ if } u_1 \neq u_2, d_1 \neq d_2 \\ \text{Cov}(e_{u_1,d_1}, e_{u_2,d_1}) &= \psi_2, \text{ if } u_1 \neq u_2 \\ \text{Cov}(e_{u_1,d_1}, e_{u_1,d_2}) &= \psi_3, \text{ if } d_1 \neq d_2 \\ \text{Var}(e_{u_1,d_1}) &= \psi^2.\end{aligned}$$

One can use the properties of linear changes of variables for multivariate normal distributions to algebraically derive the distribution  $F(B)$  of unobserved complementarities.  $F(B)$  is itself exchangeable in agent indices (as Lemma 5.3 states) and is multivariate normal with a variance matrix with diagonal and off-diagonal terms

$$\begin{aligned}\text{Cov}(b_{u_1,d_1}, b_{u_2,d_2}) &= \frac{1}{4}\nu^2, \text{ if } u_1 \neq u_2, d_1 \neq d_2 \\ \text{Cov}(b_{u_1,d_1}, b_{u_2,d_1}) &= \frac{1}{2}\nu^2, \text{ if } u_1 \neq u_2 \\ \text{Cov}(b_{u_1,d_1}, b_{u_1,d_2}) &= \frac{1}{2}\nu^2, \text{ if } d_1 \neq d_2 \\ \text{Var}(b_{u_1,d_1}) &= \nu^2,\end{aligned}$$

where the new parameter  $\nu^2 = 4(\psi^2 + \psi_1 - \psi_2 - \psi_3)$ . This example shows the reduction of information from considering unobserved complementarities instead of unobserved match characteristics. In this example,  $G(E)$  is parameterized by four parameters while the induced  $F(B)$  has only one unknown parameter.

## 4.2.6 Identification of Unobserved Complementarities

We have shown that  $H(\tilde{S})$  is identified, where recall  $\tilde{S} = (\tilde{s}(A_2, E), \dots, \tilde{s}(A_{N!}, E))$ . We now show that identification of  $H(\tilde{S})$  gives the identification of  $F(B)$ , the distribution of unobserved complementarities.

Let

$$\tilde{r}(A; B) = \sum_{\langle u, d \rangle \in A} b_{u,d} - \sum_{\langle u, d \rangle \in A_1} b_{u,d}, \quad (4.2.10)$$

where for notational compactness we define  $b_{u,1} = b_{1,d} = 0$  for all  $u$  and  $d$ . The term  $\tilde{r}(A; B)$  gives the sum of the unobserved complementarities in  $B$  corresponding to the indices of the matches in  $A$  minus the same sum for  $A_1 = \{\langle 1, 1 \rangle, \dots, \langle N, N \rangle\}$ .

One of the main results of the paper is that the distribution  $F(B)$  of unobserved complementarities is identified.

### Theorem 7.

1.  $\tilde{s}(A; E) = \tilde{r}(A; B)$  for any  $A$  and where  $B$  is formed from  $E$ .
2.  $\tilde{r}(A; B_1) = \tilde{r}(A; B_2)$  for all  $A$  if and only if  $B_1 = B_2$ .
3. Therefore, the distribution  $F(B)$  is identified because the distribution of  $H(\tilde{S})$  is identified.

The proof is in the appendix. The first part of the theorem states that the sum of unobserved match production for an assignment can be computed using the elements of  $B$ . Therefore, knowledge of  $B$  can be used to compute pairwise stable assignments, for example for counterfactual analysis. Likewise, knowledge of  $F(B)$  lets one calculate assignment probabilities

$\Pr(A | Z; F)$ . The second part of the theorem states that there is a one-to-one mapping between the sums of unobserved assignment production for assignments and values of  $B$ . Therefore, as the distribution  $H(\tilde{S})$  of the sums of unobserved match production for assignments is identified, so is the distribution  $F(B)$  of unobserved match complementarities.

**Example. 4** ( $N = 3$ ) By definition,

$$\begin{pmatrix} \tilde{r}(A_2; B) \\ \tilde{r}(A_3; B) \\ \tilde{r}(A_4; B) \\ \tilde{r}(A_5; B) \\ \tilde{r}(A_6; B) \end{pmatrix} = \begin{pmatrix} b_{1,2} + b_{2,1} + b_{3,3} - (b_{1,1} + b_{2,2} + b_{3,3}) \\ b_{1,3} + b_{2,2} + b_{3,1} - (b_{1,1} + b_{2,2} + b_{3,3}) \\ b_{1,2} + b_{2,3} + b_{3,1} - (b_{1,1} + b_{2,2} + b_{3,3}) \\ b_{1,1} + b_{2,3} + b_{3,2} - (b_{1,1} + b_{2,2} + b_{3,3}) \\ b_{1,3} + b_{2,1} + b_{3,2} - (b_{1,1} + b_{2,2} + b_{3,3}) \end{pmatrix} = \begin{pmatrix} -b_{2,2} \\ -b_{3,3} \\ b_{2,3} - (b_{2,2} + b_{3,3}) \\ b_{2,3} + b_{3,2} - (b_{2,2} + b_{3,3}) \\ b_{3,2} - (b_{2,2} + b_{3,3}) \end{pmatrix}, \quad (4.2.11)$$

where the second equality uses  $b_{u,1} = b_{1,d} = 0$  for all  $u$  and  $d$ . Then using (4.2.9) for each of the four  $b_{u,d}$ 's and (4.2.5) for each of the five  $\tilde{s}(A; E)$ 's allows one to algebraically verify Theorem 7.1 for  $N = 3$ . The interesting direction of Theorem 7.2 for  $N = 3$  states that  $B_1 = B_2$  whenever  $\tilde{r}(A; B_1) = \tilde{r}(A; B_2)$  for all  $A$ . This direction can be verified because  $\tilde{r}(A_2; B)$  through  $\tilde{r}(A_5; B)$  can be easily solved for the four elements of  $B$ . The less interesting direction of Theorem 7.2 always holds by the definition of  $\tilde{r}(A; B)$  to be a function of  $B$ . Given that we previously showed that  $H(\tilde{S})$  is identified,  $F(B)$  is also identified.

### 4.2.7 Overidentification

The distribution of unobserved match characteristics  $G(E)$  is not identified. Despite the model primitive  $G(E)$  not being identified, the distributions  $H(\tilde{S})$  and  $F(B)$  are not only identified, they are overidentified. The proof of Lemma 3 works by setting  $H(\tilde{S}^*) =$

$\Pr(A_1 | Z^*)$ , where  $\tilde{S}^*$  is the point of evaluation of the CDF  $H$ ,  $A_1$  is the diagonal assignment  $\{\langle 1, 1 \rangle, \dots, \langle N, N \rangle\}$  and  $Z^*$  is a specific value of  $Z$  chosen based on the value  $\tilde{S}^*$ . One can identify the entire model if one only observes, in each market, whether assignment  $A_1$  occurs or not. The assignment  $A_1$  is just one of  $N!$  assignments. Given the adding up constraint that the sum of probabilities of assignments is always 1, there are  $N! - 2$  other probabilities  $\Pr(A | Z)$  for each  $Z$  available to overidentify the model.

The necessity of using only one assignment probability in a proof of identification is analogous to the identification arguments for the single-agent multinomial choice model in the frameworks of Thompson (1989) and Lewbel (2000). In such multinomial choice models, only the probability of a single choice is necessary for identification. Given that choice probabilities sum to one, all but two choices provide overidentifying restrictions. Overidentification in the semiparametric multinomial choice model has not been formally exploited to form an operation testing procedure in finite samples. Given that the simpler multinomial choice model should be explored before matching models, we leave the formal exploitation of overidentification to further research.<sup>60</sup>

### 4.3 Generalizations of the Baseline Model

We consider two strict generalizations of identification result for the one-to-one matching game where all agents are matched.

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<sup>60</sup>Another source of overidentification arises if the researcher imposes that  $G(E)$  and hence, by Lemma 5.3,  $F(B)$  are exchangeable in agent indices. Exchangeability in agent indices is a restriction of the class of allowable  $F$ 's but  $F$  is identified without assuming such a restriction.



### 4.3.1 Other Observed Variables $X$ and Random Preferences

In addition to the large support match-specific characteristics  $Z$ , researchers often observe other match-specific and agent-specific data, which we collect in the random variable  $X$ , which we think of as a long vector. We also include in  $X$  the number of agents on each side,  $N$ , to allow the size of the market to vary across the sample. An example of a production function augmented by the elements of  $X$  is

$$(x_u \cdot x_d)' \beta_{u,d,1} + x'_{u,d} \beta_{u,d,2} + \mu_{u,d} + z_{u,d}, \quad (4.3.1)$$

where  $x_u$  is a vector of upstream firm characteristics,  $x_d$  is a vector of downstream firm characteristics,  $x_u \cdot x_d$  is a vector of all interactions between upstream and downstream characteristics,  $x_{u,d}$  is a vector of match-specific characteristics,  $\mu_{u,d}$  is a random intercept capturing unobserved characteristics of both  $u$  and  $d$ , and  $\beta_{u,d,1}$  and  $\beta_{u,d,2}$  are random coefficient vectors specific to the match. The two random coefficient vectors can be the sum of the **random preferences** of upstream and downstream firms for own and partner characteristics. In a marriage setting, we allow men to have heterogeneous preferences over the observed characteristics of women, which is one explanation for why observationally identical men marry observationally distinct women.

In this example,

$$X = \left( N, (x_u)_{u \in N}, (x_d)_{d \in N}, (x_{u,d})_{u,d \in N} \right).$$

Now we define

$$e_{u,d} = (x_u \cdot x_d)' \beta_{u,d,1} + x'_{u,d} \beta_{u,d,2} + \mu_{u,d}$$

and, as before notationally,

$$c_{u_1,d_1,u_2,d_2} = e_{u_1,d_1} + e_{u_2,d_2} - (e_{u_1,d_2} + e_{u_2,d_1}).$$

Using the same notation as before, we define the array of unobserved complementarities as (4.2.7). This definition of  $C$ , and similarly of  $B$ , now depends on the realizations of  $X$ . Our previous identification argument in Theorem 7 does not use  $X$ . Therefore we can condition on a realization of  $X$  to identify the conditional-on- $X$  distribution of unobserved complementarities  $F(B | X)$ . We of course require variation in  $Z$  as before, but now  $Z$  must have full support conditional on each realization of  $X$ . We do not require that  $C$ ,  $B$  and  $E$  are independent of  $X$ , but all unobservables must still be independent of  $Z$  conditional on  $X$ .

**Corollary 8.** *The distribution  $F(B | X)$  of market-level unobserved complementarities conditional on  $X$  is identified.*

Our identification of distributions of heterogeneity conditional on  $X$  follows arguments in the multinomial choice literature (Lewbel, 2000; Matzkin, 2007; Berry and Haile, 2009). This is a standard object of identification in the cited literature

We could further attempt to unpack the identified  $F(B | X)$  into the distribution of individual random coefficients and additive unobservables, such as the vectors  $\beta_{u,d,1}$  and  $\beta_{u,d,2}$  and the unobserved complementarities induced only by the scalar  $\mu_{u,d}$  in the example production function (4.3.1). We would need to assume full independence between the primitive unobservables and the elements of  $X$ . Using (4.3.1), we can think of the definition of  $b_{u,d}$  (4.2.8) as defining a system of  $(N - 1)^2$  seemingly unrelated equations, relating  $b_{u,d}$  to the elements of  $X$ , the random coefficients and the additive unobservables. Masten (2015) studies in

part seemingly unrelated regressions with random coefficients and shows that the marginal distribution of each random coefficient or additive unobservable is identified but the joint distribution of the random coefficients and additive unobservables entering all equations is sometimes not identified. One intuition is that the number of elements of  $X$  must weakly exceed the number of random coefficients and additive unobservables. Once  $F(B | X)$  is identified and the problem of unpacking  $F$  into the joint distribution of random coefficients and additive unobservables is placed in the framework of Masten (2015), the remaining identification issues are less specific to matching and so are not considered further here.

### 4.3.2 Heterogeneous Coefficients on Match Characteristics

We now define the production to a match  $\langle u, d \rangle$  to be

$$e_{u,d} + \gamma_{u,d} \cdot z_{u,d}, \tag{4.3.2}$$

where  $\gamma_{u,d} \neq 0$  is a match-specific coefficient. The coefficients  $\gamma_{u,d}$  vary across matches within each matching market but not across markets. Therefore, the  $\gamma_{u,d}$  are fixed parameters to be identified and not random coefficients. Fixing coefficients across markets but not within markets makes sense in a context where firm indices like  $u$  and  $d$  have a consistent meaning across markets. For example, the same set of upstream and downstream firms may participate in multiple matching markets, as in Fox (2016), where each market is a separate automotive component category.<sup>61</sup> As we need the  $z_{u,d}$ 's to identify  $F(B | X)$ , we rule out

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<sup>61</sup>In a marriage setting with different individuals in each market, we could assume that  $\gamma_{u,d}$  is the same for all matches where the men are all in the same demographic class (such as college graduates) and the women are all in the same demographic class (such as high-school graduates). This suggested use of demographic classes is partially reminiscent of Chiappori, Salanié and Weiss (2015), who use data over time on the US marriage market to estimate a different variance of the type I extreme value (logit) utility errors in a Choo

the case where any  $\gamma_{u,d} = 0$ .

We apply a scale normalization on production by setting  $\gamma_{1,1} = \pm 1$ . Because of transferable utility, we can identify the relative scale of each match's production. We use the matrix  $\Gamma = (\gamma_{u,d})_{u,d \in N}$ . It is first important to note that parts 1 and 2 of Theorem 7 are only about the random variable unobservables  $B$  and  $E$  and so do not involve whether  $z_{u,d}$  has a parameter  $\gamma_{u,d}$  on it or not. So those statements in Theorem 7 still hold in this more general setting. Next we state that the analog to the identification claim in the third part of Theorem 7 holds in the setting with fixed parameters  $\gamma_{u,d}$  on  $z_{u,d}$ .

**Theorem 9.** *The distribution  $F(B | X)$  and the fixed matrix of parameters  $\Gamma = (\gamma_{u,d})_{u,d \in N}$  are identified.*

The proof is in the appendix of the paper version of the chapter.

We could also study the production function (4.3.2) when each  $\gamma_{u,d}$  is a random coefficient such that the random matrix  $\Gamma = (\gamma_{u,d})_{u,d \in N}$  has some joint distribution  $J(\Gamma)$  that describes how  $\Gamma$  varies across markets. Again citing the results in Masten (2015) on seemingly unrelated regressions, we conjecture that the marginal distribution of each scalar  $\gamma_{u,d}$  is separately identified but that the joint distribution  $J(\Gamma)$  is not identified. Exploring estimation under partial identification of a distribution such as  $J(\Gamma)$  is outside the scope of the paper.

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and Siow (2006) style model for each male demographic class and for each female demographic class. Our suggested approach in this footnote lets  $\gamma_{u,d}$  vary by the intersection of male and female demographic classes, instead of agents. For example, there could be a  $\gamma_{\text{age 40 men, age 30 women}}$  specific to the listed demographic groups.

## 4.4 Data on Unmatched Agents

Up until this point, we have considered matching games where all agents have to be matched. We infer  $F(B | X)$  from sorting patterns in the data. This approach is reasonable when only data on observed matches are available. For example, it may be unreasonable to assume that data on all potential entrants to a matching market exist. In some situations, however, researchers can also observe the identities of unmatched agents. Data are available, for example, on potential merger partners in some industry that do not end up undertaking mergers or on single people in a marriage market.<sup>62</sup> When data on unmatched agents do exist, we can go beyond unobserved complementarities  $B$  and identify the distribution of match-specific unobservables  $E$ .

Here,  $X$  can contain separate numbers of downstream firms  $N_d$  and upstream firms  $N_u$ . Let

$$E = \begin{pmatrix} e_{1,1} & \cdots & e_{1,N^d} \\ \vdots & \ddots & \vdots \\ e_{N^u,1} & \cdots & e_{N^u,N^d} \end{pmatrix}.$$

Use  $\langle u, 0 \rangle$  and  $\langle 0, d \rangle$  to denote an upstream firm and a downstream firm that are not matched. An assignment  $A$  can be  $\{\langle u_1, 0 \rangle, \langle u_2, d_2 \rangle, \langle 0, d_2 \rangle\}$ , allowing single firms. We do not require match-specific characteristics  $z_{u,0}$  and  $z_{0,d}$  for unmatched firms; they can be included in  $X$  if present.

The data generating process is still (4.2.3). One difference is that a pairwise stable assignment needs to satisfy individual rationality: each non-singleton realized match has production greater than 0.

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<sup>62</sup>For example, Uetake and Watanabe (2012) study mergers between rural banks, where each county is a separate matching market.

**Theorem 10.** *The distribution  $G(E | X)$  of market-level unobservables is identified with data on unmatched agents.*

The proof shows that the distribution  $G(E | X)$  for some  $X$  can be traced using the probability that all agents are unmatched, conditional on  $Z$ . The individual rationality decision to be single identifies  $G(E | X)$  while the sorting of matched firms to other matched firms identifies only  $F(B | X)$ . Using an individual rationality condition is more similar to the utility maximization assumptions used in the identification of single-agent discrete choice models and discrete Nash games (Lewbel, 2000; Matzkin, 2007; Berry and Haile, 2009; Berry and Tamer, 2006). An agent can unilaterally decide to become unmatched.

## 4.5 Agent-Specific Characteristics

Return to the case with only unmatched agents in the data. Match specific  $z$ 's with full support are not always available in datasets. For example, in our venture capital work say we observed only patents for startups and the total experience in past deals for venture capitalists (in reality, we do observe match characteristics). Also say, for example, that the induced match specific characteristic  $z_{u,d} = z_u \cdot z_d$ , where  $z_u$  is the upstream firm characteristic experience and  $z_d$  is the downstream firm characteristic patents. Say the long vector of agent-specific characteristics  $((z_u)_{u \in N}, (z_d)_{d \in N})$  has support in  $\mathbb{R}^{2N}$ . Even this product support on agent characteristics will not allow the matrix  $Z$  with induced match-specific characteristics  $z_{u,d} = z_u \cdot z_d$  to have support on say  $\mathbb{R}^{N^2}$ .

This is a problem for identification in the baseline model because the matrix of match-specific unobservables  $E$  has  $N^2$  elements and the matrix of unobserved complementarities

$B$  has  $(N - 1)^2$  elements. There needs to be some symmetry between the distributions of observables and unobservables in the model. More prosaically, we cannot prove Lemma 3 when  $Z$  has limited support, such as when its constituents  $((z_u)_{u \in N}, (z_d)_{d \in N})$  vary only in  $\mathbb{R}^{2N}$  while the match-specific unobservables  $E$  vary over  $\mathbb{R}^{N^2}$ .

The solution is to restrict attention to a production function where, in the primitive model, unobserved characteristics enter symmetrically to observed characteristics. If observed characteristics vary at the agent level, then unobserved characteristics should also be restricted to only vary at the agent level. Taking the example of  $z_{u,d} = z_u \cdot z_d$  above, consider the production function

$$e_u \cdot e_d + z_u \cdot z_d, \tag{4.5.1}$$

where  $e_u$  and  $e_d$  are unobserved agent-specific characteristics.<sup>63</sup> Unobserved agent characteristics enter symmetrically to the observed agent characteristics in the production function. Say the unobserved agent characteristics  $((e_u)_{u \in N}, (e_d)_{d \in N})$  take on support on  $\mathbb{R}^{2N}$ . Then observed agent characteristics  $((z_u)_{u \in N}, (z_d)_{d \in N})$  also should vary on  $\mathbb{R}^{2N}$  in order for slight extensions to the previous identification arguments to go through. Specifically, one can alter the first lines of the proof of Lemma 3 and the lemma and hence the remainder of the identification machinery leading up to and including Theorem 7 will apply to the agent-specific case.

Define  $e_{u,d} = e_u \cdot e_d$  and define unobserved complementarities using (4.2.6). We show that the distribution of the matrix  $B$  of unobserved complementarities is identified.

**Corollary 11.** *The distribution  $F(B | X)$  of unobserved complementarities  $B$  is identified*

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<sup>63</sup>All assignments would be pairwise stable if the match production was instead  $e_u + e_d + z_u + z_d$ .

*in the agent-specific case.*

The short proof is in the appendix of the paper version of the chapter.

## 4.6 Many-to-Many Matching

Venture capitalists can make multiple investments during the same year. Likewise, startups often contract with multiple venture capitalists, although for simplicity our application considers only the lead venture capital investor in a startup. It is important to extend the previous results to many-to-many, two-sided matching.

Consider a two-sided matching game where upstream firm  $u$  can make a quota of  $q_u$  possible matches and downstream firm  $d$  can make  $q_d$  possible matches. The researcher has data on  $q_u$  and  $q_d$  and the quotas can vary across firms in the same market and across markets. The previous case of one-to-one matching is  $q_u = q_d = 1$  for all firms. Leaving a quota slot unfilled gives production of zero for that slot. The number of upstream firms  $N^u$  may differ from the number of downstream firms  $N^d$ .

Let the production function for an individual match still be (4.2.1) and let the production of the matches of the single upstream firm  $u$  with the pair of downstream firms  $d_1$  and  $d_2$  be equal to

$$z_{u,d_1} + e_{u,d_1} + z_{u,d_2} + e_{u,d_2}.$$

This *additive separability* in the production of multiple matches involving the same firm yields the many-to-many matching model of Crawford and Knoer (1981) and Roth and Sotomayor (1990); Sotomayor (1999). Like in the one-to-one case, a pairwise stable assignment is proven to exist, to be efficient and to be unique with probability 1. Redefine the following objects



to allow  $N^u \neq N^d$  :

$$E = \begin{pmatrix} e_{1,1} & \cdots & e_{1,N^d} \\ \vdots & \ddots & \vdots \\ e_{N^u,1} & \cdots & e_{N^u,N^d} \end{pmatrix}, Z = \begin{pmatrix} z_{1,1} & \cdots & z_{1,N^d} \\ \vdots & \ddots & \vdots \\ z_{N^u,1} & \cdots & z_{N^u,N^d} \end{pmatrix}, B = \begin{pmatrix} b_{2,2} & \cdots & b_{2,N^d} \\ \vdots & \ddots & \vdots \\ b_{N^u,2} & \cdots & b_{N^u,N^d} \end{pmatrix}.$$

We also extend to many-to-many matching the model in Section 4.3.2, where the production of a match between  $u$  and  $d$  is  $\gamma_{u,d} \cdot z_{u,d} + e_{u,d}$ . The matrix of homogeneous parameters is  $\Gamma = (\gamma_{u,d})_{u \in N^u, d \in N^d}$ .

Say first that the number of firms, quotas and production functions are such that all firms make a number of matches equal to their quotas: there are no unused quota slots in equilibrium. Leaving no unused quota is feasible if

$$\sum_{u=1}^{N^u} q_u = \sum_{d=1}^{N^d} q_d.$$

In this case, every mathematical argument for the baseline model in Section 4.2 and many of the subsequent models extends to many-to-many matching. In particular, the distribution of unobserved complementarities  $F(B | X)$  is identified using the sorting patterns in the data. Likewise, if unmatched firms are in the data and so quota slots can be left unused, the same analysis as Section 4.4 applies.

**Corollary 12.** *Consider the many-to-many matching model.*

1. *The distribution of unobserved complementarities  $F(B | X)$  and the coefficients  $\Gamma$  (if included) are identified from data on matches only.*
2. *The distribution of match-specific unobservables  $G(E | X)$  is identified if unmatched*

*firms are in the data.*

The proof is omitted as it just checks previous mathematical arguments to see that properties unique to one-to-one matching with  $N^u = N^d = N$  are not used.

## 4.7 Matching with Trades

### 4.7.1 Simple Matching with Trades

Investigating a fairly general matching model is useful because many models of empirical interest will be special cases. Consider matching with trades in the so-called trading networks model in Hatfield, Kominers, Nichifor, Ostrovsky and Westkamp (2013), which is a significant generalization of Kelso Jr and Crawford (1982). In matching with trades, agents engage in trades  $\omega$  from some finite set  $\Omega$ . A trade indexes the name of the buyer and the name of the seller and can specify other aspects, such as the quality or other specifications of the goods in question. In a labor market, trades could specify benefits such as health care plans and vacation time. In venture capital, a trade could specify the number of board seats a startup gives a venture capitalist. Trades generalize our previous notion of a match. We require data on all aspects of the trade; if quality is part of a trade then the qualities for all trades in the set  $\Omega$  must be measured. The price of trade  $\omega$  is  $p_\omega$ , although, as before, we study identification when prices are not observed in the data. Prices play the same role as transfers in the earlier matching models.

Firms are not necessarily divided into buyers and sellers ex ante; a firm can be a buyer on some trades and a seller on other trades. In a model of mergers, a firm is not ex ante either a target or acquirer; these roles arise endogenously as part of a pairwise stable outcome.

Two-sided, many-to-many matching is a strict special case of trading networks where the profits of an upstream firm undertaking trades as a buyer are  $-\infty$  and, likewise, the profits of a downstream making trades as a seller are  $-\infty$ .

As there are no ex ante upstream and downstream firms, index a firm by  $i$ . Consider first the case where the production of a trade  $\omega$  between buyer  $i$  and seller  $j$  is

$$z_\omega + e_\omega. \tag{4.7.1}$$

Notationally, the indices of  $i$  and  $j$  are subsumed into the trade  $\omega$ . If a trade should give production of  $-\infty$ , we notationally remove it from  $\Omega$ . This matching with trades game is a generalization of the two-sided many-to-many matching game in Section 4.6. In this simple setup, trades that give positive production occur and trades that give negative production do not occur. We observe the entire set of trades  $\Omega$  for each market, so the data measures whether a trade occurs or does not occur; firms that make no trades are therefore observed as well. Let the vector  $Z = (z_\omega)_{\omega \in \Omega}$  and, similarly,  $E = (e_\omega)_{\omega \in \Omega}$ . Let  $X$  collect observables entering  $E$ .

**Theorem 13.** *Consider a trades model where the production of trade  $\omega$  is  $z_\omega + e_\omega$ . Then  $G(E | X)$  is identified.*

### 4.7.2 Matching with Trades Under Submodularity

We now consider matching with trades where firms have profit functions defined over portfolios of trades. Let  $\Omega_i \subset \Omega$  be the set of trades where  $i$  is either a buyer or a seller. The

**individual profit** of a firm  $i$  undertaking the trades  $\Psi_i \subseteq \Omega_i$  at prices  $p_\omega$  for  $\omega \in \Omega$  is

$$u(i, \Psi_i) + \sum_{\omega \in \Psi_{i \rightarrow}} p_\omega - \sum_{\omega \in \Psi_{\rightarrow i}} p_\omega, \quad (4.7.2)$$

where the set  $\Psi_{i \rightarrow}$  is the trades in  $\Psi_i$  where  $i$  is the seller and  $\Psi_{\rightarrow i}$  is the trades in  $\Psi_i$  where  $i$  is the buyer. Hatfield, Kominers, Nichifor, Ostrovsky and Westkamp prove that a pairwise stable assignment (here a set of trades) exists and is efficient (and therefore unique with probability 1) under a condition on preferences called **substitutes**. A companion paper shows that the substitutes condition is equivalent to the indirect utility (profit) version of the direct utility (profit) in (4.7.2) being **submodular** for all vectors of prices,  $p_\omega$  for  $\omega \in \Omega$  (Hatfield, Kominers, Nichifor, Ostrovsky and Westkamp, 2015, Theorem 6). See the cited paper for a definition of submodularity. Submodularity of the indirect utility function is restrictive for many empirical applications. However, submodularity is only a restriction when the profit from a set of trades is not additively separable across the trades. Therefore, the underlying direct utility firm profits justifying the production-of-a-trade (4.7.1) in Section 4.7.1 imply that the corresponding indirect utility functions are submodular.

For all firms  $i$  and trades  $\psi_i \subseteq \Omega_i$  let the **pre-transfer profit** (or **valuation**) be

$$u(i, \Psi_i) = z_{i, \psi_i} + e_{i, \Psi_i},$$

where  $z_{i, \psi_i}$  is an observable specific to firm  $i$  and the set of trades  $\psi_i$  and  $e_{i, \Psi_i}$  is an unobservable specific to firm  $i$  and the set of trades  $\Psi_i$ . Let  $Z = (z_{i, \psi_i})_{i \in N, \psi_i \subseteq \Omega_i}$  be the array of observables corresponding to pairs of firms and sets of trades and let  $E = (e_{i, \Psi_i})_{i \in N, \Psi_i \subseteq \Omega_i}$  be a similar array for unobservables. The arrays  $Z$  and  $E$  are typically large but are always finite as the set of trades  $\Omega$  is finite. For identification, the support of  $-Z$  must be a weak

superset of the support of  $E$ . Further, the supports of  $E$  and  $Z$  should be restricted so that the corresponding indirect utility functions are submodular for all players for all realizations of  $E$  and  $Z$ . We leave to other work the question of how to enforce submodularity in empirical applications.<sup>64</sup> As before, observable characteristics other than the  $z_{i,\psi_i}$  are collected in a long vector  $X$  and can enter  $e_{i,\psi_i}$ .

Say that the profit from making no trades is zero and that the researcher observes data on firms that make no trades. Then the following identification result holds.

**Theorem 14.** *The distribution of unobservables  $G(E | X)$  is upper bounded by an identified function  $\bar{G}(E | X)$  in the trading networks model.  $\bar{G}(E | X) < 1$  if, for each  $X$  and  $Z$ , there exists  $E$  with positive probability where trades occur.*

The statement means that we can identify a function  $\bar{G}(E | X)$  such that  $G(E | X) \leq \bar{G}(E | X)$  for all arrays of unobservables  $E$  and conditioning observables  $X$ . A distribution function is a probability, so the trivial bound  $\bar{G}(E | X) = 1$  satisfies this property. However, if some assignment other than the assignment with no trades occurs with positive probability, then  $\bar{G}(E | X)$  is a tighter bound than the trivial bound of 1.<sup>65</sup>

The bound  $\bar{G}(E | X)$  in the proof Theorem 14 is actually  $\Pr(A_0 | Z^*, X)$  for some  $Z^*$ , where  $A_0$  is the assignment where no trades are made. The proof of Theorem 14, in the appendix of

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<sup>64</sup>If the submodularity condition fails, a stable assignment or a competitive equilibrium as defined in Hatfield, Kominers, Nichifor, Ostrovsky and Westkamp (2013) may fail to exist. One practical but perhaps inelegant approach is to ignore the lack of existence. As an example of this approach, the important contribution by Ciliberto and Tamer (2009) estimates a Nash game of finite actions and restricts attention to pure strategy equilibria, even though Nash's existence theorem applies to mixed strategy equilibria. Note that Hatfield, Kominers, Nichifor, Ostrovsky and Westkamp use a stability definition that is stronger than *pairwise* stability. Under the models discussed earlier in the current paper, *pairwise* stability implies the stronger notion of stability, while the equivalence does not hold in Hatfield, Kominers, Nichifor, Ostrovsky and Westkamp. Theorem 14 uses the stability definition in Hatfield, Kominers, Nichifor, Ostrovsky and Westkamp.

<sup>65</sup>The bound is likely not sharp. Indeed, it is possible  $G(E | X)$  is point identified and we do not know the proof.

the paper version of the chapter, extends the argument in the proofs of Theorems 10 and 13. In the proofs of Theorems 10 and 13,  $G(E | X)$  itself and not a bound equals  $\Pr(A_0 | Z^*, X)$ . This is because the unobservables  $e_{u,d}$  in Theorem 10 and  $e_\omega$  in Theorem 13 correspond to the production of a match or trade, which is the sum of profits of the two firms for the match or trade. In Theorem 14, the unobservables  $e_{i,\psi_i}$  correspond to the profit of an individual firm  $i$  and not the production of all firms in the trades. The theorem shows that it is possible to identify bounds on distributions of aspects of individual firm profit functions (up to scale) and not just aspects of production functions for matches, as in earlier results. The reason is that the individual profit functions are not additively separable across individual trades, leaving no role for the concept of the production of a trade.

## 4.8 Biotech and Medical Venture Capital

We estimate the roles of observed match and firm characteristics as well as the distribution of unobserved complementarities in the biotech and medical/health venture capital industries. In these industries, investment firms known as venture capitalists contribute money to entrepreneurial startups. We seek to understand the role of venture capitalists in the productive surplus of an investment; this contribution to match production is only present if the venture capitalist adds value over and above offering financing. We present separate parameter estimates for the biotech and medical industries.

In most cases, the first round of venture capital funding is secured well before an entrepreneur is ready to market its products to consumers or even to undertake a final round of testing. Therefore, the first round venture capital funding is essential to nurturing the entrepreneur during a period where the entrepreneur has low or no revenues of its own. We study only the

identity of the lead venture capital investor in the first round of funding. This lead venture capitalist often takes more of an active management role in the startup than other investors.

We model each life science venture capital industry as a many-to-one, two-sided matching market where each entrepreneur is funded by the lead venture capitalist and each venture capitalist can fund multiple startups. We lack data on unfunded startups and venture capitalists who make no investments. Because we focus on many-to-one matching and consider only matched firms, the appropriate nonparametric identification result is the many-to-one special case of the many-to-many result in Corollary 12.1.

Sørensen (2007) uses a structural approach to estimate a matching game between entrepreneurs and venture capitalists. Sørensen (2007) estimates a matching game where matched agents could not exchange transfers, unlike the transferable utility matching game we study. His assumption is that the unobservables are match-specific, normally distributed and independent across matches involving the same or differing firms. By assuming independence across matches involving the same firm, Sørensen (2007)'s model rules out that matches for many firms tend to be unobservedly more productive than matches for many other firms. Allowing for correlation in the matches involving the same firm may be important in venture capital, as such a correlation structure allows certain firms to contribute more to match profit than other firms. These are the “high type” firms: highly capable venture capitalists or entrepreneurial firms with great prospects in the life sciences industries.

#### **4.8.1 Data and Observed Characteristics**

We start with a carefully collected dataset on, ideally, all venture capital transactions in the biotech and medical industries. The data come from ThomsonOne. We then merge the

venture capital data with data from the US Patent and Trademark Office on the stock of patents held by the entrepreneurial firms at the time of the first round investment that we model. Our data showed that the number of venture capital deals increased substantially after 1996 and our patent data have missing records after 2008. Therefore, we use data on venture capital deals between 1997 to 2007, although we use earlier years of data to compute venture capitalist experience.

## 4.8.2 Matching Markets

For computational reasons to be discussed, our method of simulated moments estimator can handle only what we describe as medium sized matching markets. Therefore, our matching market definition is made to keep the number of matches medium sized: small compared to the entire biotech and medical venture capital industries but still large compared to the number of potential entrants in the entry literature in industrial organization (Bresnahan and Reiss, 1991; Berry, 1992; Ciliberto and Tamer, 2009). We define a matching market to be one of the eleven two-digit biotech sectors in Table 4.24 in a particular calendar year of the data. Based on our matching market definitions, in our model venture capitalists consider matching only with the set of entrepreneurs in the same two-digit biotech/medical sector and year that, in the data, the venture capitalist's match partner was in.<sup>66</sup> The assumption that venture capitalists pre-commit to a two-digit sector is strong; relaxing the restriction to two-digit sectors requires addressing computational concerns.<sup>67</sup>

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<sup>66</sup>Our market definition differs from Sørensen (2007), who defines a matching market as a six month period in one of two US states, California and Massachusetts. We use worldwide data and impose no limits that venture capitalists and entrepreneurs look for partners only within one narrow geographic region, which corresponds with our data.

<sup>67</sup>Sheng (2014) is an estimator for large network games that could likely be applied to matching games in future work. The computational savings of the estimator result in set instead of point identification in the limit, which is not in the spirit of our paper's theoretical results about point identification.



Table 4.24: Two Digit Biotech and Medical Sectors Used to Define Matching Markets

<b>Biotech</b>	<b>Medical</b>
Human	Diagnostics
Agricultural and animal	Therapeutics
Industrial	Health products
Biosensors	Health services
Research & production equipment	Pharmaceuticals
Research & other services	

We only consider two-digit sectors with fewer than 30 startups in a year. We estimate the model parameters separately for the biotech and medical industries. For biotech, we have 38 matching markets. The mean number of startups per matching market is 7.2 with a maximum of 27. The mean number of venture capitalists per matching market is 6.8 with a maximum of 25. The 6.8 venture capitalists per matching market is only a little lower than 7.2 startups because there are only a small number of venture capitalists making multiple matches in the same two-digit biotech sector and year. Now consider the medical industry, which has larger numbers of startups per two-digit sector and year. There are 15 matching markets with fewer than 30 startups. Among those 15 markets, the mean number of startups is 22.3 with a maximum of our chosen upper bound of 30. The mean number of venture capitalists is 21.5 with the maximum, in the estimation sample, of 30.<sup>68</sup>

Again, we model the lead (largest) venture capital investor in the first round of venture capital funding. Each entrepreneur appears once in the data, reflecting this first round. Each venture capitalist can occur multiple times. A venture capitalist can be engaged in multiple investments within a year and can be observed in multiple years. If a venture capitalist makes multiple first round investments as the lead investor in a given year and

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<sup>68</sup>We use servers with up to twenty cores for estimation. Using a cluster of multiple servers could allow us to increase the number of agents per matching market some, but not tremendously so because of the curse of dimensionality with computing a pairwise stable assignment to the matching game.

two-digit sector, it is treated as a single firm with a quota (the maximum number of matches it can make) equal to the number of matches that venture capitalist made in the data for that year and two-digit sector. A venture capitalist with a single match in the data has a quota of one. Our model's use of these quotas focuses on time constraints as the reason a superstar venture capitalist is not the lead investor in all entrepreneurial startups. Indeed, a lead venture capital investor takes seats on the board and an active management role in its entrepreneurial firms. The venture capitalist has scarce time to do this and so carefully selects a small number of investments, in the context of a matching model where it competes with other venture capitalists for these deals.

We do not model synergies between multiple entrepreneurs matched to the same venture capitalist; the production of a set of matches involving the same venture capitalist is equal to the sum of the production of the individual matches, as in our Corollary 12 but not our Theorem 14. Therefore, we do not study venture capitalist financial strategies such as portfolio diversification. We also do not model post-matching externalities in match production caused by, say, multiple entrepreneurs competing to treat the same, narrowly defined medical condition. It is rare for entrepreneurs in the same two-digit sector and year of the initial investment to be directly competing in the sense of treating, say, the same, narrowly defined medical condition. Therefore, in our model an entrepreneur cares about the outcome of the venture capital market for matches only because it affects the entrepreneur's own final venture capital match and corresponding transfer, not because a rival's match with a top venture capitalist could create a fierce competitor for consumers.

Unlike, say, a dataset on mergers, there are no unmatched firms in our data. While presumably there are entrepreneurial firms that fail to secure a first round of venture capital funding and venture capitalists with the equivalent of free quota slots (say spare time to help manage

an additional startup), our data do not cover them. In what follows, our model operates as if these unmatched entrepreneurs and venture capitalist quota slots do not exist.<sup>69</sup>

### 4.8.3 Observables in the Match Production Function

The production function for the output of a match involving one entrepreneur  $d$  and one venture capitalist  $u$  is

$$\pm 1 \cdot \text{Distance}_{u,d} + \beta_{\text{Sector}} \cdot \text{SectorExper}_{u,d} + \beta_{\text{ExperPatents}} \cdot \text{TotalExper}_u \cdot \log(\text{Patents}_d + 1) + e_{u,d}. \quad (4.8.1)$$

In this section, we discuss the contribution of each of the listed observable (in the data) variables; we postpone a discussion of the unobservables until the next section. Overall, we feel we have collected close to the best firm and match characteristics on a broad portion of the VC sector that academic researchers could have access to. As we will see, even these rich characteristics will leave room for unobservables.

Table 4.25 reports the means and standard deviations across realized matches for the key observable characteristics in the production function. Estimating the matching model requires us to compute these characteristics for both actual and counterfactual matches. In estimation, we rescale many of the variables as discussed separately for each variable; the table reports the variables before and after rescaling.<sup>70</sup>

The scale normalization of match production is in terms of the match-specific variable distance. Distance is measured as the distance on the surface of the Earth from the headquarters

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<sup>69</sup>The matching maximum score estimator of Fox (2016) is robust to missing data on quotas in part because it does involve computing pairwise stable assignments as part of a nested fixed point procedure.

<sup>70</sup>The table reports summary statistics by realized matches but some rescalings are based on other samples, like all venture capitalists or all startups for agent-specific characteristics.

Name	Pre-Rescaling				Rescaled			
	Biotech		Medical		Biotech		Medical	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
log (Patent count + 1)	0.45	0.85	0.46	0.87	-0.04	0.97	0.01	1
VC overall experience	0.06	0.12	0.07	0.14	0.22	1.19	0.25	1.36
VC four-digit sector specific experience	0.19	0.26	0.13	0.18	0.04	1.05	-0.02	0.98
Distance: km/1000	1.74	3.08	2.07	3.2	0.05	1	-0.02	1
# of Patents (no logs)	2.12	7.48	2.35	11.16				
Interaction term: patents * log(patents)					0.02	1.08	0.18	1.62

Table 4.25: Summary Statistics for Realized Matches

of the venture capitalist to the headquarters of the entrepreneur. We have worldwide data so some of these distances are quite large: from Europe to Australia, say. Table 4.25 measures distance in thousands of kilometers. Hence, the table reports that the mean distance across realized matches is around 1700 kilometers. We allow the coefficient on distance to be either positive or negative, estimating the model once for a positive coefficient and a second time for a negative coefficient, taking the parameter estimates with the lowest objective function value.<sup>71</sup> Not surprisingly, we will find that the coefficient on distance is indeed negative. We rescale distance to have a mean of zero and standard deviation of 1.

Distance plays an important role in the venture capital literature. The literature has argued that geographic proximity helps investors and entrepreneurs find out about each other, thereby increasing investment likelihood (Sorenson and Stuart, 2001). Furthermore, Lerner (1995) finds that VCs are more likely to sit on boards of their portfolio companies the closer are the companies, a finding consistent with lower governance costs associated with geographically proximate investments. As a result, geographic proximity is likely to be a factor

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<sup>71</sup>As discussed in Section 4.3.2, the assumption that distance is valued the same across all matches is more than a mere normalization. Using the fact that each venture capitalist appears in multiple markets, Theorem 9 allows us to identify a coefficient on distance specific to each venture capitalist. However, all of the parameters in (4.8.1), not just distance, have a homogeneous coefficient.

in selecting investment opportunities. Such governance considerations are likely to be especially important in empirical settings like ours in which startup assets are mainly intangible, the length of product development can be decades, and product development often costs hundreds of millions of dollars (Lerner, Shane and Tsai, 2003). Co-location can help for governance and monitoring reasons, as well as for facilitating the provision of value-added business development services such as organizational professionalization (Hellmann and Puri, 2000).<sup>72</sup>

While VCs tend to specialize by factors such as the startup lifecycle stage of development, perhaps the most-mentioned aspect of specialization is industry experience (Hsu, 2004; Sørensen, 2007). Industry domain experience may be important in both assessing investment opportunities as well as intermediating startup business development services (such as connecting to executive-level managers in an industry domain or striking alliance relationships with industry incumbents) – indeed Sørensen estimates that the former factor is twice as important as the latter in explaining the likelihood that a startup goes public. Furthermore, Sorenson and Stuart (2001) find that industry domain experience can allow VCs to invest in more geographically distant portfolio firms.

Our measure of a venture capitalist’s past experience in all biotech and medical sectors is equal to the number of past deals where that venture capitalist was the lead investor in the first round, which typically corresponds to knowledge in a startup domain. Experience is constructed using the complete history of our data, which starts in 1960. We do not wish for our measure of experience to trend over time, so we normalize experience each year

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<sup>72</sup>Our estimates are in units of distance instead of monetary units, such as dollars. Therefore, we cannot definitively say that the differences we estimate in match production correspond to large differences in monetary values. Nevertheless, the prior importance of distance in academic research on venture capital suggests that distance is economically important in monetary terms.

to be between 0 and 1, with 1 being the venture capitalist with the most experience that year and 0 being the venture capitalist with the least experience. Table 4.25 reports that the mean level of the venture-capitalist specific variable experience across realized matches (not venture capitalists) in the estimation samples are 0.06 for biotech and 0.07 for venture capital.<sup>73</sup> This means the venture capitalist with the most experience in a given year typically has around 20 times the past deals as the mean venture capitalist. In estimation, we scale venture capitalist experience to have a mean of zero and a standard deviation of 1. This makes the standard deviation of experience similar to the standard deviation of distance.<sup>74</sup>

We also compute the venture capitalist's experience in the specific four-digit sector of an entrepreneur. There are 85 four-digit sectors in our biotech and medical data. Again, we normalize sector-specific experience to be between 0 and 1 for each year of data by making the venture capitalist with the most sector-specific experience have a value of 1. The mean level of four-digit, sector-specific experience across realized matches for biotech is 0.19, or about 1/5 of the past deals of the venture capitalist with the most sector-specific experience. For the medical industry, the mean sector-specific experience is 0.13. In estimation, we rescale sector specific experience to have close to a mean of zero and a standard deviations of 1. Note that for a given venture capitalist, four-digit experience varies across entrepreneurs within the same two-digit sector. Sector experience is therefore a match-specific characteristic. We

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<sup>73</sup>Table 4.25 reports summary statistics for realized matches, so a venture capitalist's overall experience is counted twice if the venture capitalist makes two matches in a given year.

<sup>74</sup>Total experience is a count of past matches and so could be considered to be a function of a lagged dependent variable in another matching model where venture capitalists were allowed to be unmatched or have vacant quota slots and so the number of matches of each venture capitalist (in addition to the identity of the startup partners) was an outcome of the matching model. Past observed agent and match characteristics are statistically exogenous shifters of the outcomes of past matching games and so provide exogenous variation in past matches and hence lagged dependent variables. While this variation could lead to an approach that distinguishes venture capitalist serially correlated (over time) unobserved heterogeneity from true state dependence from experience, we do not explore that here as we assume that unobserved complementarities are statistically independent of experience.

do not measure an entrepreneurial firm's experience because we observe no information on the founders of the startup.

The most important asset of an entrepreneurial startup is likely its intellectual property. Perhaps the only way for an outside researcher to directly measure this intellectual property is to look at patents. We have data on the number of patents held by the entrepreneurial firm at the time of the first round of venture capital investment. The literature has identified at least two distinct roles of patenting for startups: as a legal instrument to exclude others from using intellectual property in the product market (or to license those rights to others in the market for technology) and as signaling devices to capital providers. Patents are important in the biotechnology and medical industries for both reasons (Levin et al., 1987). Table 4.25 shows that the mean number of patents is just over 2 for both biotech and medical, with high standard deviations of 7.5 for biotech and 11.1 for medical. 32% of entrepreneurs have zero patents at the time of the initial round of venture capital investment. The production function uses the logarithm of the patent count plus one, rescaled to have a mean close to zero and a standard deviation close to 1. We recognize that patent counts are not a perfect measure of intellectual property; this partly motivates this paper's focus on unobservable characteristics.

As discussed in the identification sections, complementarities between matched firms drive matching. Therefore, the firm-specific but not match-specific characteristics venture capitalist experience and startup patent count would drop out of the calculation of the production maximizing assignment if included without interactions. The production function includes the interaction between overall venture capital experience and the log of the patent count plus one. There may be positive complementarities between startups' patent position and more experienced (and therefore more reputable) venture capitalists. In patenting's exclu-

sionary role or in patenting's role of facilitating markets for technology, startups with more patents may wish to match with more experienced venture capitalists. Similarly on the signaling side, more experienced venture capitalists may value startup patents more highly (Hsu and Ziedonis, 2008). Table 4.25 reports sample statistics for the interaction term across actual matches (not hypothetical matches). For biotech, the interaction term has a mean of around zero and a standard deviation of 1.1. For medical, the mean is 0.2 and the standard deviation is 1.6.

#### 4.8.4 Distribution of Unobserved Complementarities

In our empirical work, we will treat firm indices as having no common meaning across matching markets. For entrepreneurs, firm indices do in fact lack meaning, as each startup appears in exactly one matching market. Venture capitalists may appear in multiple matching markets, but we still treat the same venture capitalist in different matching markets as being a different firm.

The production function in (4.8.1) has match-specific unobservables. The identification argument in Corollary 12.1 uses match-specific characteristics, which in (4.8.1) are the distance between the headquarters of the entrepreneur and the venture capitalist and the venture capitalist's experience in the entrepreneur's four-digit sector. We impose that unobservables and observables are distributed independently.

Given our data and approach of treating firm indices as irrelevant within a market, we impose that the joint distribution of unobserved complementarities is exchangeable in firm indices. Lemma 5.3 states that if  $G(E)$  is unobservable in agent indices, then so is  $F(B)$ . This result extends naturally to many-to-one matching. Even though the number of venture capitalists



can be less than the number of entrepreneurs, we treat each venture capitalist as a single firm and not a number of synthetic firms equal to the venture capitalist's quota.<sup>75</sup>

We operationalize our ideas about exchangeability using the following parametric structure. We assume the joint distribution  $F$  of the unobserved complementarities in  $B$  will be multivariate normal with the following properties

$$\begin{aligned}\text{Corr}(b_{u_1,d_1}, b_{u_2,d_2}) &= \rho_1, \text{ if } u_1 \neq u_2, d_1 \neq d_2 \\ \text{Corr}(b_{u_1,d_1}, b_{u_2,d_1}) &= \rho_2, \text{ if } u_1 \neq u_2 \\ \text{Corr}(b_{u_1,d_1}, b_{u_1,d_2}) &= \rho_3, \text{ if } d_1 \neq d_2 \\ \text{SD}(b_{u_1,d_1}) &= \sigma.\end{aligned}$$

There are different correlations between pairs of unobserved complementarities depending on whether the venture capitalists  $u_1$  and  $u_2$  are the same and whether the startups  $d_1$  and  $d_2$  are the same. Keep in mind that the unobserved complementarity  $b_{u,d}$  always involves the upstream firm 1 and the downstream firm 1, in addition to the listed firms  $u$  and  $d$ . We will estimate  $\rho_1$ ,  $\rho_2$ ,  $\rho_3$  and  $\sigma^2$  in addition to the parameters in the production function (4.8.1). We collect these parameters into the vector  $\theta = (\pm 1, \beta_{\text{Sector}}, \beta_{\text{ExperPatents}}, \rho_1, \rho_2, \rho_3, \sigma)$ , where the  $\pm 1$  corresponds to the coefficient on distance.

By Lemma 5.2, there exists a distribution  $G(E)$  for the unobserved match characteristics that induces our multivariate normal  $F(B)$  by the transformation (4.2.8). By Example 6, the underlying distribution  $G(E)$  giving our  $F(B)$  is not in general the simple multivariate normal distribution in the text of the example.

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<sup>75</sup>A venture capitalist firm typically has multiple employees known as venture capitalists. Our model and data focus on a VC firm, not its individual employees. One interpretation of  $e_{u,d}$  might be that it includes information on the VC employee that would be assigned to the startup if the match forms.

### 4.8.5 Estimator

The many-to-one matching model is a special case of Sotomayor (1999), and therefore has a unique pairwise stable assignment with probability 1. Furthermore, this pairwise stable assignment can be computed with the linear program described in that paper. Therefore, a simulated nested fixed point estimator is appropriate, where the objective function involves an integral over the unobserved complementarities  $B$  and the corresponding integrand involves solving the linear program for each realization of  $B$ .

A likelihood exists because we have fully specified the data generating process. Let the object  $X_m$  collect all the match and firm-specific observables (including the number of firms on each side of the market and the elements that in the discussion of identification would instead be in  $Z$ ) for realized and counterfactual matches for market  $m$  and let  $A_m$  be the assignment in the data for market  $m$ . The likelihood contribution for market  $m$  involves the computation of  $\Pr(A_m | X_m; \theta)$ , or

$$\Pr(A_m | X_m; \theta) = \int_B 1[A_m \text{ stable} | X_m, B; \theta] \tilde{d}F_B(B; \theta). \quad (4.8.2)$$

The indicator function  $1[A_m \text{ stable} | X_m, B; \theta]$  is equal to 1 if  $A_m$  is computed to be the pairwise stable assignment for market  $m$  with draw  $B$ , using the linear program in Sotomayor (1999). The symbol  $\tilde{d}$  in  $\tilde{d}F$  stands in for the common integration symbol “ $d$ ” from calculus, to distinguish this use from our notation  $d$  for a downstream firm.

A computational challenge with the likelihood contribution (4.8.2) is that  $\Pr(A_m | X_m; \theta)$  will typically be intractably close to 0. Consider the simple example of one-to-one matching without the option of being unmatched. There are  $N!$  possible assignments with  $N$  upstream and  $N$  downstream firms. Typically,  $\Pr(A_m | X_m; \theta)$  will be on the order of  $1/N!$ . As  $N!$  can

be close to the number of atoms in the universe with  $N = 50$ , the likelihood will involve the computation of intractably small numbers. The same issue with intractably small numbers will apply to generalized method of moments (GMM) estimators using the efficient choice of moments, which are based on the scores of the likelihood (Hajivassiliou and McFadden, 1998; McFadden, 1989).

Instead of attempting to compute  $\Pr(A_m | X_m; \theta)$  directly, we work with a simulated moments estimator that uses moments that are easier to compute. Our chosen estimator is statistically inefficient but is tractable to compute. Let  $g(A, X)$  be a function of an assignment  $A$  and agent and match characteristics  $X$  that gives some market-level output. Let  $A(X, B; \theta)$  be the pairwise stable assignment of a market with observables  $X$  and unobserved complementarities  $B$ , evaluated at the parameter  $\theta$ . With data on  $M$  markets, an empirical moment as a function of  $\theta$  is

$$Q_{g,M}(\theta) = \frac{1}{M} \sum_{m=1}^M \left[ \int_B g(A(X_m, B; \theta), X_m) \tilde{d}F_B(B; \theta) - g(A_m, X_m) \right]. \quad (4.8.3)$$

The moment condition is that  $Q_{g,M}(\theta) = 0$ . Each choice of  $g$  indexes a separate empirical moment  $Q_{g,M}(\theta)$ .<sup>76</sup> The exact choices of  $g(A, X)$  are described in the appendix of the paper version of the chapter.<sup>77</sup> We have separate moments based on agent-specific and firm-specific moments. For example, a moment might be based on the quantiles of the match-specific characteristics for only the matches in the pairwise stable assignment. These selections for the moments  $Q_{g,M}(\theta)$  provide an estimator that uses only the sorting patterns captured by

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<sup>76</sup>For readability, the above notation suppresses the reality that the dimension of the random matrix  $B$  depends on the numbers of entrepreneurial and venture capitalist firms in  $X_m$ .

<sup>77</sup>The outcome  $A(X, B; \theta)$  of an assignment being pairwise stable is discrete and hence induces a non-differentiability in the simulated GMM objective function. We use a non-gradient based, global optimization routine known as a genetic algorithm to maximize the function. We vary optimization routine settings, such as the population size of points, in order to check that our estimates appear to be a global minimum.

the choices for  $g$ . Inside the integral over the random matrix  $B$  in each moment  $Q_{g,M}(\theta)$ , the portion of the integrand  $g(A(X_m, B; \theta), X_m)$  is typically nonzero. This is unlike the integrand of the likelihood contribution in (4.8.2), which involves an indicator function that will be 0 for an intractably large number of realizations of  $B$  for matching markets with the numbers of agents in our data. Intuitively, our moments work only with agent characteristics while the likelihood contribution (4.8.2) exploits agent indices fully to compute the exact probability of the assignment in the data. Our estimator is statistically inefficient but easier to compute. We use the usual optimal weighting matrix from two-step GMM.

The integral in each moment  $Q_{g,M}(\theta)$  is approximated on the computer using simulation over the random matrix of unobservable complementarities  $B$  (McFadden, 1989; Pakes and Pollard, 1989). The method of simulated moments estimator is consistent for  $\theta$  as  $M \rightarrow \infty$  for a fixed number of simulation draws for the matrix  $B$ . In practice, we use Halton sequences to sample  $B$  while reducing simulation error at the risk of introducing some small bias from the deterministic simulation draws. The number of draws of the entire matrix  $B$  is 1000 for biotech and 500 for medical. The integral that is simulated has a dimension equal to the number of elements in the matrix  $B$ ,  $(N_m^u - 1) \cdot (N_m^d - 1)$  for market  $m$ . In the estimation sample, our market size cap means that the maximum number of elements of  $B$  is 841.

The standard errors are adjusted for simulation error. The standard errors use the usual sandwich formulas with numerical derivatives approximating the actual derivatives, as the assignment outcome  $A(X, B; \theta)$  is discrete and hence not differentiable in the simulation estimator (it is smoothed by the integral over the multivariate normal distribution for  $B$  in an estimator without numerical integration error). We calibrate the stepsize of the numerical derivatives to achieve somewhat decent confidence interval coverage in the Monte Carlo studies to be discussed now.

### 4.8.6 Monte Carlo Study

We conduct a Monte Carlo study at the reported point estimates to ensure that our chosen moments are informative in that they lead to low bias in the estimates of  $\theta$ , as done, for example, in Eisenhauer, Heckman and Mosso (2015). We also use the Monte Carlo study to calibrate the stepsizes of the numerical derivatives used for the standard errors.

We perform one Monte Carlo study based on the biotech industry and a separate Monte Carlo study based on the medical industry. For each industry, the true parameters are taken from the actual point estimates described below in Table 4.27. Each replication proceeds as follows. We randomly sample 35 matching markets for biotech and 15 matching markets for medical; these are similar to the numbers of matching markets used in the real data estimation. Sampling a matching market means using the number of startups, the number of venture capitalists, the match characteristics, and the agent characteristics from the data for that market. We then also sample a matrix  $B$  of unobserved complementarities and compute the pairwise stable assignment. We then use the data on assignments and observable characteristics to estimate the parameter vector  $\theta$ . We conduct 100 Monte Carlo replications for the biotech industry and, for computational reasons, 45 Monte Carlo replications for the medical industry.<sup>78</sup>

Table 4.26 reports the two Monte Carlo studies, one for biotech and one for medical. For each parameter in  $\theta$ , the table reports the bias, the root mean squared error (RMSE) and the coverage of the nominal 95% confidence intervals, adjusted for simulation error. RMSE

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<sup>78</sup>We use a non-gradient based, global optimization routine known as a genetic algorithm to minimize the method of simulated moments objective function. We vary optimization routine settings, such as the population size of points, in order to check that our estimates appear to be a global minimum.

is calculated as

$$\sqrt{\frac{1}{I} \sum_{i=1}^I \left( \hat{\theta}_i^{\text{syn}} - \theta^{\text{true}} \right)^2},$$

where  $\hat{\theta}_i^{\text{syn}}$  is the estimator using synthetic data for the  $i$ th out of  $I$  Monte Carlo replications. Table 4.26 shows that our choice of moments do lead to relatively low bias and RMSE; the absolute value of bias is always less than or equal to 0.05 for all parameter values.<sup>79</sup> For matching markets with much larger numbers of firms, the same number of simulation draws and the same number of markets in the data, unreported Monte Carlo studies indicate that the finite-sample bias from simulation error will be more substantial. Table 4.26 shows that the coverage of the nominal 95% confidence intervals are above 90% for all but one parameter for the biotech industry and two parameters for the medical industry. The RMSEs are small so we are not worried about the undercoverage on these three parameters leading to falsely rejecting the hypothesis that a parameter is zero.

Table 4.26: Monte Carlo Study

	Biotech				Medical			
	True	Bias	RMSE	Cover.	True	Bias	RMSE	Cover.
Sector experience	1.28	0.04	0.15	0.98	0.31	0.03	0.10	0.80
Total experience * log(patents+1)	0.02	0.01	0.06	0.88	0.67	0.05	0.18	0.84
Standard deviation of UC	2.08	0.01	0.03	0.94	1.96	0.02	0.04	1.00
Correlation, no common firm	0.17	0.00	0.02	0.94	0.74	-0.02	0.04	0.93
Correlation, common startup	0.23	0.01	0.02	0.94	0.74	0.00	0.04	0.98
Correlation, common VC	0.94	-0.01	0.02	0.91	0.77	-0.02	0.04	0.96

Both studies uniformly sample matching markets with replacement from the corresponding real-data matching markets.

Biotech: sample size of 38 markets, 100 Monte Carlo replications.

Medical: sample size of 15 markets, 45 Monte Carlo replications

<sup>79</sup>As discussed in the section on the point estimates, the economic magnitude of Monte Carlo's true value on the interaction term between total experience and patents is close to zero (0.02) for the biotech industry. Therefore, the bias of 0.01 is also small in economic magnitude even it is large compared to the true parameter in the Monte Carlo.

### 4.8.7 Estimates

Table 4.27 reports the estimates and standard errors. There are four separate sets of estimates: two for the biotech industry and two for the medical industry. For each industry, we report estimates without and with the term interacting startup patents and venture capitalist total experience. The parameters on the match-specific characteristics are not overly sensitive to the inclusion of the agent-specific characteristics, so we focus on the estimates with the interaction term. The top half of Table 4.27 reports the estimates of the production function parameters and the bottom half reports the estimates of the parameters of the multivariate normal distribution of unobserved complementarities.

Table 4.27: Venture Capital Parameter Estimates

Measure	Biotech		Medical	
Distance	-1 (-)	-1 (-)	-1 (-)	-1 (-)
Sector experience	1.32 (0.30)	1.28 (0.20)	0.22 (0.03)	0.31 (0.06)
Total experience * log(patents+1)		0.02 (0.02)		0.67 (0.11)
Standard deviation of unobserved complementarities	2.52 (0.19)	2.08 (0.04)	2.36 (0.17)	1.96 (0.10)
Correlation no common firms	0.41 (0.06)	0.17 (0.02)	0.66 (0.13)	0.74 (0.06)
Correlation same startup	0.45 (0.06)	0.23 (0.05)	0.66 (0.12)	0.74 (0.06)
Correlation same venture capitalist	0.96 (0.38)	0.94 (0.01)	0.79 (0.08)	0.77 (0.04)

We first consider the production function parameter estimates for the biotech industry in Table 4.27. Both distance and sector experience are match characteristics that have been normalized to have a standard deviation of around 1. The point estimate of 1.28 on VC experience in the four-digit sector of the startup indicates that sector experience is more

important than distance, although the confidence interval for the sector experience parameter contains the absolute value of the normalized coefficient on distance, 1. The coefficient on the interaction between startup patents and venture capitalist experience is 0.02. A change in the number of patents from 1 to 2 is equivalent to a 0.18 change in the rescaled log of patents measure. At say a value of the rescaled VC total experience of 0.5, this change in patents results in a change in match production of  $0.02 \cdot 0.18 \cdot 0.5 = 0.002$  units of the standard deviation of distance, an economically tiny effect. Another way to interpret the coefficient on the interaction term is to look at the standard deviation in the value of the interaction across realized matches, which is 1.1 in Table 4.25. An one standard deviation increase in the interaction term then results in a  $0.02 \cdot 1.1 = 0.02$  change in production in units of the standard deviation of distance. Patents and total experience do not seem to play a role in match production in biotech.

Now consider the production function parameter estimates for the medical industry in Table 4.27. The parameter of 0.31 on sector experience, with a low standard error, is economically and statistically lower than the normalized coefficient of 1 on distance. So distance is more important than sector experience in the medical industry. The coefficient on the interaction between patents and VC total experience is 0.67, 34 times the coefficient of 0.02 in biotech. Still, the change from 1 to 2 patents at a rescaled total VC experience of 0.5 results in a change of production of 0.06 distance standard deviations, still not a large effect in economic magnitudes. Table 4.25 reports the standard deviation of the interaction term across realized matches is 1.6. A one standard deviation change in the interaction term then results in production increasing by  $0.67 \cdot 1.6 = 1.1$  distance standard deviations. Interpreted using the standard deviation of the interaction term across realized matches, the effect of the interaction term is about the same as distance and greater than the effect of sector experience.



Next, we interpret the parameters of the multivariate normal distribution of unobserved complementarities. Recall the definition in (4.2.6): an unobserved complementarity is indexed by four firms and is the sum and difference of unobserved match characteristics involving four matches. For the unobserved complementarities  $b_{u,d}$  from (4.2.8), two of the four firms are always fixed at upstream firm 1 and downstream firm 1, although firm indices have no meaning across markets in our analysis. The other two firms, the  $u$  and  $d$  in  $b_{u,d}$ , vary across the unobserved complementarities in the matrix  $B$ . Interpreting the distribution of unobserved complementarities requires subtlety; it is easier to think in terms of unobserved match characteristics. Unfortunately, the distribution of unobserved match characteristics is not identified.

Consider the standard deviations of the unobserved complementarities. For biotech, the standard deviation is 2.08, with a small standard error. The standard deviation can be interpreted to mean, in a loose sense, that unobserved complementarities are twice as important as distance, which has a coefficient of -1 and a standard deviation of 1. In the same loose sense, the point estimate is that unobserved complementarities are  $2.08/1.28 = 1.6$  times as important as sector experience. For the medical industry, the standard deviation of unobserved complementarities is 1.96, meaning that, again in a loose sense, that unobserved complementarities are twice as important as distance and  $2.08/0.31 = 6.3$  times as important as sector experience.

There are three correlation parameters in the multivariate normal distribution. The first correlation is  $\text{Corr}(b_{u_1,d_1}, b_{u_2,d_2})$ , the correlation between two unobserved complementarities when neither of the upstream firms or downstream firms (other than upstream firm 1 and downstream firm 1) are the same. For biotech, this correlation is positive and low, at 0.17, with a small standard error. Presumably the positive correlation reflects the presence of

upstream firm 1 and downstream firm 1 in all unobserved complementarities  $b_{u,d}$ . For the medical industry, the correlation is much higher, at 0.74, also with a low standard error. When upstream firm 1 and downstream firm 1 contribute to unobserved complementarities in one set of four matches, they do so in others.

The second correlation is  $\text{Corr}(b_{u_1,d_1}, b_{u_2,d_1})$ , the correlation between two unobserved complementarities when the startup is the same. The correlation for biotech of 0.23 has the same sign but is lower than the correlation for the medical industry of 0.74. The third correlation is  $\text{Corr}(b_{u_1,d_1}, b_{u_1,d_2})$ , the correlation between two unobserved complementarities when the venture capitalist is the same. For biotech, this correlation of 0.94 is quite high. For the medical industry, the correlation of 0.77 is a little lower. For biotech, the correlation between two complementarities involving upstream firm 1, downstream firm 1 and the same venture capitalist is much higher than the correlation between complementarities involving upstream firm 1, downstream firm 1 and the same startup as well as the correlation when only upstream firm 1 and downstream firm 1 are the same. This seems to mean that venture capitalists are playing more of systematic role in unobserved complementarities across a variety of sets of four matches than the more idiosyncratic role of startups. In the medical industry, the three correlations are all high and about the same. The distribution of  $B$  is close to being equicorrelated and hence exchangeable in all the elements of  $B$ , not just exchangeable in the agent indices (which is imposed by our choice of distribution).

#### 4.8.8 Discussion of Unobserved Complementarities

The somewhat subtle interpretation of the estimated distribution of unobserved complementarities showcases the loss of information from using data on only matched agents, which

is common in empirical work on matching. Nevertheless, the distribution of unobserved complementarities is in the same units as the contribution of observables to the production function and the two can be compared. For both the biotech and medical industries, we found that the standard deviation of unobserved complementarities was greater than the individual contributions of distance, VC sector experience, startup patents and VC total experience. However, combining the contributions of all observables in the production function, particularly sector experience and distance, suggests that the role of unobserved complementarities is roughly the same as the role of all observables.

Reporting the standard deviation of the unobserved complementarities has some analogs to reporting the standard deviation of error terms in other empirical literatures. For example, the standard deviation of wage regression residuals is often thought of as representing the dispersion in unobserved worker ability. The standard deviation of production function residuals is typically called the dispersion in total factor productivity. There are thousands of papers on understanding the often unmeasured factors that affect worker ability and firm total factor productivity. Likewise, our estimates of large standard deviations of unobserved complementarities in venture capital suggest that there is good motivation for additional academic research on the factors making venture capital investments more productive.

## 4.9 Conclusion

Matching models that have been structurally estimated to date have not allowed rich distributions of unobservables. It has been an open question whether data on who matches with whom as well as match or agent characteristics are enough to identify such distributions of unobservables. In this paper, we explore several sets of conditions that lead to identification.

Using data on only matched firms, one can identify distributions of what we call unobserved complementarities but not the underlying primitive distribution of match-specific (or agent-specific) unobservables. The distribution of complementarities is enough to compute assignment production levels and therefore counterfactual assignment probabilities. In extensions, we can include other covariates  $X$  and identify distributions of unobservable complementarities conditional on  $X$ . We show that it is possible to identify heterogeneous-within-a-market coefficients on the large support match characteristics. The results extend naturally to two-sided, many-to-many matching.

If the data contain unmatched firms, the individual rationality decision to not be unmatched helps identify the distribution of primitively specified unobserved match characteristics, not just the distribution of unobserved complementarities. We extend this result to the fairly general case of matching with trades, as in Hatfield et al. (2015).

Our empirical work studies biotech and medical venture capital. We estimate the degree to which venture capitalists change the production of matches. Among many other empirical results, we find that the standard deviation of unobserved complementarities is roughly of the same order of magnitude as the contribution to production from the observables match and agent characteristics.

## CHAPTER V

### Conclusions

This dissertation develops new methods to analyze firm behaviors and provides estimates and predictions that inform antitrust and innovation policies. The first chapter shows that horizontal merger policies may be tougher when taking into account a merger's effects on the composition of product offerings in addition to the merger's price effects. The second chapter quantifies and decomposes the effects of vertical integration. I show that the investment coordination effects are pro-innovation and dominate the price effects. The results suggest that vertical integration policies should fully consider the potentially positive dynamic implications of a vertical merger. Both chapters are empirical studies in the context of the US smartphone industry. The third chapter develops identification strategies for a general class of matching games and estimates the formation of investment relationships between venture capitalists and biomedical startups. The estimates show that unobservables may be as important as observables in determining which VC invests in which startup firm, and understanding these unobserved factors is important for innovation policies.

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