Teaching to “The Good Ones”? Examining the Relationship Between Inequity and the Practice of and Preparation for Postsecondary Mathematics Instruction

by
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DEDICATION

This dissertation is dedicated to my grandparents, parents, and brother. Each of you have taught me about embracing difference and fighting for equity in your own ways. You have taught me to pay attention to how people tell their own stories and histories. From my grandparents, I have learned about resistance and survival. Your courage, strength, and capacity to love in the face of unimaginable evil is a legacy I cherish that fuels and sustains me in fighting for a brighter future. You taught me to reach out to others and value life, people, and relationships above all else. From my parents, I have learned that education is a worldview, a way of being, and a mindset, not a credential on a piece of paper. Education is something to seek out in unexpected places with curiosity about the world and people as my motivation. You have taught me that love grows exponentially, and we can grow our capacity to love by acting out of love. You have taught me to be more thoughtful, cautioned me to be slower to anger, and to stand up for myself and others. From my brother, I learned about the truest form of friendship. You have taught me to see the world through another's eyes and to withhold judgement. You have taught me that listening happens with one's heart as well as one's ears. All of you have shaped me as a person and scholar in unmeasurable ways, and thus, share in the creation of this work. I thank God for blessing me with such wonderful family and sustaining me in my journey. May this work be worthy of you and all you have given me and others.

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The etymology of the word *acknowledge* is likely from the Middle English word *aknou*, meaning to recognize, and the Old English suffix *–leche*, meaning knowledge.[1] Therefore, to acknowledge is to recognize knowledge, which is a very fitting way to begin a dissertation manuscript. This dissertation project is a product of not only the knowledge others have shared with me along the way, but also many people's efforts to teach me about the creation of knowledge in the social sciences. As such, I begin this dissertation manuscript by acknowledging as many of these influential people as I can for they have shaped this work, me as a scholar, and my future endeavors.

I want to start by acknowledging five scholars who have not only served as members of my dissertation committee, but have also been incredible mentors during this journey. I love words, and have always enjoyed using them in communication, yet I find that I simply do not have the appropriate words to thank these women for what they have done for me and been to me these past years. Lisa Lattuca and Anna Neumann have taught me about the landscape of higher education, not only dwelling on its current topography but the field's potential, especially in teaching and learning. Their commitment to that future is inspiring and continues to pave the way for early career scholars, like myself, to stretch the field to consider the power dynamics and politics of opportunity within postsecondary instruction. Allison Ryan introduced me to the field of motivation theory and the stereotype threat literature. She helped me to think expansively about the application of motivation theory in education. Allison also motivated me to continue to push on the boundaries of the fields I was working across in moments when I doubted myself and my work. Carla O'Connor helped me see the field of education through a sociologist's eyes, particularly in our conversations about power dynamics in instruction. Inside and outside the classroom, Carla pushed me to continually consider my own positionality in my teaching and research across identity difference. She has deeply shaped the way I think, speak, and write about

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[1] This information was accessed on June 9, 2016 from the word origin information from http://www.dictionary.com/browse/acknowledge

my own and other affinity groups, making me not only a stronger scholar but also a better person.

In particular, I want to thank Deborah Loewenberg Ball with all my heart for what she has given me as my dissertation chair, advisor, mentor, and most importantly, friend. The different components of the theoretical framework I developed for my dissertation emerged out of our conversations over the years. One question Deborah posed regularly, which I struggled with until recently, was "how do you see these different ideas fit together?" This question asked me not only to stretch myself in how I was articulating my ideas, findings, and interpretations of data but also to make transparent to others my understanding of their interrelatedness. The generosity of this particular question, as well as many others, always struck me. The implication was that my thinking was valuable and worth understanding. Not only were my thoughts about others work valued, but the knowledge I was creating in my field through my own insights and understanding had value. While this might be obvious to many students moving through a doctoral program, it was never apparent to me. So while I could write extensively about what Deborah has taught me about instruction, teacher education, theory, writing, and many other areas of scholarship in our field, the most precious lessons I learned from Deborah were about mentorship as a form of teaching and about my own potential as a scholar receiving mentorship and a faculty member who will mentor others. She has shaped me so many ways as a scholar and person. I am immeasurably grateful to Deborah for the opportunity to learn from and with her over the past five years and excited for more opportunities in the years to come. The work I have and continue to do is a result of the generosity, knowledge, and skill of Anna, Lisa, Allison, Carla, and Deborah; however, I claim the missteps as my own and hope as a reader you will be generous about them as this is my first foray into publishing these ideas.

I am grateful to have many others to thank beyond my dissertation committee members, who have supported me in many ways during this journey. I had never seriously considered pursuing graduate studies, let alone a doctorate. I wanted nothing more than to teach middle and high school in the Jerusalem public schools – where I was living. I had an opportunity to do a fully subsidized Master's degree in political science at the Hebrew University of Jerusalem and decided to go back to school. I was more interested in the experience of studying in one of my non-dominant languages, than the degree itself. I thought the experience would make me a better teacher (I was teaching English as a foreign language at the time). Midway through my degree a friend of mine from the department, Aviv Cohen, began pushing me to do a doctorate. I remember laughing at the idea.

Years later, MA degree in hand, I took a job working with the Rothschild Foundation (*Yad Hanadiv*) on a literacy project leveraging communities of practice with Josh Glazer. Josh would stop by my office regularly to discuss issues of educational policy and soon began a campaign to convince me to do a doctorate degree. He saw my passion for teaching and drive to learn more about school improvement as a strong indication I would find outlets for those interests in the academy. Over time (roughly two years of these visits), Josh honed his argument to why I should do a PhD in education at the University of Michigan with his former advisor, David Cohen. Josh won that argument in 2010 at a conference in Jerusalem when he introduced me to David. After a week of fascinating conversations with David, I decided that maybe I could successfully navigate this journey called a doctorate. David supported me throughout the first three years of my PhD, taking up and considering my ideas with generosity and patience. When I decided to look at the systemic problems of opportunity in mathematics education, David counseled me to continue my dissertation work Deborah Ball as my chair, concerned only that I would have the support to ask my own questions. Without Aviv, Josh, and David's unwavering confidence in my potential and determination to share those visions with me, I can say with absolute conviction that I would not have taken the path I did. They taught me to see beyond what I had been taught and believed about my potential. They taught me to do for myself, what I had learned to do long ago for my students. They also were constant sources of support, equal parts encouragement and constructive feedback, which are so important during a doctorate. I will forever be grateful to them for the lesson and opportunity to contribute to the field of education through my scholarship.

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One remarkable aspect about the School of Education at the University of Michigan that I discovered during my five years there was how generous the faculty are with their time and mentorship, even with their heavy teaching, research and service loads. In addition to my wonderful relationships with my dissertation committee members and advisors, I have had the opportunity to learn from and connect with many of our wonderful faculty. Among many others, I want to especially thank Elizabeth Moje, Henry Meares, Rob Jagers, Stephanie Rowley, Hyman Bass, Annemarie Palincsar, Matt Ronfeldt, Don Peurach, Donald Freeman, Betsy Davis, Nell Duke, Leah Bricker, Gina Cervetti, Betty Overton-Adkins, Julie Posselt, Mike Bastedo, Matt Diemer, Tim Boerst, Kendra Hearn, Bob Bain, Maisie Gholson, Chandra Alston, Pat Herbst, Debi Khasnabis, Simona Goldin, Vilma Mesa, Chauncey Monte-Sano, Jeff Mirel, Mary Schleppegrell, Kara Suzuka, Pamela Moss, Meghan Shaughnessy, and Mark Hoover for their kindness and mentorship. Thank you to the faculty beyond Michigan as well who have taken the time to share ideas and teach me about their fields and work, especially Diana Hess, Estela Bensimon, Kent Harber, Geoffrey Cohen, David Yeager, Matt Kloser, Brittany Aronson, and Conra Gist. I know that I have missed some names and hope those individuals will accept my apologies and gratitude.

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I want to thank my family again for all their love, support, and what they have taught me over the years. I have no words sufficient to this task. I thank God for placing me in this life with you; I am grateful for all of it, the hard and the easy parts. Thank you to my friends across the globe for their love, laughter, and surprises. I am so blessed and cannot begin to try to list your names. You know who you are and how deeply I love and honor you. And thank you to my little LoLa, my silent partner in this journey. I could not ask for a better dog. Describing you as a friend, or family, or a pet does not do you justice. You are all of that and more. You taught me more about myself than anyone else has or could. Your love knows no bounds. Your company is always willing and joyous. You are not only a life saver, little girl, but a life maker. And I thank you with all my heart for sharing your life with me. You have heard every word of this dissertation and seen every victory and setback along the way. Thank you for warming my lap and heart during this process. Thank you for reminding me to raise my head from the work and find joy and peace walking the world with you. *Eim hamon ahavah v'rav todot!*

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LIST OF ABBREVIATIONS

Association for the Study of Higher Education (ASHE)

Center for the Study of Higher and Postsecondary Education (CSHPE)

Doctoral Student Instructor (DSI)

Legitimate Peripheral Participation (LPP) (Lave & Wenger, 1991)

Mathematic Teaching Laboratory (MT Lab)

Science, Technology, Engineering, and Mathematics (STEM)

“Very High Research Activity” (RU/VH) (Carnegie Designation)

ABSTRACT

This dissertation focused on inequity in calculus instruction through a two-part study that built on the findings from an earlier exploratory study. The exploratory study, conducted in the same department, revealed connections between personal theories of mathematics intelligence that doctoral student instructors (DSIs) held for themselves and those that they held for their students. The first component of the dissertation project was a design intervention study that examined a practice-based approach to preparing DSIs to give students equitable feedback, a core instructional practice, in their postsecondary calculus instruction. The second component was a comparative investigation of teacher/student interactions across identity difference in postsecondary calculus instruction. Four of the instructors from the intervention study were observed and interviewed throughout their first semester of teaching to examine their interactions with their undergraduate students across identity difference. The three articles in this dissertation focus on the findings from this second study.

The findings suggested that the DSIs, who were members of overrepresented groups (i.e., majoritized students identifying as men and Asian or White), held some common understandings about what in meant to do mathematics well, which they used as lenses for gauging their own and others' potential to successfully navigate mathematics as a discipline. Moreover, evidence from this study indicated that when the DSIs viewed students through these lenses that they noticed different characteristics for minoritized and majoritized students, even when they exhibited similar behaviors. These impressions formed the DSIs' opinions about the potential of their students, which systematically disadvantaged women, especially those identifying as Latina and Black. Finally, the findings suggested that the DSIs acted on their ideas about intelligence through their teaching practices, creating differentiated access to learning opportunities and marginalizing minoritized students. The resulting inequitable approaches to instructional practices may reduce domain identification and motivation, create lower expectations, and depress performance for minoritized students in mathematics classrooms as explored in the pre-calculus case presented in the third article. These findings support the need for the design of

equitable approaches to mathematics instructional practices and the explicit preparation of postsecondary instructors to engage in them.

Overview of the three articles

The first article argues for the explicit preparation of future mathematics faculty to engage in equitable instructional practices in their teaching to create access for students historically underserved. Moreover, the article illustrates how those instructional practices should be chosen from practices that exist at the intersection of high-leverage practices (HLPs) and wise schooling practices (WSPs). These intersectional practices disrupt key threats to minoritized students learning in mathematics instruction and should be integrated into postsecondary mathematics teaching. Selecting practices from that intersection ensures that the limited time and resources allocated to the preparation of doctoral students for instruction are utilized to their best advantage with the aim of improving undergraduate mathematics instruction for students from historically minoritized groups.

The second article argues that very rarely do higher education scholars address issues of inequity within higher education by looking directly at the core enterprise of higher education - instruction. This article argues that postsecondary mathematics instructors may be unconsciously (re)producing inequity through their instructional practices. This study examines calculus instructors' differentiated approaches to giving students feedback verbally during instruction when the teacher and student(s) receiving the feedback share gender and racial identities and when they do not. Data from a cross-analysis of four cases studies suggest that instructors think about and interact in different ways with students whom share similar demographic profiles with the instructor. These instructors' difficulties in teaching equitably across difference are shaped by three main factors. First, this cross-analysis show robust patterns in how these instructors construct ideas of what it means to do mathematics well and who is "good" at mathematics. Second, these ideas are represented in what these instructors notice about their students and how they interpret that information. Third, this information is taken up in their interactions with students around content, particularly in how they give students critical feedback. Persistent, inequitable instruction and the resulting barriers to learning for minoritized students raise questions about how we can better prepare instructors to enact equitable instructional practices in their teaching, which would support their interactions across difference with historically marginalized students.

The third article builds on the previous conceptual and empirical work by looking inside instruction at how core practices are used to position minoritized students on the margins of full participation in classroom mathematics discourses. The comparative case study showed that the instructors studied thought about and interacted in different ways with minoritized and majoritized students. The data suggest that instructors' difficulties in teaching equitably across difference were shaped in part by three phenomena: how instructors construct ideas of what it means to do mathematics well and who is "good" at mathematics, what they notice about their students and how they interpret that information, and how this information is taken up in their interactions with students around content. This study looks inside an episode of postsecondary calculus instruction to see how these phenomena result in the marginalization of three, STEM-identified Black undergraduate women in a pre-calculus course. Observational data show that when the majoritized instructor avoided interacting with the Black undergraduate women in his classroom, resulting in fewer opportunities for them to learn content and receive critical feedback. As a result, these behaviors further marginalized those students in the mathematics discourse community and creating barriers to for learning. Finally, observational data show how these Black women disassociate with mathematics and re-imagine their futures outside of STEM as they deal with these barriers to full participation. These three articles underscore the need to better serve minoritized students in entry-level mathematics courses through the explicit design of equitable teaching practices, the preparation of instructors to enact those practices in their teaching, and additional research on minoritized students' experiences with instruction.

CHAPTER ONE – INTRODUCTION

Personal Background and Motivation for This Dissertation Study

As a child up until seventh grade, I wanted to study the cosmos. I loved anything that had to do with the galaxy. I knew the names of individual stars and constellations. I spent hours reading about the science of studying space and celestial objects. In school, this translated into a passion for science and mathematics. I liked to learn about systems, their rules, and how to use those rules to learn more about the world around me. Until I changed schools and began seventh grade, then all I learned was that I was simply not a math person.

This was not a lesson I learned all at once, but a lesson I had a jarring introduction to and a constant, effective repetition of throughout high school. I moved into a school in the Midwest that had a specialized mathematics curriculum. The ways they learned and did mathematics were very different than the more traditional approaches of the 1980s and 1990s in my other K-12 schools (three schools to be precise in two states). The shift in my perception of myself as a mathematics learner and value of the subject led to a more gradual change in my academic identity. These changes were not because of the progressive curriculum but because of the messages I received from my seventh grade mathematics teacher. Instead of teaching me the new system, which was filled with alien rules, such as number roads, story books, and mini computers, my teacher told me I was struggling because I had no conceptual understanding of mathematics. She constantly used me as an example for why the school's progressive curriculum was for students' learning than my procedural knowledge of mathematics. Whether or not her assessment of my understanding of mathematics was accurate or not, the daily abuse sent me down a rabbit hole of self-doubt. I learned to hate math class; something that had always brought wonder and joy into my school day.

High school for me was a new beginning. I had studied with that same teacher for seventh and eighth grade and was eager to start fresh with a new school and teachers. My freshman year I signed up for an advanced geometry class (which I qualified for because I placed out of Algebra I). I planned to work hard and see if I could regain my lost affinity for the subject.

I struggled with proofs. It always seemed like we were being asked to do things without being taught how to go about doing them. I came in to work with the teacher after school a few days a week. I was determined to understand the content. My geometry teacher said to me one day a few months into the year (I am paraphrasing), “I’m really impressed with you! When you find something you’re actually good at your work ethic will take you far.” I walked away from that interaction understanding two things: my teacher recognized my hard work and my hard work did not matter because I was simply not a math person. I knew science was off the table too. You need to be able to do mathematics to study science. By the end of high school, I had finished two years of calculus, taken the AP Calculus BC examination, and placed out of my quantitative requirements at the University of Wisconsin, Madison. I breathed a sigh of relief because I was done with mathematics forever. I could choose to do something I was good at instead.

But now I see, I was not doing the choosing. Certain teachers (along with other factors in my environment) chose for me. What teachers at all levels of school believe about their students, say to them, and how they interact with them matters. All that matter for student learning, because teachers matter. I truly believe that none of the teachers I just wrote about had malicious intentions; they might have even had good ones. How they went about interacting with me and others, especially when the comparisons were obvious to me, influenced my path. When the boys in my mathematics classes were given more opportunities than the girls were to share their ideas about the mathematics and those ideas were taken up with enthusiasm by the teacher, that reinforced the myths out there that girls are not as good as boys at mathematics. When the calculus teacher complemented me on my haircut, and then praised my friend for his hard work and improvement over the course of the last unit, those myths about who was meant to succeed in mathematics were being reinforced.

Those myths, or stereotypes, permeated our mathematics classrooms. The stereotypes were “in the air” (Steele, 1997). As previously explored, teachers and students bring those stereotypes into classrooms simply by being aware of them and often by acting on those stereotypes. Our textbooks also bring these stereotypes into our classrooms by referencing centuries of (White) men as the forefathers of the discipline, such as Euclid, Descartes, Pythagoras, Pascal, Fibonacci, Napier, de Fermat, and Nash, whose names and stories filled the glossy pages. Those were the mathematician superstars, known by one name, like Madonna or Prince. We did not learn about the women who have been successful mathematicians, such as

Hypatia, Sophie Germain, Ada Lovelace, or Emmy Noether. We certainly did not learn that women of color have been pillars of mathematics, such as Marjorie Lee Browne, Katherine Johnson, and Annie Easley. Men of color were excluded as well, and we could have learned a great deal from stories of self-taught mathematicians, like Benjamin Banneker.

I learned much later that stereotypes about women as well as Black, Latinx, and American Indian students in mathematics can pose threats to students holding those salient identities, a phenomenon called stereotype threat (Steele, 1997, 2010). I fell prey to those threats in classrooms where teachers were unprepared to mitigate those threats through their actions, or instructional practices. What I had not learned yet was that I only suffered from stereotype threats and differentiated treatment because of my gender. My Whiteness had protected me from other race-based identity threats as well as harmful messages and treatment.

I studied history and other social sciences and became a credentialed secondary, social studies teacher in college. I took many courses in the African American studies department. Taking courses in the mainstream history department, I recognized early on how twisted and misrepresented my own history was as a Jewish woman, the child of two refugees and grandchild of four Holocaust survivors. Assuming this was true for other groups, I went in search of history classes being offered in ethnic studies departments. Yet, studying about historical trends did not show me the systematic oppression and persistent inequities of schooling. Perhaps those classes gave me lenses through which to begin to notice, name, and attend to racism in schooling, but it was not until I began teaching in the public schools that I began to grasp my own privilege as a White woman.

A month into my first semester teaching social studies in a public school, the principal asked me to teach mathematics in addition to my other courses. I was not credentialed to teach algebra or geometry, but the school was in a bind. According to the mathematics teachers (both White men), there were four students who "needed remediation" in mathematics, and the teachers felt that with the low level of material that my high school mathematics background was sufficient enough. The teachers claimed that they simply did not have time to help those students, who needed more hand holding and were becoming behavioral problems. (I learned later that they were labeled thus because they asked for explanations and extra examples without being called on – which made sense since they were never called on when their hands were raised).

I agreed to take on the class (as if I could really refuse) when I was told that the "remedial students" were also students in my history classes. I was intrigued, but certainly not prepared to walk in the classroom and find some of my most engaged students sitting silently around a table with angry faces. The mathematics teachers had sent the only four, Black boys in the entire school for remedial mathematics. When one of my favorite students looked up at me and said (again I am paraphrasing), "This isn't history class, and you're not going to teach us math. We hate math." I said the only thing that came to mind, "I hate math too." And those wonderful students learned mathematics; in fact, they completed their algebra I, geometry, and algebra II requirements (three years of high school mathematics) during that year. They also taught me that my experience as a White woman in mathematics was at once unique and a shared experience of exclusion. It was unique since I was an individual with my own constellation of identities and experiences that were situated in particular spaces and times. What I did not see yet was that elements of those experiences were common across White women due, in part, to stereotype threats and differential treatment. But my experiences with exclusion were not the same as theirs. Those boys taught me that navigating schooling, in general, and mathematics, in particular, can be treacherous in different ways for people with different intersecting identities. Those remarkable, Black boys planted the seeds for this dissertation project, a decade before the research began. I hope this work provides a useful lens through which to view the differences across students in our mathematics experiences and the relationship those differences have to how mathematics is taught so teachers at all levels can learn to teach equitably across difference.

The Dissertation Project

This dissertation project focuses on how and why instructional practices should be explicitly designed to create equitable access to and success in mathematics learning for minoritized students. Moreover, this work argues for the explicit preparation of graduate student instructors to teach equitably in their current instructional roles and future faculty positions. Only with instructors' preparation for and enactment of equitable instructional practices will it be possible to break the cycle of inequity perpetuated in postsecondary mathematics classrooms illustrated in this research. The dissertation study included two stages.[2] The first stage was an intervention that took place during a mathematics department's teaching orientation for their doctoral student instructors (DSIs). The intervention was a series of four, consecutive one-hour sessions in which the DSIs were taught an equitable approach to the high-leverage practice of giving student feedback. The department made the sessions mandatory for all the DSIs (n=28). During the second stage, I observed and interviewed a subset of those DSIs' teaching undergraduate calculus. I wrote the dissertation outlined previously on the data from this second stage.

The Dissertation Research Project Design

This dissertation project investigated inequity in calculus instruction through a two-part study, which built on the findings from an earlier exploratory study. The exploratory study, conducted in the same department, showed correlations between personal theories of mathematics intelligence doctoral student instructors (DSIs) held for themselves and those they held for their students. The findings suggested that the idea that intelligence is innate and static could be a common belief that mathematics doctoral students from overrepresented groups (i.e., majoritized students identifying as men and Asian or White) hold about themselves and others. When instructors act on such ideas about intelligence, or entity theory, the resulting instructional practices can reduce motivation, create lower expectations, and depress performance for minoritized students in mathematics, especially those identifying as women, Black, Latino, or American Indian (Rattan, Good, & Dweck, 2012). This dissertation project examined the complex reasons for the enactment of inequitable teaching practices in postsecondary calculus instruction as well as explored how it might be possible to introduce equitable approaches to practice into a disciplinary domain rife with barriers for minoritized student to full participation.

Interested in department-situated approaches to change, the first component of the dissertation project was a design intervention study that examined a practice-based approach to preparing DSIs to give students equitable feedback, a core instructional practice, in their

[2] The dissertation research built on a background study conducted in a previous year. The background study investigated DSIs' tacit theories about intelligence in specific domains. This phenomenological study examined the DSIs' understandings of their own capacity to learn mathematics and how to teach mathematics as well as their beliefs about their students' capacity to learn mathematics. This study raised important questions about the potential relationships between these novice instructors' beliefs about doing mathematics well, how they viewed students similar and different to them, and how they interacted with students around content.

postsecondary calculus instruction. The four-hour intervention, called the Mathematics Teaching Laboratory (MT Lab), used a practice-based approach to professional development with the aim of teaching these new calculus instructors how to enact equitable approaches to giving students feedback. Taking up the feedback gathered in the exploratory study, the MT Lab provided a new cohort of doctoral students with multiple opportunities to watch simulations of teaching, talk to undergraduate students about the importance of feedback in teaching and learning, and practice giving feedback with those undergraduates. While there was evidence of instructors' learning, the intervention did not create substantial and lasting change in their practice during the four-hour intervention. Even with a longer intervention, however, it could be difficult to shift teachers' feedback practices without intervening on their own beliefs about their personal intelligence because of the relationship between the teachers' lay theories of their own intelligence and the ones they hold for their students. Without shifting their theories about themselves, it may be very difficult to change beliefs about their students' potential to learn and grow, and as a result, their communication of those beliefs to students.

The second study was a comparative case study of teacher/student interactions across identity difference in postsecondary calculus instruction. Four of the instructors from the intervention study were observed and interviewed throughout their first semester of teaching to examine their interactions with their undergraduate students across identity difference. A more detailed account of the design and implementation of this study is included in Articles Two and Three of this dissertation. The three articles in this dissertation focus on the conceptual framework and findings from this second dissertation study.

My positioning as the researcher and faculty developer

I taught the MT Lab sessions, thus positioning myself as the instructor and researcher for the first study. This was a deliberate move that allowed me to examine teaching "from the inside" (Ball, 2000), or in other words, "from the perspective of practice" (Lampert, 1998). The choice to position myself in that dual role had many affordances as well as constraints. According to Ball, this dual role allowed me to "blend the construction of practice with its analysis" (p. 366). Due to this positioning, I was able to collect insider data on the instructional decisions, moves, dilemmas, and thinking that took place during the design of the sessions as well as their enactment. This positioning enabled me to point to the aspects of instruction that as an educator I felt should be fore fronted in analysis. In other terms, I was able to make aspects of instruction visible in the analysis of my teaching that might go unseen or discarded as peripheral to a researcher from a more traditional, participant-observer perspective (Ball). Particular to this study, designing, teaching, and analyzing the sessions myself reduced the number of variables for the study. To my knowledge, in the higher education literature, there has been no research on practice-based graduate student preparation for collegiate instruction in mathematics in which the researcher designs an intervention around giving "wise" feedback and investigates the GSIs' implementation of the strategy in their classroom. Since I know of no one doing this specific type of work in faculty development, I would have needed to teach someone the design as well as the focal practice in order for them to be able to teach the intervention sessions.

From a research perspective, teaching someone else to do this work added complexity to the intervention. I would have had to determine that the trained instructor understood the intent of the intervention design and if they had implemented it with fidelity. These steps would have added a whole separate layer to the research design and analysis. If not implemented with fidelity, the entire study could have been affected, making it very difficult to study what I identified as important in the original design. From an educator's perspective, my nuanced understanding of the feedback practice and intervention design positioned me to make instructional decisions in the moment during the sessions that a trained instructor would not be empowered to make. For instance, in this dual role, when I noticed that the DSIs picked up a technique more quickly than anticipated, I had the freedom to decide in the moment to transition to another task. Moreover, in a situation where a task I designed did not serve the pedagogical goal I intended, I improvised a new task drawing from my specialized knowledge of the design, intentions, and research driving the project. Only within this dual role was I positioned to make the instructional decisions necessary in the moment, while maintaining the integrity of my research.

Yet, positioning myself as both instructor and researcher also posed challenges. Whereas I claim certain areas of specialized knowledge due to my experience teaching, prior research in the department being studied, and familiarity with the literature, I do not posses a strong foundation in mathematics[3]. To help rectify this gap in my knowledge, I consulted on the

[3] I have taken pre-calculus and calculus I as well as taught a semester of algebra and geometry at the high school level. Whereas I can recognize the concepts being taught in the calculus courses, I am not positioned to teach those courses myself with my current knowledge of mathematics.

mathematics content with an advanced doctoral student from the department who taught in as well as coordinated the calculus sequence at the research site. The doctoral student provided content expertise as well as support with data collection and small group work during the sessions. I consulted with him in the moment when questions arose regarding the mathematics content. I felt that the affordances of this dual role outweighed the constraints.

The Structure of the Dissertation Manuscript (Dissertation)

The dissertation is made up of this introduction and three scholarly articles. The dissertation focuses on how and why instructional practices should be explicitly designed to create more equitable access to and success in mathematics learning for minoritized students. Moreover, this work argues that graduate student instructors need explicit training for their current instructional roles and future faculty positions in order to break the cycle of inequity perpetuated in postsecondary mathematics classrooms illustrated in this research. The first article is a conceptual/theoretical piece and the other two articles are empirical papers. The articles are intended to be standalone pieces of scholarship that also build on one another. As such, they can be read in any order.

I chose to write a three-article dissertation after collecting and analyzing my data. The stories that surfaced during the data analysis process did not lend themselves to a traditional linear narrative. This format allowed me to draw from different theoretical and conceptual models to frame the narratives. Moreover, using the data to develop discrete articles meant that I could tie those frameworks together with a specific set of analyses, findings, and implications to make a series of focused arguments about equity in postsecondary mathematics instruction. Although these arguments build on one another, they also standalone allowing me to speak to multiple audiences.

As such, readers may read through this dissertation in a variety of ways to serve different purposes. For example, a reader interested in the development and use of the conceptual framework would read through the dissertation in a linear way (i.e., Article One, Article Two, Article Three). Someone primarily interested in the problem space – the inequitable access to and opportunities for success in mathematics learning for minoritized students – could reverse that linear order in order to begin with Article One’s empirical investigation of inequity in entry-level mathematics classrooms. Then, the reader could explore possible explanations for those inequitable interactions by moving to Article Two (an empirical examination for why and how inequity is produced through instructional practice) and finishing with Article One (a conceptual framework for thinking about disrupting that inequity). For a final example, a reader interested in the preparation of future or existing faculty to engage in equitable instruction might start with Article Two, which would allow the reader to see the disciplinary, departmental, and personal influences at work for new instructors interacting with undergraduate students across identity difference. Then, the reader could read Article Three to get a sense of minoritized undergraduate students' experiences with inequitable instruction and Article One to understand the role preparation could play in reducing these inequities.

My dissertation project was much broader than the slices of data I chose to write about in my dissertation. This was for three main reasons. First, I wanted to experiment with and demonstrate my capacity to engage in a variety of research methods. My dissertation research offered me an opportunity to explore comparative case study, design-based research, teacher self-study, ethnographic, and participant interview methods. This exploration gave me a clear understanding of what approaches I would like to pursue in my next few projects. Second, writing about the data from one component of the larger project allowed me to make more focused arguments. The three studies I conducted, while conceptually linked, did not necessarily have strong analytical connections. Writing on one set of data removed the distraction of having to explain the nature of those analytical connections. Third, the data from the classroom observations were the most compelling and most worth sharing. In part, this was because the first part of the project – the design-based intervention – was a weak treatment due to departmental constraints. The department only allowed a four-hour intervention, which was not a sufficient amount of time to shift the GSIs' beliefs about instruction or change their feedback practices. Also, the cycle of a design-based research study is usually at a minimum three iterations, which was not feasible to take on for a dissertation project. The data from this first stage of research is not sufficient for analysis, since it only represents one iteration of the design cycle. I plan to use the pilot data for a true design-based research study on practice-based preparation focused on teaching new instructors to use equitable approaches to core practices.

The broader dissertation project was centered on these research questions:

- What is involved in an instructional sequence to teach new mathematics GSIs the practice of giving "wise" feedback?

- In what ways, if any, are new GSIs able to take up this particular approach to giving feedback in their own teaching after being introduced to the strategy during their GSI preparation program?

My dissertation writing focuses on the second question by investigating related sets of questions in three articles:

Summary of Article One.

The first article is entitled *Locating equitable instruction at the intersection of high-leverage and wise schooling practices: How (re)focusing the preparation of future faculty could reduce inequity in postsecondary mathematics instruction*. This conceptual piece calls for the explicit preparation of future mathematics faculty to engage in equitable instructional practices in their teaching to create access for minoritized students. Moreover, the article illustrates how those instructional practices should be chosen from practices that exist at the intersection of high-leverage practices (HLPs) and wise schooling practices (WSPs). These intersectional practices disrupt key threats to minoritized students learning in mathematics instruction and should be integrated into postsecondary mathematics teaching. Selecting practices from that intersection ensures that the limited time and resources allocated to the preparation of doctoral students for instruction are utilized to their best advantage towards expand access to and success in undergraduate mathematics instruction for minoritized students.

Research Questions for Article One:

- How might a cultural-ecological model of instructional practice explain persistent inequities in access to and success in postsecondary mathematics for minoritized students?
- How might we begin to remedy these inequities in instruction by leveraging wise approaches to high-leverage practices?

Summary of Article Two.

The second article is entitled – ***Looking for "the good ones" … How doctoral student instructors' beliefs about being good at mathematics further position undergraduate women***

on the margins. This empirical article calls for higher education research to expand the current discourse on inequity by examining the complex interactions within instruction. Drawing from evidence from a comparative case study of postsecondary calculus instruction, this article argues that the instructors, positioned as experts, unwittingly play a gatekeeping role in entry-level mathematics courses. This gatekeeping either erects barriers for or fosters students' participation in mathematics and mathematics-intensive fields. First, this cross-analysis shows robust patterns in how these instructors construct ideas of what it means to do mathematics well and who is "good" at mathematics. Second, these ideas are represented in what these instructors notice about their students, whom share racial and gender identities with the instructor than those who do not, and how the instructors interpret that information. These findings suggest that these instructors might be primed to unconsciously (re)produce inequity through their instructional practices because of how they take up this information in their interactions with students around content, particularly in how they give students critical feedback. Persistent, inequitable instruction and the resulting barriers to learning for minoritized students raise questions about how we can better prepare instructors to enact equitable instructional practices in their teaching, which would support their teaching across identity difference.

Research Questions for Article Two:

1. What characteristics do the calculus instructors value for doing mathematics?
2. What characteristics relevant to doing mathematics do the calculus instructors notice in their students?

Summary of Article Three.

The third article is entitled – *Invisibility isn't a superpower: Positioning Black undergraduate women on the margins through core instructional practices in mathematics classrooms*. Drawing from a comparative case study, this article investigates how the enactment of core teaching practices can create inequitable access to and success learning postsecondary calculus for minoritized students. Specifically, this article examines instructional interactions across identity difference between four mathematics instructors, who identify as White or Asian men, and their students holding different racial and gender identities. An episode of instruction from one of the cases illustrates how the instructor's varied enactment of core practices creates differentiated access to learning for minoritized students in the class, particularly for Black undergraduate women. This research contributed to the empirical data on the experiences of undergraduate women, especially, Black women, by examining instruction from the inside in an identity-threat-rich domain – mathematics. Moreover, this study uncovered how inequitable enactments of core instructional practices can position Black undergraduate women on the margins of the classroom discourse community, impacting access to learning as well as feelings of belonging.

Research Questions for Article Three:

- How does the instructor's enactment of core instructional practices position undergraduate students with different gender and racial identities, particularly Black women, to participate in a mathematics classroom’s discourse community?
- How might instructors’ perceptions of students with different gender and racial identities inform and be informed by their interactions with students, particularly Black women?
- How (if at all) do relationships between perception and interaction vary in instructors’ interactions across students who hold similar or different gender and racial identities than their instructors, particularly Black women?

CHAPTER TWO – LOCATING EQUITABLE INSTRUCTION AT THE INTERSECTION OF HIGH-LEVERAGE AND WISE SCHOOLING PRACTICES: HOW (RE)FOCUSING THE PREPARATION OF FUTURE FACULTY COULD REDUCE INEQUITY IN POSTSECONDARY MATHEMATICS INSTRUCTION

Abstract

This conceptual article calls for the explicit preparation of future mathematics faculty to engage in equitable instructional practices in their teaching to create access for minoritized students. Moreover, the article illustrates how those instructional practices should be chosen from practices that exist at the intersection of high-leverage practices (HLPs) and wise schooling practices (WSPs). These intersectional practices disrupt key threats to minoritized students learning in mathematics instruction and should be integrated into postsecondary mathematics teaching. Selecting practices from that intersection ensures that the limited time and resources allocated to the preparation of doctoral students for instruction are utilized to their best advantage towards expand access to and success in undergraduate mathematics instruction for minoritized students.

Key Words: Mathematics education; postsecondary instruction; critical feedback; access, equity and excellence in higher education; STEM pipeline; core instructional practices; stereotype threat

Persistent, systemic inequities of access, opportunity, and outcome exist in higher education in the United States. These inequities stem from pervasive "hierarchies of power, communication, and opportunity" (Charmaz, 2006, pgs. 130-131). As a system, higher education differentially serves communities within society along racial, ethnic, and class lines. As a result, this differential service creates conditions under which students from these underserved communities become *minoritized*. Unlike with the label *minority*, being minoritized acknowledges that a community is underrepresented in a specific space and time due to systemic oppression and barriers to entry and full participation. For example, students identifying as Black or African American could find themselves in the minority on campus at a Predominately White Institution (PWI), yet if they decided to attend a Historically Black College or University (HBCU) with a mostly Black or African American student body, they would be members of the majority group. By extension, students from communities who have benefited from higher education underserving others are considered majoritized students in those spaces they benefit from that unearned privilege conferred due to their identities.

Minoritized students face not only barriers to entry into and successful navigation of higher education, but also stereotypes and stigmas associated with one or more racial, ethnic, and class identities they hold. Scholars write about problems of inequity for minoritized students in college readiness, admissions processes, student retention, access to funding, debt loads, curriculum, student support services, campus culture, and many other areas of higher education. Rarely, however, do scholars investigate differential treatment of minoritized students by looking inside the central enterprise of higher education— that is, at instruction, or at what instructors do as they enact core practices of teaching with students.

This article focuses on the problem of pervasive inequities by examining the case of postsecondary mathematics instruction. Since the problem is that of pervasive inequity, the remedy for this problem must also target inequity in way that permeates instruction. One mechanism available to combat inequity is instructional practice, or the work that teachers do in the classroom with students. Of particular interest are the specific practices, the actions instructors *routinely* take while doing the work of instruction (Lampert, 2009).

Nowhere in higher education is the need for equitable instructional practices more clear and urgent then in in science, technology, engineering, and mathematics (STEM) fields. In many mathematics-intensive fields in STEM, women as well as Black, Latino, and American Indian students are minoritized. For the purposes of this article, references to minoritized students henceforth speak to undergraduate students who hold one or more of those identities in the present-day context of mathematics and mathematics-intensive (MM-I) fields. Recent retention data show, for example, that women are much more likely to switch their degree to non-STEM majors than men (NCES, 2014, Table 2)[4]. Research also suggests that women leave STEM fields

[4] NCES 2014 data accessed on June 10, 2015 from onhttp://nces.ed.gov/pubs2014/2014001rev.pdf

relatively early in their postsecondary studies (Griffith, 2010; Ma, 2011). This suggests that there are serious problems with retaining women in STEM majors even after they are successfully recruited.

Persistent inequity in STEM extends beyond the inclusion and treatment of women to other marginalized identity groups. For example, Black students are much more likely to leave a STEM degree for a non-STEM major than their White and Asian[5] peers (NCES, 2014, Table 2)[6]. Moreover, research shows that PWIs are less successful than HBCUs in recruiting and retaining Black undergraduate students in a variety of STEM majors. HBCUs confer a disproportionate percentage of bachelor degrees to Black students relative to the number of students they enroll. Over a third of Black graduates in astronomy, biology, chemistry, mathematics and physics and nearly a quarter of all Black graduates in engineering earned their degrees from HBCUs, while only enrolling 11% of all Black undergraduate students (Nelms, 2011). Similarly, a study by *Excelencia* in Education looked at data from 2012-13 and found that, while the numbers of Latino undergraduate students earning STEM credentials were rising, only a very small number of institutions were responsible for granting those credentials. In fact, two percent of all higher education institutions enrolled a third of all Latino students graduating with STEM credentials and more than half of the top 25 institutions conveying STEM degrees to Latino graduates were Hispanic-Serving Institutions or HSIs (Santiago, Taylor, & Calderón, 2015, p. 6).

Although we have data on rates of participation, attrition, and completion, we know little about the experiences of minoritized students in STEM courses. While we need more research on all minoritized student populations in STEM, there are particular gaps in the literature. There is little research on Latino students' experiences and outcomes navigating STEM and almost no research on American Indian students. Notable exceptions include the research done on Latino students in STEM at *Excelencia* in Education (e.g., Santiago & Soliz, 2012; Santiago, Taylor, & Calderón, 2015). Research is also needed to examine in-group differences, investigating the unique experiences within STEM of students with multiple, marginalized identities, including

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[5] It is important to acknowledge the problematic nature of using *Asian* as a racial or ethnic identification. Using a single racial identification, which includes dozens of distinct ethnic identities, obscures important intergroup ethnic differences and promotes harmful model minority myths (Museus et al., 2011). One reason "Asian" is used in this specific article is because many of the participants in the research study that informed this article self-identified as Asian in the settings where the research occurred. Also, while this language is highly problematic, it is important in mathematics to distinguish between underserved racial and ethnic populations, and populations who on the aggregate historically and currently do not have the same persistent barriers of access to and success in mathematics.
[6] NCES 2014 data accessed on June 10, 2015 from onhttp://nces.ed.gov/pubs2014/2014001rev.pdf

Black women (e.g., Chavous, Harris, Rivas, Helaire, & Green, 2004) and Latina students (e.g., Rodriguez, 2015).

Inequity in mathematics is particularly troubling given the important gatekeeping role the discipline plays in preparing students for a plethora of mathematics-intensive fields as well as to enter the workforce. Problems of inequity in mathematics instruction ripple out and affect access to other fields. For this reason and others, we need to understand students' experiences in mathematics as a discipline, independent of studies conducted on STEM fields. Writing about STEM as one conglomerate can obscure some of the critical differences among those fields. For example, introductory or "gateway" courses in mathematics, such as calculus, are high stakes courses that serve as requirements for admission to a variety of professional programs, such as pre-medicine, business, pharmacy, and engineering (Gainen, 1995; Secada, 1989). Students must succeed in these gateway courses in order to gain admission to such programs, which means that equity in instruction in those courses is also an issue of equity of access to professional programs and careers. Moreover, studying STEM can cover up trends within fields, such as the underrepresentation of women in mathematics and mathematics-intensive fields, which is no longer the case in other STEM domains (Ceci et al., 2014).

Minoritized students also suffer long-term repercussions as a result of these inequitable barriers of access to and success in MM-I fields. Research has shown there are significant economic advantages for those who major in STEM (e.g., Jones, 2014; Webber, 2014; Langdon, McKittrick, Beede, Khan, & Doms, 2011; Arcidiacono, 2004) as well as non-monetary ones, such as prestige (e.g., Herzig, 2004). Since mathematics as a major has been ineffective at recruiting, retaining, and graduating minoritized students and those effects can compound access to other mathematics-intense fields, minoritized students are suffering long-term disadvantages due to barriers of access.

Discourse on these inequities is problematic for two reasons. First, the focus of the discourse is too narrow, concerned largely with the number of minoritized students represented the field, often framed as underrepresentation, and their performance outcomes, often framed as underperformance. Second, research on underrepresentation and underperformance of minoritized students in MM-I fields invoke deficit frameworks that shift the onus of these inequities to the students. Deficit constructs, such as grit and persistence, seek to understand why minoritized students fail to successfully navigate higher education spaces. Rather than shifting this burden to students, researchers, practitioners, and policymakers need to ask why higher education institutions, departments, and programs systematically fail to support and retain students along clear identity lines.

The narrow discourse on persistent inequities in MM-I fields needs to pay greater attention to the experiences of minoritized students navigating higher education institutions, in general, and STEM pathways, in particular. For students, their experiences on campus are interconnected and inform a myriad of decisions, including their course enrollment, modes of engagement, and major selection. Minoritized students' time in classrooms is central to their experiences on campus. Yet, little research has looked directly at postsecondary instructional practice as something experienced by minoritized students. A key exception is research on wise schooling practices, which will be examined in detail in the discussion of environmental influences on instructional culture.

Given existing inequities, not any instructional practice will support faculty in creating equitable interactions with their students, particularly when teaching students across socio-cultural differences. Faculty need to integrate equitable instructional practices into their postsecondary teaching that afford them enough *leverage* to create access to learning for *all* students. I examine the instructional practice of giving students "wise" feedback (adapted from Yeager et al., 2013) as a powerful example of just such an equitable practice. This article makes a case for teaching faculty to integrate wise feedback into their instruction. Moreover, this article argues that in order to integrate these practices systematically into instruction across institutions, departments, and faculty classrooms, educational developers should explicitly prepare graduate students aspiring to faculty positions to utilize these equitable practices to interact with students, while these future faculty are still in their doctoral programs.

I begin by reviewing the research on the environmental influences that shape instructional culture differently for minoritized and majoritized students. Next, I present instruction as a complex ecology and discuss the importance of this interactional framework in thinking about teaching practices. Then, I present these ecological and cultural frameworks together in a new cultural-ecological model of instructional practice. This new model makes the differentiated learning contexts for minoritized and majoritized students explicit in postsecondary mathematics instruction and underscores the role instructional practice can play in promoting or resisting inequity. I examine critical feedback, an important instructional practice, using the new model. Finally, I highlight the implications of this cultural-ecological model for research and practice.

Environmental Influences on the Culture of Instruction

While there are many environmental factors that shape minoritized students' learning contexts, this article focuses on the damaging presence of stereotypes in mathematics instruction. In this next section, I review the research on stereotype threat and show how those threats are of particular importance to minoritized students' learning and outcomes in the identity-threat-rich environment of mathematics. Then, I examine a promising vein of research on equitable approaches to practice that can disrupt stereotype threats under specific conditions.

Stereotype Threats "In the Air"

Stereotype threat theory (Steele, 1997; 2010) attributes underperformance of minoritized students under particular circumstances to environmental factors, rather than student deficits. These threats systematically disrupt learning for minoritized students, but not for majoritized students. A form of identity threat, stereotype threat can be triggered when an individual with an identity salient to a negative stereotype is attempting to perform a challenging task.

There are several critical components of this definition. First, stereotype threat is domain specific, meaning that the stereotype needs to be relevant to performance in that domain. For example, the stereotype that women are not as good at mathematics as men could threaten women working on a high-stakes assessment in mathematics, but would not affect them in a history examination. In this article, the domain is postsecondary mathematics. Second, context, an individual's experience of an environment, can differ across individuals within the same environment. In other words, that minoritized students are contending with the additional challenge of stereotype threats in the instructional environment means that they experience the learning context in ways that are very different than their majority peers. Third, the stereotype "in the air" is negative and so is being identified with it. Fourth, the stereotype is threatening, not because the individual believes the stereotype, but because there is potential of being identified with it. Finally, in addition to being challenging, the task and/or domain is meaningful to the individual under the threat. The high-stakes nature of mathematics courses and reliance on examinations as evaluative tools fulfill this condition.

Research on stereotype threat demonstrates that the preoccupation with being identified with the stereotype is the mechanism that causes the individual under threat to underperform (e.g., Bonnot & Croizet, 2007; McGee & Martin, 2011; Taylor, 2000; Taylor & Antony, 2000; Steele, 1997; 2010). This preoccupation adds to a student's cognitive load, making memory recall and processing more difficult during an already challenging task. This additional cognitive burden depresses a student's performance unless something is done to intervene.

Wise Schooling Practices as Stereotype Threat Interventions

Within the stereotype threat literature, there are a plethora of strategies to mitigate the effects of stereotype threat, such as value affirmations to reduce stress and threat (e.g., Cohen et al., 2006, 2009; Martens et al., 2006; Miyake et al., 2010), teaching growth mindset about intelligence (e.g., Aronson et al, 2002; Blackwell et al., 2007), creating fair tests that serve learning purposes (Good et al., 2008; Spencer et al., 1999; Steele & Aronson, 1995), conveying that diversity is valued (Purdie-Vaughns et al., 2008) and others. Wise schooling practices are one promising line of investigation (Steele, 1997; 2010). Wisdom, or the act of being wise, in a schooling context means recognizing the whole person when interacting with a student from stereotype threatened groups.

According to Steele (1997), **"**wise**"** is…

> *A term borrowed from Erving Goffman (1963), who borrowed it from gay men and lesbians of the 1950s. They used it to designate heterosexuals who understood their full humanity despite the stigma attached to their sexual orientation: family and friends, usually, who knew the person beneath the stigma. So it must be, I argue, for the effective schooling of stereotype threatened groups. (p. 624)*

For students under stereotype threat, knowing that they are seen as a whole person, rather than being associated with a stereotype, allows them to perform as they would have without the threat. Wise schooling practices are interventions that disrupt the effects of stereotype threat embedded in specific approaches to instructional practices (e.g., leading a discussion) or other schooling practices (e.g., taking a test). Research shows that wise approaches to practices reduce stereotype threats for students who are invested in a specific domain (e.g., Cohen & Steele, 2002; Cohen, Steele, & Ross, 1999; Steele, 2010; Yeager et al., 2013).

Steele (1997) suggests,

> *But for the identified of these groups, who are quite numerous on college campuses, the news may be better than is typically appreciated. For these students, feasible changes in the conditions of schooling that make threatening stereotypes less applicable to their behavior (i.e., wisdom) may be enough. They are already identified with the relevant domain, they have skills and confidence in the domain, and they have survived other barriers to identification. Their remaining problem is stereotype threat. Reducing that problem, then, may be enough to bring their performance on par with that of non-stereotyped persons in the domain. (p. 624)*

For these high academically preforming students under stereotype threat, whom Steele calls the "vanguard", he proposes two approaches: (1) make situational changes in the environment and (2) modify internal processes (Steele, 1997, p. 624). In other terms, stereotype threats could be reduced or even neutralized by changing certain situational factors in a schooling environment, such as supporting students' senses of belonging (e.g., Walton & Cohen, 2007; 2011). Steele claims that making situational changes (i.e. wisdom) is far easier than affecting internal states.

Yet, the question remains, how does one modify "the situational design of schooling" to incorporate "wisdom" into higher education? What in the environment can we reasonably expect to change and how can we prepare faculty to support those changes? Implicit in Steele's argument is the idea that these wise beliefs and practices need to be embedded in the faculty's daily work of instruction. In other words, we need to embed wisdom in forms of professional practice, such as instructional practice, in order to support faculty in delivering those needed wise messages to minoritized students. To explain this logic of change, I turn next to the instructional triangle.

Understanding Stereotype Threat Through a Cultural-Ecological Model Of Instructional Practice

To be clear, differentiated service across minoritized and majoritized student groups in postsecondary mathematics instruction has roots in a variety of problems. These problems include pre-college factors, such as socio-cultural messages regarding student potential, access to skilled instruction or advanced courses in K-12, and other problems with the K-16 pipeline. Postsecondary instruction is not the cause of all inequity in mathematics, yet instruction plays a substantial role in reproducing existing inequities as well as generating new ones (e.g., Enright, in progress b). A foundational assumption of this article is that instructors bear the ethical responsibility for making learning opportunities accessible for *all* students in the classroom. Equitable approaches to teaching practices create conditions for access to and success in learning for minoritized students by disrupting the particular set of challenges that arise due to identity-based discrimination and oppression. Meeting these learning needs that are a direct result of systems of inequity and oppression in our society, at large, and in postsecondary mathematics, in particular, is an ethical obligation of teaching. This section draws on the instructional triangle as an analytical tool through which instructional practices can promote and disrupt stereotype threats.

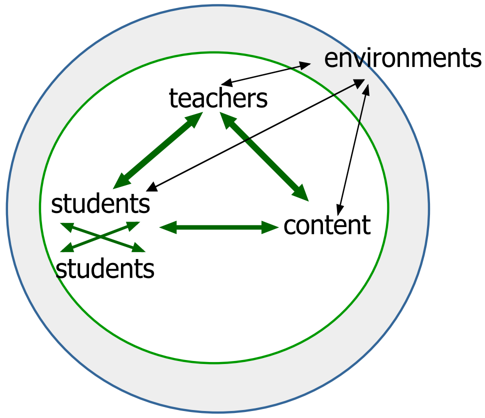
Looking at Stereotype Threat Through the Instructional Triangle

Leveraging the empirical research on stereotype threat, this article advances the claim that majoritized students, not under stereotype threat, learn in a different instructional context (Figure 1) than their minoritized peers under threat (Figure 2). The instructional triangle (Cohen, Raudenbush, & Ball, 2003) offers a lens through which to analyze minoritized students' learning under stereotype threat as well as the moves that instructors make that can either mitigate or exacerbate that threat. The arrows in the instructional triangle represent the interactions between the teacher, students, content within the environments. As illustrated, the stereotypes (and other cultural artifacts) move in from broader socio-cultural and political environments as depicted by one set of arrows into the classroom through the medium of the teachers, students, and content[7].

[7] It is important to note that this is a bidirectional relationship. The arrows connecting the instructional environment with the broader environments outside instruction are double sided, indicating that elements of the classroom culture also travel back to the broader environments through the teachers, students, and content.

Figure 1

The instructional triangle (Cohen, Raudenbush, & Ball, 2003)

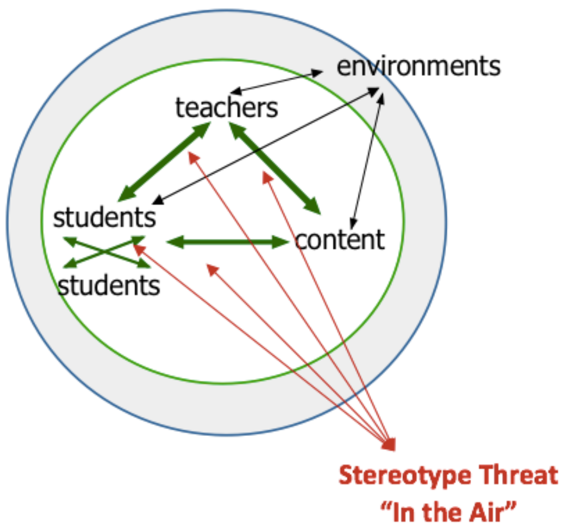


Stereotypes enter the instructional environment through teachers, students, and content and if left unchallenged can shape interactions in instruction, negatively affecting minoritized students. Evidence exists that these stereotype threats can positively impact majoritized students' performance through a phenomenon called stereotype lift (Walton & Cohen, 2002), creating an additional layer of unearned advantage over their minoritized peers. Stereotype threats in essence travel along the bidirectional green arrows disrupting other activities, such as instructional practices that also move along those arrows among the teachers, students, and content (Figure 2). Once "in the air" (Steele, 1997), the stereotypes create threats for students who identify with a salient identity group being stereotyped.

Students determine what identities of theirs are salient in a given context. To use an example related to my own gender identity, the stereotype that women are not as good as men at mathematics exists in classroom environments (as well as in environments outside the classroom) and threatens women's learning and performance in mathematics (e.g., Spencer et al., 1999). The identity in question must both be relevant to the stereotype being evoked and salient to the student potentially under threat. Racial identities have been shown to be much more salient identities for people of color than White people (e.g., Phinney, Ferguson, & Tate, 1997; Crocker & Luhtanen, 1990). Studies have also shown that gender identity is much more salient for women than men in schooling contexts (e.g., Eccles, 2009). Students targeted by multiple stereotypes, such as Black women in mathematics, might perceive one or more of their identities as most salient in a given context. For example, in one study, Black women experienced stereotype threats differently at HBCUs, where their gender identity became more salient to them, than at PWIs, where their racial identity was instead more salient (Chavous, Harris, Rivas, Helaire, & Green, 2004).

Figure 2

Stereotype threat "in-the-air" (Steele, 1997) mediating interactions among teachers, content, and students in the mathematics instructional ecology (adapted from Cohen, Raudenbush, & Ball, 2003)



Within the classroom, other sets of arrows represent interactions between the teacher, content, and students in a set of nested environments. The instructional moves or practices the teacher makes also rest on the arrows that form the triangle. When a relevant stereotype is in the air, a teacher's moves can reinforce, magnify or interrupt the threat imposed on minoritized students (Figure 2). It is important to point out that stereotype threat interrupts all of these interactions through different mechanisms. To begin, research has shown that building trust between majoritized teachers and minoritized students is critical to creating conditions under which minoritized students can learn to their fullest potential in identity-threat-rich domains (Cohen, Steele, & Ross, 1999), such as mathematics. Lack of trust interrupts the activity along the bidirectional, green arrow between teachers and minoritized students. Next, evidence also demonstrates how stereotype threat creates a cognitive burden for minoritized students to contend with when asked to perform on high-stakes tasks (e.g., Aronson, Fried, & Good, 2002; Steele, 2010), just one form of disruption of activity along the arrow between minoritized students and content. Finally, teachers' packaging of content for students is also impacted by stereotype threat. A key example of the disruption of along the arrows connecting teachers to content to students is positive feedback bias, in which majoritized teachers are more focused on protecting their own egalitarian self-images than providing minoritized students with critical feedback (Harber, 1998; 2004; Harber, Stafford, & Kennedy, 2010). In this research, White teachers gave more ambiguous feedback that is less usable when asked to give critical feedback to Black students. These different mechanisms disrupting instructional interactions at every point of the instructional triangle demonstrates that stereotype threat is a complex problem in need of multifaceted interventions that must also include changing teaching practices in order to combat the threat and equitably meet the learning needs of minoritized students.

Research shows that certain instructional practices, such as giving student feedback, have the power to trigger or interrupt stereotype threats. This research, in particular the work on wise schooling practices (Steele, 1997; 2010), will be explored in detail in the next section. Wise practices that disrupt stereotype threats are part of a larger family of equitable teaching practices. These equitable practices are the practices that should be explicitly taught to faculty in order to disrupt the (re)production of inequity through instruction in postsecondary mathematics. Next, the role instructional practice plays in combatting stereotypes the instructional ecology is examined through the case of wise feedback in mathematics instruction.

Feedback Through the Cultural-Ecological Model

We have known for several decades that learner-centered approaches to instruction (e.g., inquiry-based learning) are important to student learning (e.g., Nelson, 1991; Smith, Byrd, Nelson, Barrett, & Constantinides, 1992; Treisman, 1992). Yet, as postsecondary mathematics instruction shifts from more teacher-centered to learner-centered models, in which creating opportunities for students to learn by engaging in activities that have been shown to enhance learning, we need to be mindful of the changing classroom dynamics. In more learner-centered approaches to instruction, there is increased interaction not just between the instructor and students but also among the students themselves. In many ways, this creates a new type of work for faculty for which they have not necessarily been prepared. As the work changes, the student population is changing as well, becoming increasingly diverse (Austin, 2002; Keller, 2001; Syverson, 1996). As a result, mathematics instructors are increasingly in the position of where they need to teach across identity difference.

Increased teacher/student interaction, when done skillfully, generates more and richer opportunities for student learning, particularly women and students of color. Yet, the additional interaction in the classroom creates certain risks, including an increase in opportunities to trigger stereotype threat.[8] For example, well-meaning instructors often give differentiated feedback to minoritized and majoritized students, resulting in ambiguous or no critical feedback for minoritized students (Enright, in progress b). To be clear, this is not a criticism of the shift towards learner-centered teaching in mathematics. This is an argument for ensuring that the learner-centered instructional methods are centered on equitable instructional practices. Better instruction should not simply mean higher quality instruction for some; it should mean high quality instruction for all students. Specifically, in the case of feedback, giving feedback equitably to minoritized students requires the disruption of stereotype threats putting relationships, interactions, and the content of the feedback at risk of corruption.

Defining Feedback as a Core Practice

Feedback has been defined as broadly as "information provided by an agent (e.g., teacher, peer, book, parent, self, experience) regarding aspects of one's performance or understanding" (Hattie & Timperley, 2007). Feedback takes the form of a range of messages given by the teacher to the student, including offering praise, critical information about performance, as well as ideas about potential, intelligence, or ability. Yet, such broad definitions mask conflicts in the literature regarding the nature of feedback. It is important to understand these conflicts in order to frame the conceptualization of feedback being used in this project.

Giving student feedback is a core instructional practice, one of the coordinated activities performed by individuals or groups within a profession and that draw meaning from being situated in the profession (Lampert, 2010). Teachers give student feedback, for instance, in ways that would seem "unnatural" (Ball & Forzani, 2009) for others in different contexts, yet serve a myriad of instructional purposes when utilized by professional educators. The practice of giving student feedback rests on the bidirectional arrows that travels from the teacher through the

[8] These risks exist for minoritized students in interactions with other students as well.

content to the students (Figure 1). Yet, for minoritized students, the critical feedback students need to reflect and improve on their work gets corrupted at multiple points along the triangle.

Feedback is also a core practice because is not only used frequently in instruction, but also, cannot be avoided since students gather information from teachers' actions and also lack of action. For example, in one study, Black undergraduate women in a pre-calculus course interpreted their instructor's almost complete lack of interaction with them around content as feedback that they did not belong in that course (Enright, in progress c). The interpretation inherent in receiving feedback exacerbates attributional dilemmas that minoritized students face in choosing between different explanations for a particular behavior, outcome, or message. Feedback, as a ubiquitous practice in instruction, allows instructors a significant amount of leverage, either negative or positive, to shape opportunities for student learning.

Locating Wise Feedback in the Literature

Given this increase in teacher/student interaction, it is important to underscore that not all feedback is the same. Wise feedback, while a relatively new construct, is part of a much more mature research tradition. Feedback has been the subject of research for nearly a century, in which the earliest investigations focused on feedback as a motivational tool (e.g., Brown, 1932; Symonds & Chase, 1929). Many of the early studies and those conducted through the 1960s drew heavily on behaviorist notions, such as Thorndike's law of effect (1911; 1913; 1932) and later Skinner's theory of reinforcement (1954). In other words, early research thought of feedback as stimuli that could change behavior. Early behaviorist studies did not consider feedback as part of an individual's cognitive process or as a social phenomenon as much research does today.

This almost complete focus on affecting changes in individuals' behavior led to studies on how feedback is used to deliver information (e.g., Angell, 1949; Curtis & Woods, 1929; Peterson, 1931; Pressey, 1926) and motivate changes in or reinforce behavior, such as increased response rates (e.g., Brown; Symonds & Chase) or greater accuracy (e.g., Amsel, 1960). These studies still conceptualized feedback as stimuli provided by a teacher to a student that may or may not lead to behavior changes in that student, primarily on their learning outcomes. Beginning in the 1960s, the research on feedback as reinforcement demonstrated weak, if any, impact on student learning (e.g., Feldhusen & Brit, 1962; Hough & Revsin, 1963; Krumboltz & Weisman, 1962; Lublin, 1965; McDonald & Allen, 1962; Moore & Smith, 1961, 1964; O'Day, Kulhavy, Anderson, & Malcynski, 1971; Rosenstock, Moore, & Smith, 1965; Sullivan, Baker, & Schutz, 1967; Sullivan, Schutz, & Baker, 1971). Without demonstrable effects on student learning, researchers began to turn away from looking at feedback through the lens of operant conditioning (controlling behavior through consequences), such as reinforcement, in search of other models.

With that opening, the behaviorist models for feedback that were criticized for being performance driven and detrimental to learning, have been challenged by newer, constructivist models that are increasingly learning-oriented (e.g., Dweck, 1975; Hoska, 1993; Schunk, 1982). These newer models introduced student cognition to the feedback process. In other words, researchers moved away from trying to control student behavior to better the relationship between the content of the feedback and student performance. Understanding these relationships meant thinking about students as actively making sense of the content of the feedback they were receiving and how that impacted student achievement. Some researchers found that the more information-rich the feedback was the higher the student achievement (e.g., Albertson, 1986; Collins, Carnine, & Gersten, 1987; Grant, McAvoy, & Keenan, 1982; Hannafin, 1983; Roper, 1977), whereas others found that the amount of information embedded in the feedback had no significant correlation to student achievement (e.g. Corbett & Anderson, 1990; Gilman, 1969; Hodes, 1985; Kulhavy, White, Topp, Chan, & Adams, 1985; Merrill, 1987). These studies all tried to link the content of the feedback to specific measures of student achievement, treating cognition as a black box of sorts. The research acknowledged that cognition occurred but did little to try to understand the process itself. In short, no consensus emerged from this line of work regarding the significance of the quantity of feedback for student achievement.

Another line of research investigated how feedback can be tailored to address common types of student error (e.g., Anderson, Boyle, & Reiser, 1985; Andrews & Uliano, 1985; Birenbaum & Tatsuoka, 1987; Brown & Burton, 1978; Fischer & Mandl, 1985). Research found that the feedback strategies were only effective on minor errors, not substantial ones. Yet, the research arrived at some near consensus that feedback is beneficial to student learning, serves as a mechanism for delivering information, and provides an opportunity for students to evaluate their own learning processes and outcomes (e.g., Kulhavy & Wager, 1993). This consensus was later challenged by researchers who uncovered conceptual and empirical issues with the body of research.

Researchers took another step in the development of the construct of feedback and examined the actual mechanisms at work in studies that were increasingly learner-centered. This line of research on information-rich, attributional feedback does not suffer from the empirical or methodological problems that existed in the earlier studies. These newer studies regularly used control groups, were explicit about the populations they studied, and utilized more robust quasi-experimental designs. The newer studies examined how certain forms of feedback guide students' interpretations of their performance outcomes. According to attribution theory, students ascribed the success or failure of their performance on an academic task to different internal and external factors, such as ability, luck, effort, or even the nature of the task (Weiner, 1985; 1990). Feedback, when viewed through the lens of this theory, can be leveraged as a tool to change students' perceptions of or performance on a given task. When feedback attributes the performance to the amount of effort put into the task, students' achievement improves (e.g., Dweck, 1975; Hoska, 1993). This focus of understanding the power of feedback and its relationship to learning represented a significant shift from earlier studies focused solely on the nature of the content or amount of information in feedback in relation to student achievement.

The Power of Wise Feedback

For the purpose of this article, I focus on attributional feedback, in which a teacher offers information to a student about the student's performance or understanding of academic content that allows the student to connect their effort to the current state of that performance or understanding. The meaning of academic content is broad, including tasks and processes. Giving students information-rich, attributional feedback is promising to be among the first practices taught in to faculty, because how an instructor delivers feedback to students impacts their access to learning opportunities. The first study conducted on wise feedback in an authentic school environment showed that when feedback is given to students in equitable ways, the effects are very positive and long-lasting (Yeager et al., 2013). Yet, when delivered in ways that do not attend to stereotype threats, feedback can erect barriers to students' learning while they are contending with those threats (Steele, 1997). Part of the problem with feedback is that it is often equated with praise or telling a student they did well or are good at something without providing information on what, how, or why (e.g., Brophy, 1981; Wilkinson, 1981). Praise is reminiscent of behaviorist approaches thinking of feedback as a motivational tool. Providing praise can actually prove to be damaging under certain conditions[9] (Mueller & Dweck, 1998), particularly when for minoritized students if there is a lack of trust in the teacher (Cohen, Steele, & Ross, 1999). Wise feedback strategies have been shown to increase trust and improve student achievement, particularly for students under stereotype threat (e.g., Cohen, Steele, & Ross, 1999; Yeager et al., 2013). As a result, building trust through feedback interactions between an instructor and students is critical for teaching equitably in the classroom.

While there was a wealth of research on praising children, little was known about effectively delivering critical information to students or critical feedback (Cohen, Steele, & Ross, 1999; Kluger & DeNisi, 1996; Yeager, Purdie-Vaughns, Garcia, Apfel, Brzustoski, Master, Hessert, Williams, & Cohen, 2013). To investigate critical feedback, Cohen, Steele, and Ross (1999) developed an approach to wise feedback, claiming that stereotype threat interventions should be embedded in common teaching practices. Under laboratory conditions, they found that Black students under stereotype threat needed to receive messages that there were high academic standards and that they had the capacity to meet those expectations. White students did not need those wise messages. As previously mentioned, feedback is a practice that is used frequently in instruction, and thus, allows the instructor a significant amount of leverage, either negative or positive, to shape opportunities for student learning. Through their experiments, Cohen and colleagues demonstrated how wise feedback provided more leverage for an instructor when teaching minoritized students because the messages disrupted the stereotype threat.

Yeager and colleagues (2013) drew from this wise feedback research in their experiments in a middle school. They asked, "How can one convey criticism that could lead to improvement without undermining motivation and self-confidence," known as the "mentor's dilemma" (Yeager et al., 2013, p. 1-2). They conducted a set of three experiments focused on the connection between trust and criticism. Yeager and colleagues argue that ambiguity in a feedback interaction can have negative effects on motivation (Yeager et al., 2013). A lack of trust between students and a teacher could make feedback and the teacher's intent in delivering that feedback ambiguous, thus negatively affecting student motivation (Yeager et al., 2013). The

[9] I do not mean to suggest that instructors should never offer praise; only that one should consider the praise carefully before offering it to a student. Research suggests that when given praise should focus on a process, not the product, outcome or individual (Mueller & Dweck, 1998). For example, an instructor might say, "I can tell you worked very hard on this problem set" as opposed to "this problem set is great" or "you did great".

research shows that Black students, on average, have less trust in teachers; therefore, as a group, Black students are more vulnerable, relative to white students, to suffering negative effects as a result of ambiguity in feedback interactions (Yeager et al., 2013). In sum, the socio-cultural context plays a significant role in feedback interactions, particularly with students from social identity groups who have difficulty trusting teachers from the dominant cultural group.

Yeager and colleagues claimed that using "wise strategies" reduces the ambiguity of the feedback interaction by assuring the student that s/he is being treated as an individual and not judged with prejudice as a member of a stereotyped group. As in the laboratory experiments, in these school-based experiments the wise feedback practices communicate the teacher's high standards and belief that the student can meet them as well as provide appropriate resources to succeed (Yeager et al., 2013, p. 3). With these messages and resources, students can interpret the teacher's feedback attributing it to his/her high expectations of the student rather than any stereotype-driven bias. Yeager and colleagues demonstrated that the wise feedback practices had significant positive effects on African American students who had little trust in schooling, halting the recursive cycle identified amongst their low-trust, African American peers given standard feedback. Stated differently, the wise feedback increased performance and thus prevented distrust in this student population from deepening, which often results in declining academic performance over time. Yeager and his colleagues also demonstrated that using wise feedback strategies had long-term, positive effects on students' academic achievement (Yeager et al., 2013).

Supporting Mathematics Faculty in Giving Wise Feedback

Giving student feedback supports teacher/student relationship building around disciplinary content. Feedback, when given skillfully allows the instructor to negotiate not only the teacher/student relationship in equitable ways, but also, supports the ongoing adjustments the instructor must make between the student and the content. For instance, an instructor might notice a misconception in a student's understanding of what it means to take a second derivative of a non-linear function. If that instructor is skilled in delivering feedback in equitable ways, she could make those adjustments by pointing out the error and communicating the specific messages and information that student needs to recognize the misconception and work strategically to relearn the concept.

What is clear from the research is that feedback, when poorly done, can create risks for students, particularly students under stereotype threat. Inequitable strategies for giving feedback could potentially harm the relationship, between instructors and students, and could also inhibit learning and corrupt class culture. Because feedback is a practice in which instructors will inevitably engage, equitable strategies for giving student feedback need to be taught to mathematics faculty early in their careers. This focus in preparation is particularly critical given how different graduate students experiences are from many of their students[10].

Research suggests that many collegiate mathematics instructors believe that an individual's capacity to learn mathematics is predetermined and fixed (Rattan, Good, & Dweck, 2012). This belief is at odds with the content of wise feedback that students have potential to learn and grow to meet high expectations. In other words, research shows that the instructors who are best positioned to deliver wise feedback to students might not be prepared to do so without preparation.

So while wise feedback is a promising strategy for faculty to use in the shift into more learner-centered instruction, they need to be taught to deliver it in their instructional contexts. This approach to giving students feedback is both equitable, belonging to the family of wise schooling practices, as well as high-leverage. Both of these attributes are important. To begin, wise feedback is a practice that has been developed and tested in order to give students under stereotype threat the messages they need to flourish when learning under that threat. As such, it is an equitable practice and a likely weapon when battling inequity in the classroom.

Research also shows that instructors need to communicate that they have high academic standards for all students as well as their belief in the students' potential to reach those standards in order to create the conditions under which all students can succeed (e.g., Steele, 2011; Yeager et al., 2013). Yeager and colleagues discovered that there were long-term increases in performance for students, especially those under stereotype threat, when feedback communicated three key messages: high academic standards for all, the instructor's belief in the student's potential to reach those standards, and precise information to guide future efforts.

[10] For a detailed discussion of these differences, please see Enright (in progress a) - Looking for "the good ones" … How doctoral student instructors' beliefs about being good at mathematics further position undergraduate women on the margins.

In addition to being equity-oriented, wise feedback is also high-leverage. Introducing "wise" feedback practice into instruction is one such situational change. Steele (1997) claims, however, that some "wise" strategies have been shown to affect students who identify with or do not identify with the domain (i.e. mathematics). For instance, specific "wise" strategies for instructor to student feedback have been shown to be effective in reducing stereotype threat when the teacher student relationship is "optimistic" (Steele, p. 624). Characterizing the instructor student relationship as optimistic means that the instructor represents a trusted authority in the eyes of the student, and therefore, when she expresses confidence in the students' capacity to grow and improve in a given domain to reach the instructor's high academic standards, the student trusts the narrative, reducing the threat of competing narratives generated by stereotypes (Yeager et al., 2013). These optimistic relationships need to develop over time. We need a practice at the core of instruction, integral to teacher/student interactions, and that can support the development of these optimistic relationships through repeated use over time. Since it is very difficult to avoid giving feedback when interacting with students, preparing faculty to give feedback in wise ways becomes imperative to creating equitable conditions for access to and success in mathematics for minoritized students, conditions their majoritized peers already enjoy.

Implications for the Preparation of Future Faculty to Teach Equitably Across Difference

Higher education scholars have increasingly nominated the preparation of graduate students for postsecondary instruction as a clear pathway for improving the quality of undergraduate education (e.g. Austin 2002; Boyle, 1990; Brint, 2011; Cook & Kaplan, 2011; Gibbs & Coffey, 2004; Pfund, Mathieu, Austin, Connolly, Manske & Moore, 2012; Chism, 1998). Research on graduate student instructors' readiness for instruction has demonstrated their need to be better prepared to teach (Golde & Dore, 2002) and teach in equitable ways (Wlodkowski & Ginsberg, 1995). Other reform efforts have had few documented effects on improving university teaching practice (Brint, 2011). While I am not aware of any rigorous studies connecting the preparation of new instructors with improved instruction, scholars believe that robust preparation could have significant effects (Austin, 2002).

This content of preparation is as important as its effectiveness. Teaching wise practices in preparation would build capacity in new faculty to successfully teach across difference. Without previous teaching experience and exposure to scholarship on teaching and learning, many new instructors are left to draw upon their own experience as students and outside the academic world, such as experiences with family (Oleson & Hora, 2014). They have a tendency to cling to their "personal visions" of instruction (Brookfield, 1990; Nyquist, 1993; Wulff, 1993). Leaving new instructors to depend on those idiosyncratic experiences without explicit preparation to expand their teaching repertoire could replicate ineffective and inequitable teaching practice, especially given the differences in experience and identity between them and their students. For instance, in a study I conducted on graduate student instructors' beliefs about teaching, the participants reported experimenting with different teaching moves that helped them learn (Enright, 2015). They reported often relying on lecture in order to make it through all the required content, which has been shown to be less productive for minoritized students. Supporting new instructors in developing and utilizing wise feedback and other wise schooling practices would offer them additional tools to reach for in their instruction that would fulfill minoritized students' learning needs.

These general differences between instructors and students are compounded by socio-cultural differences, particularly in mathematics in which Black, Latino, and American Indian students as well as women are underrepresented (Austin, 2010). Socialized with different messages, which might not be visible to a majority of collegiate instructors (e.g., Taylor & Antony, 2000). New faculty may be unaware of environmental factors, such as stereotype threat, which affect minoritized students' experiences in the classroom. If they are unaware, they are less likely to know to mitigate those threats through their instruction.

Given the need to prepare instructors to enact equitable instruction, the question becomes when should that preparation begin? This article argues that the most effective time to intervene on postsecondary teaching is during faculty's early socialization into the profession or their doctoral education. In the short-term, preparing graduate students to teach results in better-prepared undergraduate instructors and better student outcomes, since many introductory mathematics courses are taught by graduate students. In the long-term, this preparation creates a cadre of faculty who are better able to cope with the challenges of teaching and able to take on more effective and equitable approaches to instruction. An underlying assumption of this argument is that it is more effective to teach new approaches to instruction to novices who are more or less clean slates as instructors. During the period of doctoral socialization, graduate students (as future faculty) are being introduced to the habits, values, norms and behaviors of the profession they wish to join (Austin, 2010; Austin & McDaniels, 2006). This socialization process has the power to affect novices as well as the academic communities that they are joining (Tierney & Rhoads, 1994; Weidman, Twale, & Stein, 2001). Future faculty would be better prepared to serve as agents of change in the institutions that train as well as hire them.

Preparing doctoral students for teaching as part of their doctoral curriculum also avoids the problem of volunteerism (where only the minority of faculty who make time to learn to teach more equitably pursue professional development). Since over 80% of mathematics faculty are trained in just 95 doctoral programs (American Mathematical Society, 2011), embedding preparation in their doctoral training could potentially change the instructional practices of nearly an entire cohort of faculty across the discipline. In doctoral granting institutions, where research has traditionally been valued over teaching (Marincovich, Prostko, & Stout, 1998), helping novices to value and thoughtfully engage in the work of teaching as they are taught to do with research could affect both the novices as well as the institution (Austin, 2010). In other words, doctoral education is a bidirectional socialization process[11]. While novices are deeply influenced by interacting with peers, faculty and other actors within their professional and personal communities, these novices in turn also influence those communities through those interactions (Weidman, Twale, & Stein, 2001).

Preparation is also an opportunity to begin learning about instruction for many future faculty. If part of the work of faculty is to engage in scholarly learning (Neumann & Pereira, 2009), then doctoral programs need to begin socializing future faculty to consider learning about instruction in their field as part of their scholarly learning. Integrating learning about teaching into graduate students' scholarly learning in systematic ways sends the message that there is "a structure of knowledge unique to the field…of which is it a part" (Neumann, 2011 p. 195). This point in the career development of faculty is a crucial time to equip them with skills, awareness and knowledge needed to engage in equitable instruction. Early intervention is particularly important since scholarly learning is understood to be a personal and emotional process that can shape faculty perceptions and actions for the course of their careers (Neumann, 2011). In other

[11] This bi-directionality does not imply equal effects in both directions.

words, preparation is not simply an opportunity to teach future faculty about equitable teaching, it is also a mechanism through which they can be shown the depth and possibilities for learning in the field of education, particularly within their academic domain.

Conclusion

Attention needs to be paid to what is being taught to future mathematics faculty. There are devastating and persistent inequities in higher education, in general, and mathematics, in particular, which perpetuate historical barriers to learning opportunities for women and students of color. These inequities are, in no insignificant part, produced and reproduced by instruction in postsecondary mathematics courses. Faculty need to be taught instructional practices that help them communicate across those differences in ways that support all students' learning. The selection of these equitable instructional practices must be deliberate and informed by rigorous research on student outcomes ensuring greater equity. In order to improve access to postsecondary mathematics teaching for minoritized students faculty need preparation that explicitly teaches them the importance and enactment of wise approaches to high-leverage teaching practices.

CHAPTER THREE – LOOKING FOR "THE GOOD ONES" … HOW DOCTORAL STUDENT INSTRUCTORS' BELIEFS ABOUT BEING GOOD AT MATHEMATICS FURTHER POSITION UNDERGRADUATE WOMEN ON THE MARGINS

Abstract

Rarely do higher education scholars address issues of inequity within higher education by looking directly at the core enterprise of higher education - instruction. Drawing from evidence from a comparative case study of postsecondary calculus instruction, this article argues that the instructors, positioned as experts, play a gatekeeping role in entry-level mathematics courses. This gatekeeping either erects barriers for or fosters students' participation in mathematics and mathematics-intensive fields. First, this cross-analysis shows robust patterns in how these instructors construct ideas of what it means to do mathematics well and who is "good" at mathematics. Second, these ideas are represented in what these instructors notice about their students, who share racial and gender identities with the instructor than those who do not, and how the instructors interpret that information. These findings suggest that these instructors' enactment of instructional practices might be (re)producing inequity because of how they take up this information in their interactions with students around content, particularly in how they give students critical feedback. Persistent, inequitable instruction and the resulting barriers to learning for minoritized students raise questions about how we can better prepare instructors to enact equitable instructional practices in their teaching, which would support their teaching across identity difference.

Key Words: Mathematics education; postsecondary instruction; critical feedback; access, equity and excellence in higher education; STEM pipeline; gateway courses

Midway through fall semester, the director of an undergraduate mathematics program at a research university sent an email out to all of the instructors of the entry-level calculus courses (doctoral students, postdoctoral fellows, and faculty). He wrote,

> *My usual request: As the students begin to think about their winter term courses, please identify the top few undergraduate students in your classes and encourage them to take another math class. A little encouragement can make a huge difference to a student who is trying to decide what to do with her/his life; see the student quotes below my signature for evidence in this direction. I'd like for the good ones to give math a chance.*

But who are "the good ones" and how do new instructors, doctoral students themselves, make those determinations? What beliefs do they hold about what it means to do mathematics well? And most importantly, how do those beliefs inform instructors' activity in entry-level courses, particularly when asked to teach students across identity difference? Given the persistence of inequity in mathematics and mathematics-intense fields for women, particularly women who identify as Black, Latina or American Indian, the individuals positioned as experts at the core of the discipline of mathematics wield enormous power (and responsibility) as gatekeepers to those fields.

In the United States, historically women as well as Black, Latino, and American Indian students, in particular women within those identity communities, have experienced substantial barriers to full participation in mathematics and mathematics-intensive fields. As a result, these students are underrepresented, or minoritized, in those fields. Barriers include overt displays of racism and sexism, institutionalized policies of exclusion, too few faculty and other figures in the fields who share their identities, curricula that fail to include relevant aspects from their identity communities, cultures that promote microaggressions, ineffective instructional approaches, and many others. These barriers are often discussed in terms of underrepresentation and underperformance across a range of academic outcomes in mathematics for these *minoritized* groups (e.g., CITE). Moreover, underpreparation in K-12 is named as the key factor perpetuating these systemic problems of inequity for women in science, technology, engineering, and mathematics (STEM) (e.g., Ceci, Ginther, Kahn, & Williams, 2014). Very rarely, however, do we address issues of inequity within higher education by looking directly at the day-to-day interactions that occur among instructors and students in core enterprise of higher education - instruction.

Traditionally, discussions of the marginalization of women, particularly women of color, in STEM education discount these day-to-day experiences navigating gendered and racialized mathematics spaces in higher education (Enright, in progress b). Even if claims that overt discrimination against women has declined in STEM are true (Ceci, Ginther, Kahn, & Williams, 2015), few studies have examined the role differentiated enactment of core instructional practices play in discouraging women from pursuing mathematics-intensive pathways. Given the overrepresentation of White and Asian men (*majoritized* groups) in postsecondary mathematics instruction (CITE) and the relative diversity of the undergraduate student population across gender and racial lines (CITE), educational research needs a better understanding of how to support teaching across identity difference. This support is critical to socialize and prepare postsecondary mathematics instructors from majoritized groups to teach equitably in critical gateway courses, required for mathematics-intensive fields (Gainen, 1995; Secada, 1989).

Bass (2015) described the problem of inequity on which this study focuses:

> *Mathematicians have an excellent tradition of nurturing students of talent. What they are less good at is identifying potential talent. The usual indices, high test scores and precocious accomplishment, are easy enough to apply, but these will typically overlook students of mathematical promise whom the system has not encouraged or given either the expectation of or opportunity for high performance. (p. 634)*

Part of a larger research project, this comparative case study investigated an explanation for why the potential of minoritized "students of mathematical promise" is often not recognized or fostered in entry-level university mathematics courses. This study draws on Lave and Wenger's (1991) theory of social practice to add complexity to how higher education thinks about teaching across identity difference. This research focused on the following questions:

3. What characteristics do the calculus instructors value for doing mathematics?
4. What characteristics relevant to doing mathematics do the calculus instructors notice in their students?

This article begins by examining why these questions are crucial to ask in research on higher education instruction. Second, I present the design of the larger project and the methods this particular study used to collect and analyze data. Third, I present two sets of findings that illustrate the characteristics of doing math "well" that are valued by instructors as well as those characteristics they notice in their students. Fourth, the findings are discussed as part of a larger challenge about the socialization of future mathematics faculty, in general, and their preparation to teach in particular. The discussion raises concerns about how instructors' interactions with students might be shaped by their beliefs about doing mathematics and racial and gender biases commonly held and perpetuated within the mathematics community. Finally, I examine the affordances and limitations of this study and propose future avenues for research.

Literature Review

Access and equity to higher education has typically been defined as an issue with admission processes, tuition affordability, curriculum, policy, or the environment in higher education, rarely discussed as a problem of teaching practice. In one notable exception, in her presidential address at the 2012 Association for the Study of Higher Education (ASHE) conference, Anna Neumann argued "once 'in,' a student must be positioned, by way of teaching, to access the skills, knowledge, and ways of knowing likely to lead to deep substantive understanding and insight" (Neumann, 2014). In other terms, once "in-the-door" of higher education, students' access to learning depends on their instructors knowing and enacting instructional practices that generate learning opportunities in equitable ways. For example, certain teaching practices, such as the "wise" feedback strategy (Cohen, Steele, & Ross, 1999; Yeager et al., 2013), have been shown to expand that access for historically marginalized students, whereas other "praise" based feedback strategies have been shown to repress learning and achievement (e.g. Mueller and Dweck, 1998; Rattan, Good, & Dweck, 2012). To respond to calls for the reform of undergraduate teaching, particularly in STEM fields (Henderson, Finkelstein & Beach, 2010; Graham, 2012), higher education needs more robust research to support the design and enactments of equitable teaching practices. Without equitable instruction, it is unclear how access to participation in mathematics and mathematics-intensive fields for minoritized students can be expanded.

The Need for Access to Participation in a Discipline

Lave and Wenger (1991) introduced the theory of legitimate peripheral participation (LPP) to illustrate how learning is not simply situated in a context, but also, embedded in social practice. According to Lave and Wenger, learning does not take place solely within the mind but is a result of interactions within a particular context in which a learner is striving for membership, such as a doctoral student seeking to become a mathematician or an undergraduate student wanting to work as a STEM professional. The idea of learning as changing participation in community of practice is very relevant to classroom instructional contexts. In fact, "Wherever people engage for substantial periods of time, day by day, in doing things in which their ongoing activities are interdependent, learning is part of their changing participation in changing practices" (Lave, 1996, p. 150). In other words, learning is a form of participation or social practice, which over time, allows a novice to become a member in a community of practice.

Becoming a member in a community of practice involves different levels of participation, being at the margins or "periphery" in Lave and Wenger's language. Periphery is understood as "an opening, a way of gaining access to sources for understanding through growing involvement" (Lave & Wenger, 1991, p. 37). Participating peripherally is not intended to be negative; it is a stage novices pass through on their way to full participation in a community of practice – when they are enabled.

The importance of being enabled to navigate from peripheral to full participation raises questions for mathematics education. Specifically, how to experts who fully participate in mathematics communities of practice enable, or fail to enable, novices' negotiation of membership? This study at once takes up Lave and Wenger's framework of legitimate peripheral participation as well as problematizes the framework by underscoring the need to look at how intersectionalities of identity of those at the periphery and center impact this process. What do the members closer to the center of expertise believe and how do those beliefs than affect who they recognize as potential members?

Central to Lave and Wenger's (1991) theory of LPP are the experts who practice at the core of their community and are the gatekeepers of their community of practice. In postsecondary mathematics, faculty and instructors are core members who play critical gatekeeping roles – guarding who has access to participation, and thus, membership. As such, it is important to ask, who are these experts in a postsecondary mathematics space? How do the experts' identities add complexity to the process of a novice becoming a peripheral, and possibly full participant? The doctoral student instructors who participated in this study were both experts and novices. They were experts, or at least becoming experts, as doctoral students in a mathematics department. Even if they were not at the core of their disciplinary community yet, they were on their way to becoming part of that core. The doctoral student instructors were also novice instructors, teaching undergraduate mathematics as instructors of record for the first time. The role of instructor positioned them as gatekeepers for their discipline (and other mathematics-intensive fields). As such, they were responsible for bringing students, novices in mathematics, into the community by helping them to participate on its periphery. But what resources did these instructors have to inform their teaching, in general, and their teaching across identity difference, in particular?

Doctoral Student Instructors' Limited Resources for Teaching Across Difference

Novice postsecondary mathematics instructors have few resources to inform their teaching. Research, including this study, suggest that they receive little, if any, effective preparation for instruction (e.g., Austin, 2010; Golde & Dore, 2002; Speer, Gutmann, & Murphy, 2005). This limits their resources for teaching to their own experiences as students observing teachers and what the institution or department provide.

Apprenticeships of Observation.

Novice instructors learn about teaching from their own experiences as students and their observations from that perspective of teaching. Lortie (1975) referred to this phenomenon as the "apprenticeship of observation", in which students develop their ideas about teaching by observing the parts of the work visible or inferred from their experience as students. In the absence of strong professional education that would intervene on these everyday commonsense experiences of teaching, the "apprenticeship of observation" becomes the first stage of the socialization process into teaching. This apprenticeship means that novices' idiosyncratic experiences become resources on which they draw in their practice as teachers. Novices might develop ideas about effective teaching practices based on their own individual experiences and shaped by their identities in a particular set of contexts. Importantly, this apprenticeship of observation does not prepare novices to teach students who have different experiences, preferences, and needs.

Moreover, Lortie warned that this apprenticeship of observation does not "lay the basis for informed assessment of teaching technique or encourage the development of analytic orientations toward that work" (p. 67). Moreover, these partial images of practice tend to be stubborn and resistant to change for those novice practitioners in their own teaching. Without rigorous professional training, changing these images of practice can be difficult, if not impossible.

Departmental Resources.

Relying on departments at research institutions for instructional resources can also be problematic, since undergraduate teaching has not historically been a priority in mathematics departments (Bass, 2003). With persistent problems with inequity in mathematics departments, this reliance on departments to prepare instructors to teach minoritized students without preparation to teach across identity difference is especially troubling.

Research has found that graduate student instructors, responsible for a large percentage of undergraduate instruction, are often sent largely unsupervised and underprepared to their teaching assignments (Speer, Gutmann, & Murphy, 2005). The few existing approaches to instructional preparation tend to be mostly institutional or departmental orientations lasting between one to five days with a heavy emphasis on logistics and policy (Speer et al., 2005), as seen in the department in this study. Typically, novice instructors are presented information about policies and procedures of the specific course they will be teaching, and sometimes given an opportunity to practice delivering a brief mini-lecture with limited feedback (Pruitt-Logan et al., 2002). In essence, novices are asked to practice something they have not been formally taught to do.

Moreover, departments often assume that graduate student instructors have the requisite content knowledge needed to teach a course (Neumann, 2001). This assumption might not hold true given the challenges inherent in "decompressing" mathematical knowledge to teach foundational ideas to students (Bass, 2015). The lack of pedagogical knowledge is also a problem. When formal preparation for teaching is available through orientations, those programs rarely cover pedagogy (Marincovich, 1998), and in the rare instances they do, opportunities for discussion of practice are usually too idiosyncratic, focusing on personal experiences with teaching (Neumann, 2001). The lack of preparation for teaching also prevents the development of a technical core for instructional practice. In other words, the practice of teaching becomes too idiosyncratic to specific departments and courses eliminating the ability for it to be discussed, compared, and evaluated according to any professional standards.

The lack of preparation for and resulting weak technical core for postsecondary mathematics instruction also pose challenges for understanding what teaching practices can be leveraged to teach across difference. Nor are there efforts to disrupt the use and communication of biases in instruction. Yet, research has shown that instructors need to be prepared to enact teaching in equitable ways (e.g., Steele, 2011; Wlodkowski & Ginsberg, 1995). Novice instructors are left to rely on their apprenticeships of observation and weak, or non-existent, departmental resources. Neither source provides support for novices to teach students across identity difference, who experience a very different learning context and have different needs than their instructors. So what are these identity differences between instructors and students?

Layers of Identity Difference Between Instructors and Students

In general, there are some very important differences between instructors and their undergraduate students' experiences with formal schooling in mathematics. Instructors are experts — or graduate students becoming experts — and often have very different learning trajectories than the typical undergraduate student in a beginning mathematics class. As discussed with the apprenticeships of observation, a graduate student may have intimate knowledge of his/her own learning needs and preferences but rarely have an awareness or understanding of how others learn without training. Moreover, graduate student instructors have a tendency to cling to these "personal visions" of instruction (Brookfield, 1990; Nyquist, 1993; Wulff, 1993). Without previous teaching experience and exposure to scholarship on teaching and learning, many graduate student instructors are left drawing from their own experience as a student and outside the academic world, such as experiences with family (Oleson & Hora, 2014). Leaving graduate student instructors to depend on those idiosyncratic experiences without explicit preparation to expand their teaching repertoire seems unwise, especially given their potentially different learning trajectories.

In addition to instructors potentially having radically different learning trajectories, they also have different socio-cultural experiences with formal schooling. Students are socialized with different messages, which might not be visible to a majority of collegiate instructors (e.g., Taylor

& Antony, 2000). General differences between instructors and students are compounded by socio-cultural differences, particularly in STEM fields, in which students from Black, Latino, and American Indian communities as well as women across racial and ethnic groups are underrepresented (Austin, 2010) or become minoritized in that space. White or Asian men are over-represented in mathematics departments or become majoritized in the same space. By extension, the graduate students, lecturers, and faculty teaching mathematics courses tend to be White or Asian men, while the demographics of the undergraduate student population taking the gateway mathematics courses is becoming increasingly diverse across gender and racial lines. As a result, majoritized instructors are left unprepared, not just to teach, but to teach minoritized students with potentially different experiences and needs than their own. Yet, why do these identity differences matter in mathematics instruction?

The Importance of Instructors' Beliefs About Student Learning and Ability

Research shows that many collegiate mathematics instructors hold counterproductive beliefs that an individual's capacity to learn mathematics is predetermined and static, "an entity (fixed) theory of math intelligence" (Rattan, Good, & Dweck, 2012). Two studies I conducted in the mathematics department examined in this article also found that doctoral student instructors (DSIs) entered the mathematics doctoral program with strong ideas about the nature of their own and their students' intelligence (Enright, 2015, in progress b). In particular, the seven White or Asian men[12] interviewed believed that an individual is born with limits to his/her potential to learn mathematics and effort can help them reach those limits but not exceed them – evidence they held an entity theory of mathematics intelligence. The DSIs held these entity beliefs for themselves as well as their students, demonstrating that the social psychology research on entity beliefs is relevant to this specific department.

These entity beliefs are important because instructors transmit their beliefs about student intelligence within a domain (e.g., mathematics) to their students through their teaching practices

[12] A convenience sample of nearly 15% of two consecutive doctoral cohorts were interviewed. The demographic profiles of doctoral students who participated in the studies (especially along gender and racial identities, age, nationality, and education) were representative of their cohort. There were so few women (less than 20%) and almost no Black and Latino doctoral students in the department, which made recruiting a diverse sample difficult. It is interesting to point out that the only woman (an additional participant in the first study) held a much more malleable theory of intelligence for herself and her students (Enright, 2015). Since she was the only woman who volunteered for the study, it is impossible to know if she is an outlier in the department or representative of other women. Further research should investigate potential differences in minoritized and majoritized doctoral student instructors theories of mathematics intelligence.

(e.g., Dweck, Davidson, Nelson, & Enna, 1978; Dweck, 2010; Kamins & Dweck, 1999; Mueller & Dweck, 1998; Rattan, Good, & Dweck, 2012). Evidence that collegiate mathematics instructors hold entity theories is particularly concerning because it is at odds with research on effective teaching from experimental psychology. Research shows that instructors need to communicate "wise" messages that they have high academic standards for all students as well as their belief in the students' potential to reach those standards in order to create the conditions under which all students can succeed, particularly those under stereotype threat (Cohen, Steele, & Ross, 1999; Yeager et al., 2013). If the instructor believes that ability or intelligence in a given domain are fixed, and therefore limited, transmitting those beliefs to students can actually serve to depress academic achievement, particularly for those who feel threatened by existing stereotypes of an identity group with which they associate (Steele, 1997; 2010). For example, the stereotype of women not being as good at mathematics as men, although false, could still create threatening conditions that enable those fixed messages about ability to inhibit women's academic performance on high stakes mathematics tasks. Entity beliefs emphasize an individuals' limits, rather than potential to learn, which can make communicating those wise messages even more challenging.

Moreover, individuals holding fixed mindsets tend to diagnose others' abilities sooner (Butler, 2000; Plaks, Stroessner, Dweck, & Sherman, 2001), even teachers (Rattan, Good, & Dweck, 2012). In other words, instructors with entity beliefs not only transmit those beliefs through their teaching to their students, but also leverage those beliefs to make premature decisions about their students' potential. For these reasons and others, graduate student instructors' beliefs about intelligence and ability are relevant to their students' learning (Dweck, 2006). This is particularly true for minoritized students under stereotype threat (Steele, 2010), since we know instructor beliefs and the messages they transmit shape minoritized students' access to opportunities to learn and succeed in mathematics.

This study examines what those beliefs about doing mathematics are and how they shape how instructors view and interact with their students as novice learners of mathematics. The next section presents the research design, data collection and analysis. Then, seven findings are outlined. These findings detail the instructors' beliefs about what characteristics are critical for doing mathematics well. Moreover, the findings highlight how these beliefs are used as lenses that inform the instructors' perceptions of their students and interactions with them. Finally, the discussion examines how these instructors' beliefs differentially position minoritized and majoritized students to participate in mathematics learning.

Data Collection and Analysis

Research Context[13]

This study of calculus instruction was conducted in a mathematics department at a public university with a Carnegie classification of "very high research activity" (RU/VH) in the United States. The institution[14] grants at least four doctoral degrees in mathematics a year, and therefore, is one of 95 institutions responsible for training over 80% of future mathematics faculty holding doctorates (American Mathematical Society, 2011). Although there are many more institutions that have mathematics departments and offer the entry-level calculus courses, this study is situated in a particular institutional context that I argue has an especially large footprint in postsecondary mathematics education, given its role in credentialing future faculty. In turn, the future faculty go out to teach at a range of different institutions, often with little understanding of how those institutional types differ from the institution that trained them (Austin & McDaniels, 2006; Nerad, 2008). Furthermore, due to the size of the university, undergraduate student enrollment in the entry-level calculus courses is very high. This means that not only are future students impacted by these future faculty, but also their students while completing their doctorates. This is especially important since explicit preparation for postsecondary STEM instruction is rare, and when offered, it is often the only training received by faculty over the course of their careers (Austin, 2009). These factors point to the importance of examining doctoral student instructors' and their students' experiences with instruction in departments with a disproportionate impact on postsecondary mathematics instruction, such as the department in this study.

13 This is a general assessment of the faculty, doctoral students, and undergraduate students' gender and racial/ethnic demographic data, providing more specific information could make the department identifiable and potentially compromise the anonymity of the research participants. The geographic location of the university is also withheld to protect anonymity.
14 The impact of this department on the training of future mathematics faculty is much greater than this statement indicates, but more specific data could potentially allow for the identification of the institution and is withheld to protect the anonymity of the participants.

Site Selection

Since there are few mathematics departments with this size footprint, negotiating access to the department was critical. I began attending departmental talks and having informal conversations with faculty, particularly those involved with the entry-level calculus course series. After developing those relationships, I requested and was granted permission to conduct an exploratory study of their doctoral student instructors' experiences with the department's calculus teaching orientation week and first semester teaching, which ended up being the first of three studies I conducted in the department (Table 1). I continued to attend departmental talks and the calculus course series leadership meetings that the instructional team conducted to plan the orientation week and subsequent course meetings. After the exploratory study was complete, the director invited me to teach a session during the orientation week in the following year, and after a series of conversations, I negotiated access to the DSIs for a four-hour intervention during orientation, named the Mathematics Teaching Laboratory (MT Lab), and permission to invite the DSIs to continue participating in the research project during their first semester of teaching. The MT Lab intervention and subsequent investigation of teaching provided the data for this current article. The director required that I offer all the DSIs the same resources and opportunities. Furthermore, in exchange for access, I investigated the preparation needs of the DSIs and reported my aggregated findings to the director.

Table 1

An overview of the research studies conducted in the Mathematics Department

	Description	**Year**	**Participants**
Study A	An exploratory study that utilized ethnographic observational methods and phenomenological interviewing to understand the DSIs' preparation for instruction and conceptualization of mathematics teaching and learning	Year I (summer orientation & fall semester)	• Observed entire DSI Cohort I (N=30) during orientation • Interviewed four DSIs from Cohort I during fall semester (convenience sample)
Study B	An intervention (the first design cycle) in a design-based research project that examined a practice-based approach to teaching DSIs to give feedback to students equitably in teaching, following	Year II (summer orientation)	• Observed entire DSI Cohort II (N=28) during orientation • Taught Cohort II in the MT Lab for four hours over the course of the week

	the DSIs' learning trajectories over the four, one-hour sessions		
Study C (reported on in this article)	A comparative case study that investigated the DSIs' interactions around academic content with students holding different gender and racial identities during the DSIs' first semester teaching	Year II (fall semester)	• Interviewed and observed four DSIs from Cohort II during fall semester (volunteer participants)

The research site was selected because the department has a new cohort of doctoral students appointed to be first-time calculus instructors each year, making the study of interactions between teachers and students in beginning, undergraduate calculus teaching possible. Moreover, this site has an established five- day long preparation program (an orientation week) for new, non-faculty calculus instructors[15] in the department, which occurs just prior to the start of the fall academic semester. This orientation week provided a stable, departmental structure in which to embed the larger design study. Finally, the gender and racial/ethnic demographics of the department created a research setting in which the phenomenon of teaching across socio-cultural difference could be studied. Of the faculty and doctoral students in the department, nearly all identified as White, Asian or Indian (from the subcontinent) and most identified as men. Nearly half of the doctoral students were designated as international students by the university. In comparison, the undergraduate student population enrolled in the entry-level calculus courses (i.e., pre-calculus and the first semester of differential calculus, or Calculus I) studied was much more racially and ethnically diverse and had gender parity. Additionally, only a handful of the undergraduate students were designated international students. These undergraduate student demographics shifted dramatically after the entry-level courses, beginning with the second semester of calculus (i.e., integral calculus or Calculus II) mirroring the faculty and doctoral student gender and racial/ethnic demographics. The dramatic change signaled that there was something to be learned from studying the teacher/student interactions in the entry-level courses where teaching across difference required interacting with individuals with other racial/ethnic or gender identities.

15 It is interesting to note that the department made it mandatory for all new graduate students and post-doctorate fellows to take part in the orientation, although they were in separate, parallel sessions for the practice teaching sessions. New lecturers and tenure-track faculty were not invited to the orientation. This study focused on the training the doctoral student instructors received during the orientation.

The orientation week served multiple functions in the department, including doctoral student class scheduling, cohort building, introducing doctoral students to the department facilities and key faculty, and of course, preparation for teaching. The preparation for teaching component involved introducing these doctoral student instructors (DSIs) to the instructional materials (e.g., lesson plans, assessments, etc.), technology (e.g., graphing calculators), course structure and policies (e.g., grading), profiles of the incoming undergraduate students (e.g., academic and demographic information), as well as to the departmental norms around instruction in the entry-level courses. These sessions also required the DSIs do some practice teaching (10-12 minutes) for which they received feedback from a faculty member and their cohort-mates. DSIs received elective course credits from the mathematics department as compensation for their participation in orientation week, including for their participation in the MT Lab.

As previously discussed, the MT Lab, which preceded the study of calculus instruction, was embedded in the department's orientation week. The department mandated participation in the MT Lab for all new DSIs teaching that academic year (n=28). The MT Lab consisted of a series of four, consecutive one-hour sessions in which the DSIs were taught the pedagogical practice of "wise" feedback, shown to be an equitable instructional practice (Cohen, Steele, & Ross, 1999; Yeager et al., 2013). The MT Lab was designed within the time constraints and other parameters established by the department (e.g., including time for social ice breaker activities and to eat lunch during the hour-long sessions). For these reasons, The MT Lab is considered a weak treatment to create meaningful change in the DSIs' teaching practice. The empirical data collected during that phase of the study supports that assessment. The MT Lab, however, served three key research functions, by creating opportunities to (1) begin a conversation about equity in instruction with the incoming cohort before they began teaching, (2) recruit participants for the study on calculus instruction (Table X, Study C), and (3) gather data on DSI learning that informed the design of the semi-structured interview protocols for the study of calculus instruction.

Participants in the Study of Calculus Instruction

I followed four DSIs who participated in the MT Lab during their first semester of teaching. All the 28 DSIs who participated in the MT Lab were recruited to participate in the second phase of the study, which was voluntary due to the additional time commitment. After the MT Lab was complete, 16 of the DSIs expressed interest in continuing with the study citing their interest in learning more about teaching and becoming effective instructors. These motivations were the first of two noticeable differences between the 16 DSIs who expressed interest in participating in the study on calculus instruction and their 12 peers who did not. Second, in pre- and post-surveys conducted in the MT Lab (Appendix A), the 16 interested DSIs on average showed a stronger belief that teaching could have an impact on learning, whereas on average, their 12 peers scored nearly a point lower on a five-point Likert scale. In short, this signaled that the 16 DSIs interested in participating in the study believed what instructors did in the classroom mattered to students' learning.

When the 16 DSIs were invited to participate after the first week of teaching, only four of those DSIs volunteered to commit to participating in the study. Those who initially volunteered but withdrew cited lack of time for the extra commitment given the difficulty balancing their course work with the demands of teaching. Some reported being warned by their faculty advisor and advanced doctoral students not to spend too much time on teaching because research was valued more in the department and field. The DSIs reported receiving the same messages as their peers who declined to participate; however, they also said they wanted someone they could talk to about their teaching and felt it would help them be better at it, which would save time in the long-term. The four DSIs viewed participating in the study as an investment that would hopefully help them become better instructors, which they hoped would help them compete for positions in their current department as well as faculty jobs. Other than this long-term outlook, I know of no distinctions between the four DSIs who volunteered and the 12 others who declined to participate after originally volunteering. Moreover, there were no patterns in the survey results or other data from the MT Lab that suggested that the four DSIs who volunteered were qualitatively different as a group than the rest of their cohort (other than the previously mentioned differences).

Table 2

Characteristics of the four DSI Research Participants

Name of DSI[16]	Racial and Gender Identities	1st Time Teaching	Countr(ies) K-12 Education	Countr(ies) Undergraduate Education	Language(s) Used in Formal Education
Dan	White Man	Yes	USA	USA	English

16 Pseudonyms used for the participants to protect their identities.

Hiroto	Asian Man	Yes	Japan/USA	USA	Japanese and English
Jakob	White Man	Yes	Germany	Germany	German
Ming	Asian Man	Yes	Singapore	USA	Chinese and English

All four DSIs, who volunteered to continue (to be referred to henceforth as "the DSIs"), participated fully in the study, which included three classroom visits with interviews before and after class as well as a final interview at the end of the semester. The DSIs shared some interesting characteristics as well as had some important differences in background (Table 2). The DSIs were compensated for their participation with refreshments during the interviews and a 50 Dollar Amazon gift card upon completion of the study.

Data Collection and Analysis

I used multiple methods to collect data on these cases of calculus instruction and the meaning the DSIs were making of their interactions with students around the content in order to triangulate the findings. These data examined the DSIs' perspective on teacher/student interactions around content in general, and their enactment of the practice of giving students verbal feedback in particular.

Mathematics Teaching Laboratory Data.

Data on the DSIs was collected in several forms during the MT Lab to inform the data collection process in the study of calculus instruction. I conducted a preliminary analysis of the DSIs' learning during the MT Lab, drawing on video records of the sessions, the DSIs' professional development journals, and instructional log. In this analysis, I drew from the practice-based preparation literature (e.g., Boerst et al., 2011; Grossman et al., 2009) in order to flag key characteristics of the phenomenon of learning practice in order to conduct a preliminary round of coding. I used that preliminary coding to develop the semi-structured interview protocols for study of calculus instruction.

Additionally, the DSIs completed a survey contained questions about their background, formal schooling, and teaching experience as well as their beliefs about the nature of intelligence and the potential of teaching to impact learning. The survey was composed of items on a five-point Likert scale from three different externally developed instruments (Appendix A). The first eight items were adapted from an instrument developed by Carol Dweck (2000) to measure adults' beliefs about the nature of intelligence (p. 178). Drawing from other researchers' work with this instrument (Gibson & Dembo, 1984; Riggs & Enochs, 1990; Ryan, personal communication), the items were adapted from general measures of beliefs about intelligence to measure the DSIs' domain specific beliefs about their own and students' mathematics abilities.

Classroom Observation Data.

First, I used a semi-focused, ethnographic approach to collect observational data from each of the four instructors' classrooms. The strategy allowed me to focus the observation on teacher/student interactions without superimposing a structure to those observations. I observed each DSI teach three 90-minute periods spaced at least three weeks apart over the course of the semester. In addition to the 270 minutes of instruction, I observed interactions prior to and after the instructional period, adding up to an additional 90 minutes of classroom discourse surrounding formal instruction. During the classroom visits, with the permission of the students, I sat at one of the eight small tables in the classrooms with three or four students. To put the students at ease, I took ethnographic fieldnotes (Emerson, Fretz, & Shaw, 2011) by hand in a basic lined notebook and drew whatever the instructor put on the board, just as they were doing. The copied text and images from the board were also important as a point of reference later for the stimulated recall interviews. Moreover, I dressed in jeans and university branded shirts, similar to many of the students, in order to be less intrusive. The students usually greeted me and occasionally asked me about my clothes, accessories, or which department I was from, but otherwise did not react to my presence in any noticeable way.

I audio recorded the classes and the verbal interactions before and after instruction. I transcribed all the audio records with the exception of the instructors' long turns of talk or lectures in which there was no interaction with students. From the transcription data, I identified episodes of interaction between an instructor and one or more students. I analyzed the instructor and student interactions from each instructor's classroom separately in order to determine if there were different patterns across the instructors. First, I identified three common types of instructor/student interactions in the data around student-generated questions, teacher-generated questions, and small group work, which became the three primary codes for those data. Over 95% of all instructor/student interactions in the classroom occurred as a result of one of those activities. An interaction was coded as initiated by a student-generated question when a student posed a question, with or without an explicit invitation for questions by the instructor. The teacher generated question code was applied when the teacher asked a question that led to an interaction with one or more students. The code was not used when the instructor did not leave any wait time for the question to be answered. Finally, the small group work tag was applied for interactions between the instructor and at least one student when the students were working on problem sets with their tables. This code often overlapped with the teacher- and student-generated question codes, but not always.

Second, I identified the student(s) interacting with the instructor. I coded these interactions as individual (the instructor and one student) or small group interactions (the instructor and two or more students). Additionally, I coded the gender and racial identities of students (Table 3). These student data are limited in that I could not collect data directly from the students beyond what they shared voluntarily during the observations. All the students across the courses with one known exception[17], signaled cis-gender identifications as men or women through their use of clothes, pronouns, and other verbal references. Determining the students' racial identities was more difficult. Most students I identified in the data as Asian, Latinx, or Black self-identified in conversations with the instructor or other students. For example, in his first planning interview, Hiroto reported a conversation with three students in which they identified themselves as Korean. In Dan's class, I observed two Latina students discussing their family histories, one student with roots in Cuba and the other in Mexico. Most of the White students did not self-identify their race. I identified most of the students who did not share information about their ethnic or racial identities as White based on a phenotypic cue, skin color, as a proxy for racial identity. As a result, it is possible that I have misidentified a student as White, and a clear limitation of the data collection methods in this study.

Table 3

Undergraduate student gender and racial identity demographics by instructor's class

	Dan's Class (N=29)	**Hiroto's Class (N=30)**	**Jakob's Class (N=30)**	**Ming's Class (N=29)**
Gender Identity	12 Women 17 Men	12 Women 17 Men 1 Queer	13 Women 17 Men	16 Women 13 Men
Racial Identity	1 Black	2 Black	1 Black	3 Black

[17] One student identified their gender as queer, neither as a man or women, adopting the pronouns they, them, and their.

	4 Latinx 7 Asian 17 White	3 Latinx 9 Asian 16 White	2 Latinx 7 Asian 20 White	5 Latinx 6 Asian 15 White

Third, I coded the instructional moves in the interaction. The most common moves were instructors asking questions, giving directions, offering mathematical explanations, modeling mathematical work, and providing student feedback. Since the practice of giving student feedback was especially important to this study, I coded the three components of feedback the DSIs were introduced to in the MT Lab, which involved communicating high academic expectations, the belief that a student could reach those expectations, and usable information. For feedback, in addition to documenting on teacher/student interactions in which one or more elements of the feedback practice was enacted, I also documented potential opportunities instructors had to give student feedback but did not. Those opportunities were identified by looking at interactions in which feedback was given and locating similar scenarios that ended abruptly without feedback being offered.

Instructor Interview Data.

During each classroom visit, I also collected three types of interview data: planning, retrospective recall (or reflective), and stimulated recall interviews (Appendix B). In addition, I conducted a cumulative retrospective recall visit at the end of the semester with each of the DSIs. The main objective of these interviews was to give the instructors a forum to talk about and reflect on their interactions with their students. Before each observation, I conducted a brief, five to ten-minute planning interview in which I primarily asked about their plans for the lesson they were about to teach. During the second planning interview, I also asked the participants about the challenges they faced interacting with students in the classroom. They were given two examples of parts of teaching that newer instructors sometimes find challenging: (a) giving clear directions and (b) giving students clear signals that they are capable of doing challenging work. The first challenge was a control; however, the second challenge was a key element of the feedback practice they were taught during the MT Lab. This prompt was added after the other components of the feedback practice were observed during the first round of teaching observations with the exception of communicating to students that they were capable of doing challenging work. The DSIs were asked again about both challenges during the third planning interview as well as in the final interview. The prompt was added in order to uncover the reason the DSIs were not communicating that message during the classes that were observed.

Additionally, after each classroom observation, I conducted a two-part interview with the DSI, which included a reflective interview followed by a semi-structured stimulated recall interview. For the reflective component, I asked them to talk about how they felt the class went and if they noticed anything surprising or out of the ordinary. Giving the DSIs the opportunity to reflect on their lesson served multiple purposes. First, reflecting on the class helped put them back into instruction, which was an important primer needed to access their thinking after for the stimulated recall component. Second, the DSIs communicated a need to reflect on how they felt the lesson went over all before they examined their thinking and decision-making in the moment, which was central to the next part of the interview. Third, the reflective interview provided the DSI with an opportunity to nominate certain interactions from the class for examination during the stimulated recall interview. Fourth, the reflection questions were aligned with the planning questions to gather data additional data on their planning and their perceived outcomes of that planning. Finally, the reflection questions allowed me to collect data on how the DSIs perceived the class was similar or different to others. This gave me a sense of whether they thought something was new, surprising or different about the class on the day I observed.

To prepare for the semi-structured stimulated recall component, I selected approximately 15 minutes (not necessarily continuous) of the audio recording from the observed class to listen to with the DSI. I chose clips in which there was a great deal of teacher/student interaction around content or I identified an opportunity for teacher/student interaction that did not occur. I planned at least three places to stop the recording and give the DSI a recall prompt (e.g., What are you paying attention to at that moment?), and in addition, I included any clips that the DSI mentioned during the reflective interview. The DSI was urged to stop the recording at any point if there was something he wished to talk about as well. The DSIs all took advantage of this option and stopped the recording at least once every recall interview. I prompted them to talk about five key components of their "in the moment" instructional experience: decision-making, thinking, observations, feelings, and meaning making. The prompts were identified as connected to categories of "in the moment" instructional experience based on the content of the DSIs' responses to the prompts (e.g., What are you paying attention to at that moment?). These prompts helped uncover the emic perspective of the instructor in the moment of instruction.

All interview data were transcribed by a professional and then refined by me. This served as a check in the analysis process, having two people create the transcripts increased their reliability. I conducted a three stage coding process with the transcript data. To begin, I identified any mention of being good or bad at doing mathematics. Next, I looked at those instances and labeled the object of the utterance if there was one: the instructor as a student, the instructor's students, the instructor's peers, or other individuals. Very few of the instructors' comments about being good at mathematics were abstract to the point of not having some type of object (3 comments across all four DSIs). Finally, I added descriptors of the quality being discussed to the instances and grouped the comments by that content. I refined the codes and repeated the process. After three cycles of testing and revising, the codes reached saturation (Small, 2009). Seven codes emerged from this process, which were then applied to the observation data as well.

Findings

Seven personal[18] characteristics the instructors valued for doing mathematics emerged from the planning, retrospective recall, stimulated recall, and end-of-semester (final) interview data. The four instructors talked about each of these values at least once over the course of the semester, although their emphases were different as explored next. This high level of consensus was not surprising, given the instructors were enrolled in the same pure mathematics doctoral program and worked for same instructional leadership team. They were undergoing similar socialization processes, and as a result, they recognized a common group of characteristics as valuable for doing mathematics. The instructors often represented these characteristics as "qualities that make someone inherently successful as a mathematician" (Dan, final interview) or as "abilities that make you one of the good ones" (Ming, final interview). In this section, each construct is presented as conceptualized by the instructors. Then, the ways the instructors use the constructs as lenses through which they view their students are illustrated.

18 The researcher also looked for characteristics instructors raised that would be considered collective. Only one instance was found. During the second visit, Hiroto talked about mathematics as a spiritual or religious act in ancient Japanese society. This was coded as a collective value. Since Hiroto did not mentioned this value in connection to his own research or instruction but as an interesting fact, the researcher decided not to include spirituality as a valued characteristic in the findings (Table x).

Accuracy and Precision as a Construct and Lens

Accuracy and Precision.

First, accuracy and precision made up the construct talked about most frequently across all the instructors. Although this construct is actually made up of two distinct concepts, the instructors wove the values of accuracy and precision together. This intertwining not only made it nearly impossible to code accuracy and precision separately, but also signaled that they were thinking about them as paired characteristics. The instructors framed the construct of accuracy and precision as the ability to represent a mathematical idea as exactly as possible (precision) so that the idea is also correct relative to the canon of mathematical knowledge (accuracy).

Although all the instructors referred to these characteristics as important in their own research and instruction, Dan spoke about accuracy and precision the most often and in greatest depth. During his first retrospective recall interview, Dan said,

> *You know there's a lot of different ways to explain this [the value of a derivative at a point]. I have to be careful because I don't want to say it too precisely and using the vocabulary we're supposed to avoid. But making sure that the imprecise statements I want them to say are precise enough that they're always true.*

Dan's comment surfaced the value of being exact (precision) as well as being correct (accuracy). More importantly, however, his statements exemplified an underlying relationship between precision and accuracy that Dan and the other instructors frequently talked about encountering in instruction. As Dan hinted in his comment, at orientation and weekly course meetings, the instructional leadership team explicitly directed the instructors not to use specialized terms that did not appear in the textbook. Dan viewed specialized vocabulary as critical to speaking about mathematics with precision. Yet, Dan and others acknowledged that specialized terms were not accessible to beginners and best avoided in instruction. Those specialized terms help to compress the vast mathematical knowledge of the discipline (Bass, 2003; 2015). Avoiding specialized terms, however, concerned the instructors. They felt that the lack of precision in vocabulary put the accuracy of the statements they were making and teaching their students to make at risk.

This concern about sacrificing accuracy by being less precise arose in another way as well among the three multilingual instructors. While fluent in English, Jakob, Ming, and even Hiroto each talked about being able to communicate mathematical ideas in another language with more precision, which they often felt compromised their capacity to explain concepts accurately in multiple ways to peers and students. Jakob acknowledged that this limitation likely prevented him from using specialized language that his students would not understand anyhow. Although Dan only spoke English, this theme emerged in a subtle way in his interviews as well. He often expressed frustration about not being able to use "mathematical language" in the classroom, which he felt forced him out of his comfort zone and required that he "break down the ideas" instead. Bass (2003) acknowledge that deconstructing and explaining ideas is often "an obstacle to mathematicians as teachers, in which role they must 'decompress' the subject matter in order to connect with their less-initiated students" (p. 769). Avoiding specialized language was a key example of how the instructors struggled to balance precision and accuracy while communicating ideas in accessible ways to their students.

Noticing Student Accuracy and Precision.

Rarely did any of the four instructors make positive comments on students' accuracy and precision. There was a clear consensus among the instructors that their students were not as accurate or precise in their communication of their mathematical thinking as their instructors wanted. For example, Jakob reported, "the students talk around their ideas a lot instead of using the vocabulary from the book and lecture" (Jakob, visit 2, stimulated recall interview). Already frustrated by the restrictions on specialized vocabulary, the instructors placed a great deal of emphasis during instruction on using the terms they were allowed to reference from the textbooks.

Dan, for instance, insisted on his students using the terms *local and global minima and maxima* in several lessons. He even explicitly said to a Latina student who described the global maxima as "the highest point" when answering his question, "your answer would be more precise if you used the real mathematical terms, rather than your everyday language" (Dan, visit 2). It is important to note that Dan did not model the use of the specialized language or even explicitly state which term he was referring to in the interaction. Dan offered the criticism without providing any useful information. In comparison, when an Asian man responded to a question later in that same lesson and said "that's the lowest point for that part of the function", Dan responded by saying the answer was right and then offered "a slight adjustment" by revoicing what the student said using the specialized term, *local minimum* (Dan, visit 2). Not only did Dan communicate that the answer was correct, but he also modeled the use of the specialized vocabulary. The data show that the instructors' emphasis on specialized vocabulary was uneven across majoritized and minoritized students, especially when interacting with students one-on-one or in small groups. This uneven emphasis is particularly important because, while the instructors framed accuracy and precision as a characteristic that could be learned with effort over time, they did not intervene in systematic ways across students to support learning.

When the instructors commented on accuracy and precision, the instructors mostly noticed the lack of this characteristic in minoritized students' written and verbal communication about mathematical ideas. For instance, Ming explicitly re-taught vocabulary to a group of majoritized students, but did not do so with a group of minoritized students. Ming modeled the use of and then defined the term *optimal price* to a group of four, White undergraduate men in his class. He said, "So the price before they start losing sales is called the optimal price" (Ming, visit 1, observation transcript). In Ming's interactions during the same group work task with a group of three Black undergraduate women, however, he did not correct their use of everyday language or model the use of *optimal price*. Later, during the retrospective recall interview, Ming talked about the the Black undergraduate women's lack of understanding of key terms, but made no mention of that same problem with the four, White men. In fact, rather than remarking on these students' lack of precision and accuracy, Ming instead talked about the surprising discrimination they demonstrated in their approach to the problem.

When speaking about the whole class, all four instructors expressed dissatisfaction with students' accuracy and precision; however, when individual incidents occurred in which the instructors noticed problems with accuracy and precision, they tended to comment on minoritized students' lacking of this characteristic, but not on majoritized students. More concerning yet, as in the example, the instructors tended to intervene less when minoritized students failed to use precise and accurate language.

Communication as a Construct and Lens

Communication.

Second, each of the instructors talked about communication, which they understood to be the ability to represent mathematical ideas in words so that even non-experts can understand. Communication was discussed almost as often as precision and accuracy. While communication came up in the instructors' discussions of precision and accuracy, the construct was also discussed separately as a highly valued characteristic for doing mathematics. Communication was one of three constructs (i.e., accuracy/precision and interpretation) that the instructors framed as an ability that can be grown and improved with effort; evidence of a growth mindset in respect to this characteristic (Dweck, 2007).

Hiroto talked about the value of communication more frequently than the other instructors and the other constructs. During his first retrospective recall interview, Hiroto said, "We want them [the undergraduate students] to be able to say a sentence about it [the mathematical idea] to someone who doesn't know calculus; something that person could understand and pull away meaningful information from it." Hiroto and the other instructors framed improving students' communication as an important learning goal. Jakob, Dan, and Hiroto explicitly linked teaching students to use mathematical vocabulary to building their communication skills. During his first retrospective recall interview, Jakob shared a personal example that illustrated this connection: "I think I have all the vocabulary to communicate my ideas but sometimes if your vocabulary is bigger you can communicate your ideas better."

In the case of communication, the instructors were not simply referring to knowing and using specialized vocabulary, but the ability to talk about those mathematical concepts in accessible language for a given audience. The instructors viewed this ability to communicate mathematical ideas in everyday language as not only demonstrating the understanding of the terms, but also, the ability to share those ideas with others. The instructors also talked about improving their own communication in research and instruction. This was particularly important to Hiroto, who spoke about practicing his communication when he designs problems for his students as well as in his own research. The idea of practicing communicating skills came up with all the instructors and informed the claim being advanced here that they saw this characteristic as learnable.

Noticing Student Communication.

As with accuracy and precision, the instructors' perspective that communication was a learnable trait underscores the importance of how they used the construct as a lens through which they view their students. The instructors talked about students' communication frequently in their interviews. The same pattern seen for accuracy and precision was observed for communication. The instructors noticed and commented on instances they believed majoritized students communicated effectively and minoritized students fell short of effective communication. For instance, during every retrospective recall interview, Hiroto made unprompted comments on the lack of clarity of a verbal or written response of one of the women in the class. When pressed to explain what the difficulty was, Hiroto admitted that his question probably was not as clear as it should have been. When prompted with a similar instance with a man where there was clearly confusion about the response, Hiroto immediately took responsibility saying, "I think I asked a poor question there so there was no good way to answer that question" (Hiroto, visit 2). Furthermore, there were nearly twice as many negative comments about minoritized students' communication than positive comments on majoritized students' communication.

The instructors' often commented on the clarity of student-posed questions as well. This focus was particularly compelling, since those questions were the topic of the vast majority of positive comments on student communication. Dan and Hiroto, in particular, noted not only the clarity of majoritized students' verbal communication in their questions, but also talked about facial expressions and other body language informing their understanding of their questions. Unfortunately, the instructors called on White or Asian men nearly three times as often to ask questions than women identifying as White, Asian, or Latina. Only rarely were Black women called on when they raised their hands to ask questions.

These patterns in student-posed questioning are significant to the findings on communication, since they highlight how few opportunities minoritized students, in general, and Black women, in particular, had to communicate mathematical ideas in class. Given the lack of opportunities to practice this skill (as well as ask their content questions) relative to their majoritized peers, they naturally had fewer opportunities to develop and demonstrate their communication. Additionally, the observation and interview data show that the instructors drew from additional sources of information when listening to majoritized students' questions[19], including body language. Although no conclusions could be drawn from the data, the researcher hypothesized that the additional information instructors had for majoritized students improved their capacity to understand their questions, whereas the lack of information on minoritized students complicated their capacity to understand.

The lack of opportunities for minoritized students to pose questions in class, left the instructors with their web homework and quizzes as the only consistent evidence of the development of communication. Since the instructors consistently complained about how weak most of the explanations were across those assessments, minoritized students, by default, received mostly negative comments and were seen to have deficits in communication. Finally, since the instructors viewed communication as a learnable skill, minoritized students, particularly Black women, were viewed as making less progress as learners, attracting more negative attention than their majoritized peers.

Curiosity as a Construct and Lens

Curiosity.

Third, the instructors framed curiosity as the desire to understand how and why something works mathematically or can be manipulated. This was the only construct that the instructors understood as a disposition or attitude, rather than as an ability. As such, the instructors viewed curiosity mostly as a fixed trait that one possesses that could not be learned or grown (Dweck, 2007). While this trait was mentioned by every instructor, Dan and Hiroto spoke about curiosity the most frequently and with greatest depth. For both instructors, curiosity, more than any other characteristic, was why a handful of students had the potential to be mathematicians.

During the third retrospective recall interview, Dan talked about curiosity as one of the characteristics that made a student "one of the good ones" in his mind. The researcher asked Dan how he responded to the director's email request to encourage students to take one more mathematics course. Dan explained that he recommended to "the good ones" that they take a proof-based course. He explained,

19 For a more detailed discussion of the differentiated information instructors had for majoritized and minoritized students, please see Enright (in progress b) - Invisibility isn't a superpower: Positioning Black undergraduate women on the margins through core instructional practices in mathematics classrooms.

Getting to see how these things work really opens everything up for some people. They think, 'Oh well, this is really awesome. Now I can start to show why these things are true. I understand it all.' With other people the added formalism and rigor and proof, it is just like, 'Why are we doing this? I already knew these facts.' They just don't have the curiosity to be a mathematician.

Both Dan and Jakob talked about enjoying interacting with others who show curiosity in mathematics. Jakob admitted that when his students demonstrated curiosity that made his job as an instructor easier, which he appreciated. Each of the four instructors talked about the importance of being enthusiastic about mathematics in the classroom, because they believed that motivated students. Not a single instructor connected their enthusiasm, in particular, or teaching, in general, to helping students learn to be curious. Curiosity was simply a trait a student either possessed or lacked.

Noticing Student Curiosity.

Curiosity was a characteristic that the instructors noticed a handful of majoritized students having and many minoritized students lacking. Although the data on instructors' out-of-class interactions with students was limited to the information instructors volunteered during interviews, the interview data suggested there was a correlation between the students who had out-of-class conversations with the instructors and the instructors' determination that those same students had curiosity about mathematics. If this pattern was an accurate representation of out-of-class conversations with students, this would suggest that the instructors were limited in their capacity to notice curiosity. The instructors seemed to need to build a relationship with a student in order to see a student as exhibiting curiosity.

In the interview data, the instructors interpreted students who posed questions as curious because they had additional background information. For instance, Dan described not only the modes of participation Joshua, a White man, engaged in, but also attributed specific motivations to those actions (Dan, visit 2, retrospective interview transcript):

I mean clearly he [Joshua] was the most involved. He had his hand up for many of the questions, asked his own questions. And the thing is it was obvious that he was -- I don't want to say struggling with the concepts but it wasn't that he knew everything. He was actively in the process of learning, tackling the ideas, asking *the questions that he was getting stopped at. It was very much a learning process for him and I think that was very important.*

Dan and the other instructors talked about being an active learner as a sign of curiosity, a trait critical to being a mathematician. Yet, when Dan described the participation of Kristine, a White woman, he did not attribute her actions to any specific motivation:

Also the girl, Kristine. She was up on my right, the closest student there. She was very active as well, both asking questions and answering questions.

According to the observation data, Kristine participated in similar ways and frequency as Joshua. Her questions were also similar to the ones Joshua posed. Dan, however, did not attribute her engagement with the content as a sign of curiosity, as he did with Joshua. This pattern of attributing curiosity as the motive behind majoritized students' question-asking but not minoritized students held true for all four instructors.

Although the instructors did not recognize most minoritized students as being curious about mathematics, they did often interpret their question-posing to other motivations. Wanting to know if something was going to be on the exam was the most common motivation the instructors attributed to minoritized students' questions. Dan and Jakob linked the motivation of learning only for the exam explicitly to a lack of curiosity, while Ming and Hiroto only implied that connection. While the instructors occasionally made this claim about a majoritized student as well, the instances of the positive attribution to curiosity outnumbered the negative attribution to only learning for the exam by two to one. The instructors talked about being enthusiastic about mathematics to create a space in which curiosity was encouraged, but stopped short of suggesting that one could learn to be curious.

Discrimination as a Construct and Lens

Discrimination.[20]

Fourth, the instructors understood discrimination as the ability to recognize the difference between a better approach to a problem and other, less desirable ones. Although discrimination

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20 Thank you to Hyman Bass for helping name and clarify the construct of *discrimination*. While this term has many different meanings, particularly in discourses on access and equity, discrimination is only used for the purposes of this article to conceptualize the ability to decide on the most reasonable approach when there are multiple options. The use of this term is specific to the context of mathematics.

was not a common theme in the data, each instructor talked about the characteristic at least once and emphasized its importance in problem-solving. For the instructors, discrimination was really about making the most reasonable choice among several options. As a result, discrimination implied the ability to recognize multiple options as well as recognizing a better one among them (often times there is no best one). Over personal correspondence, Bass (2016) described discrimination:

> *It is a widely shared disposition among professionals having to do with good sense and judgment about the use of resources. "Don't kill a mosquito with a canon." "Don't repair a watch with a hammer." For computation, mathematicians view technology as but one of a repertoire of tools and methods, and the one chosen is adapted to the individual problem at hand. It is a discriminating decision. The problem with many students is their lack of discrimination, making the calculator the default choice in all cases.*

In fact, the most common reference to discrimination by the instructors was in connection to the use of calculators. Jakob and Hiroto most often talked about how students struggled to discriminate between problems in which a calculator would be a useful tool, and those for which there were better alternatives. Jakob said,

> *Well I try to give them different perspectives on the material. Today we had this sine function where we had the zero slope and they could have of course just put that into their calculators. But I wanted to make clear that there sometimes are smarter ways to do that and that there are things they probably haven't learned in high school.*

While the instructors did not explicitly talk about discrimination as a fixed trait, they never talked about teaching students to discriminate or students learning or improving that ability. As in the quote, several times Jakob commented on the need to teach students in high school to discriminate between approaches to solving a problem. When asked how they learned, Jakob and Hiroto could not recall being taught how to discriminate, nor could they explain how they developed the skill. It is possible that the instructors' lack of awareness of developing their own skill with discrimination made it difficult for them to imagine teaching it to others.

Noticing Student Discrimination.

In all but one instance, the instructors commented only on minoritized students' lack of discrimination. In the one exception, Jakob talked about a White undergraduate man not understanding that he could answer a quiz question without a calculator. This exception is interesting in that one student also frequently missed class and quizzes as well as showed up unprepared. The comment, while about the student's lack of discrimination, was embedded in a broader complaint about his lack of effort in the course. All the remaining instances in which the instructors commented about students' lack of discrimination in the use of resources (i.e., calculators) or failure to choose the best method of solving a problem focused on minoritized students. Hiroto, for example, only commented on the lack of discrimination in the use of resources for three undergraduate women, identifying as Latina, Asian, or White. He never attempted to intervene and explain to these students that there were better alternatives to problem-solving.

While there was only one instance of critique, the instructors commented occasionally on majoritized students' ability to discriminate between alternatives. As noted in the findings on accuracy and precision, Ming highlighted the discrimination that four, White undergraduate students demonstrated in their approach to a problem assigned during group work. Ming believed that the White men wanted to see if another method would be a better approach to solving the problem. He focused on the trait that the majoritized students demonstrated, rather than comment on their failure to use specialized vocabulary. Ming, however, commented on a similar lack of accuracy and precision of a group of minoritized students. Also striking about this comparison is that the Black women were attempting to solve the problem using a different approach than the steps suggested by the instructor as well. Rather than identifying these efforts as evidence of discrimination, Ming interpreted that activity as a lack of attention to the design of the problem, and thus, as a lack of understanding of that the problem was asking. He did not interpret their efforts to find an alternative method as evidence of discrimination. The instructors consistently noticed and commented on the lack of discrimination shown by minoritized students, and even occasionally praised a handful of majoritized students for demonstrating the trait.

Table 4

Constructs Instructors Valued in Doing Mathematics

Construct Valued in Doing Mathematics	Description of the Construct That Emerged From the Data	Example of Instructors' Framing of the Construct From the Data
Accuracy & Precision	The ability to represent a mathematical idea as exactly as possible (precision) so that it is also correct relative to the canon of mathematical knowledge (accuracy).	"You know there's a lot of different ways to explain this. I have to be careful because I don't want to say it too precisely and using the vocabulary we're supposed to avoid. But making sure that the imprecise statements I want them to say are precise enough that they're always true." - Dan
Communication	The ability to represent mathematical ideas in words so that even non-experts can understand.	"We want them to be able to say a sentence about it [the mathematical idea] to someone who doesn't know calculus; something that person could understand and pull away meaningful information from it." - Hiroto
Curiosity	The desire to understand how and why something works mathematically or can be manipulated.	"Getting to see how these things work really opens everything up for some people. They think, 'Oh well, this is really awesome. Now I can start to show why these things are true. I understand it all.' With other people the added formalism and rigor and proof, it is just like, 'Why are we doing this? I already knew these facts.' They just don't have the curiosity to be a mathematician." - Dan
Discrimination	The ability to recognize the difference between a better approach to a problem and other, less desirable ones.	"Well I try to give them different perspectives on the material. Today we had this sine function where we had the zero slope and they could have of course just put that into their calculators. But I wanted to make clear that there sometimes are smarter ways to do that and that there are things they probably haven't learned in high school." - Jakob
Independence	The ability of a group or individual to do mathematical work without being reliant on an expert, in particular the instructor.	"I'd rather they [the small group] talk about this and learn from an experience rather than me just guiding them toward the answer." - Hiroto
Interpretation	The ability to understand or explain mathematical ideas.	"Interpreting mathematics is very uncommon in these lower level courses but it is very important. I don't just mean here [the University] but in general especially in high school. You're not expected to explain how you got an answer very often if you get the right answer…. Any interpretation that goes on is usually internalized without much written or spoken discussion about it." - Dan
Being Systematic	The ability to work methodically through a procedure, especially when solving a mathematical problem.	"The students, a lot of them, kind of have a grasp of the concepts but when they work through problems they mix things up sometimes because they don't work things out systematically. They start doing one thing and half way through they get sidetracked by a related approach then they end up with the wrong answer. It wasn't that they didn't understand things but they were not working through things systematically." - Ming

Independence as a Construct and Lens

Independence.

Fifth, the instructors described independence as the ability of a group or individual to do mathematical work without being reliant on an expert, in particular the instructor. Although each of the instructors mentioned this characteristic frequently, none of them discussed what it meant to be independent in depth. Independence was framed by the instructors as a choice; students either choose to exercise their independence or not. Through this frame, the instructors implied that everyone could do mathematics independently, but some chose not to for a variety of reasons, including laziness and lack of confidence.

Hiroto and Ming discussed independence more often than the other instructors. For Hiroto, independence was central to his philosophy about the best way to learn mathematics. He said, “I’d rather they [the small group] talk about this and learn from an experience rather than me just guiding them toward the answer.” Both Hiroto and Ming acknowledged that this characteristic was particularly important for their students to have, because as instructors, they did not have enough time to work with all their students during small group work. Having students work independently reduced the pressure they felt to circulate to all their students during group work.

Noticing Student Independence.

While the instructors had little to say about the construct of independence, they often reported that many of their students worked fairly independently, citing examples of students working in small groups and individually for significant periods of time. These reports, however, referred almost exclusively to majoritized students or small groups where at least half of the group members identified as White or Asian men. In fact, Dan and Hiroto expressed frustration with some students – almost exclusively White or Asian men – being too independent, working alone during group work instead of with their tablemates. Although this could have been interpreted as a negative quality for these students, both Dan and Hiroto reflected on those situations as evidence of their need to improve their facilitation of group work, not as negative characteristics of those students.

In contrast, when the instructors had complaints about students not working independently enough, those students were almost always Black women or Latina students. For example, Hiroto talked about how some students wanted too much help during small group work, which he saw as evidence that they did not want to work independently (alone or with their peers). It is interesting to note that he made these comments about the tables that were entirely women. When the tables were made up of some or all men, Hiroto described the students as "engaged" when they asked him questions. Similarly, Ming commented on the lack of independence of the women in his class, particularly of the three Black women – Maya, Krystal, and Daeshai. The observational data did not support Ming's claims that they asked for help more often or in different ways than groups of majoritized students. In fact, Maya, Krystal, and Daeshai stopped asking for help two-thirds of the way through the semester, yet Ming continued to describe them as too dependent on him for help with content throughout the semester.

Table 5

Constructs Instructors Noticed Students Have or Lack

Construct Valued in Doing Mathematics	Noticed About Majoritized Students	Noticed About Minoritized Students
Framed as Malleable Characteristics		
Accuracy & Precision	SOMEWHAT LACK	MOSTLY LACK
Communication	SOMEWHAT LACK	MOSTLY LACK
Interpretation	SOMEWHAT LACK	MOSTLY LACK
Framed as Fixed Characteristics		
Curiosity	SOMEWHAT HAVE	MOSTLY LACK
Discrimination	SOMEWHAT HAVE	MOSTLY LACK
Independence	MOSTLY HAVE	SOMEWHAT LACK
Systematic	SOMEWHAT HAVE	SOMEWHAT LACK

Interpretation as a Construct and Lens

Interpretation.

Sixth, the instructors framed the construct of interpretation as the ability to understand or explain mathematical ideas. As with accuracy/precision and communication, teaching students to interpret mathematical ideas was one of the instructors' learning goals for their students. This construct was only mentioned once by Ming and three times by Hiroto and Jakob over the course of the semester. Dan, however, talked about the concept more frequently, and like the others, he framed interpretation as something one could learn over time with effort. Dan said,

> *Interpreting mathematics is very uncommon in these lower level courses but it is very important. I don't just mean in Michigan but in general especially in high school. You're not expected to explain how you got an answer very often if you*

get the right answer…. Any interpretation that goes on is usually internalized without much written or spoken discussion about it.

Dan recognized that interpretation was not often required of students in high school and entry-level college mathematics courses, but that students were likely interpreting mathematical ideas without vocalizing them. He made an effort in his teaching to create opportunities for students to explain mathematical ideas to each other. This related to Dan's philosophy of learning in which people needed opportunities to talk mathematics to make sense of what they are learning.

Noticing Student Interpretation.

The instructors saw interpretation as something that most of their students needed to improve upon, and as such, framed interpretation as learnable. Rarely, did any of the instructors make positive comments about students' ability to interpret mathematical ideas. Most of the opportunities to interpret mathematical ideas were embedded in team and web homework, quizzes, and the examinations. The instructors talked about students internalizing too much of their explanation, assuming that their thinking behind their work was clear to the instructor or examination graders. For example, Jakob noted about a White woman's exam response: "she obviously knew how to do the problem but she didn't show us the steps she did which were probably in her head so she lost a lot of points" (Jakob, visit 2, retrospective interview). When asked explicitly how many students he felt made this mistake, Jakob said, "too many, nearly all of them do it", yet in this interview, all of the examples he volunteered were of women (Jakob, visit 2, retrospective interview). When talking about the lack of explicit interpretation, the instructors cited examples of White women more than twice as often as White or Asian men. There were no examples offered of women identifying as Asian, Latina, or Black.

Being Systematic as a Construct and Lens

Being Systematic.

The seventh and final characteristic the instructors valued for doing mathematics was the ability to work methodically through a procedure, especially when solving a mathematical problem. This construct was also framed as something students either chose to do or not. If one wished to work slowly and carefully, then one worked systematically through the steps given by the instructor. Being systematic was the construct that the instructors spoke about least often during the course of the semester. Ming mentioned the construct three separate times, but the rest of the instructors only mentioned it once.

Ming conceptualized being systematic as following a series of steps carefully in order to arrive at an answer. During his second retrospective recall interview, he said,

> *The students, a lot of them, kind of have a grasp of the concepts but when they work through problems they mix things up sometimes because they don't work things out systematically. They start doing one thing and half way through they get sidetracked by a related approach then they end up with the wrong answer. It wasn't that they didn't understand things but they were not working through things systematically.*

Ming saw working systematically through a problem largely as a matter of concentration. He also talked about working carefully through procedures in his own research. Ming believed this allowed him to advance in his work. As a result of this belief, Ming often gave his students explicit steps to follow during lecture or in the problems that he assigned them during group work. An unintended side-effect of this focus on being systematic was that the students were pushed to think procedurally, rather than conceptually when working on those tasks. For Ming at least, the value he placed on being systematic seemed to be at odds with his emphasis on interpretation.

Noticing Students Being Systematic.

Of the four instructors, Ming commented the most on students either being or not being systematic. When commenting on specific students having or lacking this characteristic, Ming had a clear division in what he noticed about his majoritized and minoritized students' work. In interviews, he attributed careful, systematic work only to majoritized students. In contrast, he only commented on the lack of this characteristic with minoritized students. In fact, during the interviews, Ming often diagnosed minoritized students' errors on homework, quizzes, and exams as "careless", rather than as evidence of misconceptions or gaps in mathematical knowledge. In one group work period, Ming praised a group of majoritized students for "figuring out the problem their own way", but when confronted with a group of minoritized students taking a similar approach, he told them they needed to follow the steps at the bottom of the problem. This does not suggest that Ming saw all majoritized students as systematic, and minoritized students as lacking this characteristic. It shows that Ming only commented on this characteristic when he noticed its presence in majoritized students' work and its absence in minoritized students' work. This pattern held true for the few instances the other instructors also commented on students being or not being systematic.

These seven characteristics were valued by each of the instructors for doing mathematics; however, these values often existed in tension with one another. Moreover, instructors were inconsistent in recognizing these characteristics in their students, particularly when teaching across identity difference. Next, the discussion takes up the implications of the relationship between instructors' conceptions of being good at doing mathematics and what they notice about their students. Additionally, this section raises questions about how these values and lenses could shape the instructors' enactment of teaching practices differently when interacting with majoritized and minoritized students.

Revisiting "The Good Ones"

Given the doctoral student instructors' (i.e., doctoral students; instructors)[21] minimal professional preparation for instruction and lack of instructional experience, they drew heavily on the resources at their disposal to inform their teaching in the entry-level calculus courses. The resources they reported utilizing mostly belonged to one of two categories: their apprenticeships of observation and departmental resources. These resources informed what the doctoral students learned to value. The findings shed light on what characteristics the instructors, as future mathematicians, valued for doing mathematics well and how those values were used as lenses when looking at their students' capacity and potential to do mathematics. Finally, these findings raise critical questions about how these values when used as lenses could lead to more robust interactions around content between instructors and students who share their gender and racial identities, and weaker or non-existent interactions with students when teaching across identity difference.

What Does It Mean to Be "One of the Good Ones"?

As previously discussed, the doctoral students' past experiences (i.e., their "apprenticeship of observation," Lortie, 1975) offered them a limited, idiosyncratic view of the relationship

21 In the discussion section, the doctoral student instructors will be referred to as doctoral students when their role as students in the department is fore fronted, and as instructors when their role as teachers of undergraduate students is the focus.

between teaching and learning. By extension, these apprenticeships also shape what characteristics instructors value for doing mathematics. In other terms, the doctoral students drew from their own success in mathematics as students to conceptualize who the "good ones" were in director's email, introduced at the beginning of the article. For example, all four future mathematicians mentioned enjoying and learning well from lectures, whereas research shows that lecture as a mode of instruction is not effective for most students (Armbruster, Patel, Johnson, & Weiss, 2009; Henderson, Finkelstein & Beach, 2010; Graham, 2012; Preszler, 2009), particularly women and students of color. The doctoral students talked about how much they enjoyed working independently puzzling over mathematics problems that came up in lectures and books. For them, independence and curiosity were innate traits they possessed that drove their success navigating mathematics. As such, the instructors believed that the "good ones" would demonstrate those traits in ways that they would recognize from their own experiences, not realizing the limitations of those experiences. Not one of the doctoral students mentioned the environments, systems, or other structures that shaped their opportunities to engage in mathematics, enabling them to rely on these individualistic traits. They shared anecdotal examples of opportunities to explore mathematics and how they relied on their independence and curiosity. The researcher saw no evidence that they were aware that others had different opportunities and experiences navigating mathematics, including many of the students they were teaching.

During their own experience as students, the doctoral student instructors saw interaction with others as opportunity to communicate about mathematical ideas. As students, talking about mathematics allowed the doctoral students to develop their ideas further, test the soundness of their interpretations, and share their interest in mathematics. These motivations for communicating with others around mathematics are highly individual. In addition to reinforcing the instructors' value of independence, these motivations likely shaped how they conceptualized communication, interpretation, and being systematic. The doctoral students discussed these constructs in terms of activities that were important to be able to do independently so one could then contribute to the construction of knowledge in the discipline. That construction of knowledge in classes might occur in small groups, but contributions are made individually, driven by one's skill in the seven characteristics the future mathematicians highlighted. The doctoral students drew from their apprenticeships in their conceptualization of what participation in mathematics looked like as well as how one learns to do it well – becoming "one of the good ones."

The doctoral student instructors who participated in this study studied and taught in the same mathematics department, and as a result, were exposed to common socialization processes and messages. They reported that their experiences as students as well as instructors in the department taught them a great deal about what it meant to be "one of the good ones" in this particular institutional context. On a general level, these doctoral students talked about being told by faculty and more advanced doctoral students that research was valued more highly than teaching. They all reported being warned that spending too much time working on their teaching would signal that they were not serious mathematicians. These messages about research and teaching had specific ideas about what doing mathematics well-meant in the department. These ideas focused on valued characteristics, such as accuracy and precision, discrimination, and being systematic. The doctoral students frequently mentioned feeling pressure to demonstrate those characteristics in preliminary examinations, interactions with faculty in and out of class, and to their peers. To these doctoral students, showing these skills meant proving their potential to contribute by constructing knowledge in their sub-field. They conceptualized accuracy and precision, discrimination, and being systematic as important traits individuals needed to be acknowledged as "one of the good ones" and successfully compete for resources in the discipline.

This is not to suggest that these future mathematicians' motivations are any way undesirable; however, they are limiting. Individualism is characteristic of the norms and values of a subset of cultures, in particular of White, middle-class and affluent communities in the United States, whereas more collectivist orientations are traditionally attributed to Black and Latino communities in the United States (Vandello & Cohen, 1999). While this study did not explicitly collect data on students' values and orientations, the idea of collaboration and other collectivist-orientations surfaced often during student conversations before, during, and after classes.[22] It is important to note that the more individualistic communities correspond to overrepresented (i.e., majoritized) populations in mathematics and mathematics-intensive fields. The individualist orientation of these overrepresented communities also dominated the conceptualization of the characteristics the doctoral students valued in doing mathematics. As a

22 For an example of these collectivist-orientations among students, please reference Enright (in progress b) - Invisibility isn't a superpower: Positioning Black undergraduate women on the margins through core instructional practices in mathematics classrooms.

result of this correlation, one must ask, whose values were not being recognized as a result? And how did the instructors' use of these values as lenses cause them to be much more likely to notice the potential to do mathematics well of students who shared gender and racial identities with them?

How the "Good Ones" Recognize Other Potential "Good Ones".

The department was not only a shared context for the doctoral student instructors' work, but also, a source of information and other resources about the work of teaching mathematics, particularly entry-level calculus. Messages from the mathematics department given to the instructors about undergraduate students and their potential to be part of the mathematics community were also impactful, especially when the instructor did not have other information about a student. In the data, instructors frequently demonstrated a lack of information about their minoritized students, which could mean that they were relying on messages, such as the one from the department, to inform their interactions with and assessments of those students (Enright, in progress b). For example, Ming reported, "We [pre-calculus instructors] were told coming into the class that most of our students weren't really quantitative people or they weren't really looking to do quantitative majors. But I think some of my students are, so that was a little bit surprising" (Ming, end of semester interview). In other words, the instructors were primed to notice and think about their students, and specific groups of students, as not likely to be interested in mathematics. Whether this became a self-fulfilling prophecy or not is beyond the scope of this study. The data, however, do show that instructors noticed students as having potential who reminded them of themselves as students.

The bias the departmental messages conveyed to these instructors about their students being disinterested and unlikely to pursue mathematics was interrupted by instructors' being able to identify with and value specific traits they saw a few majoritized students as possessing. For example, the instructors complained that many students were unwilling to volunteer to ask or answer questions in whole group. They were also frustrated that only a few students came to office hours, except for right before an examination. The instructors said that this was evidence that many of the students were not curious about the mathematics or "motivated to learn for learning sake" (Dan, visit 2, retrospective interview). The instructors also offered examples of how as students they participated actively as learners by asking and answering questions and reaching out to their teachers. The instructors found it easier to connect with and learn about students who participated in the same ways as they did.

The instructors' apprenticeships of observation provided them with a limited, non-technical view of teaching practice. With this limited view, the instructors had no preparation to think about students across lines of difference. Having been students, the instructors had resources they could use to consider their roles as experts in content, relative to the students they were teaching. Yet, those resources and resulting perspective seemed to set them up to teach beginners who performed that student role in ways similar to their own experiences. Each of the instructors had a few students who they reported identifying with, relating to, or empathizing with in their classes. These few students, with one exception, were White or Asian men.

One of Dan's students, Melissa (a White woman) proved to be the exception. Over the course of the semester, he spoke about Melissa often, but not in detail, as an engaged and capable student. When Dan received the director's request to identify "the good ones" and encourage them to take another mathematics class, he thought of Melissa. Dan said that when he decided to talk to Melissa that he was making a deliberate decision to "do his part to help fix the problem of underrepresentation of women in mathematics" (Dan, visit 3, retrospective interview). Unexpectedly, when he approached Melissa to talk about her course selection for the following semester, she told him that her father was a mathematics professor, and she was considering mathematics or physics as her major. While there was nothing wrong with Dan's decision to approach Melissa, her exposure to mathematics and mathematicians made it likely that she was socialized to navigate mathematics spaces in ways that Dan could recognize and value. In looking to reach out across difference, Dan (also with a father who taught mathematics) ended up approaching a student with similar opportunities and experiences to him.

Beyond such exceptions, the instructors struggled to teach across identity difference. The instructors regularly recognized majoritized students for demonstrating characteristics or actions the instructors recognized as connected to what they value in doing mathematics. They then interpreted similar actions by minoritized students as evidence that they lacked a valued characteristic. For example, as discussed in the findings, Ming recognized a group of majoritized students for their ability to discriminate between approaches to a problem, but critiqued a group of Black women for not being systematic in their approach to the same problem when they attempted a similar, alternative approach.

Also, instructors more often interpreted majoritized students' actions as exemplifying a value instead of lacking one. From the same example, Ming recognized the group of majoritized students for their creativity instead of criticizing them for not being systematic in their approach to the problem. (The students did not have a systematic approach to their trial and error; they were simply plugging numbers in to see if they worked.) Another example of this pattern was observed in the motivations the instructors attributed to majoritized and minoritized students when they asked questions about the content. The instructors viewed majoritized students as demonstrating curiosity, which was a valued characteristic for doing mathematics. Yet, for similar questions, they saw minoritized students, particularly Black women, as lacking independence as learners and attributed the questions to their desire to know if a topic will be on the test. The instructors seemed to be unable to notice positive traits in minoritized students, almost as if their lack of capacity to identify with students across gender and racial difference prevented them from seeing those students as potential mathematicians.

Why Gatekeeping in Mathematics Is at Its Heart an Issue of Equity.

This study sheds light on ways in which the process of novices moving from the periphery to the center of a domain of practice through social participation depend in large part on experts' gatekeeping activity. Practitioners with different levels of expertise between the periphery and core play different roles in the induction of potential members. Gatekeeping activity is understood as the creation of barriers for the individuals whom experts perceive as not belonging or as the mathematics department framed as not "one of the good ones".

Examining instructors' beliefs and how those beliefs shape their perspective of students (lenses) does not address the persistent inequitable access to learning opportunities and success for minoritized students in mathematics and mathematics-intensive fields. To better understand the production of inequity, an investigation of how those beliefs and lenses inform instructors' differentiated enactment of instructional practices across minoritized and majoritized students is necessary. This study was limited in its scope, and therefore, generalized conclusions cannot be drawn from the data explicitly connecting beliefs and lenses with the enactment of practice. Nevertheless, evidence from the data on this small sample of instructors demonstrated the existence of differentiated practice along students' identity lines and offered possible insights into why this inequitable enactment of practice might occur.[23]

The data showed that instructors consistently enacted core instructional practices differently across minoritized and majoritized students. Instructors held robust dialogues on content with majoritized students in whole class, small group, and one-on-one interactions. In these dialogues, the instructors called on majoritized students to ask and answer questions on the mathematics. Additionally, the instructors used a variety of teaching moves to check in with majoritized students to see if they got the information they needed in the interaction. For example, the instructors often used facial expressions and other body language to determine if they had responded to a majoritized student's question. The instructors also checked in with the student verbally to ask if the response addressed the student's question, and often asked the student to ask the question again. In contrast, when responding to questions from White or Asian women, and occasionally Latina students, the instructors held weak dialogues with the students, answering the question quickly or dismissing the question as something outside of the scope of the lesson. The instructors very rarely called on Black women to ask or answer questions, leading to many disrupted dialogues. When the instructors did call on them, the question the Black woman posed was not the question the instructor chose to answer. Jakob explained in one instance, "I didn't understand what she was asking, so I just answered a different question so I didn't embarrass her" (Jakob, visit 2, stimulated recall interview). The instructors did not demonstrate the same concern when interacting with majoritized students. The difference was not in the questions asked, which the instructors often remarked were similar types of questions and had comparable legitimacy. The difference was in the gender and racial identities of the students.

Whether these biases are unconscious or conscious impulses, research on positive feedback bias illustrate how biases inform interactions between White instructors and Black students, in which the instructors' concern about not looking bigoted is privileged over those students' need for valuable information about their work (Harber, 1998; Harber, Stafford, & Kennedy, 2010). Evidence of acting on biases also arose in this study with instructors abruptly

23 For a detailed description and further discussion of these findings, please see Enright (in progress b) - Invisibility isn't a superpower: Positioning Black undergraduate women on the margins through core instructional practices in mathematics classrooms.

ending interactions with Black women during group work without providing the critical feedback they offered to all the other students they interacted with in the class. The instructors dismissed the need to interact with their Black undergraduate women by painting them as not independent enough. Yet, these Black women asked for the same type of help with the same frequency as many other students who received the instructors' time and help with the content. These and other incidents raise questions about instructors' capacity to recognize and foster the potential to participate in mathematics communities of minoritized students, especially Black women, in the ways they do for majoritized students. While not every student will and should go into mathematics and mathematics-intensive fields, every student should have the choice to pursue those pathways. That choice should not be limited by the gatekeepers in the discipline in systematic ways along identity lines, regardless of whether that gatekeeping is intentional or not.

Implications for Research

This study was an exploration of the relationship between instructors' beliefs, lenses, and enactment of practice across majoritized and minoritized students using four cases of beginning instruction in a mathematics department. The design of this qualitative study created many affordances as well as constraints in terms of contributions the research can make to mathematics and higher education. This study examined four cases of calculus instruction situated in a single mathematics department. Although that department plays an important role in the field by producing future faculty, more research is needed to determine the state of access and equity in instruction in other departments. Future studies should consider comparing teaching across difference in mathematics departments across institutional types, particularly colleges and universities with diverse missions (e.g., religious, liberal arts, and minority serving institutions).

Another avenue for investigation would be to look at teaching across difference for a more diverse group of instructors. For example, researchers could examine what teaching across difference looks like in minoritized faculty's instruction or across faculty with varying levels of instructional experience.

In addition to diversifying the institutional and faculty samples, more research is needed on students' experiences with instruction, particularly for minoritized students. Due to limited resources and the agreement with the Institutional Review Board, this study did not collect student data outside of the classroom. Interviews with students would allow the research to build a more robust picture of their perceptions of and experiences with instructional practices.

Moreover, the findings from this study on within group differences for undergraduate women underscore the need for more research on experiences of students with one or more marginalized identities in classrooms across disciplines. In other words, we need a better understanding of how experiences with instruction are shaped differently across students with various intersecting identities, rather than assuming the effects of experiences are additive. Interesting differences surfaced in the data on the interactions between instructors and the Latina and Black undergraduate women in their classes. Furthermore, only a couple Black men and Latino students were in two of the instructors' classrooms, which meant that these students' experiences could not be examined across the cases. The lack of representation of Latino and Black men and American Indian students is evidence of inequity in mathematics and mathematics-intensive fields, yet this underrepresentation leads to further marginalization in research. By not investigating these students' experiences, they are being doubly underserved – first by higher education and then again by higher education research.

Conclusion

Persistent, inequitable access to learning for minoritized students raises questions about how we can better prepare instructors to enact equitable instructional practices in their teaching to support their interactions across difference with minoritized students. More research looking inside instruction is needed to understand the challenges involved and how they impact student choice. Then, research could be leveraged to intervene in instruction, preparation for instruction, departmental and institutional policy, and student support programs to build not just stronger STEM pathways but the conditions under which minoritized students are no longer marginalized and have access to full participation.

CHAPTER FOUR – INVISIBILITY ISN'T A SUPERPOWER: POSITIONING BLACK UNDERGRADUATE WOMEN ON THE MARGINS THROUGH CORE INSTRUCTIONAL PRACTICES IN MATHEMATICS CLASSROOMS

Abstract

The aim of this comparative case study is to examine the role of core teaching practices in interactions between four mathematics instructors, who identify as White or Asian men, and their students, holding different racial and gender identities, around academic content at a predominately White institution. An episode of instruction from one of the cases illustrates how an instructor's varied enactment of core practices creates differentiated access to learning for minoritized students in the class, particularly for Black undergraduate women. This research contributed to the empirical data on the experiences of undergraduate women, especially, Black women, by examining instruction from the inside in an identity-threat-rich domain – mathematics. Moreover, this study uncovered how inequitable enactments of core instructional practices can position Black undergraduate women on the margins of the classroom discourse community, impacting access to learning as well as feelings of belonging and identity development.

Key Words: Mathematics education; postsecondary instruction; critical feedback; questioning practice; access, equity and excellence in higher education; STEM pipeline; gateway courses; Black undergraduate women; STEM identity development

To build a more complete picture of barriers to access to mathematics and other Science, Technology, Engineering, and Mathematics (STEM) fields, this study examined the marginalization of undergraduate women, who were placed by the department in entry-level calculus courses at a highly selective university. One case of instruction is presented in detail to demonstrate how the inequitable enactment of core instructional practices marginalized a group of STEM-identified, Black undergraduate women. These students were academically successful and motivated– “vanguard” students (Steele, 1997) – who valued success in their current and future mathematics courses. These students went from seeing a future in STEM and related mathematics-reliant fields to de-identifying completely with these domains over a period of months. What about their experiences in and with mathematics instruction influenced this dramatic change?

Increasingly, researchers are attending to the experiences and development of students of color in higher education (e.g., Chavous, 2005; Cross, 1971, 1991; Parham, 1989; Sellers, Smith, Shelton, Rowley, & Chavous, 1998; Watt, 2015). This signals a significant departure from an inequitable, historical trend of researching and theorizing about student populations predominately consisting of White men (e.g., Chickering, 1969; Erikson, 1959; Kohlberg, 1976). Informed by Crenshaw’s (1989, 2004) foundational work on intersections of identity and systems of oppression, recent research has begun to fill long-existing gaps in the literature by examining the experiences of students holding multiple historically marginalized identities, including Black women (e.g., Porter & Dean, 2015). Of particular relevance to this study, some researchers have drawn from intersectionality theory to build robust understandings of experiences with and in postsecondary STEM fields for women of color to better understand, among other issues, persistent barriers to access, opportunity, and success (e.g., Alexander & Hermann, 2015; Rodriguez, 2015; Ross & Godwin, 2015).

One important dimension of Black undergraduate women’s experiences in higher education is shaped through interactions between instructors and students. This study reported in this paper examined how the enactment of core instructional practices in key gateway courses in postsecondary mathematics informs the interactions between instructors and students holding different gender and racial identities. In particular, a case of instruction is presented to probe Black women’s experiences with instructional interactions and consider how these experiences might shape their feelings of belonging in STEM as well as choice of college majors and career plans.

The study of instructional interactions addressed three research questions:

- How does the instructor's enactment of core instructional practices position undergraduate students with different gender and racial identities, particularly Black women, to participate in a mathematics classroom’s discourse community?

- How might instructors' perceptions of students with different gender and racial identities inform and be informed by their interactions with students, particularly Black women?
- How (if at all) do relationships between perception and interaction vary in instructors' interactions across students who hold similar or different gender and racial identities than their instructors, particularly Black women?

This article starts by situating these questions in broader discourses on gender and race in higher education, more generally, and postsecondary mathematics education, in particular. The unique positionality of Black women in these educational spaces is explored to help situate the findings for this study in systems of inequity. Next, the design of the larger project and methods for this particular study are described in detail. Then, three core findings are presented from the comparative case analysis. These findings are then illustrated by looking inside an episode of instruction from one of those cases. Implications for minoritized students of gatekeeping through the inequitable enactment of instructional practices are considered. Finally, future directions for research as well as the affordances and constraints of this study are discussed.

The Marginalization of Black Undergraduate Women

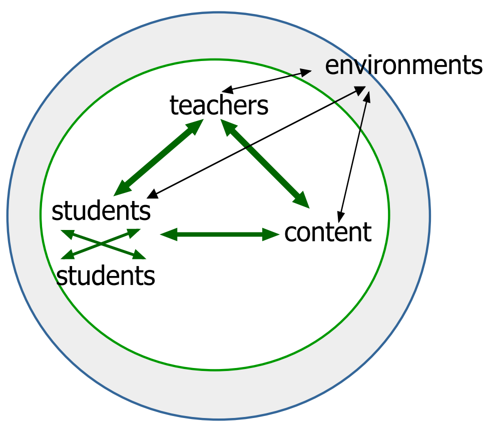
These questions examining equity in instructional interactions are important, because often times when research explores the marginalization of students from underserved populations, the focus is on representation, performance, and completion. For example, from longitudinal data on a nationally representative sample of aspiring STEM undergraduate students, Hurtado, Eagan, and Hughes (2012) discovered differences in STEM degree completion rates within a six-year period between White (43%) and Asian American (52%) undergraduate students and their Latino (29%), Black (21.8%), and Native American (24.9%) peers. These foci, while important, treat instruction as a black box in which the mechanisms, which contribute to degree completion in STEM education for underserved populations are left unexamined. Looking inside instruction at the mechanisms that contribute to students from underserved populations exiting STEM could inform change processes that need to occur in order to create more equitable environments in which minoritized students can flourish without compromising their identities or well-being. This article focuses on the experiences of undergraduate women in entry-level mathematics with a particular interest in the unique experiences of Black women. This section unpacks why research in this area is of particular importance.

Stereotype Threat in the Mathematics Instructional Ecology

This study leveraged a cultural ecological framework to examine instruction from the inside (Enright, in progress a; Enright, Hickman, and Ball, 2016). This framework has two central components: the conceptualization of the dynamics of instruction as an ecological system and the cultural dimensions of that ecology. First, this framework draws on the instructional triangle – a conceptual, analytical tool that depicts instruction as a series of dynamic, bidirectional, and interrelated relationships among teachers, students, and the subject-matter content embedded in multiple environments (Cohen, Raudenbush, & Ball, 2003, Figure 3). The black arrows show how aspects of the environment are at once drawn into the instructional ecology as well as created within that ecology and sent back out into the broader environments.

Figure 3

The instructional triangle (Cohen, Raudenbush, & Ball, 2003)

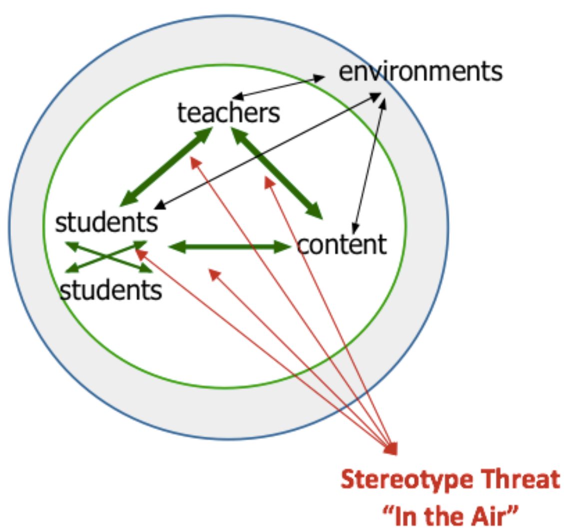


Second, while many aspects of culture from these broader environments permeate the classroom, the notion that negative stereotypes enter instructional ecologies through content and individuals is especially relevant in mathematics contexts (Figure 4). Mathematics is a stereotype-threat-rich domain, which means that there are multiple negative stereotypes minoritized individuals must contend with when learning and working in this disciplinary space. In the classroom, the instructor and students do not need to believe these negative stereotypes for them to threaten the performance on important tasks for individuals with identities salient to those stereotypes (Steele, 2010). According to Steele, that these stereotype exist "in the air" is of particular importance, since it points to the reality that their presence is unavoidable and must be dealt with in order to create an equitable learning environment (Steele, 1997, Figure 2). When left unchecked, these stereotypes can alter the instructional context for individuals under threat.[24]

The instructional triangle is a powerful tool to illustrates how these stereotypes come to be "in the air" of a classroom, and how they disrupt every dimension of instructional interactions among instructors, minoritized students, and academic content[25] (Figure 4). Stereotype threats are an explicit feature of the instructional environment that only harm the performance of students' contending with the identity threats – minoritized students. Moreover, there is evidence that these stereotype threats may actually "lift" or improve the performance of students belonging to identity groups not under threat, majoritized students, when being compared to minoritized peers (Walton & Cohen, 2003). These dynamics are important, since comparisons are made implicitly and explicitly in mathematics instruction, a stereotype-rich environment.

Figure 4

Stereotype threat "in-the-air" mediating relationships among teachers, content, and students in the mathematics instructional ecology



[24] Please read the author's conceptual article (Enright, in progress) for a more extensive description of how stereotype threat operates.
[25] Please read Enright (in progress a) for a more extensive discussion of how stereotype threats disrupt instruction interactions.

Individuals need not subscribe to these stereotypes in order for them to threaten those who hold identities salient to those stereotypes within a specific domain. Steele conceptualized this phenomenon as stereotype threat (Aronson, Quinn, & Spencer, 1998; Steele, 1997, 2010; Steele & Aronson, 1995). Research has extensively documented the harm this phenomenon has caused in mathematics contexts for women, Black, and Latino students who identify with the domain, especially Black women and Latina students[26] (e.g., Spencer, Steele, & Quinn, 1999). Moreover, research has explored a variety of manipulations that either amplify or mitigate stereotype threat (e.g., Brown & Josephs, 1999; Ganley et al., 2013; Good, Aronson, & Harder, 1997; Inzlicht & Ben-Zeev, 2000; O'Brien & Crandall, 2003; Shih, Pittinsky, & Ambady, 1999). Due to destructive stereotypes about women as well as Black students in mathematics, Black women contend with multiple forms of stereotype threat.

The Emerging Model Minority Myth

With rising levels of educational achievement over the past 20 years, researchers and administrators have increasingly positioned Black women as a new model minority group (e.g., Kaba, 2008). Model minority myths falsely depict students from specific minority groups as harder working and valuing education more highly than other minority groups (Lee, 1994). For example, the model minority myth in schooling contexts has often promoted the image of Asian American students as "academic superstars" in contrast with other underachieving minority groups (Lee, p. 413). College completion rates, however, should not be the only measure of successfully navigating higher education (Museus & Kiang, 2009), especially since they do not account for student choice, safety, any dimension of health, and many other critical outcomes.

Model minority myths promote harmful stereotypes rooted in White supremacy in that they perpetuate oppression and maintain systems that privilege Whiteness (Chou & Feagin, 2015). These myths often ignore the complexities other intersecting identities such as gender, class background, and first generation status. In essence, these myths essentialize students and their experiences by reducing them to racial stereotypes. Particularly relevant to this study, model minority myths reduce the visibility of students being targeted. For Asian American

[26] Stereotypes can threaten any individual holding a salient identity, including teachers. Black and Latino pre-service teachers experience stereotype threats in their teacher education programs (e.g., Scott & Rodriguez, 2015). White pre-service teachers have also been shown to experience identity threats to their egalitarian self-images when interacting with Black students (Harber, Stafford, & Kennedy, 2010).

college students, for example, this myth has implied erroneously that all students within this group are the same, do not need institutional supports or resources, and do not suffer from forms of race-based oppression (Museus & Kiang, 2009). This myth might also account for the negligible attention by researchers to studying Asian American and Pacific Islander populations (Museus, 2009).

Likewise, viewing Black women through the lens of the model minority myth has obscured the systematic oppression they experience navigating higher education and potentially contributed to higher education administrators' inaction in response to that oppression (Rosales & Person, 2003, p. 53). The model minority myth has perpetuated the marginalization of Black women in colleges and universities by making their struggles, victories, and experiences invisible (Zamani, 2003), and thus, ensuring that they remain underserved as a group by higher education researchers, administrators, and policy-makers. Although Asian women and Latina students are also groups who are under-researched and underserved in higher education, this study centered on the experiences of Black undergraduate women in order to understand another dimension of their experience in higher education, that of the critical "gateway" courses in mathematics.

Navigating Racialized and Gendered Mathematics Spaces

The constructs of race and gender permeate the culture and structure of schooling in the United States. Mathematics as a discipline, in particular, has been positioned "as first and foremost a White, middle-class, male domain" and perpetuates a discourse that privileges individuals with those identities (Stinson, 2013, p. 71). Educational settings were constructed around and to privilege Whiteness (Banks, 2009). Individuals do not need to be explicitly racist in their attitudes and behavior to benefit from structural and cultural racism and White privilege (Tatum, 1997). According to Chavous (2005), "on predominately white institution (PWI) campuses, issues related to race often are interwoven into academic and social domains directly and indirectly resulting in an institutional climate where race is made very salient to minority and majority group members" (p. 239). Although salient to all students, Black students might be more aware of messages about race and diversity which are communicated through experiences (Chavous; Nora & Cabrera, 1994). Since instruction is the core enterprise of higher education, Black students' experiences with instructors' enactment of instruction in classrooms would be an important source of those messages.

Universities and their departments are also gendered environments (Tierney & Bensimon, 2000), especially given that "gender is a strategy that pervades the structure and culture of a discipline" (Sallee, 2011, p. 182). Mathematics as a discipline is shaped by Western male norms (Hottinger, 2016). Mathematics and other masculine gendered STEM fields (especially Engineering) are characterized by a set of gendered values that rewards a particular performance of masculinity that encourages competitiveness with and demonstrations of competence for others (Sallee). The racialized and gendered structure and culture of a discipline can be seen in the socialization of students and faculty in the discipline (Sallee). As Black women navigate postsecondary mathematics, they are in "double jeopardy" in the sense that they must contend with both racism and sexism (Beale, 1979) as well as navigate a space in which they are not recognized within the highly racialized and gendered bias of who has the capacity to become a mathematician (Enright, in progress b).

These characterizations of the structure and culture of mathematics and other STEM departments might not hold true, however, across all institutional types. Minority serving institutions (MSIs), in particular historically Black colleges and universities (HBCUs), produce a disproportionate number of Black college graduates with STEM degrees. For instance, while underrepresented students found the hyper-competitive environments in many undergraduate science programs to be "negative and disempowering" (Chang et al., 2008), students at Xavier University (an HBCU) reported a "very supportive and collaborative culture of science" in which students could "develop their science identities without neglecting their racial identities" (Hurtado et al., 2010, p. 10). The structure and culture of STEM departments in many HBCUs might be dramatically different than at PWIs, yet even HBCUs are concerned with the underrepresentation of Black women in STEM. For example, in recent years, Spelman College launched community outreach programs (i.e. STEM Education Outreach Program and MASTERS) to raise the number of Black women in STEM fields (Arthur, 2013)[27]. Representation in STEM is important, yet it is not sufficient as a sole focus in discussions of equity in higher education.

Student choice, belonging, and experiences are also critical elements in framing the successful navigation of higher education. Chavous et al. (2004) found that specific institution

[27] Article access on April 13, 2016 from http://www.insidespelman.com/spelman-reaches-out-to-the-community-to-increase-black-women-in-stem/

and classroom contexts as well as gender identity influenced the experiences, in general, and "academic adjustment process", in particular, of Black undergraduate students (p. 14). Black women at HBCUs were more aware of the gendered culture and corresponding identity threats than were the Black women at PWIs whose experiences were centered more on the salience of race and related identity threats. Their findings pointed to the need for more research exploring the differences in experiences with higher education spaces across Black students with particular attention to intersectionality of identity. Chavous and colleagues also called for additional research that attends to Black students' perceptions of instructional practices that "may convey messages about belonging and fit" (p. 13). This paper takes up that call by looking specifically at the perspective of a group of Black undergraduate students on their instructor's differential enactment of instructional practices across students in their mathematics course at a PWI.

Positioning This Research in the Broader Discourse on Postsecondary Mathematics

Given the emergence of the new model minority myth and persistent, multidimensional nature of oppression Black women face in mathematics, attention to the social and motivational factors that inform students' choice of majors (and careers) is critical. This article explicitly rejects the common discourse on minoritized students' underrepresentation in STEM framed around deficit-oriented theories of under-preparation, as being socialized to devalue STEM pathways, or as being socialized to discount their capacity to succeed in STEM fields. The studies focus on social and motivational factors that discourage domain identification and pursuit of STEM pathways among students from underserved communities, but also erroneously imply that is the norm for these students. Research has shown a significant upward trend in the number of underrepresented students entering college aspiring to a STEM major, yet there is at the same time stagnation in the number of STEM degree completers (Eagan et al., 2014).

It is important to uncover the contextually situated mechanisms (such as instructional practices) through which social and motivational factors deter minoritized students, who enter higher education valuing STEM and recognizing their potential for success, from pursuing STEM majors. This stance is part of a larger "social turn in mathematics education research" (Lerman, 2000). Hallinger (2016) argued that this revolution expanded the focus of research to include mathematical and mathematical instructional practices, including "the meanings that both shape those practices and that are created as a result of those practices…" (p. 51). According to Hallinger, this discursive conceptualization of mathematics practice has led to research on "the ways classroom practices produce gendered, racialized, and classed mathematics identities and the ways these identities either enable or constrain student participation and success in the mathematics classroom [citing examples, such as Atweh & Cooper, 1995; Hardy, 2004; Klein, 2002; Lim 2008]" (p. 51). In other words, some research has begun to move beyond the problematic deficit-oriented frames for looking at access to and success in mathematics education to create robust understandings of how inequities are produced and reproduced through practices.

This article takes up this work by examining how minoritized students make meaning of instructors' enactment of mathematical instructional practices. That meaning-making is important in that it could influence their perceptions of their potential for success in their intended STEM field and the value they place on that pathway, shown to influence students' identity formation process (Eccles, 2009). Looking across the data on the four cases of instruction, there is a clear trend of instructors, who identify as White or Asian men, differentiating their enactment of core instructional practices when interacting with students holding gender and racial identities different from those of the instructor. I conceptualize this as "teaching across identity difference." A growing body of research on wise schooling practices (Steele, 1997) is developing tools, or approaches to teaching practices, that can be operationalized to better support teaching across difference. This work takes up the model of wise feedback as an equitable approach to a core teaching practice (Cohen, Steele, and Ross, 1999; Yeager et al., 2013).

This study documents the mechanisms by which teaching practice contributes to marginalization of minoritized students by looking across four instructors' enactment of core instructional practices. This article conceptualizes core practices as teaching moves that occur with frequency in instruction and allow the instructor to influence student learning. This conceptualization draws from a growing body of scholarship on high-leverage practices (e.g., Ball, Sleep, Boerst, & Bass, 2009; Franke, Grossman, Hatch, Richert, & Schultz, 2006; Kazemi & Hintz, 2008; Kazemi, Lampert, & Ghousseini, 2007; Kloser 2014; Sleep, Boerst, & Ball, 2007) but stops short of claiming that the two focal practices in this article, facilitating small group work and giving student feedback, meet all of the criteria to be considered "high-leverage" (Grossman, Hammerness, & McDonald, 2009).

In the next section, an episode of instruction is presented to illustrate the role the enactment of teaching practices can play in further marginalizing women, particularly those who identify as Black or Latino (minoritized students). In addition, patterns of differentiated enactment of core teaching practices are presented from a comparative case study or cross-case analysis. The episode is then used to demonstrate the ways through which an instructor's differentiated enactment of core instructional practices produced inequitable access to learning opportunities and success for three Black undergraduate women. Then, the discussion takes up Eccles's expectancy value model of identity formation to shed light on the role the messages and interactions between the instructor and Black undergraduate women around mathematics content might inform their future identification with and investment in STEM. Finally, the author offers some future directions for research and examines the affordances and constraints of this study.

Introducing an Illustrative Case

This study illustrates how the enactment of teaching practices actively marginalized Black undergraduate women in pre-calculus, a critical gateway mathematics course (Gainen, 1995; Secada, 1989). Although entry-level calculus courses are considered gateways, or courses required for many majors and professional programs, research has found that just over half of the prospective engineering majors (51%) completed basic Calculus in high school (Eagan et al., 2014). In the mathematics department from this study, this information was presented to new instructors. They were told that most students entering their courses would not be interested in learning calculus, only in passing the course. During the semester, these students shifted from identifying with and engaging in mathematics as an academic domain to seeking other pathways through the university, ones that did not require additional mathematics courses. Although the nature of the data does not support causal claims, the evidence of differentiated treatment in instruction raises important questions about the role of instructional design and instructor preparation in perpetuating the marginalization of Black women in postsecondary mathematics.

During the researcher's first observation in a pre-calculus classroom (less than three weeks into the semester), three Black undergraduate women, Krystal, Maya, and Daeshai, were engaged an animated conversation about their intended majors as they waited for class to start (Excerpt 1). They discussed the affordances and constraints as well as their motivations for choosing those majors. Finally, the theme of community arose in multiple ways throughout this conversation. An excerpt of one of their conversations about their majors is included to illustrate that, at the start of the semester, Krystal, Maya, and Daeshai valued the entry-level mathematics course as a means to gain entry to their intended major. The data showed that all three students were STEM-identified; Krystal and Maya had plans to pursue engineering majors and interested in building on their mathematics knowledge and skills, and Daeshai intended to pursue a non-STEM major that required mathematics and science coursework. All three students expressed confidence about their mathematics knowledge and skills.

Excerpt 1

Classroom Observation Transcript – Ming, Visit I, Pre-class Conversation at Table Four

Maya: Hey Daeshai, have you decided what you're going to take next semester?
Daeshai: We haven't even survived our first month and you are already worrying me about next semester?
Krystal: [*laughter*] I know! Plus, it's not like we get to make a lot of choices about what we're going to take the first year anyhow. Like if you want to major in something, that kind of tells you what you are going to need to take for a while. Engineering has my life planned out!
Daeshai: Well, I haven't really decided yet…have you, already? [*looks back and forth at Maya and Krystal*] I feel like I'm still unpacking…
Krystal: Yeah, cause you brought too many shoes! [*laughter from all three women*]
Maya: I knew coming in that I wanted to do mechanical engineering. Krys is right. We don't get much of a choice. Someone else already made those decisions and because I got placed back in this math class I'm already way behind.
Krystal: Mechanical? I thought you were doing civil like me?
Maya: [*Mostly to herself*] It's unfair. I know this stuff already. I should have been in calc one…
Krystal: Yeah, okay, so what? We'll have a smooth semester. But really, mechanical?
Maya: Mechanical is better. I really want to work for GM [General Motors] like my uncle. He said that's what they will be hiring.
Krystal: You must be crazy. You'll be the only woman in your classes and they won't know what to do with a sister with espresso skin like yours. [*Maya rolls her eyes and Daeshai laughs*] Civil is where all the action is anyways. I want to go back and tell my city all the things that they are doing wrong, fix things and make them better, you know. You should take the intro classes for civil with me, Daeshai.
Maya: You just want your friends to take all your classes with you. [*everyone laughs*] Maybe Daeshai would rather study with me and all the white boys in mechanical engineering. [*laughter*]
Daeshai: Yeah, when you put it like that, I'm all about signing up, right.
Maya: Whatever, it's not that bad. We'll rush [a sorority] and make our own little community of proud Black scholars.
Krystal: Yeah, well that's going to happen no matter what. So what are you going to do?
Daeshai: Me? I might rush. I want to look into marching band too though…
Maya: [*laughter*] No! About your major?
Daeshai: Oh! [*smiles*] I think maybe architecture or design. Those sound cool and less intense than engineering. There is actually a double major option with design, so I could do something really tight with that. Plus, then we can all work together on the other side, right?
Krystal: Oh, who wants to work with me now? [*laughter from all three women*]
Krystal: Oh, shush, he's starting class. I need to hear. It's important.

This conversation, and others over the semester, show that these students valued the successful completion of this mathematics class. All three students discuss wanting to pursue STEM or other mathematics-reliant fields, and to do so, need to successfully complete at least two (architecture), three (civil engineering), or five (mechanical engineering) mathematics courses, inclusive of their current course. Krystal and Maya explicitly identified with STEM domains, and Daeshai reported an interest in two fields that are mathematics-reliant. These three students were motivated to succeed in their mathematics course. Moreover, they were placed by the department in the course based on multiple criteria, including a placement test. Yet, by the end of the semester, Krystal and Maya changed their major plans from engineering to fields that did not require additional mathematics courses, and Daeshai had stopped coming to class. What role might these Black undergraduate women's experiences in their pre-calculus course have played in this de-identification with STEM and decision to pursue other majors? To answer this question, this article will explore a representative case of instruction looking at the instructor's differential enactment of core practices across students. Moreover, the case will illustrate how that differentiation marginalized these Black undergraduate women in the classroom discourse community and created barriers for learning.

Methodology

Research Design

This study of Black undergraduate women's marginalization through instructional practice was part of a larger research project in which the researcher collected data over a two-year period in a mathematics department of a large research university. The goal of the project was to shed light on the complexity of teaching across difference in entry-level mathematics classes. The larger project included a design intervention study examining the challenges of preparing new mathematics instructors to teach equitably across difference followed by a comparative case study examining the phenomenon of new instructors teaching across difference in their calculus classrooms. This study of minoritized students' experiences with instruction, in particular Black undergraduate women's marginalization through instructional interactions, was embedded in the comparative case study.

Research Site Selection.

The research project was conducted at a mathematics department at a university with the Carnegie classification of "very high research activity" (RU/VH). The first study the researcher conducted at the site was an exploratory study of doctoral student instructors' experiences with the department's calculus teaching orientation week and first semester teaching. The orientation week was a five-day preparation program, which introduced the DSIs to the norms and expectations of their new roles as instructors in the entry-level calculus courses (calculus series). This orientation covered everything from basic logistics to offering limited opportunities for practice teaching. The department made it mandatory for the new cohort of DSIs (N=28) to participate in the orientation and gave them course credit in exchange for their time. The findings from the exploratory study showed that DSIs entered the orientation and first semester of teaching with personal theories of their own capacity to learn mathematics as well as for their students (Enright, 2015). After the completion of the exploratory study, the researcher negotiated access for the larger research project, which included an intervention seminar during the orientation week (MT Lab) and permission to recruit the DSIs to participate in the study of calculus instruction (comparative case study). This article presents the findings from the comparative case study.

Design Intervention Study.

The MT Lab was mandatory seminar led by the researcher that was situated in the department's orientation week. The MT Lab was made up of four, one-hour sessions over a four-day period in which the DSIs were taught the pedagogical practice of "wise" feedback, shown to be an equitable instructional practice (Cohen, Steele, & Ross, 1999; Yeager et al., 2013). The researcher designed the MT Lab to meet the requirements and time constraints of the department, which required dedicated time for ice breaker activities and lunch during the sessions. Due in part to these constraints, the MT Lab was considered a weak treatment unlikely to generate substantial, lasting change in the DSIs' instructional practices, and the data collected confirms that assessment. Nonetheless, the MT Lab supported the recruitment of participants for comparative case study as well as provided data on DSI learning to inform the design of the semi-structured interview protocols for the comparative case study. Moreover, the MT Lab provided data that informed subsequent redesigns of the intervention.

At the end of the MT Lab, the researcher invited all the DSIs to participate in the next phase of the research project, which was voluntary due to the additional time commitment. Of the 28 DSIs in the cohort, 16 individuals expressed their intent to participate in the study, reporting a desire to learn more about teaching and improve their instruction. These motivations differentiated the 16 DSIs who expressed interest in participating and the remaining 12 DSIs who did not. Another difference between these two groups appeared in pre- and post-survey data from the MT Lab (Appendix A). On average, the 16 DSIs motivated to learn more about teaching showed a stronger belief that teaching can effect learning, whereas the remaining 12 DSIs scored, on average, almost a point lower on a five-point Likert scale. In other terms, the 16 DSIs believed that instructors' teaching impacted students' learning, while their peers were less certain about that relationship.

The researcher formally invited the 16 DSIs to commit to participating in the study of calculus instruction at the end of their first week of teaching. The purpose behind this delay was to give the DSIs time to settle into teaching and their own doctoral courses as well as ensure those who did commit stuck with the study, reducing the likelihood of attrition. At that point, four of the original 16 DSIs interested in the study volunteered. The remaining DSIs who declined to participate reported changing their minds, because they felt they were spending too much time on their teaching, which had costs for their own coursework. Some of the DSIs said that their advisor and other doctoral students told them not to spend too much time on teaching because research was valued more in the department and discipline. The four DSIs who volunteered to participate reported receiving similar messages, yet they believed the investment would help them become better instructors, and make them better candidates for positions in their current department and for future jobs. The research was unaware of any other differences between the four DSIs who volunteered and their 12 peers who changed their minds. Moreover, other than the previously mentioned differences in motivation, no patterns emerged from the survey data that suggested that the four DSIs who volunteered were qualitatively different as a group than the rest of their cohort.

Comparative Case Study of Calculus Instruction.

The comparative case study of calculus instruction took an interpretive, ethnographic approach to classroom observation in order to look inside instruction at the nuanced interactions around the enactment of core instructional practices between an instructor and students as well as among a group of focal students. A researcher using an ethnographic approach embeds herself in a particular social setting and utilizes participant observation to explore that setting (Atkinson et al., 2001). Specifically, an interpretive approach explores the local meanings those interactions held for the different stakeholders as understood by the researcher (Hammersley & Atkinson, 2007). Although the researcher used an ethnographic approach to the classroom observation, this was not a true ethnographic study because of resource, particularly time, constraints. The researcher visited each classroom three times over the course of the semester to watch a total of 270 instructional minutes and 90 additional minutes surrounding instruction per instructor. In a traditional ethnographic study, the researcher would be expected to spend more time in the classrooms; however, the sampling approach taken in this study provided sufficient data to uncover the phenomena presented in this article.

Furthermore, looking at instruction from the inside means that the researcher utilized records of practice (Ball, Ben-Peretz, & Cohen, 2014) and participant observation to focus on the activities of instruction and how they play out within an episode of teaching. The enactment of instructional practices is made visible and foregrounded, rather than, for instance, the research subjects' decision-making or reflection. Retrospective and stimulated recall interviews with the instructors provided the researcher with additional data on the instructor's making meaning about their interactions with students and what information they had access to inform and as a result of those interactions. All together, an interpretive approach to this ethnographic study allowed the researcher to explore the interactions among an instructor and students as well as how those actors made meaning of those actions in that setting.

The descriptive case study offers a window onto the role instructional practices can take in positioning Black undergraduate women at the margins of their mathematics discourse community. This case also sheds light on how those experiences could inform the students' identity formation in mathematics and mathematics-dependent fields.

Participants in the Comparative Case Study

Although this research focused on the enactment of core practices in mathematics instruction and not the individuals involved in these interactions, some key characteristics of the instructor and undergraduate students will be shared in this section. These subject data create a more robust picture of the interpersonal interactions around the mathematics content. Moreover, these details construct the local meaning of teaching across difference. Finally, the instructor and students volunteered this information implicitly and explicitly during the semester, and since they valued these biographical details enough to contribute them, the researcher felt an obligation to include them and present them as salient to the research.

The Instructor.

Ming[28] was just beginning his doctoral program when this research project began. He was also a first time instructor. He reported doing some one-on-one tutoring as an undergraduate student, but had never worked with a group of students (let alone a whole class). Ming identified ethnically as Chinese and racially as Asian. He also identified as a man, a mathematician, and a doctoral student. Ming mentioned these identities multiple times in the interviews, and the researcher interpreted these contributions as evidence that these identities were salient to Ming during the course of the five-month research period.

The university and department considered Ming an international student, although Ming only raised this implicitly in his descriptions of his K-12 experiences. He was born in South East Asia and was educated there in bi-lingual schools where English was one of the instructional languages. Ming attended university in the United States and reported feeling completely comfortable communicating in English in informal and formal interactions, both inside and outside the classroom. During five months of interaction with Ming and observation of his teaching, the researcher did not observe any evidence of difficulty using English to communicate.

Since Ming spent four years as a student in the U.S., he had experience with multiple mathematics classrooms in an institutional context similar to the department he joined and taught in as a doctoral student. During interviews and even occasionally while teaching, he referenced his own experiences learning mathematics as an undergraduate student. He did not, however, take any entry-level calculus courses as an undergraduate; he started his mathematics coursework in an advanced differential equations course. Ming mentioned his lack of experience as an undergraduate student in a class similar to the pre-calculus course briefly during interviews but did not elaborate on what that might mean for him in the role of instructor.

[28] This is a pseudonym. Ming was one of 60 doctoral students with whom the researcher interacted with over a two-year period, at least half of whom identified as Asian, men. The information provided in this article on Ming is intended to preserve his anonymity as a participant and might be altered in non-empirically significant ways in order to avoid identifying the university, department, and instructor. Information on the exact years the research was conducted was also withheld to preserve anonymity.

The Undergraduate Students.

There were 29 undergraduate students in Ming's pre-calculus course. The department placed these students in pre-calculus based on their high school mathematics coursework, performance in coursework and standardized tests, and how they scored on a department-specific placement test. The students self-selected into the sections of pre-calculus. All of the undergraduate students in Ming's class were in the first semester of their first year, which was typical for this entry-level course in the fall semester. For many of the students, Ming's pre-calculus seminar was the smallest course they were enrolled in and the only one in which they had regular opportunities to interact with their instructor during class. The class had an even split of students identifying as men and women[29] and was racially/ethnically diverse. One third of the students self-identified as Black, Latina/o or Asian during the course of the semester. Several of the Asian and Latina/o students also identified with specific ethnic communities. Often when students talked about their ethnic identities, that identification was part of a larger conversation centered on their racial (and often gender) identit(ies). For this reason and to protect their anonymity, the researcher will only report on students' racial and gender identities.

All the data collected on these students were from classroom observations and information volunteered by Ming during interviews. At least two-thirds of the students regularly arrived before class began and stayed after instruction ended. Upon entering the classroom, the students were informed of the presence of the researcher, the audio recorders, and that she was taking notes on the class. Students' conversations with the instructor and each other were recorded before and after class as part of the research on teaching across difference. Much of the student biographical information was collected either during these conversations or from interactions during small group work.

How the Black Undergraduate Women at Table Four Represented Their Most Salient Identities in the Mathematics Classroom.

Krystal, Maya, and Daeshai frequently referred to themselves and each other as Black. Additionally, they spoke about their experiences as Black women and as members of Black communities in conversation with each other throughout the semester. These students offered a variety of subjective meanings they held regarding their Black identities (Chatman, Eccles, &

[29] During the study, no students identified outside of this gender binary to the knowledge of the researcher. This was not the case for other classes observed.

Malanchuk, 2006) in conversation with each other before, during, and after class sessions over the course of the semester. The students often referenced the importance of giving back to their Black communities, especially when discussing their anticipated academic majors. Carlone and Johnson (2007) conceptualized a student's identification with a domain as stemming from a drive to better social conditions as the development of an altruistic disciplinary identity. Maya and Krystal were developing altruistic engineering identities, and Daeshai expressed a similar set of motivations in her consideration of architecture as a major. For example, Maya talked about choosing engineering, like her uncle, in order to be a model for her community and "raise other Black children up to believe in their future" (visit 1 transcript). Krystal also talked about joining a Black sorority on campus to have a community away from home. These students explicitly referenced the color of their skin as well in their meaning-making around interactions with their instructor. There was no discussion in the researcher's presence of whether these women held ethnic identities in addition to their Black identities.

In addition, Krystal, Maya, and Daeshai frequently referenced their status as "first semester freshman" and discussed the challenges of adjusting to a new city, managing their schedules, and making friends. Discussion around these issues often centered around their experiences navigating campus as Black women and how those experiences forced them to choose between "parts of themselves" (visit 2 transcript). For example, Daeshai debated about whether to rush a Black sorority or try out for the marching band, since her demanding schedule required her to choose. She framed the choice as one between finding "Black sisterhood" on a predominantly white campus and pursuing her love of music (visit 1 transcript).

For Maya, the tension was between her identification with STEM and as a Black woman. Maya expressed her anxiety about being one of the few Black women in mechanical engineering; a concern that Krystal also repeatedly voiced. Maya had not declared her major yet because she needed to complete a few semesters of prerequisites, particularly the calculus sequence courses. She constantly talked about feeling "behind" and what it would mean to be older than most of white men with whom she would take her engineering courses. She worried about being perceived as "another struggling Black student" and feeling alienated in those courses (visit 2 transcript).

Krystal spoke about a similar tension between her identification with STEM and as a Black student. Krystal often talked about wanting to present herself as the "proud Black scholar" she was and feeling invisible in her mathematics class (visit 1 transcript). She shared her concern about being invisible to her instructors in her STEM prerequisites and what that might mean for navigating her major coursework later. She talked about the affective dimensions of this marginalization but also the practical implications of instructors refusing to talk about the content with her (e.g., Excerpt 3). The decision around "whether *it* is worth it" was present for all three students with the "it" understood as compromising on their identities as Black women to participate in curricular and co-curricular interests (visit 1, 2 transcripts). Taking these self-identifications into account, this study identifies these students as Black undergraduate women with varying levels of STEM-identified.

Data Collection and Analysis

The researcher collected first order data through classroom observations, and audio records of instruction as well as second order data through interviews with the instructor (Martin, 1981). Due to the Institutional Review Board (IRB) restrictions, no additional data were collected from the undergraduate students beyond those data that were made available through instruction. Data was gathered on the undergraduate students from their observed and audio recorded classroom discourse and activities preceding, during, and directly after the research visits.

Table 6

Characteristics of the Instructors Who Participated in the Research Study

Name[30]	Racial and Gender Identities	1st Time Teaching	Countr(ies) K-12 Education	Countr(ies) Undergraduate Education	Language(s) Used in Formal Education
Dan	White Man	Yes	USA	USA	English
Hiroto	Asian Man	Yes	Japan/USA	USA	Japanese and English
Jakob	White Man	Yes	Germany	Germany	German
Ming	Asian Man	Yes	Singapore	USA	Chinese and English

To collect the first order data, the researcher conducted three classroom visits over the semester observing a total of 270 minutes of instruction and an additional 90 minutes of

[30] Pseudonyms used for the participants to protect their identities.

classroom discourse surrounding formal instruction. The first visit occurred three weeks into the semester. The second visit was seven weeks into the semester after the first midterm examination was returned to the students. The third visit occurred during the tenth week after the second midterm examination was returned to the students, just four weeks before the final examination. An ethnographic approach was used to gather data in the form of fieldnotes that were synthesized into field memos on each individual visit and then synthesized memos across visits (Emerson, Fretz, & Shaw, 2011). Instructional artifacts in the form of handouts, seating arrangements, and images of the chalkboard work were also collected. Two audio recorders were placed in the classroom; one was attached to a microphone that the instructor wore and the other was positioned at Table Four where the researcher sat with three students. The instructor's microphone picked up his talk as well as the students' verbal contributions in his direct vicinity. The original audio recordings were transcribed by a professional as an external check on reliability. The researcher then used the audio recordings to check the transcripts for any errors or omissions during the transcription process.

Table 7

Categories of "In the Moment" Instructional Experience Examined Through Stimulated Recall Interview Prompts

Stimulated Recall Prompt	**Category of "In the Moment" Instructional Experience**
What do you notice during that interaction?	Observations, Meaning-making
What are you thinking during that interaction?	Thinking, Decision-making, Meaning-making
What do you notice about your students at that moment?	Observations, Meaning-making
What are you paying attention to at that moment?	Observations, Meaning-making
Why did you notice that?	Decision-making, Meaning-making, Thinking
What does that mean to you?	Thinking, Meaning-making
What's your reaction to that?	Decision-making, Feeling, Meaning-making, Thinking
How do you feel about that? Can you tell me why?	Feeling, Meaning-making
Were you considering any alternative strategies/moves/questions?	Decision-making, Thinking
What were you hoping students would learn from this topic/activity/strategy?	Feeling, Meaning-making, Thinking
What were you hoping to learn from that question/comment/interaction?	Feeling, Meaning-making, Thinking

To gather the second-order data, the researcher interviewed the instructor before and after each visit as well as conducted a summative interview at the end of the semester after grades were submitted. The pre-visit interviews focused on the instructor's planning and goals for the lesson as well as inquired about the instructor's perceived strengths and challenges encountered during teaching. The researcher asked questions, such as "can you tell me a little bit about what your goals are for this lesson?" After the observation, the researcher conducted a two-part interview. First, the researcher asked a series of semi-structured interview questions to prompt reflection on the lesson. These retrospective recall interviews included prompts, such as "Tell me how you felt the class went" and "Were there any students in particular you noticed during class today?"

Second, stimulated recall interviews "help to isolate particular 'events' from the stream of consciousness" (Gass & Mackey, 2000, p. 21). Audio clips from instruction together with stimulated recall prompts were used to isolate the memories of those events and to probe the instructor's thinking in the moment of instruction. It is important to note that "the theoretical foundation for stimulated recall relies on an information-processing approach whereby the use of and access to memory structures is enhanced, if not guaranteed, by a prompt that aids in the recall of information" (Gass & Mackey, 2000, p. 17). The prompts used were structured, yet open-ended in order to access data without leading the subject to the researcher's anticipated answers (Lyle, 2003). The stimuli used were the instructional episodes from the observed class in which there was verbal interaction between the instructor and at least one student. Both the interviewer and the instructor could nominate episodes to listen to during the stimulated recall interview as well as pause the recording to ask a question (the interviewer) or report something about the instruction in that moment (the instructor). The stimulated recall prompts allowed the instructor access data on the instructor's observations, meaning-making, decision-making, feeling, or thinking – or a combination of those experiences – in that moment of instruction (Table 7). Nearly all of the recall interviews were conducted within an hour or two of instruction to minimize the gap between instructional events and when the memories of those events are accessed, which could lead to "recall decay" (Gass, 2001; Lyle, 2003). Between three and six episodes were included in each stimulated recall interview depending on the length of the audio records. The audio records of the interviews were also transcribed by a professional and checked for accuracy by the researcher.

The researcher flagged all the verbal interactions between the instructor and students in the transcript data and created individual instructional episodes from the larger transcripts. This study focused on interactions aimed at a specific student in whole group, small group, or one-on-one interactions, understood as directed interactions. The data analysis focused on verbal interaction in order to address the research questions. The instructional episodes were bounded in two ways. Either the episode represented was bounded by an interaction between the instructor and student(s), or it was bounded by a specific task. The interactional episodes focused on a verbal interaction between the instructor and a student in whole-group. They, generally, provided data on the teaching practices and information used by the instructor when engaging with a particular student around a teacher- or student-posed question. None of the four DSIs engaged in cold calling (asking students to answer questions without them volunteering) during the classes with the researcher present. All the student-generated questions counted were volunteered by the students either when the students raised their hands and the instructor called on them or they called out the question without hand-raising. For the teacher-posed questions, students who raised their hands were counted as volunteering to answer the question. The task episodes typically occurred during the small group work. Since it was difficult to ascertain whether the teacher was engaging with a specific student in a group or multiple members of a group or even across groups, these episodes were bounded by task. These data showed multiple instructor/student interactions across different students within very brief periods of time around the same task. The analysis in this article draws from both sources of data.

After identifying the interactional episodes, the researcher reviewed the interview transcripts to identify the parts in which the instructor referred to the interactional episodes. Interview clips were created from those references to be analyzed with the episode data. Since there were two types of interview data analyzed, there were also two types of interview clips: retrospective recall and stimulated recall. In the retrospective recall interviews, the instructor volunteered information on various focal students in response to general, semi-structured prompts provided by the interviewer. The interviewer never asked about specific students. In the stimulated recall interviews, however, because both the interviewer and the instructor paused the audio recording, focal students were nominated by the interviewer and instructor. This is important to show that the instructor's attention to specific students was not entirely directed by the researcher.

The researcher used an iterative process of coding the episode and clip data. Patterns emerged in the interactions between the teacher and students around teacher- and student-generated questions from the episode data. These patterns will be unpacked more in the next section with the findings. The themes identified in the patterns became the preliminary set of codes. Those preliminary codes were then applied to the clip data, refined, and new ones were added that were particular to the clip data. The process was then repeated when the researcher applied the new codes to the episode data. After three cycles of testing and revising, the codes reached saturation (Small, 2009).

The coding system that emerged had four sets of codes: general codes that applied across all the data; codes specific to the episode data; codes specific to the retrospective recall clip data; and codes specific to the stimulated recall clip data. This article will report on findings from a subset of the retrospective and stimulated recall data generated by the instructor's individual/collective identification of students and knowledge of students codes. Furthermore, this article will report on a subset of the episode data generated by two of the question codes (questions asked approaching a small group and follow up questions asked when talking to a small group) and three of the feedback codes (high expectations for mathematical work, student's capacity to learn mathematics, and usable information toward improvement). The feedback codes were informed by the framework for wise feedback practice (Cohen, Steele, & Ross, 1999; Yeager et al., 2013). The researcher chose wise feedback to develop the feedback codes for two reasons. One, the four instructors received information on wise feedback and the component messages during their week-long graduate student instructor orientation. Two, the research on wise feedback shows that certain messages are critical for students learning under stereotype threat, such as the minoritized students who are the focus of this study.

The three feedback codes from the episode data were used to calculate the volume of feedback given to students holding different identities in each classroom. First, the researcher generated frequency counts for the instances of feedback given to White and Asian men and Latino, Black, White, and Asian women in each class. Second, the researcher then grouped students by gender and racial identities who received comparable volume of feedback: (1) White and Asian men, (2) White and Asian women, and (3) Latino and Black women. Third, the researcher converted the frequency codes into percentages by group for each class. Finally, the researcher compared those percentages, finding that they were fairly consistent across classes, and averaged them to generate a percentage of feedback given by group across the four classes. In the section that follows, the questioning and feedback codes are explored together with the findings uncovered through the analysis process. One of the task episodes is also presented to illustrate the analysis process and shed light on how the themes of positioning and marginalization played out in instruction when the instructor attempted to teach across difference.

Situating the Researcher in the Discussion of Identity

It is important to acknowledge and situate the researcher's own social identities in this study examining interactions, development, and meaning-making around participants' gender and racial/ethnic identities. The researcher identifies as a White (racial identity), Jewish (ethnic identity) cis-gendered woman (gender identity).[31] Additionally, while this study was being conducted, the researcher identified as a doctoral student in education. While the researcher had taught high school mathematics, she was not credentialed in mathematics or mathematics education. Her knowledge of calculus was from her own experience as a student (two years of high school calculus) and by observing calculus teaching as a researcher. The instructors and undergraduate students who participated in this study were aware that the researcher's background was in teaching and learning and not mathematics or mathematics education.

Holding common gender identities and different racial identities from the Black undergraduate women in the study hold important implications for the collection and interpretation of the data. The researcher does not belong, nor does she have access to insider knowledge of these students' communities. The data collected during this study was volunteered in her presence by these students, and it is important to consider that what information and how the students shared that information in the researcher's presence might have been influenced by the researcher's identities, specifically racial and gender identities but likely also her age and position as an outsider in the classroom.

The differences in racial and other identities between the researcher and the instructor and the students, particularly the Black women, White men, and Asian men, also hold implications for the interpretation presented in this study. The researcher did not have access to

[31] The researcher's racial identity is constructed within the framework of Whiteness in the United States. In other places, however, her Jewish ethnic identity is constructed as a non-White identity or identity of color. In those spaces, the researcher's racial identity is renegotiated and is subsumed by her ethnic identity.

student data beyond what information was volunteered in the classroom. This restriction also precluded member checking the data collected and interpretations of that data with students. To minimize the potential gaps in the researcher's interpretations, the research followed the audio records, transcriptions, and fieldnotes texts closely in her analysis and used students' own words as much as possible to guide the construction of the interpretations. Yet, these differences in identity mean that the researcher was studying experiences with instruction and identity development in mathematics *across difference*, which creates the potential for misunderstanding and misappropriating the participants' experiences.

Findings

The findings are presented in two parts. First, the patterns identified in the cross-analysis of the four cases of instruction are discussed. Second, a representative episode from one of the cases is examined in detail to illustrate how the patterns play out in instruction. The discussion examines these findings using several different theoretical lenses in order to raise important questions about the reproduction of inequity in postsecondary instruction.

Looking Across Cases of Instruction: Exploring the Use of Core Practices in Interactions with Students Holding Different Gender and Racial/Ethnic Identities

In this sub-section, the findings from the analysis of the four cases of instruction are presented. The four instructors who volunteered for this study were considered successful instructors by the department and invited to teach again in subsequent semesters (Table 6). The three findings presented below explore patterns in their interactions with students across difference during their first semester teaching.

Cross-Case Analysis Finding I: Seeing Women of Color as the "Collective Student"[32].

Across the cases of instruction, the retrospective and stimulated recall data showed that the instructors referred to the students in work groups with a majority of women of color as a

[32] The collective student is the notion that instructors considered Black women and other women of color as one collective student. This erasure of individual identities signals the lack of status these women of color have compared to their individually recognized White peers. An idea shared with the phenomenon Silva (2002, 2004) and

collective group, rather than as individuals. For instance, in interviews, Hiroto frequently referred to a group of four Korean and Korean American students as the "Korean table" where three of the students identified as women and one student identified as gender queer (neither as a man or women, adopting the pronouns they, them and their). There was another table of Asian men, however, but when Hiroto talked about them he referred to the students at that table individually by name, not as a collective. Another example, from Ming's class, will be discussed in the representative episode.

Whereas this individual versus collective identification is meaningful in its own right, the distinction also has implications for the instructors' information on students. Instructors seemed to have little information on students holding different gender identities from them and almost no information when the identity difference was across race and gender. The retrospective and stimulate recall data suggest several potential reasons for this disparity. First, the instructors tended to notice and interpret the body language of White and Asian men, but rarely reported reading the body language of women in their class. As a result, the instructors had another source of information on the men. Second, the interview data suggest that the instructors were more likely to speculate on the interests, struggles, and intended pathways for men, than women. This might be a sign that the instructor identifies with the student or at least empathizes with him, and as such, is concerning that this speculation rarely occurred with women, especially students identifying as Latina and Black women.

Moreover, while instructors noticed and commented on both men and women's behaviors in the classroom, they tended to limit their reports on women to behavior. For example, rather than describing a student's interest in a topic or struggle understanding a concept, instructors tended to focus on visible actions, such as women's modes of classroom participation (e.g., raising hands or asking questions) and negative behaviors (e.g., arriving late or using a cell phone). Instructors commented the most about negative behaviors when talking about Black women. Interestingly, this was not the case when instructors reported on a Latina student, where the focus tended to be on their modes of participation. Additionally, instructors often attributed White and Asian women's motivation for asking a question in whole group to their desire to

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later Martin (2015) named the "collective Black". This concept refers to an emerging tri-racial system where White communities occupy the top of a hierarchy, communities who are passing as or transitioning into being considered White take up the middle, and the Black collective (or non-white) communities are at the bottom.

know *what will be on the test*. For men, however, similar questions were attributed to their curiosity about the topic or desire to *understand how things work*. Unfortunately, the researcher did not document an instance where the student called on to ask a question in whole group was a Latina or Black woman, so there is no interview data on the instructors' interpretations of their questions.

For example, Dan offered the following description in response to the researcher's prompt, "were there students in particular that you notice today?" (Dan, visit 2, retrospective interview transcript):

> *Yeah. One was Joshua. I mean clearly he was the most involved. He had his hand up for many of the questions, asked his own questions. And the thing is it was obvious that he was -- I don't want to say struggling with the concepts but it wasn't that he knew everything. He was actively in the process of learning, tackling the ideas, asking the questions that he was getting stopped at. It was very much a learning process for him and I think that was very important. Also the girl, Kristine. She was up on my right, the closest student there. She was very active as well, both asking questions and answering questions.*

In the excerpt, Dan offered an extensive description of Joshua's participation, including but not limited to the modes of participation. Additionally, Dan offered an explanation of what Joshua was thinking about and his motivation for asking questions. In contrast, Dan's description of Kristine is spare, focused on what she did, not why. These differences result in the instructor having access to different types and amounts of information across students, which informs the instructor's perception and interaction with those students as seen later in the representative episode.

Cross-Case Analysis Finding II: Silencing Minoritized Students by Disrupting Dialogues.

Patterns emerged from the data illustrating striking differentiation in instructors' interactions with students across identities around student-generated questions. These patterns showed that student questions led to robust, weak, and non-existent dialogues with the instructor.

Building on the analysis of the example presented in the previous section, this next excerpt is in response to a stimulated recall prompt that followed the retrospective interview.

Dan described his interaction with Joshua, when he asked a question that Dan had not anticipated addressing in that class:

Joshua [white, male] asked a question which actually was talking about -- It ended up being a question about the second derivative. He had some extra assumption about the function. At first I didn't have a good way to answer it. I answered it. I tried to answer it. I definitely saw it did not answer his question. You know the look on his face was not that [laughs] of someone who has had their question answered so I asked him to repeat it. And I think when I answered it the second time, it was much more effective.

In his recounting of the interaction, Dan noted the subject of Joshua's question and diagnosed what caused his confusion. Additionally, Dan described his first attempt to address that confusion, and how he read Joshua's facial expressions to gauge whether he had successfully answered his question. Finally, he asked Joshua to repeat the question, so he could try to answer it again. The interaction being examined is an example of a robust dialogue between an instructor and student. The observation and interview data show that these robust dialogues almost exclusively occurred between the instructors and White and Asian men.

In contrast, Dan's interaction with Kristine is an instance of weak dialogue. Weak dialogues are interactions between an instructor and a student that is abruptly ended either with a brief response by the instructor or a refusal to respond. In this example, Dan recalls his reason for not addressing Kristine's question – he simply had not planned on covering that material during class. Interestingly, this is the same explanation he offered for why he struggled to successfully address Joshua's question; however, with Kristine, Dan refused to answer the question, ending the dialogue.

It wasn't that I had made a conscious decision not to cover inflection points that day. If I had really thought about I probably would have included inflection points. But it was because I had written [i.e. planned] my section for the next section. I said what stuff do they need and then I wrote [out what I planned to teach]…. You know it never came up; so I never really thought about it. So when *she [white, female] asked that I was just like "oh, I didn't really plan on talking about inflection points; so we won't." [laughs]*

In the representative episode, a third type of dialogue is examined, characterized by a lack of dialogue or practice of silencing. These practices of silencing exclusively occurred in instructors' interactions with Black women and Latina students. These dialogues are important for many reasons. Instructors use these dialogues to gather information about individual students. In turn, that information informs instructors' future interactions with those students, their assessment of students' work, and even their instructional planning. Abbreviated interactions, or worse avoidance of interactions, leaves the instructor with not only less information, but also fewer sources of the information they do have on students. When this occurs with minoritized students, the research suggests that instructors are more likely to let their biases inform future interactions with those students (e.g., Harber, 1998, 2004). Additionally, robust dialogues, unlike weak and non-existent ones, are opportunities for instructors to give critical feedback to students, which will be examined next.

Cross-Case Analysis Finding III: Sending Ambiguous Instead of Critical Feedback to Minoritized Students.

Across the four cases of instruction, the data showed robust patterns of differentiation in the instructors' enactment of the core practice of giving students verbal feedback. Each instructor varied the volume, clarity, and critical nature of feedback to students along gender and racial identities. There was significant variation across the individual instructors in the amount of feedback they gave students. As a result, to measure the volume of feedback, the analysis focused on in-class differentiation of enactment per instructor and then compared those patterns across the cases. For example, on the aggregate, Dan offered his students more feedback than the other instructors, yet the data showed that the proportion of feedback given to minoritized students was comparable to other three instructors.

On average across the cases (Table 8), White or Asian men[33] received the greatest volume of feedback (68%), where the instructor gave White or Asian women nearly half as much (28%). Latina and Black women received feedback only occasionally in class (4%). These proportions did not align with the demographics of students in the classrooms. For all four

[33] There were too few Latino and Black men enrolled as students in these classes to determine any pattern within or across classes.

classes, on average, there were equal numbers of men and women. In Ming's pre-calculus course, there was a slightly higher percentage of women (55% or 16 students) with about half of those women identifying as Latina (five students) or Black (three students). On average in the three calculus I courses, the percentage of women was slightly lower (42% or 12 students) with a third of those women identifying as Latina or Black. In summary, White and Asian men received more feedback from the instructors than their numbers in the class might predict, while Latina and Black women received significantly less feedback for their numbers. In comparison, only White and Asian women received a proportion of the instructor's verbal feedback in line with the percentage of students in the class.

Table 8

Average percent of verbal feedback received from instructors by students holding different gender and racial identities in comparison to classroom demographic data

	White or Asian men		**White or Asian women**		**Latina and Black women**	
	Percent students in class	Percent feedback received	Percent students in class	Percent feedback received	Percent students in class	Percent feedback received
Pre-calculus	45%	68%	27%	28%	28%	4%
Calculus I	55%	68%	31%	28%	14%	4%

The instructors also differentiated the feedback messages they gave to students across the three identity groups. These feedback messages fell along a spectrum of clear (the student understood what the instructor was communicating) to ambiguous (the student remain unsure of the meaning of the message the instructor conveyed). All three of the categories of feedback messages below could fall anywhere along this spectrum. The role of ambiguity in the differentiated enactment of feedback will be explored further in the analysis of the representative episode.

The most frequently type of feedback given to students by the instructors was usable information the students could use in the future. For example, when Hiroto noticed an error in a student's notebook during group work, he said, "remember when you are calculating the rate of change that a good first step is to identify the variables given to you in the problem" (Hiroto, visit 3, observation transcript). The information was usable in that it was specific enough to help the student solve the current problem, but also generalizable, since the suggestion could also be applied to future problems of that kind. Instructors frequently offered all their students usable information with the exception of Black women, who rarely received clear, usable information on their work. While Latina students received feedback from their instructors at approximately the same rate as Black women, the feedback Latina students received tended to be usable information, where instructors gave Black women messages about academic expectations.

The second most common feedback message was the communication of high expectations for mathematical work.[34] This feedback helped students, particularly those under stereotype threat, attribute any difficulty completing a task or lower levels of performance to its challenging nature (Cohen, Steele, & Ross, 1999). This message was broadly conceptualized as information given to a student about the level or nature of difficulty of a mathematics concept, task, or other activity. An example of feedback on high expectations is “This problem is going to be harder, because we haven’t tried to identify the local minima and maxima, only the global ones” (Jakob, visit 2, observation transcript). This message was offered consistently across students from all three identity groups, since the instructors usually delivered this message during whole group instruction. Instructors also occasionally delivered this message to individual or small groups of students, most often to Black women.

The instructors gave students the message that they have the capacity to learn mathematics much less often. This feedback message sends the message to students that the instructor believes they have the capacity to learn the content. For minoritized students, receiving the message that their instructor believes they reach the high academic standards set for the class is particularly important to mediating stereotype threats in the environment regarding salient gender or racial identities. This feedback was conceptualized in two ways: (1) a message in which the instructor explicitly told a student that he believed that the student had the capacity to increase their knowledge or improve their performance in mathematics through continued effort or (2) a message in which the instructor implied that same message about capacity by commenting on the significance of a student’s mathematical production. For instance, Dan is implicitly sending a message about John’s capacity to learn mathematics when he said, “John is raising an important point about the second derivative that we haven’t covered yet” (Dan, visit 2,

[34] Instances of the instructor giving feedback communicating high expectations for mathematical work to the whole group were not included in the frequency counts, since they were not directed at specific students. Other messages given in whole group that were directed at specific students, however, were included.

observation transcript). Instructors communicated that message to White and Asian men about twice as often than to White and Asian women. Only two instances of that message being given to Latina students were documented, and no instances were observed with Black women.

Looking Inside a Case of Instruction: Examining the Differentiation of Core Practices When Teaching Across Difference (The Case of Table Four)

This episode will use the findings from the cross-analysis to illustrate how the Black undergraduate women sitting together (Table Four) in a pre-calculus classroom were positioned as "the table of struggling students" (retrospective recall interview transcript, visit 1). The episode was chosen because all three findings from the cross-case analysis can be seen within a brief, eight-minute episode of instruction. The three Black undergraduate women in Ming's class, Krystal, Maya, and Daeshai, sat together at Table Four during the observed classes across the semester. They were the only Black students in the classroom. The classroom observation and interview data suggest that the three students at Table Four were not struggling any more or less than the four students positioned as "engaged learners" or the four students "potential future mathematicians" (Ming, visit 1, retrospective recall interview transcript). The findings presented next from the representative episode shed light on how this positioning occurred. The first finding highlights the instructor's difficulty seeing the Black women in his classroom as individuals as well as his lack of information about them that he has for other students. The second finding illustrates how the instructor's interactions with students around content vary when interacting across difference. The third finding examines the patterns that emerge around student-posed questions or other requests for support from the instructor. Then, the discussion section will take up the question of why this differential positioning occurred, and the final section will present some implications and future directions for research, instructional design, and instructor preparation.

Context for the Lesson Prior to the Episode.

There were a total of 14 questions asked by the teacher in the lecture before students began their small group work. Krystal raised her hand four different times in response to the teacher asking the whole class questions and was not called on at all. Those four questions were answered by two white male students from Table Seven (each answered four questions) and a white female student from Table Five (answered three questions), who also had their hands raised. Krystal (Table Four) had her hand raised for the two questions answered by the White men as well as for the two questions that went unanswered. Krystal's tablemates noticed that she was not being called on and started making comments, such as "she must be invisible today" and "she's wearing her Black camo[uflage]" (Ming, visit 1, observation transcript). Other students at Tables One and Six glanced at Krystal's raised hand and back at the instructor with expressions signaling confusion and discomfort, but the students did not intervene.

Framing the Mathematics Task.

Thirty minutes into the lesson, each student group received a single copy of the handout with the pizza slice word problem (Table 9). The word problem prompted students to solve a system of equations in order to find the optimal price. The instructor discovered this problem in the department's pre-calculus course materials repository, where former course coordinators and instructors save problems, practice tests, and lesson plans for reuse. The instructor gave the students ten minutes to work on the problem before the solution would be presented by one of the groups to the whole class.

Table 9

The Pizza Slice Word Problem Worked on during Small Group Work

The current price of a pizza at a local pizza joint is \$8. At this price, 1000 pizzas are sold daily. The manager wonders if she will generate more revenue by increasing the price of the pizza. She knows that for every \$1 increase in price, she will lose 100 sales a day. What price will generate the most revenue for the pizza joint? If you don't know where to start… Find a formula for the price *P* of a pizza if the price is raised *d* dollars. Find a formula for the number of pizzas sold *T* if the price is raised *d* dollars Find a formula for the revenue earned *R* R should be a quadratic function of *d*. Put the formula for *R* in vertex form. Now answer the original question.

Two issues with the word problem emerged from the classroom observation data. First, at least half of the student groups, including Tables Four and Five, struggled to decipher the abbreviations, or *variables*, used in steps a-e of the problem. Although the variables are defined within the context of the problem, those definitions were not explicitly stated. This increased the cognitive burden of solving the problem for many students, who had difficulty figuring out what the variables stood for, while making sense of what the problem was asking them mathematically to do.

Second, the language of the prompt – "If you don't know where to start…" – was ambiguous. Table Four interpreted that part of the prompt as signaling that the following steps were optional. In comparison, Table Five interpreted the prompt as step by step directions they were meant to use to solve the problem, as the instructor had intended. Table Seven, however, interpreted the wording as a suggestion for how they might go about solving the problem. They also inferred that the instructor wanted them to present their solution in that format. They used a strategy of trial and error to solve the problem, but planned to break down their solution and record the responses to the individual prompts. Moreover, the phrasing signaled that the steps were meant to assist students who did not understand how to solve the central question, "What price will generate the most revenue for the pizza joint?" This phrasing could trigger stereotype threats in the environment for students with identities salient to those threats, such as the three Black women at Table Four. The problematic design of the task contributed to the inequitable positioning that unfolds during the episode of small group work.

Illustration of Finding I: Seeing Women of Color as the "Collective Student".

Across the cases of instruction, the data showed that the instructors referred to the students in work groups with a majority of women of color as a collective group, rather than as individuals. In other terms, the instructors saw a group of women of color as a collective student, rather than as distinct individuals. In contrast, students were mostly referred to as individuals when in groups composed of only White women. Finally, groups in which men made up at least half of the group, both men and women in those gender-mixed groups were spoken about as individuals, even when the instructors talked about group work. The racial identities of the men across these classrooms did not seem to impact this pattern.

For example, Ming referred to Krystal, Maya, and Daeshai as a collective group during the interviews, and rarely talked about these students as individuals. In fact, Krystal was the only student from this table the instructor talked about as a distinct individual. When asked during the retrospective recall interview – "were there certain students in particular you noticed today who you felt either understood things well or weren't as clear?" – Ming reported,

> *Maybe she [Krystal] just needs a little more time because I know she's kind of lost in class. When I talk to her she'll say, "I'm really confused." On the other hand, when she does her -- because there is web work in these classes I guess … She doesn't do fantastic on web work but she does fine. It seems like after a while she does pick up the material.*

In Ming's class, this trend of collective characterization of women of color is particularly visible in his assessment of Krystal, and by extension, Table Four as "struggling" students (Excerpt 2). During every visit, Ming reported on his assessment of Krystal's struggles with the material and behavior in the classroom when prompted with student-centered questions during retrospective recall interview, such as "Were there any students you noticed in particular during class today for any reason?" When asked a follow up question specifically about the assessment he volunteered of Krystal, Ming projected his perception of Krystal as a struggling student onto the rest of the Black women at Table Four. The underlined passages (Excerpt 2) illustrate how Ming leveraged his assessment of Krystal as a struggling student to characterize all the Black women at Table Four. Ming described "a few students like her [Krystal] at the same table" as being "slightly less engaged", asking for help from their classmates, not wanting to work on their own, being lost, keeping quiet, and struggling with the mathematics (Excerpt 2). Yet, when pressed for examples of what struggling looked like for Maya and Daeshai, the instructor always offered examples of Krystal's performance on assessments, attendance, and classroom activity. This created the impression that Ming saw Krystal as a proxy for her whole group, which was consistent with groups of Latina, Black, and Asian women across the classes studied.

Excerpt 2
Retrospective Recall Interview Transcript – Ming, Visit I

Researcher: Was there something about her [Krystal's] *behavior or what she said that you were noticing or do you just keep an eye on her because you know she might be struggling?*

Ming: I guess it is just usually like <u>a few students like her at the same table</u> [Table Four] *when the class is doing group work, <u>they'll be slightly less engaged</u>. Or, <u>they'll still be maybe stuck on material that I just talked about in the short lecture</u>, ten minutes or five-minutes before. <u>And they'll be asking their classmates about what happened on the board. What did we just do?</u> Which is fine but <u>I guess it is a sign that some students need a bit more time on that particular piece of material.</u>*

Researcher: Were there other students that you were noticing for different reasons?

Ming: [Continues to build on his previous answer] ***Um, I guess maybe I could talk to them more*** *[the students at Table Four]* ***because it is difficult to see what exactly is happening sometimes.*** <u>*Sometimes they don't want to work on their own*</u>*. I mean I guess that's fine to a certain extent.* ***At some point maybe I want to talk to them and ask them to explain more. Maybe have them explain what they did to one another.*** <u>*Maybe sometimes they are lost but they're just keeping quiet*</u> ***so it would be good to find out.*** *But yeah,* <u>*they are struggling*</u>*.*

Ming does not offer additional information to support his perception of Krystal, Maya, and Daeshai as a table of struggling students. Many of the actions Ming references as evidence of struggling students for Table Four are mentioned in relation to other students, who are framed as "engaged" (Tables One, Two, Three, Five, and Six) or "thinking like mathematicians" (Table Seven). In addition, Ming seems to have a great deal of information on the other students gathered from conversations in and out of class, from their body language, and contributions in class. These findings were consistent across the semester. Finally, Ming signaled that he needed to interact more with the students at Table Four and did not really understand where they were with the material when he said, "I guess maybe I could talk to them more because it is difficult to see what exactly is happening sometimes" (Excerpt 2). Moreover, Ming did have ideas about how to rectify the situation, saying "At some point maybe I want to talk to them and ask them to explain more. Maybe have them explain what they did to one another" (Excerpt 2). Yet, he never acted on those ideas to the researcher's knowledge (Excerpt 2, in bold).

Next, a task episode will be shared to illustrate the role differentiated enactment of two core practices, facilitating small group work and giving student feedback, played in the differentiated treatment of students and resulting marginalization of the three Black women in the classroom. As previously mentioned, the representative task presented is an episode of instruction from the third week of class. Data from an eight-minute episode of instruction that takes place during small group work as the students work at their tables on the pizza problem (Table 9). Although there were seven tables, data will only be presented for three of them. Tables Four and Seven were included because those interactions were qualitatively different from the interactions with other tables.

Specifically, the interactions were distinct due to the use of the two focal core practices and how the instructor positioned the students of those tables – Table Four as "struggling students" and Table Seven as "potential future mathematicians". Table Five was included as a representation of for the remaining tables, since those interactions shared similar features, particularly the instructor's use of the focal practices and how he positioned the students at those tables as "engaged students". To examine Ming's facilitation of small group work (and other instructors' enactment of this practice across the cases of instruction), the researcher decomposed the core practice of facilitating small group work into component strategies and then decomposed those strategies into teaching moves[35] (Ball, Sleep, Boerst, & Bass, 2009). The strategies highlighted in these excepts for the core practice of facilitating small group are (1) circulating between groups, (2) approaching a small group, and (3) eliciting student thinking with questions, and for the core practice of giving student feedback, they are (4) taking up student contributions in interactions with students and (5) offering usable information for students to use in future work. The specific teaching moves will be pointed out in the excerpt descriptions.

The episode will be presented in three parts – Excerpts 3, 4, and 5 – with each excerpt detailing the interactions between the instructor and students. Excerpt 3 (Table Four) contains the dialogue during this episode between the instructor and students as well as the dialogue among the students between the instructor's visits to the table. Excerpt 4 (Table Five) and 5 (Table Six) only contain the dialogue between the instructor and the students.[36] First, the episodes will be presented, and then the two central findings on the differentiated enactment of core practices will be discussed.

Looking Inside Instruction at the Engagement with Table Four.

This next excerpt presents the interactions between the Black undergraduate students at Table Four, Krystal, Maya, and Daeshai, and the instructor, Ming, during small group work. Ming asked the students to spend ten minutes working on the pizza problem (Table 4) with the other students at their tables. Ming circulated between the seven tables in the classroom during the entire ten minutes. Excerpt 3 begins two minutes into the small group work when Ming first approached Table Four.

Excerpt 3

[35] The process of decomposition and resulting component strategies and teaching moves are detailed in a related article (Enright, in progress).
[36] The undergraduate students were told about the two audio recording devices in the classroom, one next to the researcher at Table Four and the other attached to the instructor's lapel microphone. In order to honor the implicit agreement with the students that they would only be recorded when in proximity to the instructor or researcher, the data included in the article is limited to those instances.

Classroom Observation Transcript – Ming, Visit I, Group Work on Pizza Problem, Interactions with Table Four ("The Table of Struggling Students")

[*The instructor approaches the table two minutes into group work.*]
Ming: How is this table doing? What are you working on?
Maya: We are figuring out the price per slice that will make the most money.
Ming: Wait, have you read the problem? [*to the small group*] This is a harder problem than the ones we worked on all together. Do you want to try to do like problems a and b? [*walks off to Table Three*]
Maya: So that's not what this question is saying?
[*Maya picks up the copy of the problem and begins to read it silently. Maya and Daeshai are looking at their lecture notes.*]
Maya: I still think that's what the question is saying. How much can you raise the cost of pizza before enough people are like "I won't pay that" and you end up making less money because too many people start going to the place down the street instead.
Krystal: Oh! So maybe it's just a plus d?
Daeshai: It probably isn't. [*laughter*]
Maya: What about 8?
Daeshai: Wait, where did you get 8?
Maya: You know, like for a.
Krystal: [*begins rereading the problem outloud*] Find a formula for…
Daeshai: Oh, <u>8</u> plus d.
Maya: Yeah, that's it. Sorry. I wonder if we need to think about how it costs less to make less pizza. That's not in the problem but if you were making fewer pizzas wouldn't you have less like costs for materials and stuff?
Krystal: [*Krystal listens to the end of the conversation between the instructor and the table to the right of her – Table Two*] What is he talking for so long to them about?
[*Instructor moves from Table Two to Table Five and all three students at Table Four pause to listen to the conversation between the instructor and Table Five, Excerpt 3*]
Ming: What are you doing now?
Krystal: We're confused.
Ming: Oh okay. [walks away again]
[*All three students huddle together to reread/look at the one copy of the problem silently.*]
Krystal: So okay. I'm not sure what the problem is asking. Was our formula right for part a?
Maya: The questions for parts a and b look the same, right? This is frustrating. I thought I understood the math. I don't know what this problem is asking either. What's T? How is that different from P?
Krystal: I really don't know. I wish he [*the instructor*] would talk to us about the math, like at the other tables.
Daeshai: What do you think we should do next?
Krystal: I don't know. Change our skin color so he will talk to us? [*everyone looks down*] Oh, you mean about the problem… [*laughter*]
Ming: Okay. Sorry to bud in again. How are you doing on a and b? What did you get for a?
Krystal: P equals 8 plus d? [*correct formula*] But…
Ming: Okay. [*walks away*]

This excerpt illustrates two key findings in the larger dataset. First, although the instructor had stopped at Table Four multiple times during his circulating, his interactions with the three Black women at the table did not foster dialogue with the students. Ming's questions facilitated his "checking-in" (Enright, Hickman, & Ball, 2016) with these students but did not support any sustained dialogue on the mathematics. Second, Ming did not give critical feedback to Krystal, Maya, and Daeshai. Ming sent ambiguous messages to them through his questions and abrupt departures, and the students were left to interpret those messages on their own.

Looking Inside Instruction at the Engagement with Table Five.

Excerpt 4 presents the interaction between the instructor and the students at Table Five: Mike (White man), Justin (White man), Christine (Asian woman), and Katie (Asian woman). Mike and Katie were placed in the same team homework group at the beginning of the semester with two other students at Table Two. Mike knew Justin from his dormitory, and Katie knew Christine from a church group they both joined. The four students almost always sat together over the course of the semester, even after Ming changed the team homework groups mid-semester. Mike and Justin seemed to dominate most of the conversations at the table; however, from observations, Christine seemed to have the best grasp on the content. Katie was mostly silent and was never observed speaking to the instructor during group work or asking/answering questions in whole group. Excerpt 4 began five minutes into group work, when Ming approached Table Five.

Excerpt 4

Classroom Observation Transcript – Ming, Visit I, Group Work on Pizza Problem, Interactions with Table Five ("A Table of Engaged Students")

Ming: What is this table working on?
Mike: We're kinda stuck really.
Ming: What have you done so far?
Christine: We were just reading the problem. We looked at steps a and b and we weren't sure how they are different…
Ming: Oh! So P is for price and that's asking you for the general formula. Do you know what information you could use to make that formula?
Christine: Umm, is that the price per slice?
Mike: Yeah, the 8 dollars and like x for the number of slices you sell.
Ming: Right, so that's a good start. So P of d is 8 plus d. But T is a different formula. It asks you for the total slices sold; T is for total. That's kind of the way it's usually used in these kinds of problems. So what new information can you use?
Christine: Oh, so that make sense. We would use the 1000 pizza…
Mike: We would need to figure out what the formula would be when 1000 slices are sold at 8 dollars, right?
Ming: Sounds like you understand the problem and have some helpful ideas. Keep going. I'll come back and see how you are doing.

This excerpt highlights the same two findings as in the previous excerpt: creating dialogue through questions and giving students critical feedback. In Excerpt 3, however, Ming used a variety of question types, which allowed him to begin a dialogue on the mathematics with the White men and Asian women at Table Five. It is important to note that at both tables a student communicated that the group was having difficulty solving part of the Pizza Problem (Table 4), yet in the interaction with Table Five, Ming engaged with that comment as a request for help. Additionally, Ming gave these students clear and critical feedback on their mathematical work. As seen in Excerpt 3, Table Four was listening to this interaction between Ming and two students at Table Five, Mike and Christine.

Looking Inside Instruction at the Engagement with Table Seven.

Excerpt 5 presents the interaction between the instructor and the four white undergraduate men at Table Seven: Mark, John, Stephen, and Christopher. To the researcher's knowledge, these four students always sat together over the course of the semester and talked about rushing the same fraternity together. Mark and Stephen seemed to dominate most of the conversations at the table, yet from his contributions, John seemed to have the strongest understanding of the content. Christopher would talk about non-related topics, but rarely contributed to the conversation when the focus was mathematics. He also was never observed speaking to the instructor during group work or whole group. Eight minutes into group work, Ming approached Table Seven, which was when Excerpt 5 began.

Excerpt 5

Classroom Observation Transcript – Ming, Visit I, Group Work on Pizza Problem, Interactions with Table Seven ("The Table of Future Mathematicians")

Ming: How are people feeling so far? What are you guys working on?
Mark: We are figuring out the price that will generate the most revenue for the restaurant.
Ming: Can you walk me through what you've done?
John: We aren't doing the step approach to the problem [pointing to steps a-e on the worksheet]. We are experimenting with different numbers.
Stephen: Yeah, and then putting them into a chart.
Ming: Can you tell me how you are doing that? What numbers?
Stephen: You know just numbers that make sense.
Mark: We are using numbers around the eight-dollar price to see if there is a pattern.
John: You know like to see if we can land on the price that is the best.
Ming: The optimal price?
Mark: Yes. We are using the table to see when they start to lose sales.

Stephen: And when we find that and we are we going to try and to do it through the steps that are on the sheet.
Ming: So the price before they start losing sales is called the optimal price. That's really interesting. I didn't think anyone would try the problem a different way. That's good though. It shows you understand the ideas. You're thinking like mathematicians. Good work.

Again, Ming's enactment of two core practices created a significantly different set of interactions with Table Seven than with Table Four. As with Table Five, Ming asked a variety of questions that opened up dialogue on the mathematics. It is important to note that Ming's initial question and Mark's response very closely mirrored the interaction Ming had with Maya in Excerpt 3, yet the follow up question posed was significantly different. Second, he gave the four White men at this table clear feedback on their mathematical work and thinking.

Illustration of Finding II: Silencing Minoritized Students by Disrupting Dialogues

The previous three excerpts illustrated the findings from the cross-case analysis of how an instructor's enactment of questioning and feedback practices created differential access to learning opportunities for students. This section will focus on the example of Ming's varied use of questions in his interactions with groups of students and the resulting dialogues on mathematics (or lack thereof).

Ming asked the same number of questions at Tables Four, Five, and Seven (Table 5). Moreover, he posed variations of the same question when first approaching each of the tables. The types of questions asked after that initial approach, however, were quite different across the three tables (Table 10). Most importantly, there are meaningful differences in how Ming took up students' responses. He ended the interaction with the three Black women at Table Four without talking about the content, yet engaged the White and Asian students at Tables Five and Seven in dialogues on the mathematics.

For example, Ming reacted in significantly different ways to similar statements made by Maya, a Black woman, and Mark (a White man) during his first visit to each table. Ming asked Table Four, "What are you working on?", and Table Seven, "What are you guys[37] working on?" (Table 10). In response to Ming's opening question, Maya offered her interpretation of the Pizza Problem's main question: "We are figuring out out the price per slice that will make the most

[37] It is unclear whether the word "guys" was making an explicit gender reference or simply a use of slang. Ming used the word both ways over the course of the semester.

money" (Excerpt 3). As seen in his response, Ming perceived Maya's answer to his question as evidence that the students in that group had not read the entire prompt. To follow up, Ming asked another two questions that checked in on whether the students at Table Four knew what they were supposed to be doing and then walked away. The dialogue was ended abruptly when Ming left without waiting for an answer to his last question. Except for Maya's rephrasing of the prompt, there was no mathematics content in the dialogue.

Table 10

Instructor Question Types[38] Posed to Student Tables During Small Group Work on the Pizza Problem Organized by Visit[39] to Each Table

Table Four		**Table Five**		**Table Seven**	
Visit 1: Questions asked on approach	How is this table doing? What are you working on?	Visit 1: Question asked on approach	What is this table working on?	Visit 1: Question asked on approach	How are people feeling so far? What are you guys working on?
Visit 1: Follow up questions	Wait, have you read the problem? Do you want to try to do like problems *a* and *b*?	Visit 1: Follow up question	What have you done so far?	Visit 1: Follow up question	Can you walk me through what you've done?
Visit 2: Question asked on approach	What are you doing now?	Visit 1: Follow up question	Do you know what information you could use to make that formula?	Visit 1: Follow up question	Can you tell me how you are doing that? What numbers?
Visit 3: Questions asked on approach	How are you doing on *a* and *b*? What did you get for *a*?	Visit 1: Follow up question	So what new information can you use?	Visit 1: Follow up question	The optimal price?

In contrast, at Table Seven, Mark responded, "We are figuring out the price that will generate the most revenue for the restaurant" (Excerpt 5). Mark's interpretation of the question was quite similar to Maya's earlier one. Maya's response, however, was more precise in that she

[38] The question types in this table were borrowed from work done by Enright, Hickman and Ball (2016) on types of pedagogical questions in mathematics teaching.
[39] A visit is the time spent by the instructor with a group of students at a table. Each time the instructor approaches a table is considered a new visit.

stated the price for the unit ("per slice"). Moreover, she offered her interpretation of the term revenue ("make the most money"). Mark's statement, however, included more information from the prompt, including vocabulary ("revenue") and location ("the restaurant"). Mark's use of the term "revenue" might signal his understanding of the specialized vocabulary. Another possibility is that he borrowed the term from the prompt, because he did not have an interpretation yet of what that means in colloquial language. Ming followed up three times in the proceeding dialogue with questions on the mathematics to the White men at Table Seven. He asked them twice to tell him which steps they took in their problem solving, paying particular attention to the numbers they used. Then, he prompted them with a final question ("The optimal price?") to confirm that they understood how to apply the specialized vocabulary from the earlier lecture.

Although the launch of the conversation with Table Five differs from Tables Four and Seven, the questioning pattern is similar to that of Table Seven, resulting in a robust dialogue. Ming asked, "what have you done so far?" (Table 10). This is a similar question to the ones he asked Table Seven ("Can you walk me through what you've done?" and "Can you tell me how you are doing that?"). These questions are significantly different from the question posed to Table Four ("What are you doing now?"). The cognitive as well as mathematical work required of the students in these two types of questions also differ. The first two questions elicit mathematical process from the student(s); whereas, the question directed at Table Four prompted the students to say what they were doing in that moment, but not to recall and explain their process to the instructor (Enright, Hickman, Ball, 2016). In addition, Ming asked Table Five questions on their identification and use of information in their problem-solving. This type of questioning was seen to a lesser extent in Ming's dialogue with Table Seven when he asked, "What numbers?", and not at all with Table Four. These information questions allowed the instructor to check for student understanding. Furthermore, the instructor modeled important meta-cognitive questions that those students could use in future problem solving.

The conversations with Tables Five and Seven are illustrations of robust dialogues between an instructor and students on content, whereas, the interaction with Table Four was disrupted. Across the larger dataset, robust dialogues occurred almost exclusively between instructors, who identified as White or Asian men, and their students, who shared those identities. Disrupted dialogues, however, happened between instructors, who identified as White or Asian men, and their students, who held minoritized racial and gender identities. Specifically, all the documented interactions between an instructor and Black undergraduate women in the larger data set were coded as disrupted dialogues, while there were none coded for White or Asian men. This evidence shows the instructors inequitable enactment of these core practices when teaching across difference.

Illustration of Finding III: Sending Ambiguous Instead of Critical Feedback to Minoritized Students

This section focuses on the example of Ming's varied enactment of the core practice of giving student feedback across students and the resulting inequitable differentiated access to learning opportunities for minoritized students. As previously discussed, the researcher investigated three aspects of the enactment of feedback: volume, clarity, and usability. First, volume is understood as the amount of feedback given to students. Next, clarity is thought of as explicitness of the message. Finally, grounded within an explicit equity framework adopted from the wise feedback literature (Cohen, Steele, & Ross, 1999; Yeager et al., 2013), usability is conceptualized as feedback that can be taken up and used by students in their instructional context. In a stereotype-rich environment, such as a mathematics classroom, feedback must contend with the threats that minoritized students face to be considered usable. The three messages that make up wise feedback (Table 11) mitigate identity threats by resolving minoritized students' attribution dilemmas, creating a buffer from association with those stereotypes, and building trust with instructors across difference (Cohen, Steele, & Ross). In other terms, these wise feedback messages are necessary for minoritized students to make use of an instructor's feedback when under identity threat, although they were not needed for majoritized students (i.e., White and Asian men). As such, all three messages were required to determine that feedback was usable for minoritized students, while only the third message (usable information) was required for the majoritized students.

Table 11

Critical Feedback Organized by Wise Feedback Messages Given to Student Tables During Small Group Work on the Pizza Problem

Wise Feedback Message	**Table Four**	**Table Five**	**Table Seven**
(1) High Academic Expectations	This is a harder problem than the ones	none	none

	we worked on all together.		
(2) Belief in the Student's Capacity to Learn & Improve	none	Right, so that's a good start. Sounds like you understand the problem and have some helpful ideas. Keep going.	That's really interesting. I didn't think anyone would try the problem a different way. That's good though. It shows you understand the ideas. You're thinking like mathematicians. Good work.
(3) Usable Information	none	So P is for price and that's asking you for the general formula. So P of d is 8 plus d. But T is a different formula. It asks you for the total slices sold; T is for total. That's kind of the way it's usually used in these kinds of problems.	So the price before they start losing sales is called the optimal price.

Although there were differences in the thresholds for usable information between minoritized and majoritized students, the researcher found that majoritized students were much more likely (4:1) to receive all three wise feedback messages during a single class, than their minoritized peers who needed those messages to mitigate the effects of identity threat (e.g., Table 6). In the classes observed, minoritized students in double jeopardy never received all three messages.

High Academic Expectations.

As mentioned previously, all the students regularly received messages about the rigor of the work, which the researcher coded as signaling high academic expectations. Messages about the nature of the work were generally embedded in the directions they gave students in whole group (e.g., "The problem I want you to do now in your groups is harder than the one we did together on the board." Ming, visit 1, observation transcript). As seen in the cross-case data, Ming's students in double jeopardy (Table Four) also received a message about high expectations in his direct interactions with them, whereas other minoritized and majoritized students did not in this episode. Without the other two messages, however, the literature suggests that this message alone is not likely to mitigate any effects of identity threat and could potentially exacerbate them (Cohen, Steele, & Ross, 1999; Yeager et al., 2013).

As seen in the data across instructors, Ming also delivered messages about his academic expectations frequently to the whole class, messages about the rigor of the work were very inconsistent, impacting the clarity. For example, the problem that Ming introduced in his directions to the whole class as “harder” he later described as “pretty easy to figure out if you were successful on the web homework” to at least two of the small groups (Ming, visit 1, observation transcript). The reverse occurred as well as seen in the episode. Ming originally introduced the Pizza Problem as “an easier problem to practice what we’ve been doing together” to the whole class, yet in his interactions with the Black women at Table Four, he said, “This is a harder problem than the ones we worked on all together” (Ming, visit 1, observation transcript). While Maya, Krystal, and Daeshai received feedback communicating high academic expectations, the message was unclear since it could be interpreted as contradicting the earlier message about the lack of rigor of the problem. Another possibility is that the Black woman at Table Four might have interpreted this tension between Ming’s messages as a signal that he believed that the problem would be challenging for *them*. Given the attribution dilemmas students under identity threat(s) face (Steele, 1998; 2010), this message could also be implicitly communicating to these students that their instructor saw them as struggling or less capable than other students.

Belief in the Student’s Capacity to Learn and Improve.

This message was almost always observed being delivered to majoritized students across the cases. This can be seen in the episode from Ming’s classroom as well. The three Black women at Table Four received no explicit messages from Ming about their capacity to learn and improve. In contrast, the four White men at Table Seven received by far the greatest volume and clarity of feedback on their capacity. In particular, Ming said, “It [the different strategy] shows you understand the ideas. You’re thinking like mathematicians.” (Table 6). Ming not only offered them clear feedback on his belief in their capacity, but in describing them as mathematicians, he also positioned those White men in an affinity group with himself. Between those two extremes, Ming said, “Sounds like you understand the problem and have some helpful ideas” (Table 6). In this statement, Ming communicated what he thought they were doing successfully and then encouraged Table Five to continue making progress on the problem. The episode illustrates the reality that the majoritized students who least need feedback on their capacity from the instructor were the ones who received the greatest volume and clearest messages about their capacity. Unfortunately, the Black women in Ming's class under multiple, identity threats and needed feedback on their capacity in order to mitigate those threats received no such messages from Ming. The two Asian women at Table Five received feedback on their capacity as part of a small group, but it is unclear whether Ming would have given the same feedback if they were alone at the table. This feedback directed at a small group as opposed to an individual raises interesting questions about the efficacy of those messages for students under stereotype threat, on the one hand, and the true frequency minoritized students receive those messages when not working with their majoritized peers. This becomes especially important in Ming's classroom, since the Black women expressed feeling isolated and noted that their peers never sat with them even when Ming asked students to sit with new students midway through the semester.

Usable Information.

Across the cases, White and Asian women received mostly usable information in the infrequent incidences when they received feedback. Even in these cases, the feedback was abbreviated relative to the usable information offered to individual or groups of only majoritized peers. The episode in Ming's classroom illustrates this pattern. The students at Table Five received the greatest volume of usable information of the three tables in the episode. In the recall interviews, Ming described the the two White men at Table Five as engaged students but made no mention of the two Asian women; Ming interacted mostly with Mike during his visit to Table Five, not Christine (Excerpt 4). It is unclear if the Asian women at the table received the information from Ming that they needed, particularly since Mike interrupted Christine two out of the three times she spoke up. In comparison, Table Four received no clear usable information from Ming, even though Maya and Krystal (Excerpt 3) had many of the same questions as Mike and Christine (Excerpt 4). For instance, in response to Ming's request for an answer for step *a*, Krystal said, "P equals 8 plus d? But…" and Ming left for a new table before she had an opportunity to finish her question (Excerpt 3). In the recall interviews, Ming recalled that moment twice in the interviews, noting that Krystal seemed to be struggling and also reflected that she seemed to be able eventually to figure out the problems (Ming, visit 1, retrospective recall transcript). Ming gave the White men at Table Seven one clear piece of usable information about the specialized vocabulary. Given Ming's appreciation for Table Seven's "thinking like mathematicians" and his perception that Table Four was "struggling", one might expect that he would offer Table Four more usable information. Yet, in spite of making the greatest number of visits to Table Four (3), Ming spent the least amount of time at that table and offered those students the least amount of clear, usable feedback. Ming's use of these three messages, particularly his feedback on capacity and usable information, created differentiated access to learning across majoritized and minoritized students in the class, in particular for Black women.

In Ming's episode of instruction, his varying knowledge of his students, use of questions in dialogic interactions, and delivery of critical feedback created inequity in the classroom. Ming's enactment of core practices further positioned students as struggling (Table Four), as engaged (Table Five), and as future mathematicians (Table Seven). Without opportunities to ask questions and receive critical feedback, students under identity threat, especially the Black women in double jeopardy, received much less support with their mathematics learning, than their majoritized peers. This differentiated positioning and its implications for access, equity, and excellence in mathematics will be discussed in the next section.

Reflecting on the Costs of Inequitable Gatekeeping Practices for Minoritized Students

This article began by posing the question - what about Daeshai, Maya, and Krystal's experiences in and with mathematics instruction influenced the dramatic change in their plans for their majors? Daeshai dropped the class about midway through the semester, and as a result, there are no data on her coursework or plans for her major. Maya and Krystal shifted from energetically discussing the merits of mechanical versus civil engineering as a major and career to choosing majors in the humanities. Maya decided to pursue an English degree, and Krystal selected African American studies for her major. This does not suggest that those majors are in any way inferior to those in STEM fields. This shift is significant only because the data suggest that these women changed their paths through college out of survival, not choice.

More specifically, the data show that Maya and Krystal changed majors to avoid additional mathematics courses, not because the value they placed on engineering or their goals changed. Maya told Krystal midway through the semester that she had changed her mind about mechanical engineering. She said, "what my uncle does in his work is really great, but I'm just not sure that path is for me anymore. I just don’t see dragging myself through all the prereqs [prerequisite courses] and making it out whole” (Ming, visit 2, observation transcript). When pressed by Krystal, Maya admitted that she has always enjoyed literature and though English would just be a better experience. Maya said that she will probably go to law school, since she wanted “a career, not just any job” (Ming, visit 2, observation transcript). Later in the semester, Krystal announced her plan to major in African American studies, instead of civil engineering. She said to Maya, “I think I can make a difference advocating for change if I know more about our history and struggle as a people. And, it also means less math and more me!” (Ming, visit 3, observation transcript). These women framed the selection of their major as a choice between being a whole person and pursuing a STEM major.

Eccles’s (2009) identity development framework helps explain how the findings potentially informed these women’s decisions not to pursue STEM fields. As previously mentioned, Eccles frames identity development as informed by two interrelated sets of self-perceptions: an individual’s expectations of success (personal skills, characteristics, and competencies) and an individual’s perceived value of the task (personal values and goals). In particular, Eccles’s framework shows how the experiences and outcomes of Black undergraduate women’s positioning vis-à-vis Ming’s enactment of core instructional practices could inform their mathematics identity development.

First, Daeshai, Maya, and Krystal received implicit messages about their mathematical skills, characteristics, and competencies during the semester. For example, Krystal connected Ming’s lack of dialogue around content with her table to their Blackness (Excerpt 3) and interpreted that refusal to engage with them as evidence that Ming believed they could not do the mathematics (Ming, visit 2, observation transcript). Dotson (2011) conceptualized just such broken dialogues as a form of “epistemic violence” in which Black women’s “intellectual courage is undermined through routinely being taken as a ‘non-knower’ as a result of social perceptions of one’s identity” (Dotson, 2011, p. 243; Flicker, 2007). This pattern emerged in the data collected on Maya. At the beginning of the semester, she arrived confident in her mathematics knowledge and skills, but that confidence eroded over time as Ming questioned her understanding of mathematical concepts and tasks as well as abruptly ended dialogues with her (e.g., Excerpt 3). Ming also publically positioned Krystal as a non-knower, ignoring her raised hand in whole class and abruptly ending dialogues when she signaled that she wanted help or had a question (Excerpt 3). Moreover, Ming talked about Krystal as struggling to grasp the material (Excerpt 2). Clearly, Ming's interactions with these Black women positioned them on the margins of the class discourse on the content, sending messages to those women as well as their peers about their skills, characteristics, and competencies.

Similarly, the findings on critical feedback illustrated that, although other students were given messages about their capacity to learn and improve (often quite publically), Daeshai, Maya, and Krystal received no such messages. Although there are no data on how they interpreted the absence of these messages, the researcher hypothesizes that they were likely aware of the differential treatment. Research on positive feedback bias also documented resistance to giving clear, critical feedback to Black students by White individuals (Harber, 1998; 2004). The findings in this study show that White and Asian men when given the option of giving or avoiding giving feedback to Black women in their classes, not only offer more ambiguous feedback as in positive feedback bias studies, but also abruptly end interactions without offering feedback. As a result, Maya, Krystal and Daeshai never received the feedback messages they needed to mitigate the harmful effects of stereotype threat and create opportunities to learn. While both Maya and Krystal successfully completed the semester, the data suggest that their perceptions of their own capacities in mathematics were greatly diminished by the end of the class.

Second, at least for Maya and Krystal, the perceived value of the task – successful completion of pre-calculus mathematics changed over the course of the semester. At first, they highly valued the task as a means to advance their plans of declaring engineering majors. Over the course of the semester, their motivation for completing the course shifted to a desire to do well enough not to damage their grade point averages. Yet, their personal goals and values did not necessarily change. Both women expressed a desire to do work that was personally meaningful that allowed them to contribute to their home communities. What did change was Maya and Krystal's perception that they could succeed in STEM without paying too high a personal price. In other terms, they chose a major that they believed would allow them to be who they were and still reach their personal goals and maintain their values. Ming and the other instructors' differentiated enactment of core instructional practices perpetuated historical barriers to full participation in mathematics and mathematics-dependent fields for minoritized students. The instructors' lack of knowledge of their minoritized students and lack of capacity to engage with them around the mathematics created an environment that further marginalized those students. Over time, that marginalization influenced Maya, Krystal and Daeshai's eventual de-identification with STEM.

Current Research Constraints and Future Directions

This study demonstrates what can be learned when research looks beyond representation and outcomes to the experiences of students of color in higher education. Additionally, the field needs more research on within-group differences in experiences of students of color, investigating the victories and challenges students with intersecting identities face in higher education. This means broadening conceptualizations of equity, access, and excellence and look at the core activity of higher education – instruction.

It is time for higher education research to acknowledge that instruction is not merely the mechanism through which systemic forms of oppression, such as structural racism, are transported into classrooms. As illustrated through Ming's episode of instruction, teaching can also produce inequity, generating differentiated access to opportunities to learn, participate, and successfully navigate coursework. More research is needed looking inside instruction to unpack the relationships between the enactment of teaching practices and historically marginalized students' experiences and a wide range of outcomes, including less-traditional measures such as identity development and feeling of belonging. Since instructors' enactment of teaching practices contributes to inequity, higher education needs to know and do more to prepare future and current faculty to teach equitably in their classrooms. This work includes collaborating across disciplines to design equitable instructional practices. The best designed curricula, programs, and policies are only as effective as the course instructors who implement them in their classrooms.

This study advanced the development of a cultural ecological framework for looking inside instruction at the production of inequity, yet the nature of the study also limits the generalizability of the findings and applications of the framework. First, due to the small number of cases, the researcher was able to examine in depth the differentiation in micro-interactions within and across classrooms. Yet, this affordance also limited the scope of the research to a single doctoral program at a predominately White institution. These interactions could look quite different across institutional types. For example, research has shown that Black undergraduate women have different experiences in STEM courses at HBCUs than at PWIs (e.g., Chavous et al., 2004). Second, the study is limited by the demographics of the students in the instructors' classrooms. For instance, there were too few Latino and Black men in the four classes studied to see patterns in the instructors' interaction across just racial difference. Conducting a larger study of interactions across difference in mathematics instruction would allow for a more nuanced study looking at teaching across different lines of difference. First-generation college and socio-economic identities, while not visible in the way gender and racial identities are, could potentially impact instructor/student interactions as well. Third, looking at instructors that hold different racial and gender identities is important as well. This study followed doctoral students who voluntarily enrolled in the study after their orientation, and as such, was limited to the four White and Asian men who volunteered. Future studies should include women, especially from historically marginalized communities. Fourth, the researcher's own identities (as a White woman, and a doctoral student) could have impacted what information students and instructors shared with her across lines of difference. Finally, resource and IRB constraints did not allow for the collection of student data outside of classroom observations. Future research should endeavor to collect student data from individual and focus group interviews as well as outcome data from surveys and assessments. These student data would allow for the creation of a more robust picture of the effects of instructors' differentiated use of core practices across difference.

With those limitations, this study advances the field's understanding of the role instruction plays in producing inequities in the classroom. This article presents a model of what research looking inside instruction could contribute to the discourse on equity, access, and excellence in higher education. Moreover, this study advances what we know about potential influences for minoritized students' STEM identity development and related decision-making.

APPENDICES

Appendix A

GSI Questionnaire

1. **What is your full name?** ___________________________

2. **In what year were you born?** 19 ___ ___

3. **Which best describes your race or ethnicity?** (Check one)
   - O Asian or Pacific Islander
   - O Hispanic or Latino
   - O African American
   - O White
   - O Multiracial/ethnic: ___________________________
   - O Other: ______________________________

4. **Is English your first language?**
   - O Yes
   - O No If no, what is your first language? _____________________

5. **What degrees have you received (BA, MA, etc.)? From which college(s) and/or university(s) did you receive those degrees?**

   ____

   ____

6. **Have you worked as or volunteered as a math tutor?**
   - O Yes
   - O No

Please describe your experience as a math tutor:

____

____

____

____

7. **Have you worked as a math teacher in a classroom of any kind?**
   - O Yes
   - O No

Please describe your experience as a math teacher:

____

____

____

____

Questions		Please circle your response for each item				
1	You have a certain amount of ability in math and you really **can't** do much to change it.	Strongly Agree	Somewhat Agree	Neither Agree Nor Disagree	Somewhat Disagree	Strongly Disagree
2	Your ability in math is something about you that you **can't** change much.	Strongly Agree	Somewhat Agree	Neither Agree Nor Disagree	Somewhat Disagree	Strongly Disagree
3	You can learn new things, but you **can't** really change your basic ability in math.	Strongly Agree	Somewhat Agree	Neither Agree Nor Disagree	Somewhat Disagree	Strongly Disagree
4	Your ability in math is something about you that you **can** change.	Strongly Agree	Somewhat Agree	Neither Agree Nor Disagree	Somewhat Disagree	Strongly Disagree
5	Students' ability in math is **not** something that they can change very much.	Strongly Agree	Somewhat Agree	Neither Agree Nor Disagree	Somewhat Disagree	Strongly Disagree
6	Students have a certain amount of ability in math, and they **can't** do much to change it.	Strongly Agree	Somewhat Agree	Neither Agree Nor Disagree	Somewhat Disagree	Strongly Disagree
7	Students can learn new things, but they **can't** change their basic ability in math.	Strongly Agree	Somewhat Agree	Neither Agree Nor Disagree	Somewhat Disagree	Strongly Disagree
8	Students' ability in math is something that **can** be changed.	Strongly Agree	Somewhat Agree	Neither Agree Nor Disagree	Somewhat Disagree	Strongly Disagree
9	Even a teacher with good math teaching skills may **not** help many students learn mathematics.	Strongly Agree	Somewhat Agree	Neither Agree Nor Disagree	Somewhat Disagree	Strongly Disagree
10	If a student masters a new math concept quickly, this might be because the teacher knew the necessary steps in teaching that concept.	Strongly Agree	Somewhat Agree	Neither Agree Nor Disagree	Somewhat Disagree	Strongly Disagree
11	Effectiveness in mathematics teaching has **little** influence on the achievement of students with low motivation.	Strongly Agree	Somewhat Agree	Neither Agree Nor Disagree	Somewhat Disagree	Strongly Disagree

Appendix B

Interviews Classroom Visit I

PLANNING INTERVIEW I

I want to let you know about my goals for our interviews today. I am interested in many of the challenges you are facing as a new instructor but each visit I am also going to ask you some questions about instructor/student interactions in your classroom. I am not looking for anything specific. There are no right or wrong answers to any of the questions I ask. When I ask questions they are actually framed deliberately to get you to talk about what you want to talk about. Does that sound okay to you?

1. Can you tell me a little bit about what your goals are for this lesson?
2. We often focus on content when discussing learning. Were there any other messages/things you want to try to convey in this class?
3. Can you describe what a typical class is like for me? What do you do in your teaching? What types of interactions do you have with your students?
4. How do you feel about your interactions up until now with your students? Why?

POST-INSTRUCTION RETROSPECTIVE & STIMULATED RECALL INTERVIEWS I

Retrospective Recall – Overview

1. Tell me how you felt the class went.
2. Do you feel that you accomplished what you set out to do? And what makes you say that?
3. Were there any students in particular you noticed during class today? Why did they draw your attention?

Stimulated Recall - Audio Recording Review

Now we are going to listen to a few excerpts from the audio recording of the class. The purpose of this activity is for you to describe your in-the-moment thinking about your interactions with students. You can stop or ask me to stop the recording at any time.

So I'd like you to talk about anything that strikes you. For example:
• any teacher/student interaction that went particularly well and why,
• any teacher/student interaction that you feel did not going as planned and why,
• or things that you changed during the lesson and why,
• or things that you would change in the future and why.

We're going to listen to the audio clip now. Please stop it at any point if you wish to talk about something in detail. [Start playing audio recording]

Questions (for probing)
(silence) → add "**Can you tell me why?**" if not volunteered

- **What do you notice during that interaction?**
- **What are you thinking during that interaction?**

- **What do you notice about your students at that moment?**

(cues)
- **What are you paying attention to at that moment?**
- **Why did you notice that?**
- **What does that mean to you?**
- **What's your reaction to that?**
- **How do you feel about that? Can you tell me why?**

- **Were you considering any alternative strategies/moves/questions?**

- **What were you hoping students would learn from this topic/activity/strategy?**
- **What were you hoping to learn from that question/comment/interaction?**

[Stop playing class tape]

Retrospective Recall - Closing Questions

6. Ask:
(a) On a scale of 1 to 10, 1 being very poor and 10 excellent, how would you rate your teaching of this class? Why do you give it this rating?
(b) On a scale of 1 to 10, 1 being poor and 10 being excellent, how would you rate the students' learning? Why do you give it this rating?

Closing
7. Do you feel this study (interview, taping) affected your planning and teaching in any way? If yes, how?

9. Is there anything we haven't talked about that you'd like to bring up?

[Stop recording]

Schedule next observation!

Interviews Classroom Visit II

PLANNING INTERVIEW

I've been thinking about the teaching I have watched over the past few years. One thing I have noticed is that newer instructors sometimes find particular things challenging. For example, I've seen quite a few instructors struggle with giving clear directions. I have seen others find it difficult to give students clear signals that the students are capable of doing challenging work.

1. Has either of these been difficult for you so far? I'm happy to repeat them.
2. Are there other things that you have found challenging in interacting with your students that I might not have mentioned?
3. Can you tell me a little bit about what your goals are for this lesson?
4. We often focus on content when discussing learning. Were there any other messages/things you want to try to convey in this class?

RETROSPECTIVE RECALL - OVERVIEW

1. How do you feel the class went?
2. Did you try anything new or different with your students during class?
3. Were there any students in particular you noticed during class today? Why did they draw your attention?

STIMULATED RECALL - AUDIO RECORDING REVIEW

Now we are going to listen to a few excerpts from the audio recording of the class. The purpose of this activity is for you to describe your in-the-moment thinking about your interactions with students. You can stop or ask me to stop the recording at any time.

So I'd like you to talk about anything that strikes you. For example:

- any teacher/student interaction that went particularly well and why,
- any teacher/student interaction that you feel did not going as planned and why,
- or things that you changed during the lesson and why,
- or things that you would change in the future and why.

We're going to listen to the audio clip now. Please stop it at any point if you wish to talk about something in detail. [Start playing audio recording]

Questions (for probing)
(silence) → add "**Can you tell me why?**" if not volunteered

- **What do you notice during that interaction?**
- **What are you thinking during that interaction?**

- **What do you notice about your students at that moment?**

(cues)
- **What are you paying attention to at that moment?**
- **Why did you notice that?**
- **What does that mean to you?**
- **What's your reaction to that?**
- **How do you feel about that? Can you tell me why?**

- **Were you considering any alternative strategies/moves/questions?**

- **What were you hoping students would learn from this topic/activity/strategy?**
- **What were you hoping to learn from that question/comment/interaction?**

[Stop playing class tape]

RETROSPECTIVE RECALL - CLOSING QUESTIONS

6. Ask:
(a) On a scale of 1 to 10, 1 being very poor and 10 excellent, how would you rate your teaching of this class? Why do you give it this rating?
(b) On a scale of 1 to 10, 1 being poor and 10 being excellent, how would you rate the students' learning? Why do you give it this rating?

Closing
7. Do you feel this study (interview, taping) affected your planning and teaching in any way? If yes, how?

9. Is there anything we haven't talked about that you'd like to bring up?

[Stop recording]

Schedule next observation!

Interviews Classroom Visit III

PLANNING INTERVIEW

So last visit, we talked a little bit about how instructors find particular things challenging. For example I mentioned that I've seen quite a few instructors struggle with giving clear directions as well as giving students clear signals that the students are capable of doing challenging work. You mentioned that you find ________ particularly challenging but felt fairly confident about _________.

1. Has your thinking about these practices changed since we spoke last time? (Ask about the ones not mentioned.) How so?
2. Are there other things that you have found challenging in interacting with your students that I might not have mentioned?
3. Can you tell me a little bit about what your goals are for this lesson?
4. We often focus on content when discussing learning. Were there any other messages/things you want to try to convey in this class?

RETROSPECTIVE RECALL - OVERVIEW

1. How do you feel the class went?
2. Did you try anything new or different with your students during class?
3. Were there any students in particular you noticed during class today? Why did they draw your attention?

STIMULATED RECALL - AUDIO RECORDING REVIEW

Now we are going to listen to a few excerpts from the audio recording of the class. The purpose of this activity is for you to describe your in-the-moment thinking about your interactions with students. You can stop or ask me to stop the recording at any time.

So I'd like you to talk about anything that strikes you. For example:
• any teacher/student interaction that went particularly well and why,
• any teacher/student interaction that you feel did not going as planned and why,
• or things that you changed during the lesson and why,
• or things that you would change in the future and why.

We're going to listen to the audio clip now. Please stop it at any point if you wish to talk about something in detail. [Start playing audio recording]

Questions (for probing)
(silence) → add "**Can you tell me why?**" if not volunteered

- **What do you notice during that interaction?**
- **What are you thinking during that interaction?**

- **What do you notice about your students at that moment?**

(cues)
- **What are you paying attention to at that moment?**
- **Why did you notice that?**
- **What does that mean to you?**
- **What's your reaction to that?**
- **How do you feel about that? Can you tell me why?**

- **Were you considering any alternative strategies/moves/questions?**

- **What were you hoping students would learn from this topic/activity/strategy?**
- **What were you hoping to learn from that question/comment/interaction?**

[Stop playing class tape]

RETROSPECTIVE RECALL - CLOSING QUESTIONS

6. Ask:
(a) On a scale of 1 to 10, 1 being very poor and 10 excellent, how would you rate your teaching of this class? Why do you give it this rating?
(b) On a scale of 1 to 10, 1 being poor and 10 being excellent, how would you rate the students' learning? Why do you give it this rating?

Closing
7. Do you feel this study (interview, taping) affected your planning and teaching in any way? If yes, how?

9. Is there anything we haven't talked about that you'd like to bring up?

[Stop recording]

Schedule final interview!

Final Interview

Open Writing Prompt:

If you were asked to talk to a new GSi, who was going to teach the same calculus course you just taught, what would you tell him/her about teaching?

Semi-Structured Interview Protocol:

So it's been about a month since our last conversation:

1.	Tell me about the end of the semester, since I've seen you last.
2.	Did any new challenges come up in your teaching? How did they come up?
3.	Did you find anything surprising about the past month?

Let's switch gears a little bit and talk about the semester as a whole. Our previous conversations really zoomed in and looked at your thinking about and experiences teaching in the moment. Part of the work of teaching is also looking back and reflecting on teaching after the fact. I'm really interested to hear your impressions about the semester.

4.	Can you tell me a few things you have learned about your specific students this semester? Was any of it surprising?
5.	How would you characterize your students as learners of mathematics?
6.	How have you thought about or used that information about your students in your teaching? Can you think of a specific example?
7.	What do you wish you knew about them but don't? Why do you feel that information is important?
8.	What types of messages, outside of content, did you try to communicate to your students? Were there certain students which whom you felt more or less successful communicating those messages? What do you might explain those differences?
9.	Could you tell me a little bit about the different ways you tried to interact with your students in class? What seemed to work? What seemed to work less?
10.	What did you want to try to do more or at all but didn't get a chance to last semester?
11.	What about your interactions with students outside of class, like in office hours? What seemed to work? What seemed to work less? What did you want to try to do more or at all but didn't get a chance to last semester?

I'm also interested in what you think about teaching and learning mathematics more generally.

12.	Looking back on the semester, what was surprising about teaching that you might not have anticipated? Why?
13.	What have you found more and less challenging about the work of teaching this semester?
14.	What parts of teaching did you find the most challenging? Do you feel like those challenges got easier? What didn't get easier?
15.	If you were asked to define teaching for someone who has never taught before, how would you define it?
16.	How would you define learning for that person?
17.	What would you tell them is the relationship about teaching and learning?

So now that's you've taught for a semester, I am really interested in what advice you would give others about teaching undergraduate calculus. I am also interested in what you have to say about what is important to do when preparing new instructors as well.

18. If you could go back to August, before you started teaching, what advice would you give yourself? Why do you feel that's important?
19. If you could go back to this summer, what advice would you give me before orientation? Why is that important to you?
20. What supports or resources would have been helpful to have either before or during teaching? What would you ask the department for if you could?
21. What would you ask them to do or not to do during orientation? In other words, what did you feel was most useful to support your teaching? What was less useful?

So you've gotten different information and messages during this semester from different people in the department. One of those people is Steven DeBaker. He sent out an email about a month ago asking all the GSIs,

> "My usual request: As the students begin to think about their winter term courses, please identify the top few undergraduate students in your classes and encourage them to take another math class. A little encouragement can make a huge difference to a student who is trying to decide what to do with her/his life; see the student quotes below my signature for evidence in this direction. I'd like for the good ones to give math a chance."

22. Did you do anything in response to that message? If so, what?
23. How did you go about doing that?

So, to wrap up this interview, I'm going to ask you a few standard research questions.

24. On a scale of 1 to 10, 1 being very poor and 10 excellent, how would you rate your teaching of the whole course? Why do you give it this rating?
25. On a scale of 1 to 10, 1 being poor and 10 being excellent, how would you rate the students' learning? Why do you give it this rating?
26. Do you feel this study (interviews, taping) affected your planning and teaching in any way? If yes, how?
27. Is there anything we haven't talked about that you'd like to bring up?

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