Emerging Operational Contracts in Competitive Markets

by

Liang Ding

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Business Administration)
in The University of Michigan
2016

Doctoral Committee:

Professor Roman Kapuscinski, Chair
Assistant Professor Sam Aflaki, HEC Paris
Associate Professor Ozge Sahin, Johns Hopkins University
Assistant Professor Siqian May Shen
Associate Professor Xun Wu

© Liang Ding 2016

All Rights Reserved

ACKNOWLEDGEMENTS

It has been an exciting yet challenging journey to pursue the Ph.D. degree. I would like to thank my advisors, Roman Kapuscinski and Ozge Sahin, to guide me through. After days and nights we spend working together, their smart minds and rigorous attitudes hugely benefit my growth. I am also very grateful to Sam Aflaki, Siqian Shen, and Xun Wu, who serve as my committee members and have provided invaluable comments and feedback. This dissertation would have not been possible without them.

It is my fortune to spend many years at Ross School of Business, which is a wonderful place to grow both professionally and personally. All professors and students are knowledgeable and friendly. I have been lucky to know, learn from and make friends with many of them. This is the perfect place to express my gratitude to all of them who taught me or helped me over the course of my Ph.D.

Lastly, I would like to give my deepest thanks to my family and friends for their love and support. They have been and will continue to be my motivation and inspiration for living a meaningful life.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF APPENDICES

Appendix

ABSTRACT

Emerging Operational Contracts in Competitive Markets

by

Liang Ding

Chair: Roman Kapuscinski

This dissertation consists of three essays, each dealing with an emerging type of operational contracts. The first essay considers a resource exchange model where the effects of collaboration and competition are intertwined. Exchanging resources often improves utilization and is intended to increase profitability of involved firms. However, it does not guarantee success in competitive settings. More efficient use of resources might actually leads to increased competition. We explore how resource exchange contracts impact the firms and consumers. The results indicate that the resource exchange tends to benefit both firms and the consumers in most situations, except for the extreme situations where simultaneously competition is strong and the purchasing cost is either very low or very high.

The second essay focuses on vertical pricing control contracts that manufacturers use to coordinate online and offline retailers. Resale Price Maintenance (RPM) policy requires all retailers to sell at the price suggested by manufacturers. Minimum Advertised Price (MAP) policy is less strict, as it allows retailers to sell at lower prices than the manufacturer suggested, as long as these lower prices are not advertised.

This essay studies which of these two policies is more beneficial to each member of the supply chain. We show that manufacturers prefer MAP policy when the customers' valuations vary significantly and the information search requires significant effort. The MAP policy is also favorable to retailers and consumers under similar market conditions.

The third essay concerns the contractual issues when energy service companies (ESCOs) provide energy efficiency projects to residential clients. While performance based contracts have been proven successful in public, commercial, and industrial sectors, ESCOs face challenges in the residential sector. Residential clients often change consumption behavior after the project, which makes the real energy savings difficult to measure. Additionally, residential clients are much more risk averse and vulnerable to uncertain outcomes of projects. We show that piecewise linear contracts perform reasonably well. To further improve profitability, ESCOs can either reduce uncertainty of technology involved or develop the ability to verify post-project energy efficiency. We also make recommendations in monetary incentives and regulations from policy makers' perspective.

CHAPTER I

Introduction

Many companies continuously strive to reexamine their relationships with their customers, suppliers, and even competitors, in an effort to restructure their offerings of products and services. As a result, innovative contracts among players in supply chains are considered, introduced, and tested. Many of these initiatives are guided by traditional wisdom and are based on intuitive understanding of relevant forces. However, with increasing interactions among firms and more dynamic and competitive markets, some seemingly intuitive solutions may not work. My research focuses on a subset of contracts that either emerge or gain popularity in the industry and helps to extend the traditional business models and push supply chains towards more profitable or more sustainable approaches.

My dissertation consists of three essays, each dealing with an emerging type of operational contracts. Chapter II considers a resource exchange model when the effects of collaboration and competition are intertwined. Recently, an increasing number of firms are engaged or are planning to get engaged in various types of partnerships with other companies, some of the partners being their competitors. This essay studies the combined effect of collaboration and competition, where collaboration is through trade of firms' resources (predominantly inventory). While trade leads to potentially higher resource utilization and could increase firms' profitability, a number of economical and legal concerns may arise. In short term, the potential for selling to another firm creates an incentive to invest in more resources. However, higher initial investments may lead to more intense price competition between firms. In the longer term, the selling firm may also be concerned that the competitor after obtaining additional resources will not only increase sales, but also may retain some of the new customers. Independently, from legal perspective, collaboration between competitors may violate the antitrust laws. In practice the legality of such contracts is seldom challenged, but when such questions arise the effect of these agreements on consumer surplus is examined. We explore whether/when a resource trade contract can help one or both firms and how it affects consumer surplus. We show that when markets are deterministic, the firms do not have incentive to trade inventories as they are able to anticipate the demand. When firms face uncertain markets, the inventory trade is very likely to help and the effect may be quite significant. In markets that are independent, firms always benefit from inventory trade agreement. Even when competition is present, such resource exchange is often a win-win solution. This is, however, not the case when the firms face dramatically different market sizes or costs are at the extremes (either very low or very high). In such cases, either one or both firms, or the consumers may be worse off due to trading.

Chapter III focuses on pricing control contracts that manufacturers use to coordinate online and offline retailers. During last twenty years, many brick-and-mortar retailers have been facing competition from online retailers and local discounters. This influences the behavior of customers who are able to experience products in a brick-and-mortar store but purchase the products online at lower prices. With online stores effectively free-riding on brick-and-mortar retailers' demand generation effort, the sales of demand generators decrease and they have lower incentive to promote or even carry such products. For manufacturers, however, brick-and-mortar retailers play a crucial role by showcasing and advertising products to customers, so that the customers are aware of the products. Resale Price Maintenance (RPM) and Minimum Advertised Price (MAP) are two commonly used policies intended to protect retailers' margin. Under RPM policy, the manufacturer sets a minimum price for each product and requires all retailers not to price below it. Under MAP policy, manufacturers sets a "suggested" retail price to all retailers. While retailers can sell at lower prices, they are not allowed to advertise a price lower-than-suggested retail price. In this essay, we build a stylized model to study and compare the performance of RPM and MAP under various market situations. In particular, we explore which policy is more beneficial for the manufacturer, retailers and consumers. We find that MAP policy is favorable to the manufacturer when the search cost (for identifying the price and availability of the product) is high and consumers are very heterogeneous in their valuation of the product. Otherwise RPM policy would outperform MAP policy. Brick-and-mortar retailers and consumers also benefit from MAP. But they prefer MAP with even higher search cost and larger variance in consumer valuations, compared to the manufacturer. Online retailers would always prefer MAP policy over RPM policy.

Chapter IV focuses on energy service companies (ESCOs) and the contracts they engage in to provide energy efficiency projects to residential clients. Energy efficiency is one of the most efficient approaches to reduce energy cost and reduce environmental impact of energy production. Many energy efficiency projects are performed by ESCOs. A core part of ESCO's business are performance based contracts, in which payment terms are determined as a function of energy savings achieved in the underlying projects. Despite of success in public, commercial, and industrial sectors, ESCOs are involved in fewer projects in the residential sector. There are a few widely acknowledged challenges that contribute to under-developed business in the residential sector. The first challenge is that, the energy efficiency project often leads to a changed consumption behavior after the project is implemented, which makes the real energy savings difficult to measure. The second challenge is that, residential clients are much more risk averse and less willing to accept uncertain outcomes of projects. The third one is that, lack of monitoring protocols leads to ESCOs' moral hazard problem. This essay studies the contract design problem with particular attention to the residential sector. Results suggest that in the residential sector, coordinating contracts in general do not exist. That said, we show that piecewise linear contracts perform reasonably well. To improve profitability, ESCOs can either reduce uncertainty of technology involved or develop the ability to verify post-project energy efficiency. Clearly, policy makers also have an interest in promoting energy efficiency projects. We demonstrate how regulations and monetary incentives help to decrease inefficiencies in the relationships involving ESCOs and to reduce environmental cost.

CHAPTER II

Inventory Exchange: Collaboration and Competition

2.1 Introduction

Collaboration and sharing of resources are widely practiced in single firms with multiple locations or divisions. However, trading of inventory or of capacity is also a surprisingly common practice across companies, including firms competing in the same markets. Similar to a single-firm case, it has potential to improve resource utilization and to increase profitability of involved parties. However, such practices raise some hesitations due to either economic or legal reasons. From economical point of view, despite the direct benefit of pooling resources and using them more efficiently, trade of resources creates new incentives and externalities in competitive settings. For example, the firms with the prospect of selling resources to other firms may invest up front in more of these resources, which may lead to more intense price competition. Also, providing other firms with additional resources may lead to higher service level at the competing firms and result in some customers permanently switching to the competitor in the long run.

In addition to economic dis-incentives, from legal point of view, when two firms collaborate, there is a concern that their benefit may come at the expense of consumers and may violate the antitrust laws. The common violations of the antitrust laws (primarily captured in Sherman Act in the United States) are price fixing, bid rigging, and territorial allocation. These are based on law-and-economics literature, uniformly accepted as illegal and referred to as the *per se* rule. Since other situations are not clearly classified, they all need to be judged on a case-by-case basis.[1] The US Justice Department states that: “If any anticompetitive harm would be outweighed by the practice’s pro-competitive effects, the practice is not unlawful. Virtually all antitrust offenses likely to be prosecuted by a United States Attorney’s office will be governed by the per se rule.”[2]

In practice, a number of companies experiment with various forms of collaboration with other firms, one of them being resource exchange.[3] For example, in 2013, AT&T (the second largest provider of wireless services in the US) bought $1.9 billion in spectrum from Verizon (the largest provider of these services). Although it raised concerns from both regulators and customers about the concentration of spectrum among big operators, the trade was approved. Federal Communications Commission stated “This is a big win for consumers, ..., who will see more competition and more choices.” On a different scale and in a different timeframe, inventory exchange is widely practiced among car dealers selling the same brand of cars. If one dealer runs low or is out of a specific model, he/she routinely purchases cars from other nearby dealers. We observe some form of inventory or capacity trading in many other industries including cargo carriers (trade of freight capacity), providers of industrial gases (trade of industrial gas between distributors), financial institutions (financial loans), and manufacturers of the spare auto parts (trade of finished goods).

In this paper, we focus on a specific type of collaboration through trade of inven-

[1]The absence of clear rules outside the listed here per se rule was repeatedly stated by legal experts whom we interviewed at the University of Michigan. This is also consistent with general sources, such as https://en.wikipedia.org/wiki/United_States_antitrust_law#Rule_of_reason.

[2]http://www.justice.gov/usao/eousa/foia_reading_room/usam/title7/ant00007.htm

[3]Resources may include inventory or capacity, or other means of value generation.

tories, which often may also be re-labeled to trade of capacities. Our objective is to compare potential benefits with potential costs and understand the trade-offs. For that purpose, in our model, two firms operate in markets with partially substitutable products. They independently order their inventories, before uncertain demand is realized. When demand, or credible signal of demand, becomes known, the firms may trade (buy or sell) their inventory to each other. Then, they independently price and sell the products in their markets.

In order to capture the range of potential concerns, we model both short-term and long-term effects of trade that the firms need to account for. *Market dependence*, where price in one market may influence demand in another market due to partially substitutability of products, plays a direct role in short term.

In longer term, the firms are concerned with another set of externalities of their current decisions. Even when the firm selling the resources is generously compensated for them, higher service level at the firm buying the resources may have long-term externalities, such as inertia of consumers. That is, the customers may be viscous and will stay with the same firm in the future with high probability, or more customers using the product or purchasing from a firm helps to disseminate information about the firm and may translate into future demand ("word of mouth" effect). In our paper, such externalities are labeled as *reputation effect*. Reputation is one of the main drivers of consumers' inertia and defection to the competing firms, see Reichheld and Sasser (1990).In this paper, we examine the impact of both market dependence and reputation concerns on firm's decision to enter into inventory trade contracts.

In this context, we are exploring the following questions: (1) When should firms collaborate with other firms by trading inventories? (2) What is the effect of a potential trade on the initial quantity investment? (3) What is the effect of trading on consumer surplus? (4) When trading is beneficial for both the firms and consumers versus when the regulators should be concerned about negative consequences of such collaborations?

To answer the first question, our model considers several key factors, such as degree of market dependence, significance of reputation effects, relative sizes of markets, production costs as well as the uncertainties of demand in these markets. We find that the firms may be strictly worse off by entering into trade agreements when markets are deterministic, except when costs are very low. On the other hand, if the demand is uncertain and markets are of similar sizes, the firms will benefit from trading, unless the purchasing cost is either very low or high. If, however, the firms face significantly asymmetric markets, the benefits are unlikely: One firm can intentionally increase its initial investment in inventory, not primarily to sell in its own market, but hoping to increase profits primarily through the trade with other firms. As a result of such speculative purchase, either one or both firms may be worse off due to trading. High market dependence or significant reputation tend to decrease the benefits of trading.

We answer the second question by comparing the order quantity in the presence of potential trade with the order quantity without the possibility of trade. The initial investment in inventory is driven by two forces: trade allows the firms to count on availability of the resources from the other firm and, thus, get closer to centralized firm's inventory decisions. On the other hand, both market dependence and reputation concerns drive these investments away from the centralized quantity investments.

Interestingly, the consumers also prefer firms to trade, except in markets where purchasing costs are extremely low. Consumers benefit from firms trading resources in two ways: First, trade allows firms to reallocate units to meet demand and avoid the situation where one firm has leftovers while the other market still has potential to sell. Second, the trade option in most cases drives up firms' initial ordering quantity, which leads to lower prices and more consumers being served. This logic fails when the purchasing cost is extremely low. Under such situations, competitive firms would order enough inventory to completely, or nearly completely, cover their own market even without trade contract and the firms choose to trade inventory in order to restrain price competition.

Recall that firms benefit from trade contract when purchasing costs are neither very low nor very high. Thus. from a regulator's perspective, we find that the trade contract ends up being a win-win solution when the purchasing cost is in the moderate range. The cases where both the firms and consumers are better off span quite a wide range of scenarios.

2.2 Literature Review

Three sub-streams of literature are relevant to our problem and approach: (a) literature that deals with joint pricing and purchase-quantity decisions, (b) risk pooling through transshipment and resource-exchange literature, and (c) reputation models.

In our paper, the firms make both purchase quantity and price decisions. A newsvendor model with price-dependent demand is first studied by Whitin (1955). A thorough review of literature on inventory and pricing models can be found in Chen and Simchi-Levi (2012). Most of the work on this topic focuses on making inventory and price decisions simultaneously in centralized settings under demand uncertainty. Bernstein and Federgruen (2005) and Zhao and Atkins (2008) are among those who study inventory and pricing decisions in competitive markets. Van Mieghem and Dada (1999) present a model analyzing price postponement and discuss how competition, demand uncertainty, and the timing of decisions influence the results. They assume that firms are competing in quantities they bring into market, which are then sold at the market clearance price. Wang and Kapuscinski (2009) extend their model, allowing firms to set prices directly (in addition to choosing quantities) where market substitution is price-based. None of these papers, however, considers the option of resource exchange (trading).

Risk pooling among retailers/suppliers, especially in decentralized settings, is related to inventory trade in our study. The basics of risk pooling is due to decreasing coefficient of variability whenever multiple, not perfectly correlated streams of demand are combined. The recent pooling papers include Bish and Wang (2004), Chod and Rudi (2005). In our study, two independent retailers exchange inventories and thus it is most relevant to the substream of literature that focus on risk pooling through transshipment between decentralized retailers. Transshipment literature often explicitly considers transportation, pricing and coordination issues, see e.g., Rudi et al. (2001), Granot and SoŠić (2003), Hu et al. (2007), which partly decreases attractiveness of pooling. Paterson et al. (2011) provide a comprehensive review of inventory problems with lateral transshipments. Most of these papers consider centralized retailers, or decentralized retailers in non-competitive markets. In such settings collaboration (trade) is a natural choice, as it improves profitability of all participants. In this stream of papers, Zhao and Atkins (2009) is closest to our work. It considers transshipment between competing retailers. The authors find that, when transshipment price is high and competition is weak, then transshipment benefits all firms, which is consistent with our findings. This literature assumes that the transshipment prices need to be set up front and typically investigates the existence of coordinating transfer prices. Our paper does not focus on coordinating contracts. We consider the effect of collaboration on the firms and on the customers, when the trading price is determined endogenously (which leads to different behavior), and the retail price is set in response to demand realization. In this sense, our work complements Zhao and Atkins (2009). Unlike other papers in this stream, we include both short-term and long-term competitive forces by investigating immediate profits, and also the effect of current sales on future market shares through reputation effect. None of the papers, including Zhao and Atkins (2009), finds that the firms may be worse off due to trading, while we show that strategic interactions may lead to such an outcome. Most importantly, we study the effect of trade on consumer welfare, which has important legal implications for firms and regulators. Chod and Rudi (2006) consider some elements that are also relevant to our paper. They assume that the trading price is a result of the negotiation between two decentralized firms. In their paper, to determine the trading price, both price equilibrium and bargaining equilibrium are considered. The paper concludes that both price and bargaining equilibrium can lead to higher expected profits compared to no-trade case. Their results are, however, limited to independent markets and, also, based on constant-elasticity demand models. Effectively, there is no price competition and the interaction is only through the transfer of resources. Chun et al. (2013) consider bargaining equilibrium in a competitive setting. Their focus is on finding efficient algorithms for various network structures and they assume that the initial capacity is exogenously set rather than a decision variable. Also, these papers do not consider potential consequences of trade on firms' future sales and consumer welfare.

An important feature of our model is that firms' current decisions influence the firms' future market sizes and future revenues, through externalities of current decisions (reputation). This type of externality has been broadly studied in Economics literature. Kováč and Schmidt (2014) provide a review of this area. Bensaid and Lesne (1996) and Anari et al. (2010) point out that the current sales have positive influence on the future demand, and they label this effect as network externality or historical externality. With such externality, firms intend to price lower in early stages to gain market share. Similar conclusion is reached in modeling papers. Kováč and Schmidt (2014) study a market with two firms and constant number of customers. In both Caminal and Vives (1996) and Kováč and Schmidt (2014), firms compete for market share through pricing decisions and when considering future market share, they show that the current pricing decisions tend to be more competitive (lower). Reputation effects are also considered in operations settings through market-size adjustments. Hall and Porteus (2000) consider a multi-period game, where two firms make capacity decisions and compete for market share. In their model, market size is adjusted in response to the current-period sales (stockouts). Liu et al. (2007) extend their model to a general demand function and infinite horizon, while Olsen and Parker (2008) allow firms to carry inventory and to backlog customers. Our reputation model borrows the adjustment structure from this literature. While these papers model and study future market dynamics, none of them considers collaboration among competing firms.

In addition to analyzing different research questions, from technical point of view, this is the first paper, to the best of our knowledge, that incorporates both short-term trade-offs with long-term externalities (reputation) for retailers who consider inventory collaboration in competitive settings as well as study consumer welfare implications of short-term and long-term effects.

2.3 Model

We consider two firms, indexed by $i, j = 1, 2$ ($i \neq j$), operating in two possibly dependent markets. The market size for firm i is $w_i = \mu_i + \varepsilon_i$, where μ_i is the mean and ε_i is a random shock with zero mean. Both μ_i and distribution of ε_i are common knowledge. Demand d_i in each market depends on the realized market size w_i and both firms' prices p_i and p_j:

$$d_i = (w_i - \alpha p_i + \alpha\beta(p_j - p_i))^+,$$

where α reflects the sensitivity to price and β represents competition or substitution level, as defined in McGuire and Staelin (1983).

To simplify the notation, we let $a = \alpha + \alpha\beta$ and $b = \alpha\beta$ and re-write firm i's demand as:

$$d_i = (w_i - ap_i + bp_j)^+.$$

We use a and b as the market parameters throughout the whole paper except when we explicitly evaluate the effects of competition and substitution levels, β.

Firms make three decisions in the following sequence:

Ordering. First, both firms simultaneously make the ordering decisions: Firm i orders q_i units of inventory at unit cost c. The unit cost is identical for both firms.

Trading. After the market uncertainty ε_i is realized, two firms have an opportunity to trade their inventories. The trade process is modeled as a Nash-bargaining equilibrium. Let $\bar{q}_i$ be firm i's inventory level after the trade.

Pricing. Both firms independently decide their selling prices p_i's and collect revenues from customers. We denote $s_i = \min\{\bar{q}_i, d_i\}$ as the sales of firm i. The revenue from the current-period sales is $p_i s_i$.[4]

We allow the current-period decisions to influence the future profits through reputation effects. Specifically, we follow Hall and Porteus (2000) (as well as their extensions) to model the future profit as a function of the current-period demand. Denoting the future market size by $\tilde{\mu}_i$, the long-term effect of current demand is reflected as follows:

$$\tilde{\mu}_i = \mu_i + \gamma(d_i - d_j),$$

where $\gamma \in [0, 1)$ is the strength of the reputation effect (or other externalities). $\gamma = 0$ corresponds to the case where the current sales have no long-term effects, $\gamma \to 1$

[4]To reflect the reality that firms have flexibility to change price after they have observed market signal and exchanged inventory, we allow firms to set prices after the trading stage. In such situation, firms can match demand and supply by either adjusting prices or exchanging inventories. Setting prices after demand realization is an appropriate model, when for example the selling season is long enough or when the retailers do not announce the prices in advance. This is preferred by retailers although not necessarily feasible in some situations. In other situations, prices are announced upfront and cannot be changed later. Without pricing to influence demand after random market shock, such situations are very close to traditional transshipment literature and are studied in Zhao and Atkins (2008, 2009).

corresponds to a situation where current sales have strong influence in the long term. The future revenue is approximated as a function of future market size $\tilde{\mu}_i$. To keep the model tractable, we assume the future revenue is $\lambda\tilde{\mu}_i$, where λ can be interpreted as the customer lifetime value.[5]

The objective of the model is to evaluate the feasibility and benefits of collaboration, through inventory trading, to the firms and also to study the effect of trading on consumer surplus. To achieve this, we consider three scenarios. First, we establish a benchmark for our analysis *centralized scenario*, where one central controller makes all decisions. The second scenario is the fully decentralized case, *no-trade scenario*, where the competitors do not coordinate their decisions nor collaborate. The third scenario, *trade scenario*, is our focus: although the firms make ordering and pricing decisions independently, they can trade their inventory after the demand is realized but before the pricing decisions are made.

The three scenarios are formally introduced in the following subsections. We then analyze these scenarios in Sections 2.4 and 2.5. The centralized, trade, no-trade scenarios are denoted by C, T, N, respectively. Also let p, t, o represent pricing, trading, and ordering stages. We define π_i^{Xx} as firm i's revenue at the beginning of stage x ($\in \{p, t, o\}$) in scenario X ($\in \{C, T, N\}$) and define Π^{Xx} as the total revenue of firms at the beginning of stage x in scenario X.

2.3.1 Centralized Scenario

We analyze each scenario starting with the last stage, i.e., pricing, and then follow with trading (if applicable), and finally ordering. In the pricing stage, the inventory levels are already chosen and random shocks are realized. The centralized revenue in

[5]This model is supported by empirical studies such as Chevalier and Mayzlin (2006) who empirically study the word-of-mouth effect in online book industry. They conclude that more consumer reviews, which can be viewed as a proxy for previous sales, lead to higher sales in the future.

the pricing stage is given by

$$\Pi^{Cp}(\bar{q}_1, \bar{q}_2) = \max_{p_1, p_2 \geq 0} \sum_{i=1}^{2} (p_i s_i + \lambda \tilde{\mu}_i) = \max_{p_1, p_2 \geq 0} \sum_{i=1}^{2} p_i s_i + \lambda(\mu_1 + \mu_2). \tag{2.1}$$

Consider the trading stage

$$\Pi^{Ct}(K) = \max_{\bar{q}_1 + \bar{q}_2 = K} \Pi^{Cp}(\bar{q}_1, \bar{q}_2). \tag{2.2}$$

In the ordering stage, the controller chooses the total initial inventory K. The revenue function in the ordering stage is, thus,

$$\Pi^{Co}(K) = \mathbb{E}_{\varepsilon_1, \varepsilon_2} \Pi^{Ct}(K) \tag{2.3}$$

and the central controller solves

$$\max_{K} \Pi^{Co}(K) - cK.$$

2.3.2 No-Trade Scenario

In no-trade scenario, firms make decisions competitively. In the pricing stage, for given inventory and random shocks, each firm chooses its price by solving

$$\max_{p_i} \{p_i s_i + \lambda \tilde{\mu}_i\}. \tag{2.4}$$

Let the equilibrium outcome of the pricing stage be $\mathbf{p}^*(q_1, q_2)$ and let the corresponding equilibrium revenue for firm i be $\pi_i^{Np}(q_1, q_2)$. We will establish the existence of the equilibrium later. Since trading is not allowed in this scenario, firm i's revenue in the ordering stage is

$$\pi_i^{No}(q_1, q_2) = \mathbb{E}_{\varepsilon_1, \varepsilon_2} \pi_i^{Np}(q_1, q_2) \tag{2.5}$$

and each firm solves

$$\max_{q_i} \pi_i^{No}(q_1, q_2) - cq_i.$$

2.3.3 Trade Scenario

When firms are allowed to trade, the pricing stage is exactly the same as in the no-trade scenario.

Recall that q_i stands for inventory before the trade and $\bar{q}_i$ after the trade. Trade quantity is endogenously determined through Nash bargaining equilibrium, where the firms choose how to reallocate their inventory and how to allocate the benefits resulting from reallocation.

Nash bargaining game is one of the most common approaches to study decision making among independent parties that involve elements of negotiation and collaboration. In Nash bargaining game, two competitors decide the outcome of the game, given established up-front rules for dividing the benefits. Readers are referred to Muthoo (1999) for more details and further references. A number of papers in Operations Management literature use Nash bargaining solution to analyze the Nash bargaining game (Nagarajan and Šošić 2008, Chod and Rudi 2006, Kuo et al. 2011). In the Nash bargaining solution the benefits above disagreement point are divided equally among the parties. We adapt this approach. In our context, the disagreement point for each firm is the revenue the firm would collect if no trade would take place (for given quantities q_1 and q_2 owned by the firms). Consequently, Nash bargaining equilibrium in our setting will maximize the sum of the firms' revenues, anticipating the outcome of price competition in the next stage:

$$\Pi^{Tt}(K) = \max_{\bar{q}_1+\bar{q}_2=K} \sum_{i=1}^{2} \pi_i^{Np}(\bar{q}_1, \bar{q}_2). \tag{2.6}$$

Each firm's revenue function after random shock realization is

$$\pi_i^{Tt}(q_1, q_2) = \frac{1}{2}\Pi^{Tt}(q_1 + q_2) + \frac{1}{2}\pi_i^{Np}(q_1, q_2) - \frac{1}{2}\pi_j^{Np}(q_1, q_2). \tag{2.7}$$

In the ordering stage, firm i's revenue is

$$\pi_i^{To}(q_1, q_2) = \mathbb{E}_{\varepsilon_1, \varepsilon_2} \pi_i^{Tt}(q_1, q_2). \tag{2.8}$$

Thus, each firm solves

$$\max_{q_i} \pi_i^{To}(q_1, q_2) - cq_i.$$

A critical element of the analysis is evaluation of trade on consumer surplus, which is the focus of the next section.

2.3.4 Consumer Surplus

Consumer welfare is one of the most important criteria for regulators to identify collaboration agreements that violate anti-trust laws. Therefore, in addition to the firms' profits, we also evaluate the consumer welfare, in order to identify situations that might be problematic from antitrust point of view. Singh and Vives (1984) formulated the total consumer utility in a competitive market as

$$U(s_1, s_2) = \frac{1}{2(a^2 - b^2)}(2(aw_1 + bw_2)s_1 + 2(aw_2 + bw_1)s_2 - as_1^2 - as_2^2 - 2bs_1s_2),$$

where w_i is the market size and s_i is the sales. While seemingly complicated, this utility function is consistent with the linear demand function $s_i = w_i - ap_i + bp_j$ and has been routinely used in the literature (Amir and Jin 2001, Lin and Saggi 2002, Hsu and Wang 2005). That is, when consumers make decisions maximizing their consumer surplus, $U(s_1, s_2) - p_1s_1 - p_2s_2$, the resulting demand function is linear.

Expressing explicitly sales s_i as a function of prices p_i and p_j,we have[6]

$$CS = U(s_1, s_2) - p_1 s_1 - p_2 s_2 = \frac{1}{2} a(p_1^2 + p_2^2) - b p_1 p_2 - w_1 p_1 - w_2 p_2 + \frac{(w_1 + w_2)^2}{4(a-b)} + \frac{(w_1 - w_2)^2}{4(a+b)}. \tag{2.9}$$

2.4 Deterministic Market Sizes

We start our analysis with the deterministic setting, where market sizes are known up front. The deterministic setting allows us to identify and describe the critical trade-offs and establish some of the important results, which will be later used and extended in the stochastic setting. We solve the problem using backward induction and start with the pricing stage.

2.4.1 Pricing

We first present the pricing-stage outcomes for all three scenarios. The price equilibrium is the same for the two decentralized scenarios (trade and no-trade) given the same starting inventory levels at the beginning of the pricing stage, but different than for the centralized one. The following lemma shows the existence and uniqueness of the pricing equilibrium in each scenario.

Lemma II.1. *With deterministic market sizes (w_1, w_2) and given after-trade inventory levels $(\bar{q}_1, \bar{q}_2)$, we have*

(1) Centralized pricing: There exists a unique optimal price pair (p_1^{C*}, p_2^{C*}).

(2) Decentralized pricing: Assume the firms consider prices such that demand does not

[6]The definition of Singh and Vives (1984) extends the traditional definition used in independent markets. For independent markets, $b = 0$, and the total consumer surplus becomes:

$$CS = \frac{1}{2a}[(w_1 - ap_1)^2 + (w_2 - ap_2)^2]$$

which is the total consumer surplus for the linear demand function $w_i - ap_i$. Notice that this is the demand when individual consumer willingness to pay is uniformly distributed over $[0, w_i/a]$ in each market.

exceed the available quantity. There exists a unique equilibrium price pair (p_1^{X*}, p_2^{X*}), where $X = N, T$.

All of the proofs are in the appendix. Note that the condition in part (2) of the Lemma II.1 is not very restrictive, given deterministic demand. It assumes that firms do not choose extremely low prices such that the demand is larger than the available inventory.

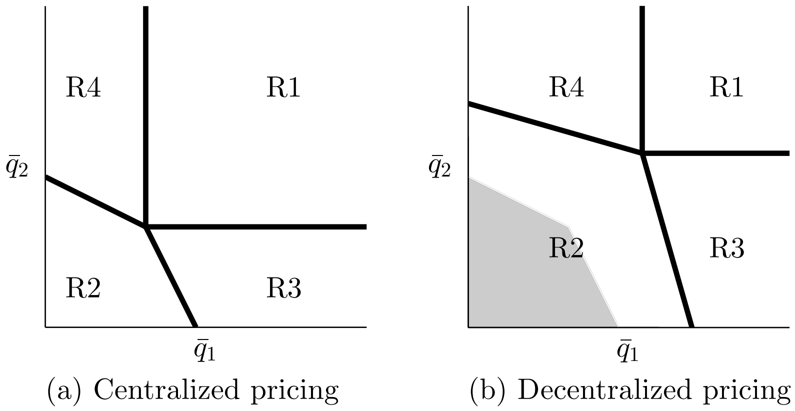


Figure 2.1: Pricing outcomes as a function of after-trade inventories. R1 is the region with left-over inventories, R2 with all inventory sold at market clearance prices, and R3 and R4 are regions where one of the products is sold at the clearance price and the other firm has leftover products.

The closed-form solutions of equilibrium and optimal prices are provided in Appendix A. Based on these solutions, Figure 2.1 illustrates pricing policies for centralized and decentralized scenarios as a function of after-trade inventory levels. In both cases, there are four regions of after-trade inventory levels $(\bar{q}_1, \bar{q}_2)$ with different equilibrium outcomes. In Region 1, both firms end up with leftovers. Consequently, the prices do not depend on after-trade inventories. In Region 2 all inventory is sold and the prices are market-clearing ones. In Regions 3 and 4, one of the firms has leftovers, while the other sells all inventory. The market clearing constraint for one market effectively determines both prices in this region.

In the decentralized pricing game the firms price weakly lower (more aggressively) than in the centralized case. Lower pricing results in higher sales and, consequently, the area where market clearance takes place (Region 2) is larger in the decentralized game. The shaded area in Figure 2.1(b) corresponds to region R2 for centralized case. In the shaded area, the decentralized prices are identical to centralized prices, while outside of this region, at least one of the decentralized prices is strictly lower.

2.4.2 Trading

Trading inventory takes place in the centralized scenario and in the trade scenario. We first describe how a central controller reallocates the total inventory, K, to the firms after random shocks are realized.

Centralized Scenario

Lemma II.2. *In centralized scenario, firm 2's after-trade (allocation) inventory, $\bar{q}_2$, is as follows*

$$\bar{q}_2 = \begin{cases} [w_2/2, K - w_1/2] & \text{if } w_1 + w_2 \leq 2K \\ 0 & \text{if } w_1 - w_2 \geq 2K \\ K & \text{if } w_2 - w_1 \geq 2K \\ \frac{2K - w_1 + w_2}{4} & \text{otherwise} \end{cases}.$$

Firm 1 obtains the remaining $(K - \bar{q}_2)$ units at the end of the trading (allocation) stage.

The resource allocation is driven by the difference of the margins in two markets. When market sizes are both large and not extremely different, the inventory is allocated in such a way that the two markets have equal margins (this is the last row defining $\bar{q}_2$ in Lemma II.2). If one of the firms has a noticeably larger market size, allocating all inventory to the larger market is beneficial as even a single unit in the smaller market would not be able to provide as high margin as customers in the larger market. But, when both market sizes are relatively small, even after the firms optimally allocate their inventory, there will be leftovers (Region 1). Since the location of leftover units does not matter, in this case there exist multiple optimal solutions, resulting in the same profit. This is illustrated in Figure 2.2 (a) for two equal-sized markets $w_1 = w_2$. In Region 2, where total inventory is small, the inventory will be allocated equally between two firms (as shown by solid line) and all inventory will be sold.

Since left-over inventory can be freely reallocated between firms, the revenue is expressed as a function of total inventory and has the following property.

Lemma II.3. $\Pi^{Ct}(K)$ *is nondecreasing and concave in* K*.*

Concavity of profit function simplifies some of the proofs of the subsequent results. While Lemma II.3 holds in centralized settings, we will see that, for decentralized trade scenario, profit function is not concave in the total inventory. Moreover, the indifference to allocation of left-over inventory does not hold anymore.

Trade Scenario

The trade option provides an opportunity for decentralized firms to adjust inventory. The adjustment involves a payment defined by Nash Bargaining Solution, which works as follows. Whenever there is an opportunity to increase the total profit, the firms will reallocate the inventory and divide the surplus equally.

The following lemma describes the total revenue (for two firms) when trade is allowed. The revenue is unimodal, but not concave in the total inventory K. First, however, define K_1 as the sum of quantities at the common point of all four regions for the decentralized case (Figure 2.2(b)).

Lemma II.4.

(1) $\Pi^{Tt}(K)$ *is continuous and unimodal in* K*. There exists* $\frac{w_1+w_2}{2} \leq K_0 \leq K_1$*, such that* $\Pi^{Tt}(K)$ *is concave for* $K \leq K_0$*, and constant for* $K > K_0$*.*

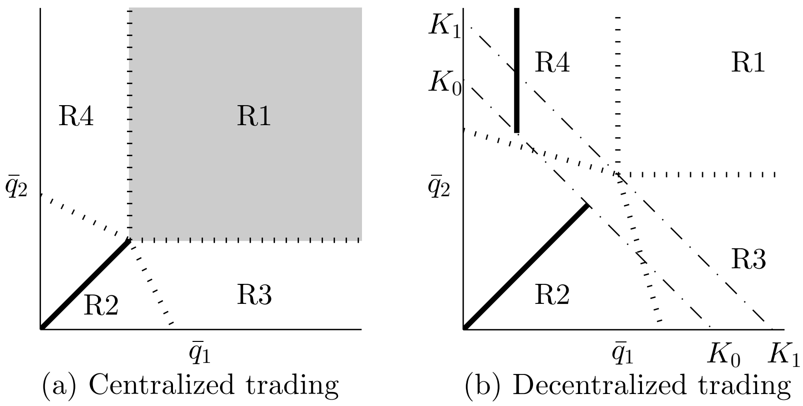


Figure 2.2: Trading outcomes. Solid line and shaded region indicate: (a) allocation of inventory in centralized scenario, (b) trading equilibrium for decentralized trade scenario.

(2) For $K \leq \frac{w_1+w_2}{2}$, the resulting inventory allocation is the same as in centralized solution.

The interesting behavior described in Lemma II.4 can be explained using Figure 2.2(b). For this illustration we assume equal market sizes $w_1 = w_2$. When the total inventory is low ($q_1 + q_2 \leq K_0$), trade results in market clearance. In this case, the firms sell equal quantities, which is exactly how a centralized firm would allocate the inventory, shown as solid line in Region 2. (When market sizes are not equal, the allocation also coincides with the centralized solution, even though the quantities are not equal.) Thus, despite the anticipated price competition, trading does not distort the allocation away from the centralized solution up to inventory level K_0. However, for $q_1 + q_2 > K_0$, the behavior changes. The firms allocate the inventory in an asymmetric manner, where one firm provides a moderate amount of inventory in its market and sells its entire inventory, while the other firm has leftovers, shown as vertical solid line in region R4. Interestingly, for all inventory levels above K_0, the sales do not change while inventory increases. For inventories $q_1 + q_2$ between K_0 and K_1, the behavior is similar to the case with leftover inventories, even though without trading, there would be no leftovers in Region R2.

If decentralized firms mimicked centralized firms and allocated their units proportionally to the market size, they would compete intensively in price and end up with fairly small profits. Instead, decentralized firms agree to transfer a portion of inventory to one market while leaving smaller inventory in the second market. As a result, the rivalry is less intensive and both firms are better off. This suggests the possibility the trade option might decrease consumer welfare, which we discuss below.

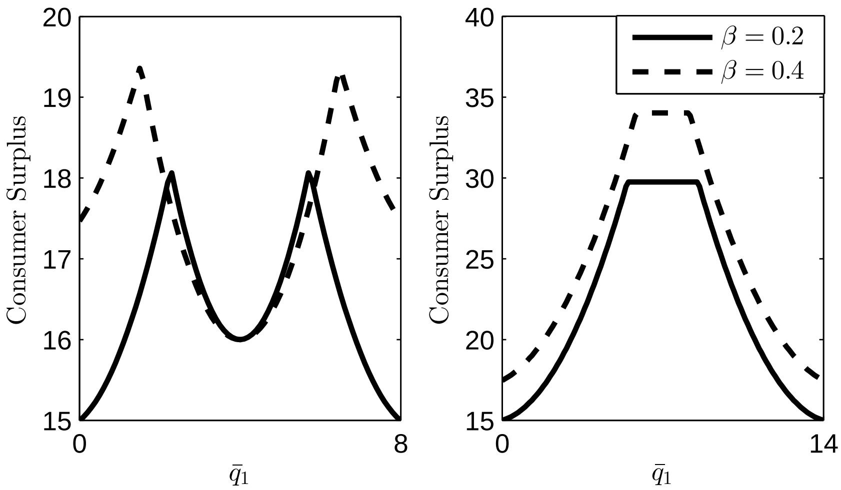
Consumer Surplus in Trade Scenario

To evaluate the effect of trading on consumer surplus, we need to understand the effect of ordering quantities (initial inventory), and also the effect of trading given the same ordering quantities. We start with the latter one and, for now, we fix the total inventory at $K = q_1 + q_2$. The following lemma describes how the surplus changes as a result of inventory reallocation.

Lemma II.5. *Let the total inventory $K = q_1 + q_2$ be fixed in the trading stage. The consumer surplus is (1) In R1, constant in $\bar{q}_1$; (2) In R2, convex in $\bar{q}_1$ and minimized at $\bar{q}_1 = K/2$; (3) In R3, decreasing in $\bar{q}_1$; and (4) In R4, increasing in $\bar{q}_1$.*

Given total inventory K, the consumer surplus as a function of firm 1's after-trade quantity is plotted in Figure 2.3(a) for low K and (b) for high K. In case (a), the potential allocations fall into regions R2, R3, and R4 in Figure 2.2(b). As long as firms do not have leftovers (region R2), the consumer surplus is convex in the allocation and reaches its minimum level (within region R2) when two markets have the same sales (Lemma II.5). This is because, any asymmetric allocation, implies lower price in the market with higher allocation (for more customers) and this effect dominates the increase of the price in the market with fewer customers, which results in a higher consumer surplus. When allocations become very asymmetric, we enter region R3 or R4. With most of inventory in one market, that firm does not use market clearance price any further, which leads to a higher price in the small market and sometimes also higher price in the large market. Thus, the consumer surplus decreases as allocations become extremely asymmetric.

For case (b) with high-total-inventory (Figure 2.3(b)), when the allocation is asymmetric, the allocation falls into region R3 or R4 in Fig.2.2(b). Similar to the low-inventory case, not attempting to sell out the inventory leads to a higher price, resulting in a decrease in consumer surplus. However, when the allocation is close to symmetric, high levels of inventory in both markets result in leftovers in both markets (region R1), where the prices in both markets and the consumer surplus are independent of the allocation.



(a) K = 8

(b) K = 14

Figure 2.3: Consumer surplus as a function of allocation of total inventory K between two firms. (a) is for low K, while (b) is for high K.

The firms, rather than consumers, choose how to allocate the inventory. While the firms become better off, the effect of trade on consumer surplus (given initial inventory) is not obvious. A stylized numerical example in Table 2.1 illustrates various possible outcomes. In symmetric setting, if firms' initial inventory is (4, 0), consumer surplus is maximized. However, firms will trade to equally divide their inventory. This decreases the consumer surplus. In asymmetric setting, suppose that initial inventories are (2, 2). If the firms do not trade, both firms and consumers suffer. If trade takes place, the allocation becomes (1,3) to match the market sizes. Consumer surplus is increased, though not to the highest possible level.

	Symmetric w_i		Asymmetric w_i	
	No trade	Trade	No trade	Trade
w_1, w_2	10,10	10,10	10,14	10,14
q_1, q_2	4,0	4,0	2,2	2,2
$\bar{q}_1, \bar{q}_2$	4,0	2,2	2,2	1,3
p_1, p_2	6,10	8,8	8,12	9,11
π_1, π_2	24,0	28,4	16,24	17,25
CS_1, CS_2	8,0	2,2	2,2	0.5,4.5

Table 2.1: Consumer surplus before and after trade.

2.4.3 Ordering Stage

Theorem II.1 below characterizes the optimal ordering policy for the centralized scenario. Theorem II.2 characterizes ordering solutions for no-trade and trade scenarios, assuming that the firms are symmetric. We discuss the asymmetric case in Section 2.5.2.

Theorem II.1. *In the centralized scenario, there exists a unique optimal inventory level K^{C*}. Assuming (without loss of generality) $w_1 \geq w_2$, we have*

$$K^{C*} = \begin{cases} \frac{w_1+w_2-2(a-b)c}{2} & \text{if } c \leq \frac{w_2}{a-b} \\ \max\left\{\frac{aw_1+bw_2-(a^2-b^2)c}{2a}, 0\right\} & \text{if } c > \frac{w_2}{a-b} \end{cases}.$$

Note that in symmetric centralized settings, the optimal solution reduces to $K^{C*} = \max\{w - (a-b)c, 0\}$.

We next consider the decentralized firms, and assume that the firms are symmetric ($w_1 = w_2 \equiv w$). We analyze symmetric equilibria.[7]

[7] In our extensive numerical study, we have not observed any asymmetric equilibria for symmetric firms.

Theorem II.2. *Consider two symmetric firms ($w_1 = w_2 \equiv w$).*

(1) No-trade scenario: there exists a unique equilibrium q_i^{N*}:

$$q_i^{N*} = \min\left\{\max\left\{\frac{(a+b)w + (a^2-b^2)(\lambda\gamma - c)}{2a+b}, 0\right\}, w\right\}.$$

(2) Trade scenario: there exist cost thresholds $c_1 \leq c_2$, such that there are two equilibria for $c \in [c_1, c_2]$ and, otherwise, the equilibrium is unique. Specifically, we have

(2)(a) (Low equilibrium:) For $c \geq c_1$, $q_i^{T*} = \max\left\{\frac{(a+b)w+(a^2-b^2)(\lambda\gamma-c)}{2a+b}, 0\right\}$ is an equilibrium.

(2)(b) (High equilibrium:) For $c \leq c_2$, $q_i^{T*} = \min\left\{\frac{(a+b)w+2(a^2-b^2)(\lambda\gamma-c)}{2a}, \frac{aw+(a^2-b^2)\lambda\gamma}{2a-b}, w\right\}$ is an equilibrium.

(2)(c) For $c \in [c_1, c_2]$, both firms obtain (strictly) higher profits in low equilibrium.

Theorem II.2(1) provides the equilibrium order levels of decentralized firms without trade. By comparison of equilibrium quantities with K^{C*}, it is easy to verify that the firms in no-trade scenario order more than those in centralized scenario ($q_i^{N*} \geq K^{C*}/2$). Due to the competition in both current and future periods, the firms behave more aggressively, first buying more inventory and, then, selling it at lower prices. The centralized firms, being aware that low price in one market will hurt the profit in the other one, price less aggressively (they set higher prices) and order more conservatively.

In the trade scenario, uniqueness and nature of the equilibrium depend on the procurement cost of the firms. There are two potential equilibria in this case: *low equilibrium* and *high equilibrium* (the order quantity in low equilibrium is lower than the one in high equilibrium, therefore the name). Except the interval where $c \in [c_1, c_2]$, the equilibrium is unique. Theorem II.2 indicates that low equilibrium order quantity is the same as no-trade equilibrium (which itself is larger than the centralized solution). High equilibrium order quantity is even higher. We can show that the low equilibrium *Pareto* dominates the high equilibrium. Therefore, in the rest of the paper, we assume the firms always choose low equilibrium for $c \in [c_1, c_2]$.

In deterministic settings the firms can fully predict their future market condition at the time they order. Thus, one might argue, that instead of relying on exchange of goods in the trading stage, they should order whatever will be needed later. This is indeed true when cost is sufficiently high, $c > c_1$ when the firms order the same quantities as in the no-trade case (and then trade does not take place). However, when cost is small enough, $c < c_1$, without trade option, firms order what they (correctly) foresee to sell and there are no leftovers. With trade option, firms order more (high equilibrium). This is illustrated in Figure 2.4(a) for $c < c_1$.

Ordering more cannot be attributed to the desire to sell the additional units to competitor. A rational firm knows that the competitor also increases her order quantity and will not need any extra inventory. Interestingly, the main driver of purchasing more inventory is increasing firm's own disagreement revenue and, thus, its bargaining power. This behavior takes place when inventory is inexpensive and the cost of leftover is relatively small.

Recall that the disagreement points are based on the no-trade outcome. Clearly, there are no leftovers in the no-trade equilibrium. Consider now the effect of a deviation from this equilibrium (which becomes relevant when trade is allowed): If one firm increases its inventory, this firm's profit decreases. If the increase in inventory is sufficiently small, the firm sells all inventory and, therefore, the price must decrease. However, the other firm, whose inventory is unchanged, is hurt as well. It has to respond by lowering price. Since the first firm is able to change both quantity and price while the second firm changes only price, the first firm suffers less from this deviation. When firms now have the potential to trade, they recognize that ordering higher inventory, decreases the disagreement point of competitor more than their own disagreement point. Thus, with low cost of inventory, "over-ordering" exists. This mechanism, based on the bargaining equlibrium, reinforces anecdotal stories, where firms choose actions that are suboptimal and hurting themselves, as long as these actions hurt even more their competitors.

Profit

When both firms adopt this strategy, their behavior resembles prisoner's dilemma: two symmetric firms would purchase extra inventory which is guaranteed to be left over. Also, despite their intent to increase the bargaining power, the firms over order by the same amount and end up having equal bargaining power. As the result of over-ordering the price competition is intensified and the firms effectively waste money on inventory, which ends up being unsold. Consequently, the existence of trade option may lead to lower profits for both firms.

Theorem II.3. *There exists a threshold c_0 such that both firms are worse off in trade scenario compared to no-trade scenario if and only if $c_0 < c < c_1$.*

Theorem II.3 is illustrated in Figure 2.4(b). For $c < c_1$ trade takes place. The firms over-order inventory and later allocate it asymmetrically. The firm with smaller inventory sells its entire inventory, while the other one has left-overs. When inventory is left over, we label this allocation as "high-inventory trade." This helps the firms to reduce the competition and benefits both of them. In deterministic settings, two firms trade inventory if and only if the high equilibrium is played. As a result, over-ordering and high-inventory trade always take place together.[8] When there are no left-overs after trading, we define this situation as "low-inventory trade." In deterministic case, low-inventory trade does not take place.

Trading benefits the firms, while over-ordering hurts them. Though over-ordering takes place for all $c < c_1$, this is not sufficient to make firms worse off. When the cost

[8]Although over-ordering and high inventory trade take place together in the deterministic case, it is useful to define over-ordering and high inventory trade separately, since in the stochastic case, they may influence firms' profits in the opposite direction And as high inventory trade can take place even though the firms order the same amount they would order in no-trade scenario (no over-ordering).

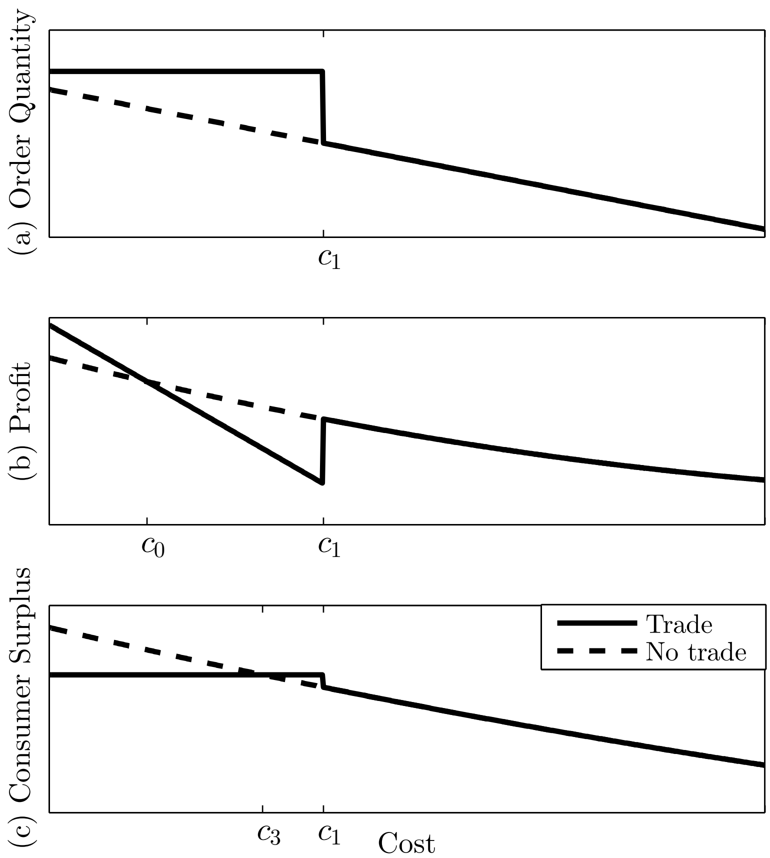


Figure 2.4: Solutions in the deterministic setting. (a) order quantity; (b) profit; (c) consumer surplus. Parameters: $w = 10; a = 1.4; b = 0.4; \lambda = 14; \gamma = 0.5$.

is very low ($c < c_0$), the extra inventory is inexpensive and the benefit from inventory coordination through trade dominates the investment in inventory and the firms are better off. However, when cost is somewhat higher ($c_0 < c < c_1$), the over-ordering becomes slightly more expensive, and then the purchase cost dominates the benefit from inventory coordination and both firms become worse off.

Consumer Surplus

Recall that if the purchasing cost $c \geq c_1$, the firms always choose low equilibrium, where they do not trade, and consequently the consumer surplus is identical in trade and no-trade scenarios. Therefore, we focus on $c < c_1$ and analyze how over-ordering and high-inventory trade influence consumer surplus. With trade option, firms will order more, which intuitively should increase consumer surplus (over-ordering effect). On the other hand, when firms trade, they want to reduce the competition in the pricing stage, which may decrease consumer surplus (high-inventory trade effect). With both effects, trade may actually lead to either higher or lower consumer surplus than no-trade scenario.

Theorem II.4. *There exists $c_3 \in [0, c_1]$ such that trade scenario has higher consumer surplus than no-trade scenario if and only if $c \in (c_3, c_1)$.*

Though it is possible that trade option increases or decreases both profit and consumer surplus, in our numerical study, we observe that the total surplus (sum of the firm profits and consumer surplus) is always lower with trade option compared to the no-trade case. In deterministic settings, trade option never benefits all parties.

Comparative Statics

As we have noted earlier, as long as there is some competition (either price competition in the current period or competition for future market share), the firms are more aggressive in the pricing stage and their profits are smaller than those for the centralized firm. In this section, we study in greater detail how the equilibrium outcome changes as a function of market substitution β (recall $a = \alpha + \alpha\beta, b = \alpha\beta$) and reputation effect λ.

Theorem II.5. *Assume a symmetric setting.*

(1) In centralized scenario: order quantity, profit, and consumer surplus are not influenced by either β or λ.

(2) In no-trade scenario: order quantity and consumer surplus are non-decreasing and profit is non-increasing in both β and λ.

(3) In trade scenario: As long as firms stay in high (low) equilibrium, order quantity and consumer surplus are non-decreasing and profit is non-increasing in both β and

λ.

As market substitution β, or reputation effect λ, or both increase, decentralized firms are involved in a fiercer rivalry compared to centralized case. Thus, in both trade and no-trade scenarios, the firms order more and offer lower prices, which leads to lower profitability and higher consumer surplus. This, however, holds only if the firms stay in the same type of equilibrium in trade scenario.

Summary of Deterministic Model

In deterministic markets, trade option may help or hurt firms or consumers and the critical factor is the cost. If cost is not very low, trade option leaves the firms with the same profit and consumers with the same surplus, because the firms order exactly what they will sell and effectively no trading takes place. When cost is very low, however, both the firms and consumers are influenced by the trade option, due to a combination of over-ordering and high-inventory trade effects. For extremely low cost, the firms over-order but due to the high-inventory trade, they can adjust inventory and prices so that the firms are better off, while the consumers are worse off. When inventory cost is somewhat higher, the same forces are in effect, but due to somewhat-higher inventory cost, the firms over-ordering becomes more expensive and the firms are worse off, while the consumers may be worse or better off. Effectively trading has bigger (positive) impact when inventory cost is very low, while over ordering has bigger (negative) effect when inventory cost is higher. Note that these two effects (over-ordering and trading) exist if and only if the firms compete either for current sales or future sales (either β or λ is positive).

2.5 Stochastic Market Sizes

Now we consider the case when the firms' market sizes are uncertain. As the trading and pricing decisions are made when market sizes are already known, these decisions are exactly the same as in the deterministic model. Since the ordering decisions are made before observing the market sizes, they are our focus.

Before analyzing the general model, we consider the effect of uncertainty in a special setting, where the two markets are independent, i.e., $\beta = \lambda = 0$. This allows us to establish a benchmark and isolate the effect of uncertainty on inventory from the effect of competition. Recall that in deterministic case without competition, the over-ordering and high-inventory trade would not take place. Therefore, trade and no-trade scenarios result in the same profits and consumer surplus.

2.5.1 Independent Markets

The existence and uniqueness of the equilibrium in the ordering stage in all three scenarios are established in Theorem II.6. We continue to state the results for the symmetric case even though the results in this section continue to hold for asymmetric settings.

Theorem II.6. *Assume $\beta = \lambda = 0$.*

(1) Centralized scenario: There exists a unique optimal solution K^{C*}.

(2) No-trade scenario: There exists a unique optimal solution q_i^{N*} for firm i.

(3) Trade scenario: There exists a unique equilibrium (q_1^{T*}, q_2^{T*}).

Let K^X be the total ordering quantity in scenario X ($X \in \{C, T, N\}$). Below we describe the relationship among these quantities.

Theorem II.7. *Either $K^{C*} \geq K^{T*} \geq K^{N*}$ or $K^{C*} \leq K^{T*} \leq K^{N*}$ holds.*

A typical relationship among order quantities is demonstrated in Figure 2.5(a), with order quantity for trade option located between centralized and no-trade order quantities. The underlying dynamics resembles *risk pooling* in the classical newsvendor model: When cost is low, we intuitively have high service level. With no trade, safety stock of a centralized firm is lower than that of two decentralized firms. With

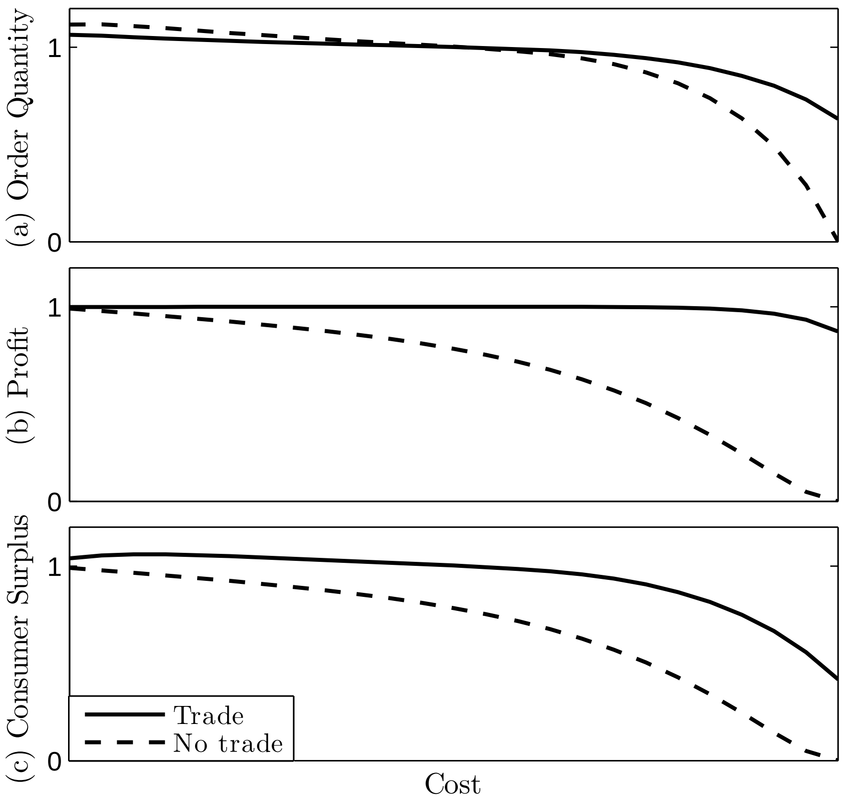


Figure 2.5: Solutions in stochastic setting with independent markets. Presented as percentage of that in centralized scenario. Parameters: $w \sim U[10, 40]; a = 1; b = 0; \lambda = 0; \gamma = 0.5$.

trade option, independent firms do not reach the efficiency of a centralized firm, but they have opportunity to "help" each other and, thus the order quantities (and the corresponding safety stock) decrease. When cost is high, similar logic applies: two decentralized firms with trade option can bring the safety stock closer to the one of centralized firm. Though not as effective as centralization, trading leads to better decisions (closer to the centralized total inventory) when markets are independent.

Profit

Theorem II.8. *In independent markets, the firms' profit with trade option is always (weakly) higher than without trade option.*

The traditional risk pooling includes two related benefits: (1) transferring inventory across locations, when needed, and (2) more appropriate investment in total inventory. The same two benefits apply in our model. Even if the orders were forced to be the same as for firms with no trade option, firms with trade option can trade their inventory to better match the market size. Additionally, the firms will order quantities closer to the quantities that a centralized firm would order. In our paper we study these two effects (ordering a different quantity and trading) separately because, when markets are competitive, they may have opposite effects on profitability. We call the first one (ordering a different quantity) as "inventory pooling" and the second one (trade of inventory without changing the original order sizes) as "inventory trade."

The following example illustrates each of these effects: Consider demand function $q_i = w_i - p_i$ and market sizes (w_1, w_2) equal to (12,20) or (20,12), with equal probabilities. The purchasing cost c is 14. In no-trade scenario, the equilibrium quantity is $q_i^{N*} = 1$ with price $p_i^N = 11$ in the small market and $p_j^N = 19$ in the large market. The revenue is 11 and 19, respectively. The expected profit for both firms is 1. To see the effect of low-inventory trade, we keep the initial inventory unchanged at one, but allow firms to trade. The trade equilibrium is to allocate two units to the large market and zero units to small market, which results in the expected profit for both firms equal to 4. In the trade scenario, firms will order slightly more, with $q_i^{T*} = 1.33$ due to risk pooling effect. Their expected profit is even higher at 4.44. In the centralized scenario, two firms will order three units in total and each firm obtains a profit of 4.5.

Numerically we observe that, when the two markets are independent, profit with trade option is very close to that in the centralized scenario. In examples we examined, it is common for the trade option to capture 98% of the efficiency loss due to decentralization. This is illustrated in Figure 2.5(b).

Summary of Four Mechanisms Effecting Profits and Consumer Surplus

We have idenfied four mechanisms that drive the changes to profits and to consumer surplus when trade option exists. The first two, strategic over-ordering (to increase bargaining power) and risk pooling influence the size of the initial orders. The other two take place in the trading phase, low-inventory trading and high-inventory trading. Obviously, these mechanisms are not completely independent. For example, due to strategic over-ordering, firms order more and as a result, high-inventory trade is more likely.

Strategic over-ordering. Having an option to trade, the firms do have an incentive to order more, solely to gain bargaining power in trading phase, rather than to increase sales. As described in Section 2.4.3, over-ordering takes place when the purchasing cost is very low and the markets are competitive (either $b > 0$ or $\lambda > 0$ or both are positive).

Inventory pooling. When cost is low (high), firms with trade option order less (more) compared to no-trade firms. This behavior takes place when there exists market uncertainty.

The two mechanisms above influence order quantity. The following two mechanisms, high-inventory trading and low-inventory trading, were described in Section 2.4.3. They take place in the trading phase, after market uncertainties are resolved.

High-inventory trade. Firms trade inventory, but after the trade there is still left-over inventory.

Low-inventory trade. Firms trade inventory, but after trade there are no left-over units.

We will use all four mechanisms to explain the firm decisions and consumer surplus in the rest of the paper.

Consumer Surplus

With no competition, for symmetric firms, consumer surplus increases due to trade option, but in asymmetric settings it may decrease.

In symmetric settings, the intuitive behavior is as follows: the quantities that two decentralized firms order are equal to each other. With trade option, firms do reallocate inventory in response to market size realizations and any deviation from the equal distribution of available inventories, that does not lead to leftovers, benefits consumers (Lemma II.5).

Interestingly, this dynamics (low-inventory trade) may decrease the expected consumer surplus when firms are not symmetric, with one firm having higher expected sales than the other one. In asymmetric settings, especially when the small firm orders very few units, for many realizations of demand, some inventory is transferred from the big firm to the small firm, which results in less asymmetric allocation and leads to lower consumer surplus compared to no-trade option.[9]

Now, let us take in consideration that the initial order quantities are not the same in trade scenario and in no-trade scenario. In trade scenario, risk pooling brings the decentralized inventory closer to the centralized one, but this means that firms with trade option may order either more or less, compared to no-trade scenario, depending whether cost is high or low. In independent market case, ordering more always increases the consumer surplus, while ordering less hurts the consumer surplus. Since higher inventory benefits the consumers in independent markets, the combined effect of low-inventory trade and risk pooling, increases the consumer surplus in most cases, when firms trade inventory.

Interestingly, the consumer surplus in the trade scenario may even be higher than that in the centralized scenario, see Figure 2.5 (c): when cost is small the firms order more than in centralized case and are able to reallocate the inventory after market sizes are observed selling to more consumers at lower prices. Since the firms' profits are almost as high as in the centralized case, trading units between decentralized firms

[9]To illustrate this phenomenon, consider the extreme case where small firm faces very low market size (say zero) with high probability and non-trivial market size with small probability. Let the cost be high enough so that the small firm would order zero units, while the large firm has large enough market size and places a positive order. In this case, the inventory trade takes place only when the small firm faces positive market realization. If the firms trade, the allocation becomes less asymmetric and, as discussed above, consumer surplus is lower.

may, in such cases, be more desirable from consumers' viewpoint than centralization.

Summary of Stochastic Model - Independent Markets

To summarize, the best candidate for collaboration is when the markets are independent and, thus, the competitive pressures due to pricing or future market share do not exist. The trade allows the firms to adjust their inventories (low-inventory trade) after uncertainty is realized according to their needs (market sizes), which benefits the firms. The firms are able, to obtain further benefits by adjusting their initial order levels through risk pooling. Both firms and consumers are better off with trade in symmetric markets, but when markets are very asymmetric, consumers may be worse off while firms are still better off.

2.5.2 Dependent Markets

Market dependence and reputation effects are reflected through $b > 0$ and $\lambda > 0$, respectively. In the general model with both market dependencies and market uncertainty, it is very challenging to characterize the equilibrium analytically. To make the analysis more tractable, we assume that the market sizes are independent random variables and follow the same uniform distribution ($w_i \sim [l, u]$). We characterize the equilibria for each of the market dependencies (one at a time) in Theorems II.9 and II.10.

Theorem II.9. *Assume that there exists short-term market dependence but no reputation effect ($b > 0, \lambda = 0$).*
(1) Centralized scenario: There exists a unique optimal solution K^{C*}.
(2) No-trade scenario: If $a \geq 1.24b$ and $u \geq 4l/3$, there exists at least one pure strategy equilibrium (q_1^{N*}, q_2^{N*}).
(3) Trade scenario: If $c \geq \underline{c}(a, b, l, u)$, there exists at least one pure strategy equilib-

rium (q_1^{T*}, q_2^{T*})*, where*

$$\underline{c}(a,b,l,u) = \begin{cases} \frac{au-al+bu}{2(a^2-b^2)}, & \text{if } u \leq 3l \\ \frac{(7a+5b)u^3-(27a+3b)lu^2+(45a-21b)l^2u-(33a-27b)l^3}{12(a^2-b^2)(u-l)^2}, & \text{if } u > 3l \end{cases}.$$

The conditions in Theorems II.9 are not very demanding. For example, the condition in Part (2) is easy to interpret. It requires the price sensitivity not to be very large ($b \leq a/1.24$) and existence of some uncertainty (upper bound and lower bound of market realizations are at least $\frac{1}{7}$ of the mean). Our numerical results suggest that even when the necessary conditions are not satisfied, there always exists at least one pure strategy equilibrium in both trade and no-trade scenarios.

Now we consider the case with the reputation effect.

Theorem II.10. *Assume that there exists reputation effect, but no short-term market dependence* ($b = 0, \lambda > 0$)*.*
(1) Centralized scenario: There exists unique optimal ordering quantity K^{C*}*.*
(2) No trade scenario: There exists a unique equilibrium (q_1^{N*}, q_2^{N*})*. Furthermore,* q_i^{N*} *is the dominant strategy for firm i (that is the value of* q_i^{N*} *does not depend on* q_j*).*
(3) Trade scenario: There exists at least one symmetric pure strategy equilibrium (q_1^{T*}, q_2^{T*})*.*

Theorem II.10 only requires symmetry of markets and it continues to hold when market distributions have general distributions and are possibly correlated.

The general model ($b > 0, \lambda > 0$) is analytically intractable. Therefore, the existence of equilibrium in a joint model ($b > 0, \lambda > 0$) and the properties of equilibria have to be tested numerically. Based on extensive numerical study, in both trade and no-trade scenarios, the equilibrium always exists (even when the conditions imposed in Theorems II.9 and II.10 are not satisfied). In order to understand how robust the effects identified in the previous sections are, we conduct the comparative statics and study profit and consumer surplus behavior through an extensive numerical study. In our study all results assume that two firms are symmetric until we explicitly relax this assumption at the end of the section.

2.5.3 Dependent Markets - Comparative Statics

We are primarily interested in understanding the effect of trading on firms' profits and consumer surplus. We use four mechanisms introduced earlier to explain both profits and consumer surplus. We first describe the effect of cost and market variability. Then, we focus on the role of competition, by looking at the effect of substitution (market dependence) and of the strength of reputation.

Effect of Cost and Variance

In Observation II.1 and corresponding Figure 2.6(a) we illustrate the change of profit of the firms as a function of purchasing cost and market uncertainty. The darker gray area denotes the cases where the profit or the consumer surplus in trade scenario is higher compared to no-trade scenario.

Observation II.1 (Effect of Cost on Profit)**.** *When markets are competitive ($\beta > 0$, $\lambda > 0$), both firms benefit from trade option, except:*
(a) when the cost is fairly low; or
(b) when the cost is very high (but not so high that the firms sell nothing) and there is strong reputation effect.

Point (a) of Observation II.1 effectively mimics the logic of the deterministic case, described in Theorem II.3: when cost is low, the firms are worse off. For the area when the firms are worse off (light gray area in Figure 2.6), over-ordering is the dominating effect and high-inventory trade only partially eliminates (decreases) the disadvantage of over-ordering. However, for super low costs, we have a different outcome: While

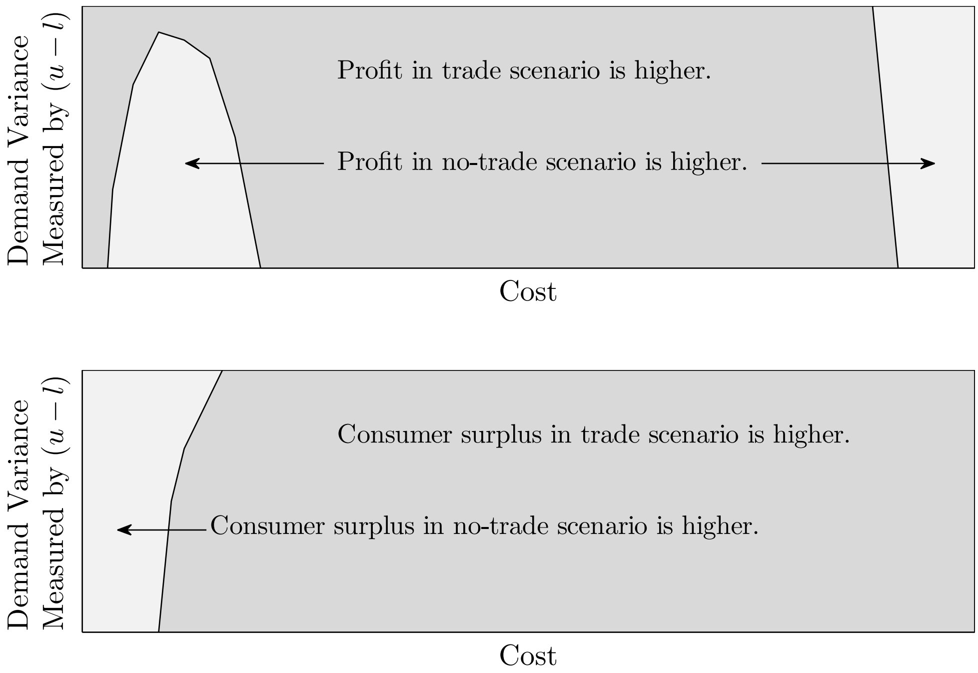


Figure 2.6: Profit(a) and consumer surplus(b) as a function of cost and demand variance.

over-ordering persists, the cost of over-ordering is not high, and the firms continue to benefit from trade.

Observation II.2 (Effect of Cost on Consumer Surplus)**.** *When markets are competitive ($\beta > 0, \lambda > 0$), consumer surplus increases due to firms having trade option, except when the purchasing cost is very low.*

Observation II.2 is illustrated in Figure 2.6(b). The behavior is driven by forces described in Sections 2.5.1 and 2.5.2. When cost is very low, the high-inventory trade effect is strong, which helps firms but drives consumer surplus down. Otherwise, the low-inventory trade is the dominating effect, which increases consumer surplus, while also helping firms.

Combining Observations II.1 and II.2, inventory trade contract is a win-win solution for firms and consumers when cost is in the moderate range. Based on our extensive numerical analysis, the results described in Observations II.1 and II.2 and shown in Figure 2.6 are consistent across all parameters we tested.

Observation II.3 (Effect of Variance)**.** *When cost is moderate (win-win for the firms and consumers), the higher the variance, the more firms and consumers benefit from trade.*

This is an intuitive outcome. When cost is moderate, the main impact is from risk pooling and low-inventory trade. While pooling (adjustment of ordering quantities) may slightly help, the main driver is frequent low-inventory trade, which benefits both firms and consumers. High variance makes the low-inventory trade occur more often.

Effect of Competition

Market dependence, β and reputation, λ directly influence ordering quantities and, consequently, the trading phase. When either β or λ increases, the competition intensifies and decentralized firms typically order more, as seen in Figure 2.7, and then lower the prices, which results in lower profits. Consumers benefit from more intensive competition and consumer surplus increases. These behaviors are consistent with those in the deterministic scenario.

Only exceptions to the monotonicity of the initial order quantity in β and λ are similar in nature to those in the deterministic setting, where firms switch from high equilibrium to low equilibrium. However, instead of a jump, in stochastic case the order quantity may smoothly decrease over a narrow range.

Below we characterize the effect of β and λ on firms' profits and consumer surplus.

Observation II.4 (Effect of Competition - β and λ)**.**
(1)(Size of Benefit for Firms and Consumers) When cost is in moderate range (win-win takes place), the benefit of trade for the firms (firm profits) and consumers (consumer surplus) shrink as β increases.[10]

[10]The firms may be worse off when cost is very high. This observation applies to the moderate range of costs.

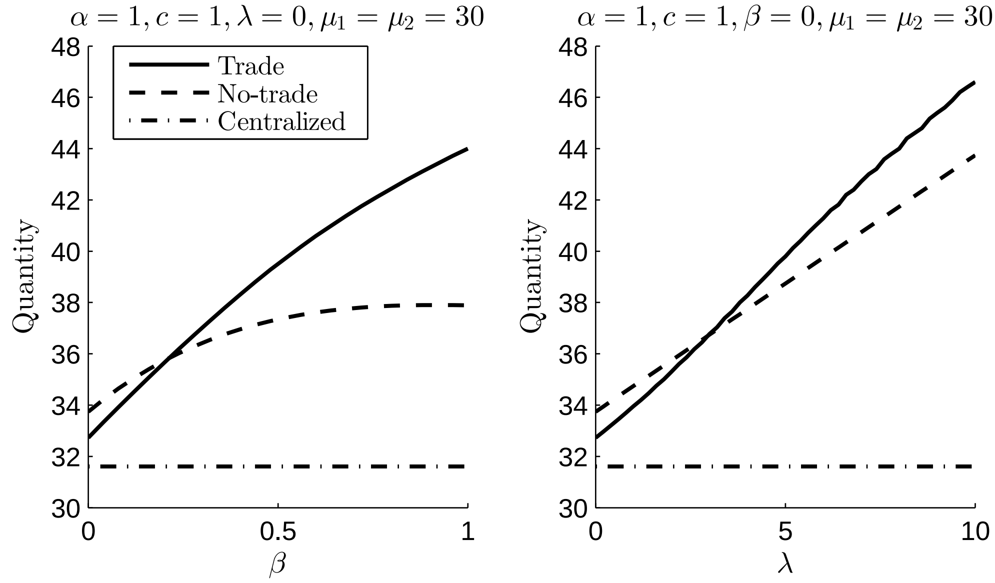


Figure 2.7: Comparative statics in β and λ.

(2)(When Firms Prefer Trading) As β increases or λ increases, the better-off (for firms) region becomes smaller.

The driving force for Observation II.4(1) is quantity ordered, as illustrated in Figure 2.8. When β increases, decentralized firms (with or without trade) tend to order more. Consequently, the benefit of trade is smaller, as firms can usually satisfy their demands using their own inventory. The consumers benefit from more intense competition, but as quantity ordered increases, the incremental benefit of trading decreases.

Part (2) expands on part (1): if the benefit of trading decreases in β, then the region when trade is beneficial also shrinks. The effect of reputation (λ) is similar. With higher λ the firms also tend to order more – they compete to gain future profits. When either of these forces (corresponding to β and λ) increases, the order quantities increase and high-inventory trade takes place more often, which lowers the consumer surplus, as firms choose to have leftovers. However, as we describe below, at the same time the prices the firms charge keep declining.

We also explicitly compare prices for trade and no-trade scenarios below.

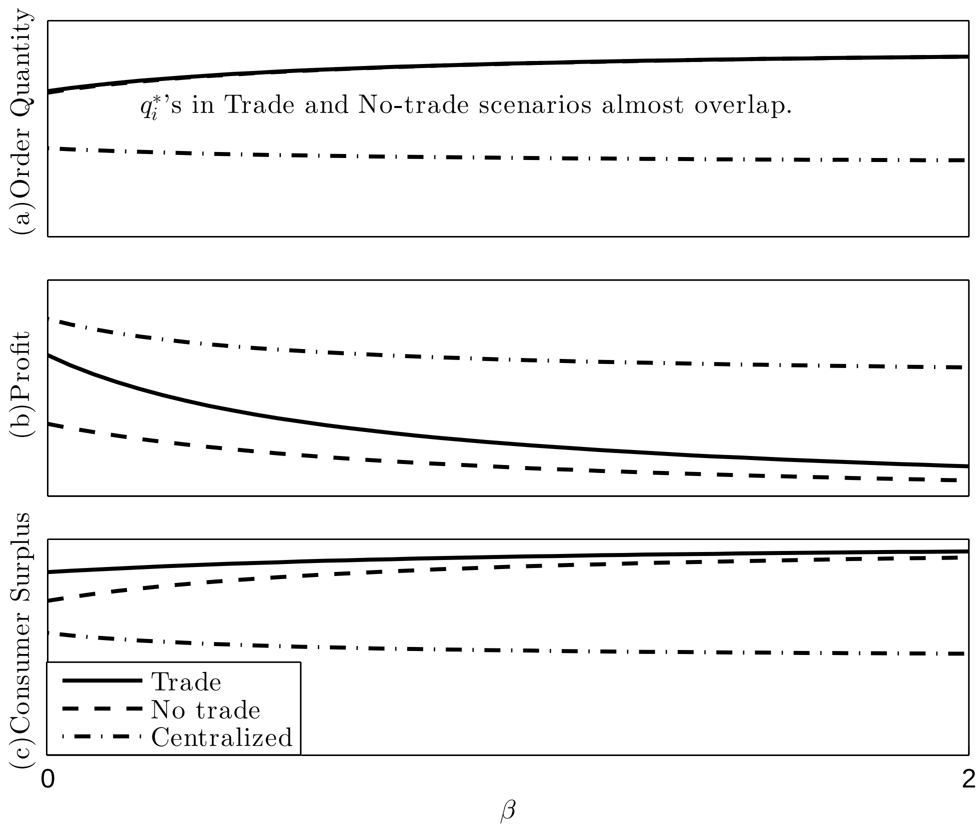


Figure 2.8: Effect of β. Parameters: $w \sim U[10, 40]; \alpha = 1; c = 20; \lambda = 10; \gamma = 0.5$.

Observation II.5 (Price).

(1) In independent markets, the average selling price in trade scenario is always less than or equal to the average price in no-trade scenario;

(2) In dependent markets, the average selling price in trade scenario is usually higher (lower) than the average price in no-trade scenario when the cost is low (high).

Recall that the trade option shifts quantity upwards (downwards) when purchasing cost is high (low). When cost is high, the order level of no-trade firms is low, but trade firms order slightly more than no-trade firms.

In independent market, since trade firms order more than no-trade firms, their prices should be even lower than the no-trade firms, reinforcing the decrease in prices. When cost is low, the inventory level is high but trade firms order slightly fewer units.

Although lower inventory may push prices to levels that are higher than no-trade firms' prices, this effect is secondary and it is dominated by that the trade firms have fewer leftovers (as explained above) and lower prices.

In dependent markets, trade firms provide lower prices for moderate to high cost for the same reason as in independent markets. When cost is low, the high-inventory trade effect matters and makes prices higher.

2.5.4 Asymmetric Markets

Now we relax the symmetry assumption. We focus on the case when two firms can have different expected market sizes. In asymmetric settings equilibrium is not guaranteed. We, therefore, numerically test how prevalent such behavior is. Three key parameters are $b \in \{0.1, 0.2, 0.4, 0.6, 0.8\}$, $\lambda \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $c \in \{0.1, 1, 5\}$. Other parameters are fixed at $\mu_1 = 10, \mu_2 = 2, a = 1, \gamma = 1$, resulting in total of 150 combinations. This case is intended to illustrate a huge asymmetry in market sizes (five to one). Equilibrium exists in 117 of combinations (78%) in no-trade scenario, and 126 combinations (84%) in trade scenario. However, if market size has 10% standard deviation, the statistics increase to 91% and 92% in no-trade and trade scenarios. To see the effect of asymmetry, we consider more moderate $\mu_2 = 4$. In such a case, equilibrium is even more likely to exist. With deterministic market sizes, equilibrium is found in 91% and 93% in no-trade and trade scenarios. With 10% standard deviation, in 100% and 97% of cases, respectively. Thus, the equilibrium may not exist, but this tends to happen with all of the following factors taking place at the same time: two markets are very asymmetric, market uncertainty is very low, and also both b and λ are large. (If either $b = 0$ or $\lambda = 0$, the equilibrium always exists.) Therefore, we are able to conduct a numerical study to compare who benefits from trade in asymmetric settings for large set of relevant parameters.

Observation II.6 (Asymmetric Market Sizes - Effect of β)**.** *When $\beta > 0$, the large* *firm benefits less (gets hurt more) in absolute terms from the inventory trade than the small firm does.*

Observation II.6 is illustrated in Figure 2.9. The x and y axis are the expected market sizes of firms 1 and 2. Firm 1 is better off with the trade option in the dark gray area, while otherwise it is worse off.

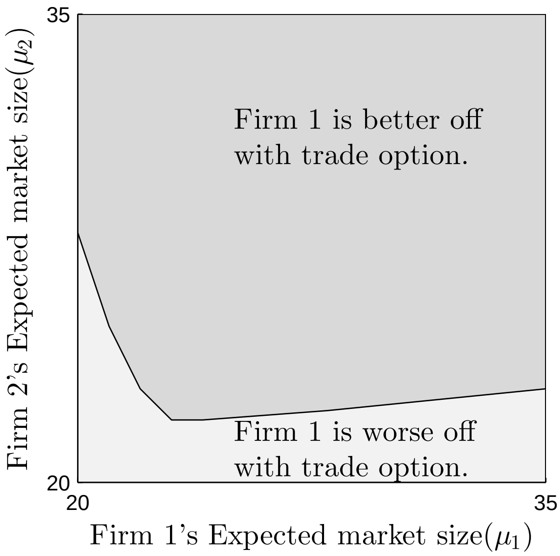


Figure 2.9: Effect of asymmetric market sizes. Parameters: $w_i \sim U[\frac{3}{5}\mu_i, \frac{7}{5}\mu_i]; a = 1.2; b = 0.2; c = 30; \lambda = 10; \gamma = 1$.

The following numerical example explains why this happens. Consider a deterministic case with the following parameters $\alpha = 1; \beta = 1; \lambda = 0; w_1 = 20; w_2 = 30; c = 8$. Equilibrium in the no-trade scenario is $q_1^N = 7.33$, $q_2^N = 10.67$, $\pi_1^N = 35.85$, $\pi_2^N = 75.85$, $p_1 = 14.89$, $p_2 = 17.11$. Equilibrium in the trade scenario is $q_1^T = 7.75$, $q_2^T = 10.25$, $\pi_1^T = 37.32$, $\pi_2^T = 74.84$. The inventories and prices after trade are $\bar{q}_1 = 6.5$, $\bar{q}_2 = 11.5$, $p_1 = 15.17$, $p_2 = 16.83$. Clearly, the large market has higher margin, which implies higher selling price. The small firm, therefore, can sell inventory to the large firm at a price higher than its purchasing cost and, anticipating this outcome, the small firm intentionally orders more than it needs. The large firm facing such a situation chooses trade with the smaller firm for the following reason: The large firm is aware that typically it will buy inventory from the small firm, but if it refused to trade, both firms would have to lower their prices. The large firm, with higher volume of sales, would suffer more and actually, the cost of buying (unnecessary) inventory may often be lower than the loss of profit due to decreased price. We observe this behavior for most parameters.[11]

Observation II.7 (Asymmetric Market Sizes - Effect of λ)**.** *When $\lambda > 0$ and the small firm's order quantity is small, the small firm benefits less (gets hurt more) in absolute terms from the inventory trade contract than the large firm does.*

The above dynamics is driven by small order quantity for the small firm in the trade scenario. In such a case, the inventory of the small firm is not threatening to the large firm. The large firm can order more and it benefits due to risk pooling (when cost is high risk pooling means ordering more). Effectively, the large firm can play a strong hand. It orders more and, thus, can satisfy more (in expectation) of its own customers. Moreover, the increased quantity that the large firm orders can be sold to the small firm (when the market realization of the small firm is high). The small firm may benefit from trade, when its market realization is high (satisfying its own demand and decreasing reputation losses), but the benefit is small since the small firm is charged high unit price for the traded inventory.

When both β and λ are positive, combined effect of the two observations above is illustrated in Figure 2.9. We see that in extremely asymmetric settings, trade rarely benefits both large and small firms, and the more symmetric the firms are, the more often both firms benefit from trade.

Throughout the analysis of asymmetric firms, we have kept here the assumption that both firms share benefit of collaboration equally, as this is the dominant model

[11] In some extreme cases, the balance of benefits may change, but it requires cost of purchase c to be very high, reputation effect λ to be strong, as well as market uncertainty to be very high.

for Nash bargaining game. However, in addition to market sizes, one may consider another source of asymmetry that comes from unbalanced bargaining powers, i.e., one firm gets a larger portion of the benefit than the other. In such cases, the less-powered firm has little incentive to engage and, thus, firms are less likely to reach an agreement to collaborate. In the extreme case, a firm who gets zero benefit will not participate in any inventory exchange.

Summary of Stochastic Model - Dependent Markets

In the last two subsections, we considered the firms that effectively compete with each other either to increase the current-period profits or future market share. We have consistently seen that the cost plays a pivotal role for both the firms and consumers. For intermediate range of costs, trading may benefit both the firms and the consumers. This is when risk pooling and low-inventory are the two dominant effects. Intuitively, one would expect that one of these parties (the firms or the consumers) should be better off. Our results indicate that both are better off. When cost is very low, the over-ordering may hurt the firms and high-inventory trade may also hurt the consumers. This indicates low potential for trading in goods with very high margin. For high costs, the consumers always benefit from trade, but when the reputation effect is very high, the firms typically are worse off. When market sizes are asymmetric the benefit for the firms decrease, compared to the symmetric case. We may see either one or even both firms worse off, suggesting that the trade is more likely between firms of similar sizes.

Regulators routinely take the consumers' point of view when deciding about legality of business practices. We see that in the overwhelming majority of situations, consumer surplus increases. Only at very low costs, the consumer surplus decreases. Thus, from point of view of competitive forces and reputation effects, this model would indicate fairly broad endorsement for allowing firms to collaborate through trade.

2.6 Conclusion

This paper studies the question whether and when companies should collaborate with other firms. We focus on a specific type of collaboration through inventory or capacity trade contracts. The expected benefits of collaboration are intuitive: the firms should be able to improve resource utilization and are expected to increase their profits. However, it is also expected that there might be negative externalities, or potential drawbacks. These may come in both short term and long term. In short term, if two markets are "dependent," where the products sold by two firms are partially substitutable, the firms compete through pricing. The potential for selling goods to competitor provides an incentive to increase initial orders and eventually leads to more aggressive pricing. Also, selling inventory to competitor may translate into a more-permanent shift of consumers from one firm to another, influencing future market shares. Combination of short-term and long-term dynamics makes it difficult to assess whether the benefit of inventory collaboration is net positive and if so, when this is the case. It is also critical whether the net benefit for consumers is positive, as consumer welfare is at the core of all anti-trust decisions. While the current antitrust law uses clear principles, current practices indicate that most of the collaborative practices fall in the area where the verdict is based on the examination of gains and losses of firms and consumer welfare.

Our paper considers a simple model that includes both short and long term effects. We show that when markets are deterministic (or very close to being deterministic), the firms do not have an incentive to get involved in inventory trade (except in the case of extremely low costs). The demand in such cases is predictable and the firms can order the needed quantities. If firms open themselves to a partnership that involves trading, they actually may get worse off, as they create new incentives to build excess inventories. Agreements in such situations, while unlikely, would lead to increase of consumer welfare.

If companies face uncertain markets, the inventory trade contract is more promising and the effects may be quite significant. In markets which are fairly independent, companies always benefit (obtain higher profits), when they trade inventory. Moreover, in such situations, the consumer surplus is also higher, except when one market is larger than the other by a very significant margin. These benefits are driven by standard inventory pooling and inventory adjustment practices.

When competition is present, firms of similar sizes often benefit from trading inventory. They are worse off only when the purchasing cost is either very low or very high. Both very low or very high costs lead to excessive inventories compared to no-trade case. The trade contract increases consumer surplus in majority of cases, as long as the purchasing cost is not very low. Therefore, the trade contract ends up being a win-win solution when the purchasing cost is in the moderate range. Market uncertainty increases the benefits of trade to both the firms and also to the consumers. When firms face significantly asymmetric markets, either one or both of the firms may be worse off due to trading. Consequently, the firms are unlikely to collaborate in such markets. Our model confirms behavior observed in practice, where we do not see many firms of dramatically different sizes being engaged in any type of inventory exchange. It also provides a more precise tool from legal point of view highlighting and, possibly eliminating the cases where consumer surplus is increased versus threatened.

CHAPTER III

Minimum Advertised Price Policy: Economic Analysis and Implications

3.1 Introduction

As technology advances, it has become unprecedentedly easy for shoppers to collect price information from different retailers. Due to the proliferation of online retailers and discount stores, many brick-and-mortar retailers suffer from eroded margins. Customers are able to experience the product in one store but make the purchase from another retailer, which offers lower price but does not provide product demonstration or auxiliary services. Brick-and-mortar retailers typically cannot match the price of low-cost competition, since they incur higher overhead costs due to higher rent, number of employees who provide in-store assistance, and advertising. As a result, brick-and-mortar retailers stop promoting or even carrying products which are involved in such price competition that results in lower profits. For manufacturers, however, brick-and-mortar retailers are an important channel through which products are showcased and promoted to customers. Low-cost retailers are not capable of playing this role due to their lack of resources such as space and service personnel. Therefore, manufacturers would have hard time reaching to a large market without brick-and-mortar retailers which invest resources into demonstrating and advertising their products.

Resale Price Maintenance (RPM) policy is a widely used mechanism in many industries including patent medicine, electronics and fashion. It can help manufacturers control retail prices and thus protect the margin from being eroded. Under RPM policy, the manufacturer sets a minimum price for the product and requires all retailers not to price below it. Since the Supreme Court ruled to judge RPM under a rule-of-reason standard rather than being per se illegal in 2007 (see Gundlach 2010, for the legal status of RPM), an increasing number of manufacturers have embraced the RPM policy. Now RPM policy can be found in many product categories from toys and electronics to fashion and home improvement. For example, Tarr (2014) indicates Sharp uses RPM policy to price some of its high-end televisions. When the authors of this article checked the price of the popular 80-inch TV (model number: 80UQ17U), it was retailing for the \$3999 (as of Feb 26, 2015) in all reputable retailers, including BestBuy, Sears, Amazon.com, etc. Sales agents at Sears (as well as Sears.com) even indicated that additional discounts did not apply to that particular television by the following statement: "Due to high levels of quality, style and performance, the price is set by manufacturer and additional discounts do not apply." The standard justification given by the practitioners and the existing literature for manufacturers' use of the RPM policy is that by setting the minimum selling price, brick-and-mortar retailers' margins are protected and they are no longer threatened by their low-cost competitors and, thus, can spend more effort on consumer acquisition. Meanwhile, the price restraint also creates a barrier for low-cost retailers to compete. As consumers expect price to be the same in all retailers (in a perfectly competitive market), they tend to make purchase at brick-and-mortar retailers where they can test the product before purchase and enjoy better customer service. Therefore, RPM policy tends to benefit manufacturers and brick-and-mortar retailers at the expense of low-cost retailers.

Recently, another vertical price restraint mechanism, Minimum Advertised Price (MAP) policy, has gained increased popularity. Under MAP policy, manufacturers set a "suggested" retail price. Retailers can sell at any price, but they are not allowed to advertise prices lower than the MAP price. To be precise, retailers cannot list any price lower than MAP price next to the product on either their catalogue or website. Instead, they may ask customers to "call for price" or "click for price." MAP was originally used as a mechanism to decrease the change of legal action (MacKay and Smith 2014): Before Supreme Court's decision in 2007, RPM was ruled as per se illegal, while MAP was not. Use of MAP provided additional legal flexibility while achieving similar outcome. The 2007 decision caused a significant shift, as both RPM and MAP are considered "legal" and, thus, to the major obstacles to their implementation disappeared.The volume of research papers and business articles referring to MAP and RPM has increased and most noticeably, several legal firms promote the use of MAP and RPM by offering advice how to use it. The authors' experience is also that we see an overwhelming increase in click/email-for-price phenomenon, which might be influenced both by its legality and by the fast growing Internet-based retail. MAP may be implemented in multiple ways. For example, one Canon lens (EF-S 17-55mm f/2.8 IS USM) is listed at \$879 (as of Feb 26, 2015) across all authorized distributors. One retailer states that lower price is available but cannot be listed due to Canon's pricing policy. One of the authors filled out a price request form and emailed this authorized distributor and was quoted a price of \$819 for exactly the same lens. While obstacles to implementing MAP and RPM effectively disappeared, it seems that the use of MAP has experienced unparallel growth. E.g., iIt is difficult to find a website with no "click-for-price." It may still be argued that MAP is easier to defend or implement, we instead focus on economic benefits of choosing one versus the other.Managers have argued whether MAP policy benefits only brick-and-mortar stores at the expense of low-cost retailers, or it is beneficial to both. Under MAP those consumers who value their time more than others would buy in brick-and-mortar retailers rather than putting forth effort to search for a lower price. Other consumers, who have more time to search, can experience the product in brick-and-mortar retailer but eventually buy from low-cost retailers.

The debate between RPM and MAP policies exists not only among managers but also among legislators, who mostly focus on the impact of price restraining policies on consumer surplus. Some legislators point out that RPM policy makes brick-and-mortar retailers more profitable, which then enables them to provide better service to consumers. But others have argued that MAP policy makes available lower prices and more options to consumers. So far there is no clear answer to which policy is more of consumers' interest.

In this paper, we build a stylized model to study the performance of RPM policy and MAP policy under various market situations. In our model, we consider one manufacturer supplying one product to two retailers. The manufacturer first chooses between RPM policy and MAP policy, and then sets the wholesale price and the retail price. One of the retailers models brick-and-mortar retailers that are able to generate demand through advertising, in-store assistance and other supporting customer services. The other retailer represents low-cost retailers who do not generate demand but serve consumers who are willing to search for the low price offered by these retailers. We call the first type the *regular retailer* or *retailer 1*. The second type is referred as the *free rider* or *retailer 2*. We also use *retailers* to refer to both of them. Under RPM policy, both retailers sell at the same public price suggested by the manufacturer. Under MAP policy, the regular retailer sells at the price suggested by the manufacturer, while the free rider offers a lower price which can be found at a cost by consumers. Consumers are assumed to be heterogeneous in their incomes. Their valuation of the product and their time value are both associated with their income level. They buy from whichever retailer offers a higher nonnegative net utility. In this context, we ask the following research questions: (1) Which policy, RPM or MAP, performs better for the manufacturer? (2) Under which policy the retailers earn higher profit? (3) Under which policy consumers have higher surplus? (4) If any party (typically the manufacturer) in the supply chain is able to contract the required search cost leading to the free rider under MAP policy, what would be the optimal search cost?

We find that there is no dominant policy for the manufacturer. MAP policy outperforms when the customer's valuations are very heterogeneous and the search cost is high. In general, the regular retailer serves high-end customers, while the free rider serves low-end ones. With high heterogeneity and high search cost, the manufacturer effectively expands the market and segments it, without losing surplus from high-end customers. In other situations, under MAP the regular retailer loses significant number of customers to the free rider and, thus, has little incentive to create demand. RPM policy is, in such situations, more profitable for the manufacturer. In terms of the retailers' preference, not surprisingly, RPM policy is never preferred by the free rider as the free rider earns zero profit under RPM while MAP policy allows the free rider to compete. Interestingly, the regular retailer also prefers MAP over RPM when customers' valuations span a large range and the search cost is high. Under MAP policy, the presence of free rider encourages the manufacturer to lower the wholesale price to increase the market share and the lower wholesale price benefits the regular retailer. Consumers benefit from MAP policy when the valuations have either very small or very large variance. When the variance is very large, MAP policy allows the free rider to serve low end customers, who would not be served otherwise. When the variance is very low, the manufacturer and regular retailer can take away most surplus under RPM policy by setting retail price close to every customer's willingness to pay. Under MAP policy, however, the customers buying from the free rider continue to enjoy a significant surplus. Finally, we find that the manufacturer, retailers even consumers prefer strictly positive search cost which enables the market segmentation.

3.2 Literature Review

This work is closely related to the research on vertical restraints. Among various types of vertical restraints, the most popular ones are franchise fees, quantity forcing, closed territory distribution, and price restraints. Highly cited (Mathewson and Winter 1984) describes the landscape of vertical restraints. More recent paper (Rey and Verge 2008) approaches the same topic from practical (legal) point of view and identifies the same types of restraints.Franchise fees are a payment of a fixed fee on top of any variable purchase cost. Quantity forcing is a provision in which-manufacturer mandates a minimum/maximum purchase quantity. Closed territory distribution specifies a geographical area that each retailer is allowed to serve. Many of the above mechanisms may achieve channel coordination in their settings, but they attempt to overcome different frictions. Franchise fees are eliminating (or in practice decreasing the effect of) double marginalization. Quantity forcing may be used for multiple reasons, but often they also enforce minimum purchase thus, again, overcoming the smaller purchase due to higher price (double marginalization). Closed territory distribution is used mostly to protect franchisees or other retailers in order to guarantee minimum profit for them. Price restraints may be the broadest among these categories. We focus on price restraints in a market with heterogeneous customers and evaluate how use of RPM or MAP may help the manufacturer. Most of the other papers use Hotelling setting with otherwise homogeneous customers and could not (at least easily) apply to settings with dramatically different delivery channels. Therefore, instead of broadly considering all types of vertical restraints, this paper focuses on and compares the two major mechanisms of price restraints - RPM and

MAP. Commonly used by manufacturers, RPM has been studied for a few decades. Telser (1960), Marvel and McCafferty (1985), Mathewson and Winter (1998) and Klein (2009) are among the papers that qualitatively analyze pro-competitive and anti-competitive effects of RPM in different historical periods. There is also a volume of theoretical literature on this topic – an insightful summary can be found in MacKay and Smith (2014). In this literature, a number of authors emphasize that RPM is intended to improve non-contractible service by restricting price competition. Mathewson and Winter (1983, 1984) consider a non-contractible service in the form of advertising effort, where only customers informed by the advertising effort make purchases. However, as a result of information spillovers, informed customers may purchase goods or services from firm other than the one investing in advertising and educating consumers. Mathewson and Winter (1983) studies RPM and models consumers' heterogeneity in search cost, which allows discount retailers to free-ride the service offered by advertising retailers with a lower but hidden price. They suggest RPM be used to eliminate the service free-riding and, thus, to improve the manufacturer's profit. In Mathewson and Winter (1984), consumers vary in their distance to the retailers, but do not search. With both information spillover and imperfect price competition, retailers tend to price lower than optimal and invest too little in the service. The paper shows that RPM, along with a fixed fee (such as a franchising fee), leads to supply chain coordination and achieves joint profit maximum. Winter (1993) interprets the service as the in-store assistance that enhances shoppers' experience. Consumers are heterogeneous in their locations (in a Hotelling framework) and also in their valuations of in-store assistance. Similar to Mathewson and Winter (1984), without any price restraints, retailers price too low compared to the optimal price for the whole channel and underinvest in service. The paper shows that RPM can correct this distortion. Other papers such as (Marvel and McCafferty 1984, Bolton and Bonanno 1988, Perry and Porter 1990) using different models also show that when retailers compete in both price and non-price attributes, RPM helps recover the supply chain from setting sub-optimally low prices.Our RPM model follows the spirit of economics literature by considering the same forces as most RPM papers. It may be interpreted as special case of Mathewson and Winter (1984), with one difference in that the manufacturer does not charge fixed fee to retailers. The fixed fee actually does not change the conclusions of Mathewson and Winter (1984) and could be included in our model as well. However, our focus is on comparison with MAP, which is not modeled in Mathewson and Winter (1984) or in the later papers that build on Mathewson and Winter (1984). Including the fixed fee would obviously complicate our analysis, without necessarily changing the insights. We confirm through numerical study that the results, conclusions, and the insights in this study continue to hold in the more general setting.

The literature on vertical price restraints primarily focuses on RPM rather than MAP. To our knowledge, there are only two analytical studies, Kali (1998) and Cetinkaya (2009), that differentiate MAP from RPM. Kali (1998) employs the same spatial demand model as in Winter (1993) but extends it by including retailers' advertising decision and manufacturer's subsidy decision. Cetinkaya (2009), on the other hand, models multiple retailers whose advertising effort has positive externality for other retailers. Regardless of the modeling choice, both papers define MAP as a version of RPM where the manufacturer subsidizes retailers' advertising expense. They show that a combination of RPM and the subsidy can maximize the channel profit, while RPM individually is insufficient. Both papers assume that any of the retailers cannot sell at lower-than-MAP price even it is not advertised. In our paper, however, we model MAP exactly as implemented in practice, i.e., retailers under MAP policy can price lower than the manufacturer suggested price but cannot advertise the low price. MacKay and Smith (2014) studies both RPM and MAP (as defined in practice) across multiple products empirically. It shows that there are observable differences between products sold under RPM and MAP policies in terms of prices and quantities sold. Our paper, to our knowledge, is the first attempt to formally model the free-riding behavior and is the first theoretical work that predicts that existence of free riders may be beneficial to manufacturers under certain circumstances.

RPM and MAP are used to increase retailers' sales effort, and often also to restrain prices. Sales effort is also studied in contexts different than vertical restraints. We describe these papers below, even though none of them deals with the central questions of our paper of manufacturer choosing between RPM and MAP contracts. Taylor (2002) and Krishnan et al. (2004) consider supply chain coordination with sales effort. Iyer (1998) and Tsay and Agrawal (2000) focus on the supply chain dynamics when a manufacturer sells through two competing retailers, both of which have to decide their effort levels. Sales effort may be in form of dissemination of product information to attract consumers. Butters (1977) models multiple sellers, who send their price to random consumers at a cost. Grossman and Shapiro (1984) and Soberman (2004) extend this model by introducing spatial differentiation of consumers and study the impact of market competition. Iyer et al. (2005) studies advertising strategy that allows competing firms to target advertising to different groups of consumers within a market. All the above papers assume that the consumers are passive (they do not search for alternative sellers) and focus, instead, on characterizing the equilibrium strategy of retailers: advertise or not advertise. In our paper, consumers actively search for price information incurring a search cost and we focus on manufacturer's choice of RPM and MAP.

The search behavior is studied by Burdett and Judd (1983), Stahl (1989), Salop and Stiglitz (1977). In Burdett and Judd (1983), Stahl (1989) consumers incur a positive cost for each additional price quote, while in (Salop and Stiglitz 1977) consumers pay a one-time fee to collect price information from all retailers. A detailed review of both approaches can be found in Baye et al. (2006). These papers focus on characterizing price competition without studying the supply chain dynamics and sales effort.

A few papers study both the sales effort of retailers and consumer search. Janssen and Non (2009) studies the role of informative advertising when consumers are able to search. They show that equilibrium advertised price can be higher than unadvertised price because it spares consumers of some search cost. In Desai et al. (2010) and Iyer and Kuksov (2012), retailers have to decide not only the advertising strategy but also service levels. While advertising is solely informational, consumers gain positive utility from the service and this is unconditional on making any purchase. Their models allow consumers to enjoy the service at one location but purchase at the other, with additional search cost incurred. Though our model combines retailers' sales effort and consumer search as well, the focus is on the manufacturer. The advertised price and advertising strategy are both determined by the manufacturer rather than retailers as assumed in papers listed above. While sales effort is well studied, our paper is the first to characterize MAP with a consumer search model and we focus on manufacturer's perspective rather than retailers'.

3.3 Model and Preliminary Results

We study a supply chain composed of a manufacturer and two retailers: a brick-and-mortar retailer and an online/low cost retailer. The brick-and-mortar retailer (regular retailer) can generate demand through activities such as advertising, in-store assistance and other supporting customer services. The free rider do not generate demand but can serve the consumers informed by the regular retailer. The sequence of events is as follows.

Stage 1: Manufacturer chooses price restraining policy: RPM policy or MAP policy.[1] Manufacturer also decides the wholesale price w and the suggested retail

[1]The manufacturer does not generate any demand. If the manufacturer does not impose price

price p in either policy. It is assumed that the manufacturer is unable to discriminate retailers through wholesale price due to either legal or economic reasons. This is a common assumption in literature studying one manufacturer supplying multiple retailers, as seen in Iyer (1998), Tsay and Agrawal (2000), Desai et al. (2010), Iravani et al. (2013).

Stage 2: Given the offering from the manufacturer, the regular retailer decides the size of the demand, denoted by θ, which is generated at a cost, $\frac{1}{2}\lambda\theta^2$. The cost includes expenses made to items such as support services, advertisement, and overhead. Other papers that study sales effort (e.g., Soberman 2004, Cachon and Lariviere 2005, Desai et al. 2010) also use similar quadratic cost functions to model the cost of sales effort. We also test other forms of convex cost structure. While analytically intractable, we confirm that all lessons in this paper continue to hold through numerical study in Section 3.6.2. Without loss of generality, we normalize λ to one. The free rider and the manufacturer are not capable of providing such services, and therefore, all the demand is assumed to be generated by the regular retailer in our model.

Stage 3: The regular retailer sells at the suggested retail price p under either policy. The free rider sells at the suggested price under RPM policy but sets its own selling price, p_2, under MAP policy.

A consumer's valuation for the product is given by $v + (1 - v)x$ where $0 \leq v \leq 1$ and x is uniformly distributed, $x \in [0, 1]$. x is the consumer's value of time which is positively correlated with the income level of a consumer. Similar to models of Janssen and Non (2009) and Iravani et al. (2013), both consumer's product valuation and time value are proportional to her income level. The implicit assumption is that the consumers' valuation of product and their value of time are perfectly correlated. The situation with imperfect correlation is considered and evaluated in Section

constraints, since the products are perfect substitutes, retailers undercut each other's price and at equilibrium the regular retailer has no incentive to generate demand. As a result, every party in this game earns zero profit.

3.6.1. Without loss of generality, we can normalize consumers' valuation between v and 1, using v to vary the heterogeneity in consumers' valuations. As v increases (decreases), the market becomes more homogenous (heterogeneous) in terms of consumer valuations for the product. If a customer, with time value x, buys from the regular retailer, she pays price p and earns a surplus of $v + (1 - v)x - p$. If this customer decides to buy from the free rider under MAP policy, she has to spend α units of time searching for the hidden price p_2. The search cost is αx, resulting in net consumer surplus $v + (1 - v)x - p_2 - \alpha x$. The consumer buys from the retailer that offers higher non-negative surplus. If both surpluses are negative, she does not buy.

With this consumer specification, the demand for each retailer can be derived. Given the regular retailer's price p and the free rider's price p_2, let $d_i(p, p_2)$ denote the percentage market share. Since the total number of consumers in the market is θ, the number of customers served by retailer i is $\theta d_i(p, p_2)$. The explicit expressions for $d_i(p, p_2)$ is presented in Table 3.1. In the rest of the paper, we simply use d_i and drop the arguments (p, p_2) unless there is any ambiguity.

Condition		d_1	d_2
$\alpha + v < 1, p \geq v + \alpha$	$p_2 \geq p - \frac{p-v}{1-v}\alpha$	$\frac{1-p}{1-v}$	0
	$p - \alpha \leq p_2 < p - \frac{p-v}{1-v}\alpha$	$1 - \frac{p-p_2}{\alpha}$	$\frac{p-p_2}{\alpha} - \frac{p_2-v}{1-\alpha-v}$
	$v \leq p_2 < p - \alpha$	0	$1 - \frac{p_2-v}{1-\alpha-v}$
	$p_2 < v$	0	1
$\alpha + v < 1, v \leq p < v + \alpha$	$p_2 \geq p - \frac{p-v}{1-v}\alpha$	$\frac{1-p}{1-v}$	0
	$v \leq p_2 < p - \frac{p-v}{1-v}\alpha$	$1 - \frac{p-p_2}{\alpha}$	$\frac{p-p_2}{\alpha} - \frac{p_2-v}{1-\alpha-v}$
	$p - \alpha \leq p_2 < v$	$1 - \frac{p-p_2}{\alpha}$	$\frac{p-p_2}{\alpha}$
	$p_2 < p - \alpha$	0	1
$\alpha + v \geq 1, v \leq p$	$p_2 \geq v$	$\frac{1-p}{1-v}$	0
	$p - \frac{p-v}{1-v}\alpha \leq p_2 < v$	$\frac{1-p}{1-v}$	$\frac{v-p_2}{\alpha+v-1}$
	$p - \alpha \leq p_2 < p - \frac{p-v}{1-v}\alpha$	$1 - \frac{p-p_2}{\alpha}$	$\frac{p-p_2}{\alpha}$
	$p_2 < p - \alpha$	0	1
$p < v$	$p_2 \geq p$	1	0
	$p - \alpha \leq p_2 < p$	$1 - \frac{p-p_2}{\alpha}$	$\frac{p-p_2}{\alpha}$
	$p_2 < p - \alpha$	0	1

Table 3.1: Market share function $d_i(p, p_2)$.

3.3.1 Centralized Supply Chain

We first model a centralized supply chain as a benchmark. In a centralized supply chain, the manufacturer chooses retailers' prices and the demand generation effort. There are three decision variables: demand generated θ, price at regular retailer p, price at free rider p_2. The manufacturer's problem is to maximize the total profit π_C, sum of revenues from both retailers minus the cost of demand generation:

$$\max_{p,p_2,\theta} \pi_C = (pd_1 + p_2 d_2)\theta - \frac{1}{2}\theta^2. \tag{3.1}$$

3.3.2 RPM Policy

Under RPM policy, both retailers' prices are set by the manufacturer at p. We assume that all customers, if they make any purchase, buy from the regular retailer when prices are the same. Given price p, the market share of the regular retailer ($p \geq v$ is assumed because pricing lower than v leaves positive surplus to consumers and the retailer can increase revenues by increasing the price at least to v) is equivalent to $d_1 = \frac{1-p}{1-v}$. The regular retailer's profit, π_1, is given by

$$\pi_1 = \theta d_1(p - w) - \frac{1}{2}\theta^2.$$

It is easy to show that the profit function is concave in θ. Solving the first order condition in θ for the regular retailer, we obtain the optimal number of customers to generate as

$$\theta^* = (p - w)d_1.$$

Finally we consider the manufacturer's problem. The manufacturer sets the wholesale price w and RPM price p such that its profit (wholesale price times the total sales to the retailers), π_M, is maximized:

$$\pi^*_{RPM} = \max_{w \geq 0, p \geq v} \pi_{RPM} = w\theta^* d_1 = \frac{w(p-w)(1-p)^2}{(1-v)^2}. \tag{3.2}$$

3.3.3 MAP Policy

Lastly we state the problem under MAP policy. We formulate the problem starting with stage 3, the free rider's pricing problem and proceed to stage 2 (demand generation) and 1 (wholesale price and suggested retail price selection) respectively.

Free Rider: p_2

At the final stage of the game, the free rider makes pricing decision given wholesale price, regular retailer's price and total consumer demand, (w, p, θ). The free rider's problem is to maximize its profit π_2:

$$\max_{p_2} \pi_2 = \theta d_2 (p_2 - w). \tag{3.3}$$

Note that θ is determined by the regular retailer earlier so it is a constant here. For different tuples of (w, p, α, v), the free rider responds in very different ways, depending on whether it competes with the regular retailer for a portion of the consumers, or whether it intends to cover the whole customer spectrum. We define 'competition' in this paper as follows: Two retailers are competing, if there exist a customer who gets strictly positive surplus buying from either retailer. According to this definition, when retailers are not competing, there may be customers who receive positive surplus when they purchase from one of the retailers but not from both retailers. Apparently the free rider's response depends on the wholesale price w and the regular retailer's price p. We first define regions in w and p and then characterize the free rider's decision, p_2^*, in each region. Regions are defined as follows (see Figure 3.1 for a graphic illustration of regions):

$$S_{11} = \{(w,p) : \alpha + v < 1, p > v + \alpha, w < 2v + \alpha - 1\},$$
$$S_{12} = \{(w,p) : \alpha + v < 1, p > v + \alpha, 2v + \alpha - 1 < w < 2p - \alpha - 1\},$$
$$S_{13} = \left\{(w,p) : \alpha + v < 1, p > v + \alpha, 2p - \alpha - 1 < w < p - \alpha - \alpha\frac{1-p}{1-v}\right\},$$
$$S_{14} = \{(w,p) : p < v + \alpha, w < p - 2\alpha\},$$
$$S_2 = \left\{(w,p) : \alpha + v < 1, p - \alpha - \alpha\frac{1-p}{1-v} < w < p - \alpha + \alpha\frac{1-p}{1-v}, w > 2v + \alpha - p - \alpha\frac{1-p}{1-v}\right\},$$
$$S_{31} = \left\{(w,p) : \alpha + v < 1, p - \alpha + \alpha\frac{1-p}{1-v} < w < p\right\},$$
$$S_{32} = \{(w,p) : \alpha + v \geq 1, v < w < p\},$$
$$S_4 = \left\{(w,p) : \alpha + v < 1, 2v - p < w < 2v + \alpha - p - \alpha\frac{1-p}{1-v}, p < v + \alpha\right\},$$
$$S_5 = \{(w,p) : \alpha + v < 1, p - 2\alpha < w < 2v - p, w < p\} \bigcup \left\{(w,p) : \alpha + v \geq 1, p - 2\alpha < w < p + 2\alpha\frac{v-p}{1-v}, w < p\right\},$$
$$S_6 = \left\{(w,p) : \alpha + v \geq 1, p + 2\alpha\frac{v-p}{1-v} < w < 2p - v + 2\alpha\frac{v-p}{1-v}\right\},$$
$$S_7 = \left\{(w,p) : \alpha + v \geq 1, 2 - 2\alpha - v < w < v, v + \frac{(1-v)(v-w)}{2(\alpha+v-1)} < p < 1\right\}.$$

In Figure 3.1, note that the regions are defined differently whether $\alpha + v > 1$ or not. A customer with time value x has product valuation $v + (1-v)x$. As x increases, consumer's valuation for the product increase as well as her search cost αx. If she buys from the regular retailer, her surplus is $v + (1-v)x - p$. If she buys from the free rider, paying p_2, her surplus is $v + (1-v)x - p_2 - \alpha x = v + (1 - v - \alpha)x - p_2$. If $\alpha + v > 1$, the search cost increases faster than the consumer's valuation in x. Therefore, the consumer surplus from the free rider is decreasing in x, i.e., the free rider attracts customers who have time value lower than the threshold, $(v - p_2)/(v +$

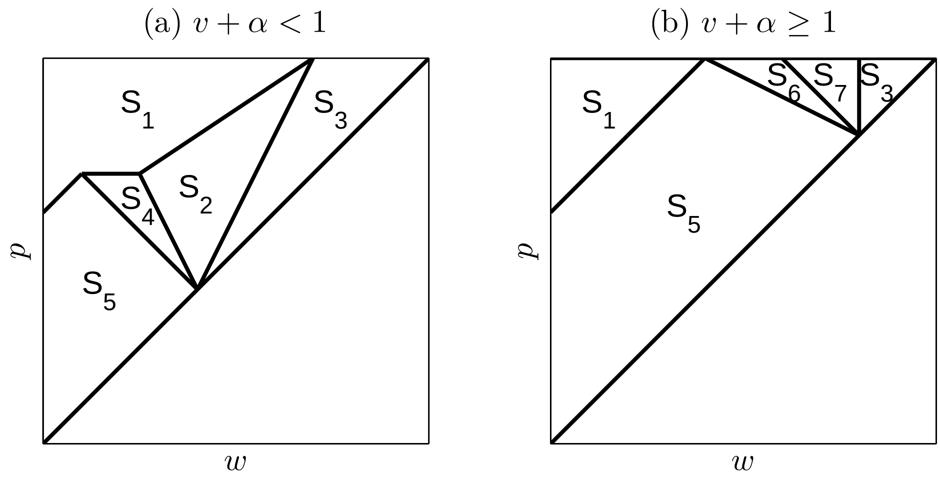


Figure 3.1: Illustration for S_i's.

$\alpha - 1$). As a result, the two retailers can be either local monopolists (customers who have medium time value are not served by either retailer) or competitors (medium time value consumers are served by one of the retailers and receive positive surplus), depending on whether customers in the middle range are served by any of the retailers. If $\alpha + v \leq 1$, the consumer's valuation of product increases faster in x than her search cost and the surplus is increasing in x. This means the free rider finds it profitable to serve customers whose time value is greater than the threshold. Therefore, high-end customers are appealing to both retailers, If the free rider stays in the market, it is in competition with the regular retailer (high-end consumers gets positive surplus from either one of the retailers).

For each region, the free rider's best response, p_2^*, is summarized in Table 3.2.[2] We define S_1 as the union of S_{1k}, $k = 1, 2, 3, 4$, because we will see later that the manufacturer's profit is zero in all four regions. Similarly, we also define $S_3 = S_{31} \bigcup S_{32}$ as they are the same from the manufacturer's viewpoint. The dynamics can be very different from one region to another and have significant impact on the manufacturer's

[2] p_2^* is derived by plugging $d_2(p, p_2)$ from Table 3.1 into its profit function (3.3).

decision. Therefore, these regions are frequently referred in both the main body of the paper and the proof. It also worth noting that all regions are defined as open sets. The p_2^* for a tuple (w, p, α, v) on the boundaries between regions are derived by taking limit from left in w.

Region	p_2^*	d_1	d_2	Characteristics
S_{11}	v	0	1	Regular retailer has no demand.
S_{12}	$\frac{1+w-\alpha}{2}$	0	$\frac{1-\alpha-w}{2(1-\alpha-v)}$	
S_{13}	$p-\alpha$	0	$\frac{1-p}{(1-\alpha-v)}$	
S_{14}	$p-\alpha$	0	1	
S_2	$\frac{w+p+\alpha v-\alpha p-pv-wv}{2(1-v)}$	$\frac{2-p-v}{2(1-v)}-\frac{p-w}{2\alpha}$	$\frac{p-w}{2\alpha}-\frac{w-v}{2(1-\alpha-v)}$	Low-end customers not served. Two retailers compete for demand.
S_{31}	$p-\alpha\frac{(p-v)}{1-v}$	$\frac{1-p}{1-v}$	0	Free rider has no demand.
S_{32}	v	$\frac{1-p}{1-v}$	0	
S_4	v	$1-\frac{p-v}{\alpha}$	$\frac{p-v}{\alpha}$	All customers served. Two retailers compete for demand.
S_5	$\frac{p+w}{2}$	$1-\frac{p-w}{2\alpha}$	$\frac{p-w}{2\alpha}$	
S_6	$p-\alpha\frac{(p-v)}{1-v}$	$\frac{1-p}{1-v}$	$\frac{p-v}{1-v}$	All customers served. Two retailers do not compete.
S_7	$\frac{p+w}{2}$	$\frac{1-p}{1-v}$	$\frac{v-w}{2(\alpha+v-1)}$	Medium range customers not served. Two retailers do not compete.

Table 3.2: Solution to the free rider's problem and resulting market share.

The free rider's price p_2^* has the following properties.

Lemma III.1.

(1) p_2^* is non-decreasing in w;

(2) p_2^* is non-decreasing in p, except when $1-v \leq \alpha, p \geq v, p - \frac{2\alpha(p-v)}{1-v} < w \leq 2p - v - \frac{2\alpha(p-v)}{1-v}$;

(3) p_2^* is non-increasing in α.

As the wholesale price w increases, the free rider has to increase its price in order to maintain a proper margin. When w is high enough ($(w, p) \in S_3$), the free rider will not participate the game and the manufacturer replicates the outcome under RPM policy. As the regular retailer's price p increases, the free rider faces less competition and is able to charge a higher price. There is an exception though. When the retailers serve all the customers but have no competition ($d_1 + d_2 = 1$), the free rider would not lower its price any more as it is very costly to compete with the regular retailer. If, however, the regular retailer raises its price and gives up some customers in the middle, the free rider would set a lower price capturing those customers until all customers are served again. The last part of the lemma says that as it gets harder to search for the free rider, free rider has to lower its price to give customers extra incentive to search.

Regular Retailer: θ

The regular retailer chooses the optimal sales effort or advertisement level. In our model, this is equivalent to choosing the level of total demand θ:

$$\max_{\theta} \pi_1 = \theta d_1(p-w) - \frac{1}{2}\theta^2. \tag{3.4}$$

Solving the first order condition, we obtain optimal number of customers $\theta^* = (p-w)d_1$.

Manufacturer: w and p

In the first stage of the game, the manufacturer chooses the wholesale price w and the suggested retail price p:

$$\pi^*_{MAP} = \max_{w,p} \pi_{MAP} = w\theta^*(d_1+d_2) = w(p-w)d_1(d_1+d_2). \tag{3.5}$$

Recall that d_i is a function of p and p_2. Substituting p_2^* from Table 3.2, we write π_M as a function of only (w, p, α, v). Depending on the region (w, p) falls in, we define

$$\pi_M = \pi_{Mi}, \text{ if } (w,p) \in S_i, i = 1, ..., 7.$$

In each region, the manufacturer's profit is given as follows:

$\pi_{M1} = 0,$

$$\pi_{M2} = \frac{w(p-w)[\alpha(2-p-v)-(1-v)(p-w)][(1-v)(2-p-w)-\alpha(2-p-v)]}{4\alpha(1-v)^2(1-\alpha-v)},$$
$$\pi_{M3} = \frac{w(p-w)(1-p)^2}{(1-v)^2},$$
$$\pi_{M4} = \frac{w(p-w)(\alpha+v-p)}{\alpha},$$
$$\pi_{M5} = \frac{w(p-w)(2\alpha+w-p)}{2\alpha},$$
$$\pi_{M6} = \frac{w(p-w)(1-p)}{1-v},$$
$$\pi_{M7} = \frac{w(p-w)(1-p)(2p+3v+2\alpha-w-2\alpha p-2pv+wv-v^2-2)}{2(1-v)^2(\alpha+v-1)}.$$

3.4 Results

In the previous section, we formulated the problem under different policies and studied the behavior of the retailers with the manufacturer's decision given. The manufacturer faces a market characterized by (α, v). In this section we first study the manufacturer's optimal decision given a certain market condition. Then we compare RPM and MAP policies from the perspective of each party within the supply chain.

3.4.1 Centralized Supply Chain

First we analyze the manufacturer's profit in a centralized supply chain. The optimal prices set by the manufacturer and the demand generated are given in Theorem III.1 and graphically shown in Figure 3.2.

Theorem III.1. *The optimal prices of the regular retailer and free rider and the total demand generation are:*

(a) If $\alpha + 2v \leq 1$, $p^* = 1/2, p_2^* = (1-\alpha)/2, \theta^* = \frac{1-\alpha-\alpha v}{4(1-\alpha-v)}$ *;*

(b) If $\alpha + 2v > 1$ *and* $\alpha + v \leq 1$, $p^* = v + \alpha/2, p_2^* = v, \theta^* = \alpha/4 + v$ *;*

(c) If $\alpha+v > 1$ *and* $(1-2v)\alpha-(1-v)^2 <= 0$, $p^* = \frac{2\alpha v-2v+v^2+1}{2\alpha}, p_2^* = \frac{3\alpha v-2v-\alpha+v^2+1}{2\alpha}, \theta^* = \frac{4\alpha v-2v+v^2+1}{4\alpha}$ *;*

(d) If $(1-2v)\alpha-(1-v)^2 > 0$, $p^* = 1/2, p_2^* = v/2, \theta^* = \frac{v^3-v^2-v-\alpha+1}{4(1-v)(1-\alpha-v)}$ *.*

The optimal profit is $\pi_C^* = \frac{1}{2}\theta^{*2}$.

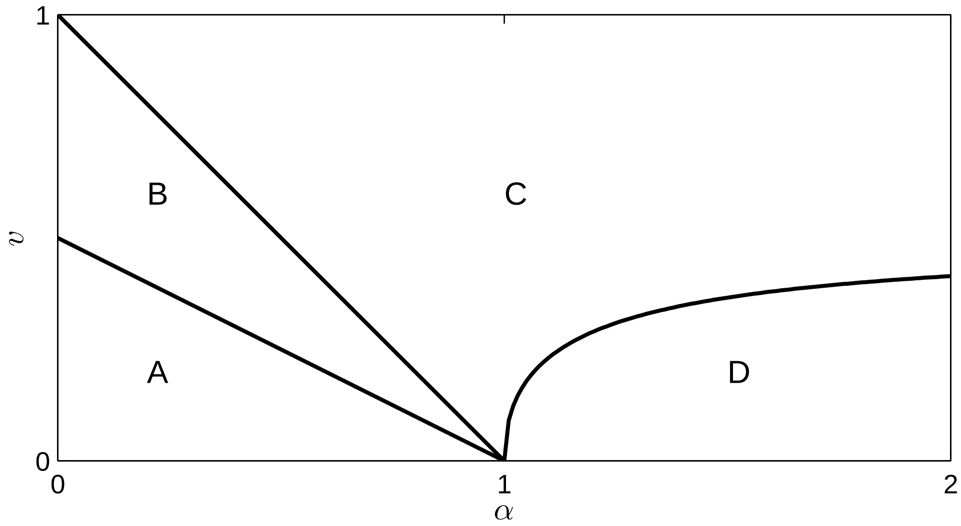


Figure 3.2: Optimal solution in centralized supply chain.

The centralized manufacturer does not exclude the free rider even though it cannibalizes the high-margin sales through the regular retailer when $\alpha + v > 1$. This is because the free rider serves additional low to middle range customers, the portion that the regular retailer cannot serve due to higher price. The search cost here is used as an instrument to segment customers. Thus the centralized manufacturer achieves higher profit than a situation when it only sells through the regular retailer.

Figure 3.2 demonstrates behavior of the manufacturer in α and v. In region A, though both retailers have positive sales, the low-end customers are not served. This is because their valuations are too low, and even the free rider cannot profitably set a price appealing to them. The other reason that prevents the free rider to price low is that small search cost, which makes it harder to differentiate customers. If the free rider lowers its price, demand of the regular retailer is cannibalized easily due to low search cost which results in low demand generation. In region B, all the customers are covered as the supply chain faces both higher customer valuation and higher search cost. In both region C and D, the manufacturer makes the two retailers local monopolists. In region C, all the customers are served because the customer valuation is relatively high. Otherwise, in region D, it is very costly to cover all the customers. As $\alpha + v > 1$, the free rider targets customers whose valuation is smaller than a threshold. Therefore, the customers in the middle range are ignored in region D.

The analysis of the centralized supply chain illustrates the key idea of how the presence of free rider can help improve the profitability. As the prices and demand generation are determined centrally, demand generation is not tied only to the regular retailer's profit and the cannibalization is not a big concern. In reality, more than often the manufacturer is able to contract only the regular retailer's price, but not the effort level (demand generation) or the free rider's price. We next study the dynamics in a decentralized setting.

3.4.2 RPM Policy

The solution to the RPM policy is given by the following theorem.

Theorem III.2. *Under RPM policy, the manufacturer's optimal suggested retail price, wholesale price and profit are*

$$p^* = \max\left\{\frac{1}{2}, v\right\}, w^* = \frac{p^*}{2}, \pi^*_{RPM} = \frac{(1-p^*)^2 p^{*2}}{4(1-v)^2} = \begin{cases} \frac{1}{64(1-v)^2}, \ \textit{if } v \leq \frac{1}{2} \\ \frac{1}{4}v^2, \ \textit{if } v > \frac{1}{2} \end{cases}.$$

One element in our model is that the regular retailer is the only party that generates demand. If the manufacturer increases w, each unit contributes more revenue but the regular retailer's margin decreases, discouraging the demand generation effort. If the manufacturer increases p, the regular retailer's margin becomes higher, but its market share becomes lower. The manufacturer chooses a price higher than the wholesale price that balance these two effects. Under MAP policy which we analyze next, multiple effects complicate the trade-offs, however the rationale on how w and p influence the manufacturer's profit remains the same.

3.4.3 MAP Policy

Under MAP policy, the manufacturer's profit functions, π_{Mi}, are prohibitively complicated so that closed form solutions are not available. In this section we focus on the manufacturer's behavior under MAP policy and its impact on each player.

Manufacturer's Decision

To understand the manufacturer's decision, first we study the special case when $v = 0$ to identify the intuition of the manufacturer's behavior and then we extend the analysis to the general case. The following Lemma shows that when the markets are very heterogeneous and the search cost is small, RPM policy strictly dominates MAP policy. For higher search costs, optimal MAP policy is equivalent to the optimal RPM policy. Indeed, Lemma III.3 shows that optimal wholesale price and suggested retail price converges to those under the RPM policy as search cost increases.

Lemma III.2 (Assume $v = 0$).
If $\alpha < \frac{1}{2}$, $\pi^*_{MAP} < \pi^*_{RPM}$; *Otherwise,* $\pi^*_{MAP} = \pi^*_{RPM}$.

Lemma III.3 (Assume $v = 0$).
w^*_{MAP} *is decreasing in* α; p^*_{MAP} *is increasing in* α.

Lemmas III.2 andIII.3 along with Theorem III.2 immediately lead to the following result.

Theorem III.3 (Assume $v = 0$).
$1/4 = w^*_{RPM} \leq w^*_{MAP} \leq p^*_{MAP} \leq p^*_{RPM} = 1/2$.

When $v = 0$, the MAP policy never outperforms the RPM policy. When $\alpha \geq \frac{1}{2}$, the search is too costly, and free rider cannot profitably charge a lower price than the suggested retail price. In other words, the volume of sales with a higher suggested retail price along with the lower wholesale price (to incentivize the free rider to enter the market) bring lower profits to the manufacturer. Therefore, when $\alpha \geq \frac{1}{2}$ the manufacturer chooses exactly the same wholesale price and retail price as those under RPM policy. When $\alpha < \frac{1}{2}$, the manufacturer decreases the margin of the regular retailer to deter the free rider from entering the market. This is because a small α allows the free rider to easily cannibalize the market share of the regular retailer. Hence the regular retailer has little incentive to generate demand. Although it also has negative impact on demand generation, narrowing the margin protects the market share of the regular retailer from erosion. Overall, it still results in lower profit than that under the RPM policy.

When v is positive, the intuition remains the same and we can state the following theorem.

Theorem III.4.

(1) $\lim_{\alpha \to 0} p^*_{MAP} - w^*_{MAP} = 0$;

(2) $\lim_{\alpha \to \infty} p^*_{MAP} = p^*_{RPM}$; $\lim_{\alpha \to \infty} w^*_{MAP} = w^*_{RPM}$.

A representative comparison between optimal w and p under the two policies is plotted in Figure 3.3. When α is small ($\alpha < 0.4$ in Figure 3.3), the manufacturer offers narrower margins to the regular retailer. As α increases to infinity, both w and p converge to those under RPM policy. Then we only need to fill the gap what happens when α is in the middle range.

As shown in Theorem III.2, under RPM policy, the manufacturer chooses $w^* = 0.25$ and $p^* = 0.5$, independent of the value of α. When α is moderate to large ($\alpha > 0.4$ in Figure 3.3), the entry of the free rider does not cannibalize much market share of the regular retailer but the free rider serves additional low-to-middle valuation customers. Instead of narrowing the margin to deter the free rider, the manufacturer widens the margin to support the free rider. This helps the manufacturer to reach more customers

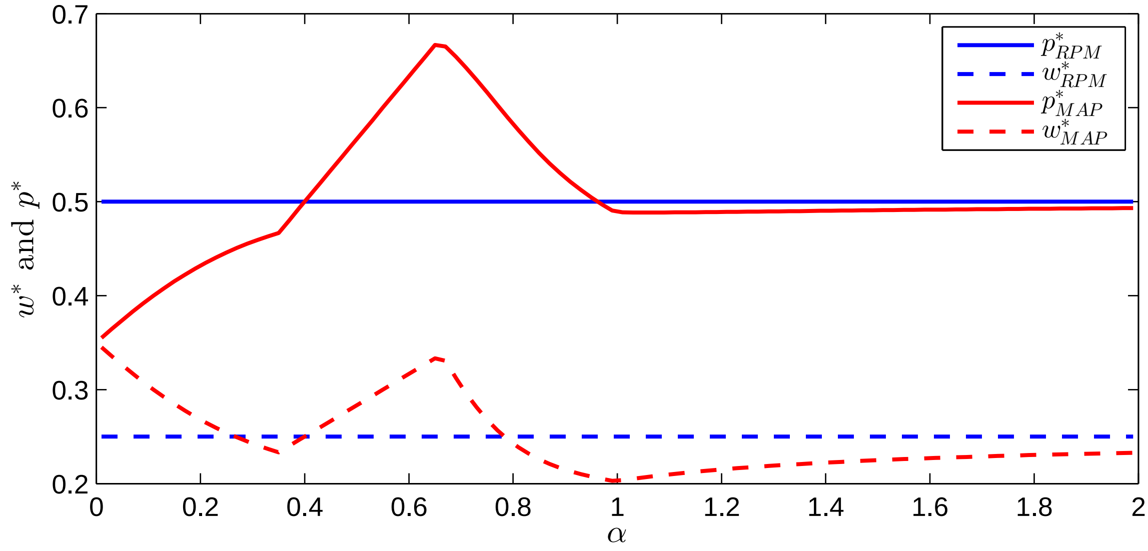


Figure 3.3: w^* and p^* under RPM and MAP policies. Parameter: $v = 0.35$.

through free rider, as well as effectively eliminate competition between the retailers if there is any. There are two ways to widen the margin: increasing retail price p and decreasing wholesale price w. When α is medium ($0.4 < \alpha < 0.9$ in Figure 3.3), there is still competition and the manufacturer would increase the retail price p. Note that in this case, the manufacturer does not decrease the wholesale price to widen the margin. When there is competition between the retailers, decreasing wholesale price allows the free rider to decrease its price and cannibalize regular retailer's market even further. On the other hand, increasing p induces the regular retailer to serve higher valuation consumers, reducing the competition. When α is high ($\alpha > 0.9$ in Figure 3.3), there is no competition between the retailers and some customers in the middle range are not served. In such situations, it would be beneficial for the manufacturer to increase the margin by decreasing the wholesale price w. Since there is no competition in this case, increasing the retail price reduces the regular retailer's market share, but does not change free rider's optimal behavior. However, decreasing the wholesale price induces free rider to serve more customers by decreasing its price.

This additional demand generates more profit compensating the loss due to lower per unit revenue.

It is crucial to fully understand the manufacturer's behavior, as many of the analysis hinge on it. The manufacturer's decision under MAP policy is summarized as follows. When α is very small, the manufacturer narrows the margin to deter the free rider from entry. As α becomes larger, the manufacturer starts to widen the margin to encourage the free rider's entry. It increases the retail price p when α is moderate, and then gradually transits to decreasing the wholesale price w as α increases. As α goes to infinity, both w and p converge to the optimal decision under RPM policy.

Manufacturer's Profit

The manufacturer's profits under MAP and RPM policies have the following two properties over the customer valuation v and the search cost α.

Lemma III.4. $\lim_{\alpha \to \infty} \pi^*_{MAP} = \pi^*_{RPM}$.

Lemma III.4 says that when the search cost is large enough, optimal MAP policy (optimal wholesale price and retail price) and the outcome are the same as RPM policy. The intuition is that when the search cost is extremely high, customers will not search for the free rider, regardless of their valuation. RPM policy behaves like a special case of MAP policy where the search cost is infinite.

Lemma III.5. *Both π^*_{RPM} and π^*_{MAP} are non-decreasing in v.*

Lemma III.5 states that as consumer valuation heterogeneity decreases, the manufacturer's profit increases. Although very intuitive, this is not a trivial result. If the manufacturer chooses a suboptimal strategy (w, p) under MAP policy, the profit could be even lower with a higher v. Consider a scenario where the manufacturer sets both w and p too high and two retailers are local monopolists ($\alpha + v > 1$). When v increases, the free rider chooses to raise the price so much that its demand is lower than before. Though the demand for the regular retailer is higher, the overall effect on the manufacturer's profit is negative. However, a profit maximizer manufacturer never faces such situations.

The following theorem answers one of our main research questions: Which one of the policies, RPM policy or MAP policy, is more profitable to the manufacturer?

Theorem III.5 (Condition: $\alpha + v \geq 1$).

(1) If $\alpha \geq 1$, there exist $\alpha_1 \leq \alpha_2$, such that:

(1a) $\pi^*_{MAP} < \pi^*_{RPM}$ if and only if $1 \leq \alpha < \alpha_1$;

(1b) $\pi^*_{MAP} > \pi^*_{RPM}$ if and only if $\alpha_1 < \alpha < \alpha_2$;

(1c) $\pi^*_{MAP} = \pi^*_{RPM}$ for other α.

(2) If $\alpha \geq 1$, there exist $v_1 \leq v_2$, such that:

(2a) $\pi^*_{MAP} < \pi^*_{RPM}$ if and only if $v_2 < v \leq 1$;

(2b) $\pi^*_{MAP} > \pi^*_{RPM}$ if and only if $v_1 < v < v_2$;

(2c) $\pi^*_{MAP} = \pi^*_{RPM}$ for other v.

(3) If $\alpha + v \geq 1$, all thresholds continue to exist, except v_1.

Figure 3.4(a) provides graphic demonstration of the policy that the manufacturer prefers. Clearly the threshold policy does not always hold when $\alpha + v < 1$.

The manufacturer's preference depends on the heterogeneity of the market and how difficult the search is for the consumers. When v is small and α is large, it is very hard for the free rider to get any customers. As a consequence, even under MAP policy, the manufacturer chooses the w and p such that the free rider is unable to profitably enter the market. The outcome is exactly the same as that under RPM policy. As v increases or α decreases, it becomes easier for the free rider to survive. Under such situations, the manufacturer accommodates the free rider to segment customers: the regular retailer serves high valuation consumers with high search cost while the free rider serves consumers with lower valuation consumers who have also low search cost. The manufacturer even widens the margin compared to the optimal under RPM policy so that free rider profits. Though competition may exist, the free rider mainly targets the low to middle range customers while the regular retailer serves the high valuation customers. If v increases (consumers are more homogenous) or α decreases (search is less costly) even further, free rider cannibalizes more of the regular retailer's market share, which results in less incentive to generate demand. Unlike the centralized supply chain, the manufacturer cannot control the free rider's price. Instead, under MAP policy the manufacturer has to narrow the margin in an effort to mitigate the regular retailer's demand cannibalization. Under such a situation, often the manufacturer earns higher profits using RPM policy than selling through the free rider under MAP policy.

Retailers

Though the price restraint policy is eventually determined by the manufacturer, the retailers should also understand the consequences of either policy on their bottom line. The retailers may be able to negotiate with the manufacturer about the contract, and in some cases they can even influence the manufacturer's decision by withdrawing their business. For example, Babies"R"Us cancelled orders from some of its suppliers as the suppliers refused imposing price restraining policies on Internet retailers which have cost advantage over Babies"R"Us ((Pereira 2008)).

It is easy to see that the free rider prefers MAP policy over RPM policy under which it always gets zero profit. Intuitively, one may believe that the regular retailer should prefer the RPM policy as it effectively elimates the competition, leaving the regular retailer as the monopolist in the market. The theorem below, however, only partially confirms this intuition.

Theorem III.6 (Condition: $\alpha + v \geq 1$)**.**
The region where the regular retailer prefers MAP is a subset of the region where the manufacturer prefers MAP.

The theorem shows the property given $\alpha + v \geq 1$, while it still holds in the rest

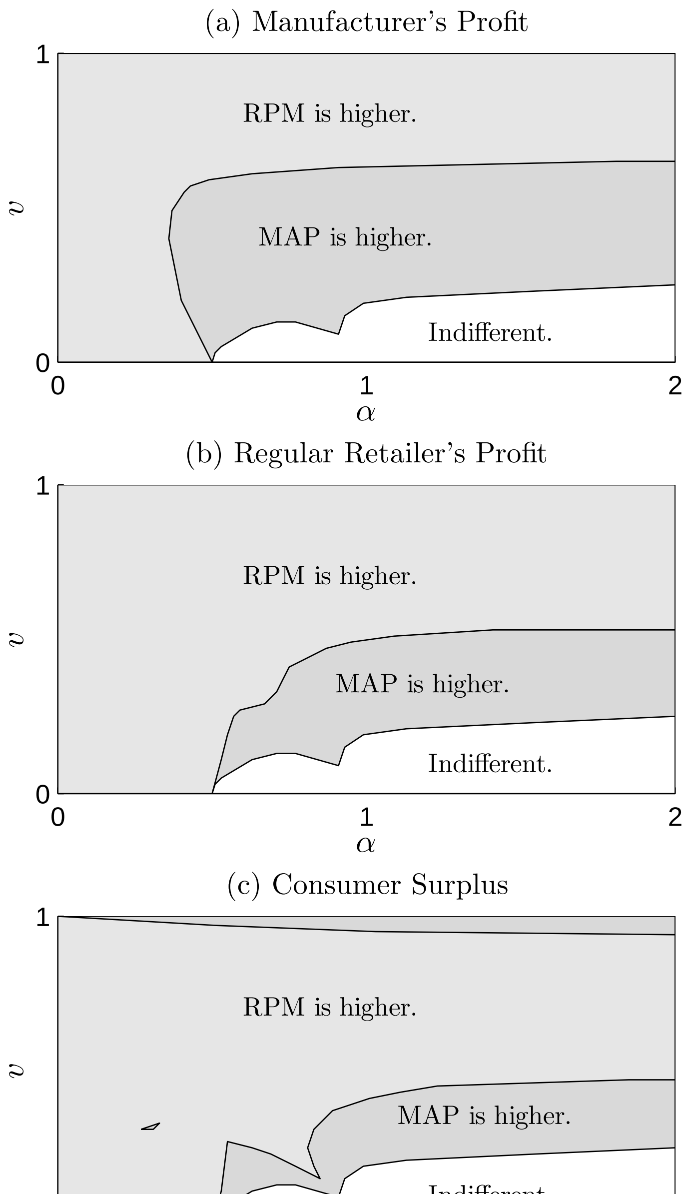


Figure 3.4: Policy preference for each player.

of the region as we observe in Figure 3.4(b). The insights come in two folds. First of all, the regular retailer is less likely to prefer the MAP policy, compared to the manufacturer. Moreover, surprisingly, the regular retailer prefers MAP policy when v is low to medium and α is large. When α is small, though the manufacturer narrows the margin to deter the free rider from entry if the supply chain switches from RPM policy to MAP policy, the profits of the manufacturer and the regular retailer still decline. As α gets larger, the manufacturer starts to prefer MAP policy and increases the retail price p to support the free rider. The loss of sales through the regular retailer is compensated by the sales through free rider. The regular retailer, however, loses sales and end up with lower profit, though the margin is slightly higher. This explains why as α increases, the manufacturer is more likely to benefit from MAP policy than the regular retailer. As α increases even more, the manufacturer starts to decrease the wholesale price w in an effort to incentivize the free rider to participate. The regular retailer also benefits from lower wholesale price and thus prefers MAP policy as well.

Consumers

The analysis of the consumer surplus provides insights for legislators to guide the market. In this part we address which policy serves consumers better based on numerical results from our model.

Figure 3.4(c) illustrates whether RPM or MAP policies result in higher consumer surplus. There are two major areas where consumer surplus is higher under MAP policy. The first area is when v is low to medium and α is large enough. Again under MAP policy the manufacturer needs to either increase p or decrease w to accommodate the free rider. It tends to increase p when α is low to medium, and to decrease w as α gets larger. As the consumer surplus is very sensitive to the selling price, a higher p typically results in smaller consumer surplus, even though more consumers may be generated by the regular retailer. On the other hand, a lower w

increases consumer surplus because it allows the free rider to charge lower price and serve more low-end customers. Moreover, lower w also leads the regular retailer to generate more customers.

We also observe another region at the top of Figure 3.4(c), where consumer surplus is higher under MAP policy. Intuition of this can be illustrated by considering the extreme case where $v = 1$. The manufacturer will set $p = 1$ under either policy. Under RPM policy, all customers buy from the regular retailer and end up with zero surplus. Under MAP policy, those customers who buy from the free rider realize positive surplus while the others still get zero. Note that this is a situation where the manufacturer strongly prefers RPM policy.

In Figure 3.4 there are significantly large regions where the manufacturer's, the regular retailer's and consumers' preferences are aligned. The crucial element is the fact that all the demand in the supply chain is generated by the regular retailer. As long as the incentive to generate demand is not hurt (when v is small and α is large), entry of the free rider is beneficial for all the parties. Otherwise, the entry of the free rider makes the total number of customers generated decrease, generally none of the parties win by itself, except in the extreme case that we discussed above where the consumer surplus increases.

3.5 Optimal Search Cost Level

In the previous section we studied the dynamics for given market condition (α, v). Very recently, some manufacturers have come up with innovative approaches to influence the search cost α. For example, accessing the price on an online retailer requires varying number of steps, ranging from simply a few clicks to multiple email exchanges. Accordingly, it may take anywhere from a few seconds to many hours or even days. The free rider may also impact the search cost but only in the direction that makes it even higher. Otherwise, it may violate the MAP policy contract in which some manufacturers specify how difficult the search should be for the consumers. However, as we will see soon, free rider will never make the search harder than the set level by the manufacturer since the search cost desired by the free rider is always smaller than the manufacturer.

In this section we focus on what the optimal search cost is for each player. Let α_C^*, α_M^*, α_{R1}^*, α_{R2}^*, α_{CS}^* be the preferred search cost of the centralized supply chain, the manufacturer, the regular retailer, the free rider, and consumers, respectively. All α^*'s are plotted as a function of v in Figure 3.5. Note that on the plot for the manufacturer, the regular retailer or consumers, there is a missing piece when v is moderate to large. This indicates that the preferred α is infinity when market heterogeneity is small to moderate. As shown in Lemma III.4, MAP policy would be equivalent to RPM policy as α approaches infinity. This implies that MAP policy with any search cost is dominated by RPM policy when v is moderate to large.

Centralized Supply Chain

Theorem III.7. *For the centralized supply chain:* $\alpha_C^* = 1 - v$.

Though the presence of search seems to make the supply chain less efficient, interestingly, the centralized supply chain prefers a positive search cost. Search cost allows the central controller to segment customers and sell to low-end customers through the free rider. Recall that the surplus of a consumer whose time value is x and buying from the free rider is $v + (1 - v - \alpha)x - p_2$. By setting $\alpha = 1 - v$, the free rider can set $p_2 = v$ and thus (i) serves all the customers not served by the regular retailer and perfectly extracts all the surplus from consumers that buy from the free rider; and (ii) does not cannibalize the high margin sales through the regular retailer because anyone buying from the free rider gets exactly zero surplus. This is an ideal situation for the central controller as he segments the market, extracts all the surplus from the lower end, and serves the remaining higher end of the market at a higher price.

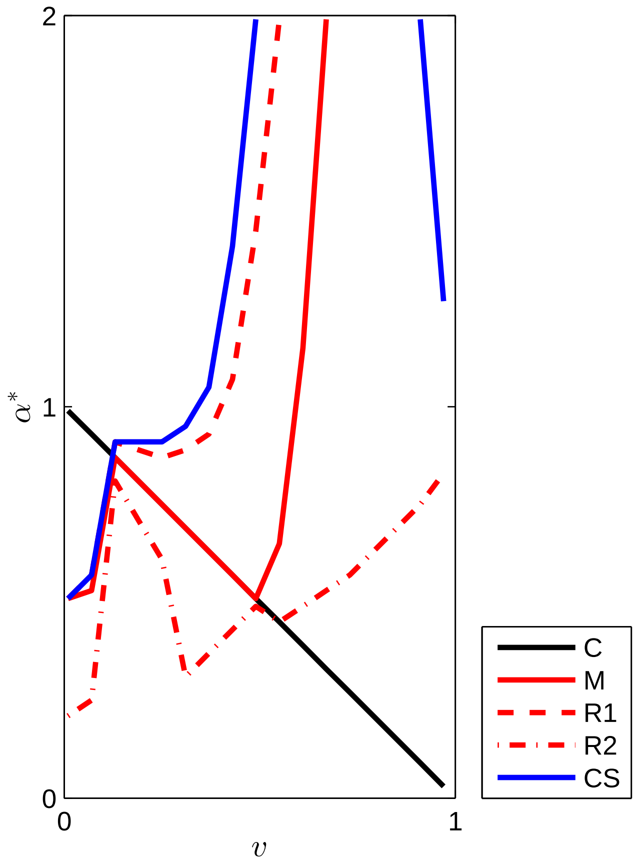


Figure 3.5: α^* for each player.

Manufacturer

Theorem III.8 (Condition:$\alpha + v \geq 1$)**.**

For the manufacturer:

$$\alpha_M^* = \begin{cases} +\infty, \ \textit{if } v \geq \frac{2}{3} \\ \textit{Any value in } \left[\max\left\{1-v, \frac{1-v}{2(2-3v)}\right\}, \max\left\{1-v, \frac{3(1-v)^2}{2(2-3v)}\right\}\right], \ \textit{if } v < \frac{2}{3} \end{cases}.$$

Though this theorem is limited to the case $\alpha + v \geq 1$, we observe in Figure 3.5 that the manufacturer would only deviate from this solution when v is very small (smaller than 0.1).

Intuitively the manufacturer should mimic the centralized supply chain because their interests are somewhat consistent. However, this strategy works only when $v \in [0.1, 0.5]$. When v is close to zero, given $\alpha = 1 - v$ the manufacturer has to offer $w \leq v$ to encourage the free rider's entry but such low wholesale price would decrease the profit even with higher sales. Instead, the manufacturer chooses a much smaller α, enabling the free rider to capture customers at the expense of regular retailer's market. When v is large (market is very homogenous), the free rider would cannibalize a significant portion of the regular retailer's market at $\alpha = 1 - v$. Unlike the centralized supply chain, the manufacturer cannot control the free rider's price to prevent such competition. As a result, the manufacturer prefers a higher, sometimes infinite, search cost to protect the regular retailer.

Retailers

The preference of α for the regular retailer, the free rider, and consumers are summarized in the following theorem and observation.

Theorem III.9 (Condition: $\alpha + v \geq 1$)**.**
For the regular retailer, $\alpha_M^* \leq \alpha_{R1}^*$.

Observation III.1.
(1) When $v < 0.9$, we have $\alpha_{R2}^* \leq \alpha_M^* \leq \alpha_{R1}^* \leq \alpha_{CS}^*$;
(2) When $v \geq 0.9$, we have $\alpha_{R2}^* \leq \alpha_{CS}^* \leq \alpha_M^* = \alpha_{R1}^* = \infty$.

We first study the retailers. Every player in the supply chain faces similar trade-offs in the α choice, but puts different weights on the market segmentation and the volume of consumers generated. A smaller α makes it easier for the free rider to compete with the regular retailer, typically resulting in higher market share for the free rider, lower market share for the regular retailer and less volume of customers generated by the regular retailer. A larger α leads to the opposite. The retailers care only about the volume of customers and their own market share. The manufacturer, instead, capitalizes on sales through both retailers. As a result, the free rider prefers a small α while the regular retailer prefers a high α. The manufacturer's choice falls in the middle.

Consumers

Intuitively the consumer surplus should decrease in search cost α. However, by Theorem III.10, we show a counterintuitive result that even consumer surplus may increase in the search cost under certain conditions.

Theorem III.10 (Condition: $\alpha + v \geq 1$)**.**
When $\max\left\{1-v, \frac{1-v}{2(2-3v)}\right\} \leq \alpha \leq \max\left\{1-v, \frac{3(1-v)^2}{2(2-3v)}\right\}$, *consumer surplus is increasing in* α.

Interplay of two effects results in an increase in consumer surplus as consumer search gets more costly. The first one is that, as α increases, the free rider has to respond with a lower price in order to compensate decrease in demand because of higher search cost. The second is that, with a higher search cost, the regular retailer faces less competition and thus generates more demand. The surplus from a larger volume of customers served can be larger than the surplus loss due to the additional search cost each customer pays.

In Observation III.1(2), the consumer's choice is driven by the fact that when v is close to one (market is almost homogenous), only those who buy from the free rider have positive surplus. This has been discussed in detail in Section 3.4.3. Now we focus on the more general result stated in Observation III.1(1). The consumers not only potentially benefit from higher search cost, but also prefer a higher search cost than the manufacturer and the retailers. Using Figure 3.3, we can explain the reason by understanding how the manufacturer changes w and p to encourage the free rider to participate under MAP policy. When α is near the value preferred by either the manufacturer ($\alpha^*_M = 0.65$) or the regular retailer ($\alpha^*_{R1} = 0.9$), the manufacturer would still increase the retail price p. At α^*_{R1}, the retail price under MAP policy is still higher than that under RPM policy. Actually the consumer surplus is very sensitive to the price. Therefore, the consumer surplus is higher when α is even greater than α^*_{R1}, where the manufacturer under MAP policy starts to decrease the retail price.

3.6 Extensions

3.6.1 Robustness of Demand Function

So far we assume a demand function where consumers' valuation and search cost are perfectly correlated. Indexed by x, consumer valuation is expressed by a line $v + x(1 - v)$. In this section we study the robustness of our model with respect to the demand function. Specifically, for consumer indexed by x, we assume the valuation is uniformly distributed in $[v+x(1-v), v+x(1-v)+d]$, where a higher d represents lower correlation between search cost and product valuation. As d increases to infinity, the two attributes become completely independent.

In Figure 3.6, we plotted the boundary, for various values of d, where the manufacturer is indifferent between RPM policy and MAP policy. For each d, the manufacturer prefers RPM above the boundary and MAP otherwise. It is clear that as the correlation between search cost and product valuation decreases (i.e., d increases), the region where MAP is preferred by the manufacturer gets smaller.

Based on our earlier discussion, the main advantage of MAP is that it allows market segmentation: the free rider serves low-end consumers that the regular retailer would not serve. If search cost and product valuation are uncorrelated, MAP becomes less effective due to the existence of high-valuation but low-search-cost consumers. This group, who purchase from the regular retailer under RPM, would defect to the free rider under MAP. This leads to less incentive for the regular retailer to create demand, and from the manufacturer's perspective this loss is likely to outweigh the additional coverage through the free rider.

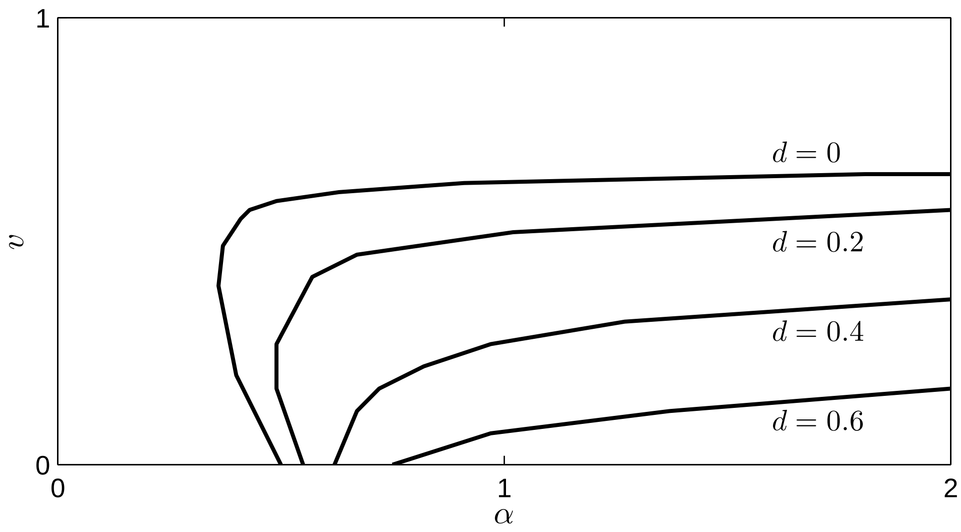


Figure 3.6: Manufacturer's policy preference with different demand functions.

3.6.2 Robustness of Effort Cost Function

While quadratic function is commonly used to model effort cost, in this subsection we numerically test the robustness of the effort function. We assume that the cost to generate θ demand is $\frac{1}{2}\theta^a$, smaller a represents close to linear cost function, while large a describes more convex cost function.

In Figure 3.7, the boundaries (above which the manufacturer prefers RPM policy) are plotted for different values of a. It shows that the area where MAP policy is preferred becomes larger as the cost function becomes more convex. To describe the intuition behind this result, first note that in any specific market situation (characterized by a combination of v and α), the demand generator would spend more effort, due to lack of competition from the free rider, under RPM policy than that under MAP policy. The profit under RPM may be higher due to higher effort, versus MAP, where the smaller effort is augmented by additional low-valuations customers and higher wholesale price. Due to convexity of cost of effort, the extra effort in RPM case will be limited. When the cost of effort is more convex, the increment of effort

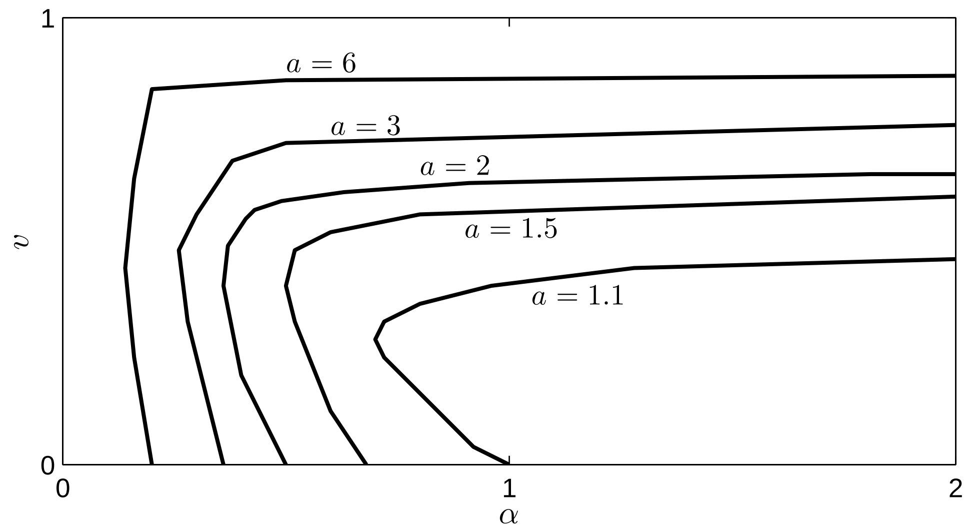


Figure 3.7: Manufacturer's policy preference with different effort cost functions.

in RPM case (compared to MAP) is smaller. Consequently, the manufacturer that chose RPM policy may switch to MAP policy with more convex effort. Despite the shift of the threshold and the corresponding decrease in the size of area where RPM dominates, the lessons and insights from the main model continue to hold for any value of a.

3.6.3 Multiple Retailers

In many businesses there are often multiple regular retailers and free riders in the market. For example, Canon cameras are carried by BestBuy and Sears stores. Both of them are considered as regular retailers because customers can test products and receive advise from store associates. On the other hand, the same cameras are available at many online retailers, such as BuyDig.com and BeachCamera.com. They do not provide much service but frequently offer hidden but lower prices under MAP policy. While only one regular retailer and one free rider are modeled in the main part, in this extension we show that all the results and intuitions continue to hold with multiple retailers.

Multiple Regular retailers

Since the regular retailer's price is dictated by the manufacturer under both RPM and MAP, there is no direct price competition. Assume that there are M symmetric regular retailers. Their demand generation efforts, θ_i, are additive, i.e., $\theta = \sum_{i=1}^{M} \theta_i$. Each of retailer gets $\frac{1}{M}$ of consumers who are willing to buy at the regular price. Then each regular retailer's problem, under both RPM and MAP policies, is

$$\max_{\theta} \ \pi_1 = \frac{1}{M}(\theta_i + \sum_{j \neq i} \theta_j) d_1 (p - w) - \frac{1}{2}\theta_i^2.$$

Solving this problem we get optimal demand generation for retailer i as $\theta_i^* = \frac{1}{M} d_1 (p - w)$. Since total number of customers $\theta^* = \sum_{i=1}^{M} \theta_i^* = d_1(p - w)$, M regular retailers collectively create same volume of demand as in our original single regular retailer model. Consequently, we find that the number of regular retailers has no impact on the manufacturer or the free rider.

Let $\pi_{R1}^{(M)}$ be each regular retailer's profit when there are M of them in the market, and $\pi_{R1}^{(1)} = \pi_{R1}$ is regular retailer's profit in our model with single regular retailer. We have (under both policies)

$$\pi_{R1}^{(M)} = \frac{1}{M}\left(1 - \frac{1}{2M}\right) d_1^2 (p - w)^2 = \frac{1}{M}\left(1 - \frac{1}{2M}\right) \pi_{R1}.$$

Because each regular retailer only creates a fraction of the demand and the effort cost is convex, the total effort cost of M retailers is smaller than that in the main model with single regular retailer, despite of the same number of customers created. As a result, the total profit of M regular retailers is greater than that with single regular retailer.

Multiple Free Riders

In our original model, the free rider sets its hidden price p_2. The implicit assumption here is that the free rider is a monopolist as the low cost retailer (but competes with regular retailer), or that there are multiple free riders but consumers only randomly pick one free rider without searching the others. One can argue that multiple free riders perfectly compete in price so that they all end up selling at the wholesale price. That is, instead of solving the maximization problem in Equation (3.3), free riders set $p_2 = w$. We are interested in how this changes the manufacturer's preference over MAP and RPM.

In order to differentiate from the original MAP policy we analyzed above, we denote the MAP policy with multiple competitive free riders as MAP2 policy.

Theorem III.11.
(1) If $\alpha + v \geq 1$, the region where MAP outperforms RPM is smaller under MAP2 policy.
(2) If $\alpha + v \geq 1$ and the manufacturer can only choose α in $[1 - v, +\infty)$, the optimal search costs lead to identical profits under MAP and MAP2, i.e., $\pi^*_{MAP}(\alpha^*_{MAP}) = \pi^*_{MAP2}(\alpha^*_{MAP2})$.
(3) If the manufacturer can choose α in $(0, +\infty)$, the optimal profit under MAP2 is weakly higher than that under MAP, i.e., $\pi^*_{MAP}(\alpha^*_{MAP}) \leq \pi^*_{MAP2}(\alpha^*_{MAP2})$.

Theorem III.11 is illustrated in Figure 3.8. With all free riders selling at w, the manufacturer sells more to customers through retailers. On the other hand, this also increases the competition between free riders and the regular retailer, and thus results in less total demand. When α is large, there is very little competition between the regular retailer and the free riders. Then under MAP2 the manufacturer benefits from free riders selling at low prices without compromising the regular retailer's incentive. This fundamental trade-off faced by the manufacturer is the same as that under MAP policy. However, under MAP2 policy the competition is more intense due to the perfect price competition among free riders. Therefore, the manufacturer is less likely to choose MAP2 policy over RPM policy (as partially proved in Theorem III.11(1) for $\alpha + v \geq 1$).

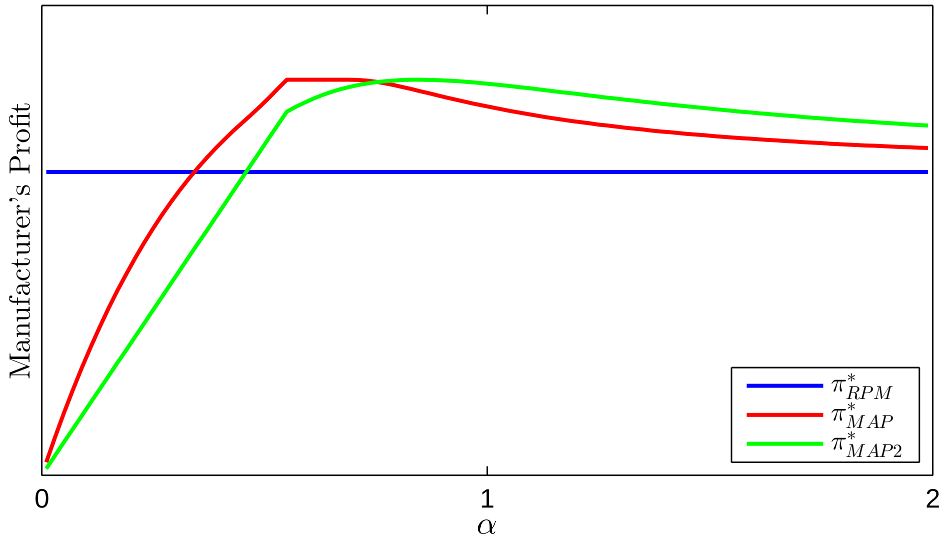


Figure 3.8: Manufacturer's profit comparison under RPM, MAP, and MAP2 policies.

Under MAP2 policy free riders set price equal to the wholesale price which leads to more intense competition with the regular retailer. This is similar to the outcome under MAP policy but with a smaller search cost. In Figure 3.8, MAP2 profit curve looks similar to a stretched version of the MAP profit curve. If the manufacturer is able to set α, the resulting optimal profits under MAP and MAP2 would be identical most of the time (as partially proved in Theorem III.11(2) for $\alpha + v \geq 1$). To summarize, while having multiple free riders changes the prices, the intuition we derived for MAP still holds.

3.6.4 New Retailer's Choice on Roles

While what role (regular retailer or free rider) a retailer will play largely depends on its infrastructure, capital investment and existing capabilities, some retailers may be able to strategically choose whether they would compete as a regular retailer or free rider. For example, many retailers, such as Walmart and Bestbuy, have both online and offline channels. They can serve as regular retailers if a certain product is carried and demonstrated at brick-and-mortar stores. Under other cases, they can sell a product only through their websites with a hidden price. In this subsection, we analyze under MAP policy what role a retailer should choose when it enters the market.

We assume there are already M regular retailers and N free riders in the market ($M, N > 0$). Additionally free riders are assumed to be local monopolists, i.e., customers only randomly pick a free rider without searching others. Let $\pi_{R2}^{(N)}$ be each free rider's profit and $\pi_{R2}^{(1)} = \pi_{R2}$ where π_{R2} is the free rider's profit in the original model. We obviously have $\pi_{R2}^{(N)} = \frac{1}{N}\pi_{R2}$.

Following the discussion of multiple regular retailers in Subsection 3.6.3, each regular retailer's profit is $\pi_{R1}^{(M)} = \frac{1}{M}\left(1 - \frac{1}{2M}\right)\pi_{R1}$. Playing regular retailer or free rider, the new retailer's profit is $\pi_{R1}^{(M+1)}$ or $\pi_{R2}^{(N+1)}$, respectively. Therefore, the new retailer should play a regular retailer if and only if

$$\frac{\pi_{R2}^{(N+1)}}{\pi_{R1}^{(M+1)}} = \frac{\frac{1}{N+1}}{\frac{1}{M}\left(1 - \frac{1}{2M}\right)} \cdot \frac{\pi_{R2}}{\pi_{R1}} \leq 1.$$

When M and N are given in a market, the new player's decision will rely on the ratio $\frac{\pi_{R2}}{\pi_{R1}}$, which is plotted in Figure 3.9. When the search cost is high and customers are heterogeneous, this ratio is large which means less free riders may be sustained in the market. The intuition is that, when search cost increases or customer valuations span a wider range, low-end customers are increasingly less likely to search for low prices offered by free riders. Consequently, it is harder for them to compete in the market. The ratio $\frac{\pi_{R2}}{\pi_{R1}}$ is highest when α is close to zero. However, this cannot be interpreted as that free riders would prefer zero search cost. As we discussed earlier, small search cost leads to small profits for both regular retailers and free riders. While the ratio is higher, the absolute value of π_{R2} is smaller than that with moderately positive search cost α.

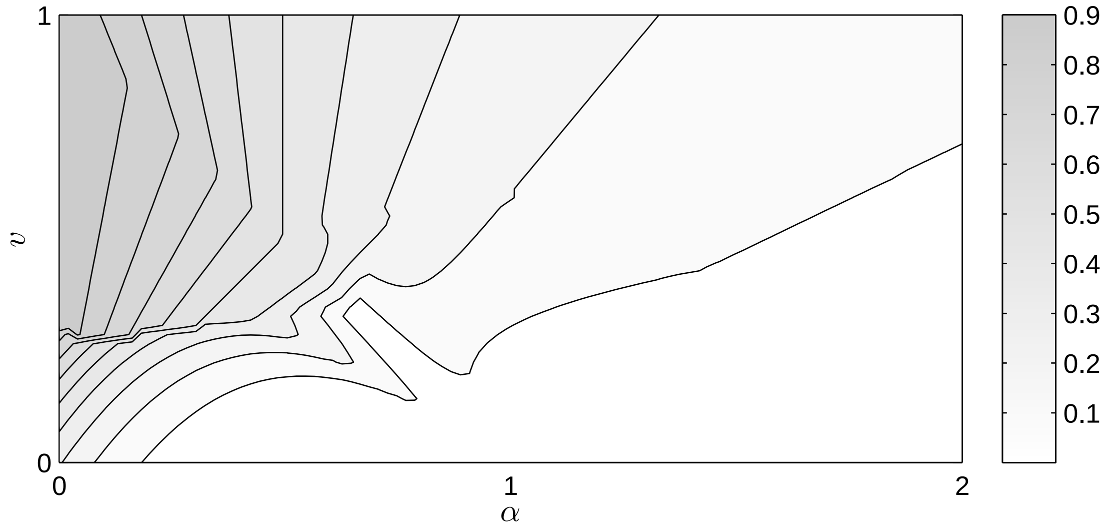


Figure 3.9: Ratio of π_{R2}/π_{R1}.

3.6.5 Manufacturer Subsidy on Sales Effort

In our model, we assume the regular retailer bears all the cost associated with demand generation. If the retailer's effort or demand outcome is observable, the manufacturer can subsidize the demand generation in order to have a higher volume of demand.

In this extension, we assume the regular retailer only needs to pay a portion, denoted by δ, of the effort cost, with the manufacturer subsidizing the rest. Thus, the regular retailer's problem (3.4) becomes

$$\max_{\theta}\ \pi_1 = \theta d_1(p-w) - \frac{1}{2}\delta\theta^2, \tag{3.6}$$

which results in

$$\theta^* = \frac{1}{\delta}(p-w)d_1.$$

The manufacturer's problem (3.5) becomes

$$\pi^*_{MAP/RPM} = \max \pi_{MAP/RPM}(w, p, \delta) = w\theta^*(d_1 + d_2) - \frac{1}{2}(1 - \delta)\theta^{*2}. \tag{3.7}$$

The general formulation for RPM or MAP policy is identical as above, while the functions of market share d_i's in (w, p) are dependent on specific policy.

Theorem III.12. *With manufacturer subsidy, MAP policy always (weakly) outperforms RPM policy, from perspectives of the manufacturer, the regular retailer, and the free rider.*

With subsidy, the manufacturer is able to set the wholesale price arbitrarily close to the retail price under RPM, and in the meantime encourages demand generation by sharing the cost. As a result, the manufacturer extracts all the channel profit, leaving the regular retailer zero profit.

Under MAP policy, for given α, the free rider cannot enter the market profitably when the margin is small. Therefore, the manufacturer can always mimic the actions under RPM policy without worrying about competition between retailers. However, this is not necessarily the best strategy. This can be best illustrated by an example. Let $v = 0.4$ and $\alpha = 0.6$. Under RPM policy, the manufacturer's decision would be $w^*_{RPM} = p^*_{RPM} = 0.5$ and $\delta^*_{RPM} = 0$. The total demand generation and manufacturer's profit are 0.417 and 0.087, respectively. Under MAP policy, the manufacturer's optimal decision is $w^*_{MAP} = 0.4$, $p^*_{RPM} = 0.7$, and $\delta^*_{RPM} = 0.316$, resulting in 0.475 units of total demand generation and 0.113 units of profit for the manufacturer. With the free rider serving low-end customers, the manufacturer may (i) decrease the wholesale price to get extra market share through the free rider; and (ii) increase the retail price to incentivize demand generation without heavy subsidy. Consequently, the manufacturer is always better off with another lever under MAP policy.

3.7 Conclusion

The RPM and MAP policies are widely used by the manufacturer to protect the margin of brick-and-mortar retailers so that they have incentive to spend effort promoting the manufacturer's products. We compares these two policies via a stylized model and analyzes when and why one policy outperforms the other for each player across the supply chain.

The manufacturer is most likely to be the player that chooses price restraining policy. We find that there is no dominant strategy for the manufacturer. The manufacturer prefers MAP policy when there are large search cost and large consumer valuation heterogeneity. Under such conditions, the free rider serves low-end customers but does not cannibalize the regular retailer's market share. The manufacturer effectively segments customers via different channels and gets higher sales. On the other hand, when the search cost is low and customers are homogeneous, it is very hard to segment customers. Contributing little extra customers, the free rider only competes with the regular retailer and makes the latter spend less demand generation effort. Therefore, the manufacturer would choose RPM policy to rule out free riders.

The retailers' perspectives are also studied. The free rider's decision is simple as it always prefers MAP policy. Under RPM policy, the free rider is unable to price lower than its brick-and-mortar competitors and, thus, gets zero market share and ends up with zero profit. Interesting, the regular retailer may also benefit from MAP policy, when the search cost is higher and customer valuations are more heterogeneous than those conditions for the manufacturer. The reason is that the manufacturer intends to embrace the free rider by either increasing the retail price or decreasing the wholesale price. This wider margin also makes the regular retailer more profitable.

The total consumer surplus is also higher under similar conditions for the regular retailer. There are two main drivers: (i) the regular retailer generates larger number of customers; and (ii) low-end customers, who are unserved under RPM policy, are now served by the free rider.

Despite of the free rider, our results indicate that preference of the manufacturer, the regular retailer and consumers is somewhat aligned. However, preference might be different in the middle range of search cost and valuation heterogeneity. This implies that the manufacturer can enforce MAP policy at the cost of the regular retailer and consumers. These results explain the different stance of brick-and-mortar retailers (typically regular retailers) and online retailers (typically free riders) about RPM and MAP on public media. They also provide a perspective to policy makers regarding to the legality of each policy.

CHAPTER IV

Performance Based Contracts for Energy Efficiency Projects

4.1 Introduction

Energy efficiency (EE) projects are often described as very attractive in economic terms and promising to provide significant environmental benefits. According to the United Nations Foundations, "energy efficiency is the cheapest, fastest, and smartest strategy available for saving money and resources and reducing greenhouse gas emissions around the world."[1] Yang and Yu (2015) estimates that, in 2020, capturing the energy efficiency opportunities will contribute 50% of the greenhouse gas abatement goal required to cap the long-term concentration of greenhouse gas in the level suggested by experts.

Notwithstanding these significant benefits, EE projects have not reached their full potential in the last decade (Yang and Yu 2015). There are a few widely recognized challenges in EE projects. The first is lack of information about expected benefits. Clients usually under-estimate the benefit provided by EE projects and, thus, hesitate to adopt them. The second challenge is that EE projects are complicated. They typically involve long span of time, major scale of construction, operations disruptions,

[1] `www.unfoundation.org/what-we-do/campaigns-and-initiatives/reduce-carbon-emissions`, retrieved April 16, 2015.

and requirement for a significant expertise. Third, EE projects tend to be expensive, with large initial investment and also uncertain outcomes (Aflaki et al. 2013). This set of difficulties has given a rise to a business model referred to as the energy services companies (ESCOs): a business that has expertise in EE projects and takes responsibilities for developing, installing and, often, also financing of the projects.

One of the challenges ESCOs face is the appropriate form of contracts with clients. Due to uncertainties of outcome and fairly big up-front investments, many clients, depending on their size and their level of risk aversion, may be hesitant to get involved in such projects. To overcome these challenges, Performance Based Contracts (PBCs) have become a core part of ESCO's business, where ESCO's compensation is linked to outcomes of a project and paid during a specified length of time (Larsen et al. 2012). This model has been used to increase energy efficiency in commercial, municipal, and industrial sectors in both developed and emerging economies (Taylor et al. 2008). Compared to fixed payment, advantages of PBCs are multi-fold. First, PBCs transfer a portion of operational risk to ESCOs, leaving clients less vulnerable to uncertain operational outcome of the project. Second, PBCs provide additional incentives for ESCOs to spend reasonable amount of effort, thereby alleviating ESCOs' moral hazard problem. Third, PBCs lead to more flexible mechanisms for the projects and remove the heavy burden of project financing from the customers' shoulders.[2]

While many variations of PBCs exist, three most common contracts seen in practice are *shared savings contract*, *guaranteed savings contract*, and *chauffage contract*.[3] In the shared savings contract, ESCO pays a portion of client's energy costs over certain period of time after the project completion, i.e, ESCO participates in the savings as well as participates in additional costs incurred by the client. NASA, for example, hired Honeywell to improve energy efficiency at the Johnson Space Flight Center

[2]Alternatively this aspect can be supported by financial institutions.

[3]Readers are referred to Bullock and Caraghiaur (2001) for a comprehensive review of the ESCO contracts.

(JSC) in Houston. Improvements ranging from energy-efficient lighting, air conditioning to water management system were implemented in more than 140 buildings at JSC. Using a shared savings contract, Honeywell will receive a portion of utility cost savings as payment for the project.[4] Guaranteed savings contract specifies a guaranteed reduction over mutually agreed period of time in client's utility bill relative to business as usual. That is, a target is established and ESCOs are responsible for the cost of any energy usage above target, or get penalized in other ways. As an example, Candelas Ltd., an Irish ESCO, provides lighting retrofit to poultry broiler sheds in Ireland and UK. Candelas Ltd. guarantees savings, typically of 65%, in direct lighting cost. If the energy savings fall short of those guaranteed, Candelas Ltd. refunds the difference between actual and guaranteed savings. In chauffage contract, clients outsource an energy related function (e.g. temperature, lighting level, air quality, etc.) to an ESCO at a flat rate. The ESCO owns, operates, and maintains all necessary equipments to provide the service. For example, Dalkia, a French ESCO, provides hospitals with heating, lighting and electricity services. While owning and financing projects, Dalkia charges hospitals monthly fee for services.[5]

Despite some success in non-residential markets, ESCOs have barely entered the residential sector. Satchwell (2010) estimates that in 2008 the residential sector represents only 9% of ESCO revenue and, additionally, most of the residential sector revenue are earned by ESCOs interacting with utility companies rather than directly contracting with households. Many potential reasons have been cited to explain the under-developed ESCO business in the residential sector (Steinberger et al. 2009, Sorrell 2009, Zimring et al. 2011, Hoyle 2013). Among them we focus on three major ones. The first one is customers' behavior and preference change after the completion of EE projects. As residential clients are only partially responsible for energy cost under any types of PBCs, they tend to choose a higher comfort level than they

[4] More information at `www.energy.gov/eere/femp/energy-savings-performance-contracts`.

[5] More case examples can be found in EEB (2011) and SEAI (2012).

had chosen prior to the project. Additionally, they also pay less attention to their own energy-saving efforts, such as closing windows or turning off lights. This is often labeled as the *rebound effect* (Greening et al. 2000) and effectively it is customers' moral hazard. Due to rebound effect, the energy usage reduction does not fully reflect the benefit of EE projects that clients receive, which undermines the effectiveness of PBCs. Thus it is one of the key elements in this paper to design appropriate contracts that overcome or at least partially manage rebound effect.[6] The second one is that individual clients are much more risk averse than businesses, which deters individual clients from adopting EE projects. The third one is ESCOs' moral hazard problem due to lack of monitoring and verification protocols.

Our study focuses on contract design issues for energy efficiency projects, with particular attention to the residential clients. The major goal is to analyze how each type of the contracts observed in practice works, taking into account ESCOs' moral hazard, clients' rebound effect, and risk aversion.

Using a game-theoretic framework, we model the interaction between an ESCO and a client and characterize the optimal contracting mechanism. Energy efficiency projects are based on the premise that the same amount of energy may result in different comfort level (or utility) of customers depending on the type of windows or level of insulation of walls. That is the client's utility is based not only on energy consumption but also on her energy efficiency level which reflects the condition of client's house. The client maximizes utility level by choosing the level of energy consumption. The ESCO offers the client an EE project, where the expected efficiency improvement depends on the ESCO's effort. A caution on both ESCO's side as well as on customers' side is not ungrounded. Both ESCO's effort in providing the EE project, and client's effort in saving energy are difficult to verify due to highly

[6] Rebound does not happen in non-residential sectors, because either the comfort level is exogenously given (e.g., plant temperature, street lighting hours) or end users of energy do not pay but their employers do.

specialized technical nature of the projects, limited observability of the inputs (e.g., type of gas used in the glass panels, or the material and its density used to create foam injected into external walls), and the client's strategic actions ex-post the project (e.g., closing windows during winter or turning off lighting when leaving home).

Within this framework, we explore two broad issues. Firstly, we investigate what contracts (shared savings contract, guaranteed savings contract, or chauffage contract) should be used in what situations, and whether first-best outcomes can be achieved. Given the existence of double moral hazard, one would expect that coordinating contracts are unlikely to emerge. Thus, we evaluate the performance of practical contracts and benchmark them against the first-best solution. Secondly, we suggest how the gap between the outcome of currently-practiced contracts and first-best solutions can be closed, both from the ESCO's and from policy makers' perspectives.

We find that, in the residential sector simple piecewise linear contracts (a general form of shared savings contract and guaranteed savings contract) work well. While it is widely believed that, in standard double moral hazard problem, the first-best outcome is not attainable even when the client is risk neutral, we show that guaranteed savings contract can achieve the first-best solution. Even when the client is risk averse, 2-rate contracts (a combination of shared savings contract and guaranteed savings contract, formally defined in Section 4.3) can still capture most benefit of performance contracting. While more complicated contracts do outperform 2-rate contracts, the improvement is very limited. This result indicates that popular shared savings contract and guaranteed savings contract can perform reasonably well in the residential sector, as long as their parameters are carefully chosen, for which our model provides useful guidance. Chauffage contract, by its nature, often requires a fixed comfort level, and does not allow clients to adjust that. Therefore, while it allows for a coordinated outcome, it is seldom applicable to residential clients.

On top of optimal contract choice and design, some further improvements in efficiency of contracts are possible. ESCO can achieve the first-best outcomes by assessing post-project energy efficiency. A certification of the ESCO's quality can effectively address the client's concern about moral hazard problem. With the additional testing/certification, a simple shared savings contract performs nearly as good as any more-complicated contracts. Information disclosure programs, usually offered by policy makers, can reduce uncertainty of EE projects, and thus increase their social benefit. Policy makers can also provide monetary incentives for EE projects, such as subsidizing such projects or charging higher utility price. While subsidy does not change the ESCO's and client's decisions, it encourages more households to adopt EE projects. On the other hand, higher utility price, such as carbon tax, would also result in greater ESCO's effort and less energy usage in addition to higher adoption rate, which leads to higher social surplus. Therefore, higher utility price is more desirable than subsidy from economic perspective.

4.2 Literature Review

This paper draws from and contributes to the general literature of sustainable operations management (Drake and Spinler 2013), in particular the sub-streams dealing with energy efficiency, contracting, and incentive coordination, in the presence of double moral hazard problem.

Energy Efficiency. During last few decades, energy efficiency projects have received increasing attention in economics and operations management literatures. Many papers qualitatively illustrate the framework of energy efficiency projects, from opportunity assessment to project execution and valuation (Sorrell 2007, Steinberger et al. 2009, Aflaki et al. 2013). Several papers also consider the theoretical and analytical aspects of energy efficiency. The focus of our paper is on interactions between ESCO and customers in presence of double moral hazard and risk aversion. The role of customers and the corresponding rebound effect is not considered in most of the papers. Eom and Sweeney (2009) and Chu and Sappington (2012) study the interaction between policy makers and utility companies. Eom and Sweeney (2009) examine the design of linear contracts that encourage utility companies to invest in energy efficiency and achieve socially optimal investment levels. Chu and Sappington (2012) extend the model and assume that ESCOs have private information in their cost structure. Thus, policy makers offer a choice of linear contracts to ESCOs, whose effort is non-contractible. They characterize the optimal menu of contracts in different market conditions. Neither Eom and Sweeney (2009) nor Chu and Sappington (2012) consider behavioral aspects of consumers. While Chun et al. (2013) generalize this framework to model consumer behavior, the paper still focuses on contract design problems between the government and utilities, rather than contracts between ESCOs and consumers. Although consumers have the energy consumption decision, they only respond to the new technology after the project but there is no financial incentive (subsidy) from either utilities or the government. Without involving consumers in PBCs, the strategic behavior of consumers after implementation of EE projects is neglected. Wirl (2000) and Wirl (2015) consider consumers implementing themselves EE projects. Thus, the papers study contracts between the government or utilities and consumers without considering ESCOs. Wirl (2000) proposes contracts that encourage consumers' effort in energy efficiency improvement. Assuming the effort is observable, fixed amount of subsidy is provided to any consumers that spend enough effort. The focus is to design such contracts for policy makers that address the negative effect of asymmetric and private information of consumers (such as discount rate). Wirl (2015) is based on the framework in Wirl (2000) but assume utilities, rather than the government, have to induce consumers to spend certain energy efficiency investment required by the government. None of the above papers considers the relationship (contracts) between ESCOs and individual consumers, which we concentrate on and which is important in practice, as these contracts change the incentives for ESCOs and consumers, increasing the difficulty of incentive alignment. Besides this, this paper also includes features that are prevalent in practice but not studied in above papers, such as piecewise linear contracts, uncertain outcome and risk aversion.

Contracting. By studying contracts between ESCOs and clients, our paper contributes to the stream of literature in contract theory dealing with moral hazard through PBCs. Readers are referred to Bolton and Dewatripont (2004) for a review of general contract theory and moral hazard problem. Our work is particularly relevant to a subgroup of literature that concentrates on double moral hazard problem, as this paper is an application and extension of general double moral hazard theory in energy efficiency industry. Double moral hazard arises when costly and un-contractible inputs (efforts) of both principal and agent have an impact on final outcomes. A few papers in the field of economics lay the foundation for analysis of double moral hazard problem within principle-agent context. In the standard setting, the monetary output, which is a function of un-contractible efforts from both sides, has to be divided by principal and agent. This literature shows that there exists no sharing rule that induces globally optimal input (effort) levels for both players (the first-best solution), even when they are risk neutral (Holmstrom 1982). Some economics papers evaluate the effect of linear contracts: Romano (1994) and Bhattacharyya and Lafontaine (1995) show that linear contracts, in which two parties share the outcome proportionally after a fixed money transfer, while non-coordinating, are as good as it gets, they weakly dominate any other possible performance based contracts (the second-best solution). Kim and Wang (1998) considers the same double-sided moral hazard problem except with risk averse agent. They conclude that linear contracts are no longer dominant. Our paper extends general theory into the energy efficiency context. We consider the ESCO's effort, as well as consumers' energy consumption decision.[7] Both of their decisions contribute to the outcome of the EE project. However, contracts are based on energy consumptions rather than any monetary outcome. Energy consumptions have different impacts on the ESCO's and client's utility: The impact on the ESCO is linear while that on the client is concave. Consequently, the general framework of zero-sum game does not apply and lessons from classic double moral hazard problem do not necessarily carry over. In fact, we show in Section 4.5.2 that when the client is risk neutral, the first-best outcome can be achieved, which is different from standard results presented in Holmstrom (1982), Romano (1994), Bhattacharyya and Lafontaine (1995).

Using PBCs to deal with moral hazard problem has many applications in OM literature. Optimal contracting mechanisms have been studied in many industries including health insurances (So and Tang 2000, Jiang et al. 2012), after-sales services (Kim et al. 2007), and call centers (Ren and Zhou 2008, Hasija et al. 2008) (see Gustafsson et al. 2010, for a comprehensive review of PBCs). A number of papers involve double moral hazard problem as well, and study contracting mechanisms within various operational and supply chain contexts. Baiman et al. (2000) models a situation, where a manufacturer and a supplier jointly invest in reducing failure rate of parts produced by the supplier. In Corbett and DeCroix (2001) and Corbett et al. (2005), a manufacturer and a supplier collaborate in reducing usage of supplies in the manufacturing process. Roels et al. (2010) focuses on contracts that maximize output of collaborative services, and advises when players should establish contractibility on additional fixed cost. In Kim and Netessine (2013), a manufacturer and a supplier work together to reduce expected production cost and its variance. While sharing similar spirit, each paper takes into consideration specifics of the applied case in different industries. They vary in the structures and performance measures that enter into payment calculus. The main difference between our paper and the double-moral-

[7] Another extension of our framework is to also include client's effort as her decision variable in Section 4.6.1.

hazard papers in OM literature is that we include the strategic behavior of consumers, rebound effect, influences both the observable benefits (energy consumption) and non-observable one (comfort level). We are not aware of papers that deal with such a situation. The rebound effect also influences the division of benefits and simple lessons from general theory do not apply and translations from other papers are non-obvious to us.[8] Also, in most other industries, the realization of effort (true effort plus random shock) is observable and thus provides a good contingency in contracts. In energy efficiency setting, contracts are often built on energy consumption, a proxy to but not exactly technology (effort) realization. To our best knowledge, this paper is the first attempt to address double moral hazard with PBCs in the energy efficiency industry.

4.3 Model and Preliminary Results

We consider a setting where an ESCO (referred to as "he") offers an energy efficiency project to a single client (referred to as "she"). The client's "comfort" function is denoted as $u(x+t)$, where x represents energy consumption and t, for technology, represents energy efficiency level at the client's house. We assume $u'' < 0$ to reflect the diminishing return in both energy consumption and technology level. In order to have analytical tractability, $u''' \geq 0$ is also assumed. Most popular utility functions in fact have this property. It is worth noting that although $u' > 0$ is a common assumption, it is not necessary for most of results unless explicitly stated.

The energy cost is p per unit, which is assumed to be exogenously determined. Let $w_c(x)$ represent the client's energy payment as a function of her energy use. Therefore, the client's net payoff is

$$v(x,t) = u(x+t) - w_c(x). \tag{4.1}$$

[8]The models are typically rooted in zero-sum game, and thus the first-best solution is not attainable. In our setting utility derived from energy consumption is concave and the first-best solution is achieved when consumers are risk neutral.

It should be noted that $w_c(x) = px$ before the EE project, but depending on the contract structure may not necessarily be the same after the project. As a result, $v(x,t)$ also depends on the contract structure.

The client's ex-ante technology and consumption levels are denoted by t_0 and x_0 respectively. t_0 is normalized to 0 without loss of generality. So the client's pre-project utility is $u(x_0) - px_0$ with no uncertainty present.

The ESCO can provide an energy efficiency project to the client by choosing the new technology level t with a convex cost of $C(t)$. t can also be interpreted as the ESCO's effort spent in the project. Given that in most cases the clients do not have the necessary expertise to enforce or inspect the technology, t is usually not contractible and, therefore, moral hazard arises. In order to have analytical tractability, we also assume $C''' \geq 0$.

The post-project technology suffers from uncertainty reflecting the unobservable circumstances in the client's building.[9] This is modeled as a random shock to the project outcome $t + \epsilon$. The random variable ϵ's support is $[\underline{\epsilon}, \bar{\epsilon}]$ with mean 0 and variance σ^2. The distribution is assumed to be common knowledge between the ESCO and the client.

The client is risk averse and is assumed to have mean-variance risk preferences:

$$\mathbf{E}_\epsilon[v(x, t+\epsilon)] - \frac{\lambda}{2} Var_\epsilon[v(x, t+\epsilon)],$$

where v is the utility defined in (4.1) and the risk aversion parameter $\lambda \geq 0$ reflects client's attitude towards payoff uncertainty. The mean-variance utility function originally stems from finance literature that studies market returns, and then widely adopted to model risk aversion in operations management literature (e.g., Chen and Federgruen 2000, Van Mieghem 2007, Kim et al. 2007).

[9]There are other sources of uncertainties, e.g., weather, and these can be easily incorporated into the model and are evaluated in Section 4.6.2.

The ESCO has to provide a take-it-or-leave-it contract before observing ϵ. To reflect the industry practice, we consider contracts contingent on the client's post-project energy consumption. We call them n-rate contracts, including terms (F, $\{z_i\}_{i=1,\ldots,n-1}$, $\{\alpha_i\}_{i=1,\ldots,n}$) with $z_0 = 0$ and $z_n = \infty$. An n-rate contract consists of two part: The first part is the up-front payment, F, from the client to the ESCO. The second part specifies the portion α_i ($0 \leq \alpha_i \leq 1$) of unit cost p that the client pays for energy usage in the range $[z_{i-1}, z_i)$. The rest of energy cost is paid by the ESCO and considered as subsidy. Our analysis focuses on 1-rate and 2-rate contracts, as they are flexible enough to capture all of the popular practical contracts we discussed earlier. For example, 1-rate contract $(F, \varnothing, \{\alpha_1\})$ describes shared savings contract, where the ESCO obtains $(1 - \alpha_1)$ of total savings as well as additional costs. 2-rate contract $(F, \{z_1\}, \{1, 0\})$ describes guaranteed savings contract, where the ESCO refunds any energy usage above threshold z_1. $(F, \varnothing, \{0\})$ is chauffage contract, where fixed payment F should be allocated over the period of contract term. In order to understand the effect of more flexible contracts, we also extend our results to 3-rate contracts through numerical study.

Under an n-rate contract, the client's energy payment (energy cost minus subsidy from the ESCO, excluding up-front payment) is

$$w_c(x) = p \sum_{i=1}^{n} \alpha_i \cdot \min\left\{(x - z_{i-1})^+, z_i - z_{i-1}\right\}. \tag{4.2}$$

The sequence of events is as follows: (1) The ESCO offers a contract $(F, \{z_i\}, \{\alpha_i\})$ to the client. (2) The client decides whether to accept the contract. If the client accepts the contract, then (3) the ESCO decides expected new technology level t. (4) The new technology, $t + \epsilon$, is observed by the client. (5) The client adjusts energy consumption of x, based on the new technology level and contract structure. The problem is formulated below starting with the client.

Client

The client's energy consumption is modeled as an optimization problem. Her pre-project energy consumption, x_0, maximizes $u(x) - px$. That is

$$x_0 = \arg\max_x \{u(x) - px\}. \tag{4.3}$$

We denote $v_0 = u(x_0) - px_0$ as the pre-project utility. After the completion of the project, the client observes new energy efficiency level $t + \epsilon$ and chooses her new energy consumption, x^*,

$$x^* = \arg\max_x \ v(x, t + \epsilon). \tag{4.4}$$

The superscript $*$ is used to denote general optimal solutions, which will be replaced by proper superscript in each scenario.

ESCO

The ESCO's payoff consists of three parts: up-front payment, cost of technology installation, and cost of energy subsidy. We denote $w_e(x)$ as cost of energy subsidy

$$w_e(x) = px - w_c(x). \tag{4.5}$$

After the contract is accepted and up-front payment is transferred, the ESCO chooses the technology (effort) level as

$$t^* = \arg\max_t \left\{-\mathbf{E}w_e(x^*) - C(t)\right\}, \tag{4.6}$$

where $x^* = x^*(t + \epsilon)$ is determined in (4.4).

In the contract design stage, the ESCO's goal is to maximize the expected payoff while making sure the contract is accepted. The main problem to solve is

$$\max_{F,\{z_i\},\{\alpha_i\}} \{F - \mathbf{E}w_e(x^*) - C(t^*)\},$$
$$s.t.\ \mathbf{E}\left[v(x^*, t^* + \epsilon)\right] - \frac{\lambda}{2} Var\left[v(x^*, t^* + \epsilon)\right] - F \geq v_0.$$

In the optimal solution the constraint must be binding, i.e. the client has strictly zero surplus. The problem reduces, therefore, to

$$V = \max_{\{z_i\},\{\alpha_i\}} \left\{ \mathbf{E}\left[v(x^*, t^* + \epsilon)\right] - \frac{\lambda}{2} Var\left[v(x^*, t^* + \epsilon)\right] - v_0 - \mathbf{E}w_e(x^*) - C(t^*) \right\}. \tag{4.7}$$

In the rest of this paper, all contracts are described as only $(\{z_i\}, \{\alpha_i\})$, while the corresponding F is implied by the above constraint.

4.3.1 Benchmark: Model with Central Planner

We first solve the model with central planner, which serves as a benchmark for the following analysis. In this part we assume that the client's payment is any continuous functions, rather than piecewise linear functions. As n becomes large, n-rate contracts are able to approximate any continuous payment structures. Further, since the technology realization $t + \epsilon$ is observable by the central planner and one-to-one correspondence exists between $t + \epsilon$ and $x^*(t + \epsilon)$,[10] the client's and the ESCO's payment functions, $w_c(x)$ and $w_e(x)$, are replaced by $w_c(t + \epsilon)$ and $w_e(t + \epsilon)$. Then, the central planner's problem is

$$\max_{t,x(t+\epsilon),w_c(t+\epsilon)} \left\{ \mathbf{E}\left[v(x, t + \epsilon)\right] - \frac{\lambda}{2} Var\left[v(x, t + \epsilon)\right] - v_0 - \mathbf{E}w_e(t + \epsilon) - C(t) \right\}. \tag{4.8}$$

The solution to the problem above is outlined in the following theorem. The superscript C, for coordination, is used to denote this optimal solution.

[10] $x^*(t + \epsilon)$ is decreasing in $t + \epsilon$, as shown in the proof of Theorem IV.1.

Theorem IV.1. *The central planner's optimal strategy is:*

(1) There exists a unique $x^C(t+\epsilon)$*, which satisfies* $u'(x+t+\epsilon)=p$*;*

(2) $w_c^C(t+\epsilon)=0$*;*

(3) There exists a unique t^C*, which satisfies* $C'(t)=p$*.*

The outcome V^C is the optimal project value for the ESCO. Clearly all uncertainty is internalized by the central planner (ESCO) and marginal utility of customers matches energy price. For analysis to come in following sections, the strategy stated in Theorem IV.1 and its outcome are used to benchmark against.

4.3.2 Rebound Effect

As discussed earlier, rebound effect is identified as one of the key barriers that prevent ESCOs from thriving in the residential sector (Greening et al. 2000, Sorrell 2009). Before examining the main problem, we briefly discuss how the rebound effect is reflected in our model, which is a fundamental driving force for the rest of analysis.

Given the same technology level, the individual client tends to consume more energy, deviating from what central planer would choose. Denote x^D (D for decentralized) as the solution to the client's problem (4.4).

Lemma IV.1. $x^D(t+\epsilon) \geq x^C(t+\epsilon)$.

When the ESCO provides any subsidy on the unit price, the client does not internalize the true cost of energy and over-consumes compared to the coordinated level – lower marginal cost leads to increased consumption. This rebound effect makes the real benefit of energy efficiency project unverifiable, and thus undermines the ESCO's profitability from PBCs. Formally stated, the rebound effect is a major difficulty in obtaining the first-best solution.

In the following sections, we first study two types of contracts that eliminate rebound effect and lead to the first-best solution in Section 4.4, although their use can be limited by practical considerations. In Section 4.5 we discuss how existence of rebound effect, combined with customer's risk aversion, leads to lack of coordination and study n-rate contracts.

4.4 Direct Control of Rebound Effect

In this section, we consider two practical contracts that have the potential to directly resolve the rebound effect. First is the case where the post-project technology is verifiable – i.e., the technology realization after the EE project can be measured with no errors – and as a consequence PBC is designed around post-project technology rather than the client's energy consumption, directly eliminating the rebound effect. Second is the case of chauffage contracts where the client's comfort level is controlled by the ESCO. In other words, the energy consumption is implicitly determined such that the specified comfort level in the contracts is delivered, therefore rebound effect is removed.

4.4.1 Verifiable Post-Project Technology

When the post-project technology level can be verified, the subsidy of ESCO is contingent on technology realization $t+\epsilon$ rather than the client's energy consumption $x^D(t+\epsilon)$. That is, the ESCO's subsidy becomes $w_e(t+\epsilon)$ instead of $w_e(x)$ in (4.5). Accordingly the client's payment is now $w_c(t+\epsilon, x) = px - w_e(t+\epsilon)$ instead of $w_c(x)$ in (4.2).

Theorem IV.2. *When the ESCO's payment is contingent on post-project technology, the contract that specifies the ESCO's payment $w_e(t+\epsilon) = M - pt$ achieves the first-best outcome. M is any constant.*

In many PBCs, the client's consumption is used as a proxy to the technology realization. As a result, the cost sharing ratio, α_i, has to play multiple and conflicting roles simultaneously. It requires the ESCO pay a large portion of variable cost to commit his effort and to share risk from the client. On the other hand, it requires exactly opposite to manage the client's rebound effect. The ability to verify the post-project technology makes it possible to totally decouple these two parts. The ESCO's subsidy driven by technology realization not only enables him to commit to certain effort level, but also fully compensates the client for uncertain outcome. Meanwhile, the client is responsible for all variable cost so that her rebound effect is completely removed. As a result, both the ESCO and client have the right incentive to make optimal decisions and the first-best solution is achieved.

While coordinating the incentives, post-project technology verification can be very costly relative to small project values in the residential sector, or sometimes even impossible. In the next subsection and Section 4.5, we assume post-project technology verification is not viable, and consider contracts contingent on energy consumption.

4.4.2 Chauffage Contract

Chauffage contract is also known as comfort contracting. It requires clear specification of service requirements. For example, a data center may outsource its air conditioning service to an ESCO, requiring a temperature below 80 Fahrenheit. A public library may outsource its lighting service to an ESCO, with guaranteed brightness of 500 lumens. In this subsection we investigate whether chauffage contracts achieve the first-best outcome.

Under chauffage contracts, energy consumption is determined by technology level and contracted comfort level. With the comfort level unchanged before and after EE project, better technology leads to smaller energy usage. As the client does not choose her consumption, rebound effect does not exist. To reflect this fact, we denote u_0 as the required comfort level. In order to keep the comfort level unchanged after the EE project, the energy consumption, x^*, has to satisfy

$$u(x^* + t + \epsilon) = u_0. \tag{4.9}$$

The client's utility (4.1) becomes

$$v(x^*, t + \epsilon) = u_0 - w_c(x^*).$$

Theorem IV.3. *(1) The 1-rate contract* $(\varnothing, \{0\})$ *is optimal; (2) There exists a unique optimal effort level* t^**; (3) The first-best outcome is achieved.*

The intuition is straightforward. Without rebound effect, the ESCO pays all the variable cost and charges the client only the fixed fee. The fixed payment can be in the form of either one time transfer or multiple installments. In such arrangement, all the risk is transferred to the ESCO and the client's utility is guaranteed. Therefore, chauffage contract addresses concerns about both risk aversion and moral hazard, and helps ESCO reach the first-best outcome.

Theorem IV.3 fully confirms practices where chauffage contract is frequently used in non-residential sectors, where the comfort level is often exogenously given. However, its use in the residential sector is limited. Residential clients often have the desire to change comfort level over time and each individual may differ in their preferred comfort level. As a result, chauffage contract is unlikely to work.[11]

To summarize, rebound effect can be resolved if the post-project technology is verifiable, or if chauffage contract is viable. In either case, a simple linear contract is optimal and achieves the first-best outcome. However, they both have limitations and often not feasible in the residential sector. Next, we study more general contracts

[11]While mostly used in non-residential sectors, chauffage contract is sometimes applied to residential clients as well. For example, in some apartments in Europe, heating is under chauffage contracts. Room temperature is fixed at 68 Fahrenheit during days and 64 Fahrenheit during nights for the whole complex. It does not suit to each individual apartment.

(shared savings contract and guaranteed savings contract) that deal with situations where rebound effect is inevitable.

4.5 Indirect Control of Rebound Effect

Facing the rebound effect, the ESCO is challenged to design appropriate contracts. We start with a case where the ESCO's effort in improving client's technology level is contractible and study how well 1-rate and 2-rate contracts perform in overcoming the rebound effect and achieving the first-best outcome (Subsection 4.5.1). Next we relax this assumption by considering the case of moral hazard, where ESCO's effort is not observable (Subsection 4.5.2).

4.5.1 Model with Complete Observability of ESCO's Effort

With contractibility of expected technology level, the ESCO's offering should include expected technology level and payment structure. The main problem stated in (4.7) becomes

$$\max_{t,\{z_i\},\{\alpha_i\}} \left\{ \mathbf{E}\left[v(x^D, t+\epsilon)\right] - \frac{\lambda}{2} Var\left[v(x^D, t+\epsilon)\right] - v_0 - \mathbf{E}w_e(x^D) - C(t) \right\}. \quad (4.10)$$

In this subsection, the superscript O/n, for observability and n-rate, is used in optimal solutions.

Theorem IV.4.

(1) Any contracts that are only contingent on consumptions (including n-rate contracts) cannot reach the first-best outcome.

(2) Under 1-rate contracts,

(2a) There exists a unique optimal effort level $t^{O/1}$, which satisfies $C'(t) = p$.

(2b) There exists a unique optimal payment rate $\alpha^{O/1}$ and $\alpha^{O/1} \in (0, 1)$.

(2c) The project value, $V^{O/1}$, decreases in risk aversion coefficient λ and project uncertainty σ^2.

As Theorem IV.4(1) indicates, contracts that are contingent on energy consumption are fundamentally limited. In order to completely remove rebound effect, the ESCO has to ask the client to take full energy variable cost. This, however, exposes the client to uncertain outcome of new technology, which decreases the overall surplus due to the client's risk aversion. Unable to overcome combined effect of rebound and risk aversion, such contracts cannot achieve the first-best outcome. Despite of non-existence of coordinating contracts, we evaluate how n-rate contracts perform, with attention to 1-rate and 2-rate contracts.

Under 1-rate contracts, the ESCO needs to decide the effort level and the single payment rate α applied to all energy usage. As Theorem IV.4(2a) shows, the effort level $t^{O/1}$ is equal to that with central planner, t^C. When the effort level is contractible, the ESCO is able to fully internalize the benefit of better technology. While ESCO only pays a fraction of the energy bill, he can charge the client up-front for her savings. Therefore, all benefit eventually goes into the ESCO's pocket, which drives him to make decision just like the central planner.

With respect to the payment rate as concerned in Theorem IV.4(2b), if the ESCO pays all variable cost (i.e., $\alpha = 0$), the client would use too much of energy without considering its cost. If the client pays all the variable cost (i.e., $\alpha = 1$), she faces too much uncertainty that generates disutility. As a trade-off, the optimal sharing ratio, $\alpha^{O/1}$, is strictly between 0 and 1. This means, both rebound effect and risk-aversion are partially mitigated but they still exist. This reinforces the fact that ESCO cannot get the first-best outcome by n-rate contract.

Theorem IV.4(2c) is intuitive. The project is more profitable when the client is less risk averse (smaller λ) or when the technology outcome is less uncertain (smaller σ^2). The project value, ESCO's technology (effort) choice, and average energy usage are plotted with respect to λ and σ^2 in Figure 4.1. The project value, $V^{O/1}$, is illustrated in Figure 4.1(a) and (d) and is compared against the first-best outcome, V^C. As the client becomes more risk averse or the project uncertainty becomes larger, the gap between $V^{O/1}$ and V^C also becomes larger. When the client is extremely risk averse, the project value may become negative, making it impossible for the ESCO to profit from such a project.

The energy usage increases in λ and σ^2, as shown in Figure 4.1(c) and (f), respectively. As the client is more risk reverse or the project outcome is more uncertain, the ESCO offers to pay a larger portion of the variable cost, in order to decrease the client's disutility associated to uncertainty. This payment structure, however, provides undesirable incentive for the client to over consume energy.

Although so far we have focused on 1-rate contracts, it is also interesting to explore how 2-rate (or n-rate) contracts compare with 1-rate contracts. While it is challenging to obtain analytical results under 2-rate contracts, we rely on numerical studies to obtain insights on how additional flexibility of 2-rate contracts improves the ESCO's profit.

As expected, the optimal 2-rate contract strictly outperforms the optimal 1-rate contract. The underlying idea is illustrated in Figure 4.2. Assuming the ESCO's effort is t, the client's energy consumption, the client's utility, and the ESCO's payment are plotted for each technology realization $(t + \epsilon)$.

The ESCO's intention is to design a contract to make the client's consumption closer to that with central planner, while at the same time reducing the uncertainty faced by the client. This is equivalent to maximizing system surplus as the ESCO can extract the whole surplus through up-front payment. Consider the ESCO starts from the optimal 1-rate contract, $(\varnothing, \{\alpha^{O/1}\})$. With the flexibility of a second rate, the ESCO would raise α_1 (while keeping $\alpha_2 = \alpha^{O/1}$ for this moment), and with appropriate z, apply it to very desirable technology realizations. Hence, when $t + \epsilon$

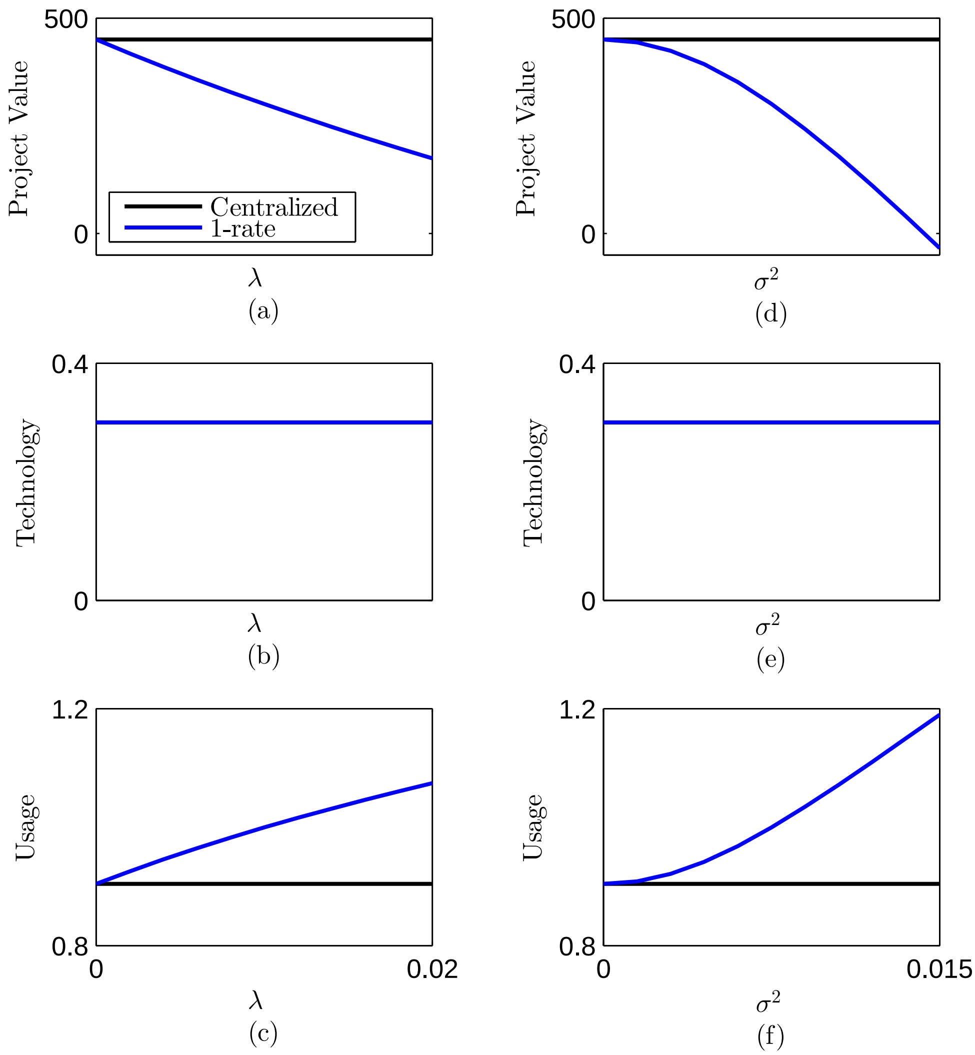


Figure 4.1: Project value, technology, and average energy usage under optimal 1-rate contract, benchmarked with those in centralized case. Parameters: $u(x+t) = 10^4 \times (1 - e^{-(x+t)}), C(t) = \frac{10^4}{2}t^2, p = 0.3 \times 10^4, \sigma^2 = 0.004$ (a-c), $\lambda = 0.01$ (d-f).

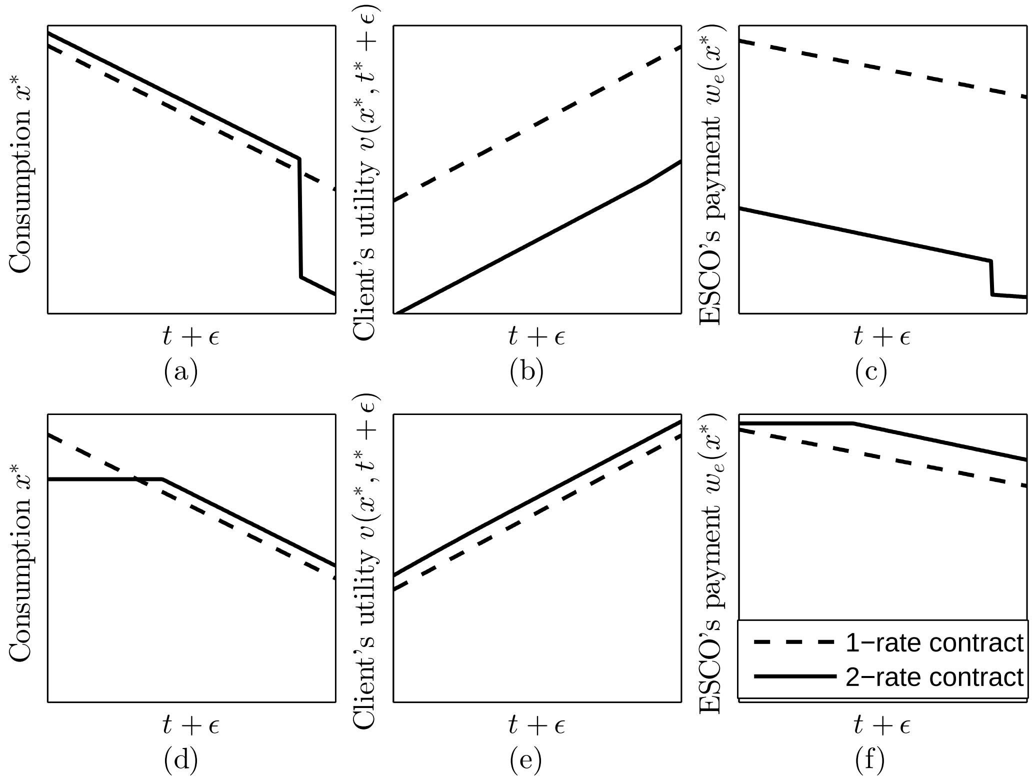


Figure 4.2: Strategies under 1-rate and 2-rate contracts. Parameters: $u(x+t) = 1 - e^{-(x+t)}, C(t) = \frac{1}{2}t^2, p = 0.3, \lambda = 300, \epsilon \sim U[-0.1, 0.1]$. Optimal 1-rate contract: $\alpha = 0.81, t = 0.3$. Optimal 2-rate contract: $\alpha_1 = 0.93, \alpha_2 = 0.79, z = 0.98, t = 0.3$.

is close to the right end, the energy consumption is significantly reduced (as seen in Figure 4.2(a)) and much closer to the centralized consumption, x^C. As a result of greater α_1, in Figure 4.2(b), the right end of the client's utility curve gets slightly higher slope, leading to higher variance. However, if only a small piece at the right end becomes steeper, the impact on overall variance is very small. In other words, consumption cost reduction is linear in α_1 while the variance increase is convex. Therefore, for the whole system, the rise of variance-related disutility is outweighed by the benefit of the smaller consumption.

As a secondary effect, the ESCO would slightly decreases α_2, which applies to the rest of technology realizations. This is because the ESCO is concerned less about client's consumption rebound due to larger α_1, while sharing more risk from the client is beneficial. Smaller α_2 makes consumption a bit higher but brings down the variance. Adjusted α_1 and α_2 together will typically lead to both smaller consumption and smaller variance, thus help mitigate the rebound effect and risk aversion further than 1-rate contracts.

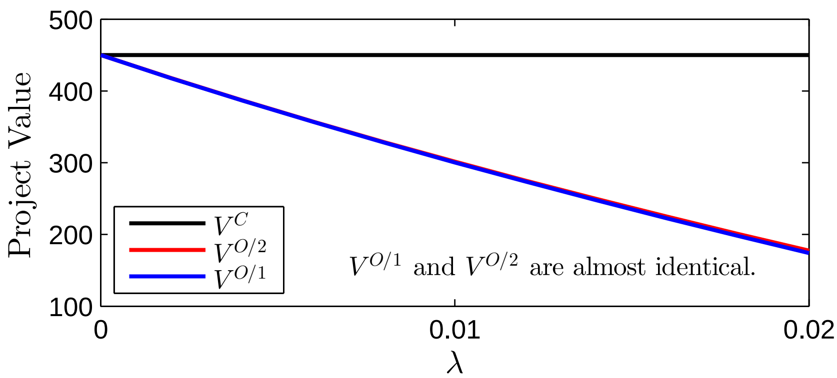


Figure 4.3: Project value under 1-rate and 2-rate contracts, benchmarked with that in centralized case. Parameters: $u(x+t) = 10^4 \times (1 - e^{-(x+t)}), C(t) = \frac{10^4}{2}t^2, p = 0.3 \times 10^4, \epsilon \sim U[-0.1, 0.1]$.

While 2-rate contracts do improve the surplus, the improvement is very limited. As shown in Figure 4.3, the optimal 2-rate contract only increases the project value by a small margin. We also numerically study 3-rate contracts $(\{z_1, z_2\}, \{\alpha_1, \alpha_2, \alpha_3\})$. While the ESCO, with 2-rate contract, can only curb energy consumptions for either very desirable technology realizations or very undesirable ones, now he works on both sides by setting α_1 and α_3 both great than $\alpha^{O/1}$. For those moderate realizations, a smaller α_2 allows the ESCO to take more risk from the client. Despite that 3-rate contracts perform strictly better, we observe diminishing return of the third rate.

The analysis of model with complete observability of ESCO's effort does not only sets an upper bound for the performance of similar contracts in the general problem with moral hazard, but also sheds some light on effective mechanisms. Practically speaking, government provides a wide range of certification programs for ESCOs.

Certified by government, an ESCO earns trust from clients much easier, which essentially makes their technology level observable. In such situations, optimal 1-rate contract (shared savings contract) works well and captures most benefit of performance contracting.

4.5.2 Model with ESCO's Moral Hazard

Without contractibility of his effort, the ESCO has to consider in the contract how to overcome moral hazard problem. That is, the payment scheme has to take care of the ESCO's incentive, additional on the client's rebound effect and her risk aversion. The superscript M/n, for moral hazard and n-rate, is used to denote optimal solution in this subsection.

Theorem IV.5.

(1) Under 1-rate contracts,

(1a) There exists a unique optimal effort level $t^{M/1}(\alpha)$, which satisfies $C'(t) = (1-\alpha)p$.

(1b) There exists a unique optimal payment rate $\alpha^{M/1}$ and $\alpha^{M/1} < \alpha^{O/1}$.

(2) Under 2-rate contracts, if $u' > 0$ and $u(\cdot)$ is bounded, we have $V^{O/1} \leq V^{M/2} \leq V^{O/2}$.

Under 1-rate contracts, the ESCO's optimal strategy is illustrated in Theorem IV.5(1). The ESCO's effort is non-contractible and determined after the contract is signed, and thus it is contingent on payment rate α. As the effort decision takes place after the contract is signed, the ESCO would only weigh his own portion of energy savings into his decision but ignore the client's. This is the failure in standard moral hazard problem and not surprisingly the ESCO spends less effort than when his effort is contractible, i.e., $t^{M/1}(\alpha) < t^{O/1} = t^C$.

The client also anticipates this failure and is willing to pay less up-front for the project. In order to convey his commitment of reasonable effort, the ESCO has to take a greater share of the variable energy cost. Therefore in the optimal strategy we have $\alpha^{M/1} < \alpha^{O/1}$.

Theorem IV.5(2) shows that $V^{M/2}$, the project value under 2-rate contracts with moral hazard, is at least as high as $V^{O/1}$. In fact, the ESCO can get exactly the same project value as $V^{O/1}$ by the 2-rate contract ($z^{M/2}$, $\{\alpha^{O/1}, 0\}$), where $z^{M/2}$ is given in the proof of Theorem IV.5(2). The intuition is illustrated in Figure 4.4. With $\alpha_2 = 0$, the client would use infinite amount of energy if technology realization is below certain level. By strategically choosing z_1 in the contract, this technology threshold can be made at $t^{O/1} + \underline{\epsilon}$, as shown in Figure 4.4(a). The ESCO needs to subsidize any energy usage above z_1, and the total subsidy also goes infinite when the technology realization is below $t^{O/1} + \underline{\epsilon}$, as shown in Figure 4.4(c). To avoid large payment under such situations, the ESCO should spend enough effort so that this does not happen even when the realization ends up with its worst possible value. That is, he commits to spend at least $t^{O/1}$ effort. Therefore, the ESCO's moral hazard problem is fully eliminated even though his effort is not contractible.[12]

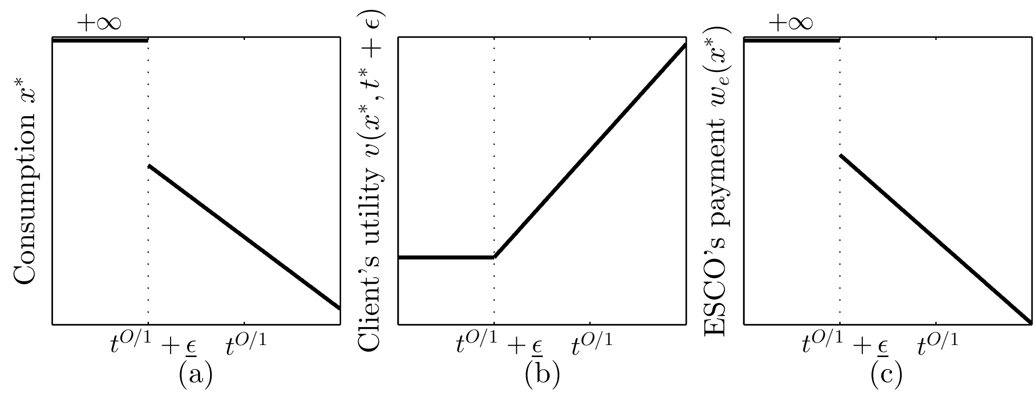


Figure 4.4: Illustration of ESCO's commitment strategy.

As discussed in the previous subsection when the ESCO's effort is contractible,

[12]We numerically test situations when $u(\cdot)$ is an general increasing-decreasing concave function. Theorem IV.5(2) and the intuition continues to hold as long as the maximizer of $u(\cdot)$ is large enough such that the ESCO would be better off to avoid paying large subsidy for poor technology realizations.

the optimal 1-rate contract performs reasonably well and the optimal 2-rate contract only marginally improves the project value. Because of Theorem IV.5(2), $V^{O/1}$ can be used as an approximate for $V^{M/2}$. The improvement of 3-rate contracts are limited by corresponding contracts with contractibility (i.e., $V^{M/3} \leq V^{O/3}$). Therefore, in practice using 2-rate contracts should be good enough to capture most benefit of performance contracting. The 2-rate contract can be interpreted as a combination of guaranteed saving contract and shared savings contract. It specifies a guaranteed usage threshold and ESCO is responsible for any cost above that. If there is additional savings below the threshold, the savings is shared by the ESCO and client.

As we have seen so far, the unattainability of the first-best solution cannot be resolved even with observability of the ESCO's actions. The following result states that customers' risk aversion, another significant barrier to EE project adoption, is mainly to blame. The superscript add-on RN is used to denote the optimal solution when the client is risk neutral.

Corollary IV.1. *If $u' > 0$ and $u(\cdot)$ is bounded, when the client is risk neutral ($\lambda = 0$), the 2-rate contract ($z^{M-RN/2}$, $\{1, 0\}$) achieves the first-best outcome. $z^{M-RN/2}$ is given in the proof.*

When the client is risk neutral, the ESCO does not have to share the risk from the client. As the client now pays all the variable energy cost, she will voluntarily choose globally optimal consumption amount, removing negative impact of rebound effect. The 2-rate contract ($z^{M-RN/2}$, $\{1, 0\}$) is a guaranteed savings contract. With a similar intuition as Theorem IV.5(2), it allows the ESCO to achieve the first-best outcome.

This result is not immediately obvious. In classic double moral hazard research, the first-best solution is not possible even when the client is risk neutral (Holmstrom 1982, Bhattacharyya and Lafontaine 1995). In their studies, the two players have to split monetary outcome of both their efforts. This zero-sum contract makes it impossible to give both players right incentives to exert efforts. In our paper, however, contracts are not designed based on any monetary outcome but on amount of energy the client uses. The critical element is that, the impact of energy consumption on the client's utility function is concave. However, the energy consumption has linear effect on the ESCO's surplus, because he pays a portion of energy cost. This difference makes the commitment strategy in Corollary IV.1 possible, which allows the ESCO to achieve the first-best outcome.

It is worth noting that a project with deterministic technology is a special case of the risk neutral model, and thus the first-best outcome can be achieved. While the client's risk attitude is difficult to change, the ESCO and policy makers can work on reducing the uncertainty of a project, in order to make EE projects more profitable. The ESCO may conduct inspections or measurement prior to a project. Policy makers also have a few levers in this field. They can provide or mandate house energy efficiency rating, or set standards for energy efficiency projects. For example, Energy Performance Certificates (EPCs) are required in UK when a property is sold or rented. EPCs include the property's current energy efficiency, recommendations about how to improve energy efficiency (e.g., plumbing or HVAC upgrade), and estimated improvement after recommended projects.[13] With more information, both the ESCO and client are more confident to predict the outcome, and consequently a project is more likely to become a success.

4.6 Extensions

We extend the model to include a number of additional frictions so that it becomes closer to reality. Throughout this section, analysis is conducted with parametric forms of comfort function and effort cost function, which allows us to accommodate additional complexities such as client's effort or policy makers' preference, but still

[13] More information at `www.gov.uk/buy-sell-your-home/energy-performance-certificates`.

have enough insights. We assume the comfort function is

$$u(x + t) = -e^{-(x+t)} \tag{4.11}$$

and effort cost function is

$$C(t) = \frac{1}{2\gamma}t^2.$$

With specific comfort and cost functions, we rewrite strategies stated in Theorems IV.1, IV.4, and IV.5 in their closed forms, so that it is easy to compare them with other results.

Corollary IV.2.

(1) The central planner's optimal strategy is: $x^C(t+\epsilon) = -\ln p - t - \epsilon$*;* $w_c^C(t+\epsilon) = 0$*;* $t^C = \gamma p$*.*

(2) Assume ESCO's effort level is contractible. Under 1-rate contracts,

(2a) The unique optimal effort level is $t^{O/1} = \gamma p$*.*

(2b) The unique optimal payment rate is

$$\alpha^{O/1} = \frac{2}{1 + \sqrt{1 + 4\lambda p \sigma^2}}.$$

(3) Assume ESCO's effort level is non-contractible. Under 1-rate contracts,

(3a) The unique optimal effort level is: $t^{M/1}(\alpha) = (1 - \alpha)\gamma p$*.*

(3b) The unique optimal payment rate is:

$$\alpha^{M/1} = \frac{2}{1 + \sqrt{1 + 4p(\lambda\sigma^2 + \gamma)}}.$$

In addition to insights discussed in Section 4.5, it is also interesting to observe in Corollary IV.2(2b) and (3b) that the optimal payment rate is decreasing in the unit energy price p. The intuition is that, when the energy is more expensive, the same level of uncertainty in energy usage translates to larger uncertainty in the client's utility. As a result, the ESCO would have to pay more of energy variable cost in order to manage the client's disutility related to risk aversion.

In the rest of this section, we first look at additional factors involved in energy efficiency projects. These include three topics: (1) the client's effort to reduce energy consumption, (2) the impact of external uncertainty (e.g., weather uncertainty), and (3) the impact of initial technology. Then we turn to other stakeholders of energy efficiency projects. Such players are (1) utility companies that own ESCOs, and (2) policy makers.

4.6.1 Client's Effort

Besides choosing energy usage, the client can also put effort into improving energy efficiency in many situations. For example, it helps if one client always closes windows when using heating or turns off lighting when leaving home. To reflect these observations, we slightly modify the model to include the client's effort in a similar structure as the ESCO's effort. The client's effort is denoted as q, with convex cost structure $\frac{1}{2\theta}q^2$. The parameter θ represents difficulty of the client's effort. As it becomes more difficult, i.e., $\theta \to 0$, the problem will converge to the original one studied in the main part. The client's comfort function and utility function become

$$u(x+t+\epsilon+q) = -e^{-(x+t+\epsilon+q)} \text{ and } v(x, t+\epsilon, q) = u(x+t+\epsilon+q) - w_c(x) - \frac{1}{2\theta}q^2.$$

After observing technology realization, the client's problem, originally stated in Equation (4.4), becomes

$$[x^*, q^*] = \arg\max_{x,q} \; v(x, t+\epsilon, q). \tag{4.12}$$

The ESCO's problem, in Equation (4.7), does not change except the client's effort q^* appearing as argument for relevant terms.

In this extension, we only consider 1-rate contracts when the ESCO's effort is contractible and 2-rate contracts when moral hazard on the ESCO side exists. As argued in Sections 4.5.1 and 4.5.2, they are good enough to capture the most of benefit from performance contracting. The superscript add-on CE, for client's effort, is used to denote optimal solutions in this extension.

Theorem IV.6.

(1) The central planner's optimal strategy is: $x^{C-CE}(t+\epsilon) = -\ln p - t - \epsilon - \theta p$, $q^{C-CE}(t+\epsilon) = \theta p$, $w_c^{C-CE}(t+\epsilon) = 0$, *and* $t^{C-CE} = \gamma p$ *.*

(2) Assume ESCO's effort level is contractible. Under 1-rate contracts, the optimal strategy is

$$\alpha^{O-CE/1} = \frac{2}{1-\theta p + \sqrt{(1+\theta p)^2 + 4\lambda p \sigma^2}} \text{ and } t^{O-CE/1} = \gamma p.$$

(3) Assume ESCO's effort level is non-contractible. Under 2-rate contracts, we have $V^{O-CE/1} \leq V^{M-CE/2} \leq V^{O-CE/2}$.

Theorem IV.6(1) is a direct generalization of Corollary IV.2(1). In central planner's strategy, both the ESCO's and client's efforts are fully leveraged to improve technology and to reduce energy consumption. That means, the marginal cost of the effort equals to the full unit cost of energy.

Theorem IV.6(2) generalizes the results in Corollary IV.2(2) to incorporate the client's effort. Similar to Corollary IV.2(2a), the contractibility of the ESCO's effort allows him to internalize all the benefit, and thus his effort decision is aligned with that of central planner. The client's share of energy cost here, $\alpha^{O-CE/1}$, is greater than $\alpha^{O/1}$ in Corollary IV.2(2b), and increases for a greater θ. Besides balancing risk aversion and rebound effect, α here also has to play the role to incentivize the client to put effort. The larger portion of unit price the client is charged for energy usage, the more effort she puts into reducing energy usage. If the client's effort is cheaper (greater θ), the ESCO will weight this incentivizing role more while compromising the ability to share risk, leading to a higher α.

Theorem IV.6(3) replicates the result in Theorem IV.5(2). The intuition is the same: Offering extremely cheap energy above an appropriate threshold, the ESCO effectively commits himself into a higher effort level to avoid huge amount of subsidy.

4.6.2 External Uncertainty

In this extension we would like to consider external uncertainties. For example, energy consumption is heavily correlated to temperature, which is uncertain ahead of time to both the ESCO and client. Different from technology uncertainty (which is labeled as internal), external uncertainty exists and makes the risk averse client suffer despite of whether a project is adopted.

The external uncertainty is introduced to the client's comfort function (4.11) and it becomes

$$u(x+t) = -e^{-(x+t+\epsilon_w)}.$$

ϵ_w represents the external uncertainty and it has mean of zero and variance of σ_w^2. ϵ_w is assumed to be independent from the technology uncertainty ϵ. With two random shocks, the problem structure remains and thus all results in the main part continue to hold. Here we are interested in how the external uncertainty influences the ESCO's and client's surpluses. We already know that if the ESCO's effort is contractible, 1-rate contracts are nearly as good as any complicated contracts but more implementable. When ESCO's effort is not contractible, 2-rate contracts are good enough. The outcome of the optimal 2-rate contract can be approximated by the optimal 1-rate contract with contractibility. Therefore, in this and all following extensions, we use 1-rate contract with contractibility to illustrate results.

Theorem IV.7. *The project value, $V^{O/1}$, increases in external uncertainty σ_w^2, while the client's utility decreases in σ_w^2.*

It is intuitive that the risk averse client suffers from additional uncertainty. However, the ESCO's surplus increases in external uncertainty while it decreases in technology uncertainty as indicated in Theorem IV.4(2c). The reason is that technology uncertainty only exists after the project, which makes the project less attractive since the client is risk averse. On the other hand, the client has to face external uncertainty both before and after the project. Through PBCs, the ESCO is able to share some external uncertainty and reduce the client's disutility. Consequently, the client is willing to pay a premium for the project.

4.6.3 Initial Technology

In the main part of this paper, the initial technology, t_0, is normalized to 0. In this extension, we study how the initial technology impacts project value. The client's pre-project comfort function is

$$u(x + t_0) = -e^{-(x+t_0)}.$$

We consider two types of improvements. In the first type, the technology is incremental. For example, the ESCO can inject additional insulation materials into walls. In such cases, the client's post-project comfort function is

$$u(x + t_0 + t + \epsilon) = -e^{-(x+t_0+t+\epsilon)}.$$

In the second type, the technology replaces the original one. For example, the ESCO can install a new window which replaces the old one. The new window will work as it is designed, despite of the quality of the old window. That is, the client's comfort function is

$$u(x + t + \epsilon) = -e^{-(x+t+\epsilon)},$$

which is independent of initial technology. Theorem IV.8 illustrates how the project value may depend on the initial technology level.

Theorem IV.8.

(1) If the technology is incremental, the project value, $V^{O/1}$, is constant in t_0.

(2) If the technology is a replacement, the project value, $V^{O/1}$, decreases in t_0.

The technology and the energy consumption are perfectly substitutable in the client's comfort function. If the technology is increment, the initial technology does not matter because any additional technology will reduce same amount of energy usage, and thus deliver same amount of value to the client. If the technology is a replacement, the post-project utility is not dependent on the initial technology. Therefore, the lousier the initial technology is, the lower utility the client has before the project, the more value the project can create.

4.6.4 Utility-owned ESCO

Energy efficiency projects are delivered not only by independent ESCOs but also by utility-owned ESCOs (Goldman et al. 2005, Larsen et al. 2012). In this extension our model is modified to accommodate utility-owned ESCOs.

The energy retail cost, p, is assumed exogenous in the main part. Now we allow utility-owned ESCO to change retail price. However, the cost to produce energy, denoted by p_0, is exogenously given. The ESCO's problem, presented in Equations (4.6) and (4.7), becomes

$$\begin{aligned} t^* &= \arg\max_t \left\{ -\mathbf{E}w_e(x^*) - \frac{1}{2\gamma}t^2 + (p-p_0)\mathbf{E}x^* \right\}, \\ \max_{p,\{z_i\},\{\alpha_i\}} & \left\{ \mathbf{E}\left[v(x^*, t^*+\epsilon)\right] - \frac{\lambda}{2}Var\left[v(x^*, t^*+\epsilon)\right] - v_0 - \mathbf{E}w_e(x^*) - \frac{1}{2\gamma}t^{*2} + (p-p_0)\mathbf{E}x^* \right\}. \end{aligned} \tag{4.13}$$

Original solutions to Problem (4.7) are functions in exogenous retail price p. In

Problem (4.13), with retail price p as a decision variable, solutions are functions in production cost p_0.

Theorem IV.9. *Let the optimal contract in Problem (4.7) be* $(\{z_i(p)\}, \{\alpha_i(p)\})$. *Then in Problem (4.13), the optimal contract is* $(p, \{z_i(p_0)\}, \{\frac{p_0}{p}\alpha_i(p_0)\})$, *where retail price p can be any value.*

While the ESCO has one more lever (energy price) to construct the contract, the problem structure turns out to be equivalent to the original one. The utility-owned ESCO now uses p_0 instead of p as the energy cost to solve the original problem, and get the optimal contract $(\{z_i\}, \{\alpha_i'\})$. The solution is independent of the retail price p, which means any $p > p_0$ can deliver the maximal project value to the ESCO. This seems a distortion to the client's marginal cost when the consumption decision is made. But, it is easily recovered by adjusted α_i's (i.e., for any retail price p, setting $\alpha_i = \frac{p_0}{p}\alpha_i'$ to achieve the same outcome). Therefore, all results shown above continue to hold.

The implication to utility-owned ESCOs is that contracts should be based on their real energy cost rather than the retail price. Since any retail price would work, the ESCO can safely keep the standard retail price unchanged and choose other contract terms accordingly. This convenience makes such contracts easy to implement.

4.6.5 Policy Implication

Energy efficiency does not only save energy cost, but also has huge potential to mitigate environmental impact of energy production, delivery and consumption. The environmental impact is often not internalized by either the ESCO or the client. Therefore, policy makers play a important role in this business. As Steinberger et al. (2009) summarizes, there exist two categories of policies relevant to the ESCO industry: regulations and monetary incentives. As each category is only briefly covered in the following, readers are referred to Ryan et al. (2011) and Cunningham and Cook

(2015) for an extensive list and discussion about energy efficiency policies. Regulations include ESCO certification, house energy efficiency grading, energy efficiency standard, etc. While in general such regulations help reduce uncertainty of energy efficiency projects or link clients to trustworthy ESCOs, modeling impacts of regulations is beyond the scope of this paper.

Monetary Incentives are the focus of this extension. Common incentives include tax credit, carbon tax, etc. Tax credit is to give certain amount or certain percentage of project cost as tax credit. For example, one gets 10% of the cost, up to \$500 as federal tax credits for insulation improvement.[14] Carbon tax is charged to all clients despite whether energy efficiency project is adopted. It is added to unit energy price by local government. For example, Boulder, Colorado implemented the United States' first tax on carbon emissions from electricity, on April 1, 2007, at a level of approximately \$7 per ton of carbon.[15]

Most monetary incentives fall into two categories: either a lump sum subsidy or carbon tax on each unit of energy consumption. With lump sum subsidy, we denote the amount of subsidy to the ESCO as G. With carbon tax policy, we denote the tax on each unit energy as r, lifting the total energy cost to $p + r$. We assume each unit energy usage incurs an environmental cost, denoted by c. It is common to quantify environmental impact in energy and environment studies. For example, Lazer and Farnsworth (2011) estimates that the emission cost of each KWH can be as high as 3 cents. Policy makers' objective is to maximize social surplus, including environmental cost. This is the most common way to model environmental externalities, as seen in many energy efficiency-focused papers and beyond (Eom and Sweeney 2009, Chun et al. 2013, Cachon 2014, Cohen et al. 2015). Same to other extensions, we use 1-rate contract with contractibility to illustrate the two policies in Theorem IV.10.

Theorem IV.10.

[14] More information at `www.energystar.gov/about/federal_tax_credits/insulation`.
[15] More information at `www.carbontax.org/where-carbon-is-taxed`.

(1) With lump sum subsidy, the optimal subsidy amount is $G^* = c(\ln \alpha^{O/1} + \gamma p)$.

(2) With carbon tax policy, the optimal tax is $r^* = c$.

(3) Compared to lump sum subsidy, carbon tax policy leads to greater technology investment, lower energy consumption, and higher social surplus.

The lump sum subsidy does not change the ESCO's effort and the client's consumption decisions. However, it allows the ESCO to charge a lower up-front fee, which in turn makes more projects happen. In fact, with the optimal subsidy amount, any socially beneficial projects would also be profitable for the ESCO. With carbon tax policy, policy makers would charge exactly the environmental cost to the client. A higher energy cost does not only make more projects happen, but also induces the ESCO and the client to make socially better decisions. That is, the client will use less energy and the ESCO will spend more effort. Therefore, carbon tax is more desirable to policy makers from economic perspective.

4.7 Conclusion

Energy efficiency is one of the smartest approaches to reduce energy cost and reduce environmental impact. Most energy efficiency projects are outsourced to ESCOs and are based on PBCs. While having thrived for decades in public, commercial and industrial sectors, the ESCO business struggles in the residential sector. This paper studies the contract design problem for energy efficiency projects. It also focuses on what are the enablers that make energy efficiency projects more successful from both ESCOs' and policy makers' perspectives.

Three most popular PBCs – shared savings contract, guaranteed savings contract, and chauffage contract – are examined. In the residential sector, if the ESCO's effort is contractible, a simple linear contract (shared savings contract) performs almost as well as other complicated contracts. The savings have to be wisely shared between the ESCO and the client, in an effort to balance the negative impact of risk aversion and rebound effect. When the ESCO's effort is not contractible, a 2-rate contract (combination of guaranteed savings contract and shared savings contract) provides reasonably good outcome and captures most benefit of performance contracting. While chauffage contract is widely adopted in non-residential sectors, requiring specific and fixed comfort level makes it less attractive in the residential sector.

When both rebound effect and risk aversion are present, any contracts contingent only on post-project energy consumption cannot achieve the first-best outcome. That said, both the ESCO and policy makers have potential approaches to increase project value. The ESCO, for example, can develop the capability of post-project technology measurement. If the ESCO is able to design contracts based on post-project technology instead of energy consumption, it is optimal to ask the client to take all variable cost and the first-best outcome can be achieved. The ESCO can also conduct pre-project inspection, in order to reduce uncertainty of a project, which in turn increases the profitability.

Government regulations can also help increase project value. For example, ESCO certification identifies trustworthy ESCOs and thus remove moral hazard problem. Informational programs, such as house energy efficiency rating and energy efficiency standard, play a role in reducing project uncertainty. Concerned about environmental cost associated to energy usage, policy makers may also want to provide monetary incentives. Both lump sum subsidy and carbon tax make more energy efficiency projects happen. Carbon tax also encourages greater ESCO's effort and less energy consumption, and leads to higher social surplus than lump sum subsidy. Therefore, carbon tax is more desirable for policy makers from economic perspective.

CHAPTER V

Conclusion

My dissertation includes three essays on operational contracts. In Chapter II, we consider inventory exchange contracts in competitive markets. Results show that while firms often benefit from exchanging their inventories, exceptions exist. They may be worse off when the purchasing cost is either very high or very low, both of which lead to excessive inventories compared to the no-trade case. Firms trading inventories also increases consumer surplus in majority of cases as long as the purchasing cost is not very low. Therefore, inventory exchange contracts are most often a win-win solution.

In Chapter III, we study vertical price constraint contracts that are used to coordinate the supply chain. The RPM and MAP policies are widely used by the manufacturer to encourage brick-and-mortar retailers to spend sales effort. We find MAP policy is preferred by the manufacturer when the customer's valuations span in a large range and the search cost is significant. Otherwise, the manufacturer would be better off choosing RPM policy. Regular retailers and consumers would also prefer MAP policy if the search cost is even higher or consumer valuations have even larger variance, compared to the manufacturers. Under such situations, MAP policy provides all parties in the supply chain a higher profit or surplus.

In Chapter IV, we focus on the design of performance based contracts between

ESCOs and residential clients. Results indicate that piecewise linear contracts perform nearly as well as any other complicated contracts. That said, due to client's risk aversion and rebound effect, the first-best outcome is not attainable. It helps fix this efficiency gap, at least partially, to reduce uncertainty of technology or to develop the ability to verify post-project energy efficiency. Appropriate regulations and monetary incentives from policy makers also make energy efficiency projects more appealing, and thus help manage environmental impact associated to energy production, delivery and consumption.

With increasingly competitive markets and constantly changing tastes of consumers, doing business is not easy. All three of my essays are inspired by practical contractual issues in different industries. My research does not only provides deep understanding of challenges in each context, but also makes recommendations to players involved, and eventually contributes to more efficient, profitable and sustainable supply chains.

APPENDICES

APPENDIX A

Proofs of Lemmas and Theorems in Chapter II

Proof of Lemma II.1.

Part (1): Centralized Pricing.

Centralized firm solves the following problem stated in (2.1).

$$\Pi^{Cp}(\bar{q}_1, \bar{q}_2) = \max_{p_1, p_2 \geq 0} \sum_{i=1}^{2} p_i s_i + \lambda(\mu_1 + \mu_2)$$

Since the term $\lambda(\mu_1 + \mu_2)$ is a constant,we drop it out in this proof as well as all following proofs. We assume the demand has to be non-negative, otherwise the centralized firm can raise p_2 arbitrarily high while not hurting demand in market 2 by setting $p_1 = \frac{a}{b} p_2$.Since it is never optimal to generate demand higher than inventory, the problem reduces to

$$\begin{aligned} &\max_{(p_1,p_2) \in S} \; p_1(w_1 - ap_1 + bp_2) + p_2(w_2 - ap_2 + bp_1) \\ = &\max_{(p_1,p_2) \in S} \; p_1 w_1 + p_2 w_2 - (ap_1^2 - 2bp_1 p_2 + ap_2^2). \end{aligned}$$

where $S = \{(p_1, p_2) : 0 \leq d_i \leq \bar{q}_i, i = 1, 2\}$. In the objective function, the only nonlinear term $-(ap_1^2 - 2bp_1p_2 + ap_2^2)$ is strictly joint concave on (p_1, p_2) because $a > b$ and S is compact and convex, so there exists a unique optimal price pair. This completes the proof of part (1). The optimal prices are given in Table A.1 and regions are illustrated in Figure 2.1(a).

Region	Conditions	Solutions
R1	$\bar{q}_1 \geq \frac{1}{2}w_1$ $\bar{q}_2 \geq \frac{1}{2}w_2$	$p_1^{C*} = \frac{aw_1+bw_2}{2(a^2-b^2)}$ $p_2^{C*} = \frac{aw_2+bw_1}{2(a^2-b^2)}$
R2	$2a\bar{q}_1 + 2b\bar{q}_2 \leq aw_1 + bw_2$ $2a\bar{q}_2 + 2b\bar{q}_1 \leq aw_2 + bw_1$	$p_1^{C*} = \frac{a(w_1-\bar{q}_1)+b(w_2-\bar{q}_2)}{a^2-b^2}$ $p_2^{C*} = \frac{b(w_1-\bar{q}_1)+a(w_2-\bar{q}_2)}{a^2-b^2}$
R3	$2a\bar{q}_1 + 2b\bar{q}_2 \geq aw_1 + bw_2$ $\bar{q}_2 \leq \frac{1}{2}w_2$	$p_1^{C*} = \frac{aw_1+bw_2}{2(a^2-b^2)}$ $p_2^{C*} = \frac{abw_1+(2a^2-b^2)w_2-2(a^2-b^2)\bar{q}_2}{2a(a^2-b^2)}$
R4	$\bar{q}_1 \leq \frac{1}{2}w_1$ $2a\bar{q}_2 + 2b\bar{q}_1 \leq aw_2 + bw_1$	$p_1^{C*} = \frac{abw_2+(2a^2-b^2)w_1-2(a^2-b^2)\bar{q}_1}{2a(a^2-b^2)}$ $p_2^{C*} = \frac{aw_2+bw_1}{2(a^2-b^2)}$

Table A.1: Solutions to prices in centralized scenario.

Part (2): Decentralized Pricing.

The pricing problems in trade and no-trade scenarios are identical and stated in (2.4). Dropping all constant terms, the problem is equivalent to the following:[1]

$$\max_{p_1} p_1(w_1 - ap_1 + bp_2 - a\lambda\gamma - b\lambda\gamma)$$

$$\text{s.t. } w_1 - ap_1 + bp_2 \leq \bar{q}_1, \ p_1 \geq 0$$

After droping constant terms, λ and γ show always together. Therefore, we normalize $\gamma = 1$, except the final numerical studies.The best response function of firm i is

$$p_i = \max\Big\{0, \underbrace{\frac{w_i + bp_j - \bar{q}_i}{a}}_{A}, \underbrace{\frac{w_i + bp_j - a\lambda - b\lambda}{2a}}_{B}\Big\}.$$

Term B is the unconstrained optimum with enough inventory. Term A reflects constrained inventory. Price 0 is optimal when the market size w_i is very small. In this case, firms are willing to give their products for free in order to earn future market

[1]Requiring demand not exceed the available quantity allows us to immediately claim existence of equilibrium.

share. Since the response function is continuous and the slope is between $[0, 1)$, there exists a unique intersection. This completes the proof of part (2). The optimal prices are given in Table A.2. $\square$

Region	Conditions	Solutions($X = N, T$)
	$-(4a^2 - b^2)\bar{q}_1 + 2a^2 w_1 + abw_2 + (2a^3 + a^2 b - 2ab^2 - b^3)\lambda \le 0$ $-(4a^2 - b^2)\bar{q}_2 + 2a^2 w_2 + abw_1 + (2a^3 + a^2 b - 2ab^2 - b^3)\lambda \le 0$ $2aw_1 + bw_2 - (2a^2 + 3ab + b^2)\lambda \ge 0$ $2aw_2 + bw_1 - (2a^2 + 3ab + b^2)\lambda \ge 0$	$p_1^{X*} = \frac{2aw_1 + bw_2 - (2a^2+3ab+b^2)\lambda}{4a^2 - b^2}$ $p_2^{X*} = \frac{2aw_2 + bw_1 - (2a^2+3ab+b^2)\lambda}{4a^2 - b^2}$
B	$aw_1 + bw_2 - b\bar{q}_2 - (ab + a^2)\lambda \ge 0$ $-a^2 w_1 - abw_2 + (2a^2 - b^2)\bar{q}_1 + ab\bar{q}_2 - (a^3 + a^b - ab^2 - b^3)\lambda \ge 0$ $bw_1 + 2aw_2 - 2a\bar{q}_2 - (ab + b^2)\lambda \ge 0$ $abw_1 + 2a^2 w_2 - (4a^2 - b^2)\bar{q}_2 + (2a^3 + a^2 b - 2ab^2 - b^3)\lambda \ge 0$	$p_1^{X*} = \frac{aw_1 + bw_2 - b\bar{q}_2 - (ab+a^2)\lambda}{2a^2 - b^2}$ $p_2^{X*} = \frac{bw_1 + 2aw_2 - 2a\bar{q}_2 - (ab+b^2)\lambda}{2a^2 - b^2}$
C	$aw_2 + bw_1 - b\bar{q}_1 - (ab + a^2)\lambda \ge 0$ $-a^2 w_2 - abw_1 + (2a^2 - b^2)\bar{q}_2 + ab\bar{q}_1 - (a^3 + a^b - ab^2 - b^3)\lambda \ge 0$ $bw_2 + 2aw_1 - 2a\bar{q}_1 - (ab + b^2)\lambda \ge 0$ $abw_2 + 2a^2 w_1 - (4a^2 - b^2)\bar{q}_1 + (2a^3 + a^2 b - 2ab^2 - b^3)\lambda \ge 0$	$p_1^{X*} = \frac{aw_2 + bw_1 - b\bar{q}_1 - (ab+a^2)\lambda}{2a^2 - b^2}$ $p_2^{X*} = \frac{bw_2 + 2aw_1 - 2a\bar{q}_1 - (ab+b^2)\lambda}{2a^2 - b^2}$
D	$w_1 - (a + b)\lambda \ge 0$ $-w_1 + 2\bar{q}_1 - (a + b)\lambda \ge 0$ $-bw_1 - 2aw_2 + 2a\bar{q}_2 + (ab + b^2)\lambda \ge 0$ $-bw_1 - 2aw_2 + (2a^2 + 3ab + b^2)\lambda \ge 0$	$p_1^{X*} = \frac{w_1 - (a+b)\lambda}{2a}$ $p_2^{X*} = 0$
E	$w_2 - (a + b)\lambda \ge 0$ $-w_2 + 2\bar{q}_2 - (a + b)\lambda \ge 0$ $-bw_2 - 2aw_1 + 2a\bar{q}_1 + (ab + b^2)\lambda \ge 0$ $-bw_2 - 2aw_1 + (2a^2 + 3ab + b^2)\lambda \ge 0$	$p_1^{X*} = 0$ $p_2^{X*} = \frac{w_2 - (a+b)\lambda}{2a}$
F	$a^2 w_1 + abw_2 - (2a^2 - b^2)\bar{q}_1 - ab\bar{q}_2 + (a^3 + a^2 b - ab^2 - b^3)\lambda \ge 0$ $a^2 w_2 + abw_1 - (2a^2 - b^2)\bar{q}_2 - ab\bar{q}_1 + (a^3 + a^2 b - ab^2 - b^3)\lambda \ge 0$ $aw_1 + bw_2 - a\bar{q}_1 - b\bar{q}_2 \ge 0$ $aw_2 + bw_1 - a\bar{q}_2 - b\bar{q}_1 \ge 0$	$p_1^{X*} = \frac{aw_1 + bw_2 - a\bar{q}_1 - b\bar{q}_2}{a^2 - b^2}$ $p_2^{X*} = \frac{aw_2 + bw_1 - a\bar{q}_2 - b\bar{q}_1}{a^2 - b^2}$
G	$w_1 - 2\bar{q}_1 + (a + b)\lambda \ge 0$ $w_1 - \bar{q}_1 \ge 0$ $-bw_1 - aw_2 + b\bar{q}_1 + a\bar{q}_2 \ge 0$ $-bw_1 - aw_2 + b\bar{q}_1 + (a^2 + ab)\lambda \ge 0$	$p_1^{X*} = \frac{w_1 - \bar{q}_1}{a}$ $p_2^{X*} = 0$
H	$w_2 - 2\bar{q}_2 + (a + b)\lambda \ge 0$ $w_2 - \bar{q}_2 \ge 0$ $-bw_2 - aw_1 + b\bar{q}_2 + a\bar{q}_1 \ge 0$ $-bw_2 - aw_1 + b\bar{q}_2 + (a^2 + ab)\lambda \ge 0$	$p_1^{X*} = 0$ $p_2^{X*} = \frac{w_2 - \bar{q}_2}{a}$
I	$\bar{q}_1 - w_1 \ge 0$ $\bar{q}_2 - w_2 \ge 0$ $-w_1 + (a + b)\lambda \ge 0$ $-w_2 + (a + b)\lambda \ge 0$	$p_1^{X*} = 0$ $p_2^{X*} = 0$

Table A.2: Solutions to prices in trade and no-trade scenarios.

Proof of Lemma II.2.

Using the pricing solution in Table A.1 in the proof of Lemma II.1(1), we can explicitly express total profit Π_i^{Cp} in (2.1) for each region i and eliminate q_1 by using

$\bar{q}_1 = K - \bar{q}_2$:

$$\Pi_1^{Cp} = \frac{aw_1^2 + aw_2^2 + 2bw_1w_2}{4(a^2-b^2)},$$
$$\Pi_2^{Cp} = \frac{-2(a-b)\bar{q}_2^2 + (a-b)(2K - w_1 + w_2)\bar{q}_2 + aKw_1 + bKw_2 - aK^2}{a^2-b^2},$$
$$\Pi_3^{Cp} = \frac{-4(a^2-b^2)\bar{q}_2^2 + 4(a^2-b^2)w_2\bar{q}_2 + a^2w_1^2 + 2abw_1w_2 + b^2w_2^2}{4a(a^2-b^2)},$$
$$\Pi_4^{Cp} = \frac{-4(a^2-b^2)\bar{q}_2^2 + 4(a^2-b^2)(2K\bar{q}_2 + Kw_1 - w_1\bar{q}_2 - K^2) + a^2w_2^2 + 2abw_1w_2 + b^2w_1^2}{4a(a^2-b^2)}.$$

$\bar{q}_1 + \bar{q}_2 = K$ corresponds to a line with slope -1 in Figure 2.2(a). As a function of $\bar{q}_2$, Π_1^{Cp} is a constant; Π_2^{Cp} is concave; Π_3^{Cp} is increasing (because $\bar{q}_2 \le \frac{w_2}{2}$ in R3); Π_4^{Cp} is decreasing (because $\bar{q}_2 \ge K - \frac{w_1}{2}$ in R4). Therefore, maximum cannot be in Region 3 or 4.

By solving the maximization problem in Region 1 and 2, we get the optimal inventory allocation and revenue as following. If $w_1 + w_2 \le 2K$, $\bar{q}_2^*$ is any value in $[\frac{1}{2}w_2, K - \frac{1}{2}w_1]$ and $\Pi^{Ct}(K) = \frac{aw_1^2+2bw_1w_2+aw_2^2}{4(a^2-b^2)}$. If $w_1 - w_2 \ge 2K$, $\bar{q}_2^* = 0$ and $\Pi^{Ct}(K) = \frac{K(aw_1+bw_2-aK)}{a^2-b^2}$. If $w_2 - w_1 \ge 2K$, $\bar{q}_2^* = K$ and $\Pi^{Ct}(K) = \frac{K(aw_2+bw_1-aK)}{a^2-b^2}$. Otherwise, $\bar{q}_2^* = \frac{2K-w_1+w_2}{4}$ and $\Pi^{Ct}(K) = \frac{(w_1-w_2)^2}{8(a+b)} + \frac{K(w_1+w_2-K)}{2(a-b)}$. □

Proof of Lemma II.3.

Without loss of generality, we assume $w_1 \ge w_2$. From the proof of Lemma II.2, we have:

$$\Pi^{Ct}(K) = \begin{cases} \frac{K(aw_1+bw_2-aK)}{a^2-b^2}, \text{ if } K \le \frac{w_1-w_2}{2} \\ \frac{(w_1-w_2)^2}{8(a+b)} + \frac{K(w_1+w_2-K)}{2(a-b)}, \text{ if } \frac{w_1-w_2}{2} < K \le \frac{w_1+w_2}{2} \\ \frac{aw_1^2+2bw_1w_2+aw_2^2}{4(a^2-b^2)}, \text{ if } K > \frac{w_1+w_2}{2} \end{cases}.$$

It is straightforward to verify that $\Pi^{Ct}(K)$ is strictly increasing and strictly concave in the first two subcases and constant in the third one. □

Proof of Lemma II.4.

All the regions mentioned in this proof refer to those in Figure 2.2(b). Let $(\hat{q}_1, \hat{q}_2)$ be the intersection of all four regions and, thus, $K_1 = \hat{q}_1 + \hat{q}_2$. The value of K_1 depends on the realizations of (w_1, w_2) and is defined in Table A.3. The first row of the table corresponds to the intuitive intersection of the regions, while rows 2 to 4 correspond to the case where both prices, or one of the prices p_1 and p_2 is 0. The solution comes from evaluating the conditions in Table A.2. For example case A is translated into the first row below, or case I is translated into row 2. In each case, we solve for q_1 and q_2 satisfying all of the boundary conditions resulting in the outcome below.

Conditions on (w_1, w_2)	value of K_1
$2aw_1 + bw_2 \geq (2a^2 + 3ab + b^2)\lambda$,$2aw_2 + bw_1 \geq (2a^2 + 3ab + b^2)\lambda$	$\frac{a(w_1+w_2)+2(a^2-b^2)\lambda}{2a-b}$
$w_1 \leq (a+b)\lambda$,$w_2 \leq (a+b)\lambda$	$w_1 + w_2$
$w_1 > (a+b)\lambda$,$2aw_1 + bw_2 < (2a^2 + 3ab + b^2)\lambda$	$\frac{aw_1+2aw_2+bw_1+2(a^2-b^2)\lambda}{2a}$
$w_2 > (a+b)\lambda$,$2aw_2 + bw_1 < (2a^2 + 3ab + b^2)\lambda$	$\frac{aw_2+2aw_1+bw_2+2(a^2-b^2)\lambda}{2a}$

Table A.3: Value of K_1.

If $K \leq K_1$, the line $\bar{q}_1 + \bar{q}_2 = K$, which is called allocation line, crosses region R2, and usually also R3 and R4. Otherwise, the allocation line crossesregion R3, R1, and R4. Define

$$\begin{aligned}
\phi_1(K) &= \max_{\bar{q}_1+\bar{q}_2=K;(\bar{q}_1,\bar{q}_2)\in \text{R1 or R2}} \sum_{i=1}^{2} \pi_i^{Np}(\bar{q}_1, \bar{q}_2), \\
\phi_3(K) &= \max_{\bar{q}_1+\bar{q}_2=K;(\bar{q}_1,\bar{q}_2)\in \text{R3}} \sum_{i=1}^{2} \pi_i^{Np}(\bar{q}_1, \bar{q}_2), \\
\phi_4(K) &= \max_{\bar{q}_1+\bar{q}_2=K;(\bar{q}_1,\bar{q}_2)\in \text{R4}} \sum_{i=1}^{2} \pi_i^{Np}(\bar{q}_1, \bar{q}_2).
\end{aligned}$$

Therefore, we have $\Pi^{Tt}(K) = \max\{\phi_1(K), \phi_3(K), \phi_4(K)\}$. We describe below the shape of $\phi_1(K)$, $\phi_3(K)$, and $\phi_4(K)$. Specifically, in step 1, we show that $\phi_1(K)$ is increasing-decreasing constant. In step 2, we show that $\phi_3(K)$ (and similarly $\phi_4(K)$)

is increasing for $K \leq K^{R3}$ and then constant for $K > K^{R3}$. To prove the lemma, it is sufficient to show that $\phi_3(K^{R3}) \leq \phi_1(K^{R3})$ (and similarly for Region 4).

Step 1. $\phi_1(K)$ is increasing in $[0, \frac{w_1+w_2}{2}]$, decreasing in $[\frac{w_1+w_2}{2}, K_1]$, constant afterwards.

When $K \leq K_1$, from Lemma II.1(2) case F, we get (assume $w_1 \geq w_2$)

$$\begin{aligned}\phi_1(K) &= \max_{\bar{q}_1+\bar{q}_2=K} \frac{-a\bar{q}_1^2 - a\bar{q}_2^2 - 2b\bar{q}_1\bar{q}_2 + aw_1\bar{q}_1 + aw_2\bar{q}_2 + bw_2\bar{q}_1 + bw_1\bar{q}_2}{a^2-b^2} \\ &= \begin{cases} \frac{K(aw_1+bw_2-aK)}{a^2-b^2}, \text{ if } K \leq \frac{w_1-w_2}{2}; (\bar{q}_2 = 0) \\ \frac{(w_1-w_2)^2}{8(a+b)} + \frac{K(w_1+w_2-K)}{2(a-b)}, \text{ if } \frac{w_1-w_2}{2} < K \leq K_1; (\bar{q}_2 = \frac{2K-w_1+w_2}{4}) \end{cases}.\end{aligned}$$

These cases are identical as in the proof of Lemma II.3 except the upper bound for the region. Therefore, $\phi_1(K)$ is concave in $[0, K_1]$ and maximized at $K = \frac{w_1+w_2}{2}$. This immediately implies part (2) of this lemma: when $K \leq \frac{w_1+w_2}{2}$, $\phi_1(K)$ reaches $\Pi^{Ct}(K)$ in Lemma II.2 and has the same allocation as well.. It also shows concavity of $\phi_1(K)$ for $K \leq K_1$.

When $K > K_1$, in region R1 both firms have leftovers. Thus $\phi_1(K)$ is constant. Formally, this involves cases (A,D,E,I) in Table A.2 and the solutions of these four cases in Lemma II.1(2) do not involve $(\bar{q}_1, \bar{q}_2)$.

Step 2. $\phi_3(K)$ is non-decreasing in $[0, K^{R3}]$, and constant afterwards.

Since, in R3, firm 2 sells all inventory while firm 1 has leftovers, $\phi_3(K)$ depends only on $\bar{q}_2$. This can also be confirmed by looking at cases (B,F) in Lemma II.1(2). Let $\bar{q}_2^{R3}$ denote the maximizer of $\phi_3(K)$. Firm 1's sales is $\bar{q}_1^{R3}$ when $\bar{q}_2 = \bar{q}_2^{R3}$. Let $K^{R3} = \bar{q}_1^{R3} + \bar{q}_2^{R3}$. Consequently, the point $(\bar{q}_1^{R3}, \bar{q}_2^{R3})$ is on the boundary of regions R2 and R3. From Figure 2.2(b) and also Table A.2 we have that$K^{R3} = \bar{q}_1^{R3} + \bar{q}_2^{R3} < K_1$.

When $K \leq K^{R3}$, $\phi_3(K)$ is non-decreasing in K because the feasible set of $\bar{q}_2$ becomes larger (while $\bar{q}_1$ does not matter). We also have $\phi_3(K) \leq \phi_1(K)$ in this range. This is because (1) when $K \leq \frac{w_1+w_2}{2}$, $\phi_1(K)$ is the same as centralized allocation and any other allocation results in lower profits; (2) when $\frac{w_1+w_2}{2} < K \leq K^{R3}$, $\phi_1(K)$ is decreasing while $\phi_3(K)$ is non-decreasing; and (3) $\phi_3(K^{R3}) \leq \phi_1(K^{R3})$ since $(\bar{q}_1^{R3}, \bar{q}_2^{R3})$ is in the boundary of R2 and R3 and thus also is taken in consideration in ϕ_1.

When $K > K^{R3}$, $\phi_3(K)$ is constant, since $(K - \bar{q}_2^{R3}, \bar{q}_2^{R3})$ is always feasible and achieves maximum in R3. We also have $\phi_3(K_1) \geq \phi_1(K_1)$ since the only feasible point for $\phi_1(K_1)$, $(\hat{q}_1, \hat{q}_2)$, is also feasible for $\phi_3(K_1)$. Therefore, $\phi_3(K)$ must intersect $\phi_1(K)$ at a point denoted as K_{03}, where $K^{R3} < K_{03} < K_1$.

$\phi_4(K)$ is symmetric to $\phi_3(K)$. Thus, there also exists $K_{04} \in (K^{R4}, K_1)$, such that (1) $\phi_4(K) < \phi_1(K)$ when $K < K_{04}$; (2) $\phi_4(K) \geq \phi_1(K)$ and $\phi_4(K)$ is constant when $K \geq K_{04}$.

Let $K_0 = \min\{K_{03}, K_{04}\}$ and proof of part (1) is completed. □

Proof of Lemma II.5.

In Region 1, the prices and sales do not change across different allocation, as there are leftovers in both markets. Obviously the consumer surplus is constant in $\bar{q}_1$. Region 3 and 4 are symmetric and thus we only show in the following results for Region 2 and 4.

Region 2. In Region 2 all units are sold. We can apply equilibrium prices (Region F in Table A.2) to the consumer surplus(Equation (2.9)), and have

$$CS_F = \frac{a\bar{q}_1^2 + 2b\bar{q}_1\bar{q}_2 + a\bar{q}_2^2}{2(a^2 - b^2)}.$$

Constrained by $\bar{q}_1 + \bar{q}_2 = K$, the consumer surplus reaches its minimum at $\bar{q}_1 = \bar{q}_2 = \frac{K}{2}$.

Region 4. Region 4 may consist of situations C, G, or both (from Table A.2). In situation C, applying the equilibrium prices from Table A.2, we get

$$CS_C = \frac{1}{2(a^2-b^2)(2a^2-b^2)^2}\Big[a(a^2-b^2)(4a^2-3b^2)\bar{q}_1^2 + 2b(a^2-b^2)[(a-b)(a+b)^2\lambda + abw_1 + a^2w_2]\bar{q}_1 + a[(a-b)(a+b)^2\lambda + abw_1 + a^2w_2]^2\Big].$$

CS_C is convex and we have

$$\left.\frac{d\ CS_C}{d\ \bar{q}_1}\right|_{\bar{q}_1=0} > 0.$$

Therefore, the consumer surplus is increasing in situation C in $\bar{q}_1$.

In situation G, we repeat the same calculation and get

$$CS_G = \frac{(a^2-b^2)\bar{q}_1^2 + (aw_2+bw_1)^2}{2a(a^2-b^2)}.$$

CS_G is convex and we have

$$\left.\frac{d\ CS_G}{d\ \bar{q}_1}\right|_{\bar{q}_1=0} > 0.$$

Therefore, the consumer surplus is increasing in $\bar{q}_1$ in situation G. Consequently, the consumer surplus is increasing in $\bar{q}_1$ in Region 4. □

Proof of Theorem II.1.

As the market sizes are deterministic, we have $\Pi^{Co}(K) = \Pi^{Ct}(K)$, which is given in proof of Lemma II.3. Taking derivative, we obtain

$$\Pi^{Co'}(K) = \begin{cases} \frac{aw_1+bw_2-2aK}{a^2-b^2} & \text{if } K \le \frac{w_1-w_2}{2} \\ \frac{w_1+w_2-2K}{2(a-b)} & \text{if } \frac{w_1-w_2}{2} < K \le \frac{w_1+w_2}{2} \\ 0 & \text{if } K > \frac{w_1+w_2}{2} \end{cases}.$$

Let $\Pi^{Co'}(K) = c$ and K^{C*} is solved. □

Proof of Theorem II.2.

Part (1): No-trade Scenario.

As the market size is known up front, without trading no firm would order more than they can sell (case F in Table A.2). Using equilibrium prices p_i^{N*} into firms' revenue function 2.4, we obtain:

$$\pi_i^{No}(q_1,q_2) = \pi_i^{Np}(q_1,q_2) = \frac{-aq_i^2 + (aw_i + bw_i + a^2\lambda - b^2\lambda - bq_j)q_i - (a^2-b^2)\lambda q_j}{a^2-b^2}.$$

Again, due to no trade and additionally due to deterministic market sizes, the revenue functions in trading stage and ordering stage are identical. The response function, thus, becomes:

$$q_i = \min\left\{\max\left\{\frac{(a+b)w + (a^2-b^2)\lambda - (a^2-b^2)c - bq_j}{2a}, 0\right\},\right.$$
$$\left.\frac{(a^2+ab)w + (a^3+a^2b-ab^2-b^3)\lambda - abq_j}{2a^2-b^2}, \frac{(a+b)w - bq_j}{a}\right\}.$$

The first term above is from the first-order condition, while the last two terms are the boundary of the region R2 in Figure 2.1(b).When neither firm has leftovers (i.e., in region R2), the response function is continuous and decreasing. The slope of the response function is greater than -1. Consequently there exists a unique equilibrium q_i^{N*}, which satisfies $0 \le q_i^{N*} \le \frac{K_1}{2}$ (see Table A.3 for K_1).

Unique equilibrium implies that the equilibrium is symmetric. In the response function above, the second term is always larger than the first, whenever $q_j \le \frac{K_1}{2}$ and, thus, can be dropped. Based on evaluation of the other two terms and extreme points, the analytical form of q_i^{N*} is obtained.

Part (2): Trade Scenario.

To simplify notation, we denote (only in this proof)

$$f(q_1 + q_2) = \frac{1}{2}\Pi^{Tt}(q_1 + q_2),$$
$$g(q_1, q_2) = \frac{1}{2}(\pi_1^{Np}(q_1, q_2) - \pi_2^{Np}(q_1, q_2)) - cq_1.$$

Referring to Figure A.1(left), $f(\cdot)$ is concave when $q_1 + q_2 \leq K_0$, and constant afterwards (Lemma II.4). $g(q_1, q_2)$ is concave in q_1 in areas (i) and (iii) (which are jointly R2), and decreases at constant slope $-c$ in area (ii) (Lemma II.1). Firm 1's profit function is $\pi_1^{Tt}(q_1, q_2) - cq_1 = f(q_1 + q_2) + g(q_1, q_2)$.

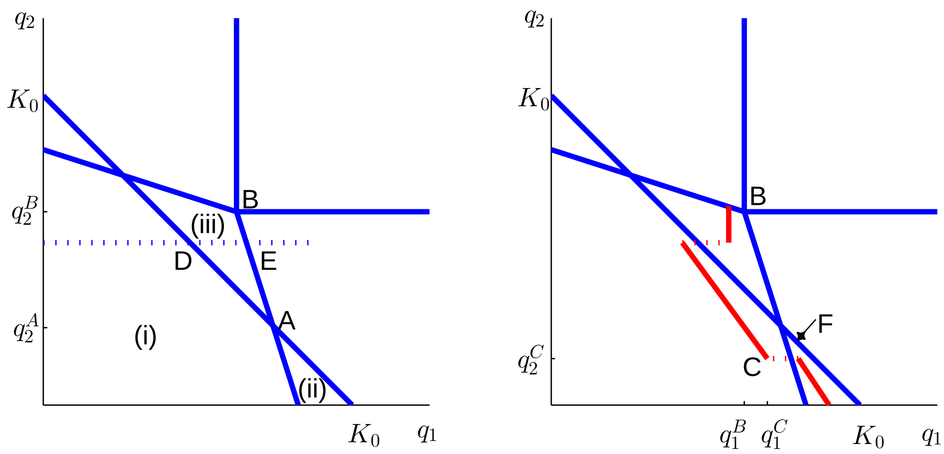


Figure A.1: Proof of Theorem II.2.

Consider now firm 1's response function for $q_2 \leq q_2^B$ (refer to the left part of Figure A.1). Note that for $q_1 + q_2 > K_0$, the total revenues of both firms, $\Pi^{Tt}(q_1 + q_2)$, do not change if firm 1 increases q_1. Also, in no-trade scenario the revenues do not change in Regions R1, R3, and R4. Thus, outside of areas (i), (ii), and (iii), the marginal effect of increasing q_1 is simply $-c$. Consequently, firm 1's best response must be areas (i), (ii), or (iii), for $q_2 \leq q_2^B$.

Step 1: Separability of $g(q_1, q_2)$.

By explicitly writing $g(q_1, q_2)$ for region R2, it is easy to see that it is separable in q_1 and q_2. Specifically, this is because, the revenue function in the pricing stage is (from case F in Table A.2):

$$\pi_i^{Np}(q_i, q_j) = \frac{-aq_i^2 - bq_iq_j + (aw_1 + bw_1 + a^2\lambda - b^2\lambda)q_i - (a^2 - b^2)\lambda q_j}{a^2 - b^2} \qquad \text{(A.1)}$$

and, thus, the terms q_1q_2 cancel out, implying that $g(q_1, q_2)$ is separable. Consequently, $g(q_1, q_2) = g_1(q_1) + g_2(q_2)$. Additionally, $g_1(q_1)$ is concave.

Step 2: At most one jump between areas (i) and (iii).

While the maximizer of $\pi_1^{Tt}(q_1, q_2)$ as function of q_2 is continuous, within each of the area (i), (ii), or (iii) individually, it may, however, jump between areas. In this step we consider $q_2 \in [q_2^A, q_2^B)$ and show that the response function $q_1^*(q_2)$ (right part of Figure A.1) has at most one jump (point of discontinuity as a function of q_2) when considering jointly areas (i) and (iii). It suffices to show that, if for a given q_2, $q_1^*(q_2)$ is in area (iii), then for any larger argument $q_2 + \delta (\delta > 0)$, $q_1^*(q_2 + \delta)$ is also in area (iii). Consider points D and E, as shown in Figure A.1, and assume that when $q_2 = q_2^D$ (the dotted line),the best response $q_1^*(q_2^D) \in (q_1^D, q_1^E)$ in area (iii). For points D' and E' on the higher lines, with $q_2^{D'} = q_2^D + \delta (\delta > 0)$, we have $q_1^{D'} = q_1^D - \delta$, and (as long as $q_2^{D'} \leq q_2^B$, we also have $q_1^{E'} > q_1^E - \delta$. For any $q_1 \in [0, q_1^{D'}]$, we have

$$\pi_1^{Tt}(q_1, q_2 + \delta) - cq_1$$

$$= f(q_1 + q_2 + \delta) + g_1(q_1) + g_2(q_2 + \delta)$$

$$= f(q_1 + \delta + q_2) + g_1(q_1 + \delta) + g_2(q_2) - g_1(q_1 + \delta) - g_2(q_2) + g_1(q_1) + g_2(q_2 + \delta)$$

$$\leq f(q_1^*(q_2) + q_2) + g_1(q_1^*(q_2)) - [g_1(q_1 + \delta) - g_1(q_1)] + g_2(q_2 + \delta)$$

the above inequality is because$q_1^*(q_2)$ is best response for q_2

$$\leq f(q_1^*(q_2) + q_2) + g_1(q_1^*(q_2)) - [g_1(q_1^*(q_2)) - g_1(q_1^*(q_2) - \delta)] + g_2(q_2 + \delta)$$

because g_1 is concave and $q_1 + \delta < q_1^*(q_2)$

$$= f(q_1^*(q_2) + q_2) + g_1(q_1^*(q_2) - \delta) + g_2(q_2 + \delta)$$

$$= f(q_1^*(q_2) - \delta + q_2 + \delta) + g_1(q_1^*(q_2) - \delta) + g_2(q_2 + \delta)$$

$$= \pi_1^{Tt}(q_1^*(q_2) - \delta, q_2 + \delta) - c(q_1^*(q_2) - \delta).$$

Thus, $(q_1^*(q_2) - \delta, q_2 + \delta)$ results in higher profit, for firm 1, than any $(q_1, q_2 + \delta)$ in area (i). Note that $(q_1^*(q_2) - \delta, q_2 + \delta)$ is in area (iii). Therefore, the best response $q_1^*(q_2 + \delta)$ is in area (iii). Consequently, there is at most one jump from area (i) to region (iii).

Step 3: Jumps between areas (i) and (ii) cannot result in symmetric equilibria.

Now consider any $q_2 \in [0, q_2^A]$. If there is any jump between area (i) and (ii), assuming the jump points are C(q_1^C, q_2^C) and F(q_1^F, q_2^F) with $q_2^C = q_2^F$, we must have $q_1^C > q_1^B$ (refer to Figure A.1).

As q_1^C and q_1^F are both the best responses for the same q_2, we must have $\frac{\partial}{\partial q_1}\pi_1^{Tt}(q_1^C, q_2^C) = \frac{\partial}{\partial q_1}\pi_1^{Tt}(q_1^F, q_2^F) = c$.

If we take into account the shape of Π^{Tt} and $(\pi_1^{Np} - \pi_2^{Np})$ (both are concave at first and then constant), we get $\frac{\partial}{\partial q_1}\pi_1^{Np}(q_1^F, q_2^F) - \frac{\partial}{\partial q_1}\pi_2^{Np}(q_1^F, q_2) = 0$. From Equation (2.7), we can express $\frac{1}{2}\Pi^{Tt}(q_1^F + q_2^F) = \pi_1^{Tt}(q_1^F, q_2^F) - \frac{1}{2}(\pi_1^{Np}(q_1^F, q_2^F) - \pi_2^{Np}(q_1^F, q_2^F))$. Thus, $\frac{1}{2}\frac{\partial}{\partial q_1}\Pi^{Tt}(q_1^F, q_2) = c - 0 = c$. As $q_1^C < q_1^F$ and Π^{Tt} is concave for $q_1 + q_2 \leq K_0$, we have $\frac{1}{2}\frac{\partial}{\partial q_1}\Pi^{Tt}(q_1^C, q_2^C) > \frac{1}{2}\frac{\partial}{\partial q_1}\Pi^{Tt}(q_1^F, q_2^F) = c$. Using Equation (2.7) again,we get $\frac{\partial}{\partial q_1}\pi_1^{Np}(q_1^C, q_2^C) - \frac{\partial}{\partial q_1}\pi_2^{Np}(q_1^C, q_2^C) < 0$. In order to compare q_1^B and q_1^C, we first identify $\hat{q}_1$that maximizes $\pi_1^{Np}(q_1, q_2^C) - \pi_2^{Np}(q_1, q_2^C) = 0$. From Equation(A.1) above, we get $\hat{q}_1 = \frac{(a+b)w + 2(a^2 - b^2)\lambda}{2a}$. From concavity of $g_1(q_1)$, we have $q_1^C > \hat{q}_1$. Also, it is easy to verify that $\hat{q}_1 > q_1^B$ ($q_1^B = K_1/2$ from Lemma II.4). Thus, the jump between areas(i) and (ii), if any, may only take place for $q_1 > q_1^B$. Consequently, for the purpose of identifying symmetric equilibria, it can be ignored.

Step 4: Characterization of equilibria.

The jump between areas (i) and (iii), described in Step 2, is positive. The existence of positive jumps still leads to existence of equilibria (possibly multiple) and for symmetric response functions, a symmetric equilibrium must exist in either area (i) or area (iii) or both.

From Lemma II.4 and its proof, we express $\Pi^{Tt}(K)$ in the symmetric setting as

$$\Pi^{Tt}(K) = \begin{cases} \frac{K(2w-K)}{2(a-b)} & \text{if } K \leq K_0 \\ \text{constant} & \text{if } K > K_0 \end{cases}. \tag{A.2}$$

In areas (i) and (iii) the profit function in trading case is given by Equations (2.7), (A.1), and (A.2). Taking derivatives and making them equal to 0 results in two necessary conditions for (low and high) equlibria. If equilibrium is in area (i), it is (low equilibrium)

$$q_i^{T*} = \frac{(a+b)w + (a^2-b^2)(\lambda - c)}{2a+b}.$$

If equilibrium in in area (iii), it is (high equilibrium)

$$q_i^{T*} = \min\left\{\frac{(a+b)w + 2(a^2-b^2)(\lambda - c)}{2a}, \frac{aw + (a^2-b^2)\lambda}{2a-b}, w\right\}.$$

Whether the high or low equilibria exist apparently depends on cost c. If for a given $\hat{c}$, low equilibrium exists, then for $\forall c \geq \hat{c}$, the response function will have lower values and, thus, the response functions will continue to intersect in area (i), resulting in existence of low equilibrium. Hence there exists a threshold c_1 such that for $\forall c \geq c_1$, low equilibrium exists. Using a similar argument, we show that there exists a thresholdc_2 such that for $\forall c \leq c_2$, high equilibrium exists. Note that we must have $c_1 \leq c_2$, because otherwise there is no equilibrium for $c \in (c_2, c_1)$, which contradicts the proved-above existence result.

Step 5: Comparison of low and high equilibria.

In the symmetric setting, the final profit is $\frac{1}{2}\Pi^{Tt}(2q_i^{T*}) - cq_i^{T*}$. In Lemma II.4, we showed $\Pi^{Tt}(K)$ is increasing and equals $\Pi^{Ct}(K)$ when $K < \frac{w_1+w_2}{2}$; decreasing when $\frac{w_1+w_2}{2} < K < K_0$; constant afterwards. While maximizers of $\Pi^{Ct}(K) - cK$, $\Pi^{Tt} - cK$ are the same $K = K^{C*}$, the equilibrium point for trade solution may be different. Assume that both equilibria exist and let q_i^H and q_i^L denote the high and low equilibria, respectively. Since the jump is between regions (i) and (iii) separated by line $q_1 + q_2 = K_0$ (and explicitly comparing with K^{C*}), we have

$$K^{C*} \leq 2q_i^L < K_0 < 2q_i^H.$$

Since $\Pi^{Tt}(K) - cK$ is decreasing for $K \geq K^{C*}$, we have $\Pi^{Tt}(2q_i^L) - 2cq_i^L > \Pi^{Tt}(2q_i^H) - 2cq_i^H$. □

Proof of Theorem II.3.

When $c \geq c_1$, two firms order the same quantities in trade and no-trade scenarios. As the market is deterministic and two firms are symmetric, in trade scenario there is actually no inventory exchange, even though it is allowed. Thus, we have exactly the same outcomes in these two (trade and no-trade) scenarios.

When $c < c_1$, let $\Delta(c)$ be the difference between each firm's profit in two scenarios (using firm 1's profit for the purpose of illustration throughout this proof).

$$\Delta(c) = (\pi_1^{To}(q_1^{T*}, q_2^{T*}) - cq_1^{T*}) - (\pi_1^{No}(q_1^{N*}, q_2^{N*}) - cq_1^{N*}).$$

When $c = c_1$, in trade scenario the low equilibrium still exists but will disappear for any smaller c. If low equilibrium is played, the profit is the same as firms in no trade scenario. If the high equilibrium is played, the profit is lower than that from low equilibrium, thus lower than that from no trade scenario. Therefore, we have $\Delta(c_1^-) < 0$. Next we show, for case of small and big w, that $\Delta(c)$ can intersect 0 at most once in $0 \leq c < c_1$.

Case 1: $w < (a+b)\lambda$**.**

We re-write the result of Theorem II.2 to differentiate between the cases listed in the theorem: We denote $c^{T1} = \frac{2(a+b)\lambda - w}{2(a+b)}$, such that

$$q_i^{T*} = \begin{cases} w & \text{if } c \leq c^{T1} \\ \frac{(a+b)w + 2(a^2-b^2)(\lambda - c)}{2a} & \text{if } c > c^{T1} \end{cases}.$$

And, similarly, to compare no-trade cases, we use the threshold $c^N = \frac{(a^2-b^2)\lambda - wa}{a^2-b^2}$. As $c^N < c^{T1}$, we have.

$$q_i^{N*} = \begin{cases} w & \text{if } c \leq c^{N} \\ \frac{(a+b)w + (a^2-b^2)(\lambda - c)}{2a+b} & \text{if } c > c^{N} \end{cases}.$$

Therefore, we have

$$\Delta(c) = \begin{cases} \pi_1^{To}(q_1^{T*}, q_2^{T*}) & \text{if } c \leq c^N \\ \pi_1^{To}(q_1^{T*}, q_2^{T*}) - \frac{[(a^2-b^2)(c-\lambda)+aw][(a^2-ab)c+(a^2-b^2)\lambda+(a+b)w]}{(a-b)(2a+b)^2} & \text{if } c^N < c \leq c^{T1} \\ \pi_1^{To}(q_1^{T*}, q_2^{T*}) + \frac{(a+b)(A_1c^2+A_2c+A_3)}{2a(a-b)(2a+b)^2} & \text{if } c > c^{T1} \end{cases},$$

where

$$A_1 = 2(3a+b)(a+b)(a-b)^2 > 0,$$

$$A_2 = -(a-b)(4a+b)(2a^2\lambda - 2b^2\lambda + bw),$$

$$A_3 = 2a(w + a\lambda - b\lambda)(a^2\lambda - b^2\lambda - aw).$$

First note that, even though quantities q_i^{T*} depend on c, we have that $\pi_1^{To}(q_i^{T*}, q_i^{T*})$ is constant in c for $K \leq K_0$:First, recall that the high equilibrium satisfies: $q_1^{T*} + q_2^{T*} > K_0$. Second, $\pi_1^{To}(q_1^{T*}, q_2^{T*}) = \frac{1}{2}\Pi^{Tt}(q_1^{T*} + q_2^{T*})$ and Π^{Tt} is constant for $K > K_0$. Thus, $\pi_1^{To}(q_1^{T*}, q_2^{T*})$ is also constant, i.e., independent of c. Therefore, we only need to consider the remaining terms in $\Delta(c)$.

From above, we have that for $c \leq c^N$, $\Delta(c)$ is a constant. For $c^N < c \leq c^{T1}$, $\Delta(c)$ is concave and decreasing. For $c > c^{T1}$, $\Delta(c)$ is convex. Hence, with $\Delta(c_1^-) < 0$ and due to its continuity, $\Delta(c)$ can intersect 0 at most once in $0 \leq c < c_1$.

Case 2: $w \geq (a+b)\lambda$**.**

Comparing again the terms in Theorem II.2, we differentiate between the relevant cases. We denote $c^{T2} = \frac{2a^2\lambda - 2b^2\lambda + wb}{4a^2+2ab-2b^2}$, such that

$$q_i^{T*} = \begin{cases} \frac{aw+(a^2-b^2)\lambda}{2a-b} & \text{if } c \leq c^{T2} \\ \frac{(a+b)w+2(a^2-b^2)(\lambda-c)}{2a} & \text{if } c > c^{T2} \end{cases}.$$

In this case, $c^N < 0$. thus, we have

$$\Delta(c) = \begin{cases} \pi_1^{To}(q_1^{T*}, q_2^{T*}) - \frac{B_1c^2+B_2c+B_3}{(a-b)(2a-b)(2a+b)^2} & \text{if } c \leq c^{T2} \\ \pi_1^{To}(q_1^{T*}, q_2^{T*}) + \frac{(a+b)(A_1c^2+A_2c+A_3)}{2a(a-b)(2a+b)^2} & \text{if } c > c^{T2} \end{cases},$$

where

$$B_1 = a(2a-b)(a+b)(a-b)^2 > 0,$$

$$B_2 = a(a-b)(2a+3b)(2a^2\lambda - 2b^2\lambda + bw) > 0,$$

$$B_3 = -(2a-b)(a+b)(w+a\lambda-b\lambda)(a^2\lambda - b^2\lambda - aw).$$

Similarly, for $c \leq c^{T2}$, $\Delta(c)$ is concave and decreasing. For $c > c^{T2}$, $\Delta(c)$ is convex. Hence, $\Delta(c)$ can intersect 0 at most once in $0 \leq c < c_1$.

If $\Delta(0) < 0$, we have $\Delta(c) < 0$ if $0 \leq c < c_1$.[2] Otherwise, we let $c_0 < c_1$ be the unique cost such that $\Delta(c_0) = 0$, which satisfies the theorem's statement. □

[2]Formally, we can denote $c_0 = -1$.

Proof of Theorem II.4.

Let $CS^{T/N}(c)$ be the consumer surplus in trade/no-trade scenario as a function of unit cost c. As long as the high equilibrium is played ($c < c_1$), $CS^T(c)$ is a constant across all purchasing costs c (even though purchased quantities change, the prices and the resulting sales are not influenced). Thus, it is sufficient to show that $CS^N(c)$ is decreasing in $c \in [0, c_1]$. In the no-trade equilibrium, both firms order q_i^{N*}. From the inverse demand function we have $p_i = \frac{aw_i + bw_j - aq_i - bq_j}{a^2 - b^2}$, and the consumer surplus can be evaluated (using Equation (2.9)) as

$$CS^N(c) = \frac{(q_i^{N*}(c))^2}{a - b}.$$

Since $q_i^{N*}(c)$, given in Theorem II.2, is non-increasing in cost c, we also have that $CS^N(c)$ is non-increasing in c.[3] $\square$

Proof of Theorem II.5.

Part (1): Centralized Scenario.

Order quantity and profit are not influenced, which is intuitive and follows directly from the proof of Theorem II.1. With the same order quantity, the price and sales do not change either (follows right away from the pricing results in Table A.1). Therefore, the consumer surplus is also constant as β or λ changes.

Part (2): No-trade Scenario.

The order quantity, q_i^{N*} is non-decreasing in both β and λ, as can be easily verified based on Theorem II.2. The consumer surplus is also non-decreasing, as it increases in q_i^{N*}, with other factors independent of β and λ, as shown in the proof of Theorem II.4.

From the proof of Lemma II.1, using solution for region F in Table A.2, the profit

[3]Note that the theorem allows consumer surplus in trade case to be higher for all $c \in (0, c_1)$. If $CS^T(c_1^-) > CS^N(c_1^-)$, then there exist c_3 such that $CS^T(c) > CS^N(c)$ for $c_3 < c < c_1$. (c_3 may be 0.) Otherwise if $CS^T(c_1^-) \leq CS^N(c_1^-)$, $CS^T(c) \leq CS^N(c)$ always holds.

function for no-trade scenario simplifies to

$$\pi_i^{No}(q_i^{N*}, q_i^{N*}) = \frac{q_i^{N*}(w - q_i^{N*})}{a - b} - cq_i^{N*}.$$

From Theorem II.2, we have that $q_i^{N*} > \frac{1}{2}(w - ac + bc)$, which immediately implies that $\pi_i^{No}(q_i^{N*}, q_i^{N*})$ is non-increasing when q_i^{N*} is non-decreasing.

Part (3): Trade Scenario.

In the low equilibrium, the ordering quantity, profit, and consumer surplus are the same as in no-trade scenario, thus monotonicity holds as shown above.

In the high equilibrium, it is easy to verify, based on Theorem II.2, that the ordering quantity is non-decreasing in both λ and β. Therefore, in the rest of the proof we focus on monotonicity of the profit and of the consumer surplus for the high equilibrium.

Recall that the profit is: $\pi_i^{Tt}(q_i^{T*}, q_i^{T*}) - cq_i^{T*} = \frac{1}{2}\Pi^{Tt}(2q_i^{T*}) - cq_i^{T*}$. $\Pi^{Tt}(2q_i^{T*})$ is independent of q_i^{T*} (because in high equilibrium we have leftovers, $2q_i^{T*} > K_0$), but it changes with λ and β. We will show that Π^{Tt} is non-increasing in β and λ. With q_i^{T*} non-decreasing, this will imply that π_i^{Tt} must be non-increasing. Below we show that: Total revenue, Π^{Tt}, is non-increasing in λ and β; and consumer surplus is non-decreasing in λ and β.

In the high equilibrium, trade always results in an asymmetric allocation, as discussed below Lemma II.4. In the symmetric setting, the final resource allocation is either in R3 or R4 (Figure 2.2(b)). Without loss of generality, we assume that it is in R4. Recall that in R4 firm 2 has leftovers so each firm's final revenue, π_i^{Np}, depends on firm 1's final inventory $\bar{q}_1$.

Two subregions C and G. Optimal allocation (bargaining outcome) $\bar{q}_1$ for given λ or β.

Even within R4, $\pi_1^{Np} + \pi_2^{Np}$ is not always concave in $\bar{q}_1$. Referring to Figure A.2, R4 is divided into two subregions, labeled as C and G corresponding to cases C and G in Table A.2. Denote the unconstrained maximizer in subregions C and G are $\bar{q}_1^{[C]}$ and $\bar{q}_1^{[G]}$.[4]

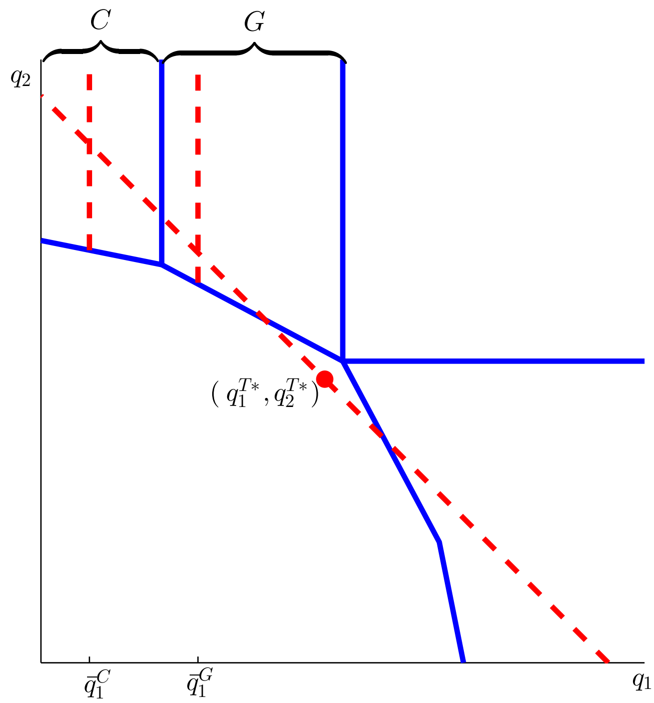


Figure A.2: Proof of Theorem II.5.

Analysis of subregion C.

From Table A.2, we obtain the total revenue function and derive its maximizer, $\bar{q}_1^{[C]}$:

$$\bar{q}_1^{[C]} = \frac{(4a^3 - 4ab^2 - b^3)w - 2b(a-b)(a+b)^2\lambda}{2a(4a^2 - 3b^2)}$$

If $\bar{q}_1^{[C]} > 0$, the optimized total revenue in subregion C, labeled as $\Pi^{[C]}$, is

$$\begin{aligned}\Pi^{[C]} &= \frac{(8a^2 + 8ab + b^2)w^2 - 4b(a+b)^2\lambda w - 4(a-b)(a+b)^3\lambda^2}{4a(4a^2 - 3b^2)} \\ &= \frac{-4\alpha(\alpha+2\beta)^3\lambda^2 - 4\beta w(\alpha+2\beta)^2\lambda + w^2(8\alpha^2 + 24\alpha\beta + 17\beta^2)}{4(\alpha+\beta)(4\alpha^2 + 8\alpha\beta + \beta^2)}.\end{aligned}$$

[4]We use [C] to denote subregion C, to differentiate it from centralized solution C.

$\Pi^{[C]}$ is clearly non-increasing in λ. We also show that it is non-increasing in β. (Formal verification requires a few additional steps.)

Using the same approach, we derive the consumer surplus in subregion C, labeled as $CS^{[C]}$,

$$\begin{aligned}
CS^{[C]} &= \frac{4(a-b)^2(a+b)^3\lambda^2 + 8aw(a-b)(a+b)^2\lambda + w^2(8a^3+4a^2b-3ab^2-b^3)}{8a(a-b)(4a^2-3b^2)} \\
&= \frac{4\alpha^2(\alpha+2\beta)^3\lambda^2 + 8\alpha w(\alpha+\beta)(\alpha+2\beta)^2\lambda + w^2(8\alpha^3+28\alpha^2\beta+29\alpha\beta^2+8\beta^3)}{8\alpha(\alpha+\beta)(4\alpha^2+8\alpha\beta+\beta^2)}.
\end{aligned}$$

$CS^{[C]}$ is again non-decreasing in λ and with a few additional steps can be shown to be non-decreasing in β.

If $\bar{q}_1^{[C]} \leq 0$, $\bar{q}_1 = 0$ is optimal in subregion C. Following the same approach, the total revenue ($\Pi^{[0]}$) and consumer surplus ($CS^{[0]}$) are as follows.

$$\begin{aligned}
\Pi^{[0]} &= \frac{(w-a\lambda)(a+b)^2(a^2\lambda+aw-b^2\lambda)}{(2a^2-b^2)^2} \\
&= \frac{(\alpha+2\beta)^2(w-\alpha\lambda-\beta\lambda)(\alpha w+\beta w+\alpha^2\lambda+2\alpha\beta\lambda)}{(2\alpha^2+4\alpha\beta+\beta^2)^2}, \\
CS^{[0]} &= \frac{a(a+b)(a^2\lambda+aw-b^2\lambda)^2}{2(a-b)(2a^2-b^2)^2} \\
&= \frac{(\alpha+\beta)(\alpha+2\beta)(\alpha w+\beta w+\alpha^2\lambda+2\alpha\beta\lambda)^2}{2\alpha(2\alpha^2+4\alpha\beta+\beta^2)^2}.
\end{aligned}$$

Again, it can be verified that $\Pi^{[0]}$ is non-increasing in both λ and β and that $CS^{[0]}$ is non-decreasing in both λ and β.

Analysis of subregion G.

From table A.2, we use optimal price for subregion G and,solving the maximization problem, we obtain the optimal quantity $\bar{q}_1^{[G]} = \frac{w}{2}$.

With the optimal $\bar{q}_1^{[G]}$, the total revenue ($\Pi^{[G]}$) and consumer surplus ($CS^{[G]}$)

become

$$\Pi^{[G]} = \frac{w^2}{4a} = \frac{w^2}{4(\alpha+\beta)},$$
$$CS^{[G]} = \frac{(5a+3b)w^2}{8a(a-b)} = \frac{(5\alpha+8\beta)w^2}{8\alpha(\alpha+\beta)}.$$

$\Pi^{[G]}$ and $CS^{[G]}$ are independent of λ and non-increasing in β.

Potential discontinuity results in at most one jump in $\bar{q}_1$. The allocation, $\bar{q}_1$, changes smoothly in λ and β except at most one jump from subregion C to subregion G.

While the jump will not effect the revenue (as revenues must be equal when switching between two forms of allocations), it may influence the consumer surplus. Therefore, we need to explicitly describe these two cases. The optimal $\bar{q}_1$ can be either $\max\{0, \bar{q}_1^{[C]}\}$ or $\bar{q}_1^{[G]}$. Since $\max\{0, \bar{q}_1^{[C]}\} < \bar{q}_1^{[G]}$ (and the revenue is concave in each subregion), the boundary between subregions C and G is never optimal. From Table A.2, when $w \leq a\lambda$, only subregion G exists (subregion C is empty); When $a\lambda < w < (a+b)\lambda$, both subregions C and G exist; When $w \geq (a+b)\lambda$, only subregion C exists. Thus, we only need to show that, when $a\lambda < w < (a+b)\lambda$, choice of $\bar{q}_1$ only has one jump from $\max\{0, \bar{q}_1^{[C]}\}$ to $\bar{q}_1^{[G]}$ as λ or β increases. In order to prove this, we show $\Pi^{[G]} - \Pi^{[C]}$ and $\Pi^{[G]} - \Pi^{[0]}$ are nondecreasing in λ and β. That is, as λ or β increases, once $\bar{q}_1$ switches to $\bar{q}_1^{[G]}$, it never returns to subregion C.

For λ the result is straightforward: Since $\Pi^{[G]}$ is constant in λ while $\Pi^{[0]}$ and $\Pi^{[C]}$ are non-increasing in λ, we must have $\Pi^{[G]} - \Pi^{[C]}$ and $\Pi^{[G]} - \Pi^{[0]}$ are non-decreasing in λ.

For β, we have:

$$\Pi^{[G]} - \Pi^{[C]} = \frac{(\alpha+2\beta)^2(\alpha^2\lambda^2 + 2\alpha\beta\lambda^2 + \beta\lambda w - w^2)}{(\alpha+\beta)(4\alpha^2+8\alpha\beta+\beta^2)},$$
$$\Pi^{[G]} - \Pi^{[0]} = \frac{1}{4(\alpha+\beta)(2\alpha^2+4\alpha\beta+\beta^2)^2}(4\alpha(\alpha+\beta)^2(\alpha+2\beta)^3\lambda^2$$

$$+ 4\beta^2 w(\alpha+\beta)(\alpha+2\beta)^2\lambda - \beta w^2(2\alpha+3\beta)(4\alpha^2+10\alpha\beta+5\beta^2)).$$

Both $\Pi^{[G]} - \Pi^{[C]}$ and $\Pi^{[G]} - \Pi^{[0]}$ can be shown to be non-decreasing in β.

The effect of discontinuity of $\bar{q}_1$ on consumer surplus.

At the jump point, the revenues are equal. The consumer surplus, however, may potentially change. Let $\bar{CS}^{[C]}(q_1)$ and $\bar{CS}^{[G]}(q_1)$ be the consumer surplus in subregions C and G, before q_1 is optimally chosen. Using optimal prices from Table A.2, consumer surplus (2.9) becomes

$$\begin{aligned}
\bar{CS}^{[C]}(q_1) = & \frac{1}{2(a-b)(2a^2-b^2)^2}(a(a-b)(4a^2-3b^2)q_1^2 \\
& + 2b(a-b)(a+b)(a^2\lambda+aw-b^2\lambda)q_1 + a(a+b)(a^2\lambda+aw-b^2\lambda)^2), \\
\bar{CS}^{[G]}(q_1) = & \frac{(a-b)q_1^2+(a+b)w^2}{2a(a-b)}.
\end{aligned}$$

Both $\bar{CS}^{[C]}(q_1)$ and $\bar{CS}^{[G]}(q_1)$ are increasing for $q_1 \geq 0$. As a result, when firm 1's inventory increases from $\max\{0, \bar{q}_1^{[C]}\}$ to $\bar{q}_1^{[G]}$, the consumer surplus also increases.

Summary for Profit and Consumer Surplus with High Equilibrium in (3) Trade Scenarios.

Re-iterating, we have shown above: The profit changes in the same direction as the total revenue. Both total revenue and consumer surplus depend on firm 1's final inventory $\bar{q}_1$. The optimal $\bar{q}_1$ can be either $\max\{0, \bar{q}_1^{[C]}\}$ or $\bar{q}_1^{[G]}$. If $\bar{q}_1$ remains in the same subregion (C or G), the total revenue is non-increasing the consumer surplus non-decreasing in β and λ. For a certain β or λ, optimal $\bar{q}_1$ jumps from subregion C to subregion G. The total revenue remains the same in the jump while the consumer surplus goes up. □

Proof of Theorem II.6.

Part (1): Centralized Scenario.

For any market realization, $\Pi^{Ct}(K)$ is concave, according to Lemma II.3. Therefore, $\Pi^{Co}(K)$ is concave as well. Thus, there exists a unique optimal ordering quantity K^{C*}.

Part (2): No-trade Scenario.

As $\beta = \lambda = 0$ and there is no trade option, the two firms are essentially operating completely independently. Their profit functions are concave and, thus, there exists a unique optimal solution q_i^{N*} for each firm.

Part (3): Trade Scenario.

Denote firm 1's best response function as $BR(q_2)$. Below we prove that $BR(q_2)$ is continuous and its slope is between $[-1, 0]$.

Since $\lambda = \beta = 0$, the pricing stage revenue π_i^{Np} is a function of only q_i (independent of q_j), and it is increasing and strictly concave when $q_i \leq \frac{w_i}{2}$, and constant when $q_i > \frac{w_i}{2}$. In Lemma II.4, $\lambda = \beta = 0$ leads to $K_1 = \frac{w_1+w_2}{2}$. As a result, $\Pi^{Tt}(K)$ is increasing and strictly concave when $K \leq \frac{w_1+w_2}{2}$, and constant when $K > \frac{w_1+w_2}{2}$.

Recall that $\pi_1^{Tt}(q_1, q_2) = \frac{1}{2}(\Pi^{Tt}(q_1+q_2)+\pi_1^{Np}(q_1)-\pi_2^{Np}(q_2))$. Therefore, $\pi_1^{Tt}(q_1, q_2|w_1, w_2)$ is increasing and strictly concave for $q_1 \leq \max\{\frac{w_1+w_2}{2} - q_2, \frac{w_1}{2}\}$, and constant otherwise. The ordering stage revenue $\pi_1^{To}(q_1, q_2) = \int_{w_1,w_2} \pi_1^{Tt}(q_1, q_2|w_1, w_2)$ is also increasing and strictly concave up to a threshold, and constant for any greater q_1. Solving $\frac{\partial}{\partial q_1}\pi_1^{To}(q_1, q_2) = c$, we get $BR(q_2)$ and at $(BR(q_2), q_2)$, π_1 must be strictly concave. Therefore the response function must be continuous.

Assume firm 2's order quantity increases from q_2 to $q_2 + \varepsilon$ ($\varepsilon > 0$). Then we have

$$\begin{aligned}
&\frac{\partial}{\partial q_1}\pi_1^{Tt}(BR(q_2), q_2 + \varepsilon|w_1, w_2) \\
&= \frac{1}{2}\frac{\partial}{\partial q_1}\Pi^{Tt}(BR(q_2) + q_2 + \varepsilon|w_1, w_2) + \frac{1}{2}\frac{\partial}{\partial q_1}\pi_1^{Np}(BR(q_2)|w_1) \\
&\leq \frac{1}{2}\frac{\partial}{\partial q_1}\Pi^{Tt}(BR(q_2) + q_2|w_1, w_2) + \frac{1}{2}\frac{\partial}{\partial q_1}\pi_1^{Np}(BR(q_2)|w_1) \\
&= \frac{\partial}{\partial q_1}\pi_1^{Tt}(BR(q_2), q_2|w_1, w_2).
\end{aligned}$$

Then we have

$$\frac{\partial}{\partial q_1}\pi_1^{To}(BR(q_2), q_2+\varepsilon) \le \frac{\partial}{\partial q_1}\pi_1^{To}(BR(q_2), q_2) = c.$$

On the other hand, we also have

$$\begin{aligned}
&\frac{\partial}{\partial q_1}\pi_1^{Tt}(BR(q_2)-\varepsilon, q_2+\varepsilon|w_1, w_2)\\
&= \frac{1}{2}\frac{\partial}{\partial q_1}\Pi^{Tt}(BR(q_2)+q_2|w_1, w_2) + \frac{1}{2}\frac{\partial}{\partial q_1}\pi_1^{Np}(BR(q_2)-\varepsilon|w_1)\\
&\ge \frac{1}{2}\frac{\partial}{\partial q_1}\Pi^{Tt}(BR(q_2)+q_2|w_1, w_2) + \frac{1}{2}\frac{\partial}{\partial q_1}\pi_1^{Np}(BR(q_2)|w_1)\\
&= \frac{\partial}{\partial q_1}\pi_1^{Tt}(BR(q_2), q_2)
\end{aligned} \tag{A.3}$$

and

$$\frac{\partial}{\partial q_1}\pi_1^{To}(BR(q_2)-\varepsilon, q_2+\varepsilon) \ge \frac{\partial}{\partial q_1}\pi_1^{To}(BR(q_2), q_2) = c. \tag{A.4}$$

Since $\frac{\partial}{\partial q_1}\pi_1^{To}(BR(q_2+\varepsilon), q_2+\varepsilon) = c$ and the partial derivative is non-decreasing, we immediately have

$$BR(q_2)-\varepsilon \le BR(q_2+\varepsilon) \le BR(q_2).$$

Thus, the slope of the response function is between $[-1, 0]$.

To guarantee the uniqueness, we next show that the two response functions cannot have slope of -1 at the same point. Let u_i be the upper bound of w_i's distribution. Firm 1's ordering quantity q_1 should never exceed $\max\{\frac{u_1}{2}, \frac{u_1+u_2}{2}\}$ because for any market realization extra units have zero marginal benefit. That is represented by regions B1, F, C1 and C2 in Figure A.3 (regions named after Table A.2).

In region B1, $\pi_1^{Np}(q_1|w_1)$ is constant for any w_1 realization, and thus equality holds in (A.3) and (A.4). Consequently, the part of response function that falls into region B1 has slope -1. In region F, C1 and C2, $\pi_1^{Np}(q_1|w_1)$ is strictly concave if w_1 realizes to be greater than $2q_1$ and constant otherwise. That is, in (A.3), strictly inequality

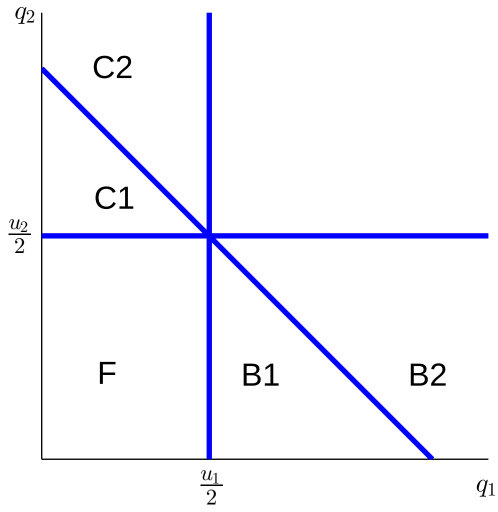


Figure A.3: Proof of Theorem II.6.

holds at least for some realization of w_1. Since π_1^{To} is the average of $\pi_1^{Np}(q_1|w_1)$ over all possible w_1, the inequality in (A.3) must hold. That is, when the response function falls into region F, C1 and C2, the slope must be strictly greater than -1.

Similarly, the slope of firm 2's response function is -1 in region C1, and strictly greater than -1 in region F, B1 and B2. Therefore, slopes of their response functions cannot simultaneously be -1. $\square$

Proof of Theorem II.7.

For any given market size realization, the revenue function in independent markets is

$$\pi_1^{Tt}(q_1, q_2|w_1, w_2) = \frac{1}{2}\Pi^{Tt}(q_1 + q_2|w_1, w_2) + \frac{1}{2}\pi_1^{Np}(q_1|w_1, w_2) - \frac{1}{2}\pi_2^{Np}(q_2|w_1, w_2).$$

Taking derivative with respect to q_1, we have

$$\frac{\partial}{\partial q_1}\pi_1^{Tt}(q_1, q_2|w_1, w_2) = \frac{1}{2}\frac{\partial}{\partial q_1}\Pi^{Tt}(q_1 + q_2|w_1, w_2) + \frac{1}{2}\frac{\partial}{\partial q_1}\pi_1^{Np}(q_1, q_2|w_1, w_2).$$

From Lemma II.4 we have that $\Pi^{Tt}(K) = \Pi^{Ct}(K)$, when $K \leq \frac{w_1+w_2}{2}$ and that these two functions are constant for $K \geq K_1$. However, in the special case when $\lambda = \beta = 0$, $K_1 = \frac{w_1+w_2}{2}$, implying $\Pi^{Tt}(K) = \Pi^{Ct}(K)$ for all K. Therefore, the derivative function in trade scenario is an average of that in centralized scenario and no-trade scenario, implying that the response function in trade scenario is always between that in centralized scenario and no-trade scenario. In no-trade scenario, response function is a horizontal line since the decision is not affected by the competitor. Let (q_1^{N*}, q_2^{N*}) be optimal decisions in no-trade scenario, K^{C*} be the optimal total inventory level in centralized scenario. Then (q_1^{T*}, q_2^{T*}) must fall into the triangle given by $x = q_1^{N*}$, $y = q_2^{N*}$, and $x + y = K^{C*}$. Therefore, we have either

$$q_1^{N*} \leq q_1^{T*}, q_2^{N*} \leq q_2^{T*}, q_1^{T*} + q_w^{T*} \leq K^{C*},$$

or

$$q_1^{N*} \geq q_1^{T*}, q_2^{N*} \geq q_2^{T*}, q_1^{T*} + q_w^{T*} \geq K^{C*}.$$

□

Proof of Theorem II.8.

Given initial inventory (q_1, q_2) and market size realization (w_1, w_2), we have

$$\begin{aligned}
\pi_1^{Tt}(q_1, q_2) &= \frac{1}{2}(\Pi^{Tt}(q_1 + q_2) + \pi_1^{Np}(q_1) - \pi_2^{Np}(q_2)) \\
&\geq \frac{1}{2}(\pi_1^{Np}(q_1) + \pi_2^{Np}(q_2) + \pi_1^{Np}(q_1) - \pi_2^{Np}(q_2)) = \pi_1^{Np}(q_1).
\end{aligned}$$

We immediately extend this result to the order stage profit by integration

$$\pi_1^{To}(q_1, q_2) \geq \pi_1^{No}(q_1).$$

Now let q_i^{T*} and q_i^{N*} be the equilibrium ordering quantities in trade and no-trade scenarios, we have

$$\pi_1^{No}(q_1^{N*}) - cq_1^{N*} \leq \pi_1^{To}(q_1^{N*}, q_2^{T*}) - cq_1^{N*} \leq \pi_1^{To}(q_1^{T*}, q_2^{T*}) - cq_1^{T*}.$$

□

Proof of Theorem II.9.

Part (1): Centralized Scenario.

For any market realization, $\Pi^{Ct}(K)$ is concave, according to Lemma II.3. Therefore, $\Pi^{Co}(K)$ is concave as well. Since the profit is $\Pi^{Co}(K) - cK$ and Π^{Co} is strictly concave until the maximum, the uniqueness follows.

Part (2): No-trade Scenario.

For given (w_1, w_2), firms sell all inventories only in region R2 (see Figure II.1(b)), which we refer to as clearance region. The intersection of the four regions is $(\frac{2a^2w_1+abw_2}{4a^2-b^2}, \frac{2a^2w_2+abw_1}{4a^2-b^2})$ (from Lemma II.1 case A). Since $w_i \sim U[l, u]$, the clearance region R2 is largest when $w_1 = w_2 = u$. Therefore, the symmetric equilibrium in the ordering stage, if it exists, has to have each of the order quantities in the interval $[0, \frac{au}{2a-b}]$.

In Figure A.4, the order quantities (q_1, q_2) are fixed, and the axis are the market realization (w_1, w_2). This is another representation of the result in Figure 2.1(b) and Regions 1 through 4 are labeled in the same way. The revenue function in the ordering stage, $\pi_1^{No}(q_1, q_2)$, is calculated by integrating the pricing-stage revenue, $\pi_1^{Np}(q_1, q_2)$, over $[l, u]^2$ (dashed square). As the regions change in (q_1, q_2), the overall revenue function consists of many cases (the regions have different analytical form and the revenue depends also on relationships between market realizations and quantities). The closed-form expression for $\pi_1^{No}(q_1, q_2)$ are fairly complicated and they are omitted here. However, the expressions are available from the authors upon request.

The general idea of the proof is as follows: When $q_2 \in [0, \frac{au}{2a-b}]$, we want to show

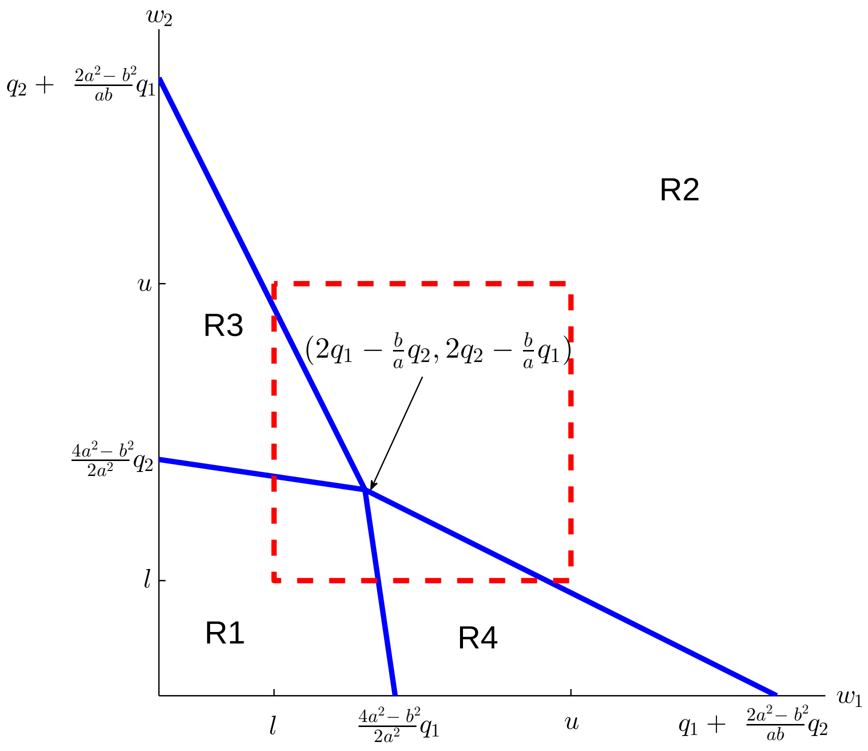


Figure A.4: Proof of Theorem II.9

$\pi_1^{No}(q_1, q_2)$ is unimodal in q_1. It suffices to show that $\pi_1^{No'}(q_1, q_2)$ is quasi-convex in q_1 and $\lim_{q_1 \to \infty} \pi_1^{No'}(q_1, q_2) = 0$. When $0 \leq q_2 \leq \frac{2a^2u+abl}{4a^2-b^2}$, this can be proved without any additional conditions. When $\frac{2a^2u+abl}{4a^2-b^2} < q_2 \leq \frac{au}{2a-b}$, we show the same property with additional conditions:

$$4a^3 - 4a^2b - 2ab^2 + b^3 \geq 0,$$
$$u \geq \frac{4}{3}l.$$

With conditions above, the response function is continuous in $[0, \frac{au}{2a-b}]$ and $q_2^*(\frac{au}{2a-b}) < \frac{au}{2a-b}$. Since two firms are symmetric, there must exist at least one symmetric equilibrium in area $[0, \frac{au}{2a-b}]^2$. The details of the proof are available on request.

Part (3): Trade Scenario.

The sufficient conditions require that the purchasing cost is sufficiently high and they effectively imply that both firms sell all inventory.[5] We show below that $c \geq \underline{c}(a,b,l,u)$ is a sufficient condition leading to no leftovers.

For given (w_1, w_2), firms sell all inventories only in region R2 (Figure II.1(b)) and the intersection of the four regions is $(\frac{2a^2w_1+abw_2}{4a^2-b^2}, \frac{2a^2w_2+abw_1}{4a^2-b^2})$ (from Lemma II.1 case A). Since $l/2 < \frac{al}{2a-b}$, if the optimal order quantity $(q_1^*, q_2^*) \in [0, l/2]^2$, firms can always sell all the inventory, even for the least favorable market-size realizations. Recall that firm 1's revenue function is $\pi_1^{Tt}(q_1,q_2) = \frac{1}{2}\Big(\Pi^{Tt}(q_1+q_2) + \pi_1^{Np}(q_1,q_2) - \pi_2^{Np}(q_1,q_2)\Big)$. In order to show existence of equilibrium, we modify the profit function for $q_1 > \frac{aw_1+bw_2}{2a}$ and replace it by a higher profit. The modified function is well behaved and easier to analyze. For the modified function, we show that the equilibrium would anyhow be $q_1 \leq \frac{l}{2}$ (that is in the region, where the values are not modified).

Specifically, we construct $\hat{\pi}_1^{Tt}(q_1,q_2)$ by replacing Π^{Tt} by Π^{Ct} and replacing $\pi_1^{Np} - \pi_2^{Np}$ by $\hat{\pi}_1^{Np}$, where Π^{Ct} is the centralized revenue of both firms and

$$\hat{\pi}_1^{Np}(q_1,q_2) = \begin{cases} \pi_1^{Np}(q_1,q_2) - \pi_2^{Np}(q_1,q_2) & \text{if } q_1 \leq \frac{aw_1+bw_2}{2a} \\ \pi_1^{Np}(\frac{aw_1+bw_2}{2a}, q_2) - \pi_2^{Np}(\frac{aw_1+bw_2}{2a}, q_2) & \text{otherwise} \end{cases}.$$

We have Π^{Ct} is non-decreasing and concave (from Lemma II.3). Additionally, $\Pi^{Ct} \geq \Pi^{Tt}$ and these two are equal when $K \leq \frac{w_1+w_2}{2}$ (from Lemma II.4). Thus $\hat{\pi}_1^{Tt}(q_1,q_2) = \pi_1^{Tt}(q_1,q_2)$ for $q_1 \leq \frac{aw_1+bw_2}{2a}$.

We also have $\pi_1^{Np}(q_1,q_2) - \pi_2^{Np}(q_1,q_2)$ is concave and increasing in q_1 when $q_1 \leq \frac{aw_1+bw_2}{2a}$, and it is constant when $q_1 > \frac{aw_1+bw_2}{2a}$. Therefore, $\hat{\pi}_1^{Np}(q_1,q_2)$ is non-decreasing and concave in q_1. Additionally, $\hat{\pi}_1^{Np}(q_1,q_2) \geq \pi_1^{Np}(q_1,q_2) - \pi_2^{Np}(q_1,q_2)$. Therefore,

$$\hat{\pi}_1^{Tt}(q_1,q_2) = \frac{1}{2}(\Pi^{Ct}(q_1+q_2) + \hat{\pi}_1^{Np}(q_1,q_2)) \geq \pi_1^{Tt}(q_1,q_2)$$

[5] If for some realization firms have leftovers after trading and pricing, the revenue function is not necessarily unimodal.

where equality holds when $q_1 \leq \min\{\frac{w_1+w_2}{2} - q_2, \frac{aw_1+bw_2}{2a}\}$. Denote

$$\hat{\pi}^{To}(q_1, q_2) = \int_{w_1, w_2} \hat{\pi}^{Tt}(q_1, q_2).$$

We immediately have $\hat{\pi}^{To} \geq \pi^{To}$, where equality holds when $(q_1, q_2) \in [0, l/2]^2$. Additionally, $\hat{\pi}^{To}(q_1, q_2)$ is concave in q_1 if $q_2 \leq \frac{l}{2}$. Let $\hat{q}_1^*(q_2)$ be the q_1 that solves $\frac{\partial \hat{\pi}^{To}(q_1,q_2)}{\partial q_1} = c$. If

$$\hat{q}_1^*(q_2 = \frac{l}{2}) \leq \frac{l}{2}, \tag{A.5}$$

$\hat{q}_1^*(q_2)$ must intersect the diagonal line at $(\hat{q}, \hat{q})$ with $\hat{q} \leq \frac{l}{2}$. In fact, $(\hat{q}, \hat{q})$ is also an equilibrium for the original problem.

The condition (A.5) is equivalent to:

$$c \geq \left. \frac{\partial \hat{\pi}_1^{To}}{\partial q_1} \right|_{(l/2, l/2)},$$

which can easily be translated into the condition stated in the theorem. □

Proof of Theorem II.10.

Part (1): Centralized Scenario.

As terms of λ cancel out in the total revenue function $\Pi^{Co}(K)$, λ does not play any role in the centralized decision. Therefore, the proof is same as the proof of Theorem II.6(1).

Part (2): No-trade Scenario.

For any given (w_1, w_2) and q_j, $\pi_i^{Np}(q_i, q_j)$ is increasing and concave in q_i. The concavity is strict for the increasing part. Therefore, $\pi_i^{No}(q_i, q_j)$ is also increasing and concave in q_i and similarly strictly concave while increasing. Therefore, there exists unique solution q_i^* satisfying

$$\frac{\partial \pi_i^{No}(q_i, q_j)}{\partial q_i} = c.$$

Note that $\lambda(\mu_i - \gamma s_2)$ is a constant for any given q_j, hence q_i^* is independent of q_j, which means it is a dominant strategy.

Part (3): Trade Scenario.

In symmetric settings, if the response function of a firm only has positive jumps, then it must intersect the diagonal line. The intersection is obviously a symmetric equilibrium. In the following, we prove the theorem by showing that the response function may only have positive jumps.

Recall (from the Model section) that the revenue of firm 1 in the trading stage is

$$\pi_1^{Tt}(q_1, q_2) = \frac{1}{2}\left(\Pi^{Tt}(q_1 + q_2) + \pi_1^{Np}(q_1, q_2) - \pi_2^{Np}(q_1, q_2)\right).$$

The terms π_i^{Np}'s, based on the proof of Lemma II.1, can be expressed as:

If $w_i - a\lambda \geq 0$,

$$\pi_i^{Np} = \begin{cases} \frac{q_i}{a}(w_i + a\lambda - q_i) + \lambda(\mu_i - s_j) & \text{if } q_i \leq \frac{w_i + a\lambda}{2} \\ \frac{1}{4a}(w_i + a\lambda)^2 + \lambda(\mu_i - s_j) & \text{if } q_i > \frac{w_i + a\lambda}{2} \end{cases};$$

If $w_i - a\lambda < 0$,

$$\pi_i^{Np} = \begin{cases} \frac{q_i}{a}(w_i + a\lambda - q_i) + \lambda(\mu_i - s_j) & \text{if } q_i \leq w_i \\ \lambda w_i + \lambda(\mu_i - s_j) & \text{if } q_i > w_i \end{cases}.$$

Taking derivative with respect to q_1, we get:

$$\frac{\partial \pi_1^{Np}}{\partial q_1} = \begin{cases} \frac{(w_1 + a\lambda - 2q_1)}{a} & \text{if } q_1 \leq \min\{\frac{w_1 + a\lambda}{2}, w_1\} \\ 0 & \text{if } q_1 > \min\{\frac{w_1 + a\lambda}{2}, w_1\} \end{cases},$$

$$\frac{\partial \pi_2^{Np}}{\partial q_1} = \begin{cases} -\lambda & \text{if } q_1 \leq \min\{\frac{w_1 + a\lambda}{2}, w_1\} \\ 0 & \text{if } q_1 > \min\{\frac{w_1 + a\lambda}{2}, w_1\} \end{cases}.$$

It is straightforward to see that $\pi_1^{Np'}$ is non-increasing in q_1 and $\pi_2^{Np'}$ is non-decreasing in q_1. For any market realization (w_1, w_2), we can easily check that

$$\pi_1^{Np'}(q_1 - \varepsilon, q_2 + \varepsilon) \geq \pi_1^{Np'}(q_1, q_2),$$

$$\pi_2^{Np'}(q_1 - \varepsilon, q_2 + \varepsilon) \leq \pi_2^{Np'}(q_1, q_2),$$

for $\forall \varepsilon > 0$, where the derivatives are with respect to K. Therefore, we have

$$\pi_1^{Tt'}(q_1 - \varepsilon, q_2 + \varepsilon) \geq \pi_1^{Tt'}(q_1, q_2).$$

By taking integration, we extends this result to the ordering stage revenue:

$$\pi_1^{To'}(q_1 - \varepsilon, q_2 + \varepsilon) \geq \pi_1^{To'}(q_1, q_2).$$

For a given q_2, let $q_1^*(q_2)$ be the optimal response. We immediately have

$$q_1^*(q_2 + \varepsilon) \geq q_1^*(q_2) - \varepsilon \text{ for } \forall \varepsilon > 0.$$

Even though we have not excluded the possibility that $q_1^*(q_2)$ may be decreasing, it cannot decrease faster than with slope of -1 and, thus, it can only have positive jumps, if any. $\square$

APPENDIX B

Proofs of Lemmas and Theorems in Chapter III

Proof of Lemma III.1.

The results immediately follow by checking the close form solution of p_2^* in Table 3.2. $\square$

Proof of Theorem III.1.

Taking first order condition on θ from Equation (3.1), we obtain

$$\theta^* = pd_1 + p_2 d_2 \text{ and } \pi_M^* = \frac{1}{2}\theta^{*2}.$$

Therefore, it suffices to choose (p, p_2) such that $pd_1 + p_2 d_2$ is maximized. The proof is approached by plugging $d_i(p, p_2)$ into $pd_1 + p_2 d_2$.

(1) When $\alpha + v \leq 1$.

It is obvious that optimal $p_2 \in [v, 1-\alpha]$, and for given p_2, optimal $p \in [v, p_2 + \alpha]$. Within the reduced set, we have two cases:

If $v \leq p \leq \frac{(1-v)p_2 - \alpha v}{1-v-\alpha}$, we have

$$d_1 = \frac{1-p}{1-v}, d_2 = 0.$$

Therefore $p^* = \max\{\frac{1}{2}, v\}$. p_2 does not matter since $d_2 = 0$.

If $\frac{(1-v)p_2-\alpha v}{1-v-\alpha} < p \leq p_2 + \alpha$, we have

$$d_1 = 1 - \frac{p - p_2}{\alpha}, d_2 = \frac{p - p_2}{\alpha} - \frac{p_2 - v}{1 - v - \alpha}.$$

From the first order condition on p, we get $p = p_2 + \alpha/2$. Replacing p, p_2 is solve as $p_2 = (1-\alpha)/2$. Recall that p_2 is bounded by v from lower end, we have

$$p_2^* = \max\left\{v, \frac{1-\alpha}{2}\right\}, p^* = p_2^* + \frac{\alpha}{2}.$$

Comparing the optimal solutions from two cases, we find that the second pricing policy is always better than the first one.

(2) When $\alpha + v > 1$.

It is obvious that optimal $p \in [v, 1]$, and for given p, optimal $p_2 \in \left[v, p - \frac{\alpha(p-v)}{1-v}\right]$. From the first order condition on p_2, we get

$$p_2^* = \begin{cases} \frac{v}{2}, \text{ if } p > \frac{v(2\alpha+v-1)}{2(\alpha+v-1)} \\ p - \frac{\alpha(p-v)}{1-v}, \text{ otherwise} \end{cases}.$$

Then we solve p^*:

$$p^* = \begin{cases} \frac{1}{2}, \text{ if } (1-2v)\alpha - (1-v)^2 > 0 \\ \frac{2\alpha v - 2v + v^2 + 1}{2\alpha}, \text{ otherwise} \end{cases}.$$

□

Proof of Theorem III.2.

Results immediately follow by taking the first order derivative of the profit function (3.2). □

Proof of Lemma III.2.

When $v = 0$, the feasible regions reduce to $S_1 \bigcup S_2 \bigcup S_3$. As $\pi_{M1} = 0$, we focus on $S_2 \bigcup S_3$, the reduced forms of which are

$$\begin{aligned} S_2 =& \{(w,p) : \max\{(1+\alpha)p - 2\alpha, (\alpha-1)p\} < w < (1-\alpha)p\}, \\ S_3 =& \{(w,p) : (1-\alpha)p < w < p\}. \end{aligned}$$

The proof consists of three parts: (1)$\alpha \geq 1$; (2)$\frac{1}{2} \leq \alpha < 1$; (3)$\alpha < 1/2$.

(1) When $\alpha \geq 1$.

When $\alpha \geq 1$, $S_2 = \varnothing$ and $\pi_{MAP}(w,p) = \pi_{RPM}(w,p)$ for any $(w,p) \in S_3$. Thus $\pi^*_{MAP} = \pi^*_{RPM}$.

In part (2) and (3), S_2 reduces to $S_2 = \{(w,p) : (1+\alpha)p - 2\alpha < w < (1-\alpha)p\}$.

(2) When $\frac{1}{2} \leq \alpha < 1$.

We show in the following that the optimal solution has to satisfy $w > (1-\alpha)p$, i.e. the maximizer is not in S_2.

(2a) If $p \leq 2/3$.

Recall that in the range:

$$\pi_{M2} = \frac{1}{4\alpha(1-\alpha)}(2\alpha - (1+\alpha)p + w)(p-w)(2 - 2\alpha - (1-\alpha)p - w)w.$$

If we let $\pi_{M2} = 0$ and solve w, we get the following four roots:

$$w_1 = 0, w_2 = (1+\alpha)p - 2\alpha, w_3 = 2 - 2\alpha - (1-\alpha)p, w_4 = p,$$

and the following inequality holds:

$$\max\{w_1, w_2\} \leq (1-\alpha)p \leq \min\{w_3, w_4\}.$$

This means, π_{M2} is unimodal in w. Then we look at the first order derivative:

$$\frac{\partial \pi_{M2}}{\partial w}|_{w=(1-\alpha)p} = p(2-3p)(1-p)(2\alpha-1)/2 \geq 0, \text{ if } p \leq 2/3.$$

As a result, when $\max\{(1+\alpha)p-2\alpha, 0\} < w \leq (1-\alpha)p$ with $p \leq 2/3$, π_{M2} is always increasing in w. Therefore they are not optimal.

(2b) If $p > 2/3$.

We rearrange $(1+\alpha)p-2\alpha < w \leq (1-\alpha)p$ as $\frac{w}{1-\alpha} \leq p \leq \frac{w+2\alpha}{1+\alpha}$. Then in this part we show that (w,p) is not optimal if $\max\{\frac{w}{1-\alpha}, \frac{2}{3}\} \leq p \leq \frac{w+2\alpha}{1+\alpha}$. (we consider when $\frac{w+2\alpha}{1+\alpha} > \frac{2}{3}$, otherwise the proof is done.)

Let $\pi_{M2} = 0$ and solve p, we get the following three roots:

$$p_1 = w, p_2 = \frac{w+2\alpha}{1+\alpha}, p_3 = 2 - \frac{w}{1-\alpha},$$

and we have $p_1 \leq \max\{\frac{w}{1-\alpha}, \frac{2}{3}\} \leq p_2 \leq p_3$.

If $\frac{w}{1-\alpha} \geq 2/3$, we evaluate

$$\left.\frac{\partial \pi_{M2}}{\partial p}\right|_{p=\frac{w}{1-\alpha}} = \frac{w(1-\alpha-w)(2-2\alpha-(3+2\alpha)w)}{2(1-\alpha)^2} \leq 0, \text{ if } \frac{w}{1-\alpha} \geq \frac{2}{3}.$$

If $\frac{w}{1-\alpha} < \frac{2}{3}$, we evaluate

$$\left.\frac{\partial \pi_{M2}}{\partial p}\right|_{p=2/3} = \frac{w[(-6\alpha-3)w^2 + (-8\alpha^2+2\alpha+8)w + 4\alpha - 4]}{12\alpha(1-\alpha)} \leq 0, \text{ if } \frac{w}{1-\alpha} < 2/3.$$

In either case, π_{M2} is decreasing in p when $\max\{\frac{w}{1-\alpha}, 2/3\} \leq p \leq \frac{w+2\alpha}{1+\alpha}$. Therefore, such $(w,p) \in S_2$ is not optimal.

Combining (2a) and (2b), we conclude that optimal solution is always contained in S_3. Solve π_{M3} and get optimal solution: $w^* = 1/4, p^* = 1/2, \pi^*_{MAP} = 1/64 = \pi^*_{RPM}$ when $\alpha \geq 1/2$.

(3) When $\alpha < 1/2$.

When $\alpha < 1/2$, we have $(1+\alpha)p - 2\alpha < w \leq (1-\alpha)p \leq \alpha p + 1 - 2\alpha$. Therefore,

$$\begin{aligned}
&[2\alpha - (1+\alpha)p + w][2 - 2\alpha - (1-\alpha)p - w] \\
&\leq [2\alpha - (1+\alpha)p + (1-\alpha)p][2 - 2\alpha - (1-\alpha)p - (1-\alpha)p] \\
&= 4\alpha(1-\alpha)(1-p)^2.
\end{aligned}$$

For any $(w, p) \in S_2$, we have

$$\begin{aligned}
\pi_{M2} &= \frac{1}{4\alpha(1-\alpha)} w(p-w)[2\alpha - (1+\alpha)p + w][2 - 2\alpha - (1-\alpha)p - w] \\
&<= (1-p)^2(p-w)w = \pi_{RPM}(w, p).
\end{aligned}$$

For any $(w, p) \in S_3$, we have $\pi_{M3} = \pi_{RPM}$. Therefore we have $\pi^*_{MAP} \leq \pi^*_{RPM}$. □

Proof of Lemma III.3.

For notational simplicity, we drop the subscript MAP. As shown in Lemma III.2, we have $w^*(\alpha) = 1/4$ and $p^*(\alpha) = 1/2$ if $\alpha \geq 1/2$. If $0 < \alpha < 1/2$, $\pi_{M3}(w, p, \alpha)$ is decreasing in w in S_3. Therefore, (w^*, p^*) must fall in $\bar{S}_2$ (S_2 is the interior). As shown in Figure 3.1, this is a triangle in (w, p) coordinate. Next we prove a sequence of properties, which lead to the result step by step.

Step 1: $(w^*, p^*) \in S_2$ (not on the boundaries of S_2).

This essentially says the optimal solution is not in the boundary of the triangle when $0 < \alpha < 1/2$. When $w = 0$ or $p = \frac{w+2\alpha}{1+\alpha}$, which define the left boundary of S_2, we have $\pi_{M2}(w, p, \alpha) = 0$ so they cannot be optimal. It suffices to show that optimal solution does not exist on the right boundary, defined by $w = (1-\alpha)p$. This is proved in the following by contradiction.

If the optimal solution satisfies $w = (1-\alpha)p$, we substitutes w with $(1-\alpha)p$ and get:

$$\pi_{M2}(w, p, \alpha) = \alpha(1-\alpha)p^2(1-p)^2.$$

So the optimal solution should be $p^* = 1/2, w^* = (1-\alpha)/2$. However, if we evaluate the first order condition with respect to w, we find

$$\left.\frac{\partial \pi_{M2}}{\partial w}\right|_{p=\frac{1}{2}, w=\frac{1-\alpha}{2}} = \frac{2\alpha - 1}{16} < 0.$$

The manufacturer can always decrease w a little (which remains in S_2) to get a higher profit. Therefore, the optimal solution does not exist on the right boundary either.

Step 2: $\pi_{M2}(w, p, \alpha)$ is quasi-concave for $(w, p) \in S_2$.

Let $p = bw + c$ (b and c are arbitrary coefficients). After replacing p, the profit function is dependent only in w. $\pi_{M2}(w, p, \alpha)$ is quasi-concave for $(w, p) \in S_2$, if and only if, for any (b, c) such that $p = bw + c$ intersects S_2, $\pi_{M2}(w, p(w), \alpha)$ is quasi-concave in w. We can prove the latter statement by dividing (b, c) and conquering case by case.

We use $b < -\frac{1}{1-\alpha}$ as an illustration of the proof. The proof for other cases is similar and thus omitted here.

After replacing p, the profit function is 4th degree polynomial in w with quartic coefficient $(1 + b - \alpha b)(1 - b)(1 - b - \alpha b)$ (which is negative in this case). The four roots are:

$$r_1 = 0, r_2 = \frac{c}{1-b}, r_3 = \frac{(1-\alpha)(2-c)}{1+b-\alpha b}, r_4 = \frac{c + \alpha c - 2\alpha}{1 + c - \alpha c - 2b}.$$

There are two subcases.

(2a) If $\frac{2\alpha}{1+\alpha} \leq c < 1 - b + \alpha b$.

Under such conditions, $p = bw + c$ intersects S_2 when $w \in (w_1, w_2)$, where

$$w_1 = \frac{c + \alpha c - 2\alpha}{1 + c - \alpha c - 2b} = r_4, w_2 = \frac{c - \alpha c}{1 - b + \alpha b}.$$

Given all the conditions, we can prove the following inequality:

$$\max(r_1, r_3) < r_4 = w_1 < w_2.$$

Therefore, $\pi_{M2}(p(w), w, \alpha)$ is quasi-concave in $w \in (w_1, w_2)$.

(2b) If $0 < c < \frac{2\alpha}{1+\alpha}$.

Now $p = bw + c$ intersects S_2 when $w \in (w_1, w_2)$, where

$$w_1 = 0 = r_1, w_2 = \frac{c - \alpha c}{1 - b + \alpha b}.$$

Given all the conditions, we can prove the following inequality:

$$\max(r_3, r_4) < 0 = r_1 = w_1 < w_2.$$

Therefore, $\pi_{M2}(p(w), w, \alpha)$ is quasi-concave in $w \in (w_1, w_2)$.

Step 3: Fix w, $p^*(\alpha)$ is increasing in α.

The first order condition with respect to p is:

$$\begin{aligned}\frac{\partial \pi_{M2}}{\partial p} = \frac{w}{4\alpha(1-\alpha)}\Big[&(3 - 3\alpha^2)p^2 + (4\alpha w - 2w - 4\alpha + 2\alpha^2 w + 8\alpha^2 - 4)p \\ &- 4\alpha^2 w - 4\alpha^2 - 2\alpha w^2 - 2\alpha w + 4\alpha - w^2 + 4w\Big] = 0.\end{aligned}$$

Define $h_1(w, p, \alpha)$ as

$$\begin{aligned}h_1(w, p, \alpha) =&(3 - 3\alpha^2)p^2 + (4\alpha w - 2w - 4\alpha + 2\alpha^2 w + 8\alpha^2 - 4)p \\ &- 4\alpha^2 w - 4\alpha^2 - 2\alpha w^2 - 2\alpha w + 4\alpha - w^2 + 4w = 0.\end{aligned}$$

By implicit function theorem, we have

$$\frac{dp}{d\alpha} = -\frac{\partial h_1/\partial \alpha}{\partial h_1/\partial p}.$$

Note that $\partial h_1/\partial p < 0$ (second order condition). We only need to prove $\partial h_1/\partial \alpha > 0$ for $(w,p) \in S_2$. We have

$$\frac{\partial h_1}{\partial \alpha} = -6\alpha p^2 + (16\alpha + 4w + 4\alpha w - 4)p + 4 - 2w - 8\alpha w - 2w^2 - 8\alpha.$$

By checking the boundary of S_2, we showed that this is negative for any $(w,p) \in S_2$.

Step 4: Fix p, $w^*(\alpha)$ is decreasing in α.

First we want to refine the candidate set for the optimal solution. Define

$$T = \{(w,p) : \frac{p}{2} < w < 1-\alpha, \frac{w}{1-\alpha} < p < \frac{w+2\alpha}{1+\alpha}\}.$$

Apparently we have $T \subset S_2$. Given $(w^*, p^*) \in S_2$, if $w^* > p/2$, we can conclude that $(w^*, p^*) \in T$.

$$\begin{aligned}\frac{\partial \pi_{M2}}{\partial w} =& \frac{1}{4\alpha(1-\alpha)}\Big[(1-\alpha^2)p^3 + (4\alpha w - 2w - 2\alpha + 2\alpha^2 w + 4\alpha^2 - 2)p^2 \\ &+ (-8\alpha^2 w - 4\alpha^2 - 6\alpha w^2 - 4\alpha w + 4\alpha - 3w^2 + 8w)p \\ &+ 8\alpha^2 w + 12\alpha w^2 - 8\alpha w + 4w^3 - 6w^2\Big] = 0.\end{aligned}$$

We define the terms in square bracket as $h_2(w,p,\alpha)$.

Substituting w with $p/2$, we obtain

$$h_2(p/2, p, \alpha) = \frac{1}{4}p^2(2-p)(1-2\alpha) > 0,$$

if $\alpha \in (0, 1/2)$. Therefore, $w^* > p^*/2$ and thus $(w^*, p^*) \in T$.

The rest of the proof is similar as that in step 3. Again we only need to show

$\partial h_2/\partial\alpha < 0$ for $(w,p) \in T$. We have

$$\frac{\partial h_2}{\partial \alpha} = 2(2-p)[\alpha p^2 + (1-2w-2\alpha w-2\alpha)p + 4\alpha w - 2w + 3w^2].$$

By checking the boundary of T, we showed that $\frac{\partial h_2}{\partial \alpha}$ is negative for any $(w,p) \in T$.

Step 5: $p^*(\alpha)$ is increasing in α.

Let $w(p)$ be the optimal w for a given p and α. The profit function is $\pi_{M2}(w(p), p, \alpha)$. The first order condition with respect p is:

$$\frac{d\pi_{M2}}{dp} = \frac{\partial \pi_{M2}}{\partial w}\frac{\partial w}{\partial p} + \frac{\partial \pi_{M2}}{\partial p} = 0.$$

By envelope theorem, we have $\frac{\partial \pi_{M2}}{\partial w} = 0$ when w is the optimal value given p. So first order condition reduces to $h_1(w(p), p, \alpha) = 0$ which is exactly same as in step (3). We already showed in step (3) that

$$\frac{\partial h_1}{\partial \alpha} > 0.$$

Note $\frac{\partial h_1}{\partial p} < 0$ (second order condition). We then have

$$\frac{dp^*}{d\alpha} = -\frac{\partial h_1/\partial p}{\partial h_1/\partial \alpha} > 0.$$

Step 6: $w^*(\alpha)$ is decreasing in α.

The proof is same as step 5. Just need to switch the roles of w and p.

□

Proof of Theorem III.3.

It holds directly from Lemma III.2 and Lemma III.3. □

Proof of Theorem III.4.

When $v + \alpha < 1$, the optimizer has to be in $S_2 \bigcup S_3 \bigcup S_4$. We have

$$\lim_{\alpha \to 0} S_2 = \lim_{\alpha \to 0} S_3 = \{(w,p) : w = p, v < w < 1\}; \lim_{\alpha \to 0} S_4 = \{(w,p) : w = p = v\}.$$

Therefore, we also have $\lim_{\alpha \to 0} p^*_{MAP} - w^*_{MAP} = 0$

The second part of the theorem will be proved in Lemma III.4. □

Proof of Lemma III.4.

Any (w,p) eventually falls into S_3, S_5, or S_7 as $\alpha \to \infty$. For any (w,p), we also have

$$\lim_{\alpha \to \infty} \pi_{MAP}(w,p) = \pi_{RPM}(w,p).$$

Therefore, we get: (1) $\lim_{\alpha \to \infty} w^*_{MAP} = w^*_{RPM}$; (2) $\lim_{\alpha \to \infty} p^*_{MAP} = p^*_{RPM}$; (3) $\lim_{\alpha \to \infty} \pi^*_{MAP} = \pi^*_{RPM}$. □

Proof of Lemma III.5.

From Theorem III.2 we immediately get π^*_{RPM} is increasing in v. We focus on the MAP policy in the following.

The idea of the proof for π^*_{MAP} is to show for any (w,p), $\pi_{MAP}(w,p)$ is non-decreasing in v. Note that S_1 thought S_7 are defined all as open sets. Given the continuity of profit function, the boundary will not influence our analysis.

It is easy to see that $\pi_{Mi}(i = 1, 3, 4, 5, 6)$ are non-decreasing in v. In the rest of the proof we deal with $\pi_{Mi}(i = 2, 7)$. We divide π_{M7} into two sets:

$$S_{71} = \left\{(w,p) : \alpha + v > 1, 1 - \alpha < w < v, v + \frac{(1-v)(v-w)}{2(\alpha + v - 1)} < p < 1\right\},$$
$$S_{72} = \left\{(w,p) : \alpha + v > 1, 2 - 2\alpha - v < w < 1 - \alpha, v + \frac{(1-v)(v-w)}{2(\alpha + v - 1)} < p < 1\right\}.$$

(1) π_{M2} is non-decreasing in v.

π_{M2} can be written as

$$\pi_{M2} = w(p-w)d_1(d_1+d_2),$$

where

$$\begin{aligned} d_1 &= \frac{1}{2}\left(1+\frac{1-p}{1-v}\right)-\frac{p-w}{2\alpha}, \\ d_2 &= \frac{p-w}{2\alpha}+\frac{1}{2}\left(\frac{1-\alpha-w}{1-\alpha-v}-1\right). \end{aligned}$$

Clearly both d_1 and d_2 are increasing in v (with $p<1$ and $w<1-\alpha$ in S_2). Hence π_{M2} is non-decreasing in v.

(2) π_{M7} is non-decreasing in v in S_{71}.

Taking derivative with respect to v, we have (positive denominator being dropped)

$$\frac{d\pi_{M7}}{dv} = -4(\alpha+v-1)^2p+2w-7v-7\alpha+7\alpha v-\alpha w-4vw+2v^2w+4\alpha^2+5v^2-v^3+\alpha vw+3.$$

For all $(w,p) \in S_{71}$, we have $\frac{d\pi_{M7}}{dv} > 0$.

(3) The optimal solution cannot exist in S_{72}.

First we show that π_{M7} is unimodal in p. π_{M7} is positive 3-order polynomial in p. Additionally, we have $\pi_{M7}(p=w) = \pi_{M7}(p=1) = 0$. For $w<p<1$, π_{M7} is unimodal, given $\pi_{M7} \geq 0$.

Then we show that for given w and v, π_{M7} is decreasing in p within S_{72}. It suffices to prove $\left.\frac{d\pi_{M7}}{dp}\right|_{p=v+\frac{(1-v)(v-w)}{2(\alpha+v-1)}} < 0$. We have (positive denominator being dropped)

$$\begin{aligned} \left.\frac{d\pi_{M7}}{dp}\right|_{p=v+\frac{(1-v)(v-w)}{2(\alpha+v-1)}} =& (2\alpha+v-1)w^2+(6\alpha v-2v-10\alpha+8\alpha^2+2)w \\ & -12\alpha^2v+4\alpha^2-16\alpha v^2+26\alpha v-8\alpha-5v^3+15v^2-14v+4. \end{aligned}$$

Checking the range of w,

$$\left.\frac{d\pi_{M7}}{dp}\right|_{p=v+\frac{(1-v)(v-w)}{2(\alpha+v-1)},w=2-2\alpha-v} = -4(\alpha+v-1)^2(2\alpha+v-1) < 0,$$
$$\left.\frac{d\pi_{M7}}{dp}\right|_{p=v+\frac{(1-v)(v-w)}{2(\alpha+v-1)},w=1-v} = -(\alpha+vl-1)^2(6\alpha+5v-5) < 0.$$

Therefore, we conclude that for $2-2\alpha-v < w < 1-\alpha$, the optimal p is not in S_{72}.

$\square$

Proof of Theorem III.5.

Part (1).

The proof for Part (1) only requires $\alpha + v \geq 1$, instead of $\alpha \geq 1$.

Recall that under MAP policy the optimal solution can only exist in regions $S_6 \bigcup S_7 \bigcup S_3$ (refer to Figure 3.1). π_{M6} and π_{M3} are clearly independent of α, while π_{M7} is decreasing in α, with other parameter unchanged. Reorganize π_{M7} to see this:

$$\pi_{M7} = \frac{w(p-w)(1-p)}{(1-v)^2}\left(1-p+\frac{(1-v)(v-w)}{2(\alpha+v-1)}\right).$$

Step 1: Only one boundary, the left one of S_6, may contain the maximizer.

The right boundary ($w = p$) and the top boundary ($p = 1$) may never contain the optimizer because those decisions result in zero profit. Only the left boundary of S_6 ($w = p + 2\alpha\frac{v-p}{1-v}$) is concerned.

Maximizing π_{M6} without constraint, we get

$$w_6^* = 1/3, p_6^* = 2/3.$$

$(w_6^*, p_6^*) \in S_6$ is equivalent to the following conditions:

$$v < \frac{2}{3} \text{ and } \frac{1}{6}\left(1+\frac{1}{2-3v}\right) \leq \alpha \leq \frac{3(1-v)^2}{2(2-3v)}.$$

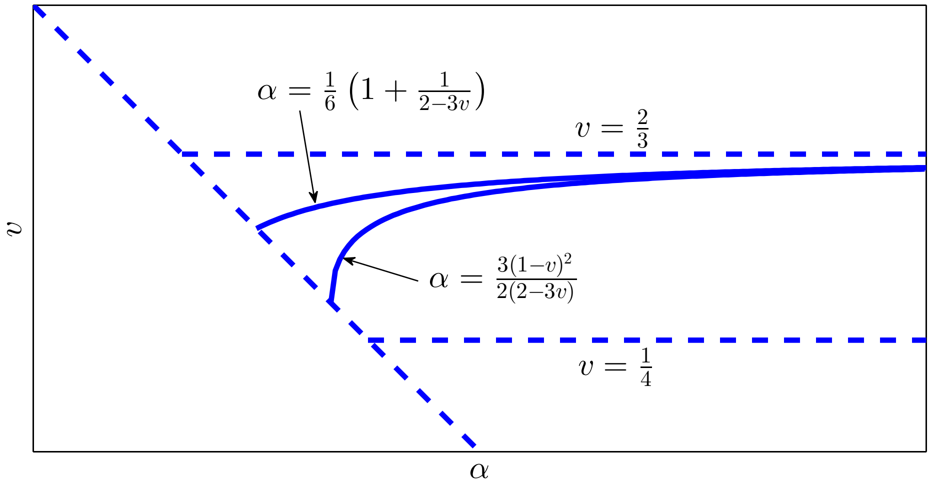


Figure B.1: Proof of Theorem III.5 part (1).

Step 2: π^*_{MAP} is increasing in α when $v \geq \frac{2}{3}$ or $v < \frac{2}{3}$ with $\alpha < \frac{1}{6}\left(1+\frac{1}{2-3v}\right)$.

Under above conditions, the maximizer in S_6, denoted as (w_6^o, p_6^o), is on the right boundary of S_6. We first show that (w_6^o, p_6^o) is also the overall maximizer in $S_6 \bigcup S_7 \bigcup S_3$.

Maximizing π_{M3} without constraint, we get

$$w_3^* = 1/4, p_3^* = 1/2.$$

As π_{M3} is unimodal, the maximizer cannot exist in S_3 when $v > 1/4$.

When $\alpha = 1 - v$, $S_7 = \varnothing$. As α increases, the boundary between S_6 and S_7 shift towards left and (w, p) pairs enter S_7 from S_6. Now we consider any $(w_0, p_0) \in S_7$. It must starts in S_6 and turn into S_7 as α increases. So we must have $\pi_{M6}(w_0, p_0) \geq \pi_{M7}(w_0, p_0)$ because both π_{M6} and π_{M7} are non-increasing in α. We also have $\pi_{M6}(w_6^o, p_6^o) \geq \pi_{M6}(w_0, p_0)$ because the unconstrained maximizer $(1/3, 2/3)$ is on the left side of S_6 and π_{M6} is unimodal in $S_6 \bigcup S_7$. Therefore, we showed that (w_6^o, p_6^o) is the maximizer over $S_6 \bigcup S_7 \bigcup S_3$.

Then we show that π^*_{MAP} increases in α within this region. As α increases, the left boundary of S_6 shifts leftwards, getting closer to the unconstrained maximizer (w^*_6, p^*_6). Combining the facts that π_{M6} is unimodal and the constrained maximizer is on the left boundary of S_6, we establish that π^*_{MAP} is strictly increasing in α.

Step 3: π^*_{MAP} is constant in α when $(w^*_6, p^*_6) \in S_6$.

As both (w^*_6, p^*_6) and π_{M6} are independent of α, it suffices to show that (w^*_6, p^*_6) is the maximizer not only in S_6 but also globally.

In this region we also have $v > \frac{1}{4}$, thus the maximizer cannot exist in S_3. Using similar logic as the last step, we also see that: $\pi_{M6}(w^*_6, p^*_6) \geq \pi_{M6}(w_0, p_0) \geq \pi_{M7}(w_0, p_0)$ for any $(w_0, p_0) \in S_7$. Therefore, (w^*_6, p^*_6) is the global maximizer.

Step 4: π^*_{MAP} is non-increasing in α when $v < \frac{2}{3}$ and $\alpha > \frac{3(1-v)^2}{2(2-3v)}$.

Within this region, (w^*_6, p^*_6) is to the right of S_6. The global maximizer is not in S_6 because π_{M6} is unimodal. Recall that for any $(w_0, p_0) \in S_3 \bigcup S_7$, π_{M3} or π_{M7} is non-increaseing in α. Hence, pi^*_{MAP} is non-increasing in α.

When $v \geq \frac{1}{4}$, S_3 does not include the maximizer and thus pi^*_{MAP} is always decreasing in α. When $v < \frac{1}{4}$, the maximizer would eventually be $(\frac{1}{4}, \frac{1}{2})$ in S_3 and pi^*_{MAP} is a constant in α once the maximizer becomes $(\frac{1}{4}, \frac{1}{2})$.

Step 5: Claim the threshold property in v.

For any given v, π^*_{MAP} is either always strictly increasing (when $v \geq 2/3$) or strictly increasing until $\alpha = \frac{1}{6}\left(1 + \frac{1}{2-3v}\right)$ and then remain weakly decreasing. As $\alpha \to \infty$, π^*_{MAP} converges to π^*_{RPM} (from Lemma III.4). Therefore, the threshold property stated in Part (1) must be true.

We can actually solve (w^o_6, p^o_6) and get

$$\begin{aligned}
\pi^*_{MAP} =& \frac{2\alpha}{27(2\alpha + v - 1)^2} \\
& \Big((8\alpha^2 + 4\alpha v - 8\alpha + 2v^2 - 2v + 2)\sqrt{4\alpha^2 + 2\alpha v - 4\alpha + v^2 - v + 1} \\
& - 16\alpha^3 - 12\alpha^2 v + 24\alpha^2 + 6\alpha v^2 + 12\alpha v - 12\alpha + 2v^3 - 3v^2 - 3v + 2\Big).
\end{aligned}$$

Then when $\alpha + v > 1$, we have $\pi^*_{MAP} \geq \pi^*_{RPM}$ if and only if

$$(128 - 192v)\alpha^3 - (12v^2 - 192v + 128)\alpha^2 - (76v^3 - 60v^2 + 48v - 32)\alpha - 27v^2(1-v)^2 \geq 0.$$

Denote the left hand side of the above inequality as $\Delta(\alpha, v)$ For $v \in [0, 2/3)$, $\Delta(\alpha, v) = 0$ defines α_1. For $v \in [2/3, 1]$, $\alpha_1 = +\infty$.

Part (2).

(2a) Existence of v_2.

The proof for existence of v_2 only requires $\alpha + v \geq 1$, instead of $\alpha \geq 1$.

The existence of threshold α_1 also implies that $\frac{\partial \Delta}{\partial \alpha} \geq 0$ when $\Delta(\alpha, v) = 0$. We only need to show $\frac{\partial \Delta}{\partial v} < 0$ when $\Delta(\alpha, v) = 0$.

Note that $\Delta(1/2, 1/2) = 0$. It suffices to show that $\frac{\partial \Delta}{\partial v} < 0$ when $\alpha \times v \in [\frac{1}{2}, +\infty] \times [\frac{1}{2}, \frac{2}{3}]$. In this region, we have:

$$\left(\frac{\partial \Delta}{\partial v}\right)'' = 324 - 648v - 456\alpha < 0,$$
$$\left(\frac{\partial \Delta}{\partial v}\right)' = -324v^2 + (324 - 456\alpha)v - 24\alpha^2 + 120\alpha - 54 < 0,$$
$$\frac{\partial \Delta}{\partial v} = -108v^3 + (162 - 228\alpha)v^2 + (-24\alpha^2 + 120\alpha - 54)v - 192\alpha^3 + 192\alpha^2 - 48\alpha < 0.$$

Thus we have

$$\frac{dv}{d\alpha} = \frac{\partial \Delta / \partial \alpha}{\partial \Delta / \partial v} \leq 0,$$

meaning $\Delta(\alpha, v) = 0$ is an increasing curve in α-v coordinate. This is equivalent to the existence of threshold v_2. For any α, v_2 is defined by $\Delta(\alpha, v_2) = 0$.

(2b) Existence of v_1.

As only $pi_{M3}(w, p) = pi_{RPM}(w, p)$, if $\pi^*_{MAP} = \pi^*_{RPM}$ holds in a non-degenerate interval of either α or v, the maximizer under MAP policy has to be in S_3. Recall that the unconstrained maximizer of pi_{M3} is $(w_3^*, p_3^*) = (\frac{1}{4}, \frac{1}{2})$. As a result, $v \leq 1/4$ is a sufficient condition for global maximizer being in S_3. In the rest of the proof we limit $v \in [0, \frac{1}{4}]$, as the other region is irrelevant here.

Also recall that the unconstrained maximizer in S_6 is $(w_6^*, p_6^*) = (\frac{1}{3}, \frac{2}{3})$. Hence for $v \leq 1/4$, (w_6^*, p_6^*) is on the right side of S_6, and thus $\pi_{M6}^* \leq \pi_{M7}^*$, where π_{Mi}^* is the constrained optimal value in S_i. The existence of v_1 is equivalent of the following statement: There exists $v_1 \in [0, \frac{1}{4}]$, such that $\pi_{M7}^* < \pi_{M3}^*$ if and only if $v \in [0, v_1)$.

We already know that

$$\pi_{M3}^* = \frac{1}{64(1-v)^2} = \pi_{RPM}^*.$$

The main task here is to study the behavior of π_{M7}^*. Denote (w_7^*, p_7^*) as the maximizer in $\bar{S}_7$.

Step 1: Show $w_7^* \geq \frac{(1-v)^2}{\alpha} + 2v - 1$.

For any policy $A(w_0, p_0) \in S_7$ with $w_0 < \frac{(1-v)^2}{\alpha} + 2v - 1$, we can define two other policies (A,B,C refer to Figure B.2): $B(w_0, \frac{1+w_0}{2})$ and $C(1+w_0-v-2\alpha+\alpha\frac{1-w_0}{1-v}, \frac{1+w_0}{2})$. B is the optimal policy within S_6 for given w_0. C is on the boundary between S_6 and S_7. Recall that for any $(w,p) \in S_7$, we have $\pi_{M7}(w,p) < \pi_{M6}(w,p)$. We also have $\pi_{M6}(B) < \pi_{M6}(C)$ because (i) π_{M6} is unimodal in w; and (ii) $w = p/2$ would be optimal for given p and both B and C are on the left side of $w = p/2$. Thus,

$$\pi_{M7}(A) < \pi_{M6}(A) < \pi_{M6}(B) < \pi_{M6}(C) = \pi_{M7}(C) \leq \pi_{M7}^*.$$

Therefore $A(w,p)$ is not the maximizer. Note that if $w > \frac{(1-v)^2}{\alpha} + 2v - 1$, B is no longer in S_6 and thus the above inequality does not hold.

Then the candidate region reduces to S_7' from S_7, where

$$S_7' = \{(w,p) : \frac{(1-v)^2}{\alpha} + 2v - 1 \leq w \leq v, v + \frac{(1-v)(v-w)}{2(\alpha+v-1)} \leq p \leq 1\}.$$

It is obvious that the top boundary of S_7', $p = 1$, cannot be the maximizer. The left

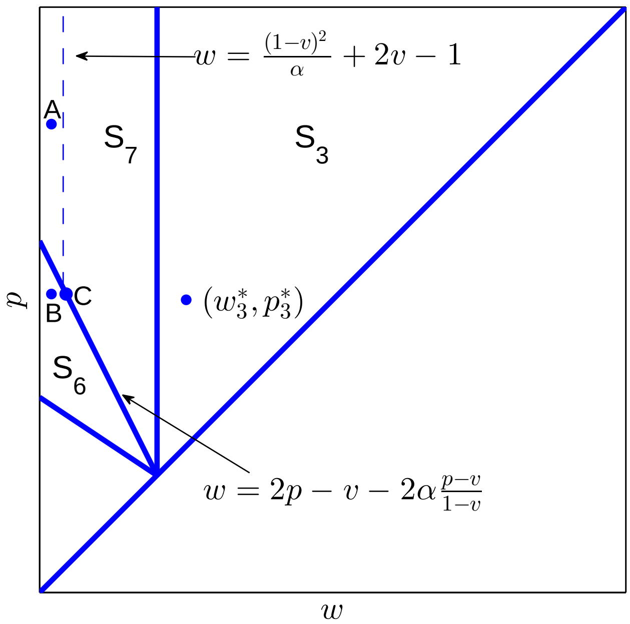


Figure B.2: Proof of Theorem III.5 part (2).

boundary, $w_7 = \frac{(1-v)^2}{\alpha} + 2v - 1$, is considered as interior points since they are included in S_7. Three subcases are discussed regarding whether (w_7^*, p_7^*) is on the other two boundaries or in the interior.

Step 2: If (w_7^*, p_7^*) is an interior maximizer.

For any (w, p) in the interior of S_7':

$$\pi_{M7}(w,p) - \pi_{M3}^* = \frac{1}{(1-v)^2}\left[w(p-w)(1-p)\left(1-p+\frac{(1-v)(v-w)}{2(\alpha+v-1)}\right) - \frac{1}{64}\right].$$

Since (All derivatives in this proof are with respect to v)

$$\left[\frac{(1-v)(v-w)}{(\alpha+v-1)}\right]' = \frac{\alpha w - (1-v)^2 - 2\alpha v + \alpha}{(\alpha+v-1)^2} \geq 0.$$

Therefore, if $\pi_{M7}(w,p) - \pi_{M3}^* > 0$ for a given v, it is also true for any greater v. This property extends to $\pi_{M7}^* - \pi_{M3}^*$ as well.

Step 3: If (w_7^*, p_7^*) satisfies $w = 2p - v - 2\alpha\frac{p-v}{1-v}$ (boundary maximizer).

Let the margin $d = p - w$ be the new variable in π_{M7}. Plug in $w = 2p - v - 2\alpha\frac{p-v}{1-v}$ and $d = p - w$, we get:

$$\pi_{M7}(d) = \frac{d(2\alpha + v - 1 - d)(2d - 2\alpha d - 2vd + v^2 + 2\alpha v - v)}{(2\alpha + v - 1)^2}.$$

Then for any d:

$$\begin{aligned}&\pi_{M7}(d) - \pi^*_{M3}\\ &= \frac{1}{(1-v)^2}\left[\frac{(1-v)^2 d(2\alpha + v - 1 - d)(2d - 2\alpha d - 2vd + v^2 + 2\alpha v - v)}{(2\alpha + v - 1)^2} - \frac{1}{64}\right].\end{aligned}$$

And for $\alpha \in [1, +\infty)$ (Note: This is the only step in the proof that requires $\alpha \geq 1$):

$$\left[\frac{(1-v)^2(2\alpha + v - 1 - d)(2d - 2\alpha d - 2vd + v^2 + 2\alpha v - v)}{(2\alpha + v - 1)^2}\right]' \geq 0$$

for any $d \in (0, \frac{(1-v)(2\alpha+v-1)}{2\alpha}]$, which is equivalent to $w \in [\frac{(1-v)^2}{\alpha} + 2v - 1, v)$.

Therefore, if $\pi_{M7}(d) - \pi^*_{M3} > 0$ for a given v, it is also true for any greater v. This property extends to $\pi^*_{M7} - \pi^*_{M3}$ as well.

Step 4: If (w_7^*, p_7^*) satisfies $w = v$ (boundary maximizer).

For any (w, p) on the right boundary, $w = v$, we have (for $v \leq \frac{1}{4}$)

$$\pi_{M7}(w, p) = \pi_{M3}(w, p) \leq \pi_{M3}(w_3^*, p_3^*).$$

Therefore, $\pi^*_{M7} \leq \pi^*_{M3}$ in this case.

Summary of part (2b).

With step 2, 3 and 4, it can be concluded that once $\pi^*_{M7} - \pi^*_{M3} > 0$ for a given v, it holds for any greater v. This completes the proof of existence of threshold v_1

Part (3).

The proof for the existence of α_1,α_2 and v_2 is same as part (1) and (2). In Table B.1 is an example that threshold v_1 does not exist when $\alpha + v \geq 1$ and $\alpha < 1$.

α	v	w^*_{MAP}	p^*_{MAP}	π^*_{MAP}	π^*_{RPM}
0.919	0.085	0.0811	0.5312	0.01870	0.01866
0.919	0.100	0.0831	0.4992	0.01925	0.01929
0.919	0.115	0.0874	0.4744	0.02009	0.01995

Table B.1: Example where v_1 does not exist.

□

Proof of Theorem III.6.

Recall from Equation (3.4) that the regular retailer's profit (under either policy) is

$$\pi_1 = \theta d_1(p - w) - \frac{1}{2}\theta^2.$$

Note the optimal decision is $\theta^* = d_1(p - w)$. Therefore, we have

$$\pi_1^* = \frac{1}{2}\theta^{*2}.$$

This means, we only need to compare θ^*, which is a perfect indicator of the regular retailer's profit.

We examine the three regions in Figure 3.4(a).

(1) The region where the manufacturer prefers RPM.

In this region, we have $\pi^*_{RPM} \geq \pi^*_{MAP}$. That is:

$$w_{RPM}\theta_{RPM}d_{1RPM} \geq w_{MAP}\theta_{MAP}(d_{1MAP} + d_{2MAP}).$$

In the proof of Theorem III.5 Part (1), we showed that π^*_{MAP} is increasing in α when $v \geq \frac{2}{3}$ or when $v < \frac{2}{3}$ and $\alpha \leq \frac{1}{6}\left(1 + \frac{1}{2-3v}\right)$. Under such conditions, the maximizer is on the left boundary of S_6 (refer to Figure 3.1). As α increases and crosses this threshold, π^*_{MAP} becomes non-increasing. Hence the boundary between RPM region and MAP region must fall into (i) $v \geq \frac{2}{3}$; or (ii) $\alpha \leq \frac{1}{6}\left(1 + \frac{1}{2-3v}\right)$ and

$v < \frac{2}{3}$. So we have $v > \frac{1}{2}$ in this region.

Under RPM policy, since $v > 1/2$, we have $w_{RPM} = v/2$ and $d_{1RPM} = 1$ (from Theorem III.2). Under MAP policy, in this region the maximizer (w, p) is on the left boundary of S_6. Therefore, we have $d_{1MAP} + d_{2MAP} = 1$ (from Table 3.2) and the left boundary of S_6:

$$p = \frac{2\alpha v - w + vw}{2\alpha + v - 1}.$$

Replacing p in π_{M6}:

$$\pi_{M6} = \frac{2\alpha w(v - w)(2\alpha + w - 1)}{(2\alpha + v - 1)^2}.$$

We have π_{M6} is unimodal in $w \in [1 - 2\alpha, v]$. Taking derivative with respect to w and plugging in $w = w_{RPM}$:

$$\pi'_{M6}|_{w=w_{RPM}} = \frac{\alpha v^2}{2(2\alpha + v - 1)^2} > 0.$$

Therefore we have $w_{RPM} < w_{MAP}$. Combining with $d_{1RPM} = d_{1MAP} + d_{2MAP} = 1$, we get

$$\theta_{RPM} > \theta_{MAP}.$$

That is, the regular retailer prefers RPM policy in this region.

(2) The region where the manufacturer is indifferent between RPM and MAP.

Since in this region, the free rider is not involved in the business and both w and p are identical under the two policies, the regular retailer is also indifferent.

(3) The region where the manufacturer prefers MAP.

The regular retailer may prefer either MAP or RPM, depending on market conditions. □

Proof of Theorem III.7.

It is straight-forward to check that, $\pi_M(\alpha)$ in Theorem III.1 is increasing when $\alpha < 1 - v$ and decreasing afterwards. □

Proof of Theorem III.8.

In Theorem III.5 Part (1), we have proved that π^*_{MAP} is increasing in α when $\alpha < \frac{1-v}{2(2-3v)}$, constant when $\frac{1-v}{2(2-3v)} \leq \alpha \leq \frac{3(1-v)^2}{2(2-3v)}$, and decreasing when $\alpha > \frac{3(1-v)^2}{2(2-3v)}$. □

Proof of Theorem III.9.

(1) When $\frac{2}{3} \leq v \leq 1$.

In this part we show that $\pi_1(\alpha)$ is always increasing in α so $\alpha^*_{R1} = +\infty$.

When $\frac{2}{3} \leq v \leq 1$, the manufacturer's decision is on the left boundary of region 6 (as shown in Theorem III.5). Its profit function is (with $w = p - 2\alpha\frac{p-v}{1-v}$ already plugged in)

$$\pi_M(p) = \frac{2\alpha(1-p)(p-v)(2\alpha p - p - 2\alpha v + pv)}{(1-v)^3}.$$

The first order condition in p is (defined as f):

$$f(p, \alpha) = (6\alpha + 3v - 3)p^2 + (-2v^2 - 8\alpha v - 4\alpha + 2)p + 4\alpha v - v + 2\alpha v^2 + v^2 = 0.$$

By implicit function theorem, we get:

$$\frac{dp}{d\alpha} = -\frac{\partial f/\partial \alpha}{\partial f/\partial p} = \frac{(p-v)(3p-v-2)}{3(1-2\alpha-v)p + 4\alpha v + 2\alpha + v^2 - 1}.$$

The denominator is positive because the second order condition $\partial f/\partial p < 0$ when p is at its optimum.

On the other hand, the retailer's profit function is (w is also substituted):

$$\pi_1(p,\alpha) = \frac{2\alpha(p-v)(1-p)}{(1-v)^2}.$$

When p is at its optimum, we have

$$\begin{aligned}\frac{d\pi_1}{d\alpha} =& \frac{2}{(1-v)^2}\left((p-v)(1-p) + \alpha(1+v-2p)\frac{dp}{d\alpha}\right)\\ =& \frac{4(1-v)(p-v)[-3p^2+(v-\alpha+4)p-v+\alpha v-1]}{3(1-v)^2((1-2\alpha-v)p+4\alpha v+2\alpha+v^2-1)}.\end{aligned}$$

We only need to verify that the terms within square brackets is positive when $\frac{2}{3} \leq v \leq 1$ and $\alpha \geq 1-v$ and p satisfies $f(p,\alpha) = 0$. This can be easily checked true. The arithmetic is omitted.

(b) When $\frac{1}{3} \leq v < \frac{2}{3}$.

Recall that α_M^* is any value in $[\max\{1-v, \frac{1-v}{2(2-3v)}\}, \frac{3(1-v)^2}{2(2-3v)}]$. When $1-v \leq \alpha \leq \frac{1-v}{2(2-3v)}$, π_1 is increasing in α, the proof of which is same as above because the maximizer is still on the left boundary of S_6. When $\frac{1-v}{2(2-3v)} \leq \alpha \leq \frac{3(1-v)^2}{2(2-3v)}$, the manufacturer's decision remains at $(w^*,p^*) = (\frac{1}{3},\frac{2}{3})$. From Table 3.2 we see that d_1 is constant in α in S_6. As a result, π_1 is also a constant in α.

Therefore, π_R is non-decreasing when $1-v \leq \alpha \leq \frac{3(1-v)^2}{2(2-3v)}$ while $\alpha_M^* \leq \frac{3(1-v)^2}{2(2-3v)}$. So we have $\alpha_M^* \leq \alpha_{R1}^*$.

(c) When $0 \leq v < \frac{1}{3}$.

In this region $\alpha_M^* = 1-v$ so we must have $\alpha_M^* \leq \alpha_R^*$ as this theorem is considered when $\alpha \geq 1-v$.

□

Proof of Theorem III.10.

Within this region, manufacturer's decision is $(w^*,p^*) = (\frac{1}{3},\frac{2}{3})$, independent of α (as shown in Theorem III.5). In the row of S_6 in Table 3.2, we see that (1) the free rider's price drops in α; and (2) d_1 and d_2 are independent of α. The number of consumers is $\theta = d_1(p - w)$, which remains unchanged in α as well. The total consumer surplus must be higher. $\square$

Proof of Theorem III.11.

Part (1) and (2): When $\alpha + v \geq 1$.

Consider MAP policy. For a given $\alpha \geq 1 - v$, let (w^*, p^*) be the optimizer. That is: $\pi^*_{MAP}(\alpha) = \pi_{MAP}(w^*, p^*, \alpha)$. We can always find $\alpha' \geq \alpha$, such that: $\pi_{MAP}(w^*, p^*, \alpha) = \pi_{MAP2}(w^*, p^*, \alpha')$. This is because: when $\alpha \geq 1 - v$, there is no competition between the regular retailer and the free rider. Under MAP2, free rider's market share, d_2, would be higher than that under MAP. When increasing α, d_2 decreases continuously. Thus there exists α' such that d_2 is same as that under MAP (with original α) and d_1 is also the same.

Consider MAP2 policy. For a given $\alpha \geq 1 - v$, let (w_2^*, p_2^*) be the optimizer. That is: $\pi^*_{MAP2}(\alpha) = \pi_{MAP2}(w_2^*, p_2^*, \alpha)$. We can always find $1 - v \leq \alpha' \leq \alpha$, such that: $\pi_{MAP2}(w_2^*, p_2^*, \alpha) = \pi_{MAP}(w_2^*, p_2^*, \alpha')$. The argument for this is similar to that above. The continuity of d_2 in α results from Table 3.2. The threshold α' is greater than $1 - v$ because if $\alpha' = 1 - v$ under MAP, the free rider must have a higher market share than original d_2 under MAP2.

In Theorem III.5 we showed that for $\alpha + v \geq 1$, under MAP there exists α_1 such that $\pi^*_{MAP}(\alpha) < \pi^*_{RPM}$ if and only if $\alpha < \alpha_1$. Then for any $\alpha < \alpha_1$ we have

$$\pi^*_{MAP2}(\alpha) = \pi_{MAP2}(w^*, p^*, \alpha) \leq \pi_{MAP}(w^*, p^*, \alpha') \leq \pi^*_{MAP}(\alpha') < \pi^*_{RPM},$$

where $1 - v \leq \alpha' \leq \alpha$. This completes proof of part (1).

Let α^*_{MAP} and α^*_{MAP2} be the optimal α for the manufacturer under MAP and

MAP2. From the inequality above, we know that:

$$\pi^*_{MAP2}(\alpha^*_{MAP2}) \leq \pi^*_{MAP}(\alpha') \leq \pi^*_{MAP}(\alpha^*_{MAP}),$$

where $1 - v \leq \alpha' \leq \alpha^*_{MAP2}$. Similarly, we also have

$$\pi^*_{MAP}(\alpha^*_{MAP}) \leq \pi^*_{MAP2}(\alpha') \leq \pi^*_{MAP2}(\alpha^*_{MAP2}),$$

where $\alpha' \geq \alpha^*_{MAP}$. This completes the proof of part (2).

Part (3): When $\alpha + v < 1$.

Consider MAP policy. For $\alpha < 1 - v$, there is competition between the regular retailer and the free rider. So changing d_2 also influences d_1. As a result, for the same pair of (w, p), in general we cannot find a α' under MAP2 such that d_1 and d_2 are identical as those under MAP with original α. For a given $\alpha < 1 - v$, let (w^*, p^*) be the optimizer. That is: $\pi^*_{MAP}(\alpha) = \pi_{MAP}(w^*, p^*, \alpha)$.

Under MAP2, we can always find $\alpha' \geq \alpha$ (may be greater than $1 - v$), such that d_1 is the same but d_2 is weakly higher compared to those under MAP with α. This is because d_1 is continuous in α under MAP2. Therefore, we obtain: $\pi_{MAP}(w^*, p^*, \alpha) \leq \pi_{MAP2}(w^*, p^*, \alpha')$. This, along with analysis for $\alpha + v \geq 1$, completes the proof of part (3).

Consider MAP2 policy. There is no analogous property. Actually, $\pi^*_{MAP}(\alpha^*_{MAP}) < \pi^*_{MAP2}(\alpha^*_{MAP2})$ may happen when both α^*_{MAP} and α^*_{MAP2} smaller than $1 - v$.

□

Proof of Theorem III.12.

First we prove the result from the manufacturer's perspective. The manufacturer's profit is (plugging θ^* into Equation (3.7))

$$\pi_{MAP/RPM}(w,p,\delta) = \frac{1}{\delta}w(p-w)d_1(d_1+d_2) - \frac{1}{2}(\frac{1}{\delta^2} - \frac{1}{\delta})d_1^2(p-w)^2.$$

We first optimize this profit over δ taking the others as given and get

$$\delta^* = \frac{2d_1(p-w)}{wd_1 + pd_1 + 2wd_2}.$$

Using δ^*, we get

$$\pi_{MAP/RPM}(w,p) = \frac{1}{8}(pd_1 + wd_1 + 2wd_2)^2. \tag{B.1}$$

Now consider the RPM policy. Recall that $d_1 = \frac{1-p}{1-v}$ (with $p \geq v$) and $d_2 = 0$. Equation (B.1) reduces to

$$\pi_{RPM}(w,p) = \frac{1}{8}\left[(w+p)\frac{1-p}{1-v}\right]^2.$$

Optimizing $\pi_{RPM}(w,p)$ with constraints $w \leq p$ and $p \geq v$ (as $p < v$ is never the optimal), we get the solution

$$p^*_{RPM} = \max\left\{\frac{1}{2}, v\right\},\ w^*_{RPM} = p^*_{RPM}.$$

Then consider the MAP policy. Under MAP policy, if $w = p$, we notice that

$$d_1(w,p) = \frac{1-p}{1-v} \text{ and } d_2(w,p) = 0.$$

Therefore,

$$\pi_{MAP}(w^*_{RPM}, p^*_{RPM}) = p^*_{RPM}.$$

That mean, the manufacturer under MAP policy can always mimic the best action under RPM policy and get the same level of profit. Consequently, we have

$$p^*_{MAP} \geq p^*_{RPM}.$$

Next we consider the profitability of the retailers under different policies. Under RPM policy, the free rider is ruled out and thus has zero profit. Since the manufacturer will set wholesale price equal to retail price, the regular retailer also end up zero profit. Therefore, both retailers weakly prefer MAP policy over RPM policy.

$\square$

APPENDIX C

Proofs of Lemmas and Theorems in Chapter IV

Proof of Theorem IV.1.

First we show Part (1). The central planner always choose x that maximizes surplus after technology realization. For any realization of ϵ, the central planner maximizes $u(x+t+\epsilon)-px$. $x^C(t+\epsilon)$ must satisfy the first order condition $u'(x+t+\epsilon) = p$. Additionally, we have

$$\frac{dx^C(t+\epsilon)}{dt} = -1 < 0.$$

That is, $x^C(t)$ is decreasing in t.

Then we show Part (2). As $v(x, t+\epsilon) - w_e(t+\epsilon) = u(x+t+\epsilon) - px$ is independent of functional form of $w_c(t+\epsilon)$, the client's payment only matters in the variance term. By setting $w_c(t+\epsilon) = u(x^C + t + \epsilon) + C$, the variance term is reduced to zero, and thus the objective function is maximized. In fact, since x^C satisfies $u'(x + t + \epsilon) = p$, $u(x^C + t + \epsilon)$ is also a constant in t and ϵ. Therefore, it is equivalent to choose $w_c(t + \epsilon) = 0$.

Now we show Part (3). In fact, for any realization of ϵ we have

$$\frac{d}{dt}\left\{u(x^C + t + \epsilon) - px^C\right\} = u'(x^C + t + \epsilon) = p.$$

$\square$

Proof of Lemma IV.1.

For any $\hat{x} \leq x^C(t+\epsilon)$, we have $u'(\hat{x}+t+\epsilon) \geq p$ from Theorem IV.1(1). In decentralized setting,

$$\left.\frac{dv(x,t+\epsilon)}{dx}\right|_{x=\hat{x}} \geq u'(\hat{x}+t+\epsilon) - p \cdot \max\{\alpha_i\} \geq u'(\hat{x}+t+\epsilon) - p \geq 0.$$

Hence, $x^D(t+\epsilon) \geq x^C(t+\epsilon)$. $\square$

Proof of Theorem IV.2.

We verify both the ESCO's and client's decisions are the same as those with central planner shown in Theorem IV.1.

First we consider the client. Now the ESCO's subsidy is independent of consumption. The client internalizes the full energy price p. Her consumption has to satisfy $u'(x+t+\epsilon) = p$, which is the same as that in Theorem IV.1(1). Let y_0 be the constant that satisfies $u'(y_0) = p$. We have

$$x^D(t+\epsilon) + t + \epsilon = y_0. \tag{C.1}$$

For any realization ϵ, the client's utility is

$$v^D(x^D, t+\epsilon) = u(x^D + t + \epsilon) - px^D + w_e(t+\epsilon) = u(y_0) - py_0 + M.$$

$v^D(x^D, t+\epsilon)$ is a constant independent of ϵ. Therefore the uncertainty related disutility, $\frac{\lambda}{2}VAR[v^D(x^D, t+\epsilon)]$, is zero.

Then we examine the ESCO. The effort decision problem (4.6) becomes

$$\max_{t} \left\{-(M-pt)-C(t)\right\}.$$

The solution t^* satisfies $C'(t)=p$, which is the same to t^C in Theorem IV.1(3).

As both the ESCO's and client's decisions are the same as those with central planner, the outcome is also the same. □

Proof of Theorem IV.3.

Under chauffage contracts, consumption x is implied by Equation (4.9). Only technology level (effort) t is chosen. Let y_0 be the constant such that $u(y_0)=u_0$. For any technology realization $t+\epsilon$, the implied energy consumption is $x^*=y_0-t-\epsilon$.

We consider contract $(\varnothing,\{0\})$ in the decentralized setting. The payments are $w_c(x)=0$ and $w_e(x)=px$. The ESCO's effort decision becomes

$$\max_{t} \left\{-p(y_0-t)-C(t)\right\}.$$

The solution t^* satisfies $C'(t)=p$, which is the same to t^C in Theorem IV.1(3). Therefore, the outcome is also the same. □

Proof of Theorem IV.4.

Part (1): N-rate Contract.

Similar to Theorem IV.1, we consider the client's payment, $w_c(x)$, is a continuous function in consumption, x. This is general enough to represent any contract that is only contingent on consumptions, including all the n-rate contracts.

In order to reach the first-best outcome, for a given technology realization $t+\epsilon$,

$x^*(t+\epsilon)$ has to satisfy (from Theorem IV.1(1))

$$u'(x+t+\epsilon) = p.$$

On the other hand, with payment structure $w_c(x)$, the client's decision will be derived from the first order condition

$$u'(x+t+\epsilon) - w_c'(x) = 0.$$

Combining the two equations above, we get $w_c'(x) = p$ or $w_c(x) = px$. This means, in order to get the client's consumption depiction right, she has to internalize all the variable cost.

If the ESCO lets $w_c(x) = px$, however, the client's utility $v(x^*, t+\epsilon) = u(x^* + t+\epsilon) - px^*$ is not a constant. That means, the disutility due to risk aversion can only be reduced but not fully eliminated. Therefore, the first-best outcome cannot be reached by this type of contracts.

Part (2): 1-rate Contract.

We first prove Part (2a) and (2b). In 1-rate contracts, the ESCO needs to decide t and α simultaneously. For given technology realization $t+\epsilon$, x^D has to satisfy $u'(x^D + t + \epsilon) = \alpha p$. Define $y(\alpha)$ be the value that satisfies $u'(y(\alpha)) = \alpha p$. Then we have

$$x^D(t+\epsilon) = y(\alpha) - t - \epsilon \text{ and } v^D(x^D, t+\epsilon) = u(y(\alpha)) - \alpha p(y(\alpha) - t - \epsilon). \qquad \text{(C.2)}$$

And we also have

$$\begin{aligned} y'(\alpha) &= \frac{p}{u''(y(\alpha))} < 0, \\ y''(\alpha) &= -\frac{p}{(u''(y(\alpha)))^2} \cdot u'''(y(\alpha)) \cdot y'(\alpha) \geq 0. \end{aligned}$$

The last inequality holds because we assume $u''' \geq 0$ in our model.

The objective function (4.10) becomes

$$\max_{t,\alpha} \left\{ u(y(\alpha)) - py(\alpha) + pt - \frac{\lambda}{2}\alpha^2 p^2 \sigma^2 - v_0 - C(t) \right\}. \tag{C.3}$$

Note t and α are separable. Obviously $t^{O/1}$ satisfies $C'(t) = p$.

Then it suffices to show the objective function, denoted as $V_{OBS}(\alpha)$, is concave in α. To see this, we have

$$\begin{aligned} V'_{OBS}(\alpha) &= -(1-\alpha)py'(\alpha) - \lambda p^2\sigma^2\alpha, \\ V''_{OBS}(\alpha) &= y'(\alpha) - (1-\alpha)y''(\alpha) - \lambda p^2\sigma^2 < 0. \end{aligned} \tag{C.4}$$

Therefore, there exists a unique optimal payment rate $\alpha^{O/1}$. To see $\alpha^{O/1} \in (0,1)$, we have

$$\begin{aligned} V'_{OBS}(0) &= -py'(0) > 0, \\ V'_{OBS}(1) &= -\lambda p^2\sigma^2 < 0. \end{aligned}$$

Next we prove Part (2c). We have

$$\begin{aligned} \frac{dV_{OBS}(\alpha^{O/1})}{d\lambda} &= -\frac{1}{2}p^2\sigma^2(\alpha^{O/1})^2 < 0, \\ \frac{dV_{OBS}(\alpha^{O/1})}{d\sigma^2} &= -\frac{\lambda}{2}p^2(\alpha^{O/1})^2 < 0. \end{aligned}$$

Therefore, $V^{O/1} = V_{OBS}(\alpha^{O/1})$ decreases in λ and σ^2. □

Proof of Theorem IV.5.

Part (1): 1-rate Contract.

Substituting x^D in Equation (C.2) into ESCO's effort decision problem (4.6), we have

$$\max_t \{-(1-\alpha)p(y(\alpha)-t) - C(t)\}.$$

Thus $t^{M/1}(\alpha)$ must satisfy $C'(t) = (1-\alpha)p$. Additionally, we also have

$$\begin{aligned} t^{M/1'}(\alpha) &= -\frac{p}{C''(t^{M/1}(\alpha))} < 0, \\ t^{M/1''}(\alpha) &= \frac{p}{(C''(t^{M/1}(\alpha)))^2} \cdot C'''(t^{M/1}(\alpha)) \cdot t^{M/1'}(\alpha) \le 0. \end{aligned}$$

The last inequality holds because we assume $C''' \ge 0$ in our model.

Under 1-rate contracts, the objective function (4.7) becomes

$$\max_\alpha \left\{ u(y(\alpha)) - py(\alpha) + pt^{M/1}(\alpha) - \frac{\lambda}{2}\alpha^2 p^2 \sigma^2 - v_0 - C(t^{M/1}(\alpha)) \right\}. \tag{C.5}$$

We need to show the objective function, denoted as $V_{MH}(\alpha)$, is concave in α. In fact, we have

$$\begin{aligned} V'_{MH}(\alpha) &= V'_{OBS}(\alpha) + \alpha p t^{M/1'}(\alpha), \\ V''_{MH}(\alpha) &= V''_{OBS}(\alpha) + pt^{M/1'}(\alpha) + \alpha p t^{M/1''}(\alpha) < 0. \end{aligned}$$

Therefore, there exists a unique optimal payment rate $\alpha^{M/1}$. We also have

$$V_{MH'}(\alpha^{O/1}) = V_{OBS'}(\alpha^{O/1}) + \alpha p t^{M/1'}(\alpha) = \alpha p t^{M/1'}(\alpha^{O/1}) < 0.$$

Therefore, $\alpha^{M/1} < \alpha^{O/1}$.

Part (2): 2-rate Contract.

The right inequality is obvious. To prove the left inequality, we show that, the optimal contract $(\alpha^{O/1}, t^{O/1})$ in Theorem IV.4(2) and its outcome can be replicated with 2-rate contracts with moral hazard.

Since comfort function $u(\cdot)$ is increasing and bounded, we denote $M = \lim_{x\to\infty} u(x+$

t). Consider the 2-rate contract $(z, \{\alpha_1, 0\})$. Under this contract, the client's utility is

$$v^D(x, t+\epsilon) = \begin{cases} u(x+t+\epsilon) - \alpha_1 p x & \text{if } x \leq z \\ u(x+t+\epsilon) - \alpha_1 p z & \text{if } x > z. \end{cases}$$

The solution to the client's problem (4.4) is

$$x^D(t+\epsilon) = \begin{cases} y(\alpha_1) - t - \epsilon & \text{if } t+\epsilon \geq \frac{1}{\alpha_1 p}[M - u(y(\alpha_1))] + y(\alpha_1) - z \\ +\infty & \text{otherwise }, \end{cases}$$

where $y(\alpha)$ is the value that satisfies $u'(y(\alpha)) = \alpha p$.

If the technology realization $t + \epsilon$ is smaller than the threshold above, the client would consume infinite amount of energy, which leads to negative infinite payoff for the ESCO. In order to avoid such situations, the ESCO would exert enough effort such that even the worst technology realization is above the threshold. Recall ϵ is bounded below by $\underline{\epsilon}$. By choosing

$$z = \frac{1}{\alpha_1 p}[M - u(y(\alpha_1))] + y(\alpha_1) - (t^{O/1} + \underline{\epsilon}), \tag{C.6}$$

we make sure that $t \geq t^{O/1}$. Then the ESCO's effort problem (4.6) becomes

$$\max_{t \geq t^{O/1}} \left\{-(1-\alpha_1)p(y(\alpha_1) - t) - C(t)\right\},$$

which is concave in t and maximizes at $t = t^{M/1}(\alpha_1) < t^{O/1}$. Therefore, the ESCO would choose $t = t^{O/1}$.

If the ESCO also chooses $\alpha_1 = \alpha^{O/1}$, the resulting outcome is the same as $V^{O/1}$. $\square$

Proof of Corollary IV.1.

First of all we prove with observability of the ESCO's effort, the 1-rate contract

$(\varnothing, \{1\})$, with $t^{O/1}$ defined in Theorem IV.4(2a), achieves the first-best outcome.

Using $\lambda = 0$ in the objective function (C.3) and its first order derivative (C.4), we get $\alpha^* = 1$. As a result, the induced client's consumption decision, x^D, must satisfy $u'(x+t+\epsilon) = p$. Both the ESCO's and client's decisions are identical to those with central planner, stated in Theorem IV.1. Therefore, the first-best outcome is achieved, i.e., $V^{O-RN/1} = V^C$.

Then we consider the case with moral hazard. From Theorem IV.5(2) we have $V^{M-RN/2} \geq V^{O-RN/1}$. Therefore, 2-rate contracts with ESCO's moral hazard also achieve the first-best solution. The optimal contract is $(z^{M-RN/2}, \{1, 0\})$, where $z^{M-RN/2}$ is given in (C.6) with $\alpha_1 = 1$. $\square$

Proof of Corollary IV.2.

Part (1): Central Planner.

The results are immediately obtained by using $u(x + t + \epsilon) = -e^{-(x+t+\epsilon)}$ and $C(t) = \frac{1}{2\gamma}t^2$ in Theorem IV.1.

Part (2): Observability of ESCO's Effort.

Under 1-rate contracts and comfort function $u(x+t+\epsilon) = -e^{-(x+t+\epsilon)}$, the energy consumption and client's utility are

$$\begin{aligned} x^D(t+\epsilon) &= -\ln \alpha p - t - \epsilon, \\ v^D(x^D, t+\epsilon) &= \alpha p(\ln \alpha p + t + \epsilon - 1). \end{aligned} \tag{C.7}$$

The objective function (C.3) reduces to

$$\max_{t,\alpha} \left\{ p(\ln \alpha p - \alpha) + pt - \frac{\lambda}{2}\alpha^2 p^2 \sigma^2 - v_0 - \frac{1}{2\gamma}t^2 \right\}.$$

The first order conditions are

$$
\begin{aligned}
&p - \frac{t}{\gamma} = 0, \\
&\frac{1}{\alpha} - 1 - \lambda p \sigma^2 \alpha = 0.
\end{aligned}
\tag{C.8}
$$

The closed form solutions to t and α are immediately obtained from the equations above.

Plugging in $u(x) = -e^{-x}$ into Equation (4.3), we get the pre-project energy consumption and client's utility

$$
\begin{aligned}
&x_0 = -\ln p, \\
&v_0 = p(\ln p - 1).
\end{aligned}
\tag{C.9}
$$

Substituting $\alpha^{O/1}$ and $t^{O/1}$ into the objective function above, we get

$$
\begin{aligned}
V^{O/1} &= p(\ln \alpha^{O/1} p - \alpha^{O/1}) + \frac{1}{2}\gamma p^2 - \frac{\lambda}{2}(\alpha^{O/1} p)^2 \sigma^2 - v_0 \\
&= \frac{1}{2} p[(\ln \alpha^{O/1} + \gamma p) + (\ln \alpha^{O/1} + 1 - \alpha^{O/1})].
\end{aligned}
\tag{C.10}
$$

The second line holds because $\alpha^{O/1}$ satisfies the first order condition (C.8).

Part (3): Moral Hazard.

From Theorem IV.5, we get $t^{M/1}(\alpha) = (1-\alpha)\gamma p$. The objective function (C.5) reduces to

$$
\max_{\alpha} \left\{ p(\ln \alpha p - \alpha) + (1-\alpha)\gamma p^2 - \frac{\lambda}{2}\alpha^2 p^2 \sigma^2 - v_0 - \frac{1}{2}(1-\alpha)^2 \gamma p^2 \right\}.
$$

The first order condition in α is

$$
p(\frac{1}{\alpha} - 1) - p^2(\lambda \sigma^2 + \gamma)\alpha = 0.
$$

$\alpha^{M/1}$ can be obtained from the first order condition above. $\square$

Proof of Theorem IV.6.

Part (1): Generalization of Corollary IV.2(1). The results are direct generalization of Corollary IV.2. Proof details are omitted here.

Part (2): Generalization of Corollary IV.2(2).

The client's consumption must satisfy $u'(x+t+\epsilon+q) = \alpha p$. Therefore, we have

$$x^{D-CE}(t+\epsilon+q) = -\ln \alpha p - (t+\epsilon+q).$$

The client's utility is

$$v^{D-CE}(x^{D-CE}, t+\epsilon, q) = -\alpha_1 p[1 - \ln \alpha_1 p - t - \epsilon - q] - \frac{1}{2\theta} q^2.$$

Hence, the client's optimal effort is $q^{D-CE} = \alpha\theta p$ and the resulting utility is

$$v(x^{D-CE}, t+\epsilon, q^{D-CE}) = -\alpha p[1 - \ln \alpha p - t - \epsilon] + \frac{1}{2}\alpha^2 \theta p^2.$$

Plugging this into the ESCO's problem (4.10), we get first order conditions

$$\begin{aligned} &p - \frac{t}{\gamma} = 0, \\ &\frac{1}{\alpha} - 1 - \lambda p \sigma^2 \alpha - \alpha\theta p + \theta p = 0. \end{aligned}$$

The closed form solutions to t and α are immediately obtained from the equations above.

Part (3): Generalization of Theorem IV.5(2).

The right inequality is obvious. We focus on the left inequality. By choosing

$$\alpha_1 = \alpha^{O-CE/1}, \alpha_2 = 0, z = 1 - \ln \alpha_1 p - \frac{1}{2}\alpha_1 \theta p - (t^{O-CE/1} + \underline{\epsilon}),$$

the ESCO effectively commits himself to spend effort $t^{O-CE/1}$. So the ESCO is able to replicate the outcome in Part (2), which gives $V^{O-CE/1} \leq V^{M-CE/2}$. The details of this proof are similar to Theorem IV.5(2), and thus omitted. □

Proof of Theorem IV.7.

Using $u(x) = -e^{-(x+\epsilon_w)}$ in Equation (4.3), we get the client's consumption and resulting utility before the project:

$$\begin{aligned} x_0 &= -\ln p - \epsilon_w, \\ v_0 &= p(\ln p - 1) - \frac{\lambda}{2} p^2 \sigma_w^2. \end{aligned} \tag{C.11}$$

Obviously, v_0 decreases in σ_w^2. The client always have the same utility before and after the project as the ESCO is able to take all surplus. Therefore, the client's post-project utility decreases in σ_w^2.

Then we consider the ESCO's surplus. To keep notations short, we drop any superscript for optimal values, or replace it with * if necessary. Based on Equation (C.10), we have

$$V(\sigma_w^2) = \frac{1}{2} p[(\ln \alpha^* + \gamma p) + (\ln \alpha^* + 1 - \alpha^*)] + \frac{\lambda}{2} p^2 \sigma_w^2.$$

where α^* is generalized from Corollary IV.2(2b) as

$$\alpha^* = \frac{2}{1 + \sqrt{1 + 4\lambda p(\sigma^2 + \sigma_w^2)}}.$$

Then, we have

$$\frac{d\, V(\sigma_w^2)}{d\sigma_w^2} = \frac{1}{2} \lambda p^2 \left(1 - \alpha^*\right) > 0.$$

Therefore, the ESCO's surplus increases in σ_w^2. □

Proof of Theorem IV.8.

Part (1): Incremental Technology.

The project value is function of p, λ, γ, and t_0. According to the objective function (4.7), we have

$$V(p,\lambda,\gamma,t_0) = e^{-t_0}V(e^{t_0}p, e^{-t_0}\lambda, e^{-t_0}\gamma, 0)$$

and the problem is equivalent to the one in the main part. From Corollary IV.2(2), we have

$$\alpha^* = \frac{2}{1+\sqrt{1+4\lambda p\sigma^2}} \text{ and } t^* = \gamma p,$$

which is actually independent of t_0. From Equation (C.10), we have

$$V(p,\lambda,\gamma,t_0) = \frac{1}{2}p(2\ln\alpha^* + 1 - \ln\alpha^* + \gamma p).$$

which is also independent of t_0.

Part (2): Replacement Technology.

In this case, t_0 only matters for pre-project utility. Generalizing Equation (C.9), we get

$$v_0 = p(\ln p + t_0 - 1).$$

v_0 increases in t_0 and post-project surplus is constant in t_0. Therefore, project value $V^{O/1}$ decreases in t_0. $\square$

Proof of Theorem IV.9.

For the clarity of this proof, we put retail price p and decision variable α_i's as arguments, e.g., $w_e(x,p,\{\alpha_i\})$ and $w_c(x,p,\{\alpha_i\})$. For any retail price p, we define

$\alpha_i' = \frac{p}{p_0}\alpha_i$. From Equation (4.2) we have

$$\begin{aligned} w_c(x,p,\{\alpha_i\}) &= \sum_{i=1}^{n} \alpha_i p \min\{\max\{0, x - z_{i-1}\}, z_i - z_{i-1}\} \\ &= \sum_{i=1}^{n} \frac{p}{p_0}\alpha_i p_0 \min\{\max\{0, x - z_{i-1}\}, z_i - z_{i-1}\} \\ &= w_c(x, p_0, \{\alpha_i'\}). \end{aligned}$$

From Equation (4.5) we have

$$\begin{aligned} -w_e(x,p,\{\alpha_i\}) + (p - p_0)x &= -px + w_c(x,p,\{\alpha_i\}) + (p-p_0)x \\ &= -p_0 x + w_c(x, p_0, \{\alpha_i'\}) \\ &= -w_e(x, p_0, \{\alpha_i'\}). \end{aligned}$$

If we use α_i''s to replace α_i's as decision variables, Problem (4.13) becomes

$$t^* = \arg\max_t \left\{ -\mathbf{E} w_e(x^*, p_0, \{\alpha_i'\}) - \frac{1}{2\gamma} t^2 \right\},$$
$$\max_{\{z_i\},\{\alpha_i'\}} \left\{ \mathbf{E}\left[v(x^*, t^* + \epsilon)\right] - \frac{\lambda}{2} Var\left[v(x^*, t^* + \epsilon)\right] - v_0 - \mathbf{E} w_e(x^*, p_0, \{\alpha_i'\}) - \frac{1}{2\gamma} t^{*2} \right\}.$$

This is exactly same to Problem (4.7), except retail price p in (4.7) replaced by energy cost p_0. Assume the optimal contract for above problem is $(\{z_i(p_0)\}, \{\alpha_i'(p_0)\})$. Then the optimal contract for Problem (4.13) is $(p, \{z_i(p_0)\}, \{\frac{p_0}{p}\alpha_i'(p_0)\})$, where retail price p can be any value. $\square$

Proof of Theorem IV.10.

We start with defining each player's utility or surplus in Table C.1. π is used to denote utilities with appropriate superscript (E/C/PM for ESCO/client/policy makers respectively). The subscript 0 or 1 indicates utility before and after the project. $v_0(p)$ and $x_0(p)$ are given in Equation (C.9). $V_0(p)$ is given in Equation

(C.10). $x^*(p)$ is given in Equation (C.7). To keep notations short, we drop any superscript for optimal values, or replace it with * if necessary.

	π_0^E	π_0^C	π_0^{PM}	π_1^E	π_1^C	π_1^{PM}
No policy	0	$v_0(p)$	$-cx_0(p)$	$V(p)$	$v_0(p)$	$-c\mathbf{E}x^*(p)$
Lump sum subsidy	0	$v_0(p)$	$-cx_0(p)$	$V(p)+G$	$v_0(p)$	$-c\mathbf{E}x^*(p)-G$
Carbon tax	0	$v_0(p+r)$	$(r-c)x_0(p+r)$	$V(p+r)$	$v_0(p+r)$	$(r-c)\mathbf{E}x^*(p+r)$

Table C.1: Utilities for each player before and after the project.

π_i^{PM} includes any subsidy, carbon tax income, and environmental cost. The total social surplus, denoted by Π_i, is

$$\Pi_i = \pi_i^E + \pi_i^C + \pi_i^{PM}.$$

Part (1): Lump Sum Subsidy.

With lump sum subsidy, policy makers cannot change the ESCO's effort and the client's consumption decisions. However, it encourages the ESCO to offer a lower price to residents and thus more projects will be adopted.

Without subsidy, a project is done if and only if

$$\pi_1^E = V(p) + G \geq 0.$$

Policy makers would execute a project if and only if $\Pi_1 \geq \Pi_0$. That is

$$V(p) + v_0(p) - c\mathbf{E}x^*(p) \geq v_0(p) - cx_0(p).$$

Policy makers want the ESCO decision to be equivalent to their own. Therefore, we have

$$G^* = c(x_0(p) - \mathbf{E}x^*(p)) = c(\ln \alpha^* + \gamma p).$$

Part (2): Carbon Tax.

In cases the project is not adopted, policy makers maximize social surplus Π_1.

That is,

$$\max_{r} \ \Pi_0 = v_0(p+r) + (r-c)x_0(p+r).$$

Taking derivative with respect to r, we get

$$\frac{d\Pi_0}{dr} = v_0'(p+r) + x_0(p+r) + (r-c)x_0'(p+r) = -\frac{r-c}{p}.$$

The optimal solution is apparently $r^* = c$.

In cases the project is adopted, policy makers solve

$$\max_{r} \ \Pi_1 = V(p+r) + v_0(p+r) + (r-c)\mathbf{E}x^*(p+r),$$

where α^* is given in Corollary IV.2(2). We then have

$$\begin{aligned} \frac{d\mathbf{E}x^*(p+r)}{dr} &= \frac{d}{dr}[-\ln \alpha^*(p+r) - \gamma(p+r)] \\ &= -\frac{1}{2(p+r)}\left(1 + \frac{1}{\sqrt{1+4\lambda(p+r)\sigma^2}}\right) - \gamma < 0 \end{aligned} \tag{C.12}$$

and

$$\begin{aligned} \frac{d\Pi_1}{dr} &= \frac{dV(p+r)}{dr} + \frac{dv_0(p+r)}{dr} + \mathbf{E}x^*(p+r) + (r-c)\frac{d\mathbf{E}x^*(p+r)}{dr} \\ &= (r-c)\frac{d\mathbf{E}x^*(p+r)}{dr}. \end{aligned}$$

Let $d\Pi_1/dr = 0$ and we get $r^* = c$.

Despite of whether the project is adopted, it is always optimal to have $r^* = c$. It is also worth noting that with $r^* = c$, $\pi_1^E \geq 0$ and $\Pi_1 \geq \Pi_0$ are equivalent, which means the ESCO's decision is perfectly aligned with policy makers.

Part (3): Comparison.

We first show that if $\pi_1^E \geq 0$ with lump sum subsidy, it is also true with carbon tax policy. Denote $\Delta\pi_1^E$ as

$$\Delta\pi_1^E = V(p+r^*) - V(p) - G^* = V(p+c) - V(p) - c(\ln\alpha^*(p) + \gamma p).$$

Note α^* is a function of p and we make it clear because it is different under two policies. When $c = 0$, $\Delta\pi_1^E = 0$. We also have

$$\frac{d\Delta\pi_1^E}{dc} = [\ln\alpha^*(p+c) + \gamma(p+c)] - [\ln\alpha^*(p) + \gamma p] > 0.$$

The inequality holds because of Equation (C.12). Therefore, the ESCO's surplus is higher with carbon tax policy. If a project is adopted with lump sum subsidy, it should also be adopted with carbon tax policy.

Next we compare two policies under three scenarios: (a) A project is adopted with both policies; (b) A project is adopted with neither policies; and (c) A project is adopted with carbon tax policy, but not with lump sum subsidy. Denote Δt, Δx, and $\Delta\Pi$ as difference in technology investment, consumption, and social surplus (value in carbon tax policy minus that in lump sum subsidy).

(3a) A project is adopted with both policies:

We have $\Delta t = \gamma(p+c) - \gamma p = \gamma c > 0$ and $\Delta x = \mathbf{E}x^*(p+c) - \mathbf{E}x^*(p) < 0$. The latter inequality holds because of Equation (C.12).

Note if $r = 0$ with carbon tax policy, social surplus Π_1 under two policies are identical. In Part (2) we showed that Π_1 is increasing in r when $r \leq c$. Since policy makers choose $r^* = c > 0$, we must have $\Delta\Pi > 0$.

(3b) A project is adopted with neither policies:

Since no project would be adopted with both policies, we have $\Delta t = 0$. Similar to Part (3a), we have Π_1 is increasing in r when $r \leq c$ (shown in Part (2)), and thus

$\Delta\Pi > 0$ also holds. To see the difference in energy consumptions, we have

$$\Delta x = x_0(p + c) - x_0(p) = -\ln(p + c) + \ln p < 0.$$

(3c) A project is adopted with carbon tax policy, but not with lump sum subsidy:

In this scenario, $\Delta t > 0$ is obvious. Next we prove $\Delta x < 0$. According to Equation (C.10), we have

$$V(p + c) = \frac{1}{2}(p + c)[(\ln\alpha^* + \gamma(p + c)) + (\ln\alpha^* + 1 - \alpha^*)].$$

Since $V(p+c) \geq 0$ (because a project is adopted with carbon tax policy) and $\ln\alpha^* + 1 - \alpha^* < 0$ (for $\alpha^* \in (0, 1)$), we must have

$$\ln\alpha^* + \gamma(p + c) > 0.$$

On the other hand, we have

$$\mathbf{E}x^*(p + c) - x_0(p + c) = -[\ln\alpha^* + \gamma(p + c)] < 0.$$

Therefore,

$$\Delta x = \mathbf{E}x^*(p + c) - x_0(p) < x_0(p + c) - x_0(p) = -\ln(p + c) + \ln p < 0.$$

To see the difference in social surplus, we have

Π_1 with carbon tax policy $\geq \Pi_0$ with carbon tax policy $> \Pi_0$ with lump sum subsidy.

The first inequality holds because a project is adopted and the ESCO's decision is aligned with policy makers, as shown at the end of the proof of Part (2). The second inequality holds because of the result in (3b). $\square$

BIBLIOGRAPHY

BIBLIOGRAPHY

Aflaki, S., P.R. Kleindorfer, V. Sáenz. 2013. Finding and implementing energy efficiency projects in industrial facilities. *Production and Operations Management* **22**(3) 503–517.

Amir, R., J.Y. Jin. 2001. Cournot and Bertrand equilibria compared: Substitutability, complementarity and concavity. *International Journal of Industrial Organization* **19**(3) 303–317.

Anari, N., S. Ehsani, M. Ghodsi, N. Haghpanah, N. Immorlica, H. Mahini, V.S. Mirrokni. 2010. Equilibrium pricing with positive externalities. *Internet and Network Economics*. Springer, 424–431.

Baiman, S., P.E. Fischer, M.V. Rajan. 2000. Information, contracting, and quality costs. *Management Science* **46**(6) 776–789.

Baye, M.R., J. Morgan, P. Scholten. 2006. Information, search, and price dispersion. *Handbook on Economics and Information Systems*. Elsevier Press, Amsterdam, 323–376.

Bensaid, B., J.P. Lesne. 1996. Dynamic monopoly pricing with network externalities. *International Journal of Industrial Organization* **14**(6) 837–855.

Bernstein, F., A. Federgruen. 2005. Decentralized supply chains with competing retailers under demand uncertainty. *Management Science* **51**(1) 18–29.

Bhattacharyya, S., F. Lafontaine. 1995. Double-sided moral hazard and the nature of share contracts. *The RAND Journal of Economics* **26**(4) 761–781.

Bish, E.K., Q. Wang. 2004. Optimal investment strategies for flexible resources, considering pricing and correlated demands. *Operations Research* **52**(6) 954–964.

Bolton, P., G. Bonanno. 1988. Vertical restraints in a model of vertical differentiation. *The Quarterly Journal of Economics* **103**(3) 555–570.

Bolton, P., M. Dewatripont. 2004. *Contract theory*. MIT press.

Bullock, C., G. Caraghiaur. 2001. *A guide to energy service companies*. Prentice Hall.

Burdett, K., K.L. Judd. 1983. Equilibrium price dispersion. *Econometrica* **51**(4) 955–969.

Butters, G.R. 1977. Equilibrium distributions of sales and advertising prices. *The Review of Economic Studies* **44**(3) 465–491.

Cachon, G.P. 2014. Retail store density and the cost of greenhouse gas emissions. *Management Science* **60**(8) 1907–1925.

Cachon, G.P., M.A. Lariviere. 2005. Supply chain coordination with revenue-sharing contracts: strengths and limitations. *Management Science* **51**(1) 30–44.

Caminal, R., X. Vives. 1996. Why market shares matter: An information-based theory. *The RAND Journal of Economics* **27**(2) 221–239.

Cetinkaya, V. 2009. Essays on vertical restraints and vertical integration. Ph.D. thesis, Rutgers University.

Chen, F., A. Federgruen. 2000. Mean-variance analysis of basic inventory models. Working paper, Columbia University.

Chen, X., D. Simchi-Levi. 2012. Pricing and inventory management. *The Oxford Handbook of Pricing Management*. Oxford University Press, 784–822.

Chevalier, J.A., D. Mayzlin. 2006. The effect of word of mouth on sales: Online book reviews. *Journal of Marketing Research* **43**(3) 345–354.

Chod, J., N. Rudi. 2005. Resource flexibility with responsive pricing. *Operations Research* **53**(3) 532–548.

Chod, J., N. Rudi. 2006. Strategic investments, trading, and pricing under forecast updating. *Management Science* **52**(12) 1913–1929.

Chu, L.Y., D.E.M. Sappington. 2012. Designing optimal gain sharing plans to promote energy conservation. *Journal of Regulatory Economics* **42**(2) 115–134.

Chun, S.Y., A. Kleywegt, A. Shapiro. 2013. Resource exchange seller alliances. Working paper, Georgetown McDonough School of Business.

Cohen, M.C., R. Lobel, G. Perakis. 2015. The impact of demand uncertainty on consumer subsidies for green technology adoption. *Management Science* Forthcoming.

Corbett, C.J., G.A. DeCroix. 2001. Shared-savings contracts for indirect materials in supply chains: Channel profits and environmental impacts. *Management Science* **47**(7) 881–893.

Corbett, C.J., G.A. DeCroix, A.Y. Ha. 2005. Optimal shared-savings contracts in supply chains: Linear contracts and double moral hazard. *European Journal of Operational Research* **163**(3) 653–667.

Cunningham, L.J., B. Cook. 2015. *Renewable energy and energy efficiency incentives: A summary of federal programs*. Congressional Research Service.

Desai, P.S., A. Krishnamoorthy, P. Sainam. 2010. Call for prices? Strategic implications of raising consumers' costs. *Marketing Science* **29**(1) 158–174.

Drake, D.F., S. Spinler. 2013. OM forum-sustainable operations management: An enduring stream or a passing fancy? *Manufacturing & Service Operations Management* **15**(4) 689–700.

EEB. 2011. *Saving energy in Europe: 15 good practice case studies*. European Environmental Bureau.

Eom, J., J.L. Sweeney. 2009. Shareholder incentives for utility-delivered energy efficiency programs in california. Working paper, Stanford University.

Goldman, C., N. Hopper, J. Osborn, T. Singer. 2005. Review of US ESCO industry market trends: An empirical analysis of project data. *Energy Policy* **33**(3) 387–405.

Granot, D., G. SoŠić. 2003. A three-stage model for a decentralized distribution system of retailers. *Operations Research* **51**(5) 771–784.

Greening, L.A., D.L. Greene, C. Difiglio. 2000. Energy efficiency and consumption - the rebound effect - a survey. *Energy Policy* **28**(6) 389–401.

Grossman, G.M., C. Shapiro. 1984. Informative advertising with differentiated products. *The Review of Economic Studies* **51**(1) 63–81.

Gundlach, G.T. 2010. Overview of the special issues: Antitrust analysis of resale price maintenance after leegin. *The Antitrust Bulletin* **55**(2) 271.

Gustafsson, A., S. Brax, L. Witell, P. Hypko, M. Tilebein, R. Gleich. 2010. Clarifying the concept of performance-based contracting in manufacturing industries: A research synthesis. *Journal of Service Management* **21**(5) 625–655.

Hall, J., E. Porteus. 2000. Customer service competition in capacitated systems. *Manufacturing & Service Operations Management* **2**(2) 144–165.

Hasija, S., E.J. Pinker, R.A. Shumsky. 2008. Call center outsourcing contracts under information asymmetry. *Management Science* **54**(4) 793–807.

Holmstrom, B. 1982. Moral hazard in teams. *The Bell Journal of Economics* **13** 324–340.

Hoyle, J.W. 2013. *Performance-based Potential for Residential Energy Efficiency*. Center for International Climate and Environmental Research - Oslo.

Hsu, J., X.H. Wang. 2005. On welfare under Cournot and Bertrand competition in differentiated oligopolies. *Review of Industrial Organization* **27**(2) 185–191.

Hu, X., I. Duenyas, R. Kapuscinski. 2007. Existence of coordinating transshipment prices in a two-location inventory model. *Management Science* **53**(8) 1289–1302.

Iravani, F., S. Dasu, R. Ahmadi. 2013. Pricing and sales effort decisions in the presence of gray markets. Working paper, University of Washington.

Iyer, G. 1998. Coordinating channels under price and nonprice competition. *Marketing Science* **17**(4) 338–355.

Iyer, G., D. Kuksov. 2012. Competition in consumer shopping experience. *Marketing Science* **31**(6) 913–933.

Iyer, G., D. Soberman, J.M. Villas-Boas. 2005. The targeting of advertising. *Marketing Science* **24**(3) 461–476.

Janssen, M.C.W., M.C. Non. 2009. Going where the ad leads you: On high advertised prices and searching where to buy. *Marketing Science* **28**(1) 87–98.

Jiang, H., Z. Pang, S. Savin. 2012. Performance-based contracts for outpatient medical services. *Manufacturing & Service Operations Management* **14**(4) 654–669.

Kali, R. 1998. Minimum advertised price. *Journal of Economics & Management Strategy* **7**(4) 647–668.

Kim, S.-H., M.A. Cohen, S. Netessine. 2007. Performance contracting in after-sales service supply chains. *Management Science* **53**(12) 1843–1858.

Kim, S.-H., S. Netessine. 2013. Collaborative cost reduction and component procurement under information asymmetry. *Management Science* **59**(1) 189–206.

Kim, S.K., S. Wang. 1998. Linear contracts and the double moral-hazard. *Journal of Economic Theory* **82**(2) 342–378.

Klein, B. 2009. Competitive resale price maintenance in the absence of free riding. *Antitrust Law Journal* **76**(2) 431–481.

Kováč, E., R.C. Schmidt. 2014. Market share dynamics in a duopoly model with word-of-mouth communication. *Games and Economic Behavior* **83** 178–206.

Krishnan, H., R. Kapuscinski, D.A. Butz. 2004. Coordinating contracts for decentralized supply chains with retailer promotional effort. *Management Science* **50**(1) 48–63.

Kuo, C.W., H.S. Ahn, G. Aydin. 2011. Dynamic pricing of limited inventories when customers negotiate. *Operations Research* **59**(4) 882–897.

Larsen, P.H., C. Goldman, A. Satchwell. 2012. Evolution of the us energy service company industry: Market size and project performance from 1990–2008. *Energy Policy* **50** 802–820.

Lazer, J., D. Farnsworth. 2011. *Incorporating environmental costs in electric rates*. Regulatory Assistance Project.

Lin, P., K. Saggi. 2002. Product differentiation, process R&D, and the nature of market competition. *European Economic Review* **46**(1) 201–211.

Liu, L., W. Shang, S. Wu. 2007. Dynamic competitive newsvendors with service-sensitive demands. *Manufacturing & Service Operations Management* **9**(1) 84–93.

MacKay, A., D.A. Smith. 2014. The empirical effects of minimum resale price maintenance. Working paper, University of Chicago.

Marvel, H.P., S. McCafferty. 1984. Resale price maintenance and quality certification. *The RAND Journal of Economics* **15**(3) 346–359.

Marvel, H.P., S. McCafferty. 1985. The welfare effects of resale price maintenance. *The Journal of Law and Economics* **28**(2) 363–379.

Mathewson, G.F., R.A. Winter. 1983. The incentives for resale price maintenance under imperfect information. *Economic Inquiry* **21**(3) 337–348.

Mathewson, G.F., R.A. Winter. 1984. An economic theory of vertical restraints. *The RAND Journal of Economics* **15**(1) 27–38.

Mathewson, G.F., R.A. Winter. 1998. The law and economics of resale price maintenance. *Review of Industrial Organization* **13**(1) 57–84.

McGuire, T.W., R. Staelin. 1983. An industry equilibrium analysis of downstream vertical integration. *Marketing Science* **2**(2) 161–191.

Muthoo, A. 1999. *Bargaining theory with applications*. Cambridge University Press.

Nagarajan, M., G. Sošić. 2008. Game-theoretic analysis of cooperation among supply chain agents: Review and extensions. *European Journal of Operational Research* **187**(3) 719–745.

Olsen, T.L., R.P. Parker. 2008. Inventory management under market size dynamics. *Management Science* **54**(10) 1805–1821.

Paterson, C., G. Kiesmüller, R. Teunter, K. Glazebrook. 2011. Inventory models with lateral transshipments: A review. *European Journal of Operational Research* **210**(2) 125–136.

Pereira, J. 2008. Price-fixing makes comeback after supreme court ruling. `www.wsj.com/articles/SB121901920116148325`, retrieved on Mar 5, 2015.

Perry, M.K., R.H. Porter. 1990. Can resale price maintenance and franchise fees correct sub-optimal levels of retail service? *International Journal of Industrial Organization* **8**(1) 115–141.

Reichheld, F., W. Sasser. 1990. Zero defections: Quality comes to service. *Harvard Business Review* **68**(5) 105–111.

Ren, Z.J., Y.-P. Zhou. 2008. Call center outsourcing: Coordinating staffing level and service quality. *Management Science* **54**(2) 369–383.

Rey, V., T. Verge. 2008. Economics of vertical restraint. *Handbook of Antitrust Economics*. MIT Press, 353–390.

Roels, G., U.S. Karmarkar, S. Carr. 2010. Contracting for collaborative services. *Management Science* **56**(5) 849–863.

Romano, R.E. 1994. Double moral hazard and resale price maintenance. *The RAND Journal of Economics* **25**(3) 455–466.

Rudi, N., S. Kapur, D.F. Pyke. 2001. A two-location inventory model with transshipment and local decision making. *Management Science* **47**(12) 1668–1680.

Ryan, L., S. Moarif, E. Levina, R. Baron. 2011. *Energy efficiency policy and carbon pricing*. International Energy Agency.

Salop, S., J. Stiglitz. 1977. Bargains and ripoffs: A model of monopolistically competitive price dispersion. *The Review of Economic Studies* **44**(3) 493–510.

Satchwell, A. 2010. *A Survey of the US ESCO Industry: Market Growth and Development from 2008 to 2011*. Lawrence Berkeley National Laboratory.

SEAI. 2012. *A guide to energy performance contracts and guarantees*. Sustainable Energy Authority of Ireland.

Singh, N., X. Vives. 1984. Price and quantity competition in a differentiated duopoly. *The RAND Journal of Economics* **15**(4) 546–554.

So, K.C., C.S. Tang. 2000. Modeling the impact of an outcome-oriented reimbursement policy on clinic, patients, and pharmaceutical firms. *Management Science* **46**(7) 875–892.

Soberman, D. 2004. Research note: Additional learning and implications on the role of informative advertising. *Management Science* **50**(12) 1744–1750.

Sorrell, S. 2007. The economics of energy service contracts. *Energy Policy* **35**(1) 507–521.

Sorrell, S. 2009. Jevons' paradox revisited: The evidence for backfire from improved energy efficiency. *Energy Policy* **37**(4) 1456–1469.

Stahl, D.O. 1989. Oligopolistic pricing with sequential consumer search. *The American Economic Review* **79**(4) 700–712.

Steinberger, J.K., J. Van Niel, D. Bourg. 2009. Profiting from negawatts: Reducing absolute consumption and emissions through a performance-based energy economy. *Energy Policy* **37**(1) 361–370.

Tarr, G. 2014. Sharp extends UPP to all Aquos Quattron and UHD TVs. `www.twice.com/news/tv/sharp-extends-upp-all-aquos-quattron-and-uhd-tvs/45384`, retrieved on Mar 5, 2015.

Taylor, R.P., Govindarajalu C., Levin J., A.S. Meyer, W.A. Ward. 2008. *Financing energy efficiency: Lessons from Brazil, China, India, and beyond*. World Bank Publications.

Taylor, T.A. 2002. Supply chain coordination under channel rebates with sales effort effects. *Management Science* **48**(8) 992–1007.

Telser, L.G. 1960. Why should manufacturers want fair trade? *The Journal of Law and Economics* **3** 86–105.

Tsay, A.A., N. Agrawal. 2000. Channel dynamics under price and service competition. *Manufacturing & Service Operations Management* **2**(4) 372–391.

Van Mieghem, J.A. 2007. Risk mitigation in newsvendor networks: Resource diversification, flexibility, sharing, and hedging. *Management Science* **53**(8) 1269–1288.

Van Mieghem, J.A., M. Dada. 1999. Price versus production postponement: Capacity and competition. *Management Science* **45**(12) 1639–1649.

Wang, L., R. Kapuscinski. 2009. Coordination and price flexibility for multiple products with price-based substitution. Working paper, University of Michigan.

Whitin, T. 1955. Inventory control and price theory. *Management Science* **2**(1) 61–68.

Winter, R.A. 1993. Vertical control and price versus nonprice competition. *The Quarterly Journal of Economics* **108**(1) 61–76.

Wirl, F. 2000. Lessons from utility conservation programs. *The Energy Journal* **21**(1) 87–108.

Wirl, F. 2015. White certificates - energy efficiency programs under private information of consumers. *Energy Economics* **49** 507–515.

Yang, M., X. Yu. 2015. *Energy efficiency: Benefits for environment and society*. Springer.

Zhao, X., D. Atkins. 2008. Newsvendors under simultaneous price and inventory competition. *Manufacturing & Service Operations Management* **10**(3) 539–546.

Zhao, X., D. Atkins. 2009. Transshipment between competing retailers. *IIE Transactions* **41**(8) 665–676.

Zimring, M., M.G. Borgeson, I. Hoffman, C. Goldman, E. Stuart, A. Todd, M. Billingsley. 2011. *Delivering energy efficiency to middle income single family households*. Lawrence Berkeley National Laboratory.