- Solar wind driven variations of electron plasma sheet
- ² densities and temperatures beyond geostationary
- ³ orbit during storm times

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This is the author manuscript accepted for publication and has undergone full peer review but has not been through the copyediting, typesetting, pagination and proofreading process, which may lead to differences between this version and the Version of Record. Please cite this article D RaAdEi: 10.1002/2016JA02294 mgust 18, 2016, 1:10am D R A F T

Х - 2 DUBYAGIN ET AL.: ELECTRON PLASMA SHEET EMPIRICAL MODEL Abstract. The empirical models of the plasma sheet electron temperature and density on the night at distances between 6 and 11 R_E are con-5 structed based on Time History of Events and Macroscale Interactions Dur-6 ing Substorms (THEMIS) particle measurements. The data set comprises bservations in the plasma sheet during geomagnetic storm $\sim 400 \text{ house}$ 8 periods. The equatorial distribution of the electron density reveals a strong 9 earthward gradient and a moderate variation with magnetic local time sym-10 metric with respect to the midnight meridian. The electron density depen-11 dence on the external driving is parameterized by the solar wind proton den-12 sity averaged over 4 hours and the southward component of interplanetary 13 magnetic field (IMF B_S) averaged over 6 hours. The interval of the IMF in-14 tegration **n** much longer than a typical substorm growth phase and it rather 15 corresponds to the geomagnetic storm main phase duration. The solar wind 16 proton density is the main controlling parameter but the IMF B_S becomes 17 of almos me importance in the near-Earth region. The root-mean-square 18 deviation between the observed and predicted plasma sheet density values 19 is 0.23 cmand the correlation coefficient is 0.82. The equatorial distribu-20 tion of the electron temperature has a maximum in the post-midnight – morn-21 ing MLT sector, and it is highly asymmetric with respect to the local mid-22 night. The electron temperature model is parameterized by solar wind ve-23 locity (averaged over 4 hours), IMF B_S (averaged over 45 min), and IMF B_N 24 emponent of IMF, averaged over 2 hours). The solar wind ve-(northward 25 locity is a major controlling parameter and IMF B_S and B_N are compara-26

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ble in importance. In contrast to the density model, the electron tempera-27 ture shows higher correlation with the IMF B_S averaged over ~45 min (sub-28 storm growth phase time scale). The effect of B_N manifests mostly in the 29 outer part of the modelled region $(r > 8R_E)$. The influence of the IMF B_S 30 is maximum in the midnight – post-midnight MLT sector. The correlation co-31 efficient between the observed and predicted plasma sheet electron temper-32 ature values is 0.76 and the root-mean-square deviation is 2.6 keV. Both mod-33 els reveal better performance in the dawn MLT sector. 34

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1. Introduction

The distributions of low energy electrons (below 200-300 keV) and their variations in the 35 near-Earth plasma sheet, at distances beyond geostationary orbit, have not sufficiently 36 been studied in detail. Yet, this population is critically important for magnetospheric 37 especially during storm times. One obvious example is their role as the seed dynamics 38 population being further accelerated to MeV energies by various processes in the Earth's 30 radiation betts. Several modeling attempts have been made [Jordanova and Miyoshi, 2005; 40 2006; Chen et al., 2006; Jordanova et al., 2014]. The electron flux at these Miuoshi et 41 low energie s argely determined by convective and substorm-associated electric fields and 42 varies significantly with geomagnetic activity driven by the solar wind [Mauk and Meng, 43 1983; Kerns et al., 1994; Liemohn et al., 1998; Ganushkina et al., 2013, 2014]. Inward 44 rort includes also radial diffusion and excites plasma wave instabilities that electron tr 45 to local electron acceleration and electron precipitation into the atmosphere. give rise 46 and loss processes are far from being understood at present. It should be also Transport noted that the electron flux at these energies is important for spacecraft surface charging [Garrett, 1981; Lanzerotti et al., 1998; Davis et al., 2008; Thomsen et al., 2013]. 49 been a number of studies on low energy electrons at geostationary orbit. There h 50 [1999]; Denton et al. [2005]; Sicard-Piet et al. [2008]; Denton et al. [2015] Korth et 51 concentrated mainly on the analysis of LANL MPA and SOPA electron data. Friedel et52 zed the electron data from the Polar Hydra instrument and Kurita et al. al. [2001] 🖪 53

[2011] the data from the THEMIS spacecraft. None of the studies produced solar wind

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driven empirical relations for electron fluxes or moments of electron distribution function

⁵⁶ which can be used easily for radiation belt modeling.

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In the near-Earth plasma sheet, continuous measurements of plasma sheet electrons 57 are not available, in contrast to geostationary orbit. Numerous studies addressed the 58 magneto-plasma transport and sources [Terasawa et al., 1997; Borovsky et al., 59 1998a, b; Wine and Newell, 2002]. There have been several statistical models for plasma 60 sheet electrons derived from GEOTAIL and CLUSTER data, such as, for example, Åsnes 61 et al. [2008]; Burin des Roziers et al. [2009]. Artemyev et al. [2013] analyzed the electron 62 temperature radial distribution in the magnetotail using THEMIS observations at r >63 These studies are not models with empirical relations which can be used for real $10R_{E}$. 64 event modeling by the wider scientific community. 65

empirical models of the plasma sheet plasma parameters have been presented Only two 66 since 2000 These models are Tsyganenko and Mukai [2003] and Sergeev et al. [2015]. The 67 Tsyganerso and Mukai [2003] model is the only model, where an analytical description of 68 \blacksquare derived for a 2D distribution of the central plasma sheet ion temperature the plas T_i , density n_i and pressure p_i as functions of the incoming solar wind and interplanetary 70 magnetic field parameters at distances of 10-50 R_E based on Geotail data. Sergeev et 71 al. [2015] presented the correlations between 1-h-averaged central plasma sheet and solar 72 wind (and AL index) parameters based on THEMIS data but they were not derived for 73 storm times 74

Ganushkina et al. [2013, 2014, 2015] modeled the electron transport from the plasma sheet to the reostationary orbit setting the boundary at 10 R_E as a kappa distribution with the parameters of number density n_e and temperature T_e in the plasma sheet given

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by Tsyganenko and Mukai [2003]. In Ganushkina et al. [2013, 2014, 2015], the electron n_e 78 is assumed to be the same as that for ions and $T_e/T_i = 0.2$ is taken into account (which 79 relation was shown, for example, in *Kaufmann et al.* [2005] and *Wang et al.* [2012], based 80 on Geotail and THEMIS data). A time shift of 2 h following *Borovsky et al.* [1998b] for 81 the solar wind material to reach the midtail plasma sheet is also introduced. Applying 82 Tsyganen o and Mukai [2003] model for boundary conditions for electrons has a number 83 of serious limitations. This model was derived from Geotail data for ions. According to 84 the studies based on THEMIS data analysis [Wang et al., 2012], the ratio T_e/T_i can vary 85 during disturbed conditions. Moreover, at distances closer than 10 R_E , it can happen 86 that the correlation between T_i and T_e does not exist at all and no certain ratio can be 87 determined [*Runov et al.*, 2015]. 88

The paper presents the empirical model of the electron plasma sheet densities and temperature derived from the THEMIS [*Angelopoulos*, 2008a] data. Sections 2 and 3 contain the detailed description of the data we have selected and analyzed. Section 4 demonstration the methodology of determining the model input parameters. Section 5 presents the empirical relations for electron plasma sheet density and temperature. The results of the study are discussed in Section 6. The goal of Section 7 is to validate the model performance and Section 8 presents the conclusions.

2. The Data Sources

This study elies on the data of the Time History of Events and Macroscale Interaction during Substorms (THEMIS) mission [*Angelopoulos*, 2008a]. The mission was launched on February 17, 2007, and it comprises five identical probes on elliptical, nearlyequatorial orbits. Each of the probes has among other scientific instruments two particle

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instruments, namely, Electrostatic Analyser (ESA) [McFadden et al., 2008a] to measure 100 the ion and electron distribution functions over the energy range from a few eV up to 101 25 (30) keV for ions (electrons) on each spin period (~ 3 sec.) and Solid State Telescope 102 (SST) [Angelopoulos et al., 2008b] to measure ion and electron distributions over energies 103 from 25 **Lev** to first MeVs on each spin period. We also used the spin resolution 104 Flux Gate Magnetometer (FGM) data [Auster et al., 1991]. All aforementioned data 105 and the calibrating procedures are publicly available at the THEMIS mission web site 106 (http://themis.ssl.berkeley.edu/index.shtml) 107

In this study we used solar wind and IMF data from the OMNI database from the GSFC/SPDF OMNIWeb interface at http://omniweb.gsfc.nasa.gov. 5-min. resolution data were used as input parameters for magnetotail neutral sheet model [*Tsyganenko and Fairfuld* 2004] and 1-min. resolution data were used for computation of the input parameters for imperature and density.

Finally, the T-min. resolution SYM-H index was downloaded from World Data Center for geonloguetism, Kyoto (http://wdc.kugi.kyoto-u.ac.jp/).

3. Selection of data intervals

¹¹⁵ We have analyzed the data from the particle detectors onboard the THEMIS probes ¹¹⁶ P3, P4, P5 (D, E, A) during geomagnetic storms which took place through 2007–2013. ¹¹⁷ All observations came from the region on the nightside at distances $r = 6-11 R_E$. The ¹¹⁸ major axes of the orbits for all probes were aligned so that the probes were clustered ¹¹⁹ closely during their apogees at $r = 10-12 R_E$. However, in this study we did not use ¹²⁰ the advantage of a multi-spacecraft mission and consider the measurements at different

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¹²¹ spacecraft as independent data records. The probe separation in this region was typically ¹²² $\sim 0.03-2R_E$ except for year 2013 when the separation varied between 2 and $8R_E$.

Storm periods were of a special interest for our study, since the solar wind driving as well as the magnetospheric plasma parameters can reach extreme values and all the dependencies well as their saturation levels can manifest more clearly. For this reason, we selected all the periods with SYM - H < -50 nT and one day before and one day after these periods for almost whole THEMIS mission lifetime 2007–2013. This selection also includes the quiet periods before the storms.

When studying the distribution of the plasma parameters in the equatorial plane, it 129 is important to make sure that a probe was in very center of the plasma sheet (near 130 the magnetotal current neutral sheet) to refer the measurements to a particular radial 131 distance. So control the spacecraft position relative to the neutral sheet we use two step 132 selection: (1) Select all periods when the probes are within 1.5 R_E from the neutral 133 sheet producted by Tsyganenko and Fairfield [2004] model; (2) Using THEMIS magnetic 134 meents we select only measurements when $|B_n| > |B_t|$, where B_n and B_t field me 135 are the magnetic field components normal and tangential to the model neutral sheet. 136 Such approach is very robust and it has been successfully applied to the THEMIS data 137 [Dubyagin et al., 2010]. This selection procedure was applied to THEMIS data when 138 P3, P4, P5 (D, E, A) probes were at $R = 6-11R_E$. 139

Although the combined distribution function covers the energy range up to 3 MeV, we only used the data in the 30 eV – 300 keV energy range. The 30 eV low energy limit is chosen so as to eliminate the possible contribution of photoelectrons in case the spacecraft potential is evaluated incorrectly. The electron and ion moments were computed

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¹⁴⁴ from combined (ESA and SST) distribution function using updated calibration proce-¹⁴⁵ dures (including ESA background contamination and SST sun contamination removal, ¹⁴⁶ software version dated December 2015). However, even after all calibration procedures ¹⁴⁷ are applied, the penetrating background may not be fully removed. As an additional test ¹⁴⁸ of the data accuracy, we compare the densities measured by the ion and electron detec-¹⁴⁹ tors. The readers are referred to *McFadden et al.* [2008b] for more details on the ESA ¹⁵⁰ performance issues.

Depending on ESA and SST mode, the combined plasma moments are available at spin 151 resolution or only at ~ 96 second resolution. When the 3-second resolution moments 152 were available, it was convenient to average them over 96 sec intervals (1.6 minute) to get 153 combined data set with uniform time resolution. It should be noted that the measurements 154 solution were not accumulated values but instant distributions (accumulated at 96-sec 155 during one spacecraft spin period). For this reason, these data are expected to reveal 156 more scatter and we do not use them for model construction. This data set was used only 157 for verificat of the models. 158

After synchronization with the solar wind data, we obtained $\sim 83,000$ data records 159 with ~ 1.6 min resolution $\sim 63,000$ of which are obtained from the spin-resolution data. 160 Since the <u>quasi-neutrality</u> holds in the magnetospheric plasma, the quality of the plasma 161 moments can be checked comparing the densities computed from electron (N_e) and ion 162 (N_i) measurements. It turned out that significant part of the data shows discrepancies 163 between N_e and N_i . In majority of these anomalous events, N_i exceeds N_e . We analyzed 164 nd found that typically cold dense plasma with energies $\leq 100 \text{ eV}$ can be seen these events 165 right above the low energy limit. It is likely that some part of this cold population is cut 166

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off by low energy limit and the fraction of the cut off population is different for the ion and 167 electron distributions. This leads to discrepancy between N_e and N_i measurements. We 168 also found that a vast majority of these events occurred in the 18–24 MLT sector during 169 the periods with very weak geomagnetic activity. This finding is also in agreement with 170 our hypethesis because a cold plasma of the plasmasphere can extend to larger geocentric 171 distances during such periods especially in the dusk-to-midnight sector. Although these 172 data potentially can be used in the future studies, if the presence of multiple populations 173 in the particle distribution is properly addressed, in the present study we discard all 174 measurements which do not satisfy the condition $N_i/1.5 < N_e < 1.5N_i$. This procedure 175 reduced the size of our statistics by one third. Although this criterion seems to be rather 176 weak, it is justified because, during storm time, the ion data are expected to be less 177 accurate in comparison to the electron data due to a contamination from heavier ions and 178 larger gap between ESA and SST energy ranges (especially for late years). 179

Finally, our data set consists of $\sim 45,000$ records obtained from the spin resolution mea-180 $\rightarrow 12,000$ obtained from ~ 1.6-min resolution measurements. Hereafter, suremen 181 we will refer to these data sets as a "primary" and "auxiliary" data sets, respectively. 182 Table 1 shows the number of samples in the data sets for every year during the THEMIS 183 mission. The primary data set includes only data starting from the year 2010 while the 184 years 2007–2009 contribute 20% to the auxiliary data set. Figure 1a shows the distri-185 bution of the points corresponding to the primary data set in the XY_{GSM} plane (only 186 every tenth point is shown). The colors correspond to different SYM-H index ranges. The 187 awn asymmetry can be seen in the figure. Although moderate asymmetry strong dusk 188 existed in the original data set (probably owning to orbital/seasonal effect), so promi-189

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¹⁹⁰ nent lack of the data points in the dusk sector is mostly due to removal of the data with ¹⁹¹ $N_i \neq N_e$. Though not immediately obvious from this dense distribution of points, the ¹⁹² dawn-dusk asymmetry exists only for the moderate SYM-H subsets and disappears for ¹⁹³ SYM-H<-50 nT.

It is weather may arring these datasets with datasets used in the previous studies. Tsyqa-194 nenko and Mukai [2003] used Geotail data and their dataset comprised 7234 1-min records 195 (~ 120 hours). Since we used 1.6-min resolution data, the size of our dataset should be 196 multiplied by factor 1.6 to compare with Tsyganenko and Mukai [2003] dataset. However, 197 we used observations onboard three probes clustered closely. For this reason, the size of 198 our dataset should be divided by 3 (this estimate is a bit pessimistic because the probe 199 separation can be as large as ~ $9R_E$). After this normalization, our dataset size corre-200 sponds to 400 hours of observations. Wang et al. [2006] apparently used the same data 201 set as Tsynaperko and Mukai [2003]. Sergeev et al. [2015] use 4500–5000 hourly averaged 202 measurements onboard three THEMIS probes on the night side 21–06 MLT $r = 9-12R_E$. 203 by 3, to take into account simultaneous measurements at three probes, the After di 204 data set size is 1500–1600 h, which is four times larger than data set used in the present 205 wer, Sergeev et al. [2015] use only data from ESA spectrometer in 5 eV-25 keV study. Hoy 206 energy range and there is no spatial dependence included in the model. 207

4. Solar wind driven model for electron plasma sheet densities and temperature: Input parameters 4.1. Methedology

The macroscopic plasma parameters in the near-Earth magnetotail are affected by multiple factors. Among them, there are the magnetic configuration change (it affects

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the plasma parameters through the adiabatic compression of the magnetic flux tubes) 210 [Borovsky et al., 1998b; Dubyagin et al., 2010; Artemyev et al., 2013], the substorm cycle 211 (arrival of a new hot tenuous plasma from the distant magnetotail during the main phase) 212 [Sergeev et al., 2015], the variations of the magnetosheath plasma parameters (since the 213 magneto-least is a source of the plasma sheet material) [Terasawa et al., 1997; Borovsky 214 et al., 1998a; Wang et al., 2010], and the variation of the magnetotail plasma transport 215 modulated by the dayside reconnection rate. To make it even more complicated, the 216 regions and mechanisms of the magnetosheath plasma penetration into the magnetotail 217 are different during periods of southward and northward IMF [Wang et al., 2010]. In 218 addition, all these factors affect the plasma sheet with different time lags and these delays 219 can be different for different regions of the magnetotail [Terasawa et al., 1997; Borovsky 220 et al., 1992a: Wang et al., 2010] 221

To inverticate the lag of the solar wind influence, every record of the plasma sheet 222 electron censity and temperature was accompanied by solar wind data containing 12 hour 223 prehisto I the OMNI database, the solar wind parameters are projected in time to 224 the moment when solar wind reaches the estimated bow shock position. We estimate the 225 shortest time for solar wind disturbance (seen in the OMNI data) to have an effect on 226 the nightside inner magnetosphere to be ~ 5 minutes. For every measurements in the 227 plasma sheet taken at time t_0 , the 12 hours period preceding the time $t_0 - 5$ min. was 228 broken into 15 minute subintervals and solar wind parameters were averaged over these 229 That is, every measurement in the plasma sheet was complemented by 48 subintervals. 230 ages of the solar wind parameters for the preceding 12 h interval. of 15-min a 231

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As a first step, we binned the THEMIS observations according to the probe location 232 in the plasma sheet. We used two discriminating parameters: a geocentric distance r =233 $(X^2 + Y^2 + Z^2)^{\frac{1}{2}}$ and an azimuth angle $\phi = \arctan(-Y_{GSM}/X_{GSM})$. We used two intervals 234 of geocentric distance: $r = 6-8.5R_E$ and $r = 8.5-11R_E$, and three sectors of the azimuth 235 angle: demoide $(-90^{\circ} < \phi < -30^{\circ})$, central $(-30^{\circ} < \phi < 30^{\circ})$, and duskside $(30^{\circ} < \phi < -30^{\circ})$ 236 90°). These birs are shown in Figure 1b. We investigated the dependence of the electron 237 plasma parameters on the solar wind parameters separately for each bin. Let P_k be a 238 plasma sheet parameter and D_{ik} be a 15-min average of a solar wind parameter. Here k is 239 the index corresponding to the plasma sheet measurements at the time t_k and i = 1, ..., 48240 corresponds to the 15-min average preceding the time t_k by $\Delta t = 5 \min + i \cdot 15 \min$. 241 For L = 1, ..., 48 and for $M \le 48 - L + 1$, we computed the following mean sums: 242

$$F(L, M, k) = \frac{\sum_{i=L}^{L+M-1} D_{ik}}{M}.$$
 (1)

Here L_{1} epresents the lag and M represents the duration over which the parameter is averaged.

²⁴⁶ These sums are equivalent to time integrals:

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$$F(t_{lag}, \Delta T, t_k) = \frac{1}{\Delta T} \int_{t_k - t_{lag} - \Delta T}^{t_k - t_{lag}} D(t) dt.$$
(2)

The derives of the plasma sheet parameter response to the changes of the solar wind can be deduced from the analysis of the correlation coefficient between P_k and F(L, M, k) for different L and M. These correlation coefficients can be plotted as function of L and M converted to the time units t_{lag} and ΔT .

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Imagine an ideal system whose parameter P responds to the changes of some other 252 parameter D with a fixed time lag t_r . The correlation between P and D would have a 253 peak at $t_{lag} = t_r$ and $\Delta T = 0$. However, the correlation would still be high for nonzero 254 ΔT as long as t_r is inside the interval of averaging $(t_{lag} < t_r < t_{lag} + \Delta T)$ and ΔT is 255 less than the autocorrelation time scale (T_{auto}) for D (that is, if an instant value of D 256 can be approximated by its mean average over the time interval ΔT). The shaded area 257 in Figure 2 shows the region satisfying the aforementioned conditions. Obviously, inside 258 this region the correlation is highest when the interval of averaging is centered at t_r , that 259 $= t_r$ (blue dashed line in Figure 2). is $t_{lag} + \Delta T/2$ 260

However, the parameters of the system do not necessarily depend on instant values (even 261 if lagged) of the external drivers. For example, the magnetic flux in the magnetotail lobes 262 better correlates with the time integrated solar wind geoeffective electric field than with 263 its instant value [Shukhtina et al., 2005]. In such a case, one can expect that correlation 264 would be ugner at some $\Delta T > 0$. In addition, in real magnetosphere the time lags 265 obvious not constant. It also leads to smearing out the correlation peak at $\Delta T = 0$ 266 and an increase of the correlation at $\Delta T > 0$. 267

4.2. Input parameters for electron plasma sheet density model

Figure 3 shows the plots for correlation between the plasma sheet and the solar wind densities (pll results in Sections 4.2–5 are obtained using primary data set). Figures 3a–f correspond to ix spatial bins shown in Figure 1b. The horizontal axis corresponds to the time larger index L in Equation 1. The vertical axis corresponds to the interval of averaging or index M in Equation 1. A color scale on the right side of each plot shows

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to $\Delta T_N = const - 2 \cdot t_N$ dependence (equivalent to blue dashed line in Figure 2).

There is an obvious similarity between these plots and Figure 2. The correlation maxima in Figures 3a, b, c, d are roughly organized along oblique lines, and the regions of enhanced period ation are delineated by lines $\Delta T_N = const - t_N$ on the left/bottom side in Figures 3a, d, f.

The plots on the left and right correspond to the dawn and dusk bins, respectively. It 279 can be seen that the maximum correlation is found for the dawnside bins $(C.C. \ge 0.70)$ and 280 is higher for the outer bins (BIN 1–3 see Figure 1b). These results are in the correlation 281 agreement with dusk-dawn asymmetry of the plasma transport from the magnetosheath 282 found by Wing et al. [2005] and Wang et al. [2010], however, it is a bit counterintuitive 283 taking interaccount the eastward direction of the electron magnetic drifts. The lag values 284 are generally in agreement with those found by *Borovsky et al.* [1998a]. 285

Table 2 presents the statistical properties of the data subsets for the different bins. First 286 three lin \blacksquare sent the bin numeration and the coordinates. Forth line shows the number 287 of 1.6-min resolution records in every bin. It can be seen that the most sparsely populated 288 bin is BIN 6. Its data set comprises 2295 records. However, this number is misleading 289 since the time-scales of the solar wind parameters variations are much longer than 1.6-min 290 resolution of our data set. Borovsky et al. [1998a] obtained the following characteristic 291 times-scales 1.5 h for IMF B_Z , ~ 10 h for solar wind density, and ~ 32 h for solar wind 292 velocity (these scales are expected to be somewhat shorter for storm periods). To evaluate 293 statistics more realistically, we searched through the database, counting the size of 294 separate 1-hour intervals containing at least one data point. We found 444 such intervals 295

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for BIN 1 and 133 intervals for BIN 6. For 5-hour characteristic period, we found 181 296 intervals for BIN 1 and only 77 intervals for BIN 6. For this reason (and may be partly 297 due to orbital/seasonal effect), the standard deviations of the solar wind parameters also 298 show some variations from bin to bin. Bottom part of Table 2 shows the ranges of the 299 standard deviations found for various lag values between 0 and 12 hours (the standard 300 deviations were computed for 15-min resolution data). It can be seen that the variability 301 of the solar wind parameters changes significantly for different time lag values inside a 302 data subset for a single bin. It means that some dependencies seen in Figure 3 could be 303 due to a limited size of the dataset since one can expect that the correlation between two 304 quantities depends on the variability of the driving one. To rule out this possibility, we 305 plotted additional figures (not shown) in the same format as Figure 3 but for a standard 306 deviation \mathbf{x} of a corresponding solar wind parameter. Analyzing these figures, we found 307 that the main features seen in Figures 3 are real (σ shows no or weak variation in that 308 part of the ingure). 309

• values of ΔT_N and t_N corresponding to the highest correlation obviously Although 310 are different from bin to bin, we need to choose fixed values for a computation of the 311 input parameters for the empirical models. We attempted to find a compromise so that 312 the model works for all MLTs in $r = 6-11R_E$ range. Keeping this in mind, $t_N = 0.5$ h 313 4 h were chosen. These values are marked by a black circle in all panels and ΔT_N 314 of Figure 2 However, it should be remembered that the confidence interval of these 315 parameters is very broad (at least ± 1 hour). 316

The mode dependence on IMF is parameterized by southward (B_S) and northward (B_N) IMF components $(B_S = -B_Z^{IMF}$ if $B_Z^{IMF} < 0$ and $B_S = 0$ if $B_Z^{IMF} \ge 0$; $B_N = 0$

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if $B_Z^{IMF} < 0$ and $B_N = B_Z^{IMF}$ if $B_Z^{IMF} \ge 0$). Figure 4 shows the plots of correlations 319 between the plasma sheet electron density and IMF B_S . The format is the same as in 320 Figure 3. In contrast to the solar wind density, the highest correlation between the B_S 321 and plasma sheet electron density is found for the near-Earth bins. Surprisingly, highest 322 correlations are found for relatively long intervals of averaging $\Delta T_{BS} = 2-6$ h. This is 323 much longer than typical substorm growth phase duration. It could be due to strong 324 variations of the lag in the real system, but in such a case one would expect weaker 325 correlation. We will discuss the possible reasons for this in Section 7. We chose the 326 $t_{BS} = 0.5$ h and $\Delta T_{BS} = 6$ h. The lag was chosen so to be the same as that for solar wind 327 density parameter (and it will be shown later that 0.5 h lags are reasonable choice for all 328 temperature model parameters too). 329

Table 3 summarizes the results presented in this section. When comparing the top 330 and bottom parts of the Table 3, it can be seen that introducing a time lag to the 331 input parameter can significantly improve the correlations. We have also checked a few 332 and IMF parameters (not shown). However, even if the correlations more so 333 were comparable to those for N_{SW} , B_S and B_N , the resulting model quality (gauged by 334 correlation between the model predictions and the data, see Section 7) was worse and 335 we discarded them in the present version of the model. For example, motivated by the 336 fact that the solar wind - magnetotail plasma transport characteristic time is different for 337 the intervals southward and northward IMF B_Z , we introduced two parameters $N_{SW}^{(S)}$ and 338 $N_{SW}^{(N)}$. $N_{SW}^{(S)} = N_{SW}$ when IMF $B_Z < 0$ and $N_{SW}^{(S)} = 0$ when IMF $B_Z > 0$. $N_{SW}^{(N)}$ is defined 339 way. Although the lag-duration plots for $N_{SW}^{(S)}$ and $N_{SW}^{(N)}$ showed plausible in an opposi 340 patterns, the resulting quality of the electron density model was worse. 341

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It can be noticed that northward component of IMF shows a bit worse correlation with the plasma sheet density than southward component (Table 3). It turned out that discarding IMF B_N from the list of input parameters leads to only minor reduction of the density model quality. For this reason, and for the sake of simplicity, we have left only two input permeneters N_{SW} and B_S in our density model.

4.3. Input parameters for electron plasma sheet temperature model

Table 4 shows the correlation between the plasma sheet electron perpendicular temper-347 solar wind parameters. It can be seen that solar wind velocity exhibits ature (T_e) and relation. Similar results have been found for plasma sheet ion temperature strongest for 349 [Borovsky et al., 1998a; Tsyganenko and Mukai, 2003]. The lowest correlations are ob-350 tained for the duskside bins. It can also be noticed that IMF B_S and B_N affect the electron 351 temperature in an opposite way. Figure 5 shows the correlations between T_e and V_{SW} for 352 in the same format as in Figure 3. The correlations show very weak desix spatial 353 **b** and ΔT_V for several bins. It is an expected result since the solar wind pendence on 354 velocity autocorrelation characteristic time scale is largest of all solar wind parameters 355 (See Figure 6 in Borovsky et al. [1998a]). We chose $t_V = 0.5$ h and $\Delta T_V = 4$ h. 356

Figure (shows the similar correlation plots for IMF B_S . There is no clear dependence on MLT. Although for some bins the correlation is rather weak, the duration and the lag at the correlation peak fit well the substorm timescales (0.5–2 hours). We chose the time lag t_{BS} 30 minutes which can be interpreted as the time needed for the lobe magnetic flucto start to influence the near-Earth magnetotail and the averaging interval $\Delta T_{BS} = 45$ minutes is close to the typical substorm growth phase duration.

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Figure 7 shows the similar plots for IMF B_N . Color scale on the right side of each plot corresponds to the absolute value of the correlation coefficient. The highest correlation is on the dawnside. Surprisingly, the correlations are even higher than those for B_S . To make sure that these correlations are not due to the mutual correlation between IMF B_N and V_{SW} , we inspected the correlation between B_N and V_{SW} for various lags t_V and t_{BN} and found no significant correlation. We chose $t_{BN} = 0.5$ h and $\Delta T_{BN} = 2$ h.

5. Solar wind driven model for electron plasma sheet densities and temperature: Empirical relations

Using the fine constants given in Table 5, we computed the input parameters for the electron density and temperature models as time integrals in the form of Equation 2. Note that the lag values in Table 5 (0.58 h) are different from those determined in Sections 4.2 and 4.3 (0.5 h). The lag constants in Table 5 just take into account 5-min offset of the solar wind parameters used in this study (See Section 4.1).

At the first tep, we use the following functional form of the plasma sheet parameter dependence on the solar wind input parameters:

$$P_{ps} = G_0(\phi, R) + \sum_{j=1,\dots} G_j(\phi, R) \cdot P_j^{SW},$$
(3)

where P^{SW} are the corresponding solar wind parameters, and $G_j(\phi, R)$ are the 2nd order polynomials of an azimuth angle ϕ and radial distance R given as

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$$G_{j}(\phi, R) = \sum_{m,n=0,1,2} C_{mnj} \cdot R^{n} \phi^{m}.$$
 (4)

The polynomial coefficients C_{mnj} were found by fitting Equation 3 to the data (primary data set). After the first set of the coefficients was found, we computed the correlation coefficient between the plasma sheet parameters and the model predictions. Using this

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correlation coefficient as a reference value, we started to remove more and more terms 383 from Equation 3 (simplifying the polynomials) seeking for a minimal set of terms which 384 still provide good model quality. That is, for every possible subset of the terms in Equa-385 tion 3, we fitted this truncated model to the data and computed the correlation coefficient 386 between the data and the model. Comparing this correlation coefficient with a reference 387 one, we checked that such simplification of Equation 3 did not lead to significant reduction 388 of the model quality. After this simplification was done, we introduced the nonlinear pa-389 rameters (exponential powers of the driving parameters) and checked if this modification 390 leads to significant improvement. The downhill simplex algorithm was used for finding a 391 minimum of the error function [Nelder and Mead, 1965]. 392

Applying this method to the plasma sheet electron density and temperature datasets, we come up with following solutions. The number density in the plasma sheet (N_{ps}) is given in cm³ as follows:

$$N_{ps} = A_1 + A_2 R^* + A_3 \phi^{*2} R^* + A_4 \phi^{*2} + A_5 N_{sw}^* + (A_6 + A_7 R^*) B_S^*,$$
(5)

where, $\phi^* = \phi/90^\circ$, $R^* = R/10R_E$ are normalized coordinates, and N^*_{sw} , B^*_S are the time-integrad and normalized parameters characterizing the external conditions and defined as $N^*_{sw}(t_0) = \frac{1}{10 \text{ cm}^{-3}\Delta T_N} \int_{t_0-t_N-\Delta T_N}^{t_0-t_N} N_{sw}(t)dt,$ (6)

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$$B_{S}^{*}(t_{0}) = \frac{1}{2 \text{ nT } \Delta T_{BS}} \int_{t_{0}-t_{BS}-\Delta T_{BS}}^{t_{0}-t_{BS}} B_{S}(t) dt.$$
(7)

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Here, N_{sw} and B_S are the solar wind density and southward IMF component. The values for t_N , ΔT_N , t_{BS} and ΔT_{BS} are given in Table 5 and the model coefficients A_i are given in Table 6. Figure 8a shows the electron density values observed by THEMIS probes versus the model predictions.

The temperature in the plasma sheet (T_{ps}) is given in keV as follows:

$$T_{ps} = [A_1 \underbrace{A_2 \phi^*}_{sw} + A_3 V_{sw}^* + (A_4 + A_5 \phi^{*2} R^*) B_S^{*A_7} + A_6 R^* B_N^{*A_8}]^{A_9},$$
(8)
where
$$O$$

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$$V_{sw}^{*}(t_{0}) = \frac{1}{400 \text{ km/s } \Delta T_{V}} \int_{t_{0}-t_{V}-\Delta T_{V}}^{t_{0}-t_{V}} V(t)dt, \qquad (9)$$

$$B_{S}^{*}(t_{0}) = \frac{1}{2 \text{ nT } \Delta T_{BS}} \int_{t_{0}-t_{BS}-\Delta T_{BS}}^{t_{0}-t_{BS}} B_{S}(t) dt, \qquad (10)$$

 $B_N^*(t_0) = \frac{1}{2 \text{ nT } \Delta T_{BN}} \int_{t_0 - t_{BN} - \Delta T_{BN}}^{t_0 - t_{BN}} B_N(t) dt.$ (11)

Here, V_{sw} , B_S , and B_N are the solar wind density and the southward and northward 412 **rem**ts, respectively. The values for t_V , ΔT_V , t_{BS} , ΔT_{BS} , t_{BN} and ΔT_{BN} are IMF com 413 given in Table 5 and the model coefficients A_i are given in Table 6. Figure 9a shows the 414 electron temperature values observed by THEMIS probes versus the model predictions. 415 that for high electron temperatures, the THEMIS measurements typically It can be 416 exceed the model prediction. This bias would be much stronger if the standard least-417 squared error function is used. In order to minimize the bias, we have modified the error 418 function as follows: 419 DRAFT August 18, 2016, 1:10am DRAFT

$$ERR = \sum_{j} W \cdot |T_{j}^{THM} - T_{j}^{model}|$$
(12)

Here, T_j^{THM} and T_j^{model} are the THEMIS measurements and model predictions, respectively, and weight coefficient W is a linear function of T_j^{THM} changing from 1 at $T_j^{THM} = 0$ to 1.5 at $T_j^{THM} = 22$ keV.

6. Solar wind driven model for electron plasma sheet densities and temperatures Results

Some properties of the empirical electron plasma models becomes evident after inspec-424 tion of Equations 5 and 8 and Table 6. The resulting density model is very simple. Only 425 terms symmetric with respect to the midnight meridian remain after the model simplifica-426 tion as des ibed in Section 5. The symmetry of the density distribution is an interesting 427 finding since the storm time inner magnetosphere is highly asymmetric (at least during the main phase). The plasma sheet density response to changes of the solar wind density is positive and uniform across whole region of the model applicability. It is a bit surprising, 430 but the plasma sheet electron density response to the southward IMF component is also 431 sygenenko and Mukai [2003] reported opposite dependence. However, it should positive. 432 be noted that the model is parameterized by B_S lagged by 0.5 h and averaged over six 433 hours, that is, this density response is not related to the substorm cycle but rather to the 434 geomagnetic storm time-scale. In addition, this response is strongest in the near-Earth 435 region and disappears at $r = 11R_E$, where Tsyganenko and Mukai [2003] model's validity 436 This IMF B_S effect can be interpreted as a result of the compression of region begins. 437 the flux tube due to inflation of the inner magnetosphere magnetic configuration caused 438 by the ring current strengthening. However, we can not be sure that this effect manifests 439

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⁴⁴⁰ only during storm-times. Figures 10a and 10b show the distribution of the plasma density ⁴⁴¹ in the equatorial plane. The corresponding input parameters are given at the top of each ⁴⁴² panel. The density increases towards the Earth and peaks at midnight. Note that the ⁴⁴³ model reveals opposite MLT dependence at the outer boundary of the region (the density ⁴⁴⁴ is highest preed the dusk and dawn meridians). This feature manifests more clearly in ⁴⁴⁵ Figure 10 and it is in agreement with *Tsyganenko and Mukai* [2003] model (see their ⁴⁴⁶ Figure 10).

Figures 10c $\,{\rm f}$ show the equatorial maps of the electron temperature distributions for four 447 combinations of the model input parameters. In contrast to the density distributions, the 448 electron temperature exhibits very strong dusk-dawn asymmetry. Figure 10c shows the 449 temperature distribution for $B_S^* = B_N^* = 0$. In fact, it is unlikely that such combination 450 of the parameters occur in reality since it implies that transverse component of IMF is 451 45 minutes (see Table 5). For these parameters, the model temperature zero for at least 452 increases monotonically from dusk to dawn meridian showing no dependence on radial 453 distance 454

⁴⁵⁵ As it follows from Equation 8 and Table 6, the near-Earth plasma sheet electron temper-⁴⁵⁶ ature increases with the solar wind velocity increase. Although there is only one coefficient ⁴⁵⁷ associated with V_{SW}^* in Equation 8, the electron temperature response to V_{SW}^* increase ⁴⁵⁸ is not uniform since the left part of Equation 8 is raised to the power of 2.3 ($A_9 = 2.3$ ⁴⁵⁹ see Table 6). It means that the response is stronger on the dawn side where the electron ⁴⁶⁰ temperature is higher.

⁴⁶¹ The electron temperature increases with the southward IMF component increase. This ⁴⁶² effect is strongest near the midnight and disappears at the dawn and dusk MLTs. It

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leads to the temperature peak localization in the midnight – dawn sector (See Figures 10d and 10f). The increase of IMF B_S leads to a shift of the temperature maximum from the dawn sector towards midnight. The post-midnight location of the electron temperature peak is probably related to the substorm activity (hot electrons drift eastward from an **injection** place in pre-midnight sector).

The electron temperature response to the northward IMF component (integrated over 2 hours) is negative and strongest at the outer border of the region. Figure 10e demonstrates the cooling of the electrons in the outer part of the region during the prolonged periods of northward IMF. It is probably related to the arrival of the cold magnetosheath plasma during the intervals of the northward IMF [*Wing et al.*, 2005; *Wang et al.*, 2007, 2010].

7. Discussion of the Model Performance

Figures 8a and 9a present the scatter plots of the model predictions versus real THEMIS 474 observations (primary data set) for electron density and temperature models, respectively. 475 The correlation coefficients between the model and the data were 0.82 for electron den-476 sity and 0.75 for electron temperature models. Table 7 shows the correlation coefficients 477 odel predictions and the real data (primary data set) computed for evbetween t 478 erv spatial bin separately. The root-mean-square deviations (RMS) and mean absolute 479 deviations (MAD) are also shown. It can be seen that both models show their best per-480 the dawnside of the region. It is not immediately clear what causes such formance on 481 Since the electrons undergo eastward magnetic drifts, their drift trajectories asymme 482 are expected to be regular on the dawnside, in contrast to the duskside where the drift 483 paths can bifurcate (especially in the near-Earth region). Substorm activity is typically 484

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⁴⁸⁵ peaked at the pre-midnight sector (and this distortion can become even stronger during
⁴⁸⁶ the storm periods) and it can also contribute to the poorer performance of the model on
⁴⁸⁷ the duskside.

However, a model performance estimation using the same data the models has been fit-488 ted to cannot be considered as an independent test. The auxiliary data set (see Section 3) 489 has not been used for the model coefficients determination. Indeed, it can be considered 490 as an almost independent data set because only 26% of its data have the "neighbours" 491 from primary data set within ± 30 min (these neighbours are typically measurements on 492 In addition, 20% of the auxiliary data set are referred to the early period other probes) 493 of the THEMIS mission (2007–2009) which is not included in the primary data set. This 494 theoretically allows us to check if there is any bias in the primary data set related to the 495 detectors degradation. On the other hand, the auxiliary data set represents unaveraged 496 \sim 3-sec resolution measurements and we expect more noise in this data set and, hence, poorer correlations. Finally, the auxiliary data set is three times smaller than the primary 498 data set e can not expect that the model coefficients obtained by fitting the model to the smaller data set are of the same accuracy level. 500

hows the correlation coefficients and the average deviations between the model Table 8 501 and the auxiliary data set. Although the correlation coefficients are lower than those for 502 the primary data set, they are still higher than 0.7 (typical correlation for the empirical 503 models of the near-Earth plasma environment [Tsyganenko and Mukai, 2003; Sergeev et 504 al., 2015). Strangely enough, the density model shows better agreement with auxiliary 505 be dusk side but it might be an effect of limited statistics. The scatter data set on 506 plots of the model prediction versus the data from auxiliary data set are presented in 507

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Figures 8b and 9b. It can be seen that during high density periods, the models tend to 508 underestimate the density values for significant number of events. Although this feature 509 can be also noticed in Figures 8a for the primary data set, it is much more prominent in 510 Figure 8b. To rule out possibility that this difference between two data sets is due to the 511 detectors degradation, we inspected the data corresponding to these problematic points. 512 It turned out that only 11% of these data are referred to the years 2007–2009, indicating 513 that there is another reason of this discrepancy. We also checked the hypothesis that 514 this bias is caused by transient processes in the plasma sheet called bursty bulk flows 515 et al., 1992; Baumjohann et al., 1990]. However, the occurrence of events Angelopoulos 516 with the ion flow velocity exceeding 100 km/s for the problematic points is similar to that 517 for the points near the diagonal of Figure 8b. 518

Finally, to test the model coefficient sensitivity to the change of the data set, we fitted 519 the models to the auxiliary data set. The resulting coefficients are presented in the bottom 520 part of **Exple 5**. It can be seen that the difference between the density model coefficients 521 fitting to the different data set can be as large as factor 3 (see A_3 , A_4 obtained 522 coefficients). However, the difference between polynomials $A_1 + A_2 R^* + A_3 \phi^{*2} R^* + A_4 \phi^{*2}$ 523 π s in Equation 5) is within 40%. The coefficients are not so different for the (first four t 524 temperature model. 525

⁵²⁶ Comparison of our models performance with other empirical models is not straightfor-⁵²⁷ ward. On one hand, our electron density model shows the best correlations between the ⁵²⁸ model predictions and the data among all existing empirical models. On the other hand, ⁵²⁹ such an evaluation of the model performance is strongly biased. The regions of applica-⁵³⁰ bility of our model and the models of other authors overlap only partly. The different

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data sets were used for the construction of the models. Our data set includes storm-time intervals. The solar wind driving parameters undergo stronger variations during storm periods and all dependencies can be tracked more easily. On the other side, these highly disturbed periods obviously add more scatter to the data.

The conduction of the Tsyganenko and Mukai [2003] ion temperature model predictions 535 with the data is comparable with that for our model for electron temperature (0.71 versus 536 0.75, respectively). The comparison of the ion and electron models seems to be justified 537 because the ion and electron temperatures are highly correlated in the central plasma sheet 538 [Baumjohann et al., 1989]. It should be mentioned that the correlations in the Tsyganenko 539 and Mukai 2003 study were computed for the whole region of the models applicability. 540 Since the Tsyganenko and Mukai [2003] model covers the magnetotail between r = 10-541 $50R_E$, and the ion temperature reveals a stable increase with distance, a simple comparison 542 of the correlations for the whole data sets puts the Tsyganenko and Mukai [2003] model in 543 the more avorable conditions. On the other hand, the highly dynamic bursty bulk flows 544 occur m uently in the distant plasma sheet [Baumjohann et al., 1990]. In addition, 545 Runov et al. [2015] found that the correlation between the ion and electron temperatures $< 12R_E$ and Artemyev et al. [2011] found that the relation between the disappears 547 electron and ion temperatures is non-linear in the mid-tail. 548

For development in the future, we foresee the following possibilities: (1) A presence of the multiple population components (cold, hot) should be addressed; (2) The inclusion of the geomagnetic activity indices as input parameters will increase the model accuracy; (3) Expansion of the dataset including non-storm periods.

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8. Conclusions

The empirical models of the plasma sheet electron temperature and density on the nightside for $6R_E < r < 11R_E$ has been constructed using the data of the THEMIS mission obtained during the geomagnetic storm periods. The models depend on spatial coordinates are well as on the interplanetary medium parameters. The reader can find the codes for hoth models as well as procedures for the input parameters computation in the supplemental materials.

The model performances have been essentially improved by using lagged and time averaged solar wind parameters as model inputs. The best time-lag and duration of averaging were different for different parameters as well as showed some dependence on MLT (the latter feature is not included in the current model version).

It was found that the plasma sheet electron density equatorial distribution is symmet-563 to the midnight meridian. It reveals a strong earthward gradient and a ric with respect 564 moderate symmetric variation with MLT. The plasma sheet density dependence on the 565 g is parameterized by the solar wind proton density (averaged over preexternal 566 ceding 4 hours) and southward IMF component (averaged over preceding 6 hours). In 567 the results of previous studies, the solar wind proton density is the main agreement 568 controlling parameter but the IMF B_S becomes of almost the same importance in the 569 near-Earth region. The model density shows a positive response to the increase of either 570 input parameter. The electron density revealed better correlation with IMF B_S averaged 571 over the time interval which is closer to the geomagnetic storm main phase (~ 6 hours) 572 we substorm growth phase (~ 45 minutes). The root-mean-square deviation rather than 573 between the observed and predicted plasma sheet density values is 0.23 cm^{-3} and the 574

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⁵⁷⁵ correlation coefficient is 0.82, the highest correlation with the data set ever obtained for ⁵⁷⁶ these kinds of empirical models.

The electron temperature model is highly asymmetric with respect to the local midnight. 577 The electron temperature maximum is located in the post-midnight – morning MLT 578 sector. The model is parameterized by solar wind velocity and southward and northward 579 components of IMF. The solar wind velocity is a major controlling parameter and the 580 importance of B_S and B_N is comparable. The plasma sheet electron temperature responds 581 positively to the solar wind velocity and IMF B_S increase and it responds negatively to the 582 IMF B_N increase. In contrast to the density model, the electron temperature shows higher 583 correlation with the southward IMF component when IMF B_S is averaged over preceding 584 ~ 45 min (substorm growth phase time scale). The effect of the northward component is 585 parameter red by ~ 2 hour average of IMF B_N . The impact of the prolonged IMF B_N 586 manifests mostly in the outer part of the modelled region $(r > 8R_E)$ while the influence 587 of the IL B_S is maximal in the midnight – post-midnight MLT sector. The correlation 588 coefficie **determine** en the observed and predicted plasma sheet electron temperature values is 0.76 and the root-mean-square deviation is 2.6 keV. 590

The both models reveal the dawn-dusk asymmetry of their performances with better accuracy achieved in the dawn MLT sector. The correlations between the model predictions and observations vary between C.C.>0.7 in the dawn MLT sector and C.C.= 0.5-0.7in the dusk sector.

595 Acknowledgments.

The plasma moments were obtained from the THEMIS mission web site (http://themis.ssl.berkeley.edu/index.shtml). The solar wind and IMF data were

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downloaded from the OMNI database from the GSFC/SPDF OMNIWeb interface at 598 http://omniweb.gsfc.nasa.gov. The 1-min. resolution SYM-H index was provided by the 599 World Data Center for geomagnetism, Kyoto (http://wdc.kugi.kyoto-u.ac.jp/). The part 600 of the research done by N. Ganushkina and S. Dubyagin has received funding from the 601 European Union Seventh Framework Programme (FP7/20072013) under grant agreement 602 606716 SFACESTORM and from the European Union Horizon 2020 Research and Inno-603 vation programme under grant agreement 637302 PROGRESS. N. Ganushkina thanks the 604 International Space Science Institute in Bern, Switzerland, for their support of the interna-605 tional teams on "Analysis of Cluster Inner Magnetosphere Campaign data, in application 606 the dynamics of waves and wave-particle interaction within the outer radiation belt" and 607 "Ring current modeling: Uncommon Assumptions and Common Misconceptions". 608

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cript Figure 1. (a) Spatial coverage of the equatorial magnetosphere by THEMIS observations. Only every tenth point is shown. Color shows corresponding SYM-H. (b) Spatial bins numeration. uthor N Sketch explaining how to interpret Figures 3–7. The horizontal axis represents the Figure time lag and the vertical axis represents duration of averaging. See explanation in the text.

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Figure 3. Sorrelation coefficients (color coded) between the plasma sheet electron density and the solar wind density for six regions of the magnetotail. Vertical and horizontal axes show the solar wind density average duration and the lag of the solar wind density observations with respect to plasma sheet measurements. The oblique lines show $\Delta T_N = const - 2 \cdot t_N$ dependencies. The black filled circles mark ΔT_N and t_N which are used for the input parameters computation.

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Figure The same as Figure 3 but for correlation coefficients between the plasma sheet electron density and the southward component of IMF B_Z .

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Figure 5 The same as Figure 3 but for correlation coefficients between the plasma sheet electron temperature and the solar wind velocity.

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Figure The same as Figure 3 but for correlation coefficients between the plasma sheet electron temperature and southward component of IMF B_Z .

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Figure 7 The same as Figure 3 but for correlation coefficients between the plasma sheet electron temperature and the northward component of IMF B_Z .

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Figure S. Plasma sheet electron density predicted by the empirical model versus that measured by the EMAIS probes. (a) The THEMIS measurements are represented by primary data set. Every tenth point is shown. (b) The THEMIS measurements are represented by auxiliary data set. Every tenth point is shown.

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Figure 9. The same as Figure 8 but for the electron temperature model.

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Figure 10 Distributions of the electron temperature and density in the equatorial plane. (a–b) density model, (c–f) electron temperature model.

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Table 1. Distribution of the number of the samples over the THEMIS mission period forprimary and auxiliary data sets.

Years	2007	2008	2009	2010	2011	2012	2013
# primary	0	0	0	7475	11347	12693	13486
# auxiliary	1992	583	38	1688	2033	2520	3317

Table 2. Statistical properties of the data sets for different spatial bins. Top part is for standard deviations of instant values corresponding to the zero lag, and the bottom part shows the range of standard deviations found for lags between 0 and 12 h.

				0			
	<u>Bin index</u>	1	2	3	4	5	6
	$r, [R_E]$	8.5 - 11	8.5 - 11	8.5 - 11	6 - 8.5	6 - 8.5	6 - 8.5
	ϕ	$-90^{\circ}30^{\circ}$	-30° – 30°	$30^{\circ}-90^{\circ}$	$-90^{\circ}30^{\circ}$	-30° – 30°	30°-90°
	#	16257	9046	4698	6780	5812	2295
	$N_{\rm CW}, [{\rm cm}^{-3}]$	5.1	3.7	5.1	6.0	4.3	3.5
	$\sigma V_{\rm cur},{\rm km/s}$	118	109	88	112	110	93
	σB_{ZMF} , nT	4.0	3.9	4.0	4.4	3.9	3.6
_	$\sigma N_{SW}, [\mathrm{cm}^{-3}]$	4.6-6.3	3.6 - 5.2	3.3 - 5.1	5.7 - 9.6	3.3 - 8.4	3.0-4.6
	σV_{SW} , km/s	117 - 121	106-111	88-95	110-118	108-114	90- 98

Table 3 Correlations of the plasma sheet electron density with the solar wind parameters. Top part is for instant values $t_0 - 45$ min. and the bottom part shows best correlations found for all 1 g and durations of averaging.

Bin index	1	2	3	4	5	6
$\overline{N_{SW}}$	0.71	0.54	0.60	0.69	0.57	0.39
IMF B_S	0.18	0.20	0.16	0.28	0.38	0.36
IMF B_N	0.24	0.33	0.35	0.16	0.13	0.08
$\overline{N_{SW}}$	0.77	0.58	0.70	0.73	0.59	0.60
IMF B_S	0.33	0.35	0.42	0.58	0.48	0.47
IMF B_N	0.28	0.35	0.46	0.18	0.20	0.22

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Table 4. Correlations of the plasma sheet electron temperature with the solar wind parameters. Top part is for instant values $t_0 - 45$ min. and the bottom part shows best correlations found for all lags and durations of averaging.

$ \begin{array}{c} V_{SW} & 0.59 & 0.63 & 0.28 & 0.59 & 0.59 & 0.31 \\ \text{IMF } B_S & 0.17 & 0.32 & 0.19 & 0.32 & 0.28 & 0.12 \\ \text{IMF } B_N & -0.36 & -0.29 & -0.17 & -0.42 & -0.38 & -0.23 \\ \hline V_{SW} & 0.60 & 0.65 & 0.40 & 0.61 & 0.60 & 0.37 \\ \text{IMF } B_S & 0.19 & 0.34 & 0.26 & 0.36 & 0.30 & 0.25 \\ \text{IMF } B_N & -0.42 & -0.31 & -0.25 & -0.53 & -0.39 & -0.32 \\ \hline \\ \textbf{MF } B_N & -0.42 & -0.31 & -0.25 & -0.53 & -0.39 & -0.32 \\ \hline \\ \textbf{MF } B_N & 0.58 & h & 4.00 & h & 0.58 & h & 6.00 & h \\ \hline \\ \textbf{Temperature} & 0.58 & h & 0.75 & h & 0.58 & h & 4.00 & h & 0.58 & h & 2.00 & h \\ \hline \\ \textbf{MF } B_N & -1.23 & -1.01 & 0.874 & -0.820 & 0.392 & 0.521 & -0.474 \\ \hline \\ \textbf{Temperature} & -0.0215 & -0.426 & 1.47 & 0.587 & -0.538 & -0.489 & 0.32 & 0.36 & 2.31 \\ \hline \\ \textbf{MF } Temperature^{\dagger} & -0.0922 & -0.390 & 1.64 & 0.767 & -1.02 & -0.395 & 0.26 & 0.52 & 2.16 \\ \hline \\ \textbf{M coefficients obtained by fitting the model to the auxiliary data set} \end{array}$		Din maex		2	3	4	\mathbf{b}	6		
$ \begin{array}{c} \text{IMF } B_S & 0.17 & 0.32 & 0.19 & 0.32 & 0.28 & 0.12 \\ \text{IMF } B_N & -0.36 & -0.29 & -0.17 & -0.42 & -0.38 & -0.23 \\ \hline V_{SW} & 0.60 & 0.65 & 0.40 & 0.61 & 0.60 & 0.37 \\ \text{IMF } B_S & 0.19 & 0.34 & 0.26 & 0.36 & 0.30 & 0.25 \\ \hline \text{IMF } B_N & -0.42 & -0.31 & -0.25 & -0.53 & -0.39 & -0.32 \\ \hline \text{Offme constants for computation of the empirical models input parameters} \\ \hline t_N & \Delta T_N & t_{BS} & \Delta T_{BS} & t_V & \Delta T_V & t_{BN} & \Delta T_{BN} \\ \hline \text{Density} & 0.58 \text{ h } 4.00 \text{ h } 0.58 \text{ h } 6.00 \text{ h} \\ \hline \text{Temperature} & 0.58 \text{ h } 0.75 \text{ h } 0.58 \text{ h } 4.00 \text{ h } 0.58 \text{ h } 2.00 \text{ h} \\ \hline \text{Temperature} & 0.58 \text{ h } 0.75 \text{ h } 0.58 \text{ h } 4.00 \text{ h } 0.58 \text{ h } 2.00 \text{ h} \\ \hline \text{Temperature} & -0.0215 & -0.426 & 1.47 & 0.587 & -0.538 & -0.489 & 0.32 & 0.36 & 2.31 \\ \hline \text{Temperature}^{\dagger} & 1.01 & -0.747 & 0.303 & -0.248 & 0.362 & 0.498 & -0.474 \\ \hline \text{Temperature}^{\dagger} & -0.0922 & -0.390 & 1.64 & 0.767 & -1.02 & -0.395 & 0.26 & 0.52 & 2.16 \\ \hline \text{Id coefficients obtained by fitting the model to the auxiliary data set} \\ \end{array}$	-	V_{SW}	0.59	0.63	0.28	0.59	0.59	0.31		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	-	IMF B_S	0.17	0.32	0.19	0.32	0.28	0.12		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		IMF B_N	-0.36	-0.29	-0.17	-0.42	-0.38	-0.23		
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$\frac{\text{IMF } B_N -0.42 -0.31 -0.25 -0.53 -0.39 -0.32}{\Delta \mathbf{O}}$		IMF B_S	0.19	0.34	0.26	0.36	0.30	0.25		
$\frac{1}{1000}$ The constants for computation of the empirical models input parameters $\frac{t_N}{\Delta T_N} \frac{\Delta T_N}{t_{BS}} \frac{\Delta T_{BS}}{\Delta T_{BS}} \frac{t_V}{\Delta T_V} \frac{\Delta T_V}{t_{BN}} \frac{\Delta T_{BN}}{\Delta T_{BN}}$ Density 0.58 h 4.00 h 0.58 h 6.00 h Temperature 0.58 h 0.75 h 0.58 h 4.00 h 0.58 h 2.00 h Temperature 0.58 h 0.75 h 0.58 h 4.00 h 0.58 h 2.00 h Temperature 0.58 h 0.75 h 0.58 h 4.00 h 0.58 h 2.00 h Temperature 0.58 h 0.75 h 0.58 h 4.00 h 0.58 h 2.00 h Temperature 0.58 h 0.75 h 0.58 h 4.00 h 0.58 h 2.00 h Temperature 0.58 h 0.75 h 0.58 h 4.00 h 0.58 h 2.00 h Temperature 0.58 h 0.75 h 0.58 h 4.00 h 0.58 h 2.00 h Temperature 0.58 h 0.75 h 0.58 h 4.00 h 0.58 h 2.00 h Temperature 0.58 h 0.75 h 0.58 h 4.00 h 0.58 h 2.00 h Temperature 0.58 h 0.75 h 0.58 h 4.00 h 0.58 h 2.00 h Temperature 0.58 h 0.75 h 0.58 h 4.00 h 0.58 h 2.00 h Temperature 0.58 h 0.75 h 0.58 h 4.00 h 0.58 h 2.00 h Temperature 0.0215 -0.426 1.47 0.587 -0.538 -0.489 0.32 0.36 2.31 Temperature 1.01 -0.747 0.303 -0.248 0.362 0.498 -0.474 Temperature 1.00922 -0.390 1.64 0.767 -1.02 -0.395 0.26 0.52 2.16 H coefficients obtained by fitting the model to the auxiliary data set		IMF B_N	-0.42	-0.31	-0.25	-0.53	-0.39	-0.32		
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Temperature [†] -0.0922 -0.390 1.64 0.767 -1.02 -0.395 0.26 0.52 2.16	CEMpirical Density Temprature	model par A_1 1.23 -0.0215	$\frac{\text{ameter}}{A_2}$ -1.01 0.426	s. A_3 0.874 1.47	A_4 -0.820 0.587	A_5 0.392 -0.538	A_6 2 0.52 8 -0.48	A_7 A_7	A_8	A_9 2.31
a coefficients obtained by fitting the model to the auxiliary data set	Consity Temprature	A_1 A_1 1.23 -0.0215 - 1.01 -	ameter A_2 -1.01 0.426 0.747	s. A_3 0.874 1.47 0.303	A_4 -0.820 0.587 -0.248	A_5 0.392 -0.538 0.362	$ \begin{array}{r} A_6 \\ 2 & 0.52 \\ 8 & -0.48 \\ \hline 2 & 0.40 \end{array} $	A_7 A_7 A_7 A_7 A_7 A_7 A_7 A_7 A_7 A_7 A_7 A_7 A_7 A_7 A_9 A_2 A_2 A_2 A_3 A_2 A_3 A_2 A_3 A_3 A_4 A_3 A_3 A_4 A_3 A_3 A_4 A_3 A_4 A_5 A_7	A ₈ 0.36	A_9 2.31
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	6 Empirical Density Temperature Temperature [†] del coefficients	$\begin{array}{c} model par \\ \hline A_1 \\ 1.23 \\ -0.0215 \\ \hline 1.01 \\ -0.0922 \\ \hline s obtained \\ \end{array}$	ameter A_2 -1.01 0.426 0.747 0.390 by fittin	s. A_3 0.874 1.47 0.303 1.64 ng the	$\begin{array}{c} A_4 \\ -0.820 \\ 0.587 \\ -0.248 \\ 0.767 \\ \text{model} \end{array}$	$ \begin{array}{r} A_5 \\ 0.392 \\ -0.538 \\ 0.362 \\ -1.02 \\ to the \end{array} $	$ \begin{array}{cccc} $	$\begin{array}{c} & A_7 \\ 21 & -0.474 \\ 89 & 0.32 \\ 08 & -0.474 \\ 95 & 0.26 \\ ary \ data \ s \end{array}$	A_8 0.36 0.52 set.	A_9 2.31 2.16
\sim	6 Exppirical Density Temperature Consity [†] Temperature [†] del coefficients	$\begin{array}{c} model par\\ \hline A_1\\ 1.23\\ -0.0215\\ \hline 1.01\\ \hline -0.0922\\ \hline s \ obtained \end{array}$	ameter A_2 -1.01 0.426 0.747 0.390 by fittin	s. A_3 0.874 1.47 0.303 1.64 mg the	$\begin{array}{c} A_4 \\ -0.820 \\ 0.587 \\ -0.248 \\ 0.767 \\ \text{model} \end{array}$	$ \begin{array}{r} A_5 \\ 0.392 \\ -0.538 \\ 0.362 \\ -1.02 \\ to the \end{array} $	$\begin{array}{c c} & A_6 \\ 2 & 0.52 \\ 8 & -0.48 \\ 2 & 0.49 \\ 2 & -0.39 \\ auxilia \end{array}$	A_7 A_7	A_8 0.36 0.52 set.	A_9 2.31 2.16
0	⁵ Empirical Density Temperature Temperature [†] lal coefficients	$\begin{array}{c} model par\\ \hline A_1\\ 1.23\\ -0.0215\\ \hline 1.01\\ \hline -0.0922\\ \hline s \ obtained \end{array}$	ameter A_2 -1.01 0.426 0.747 0.390 by fittin	s. A_3 0.874 1.47 0.303 1.64 ng the	$\begin{array}{c} A_4 \\ -0.820 \\ 0.587 \\ -0.248 \\ 0.767 \\ \text{model} \end{array}$	$ \begin{array}{r} A_5 \\ 0.392 \\ -0.538 \\ 0.362 \\ -1.02 \\ to the \end{array} $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		A_9 2.31 2.16
0	6 Empirical Density Temperature Temperature [†] del coefficients	$\begin{array}{c} model par \\ \hline A_1 \\ 1.23 \\ -0.0215 \\ \hline 1.01 \\ -0.0922 \\ \hline s \ obtained \end{array}$	ameter A_2 -1.01 0.426 0.747 0.390 by fittin	s. A_3 0.874 1.47 0.303 1.64 ng the	$\begin{array}{c} A_4 \\ -0.820 \\ 0.587 \\ -0.248 \\ 0.767 \\ \text{model} \end{array}$	$ \begin{array}{r} A_5 \\ 0.392 \\ -0.538 \\ \overline{0.362} \\ -1.02 \\ to the \end{array} $	$\begin{array}{c c} & A_6 \\ 2 & 0.52 \\ 8 & -0.48 \\ \hline 2 & 0.49 \\ 2 & -0.39 \\ \hline auxilia \end{array}$	A_7 A_7	A_8 0.36 0.52 set.	A_9 2.31 2.16
O 'CHaracteristics of the empirical models quality. Top part of the table for the	6 Empirical Density Temperature Temperature [†] del coefficients 0 7. Character	model par A_1 1.23 -0.0215 - 1.01 - -0.0922 - s obtained b	ameter A_2 -1.01 0.426 0.747 0.390 by fitting e empiric	s. A_3 0.874 1.47 0.303 1.64 ng the ical mo	A ₄ -0.820 0.587 -0.248 0.767 model	$ \begin{array}{r} A_5 \\ 0.392 \\ -0.538 \\ 0.362 \\ -1.02 \\ to the \\ nality. $	$ \begin{array}{r} A_6 \\ 2 & 0.52 \\ 8 & -0.48 \\ 2 & 0.49 \\ 2 & -0.39 \\ auxilia \\ Top pa $	A_7 A_7	A_8 0.36 0.52 set. table f	
Orman and the empirical models quality. Top part of the table for the	6 Empirical Temprature Temperature [†] del coefficients 7. Character	model par A_1 1.23 -0.0215 - 1.01 - -0.0922 - s obtained b istics of the	ameter A_2 -1.01 0.426 0.747 0.390 by fitting e empirities	s. A_3 0.874 1.47 0.303 1.64 ng the ical mo	A_4 -0.820 0.587 -0.248 0.767 model	A_5 0.392 -0.538 0.362 -1.02 to the nality.	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	A_7 A_7	A_8 0.36 0.52 set. table f	
O Characteristics of the empirical models quality. Top part of the table for the model and the bottom one is for the temperature model	6 Empirical Density Temperature Temperature [†] del coefficients 7. Character mpdel and the	model par A_1 1.23 -0.0215 - 1.01 - -0.0922 - s obtained b istics of the e bottom or	ameter A_2 -1.01 0.426 0.747 0.390 by fitting e empiriments for	s. A_3 0.874 1.47 0.303 1.64 mg the ical mo	A ₄ -0.820 0.587 -0.248 0.767 model	$ \begin{array}{r} A_{5} \\ 0.392 \\ -0.538 \\ 0.362 \\ -1.02 \\ to the \\ uality. \\ ture m $	$ \begin{array}{cccc} $	A_7 A_7	A_8 0.36 0.52 set. table f	
Characteristics of the empirical models quality. Top part of the table for the model and the bottom one is for the temperature model $\overline{\text{Bin index}}$ all 1 2 3 4 5 6	⁵ Empirical Density Temperature Ionsity [†] Temperature [†] Ial coefficients O Zenaracter mpdel and the	model par A_1 1.23 -0.0215 - 1.01 - -0.0922 - s obtained b istics of the bottom of Bin index	ameter A_2 -1.01 0.426 0.747 0.390 by fitting e empiring he is for all	s. A_3 0.874 1.47 0.303 1.64 mg the ical model r the t 1	A_4 -0.820 0.587 -0.248 0.767 model odels qu empera 2	$ \begin{array}{r} A_{5} \\ 0.392 \\ -0.538 \\ 0.362 \\ -1.02 \\ to the \\ to the \\ tality. \\ ture m \\ 3 $	$ \begin{array}{r} $	$ \frac{A_7}{6} = \frac{A_7}{21} - 0.474 \\ \frac{89}{20} = 0.32 \\ \frac{38}{20} - 0.474 \\ \frac{95}{20} = 0.26 \\ \frac{37}{20} = 0.26 \\ \frac{37}{20} = 0.26 \\ \frac{37}{20} = 0.474 \\ \frac{37}{20} = 0.$	$ \begin{array}{r} A_8 \\ 0.36 \\ 0.52 \\ set. \\ table f $	

Din index	an	T	\angle	Э	4	\mathbf{G}	0
C.C.	0.82	0.77	0.73	0.70	0.84	0.73	0.72
RMS, $[cm^{-3}]$	0.23	0.21	0.16	0.17	0.28	0.29	0.32
MAD, $[cm^{-3}]$	0.15	0.13	0.11	0.13	0.20	0.22	0.23
C.C.	0.75	0.72	0.72	0.65	0.79	0.75	0.54
RMS, [keV]	2.6	2.5	3.1	2.0	2.4	2.9	2.3
MAD, [keV]	1.8	1.8	2.1	1.3	1.8	2.1	1.7

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Table The same as Table 7 but for comparison with auxiliary data set. In addition, a number of data records for every bin is given in the second line. Bin index all 1 2 3 4 5 6

Bin index	all	1	2	3	4	5	6
#	12171	5220	1211	1069	2922	1014	689
C.C.	0.73	0.61	0.70	0.70	0.63	0.79	0.80
RMS, $[cm^{-3}]$	0.28	0.27	0.18	0.19	0.34	0.32	0.28
MAD, $[cm^{-3}]$	0.19	0.16	0.12	0.14	0.25	0.23	0.21
C.C.	0.71	0.75	0.65	0.82	0.72	0.67	0.57
RMS, [keV]	3.1	2.4	3.6	3.7	2.9	4.3	4.2
MAD, [keV]	2.2	1.8	2.5	2.2	2.2	3.0	3.0

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