Let ∞ be the place of $\mathbb{Q}$ given by the usual absolute value so that the real numbers $\mathbb{R}$ equal $\mathbb{Q}_\infty$. Of course, one knows that $\mathbb{Z} \subset \mathbb{Q}_\infty = \mathbb{R}$ discretely, and $\mathbb{R}/\mathbb{Z}$ is discrete. Similarly, one has $A \subset F_\infty$ discretely, and F_∞/A can also be seen to be compact (through the use of the Riemann–Roch theorem for instance). Remarkably, as one uses $\mathbb{R}$ and its algebraic closure $\mathbb{C}$ to construct the exponential function, elliptic curves, L-functions, and so forth, so too do F_∞ and its algebraic closure $\bar{F}_\infty$ possess a very rich collection of analytic functions and associated objects.

More specifically, in analogy with $\mathbb{Z}$-lattices in $\mathbb{C}$, an '**A**-lattice in $\bar{F}_\infty$' is a finitely generated discrete A-submodule of $\bar{F}_\infty$. To the rank 1 lattice $2\pi i\mathbb{Z}$ one attaches the usual exponential function $\exp(x)$. To an A-lattice M of *arbitrary* rank, one attaches its exponential function $\exp_M(x)$, defined by $\exp_M(x) := x\prod_{0\neq m\in M}(1-x/m)$. This function is easily seen to be entire and *additive*, thereby giving an analytic isomorphism of $\mathscr{F} := \bar{F}_\infty/M$ with $\bar{F}_\infty$. The space $\mathscr{F}$ has an obvious A-module structure on it, which can be carried over to $\bar{F}_\infty$ via $\exp_M(x)$. The resulting A-module structure on $\bar{F}_\infty$ is called a 'Drinfeld module'. Drinfeld modules are remarkably rich analogs of elliptic curves, and are the subject of much current research. They have moduli spaces with associated modular forms. They can also be given Γ-functions and L-series in analogy with elliptic curves; as the L-series of an elliptic curve is derived from the associated $\mathbb{Z}$-action, so too are the L-series of Drinfeld modules derived from their associated A-action. In particular, such L-series will naturally have values in $\bar{F}_\infty$. This allows one, for instance, to establish an analog of the famous formula of Euler on $\zeta(2n)$, where $\zeta(s)$ is the Riemann zeta function and n is a positive integer. While much is known about such L-series and modular forms, they remain highly mysterious. Rosen's book contains an introduction to this chain of ideas which will leave the reader in a good position to appreciate current work.

The interplay over the years between global function fields and number fields has been intense and extremely fruitful. *Number theory in function fields* does an excellent job of introducing these ideas. It will be a welcome resource for any number theorist, and it should become a standard text for graduate students in the area. There is a great pedagogical advantage in viewing difficult classical problems first in the function field arena, where they often have very clear, precise, and suggestive solutions.

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CELLULAR AUTOMATA: A DISCRETE UNIVERSE

By ANDREW ILACHINSKI: 808 pp., £76.00, ISBN 981-02-4623-4
(World Scientific, Singapore, 2001).

Bull. London Math. Soc. 35 (2003)

DOI: 10.1112/S0024609302241910

Cellular automata ('CAs') are discrete spatially extended dynamical systems, capable of a vast variety of behaviors. Some people study them for their own sake; some use them to model real phenomena; and some speculate that they underlie fundamental physics. The present volume is the most comprehensive single-author book on CAs to date, and provides a useful unified reference to many ideas scattered through the literature. While aimed at an audience of physicists, it should be useful and comprehensible to mathematicians and computer scientists. While no one book could exhaust such a wide subject, there are several places where this one falls short, and others where it is too generous to ideas that, while popular ten years ago in the complex systems community, have not borne fruit.

After an introduction and a lengthy chapter on formalism (mostly discrete mathematics), the author begins with a phenomenological exploration of basic CA rules. He discusses periodic domains and particles, temporal and spatial correlations, mean-field theory, and Wolfram's grouping of CAs into four somewhat ill-defined classes. He then discusses Langton's λ parameter and the 'edge of chaos' idea. Unfortunately, he repeats early claims that CAs must evolve towards the 'edge of chaos' in order to perform computational tasks, even though this was thoroughly demolished by Mitchell, Hraber and Crutchfield in 1993.

After a nice discussion of Conway's game of life and a sketch of the proof that it can perform universal computation, in Chapter 4 the author gives an introduction to the theory of continuous dynamical systems, and how notions like invariant measure carry over into CAs. Chapter 5 mainly enumerates periodic orbits and bounds transient times.

Chapter 6 gives an introduction to the theory of languages and automata, including non-regular languages. It focuses, however, on the dynamics of the regular languages generated by CAs. That CAs can give rise to context-free and context-sensitive languages is reduced to a brief mention of the work of Hurd. Also, the author misses all of the recent work of Machta *et al.* on the computational complexity of simulating CAs and physical systems, and their relationship to parallel circuit complexity.

Chapter 7 discusses probabilistic CAs and gives an introduction to scaling, phase transitions, and the Ising model of magnetism. It also explains why naive CA simulations often produce unphysical results, and how Creutz and others have designed CA rules that avoid this. It does not explain why most computational physicists still prefer traditional Monte Carlo simulations to CAs.

Chapter 8 has some excellent material on reversible CAs, and on work by Margolus, Takesue, Pomeau, Goles and Vichniac on building thermodynamics from microscopically reversible dynamics. After discussing coupled map lattices and spatio-temporal intermittency, the chapter concludes with a smorgasbord of popular complex systems, including Kauffman's Boolean *Nk* networks, random maps, and sandpiles. There is a brief section on reaction-diffusion systems, which is the only place in the book where CAs appear as models of pattern formation. (The section on quantum CAs, unfortunately, ignores work by Meyer, Watrous and others.) The chapter presents a CA model of AIDS and immune response, but is far too uncritical about whether such crude models tell us anything important about AIDS.

Chapter 9 gives a good introduction to lattice gases, discussing why CAs can, sometimes, efficiently simulate hydrodynamics and its generalizations. In general, fluid motion in a lattice gas inherits the unphysical anisotropy of the lattice. If the lattice has the right symmetry properties, however, isotropy reappears on large scales. The author's orientation towards physics comes through clearly here, since he does not explain hydrodynamic notation (for example, $\nabla \cdot \vec{v}$) at all.

Chapter 10, on neural networks, has nothing to do with the rest of the book.

Chapter 11 focuses on 'artificial life,' including agent-based models, von Neumann's and Langton's self-reproducing automata, and genetic algorithms. It presents the author's detailed model of land combat. This certainly has interesting dynamics, but we are left to wonder whether it is at all realistic.

Chapter 12 asks 'Is Nature, underneath it all, a CA?' Many have speculated that the world's apparent continuity masks a fundamentally discrete physics, among them Richard Feynman, John Wheeler and T. D. Lee. (In a sense, students of topological quantum field theories are pursuing this idea.) CAs are possible candidates for such a physics, and Fredkin, Toffoli, Margolus and Wolfram have advocated this vigorously. Many subtle issues are involved: for instance, to capture general relativity's coupling between matter and space-time curvature, the values of the cells must modify the lattice structure. Unfortunately, this chapter is simply a potpourri of speculative theories, without any exploration of whether these have led to any progress in physics. The author is entitled to philosophize, but not to indiscriminately mix field theory, mystical triads, quantum computation and Fritjof Capra. It would have been far better to work through a few models with detail and rigor, and let readers judge their worth.

The book ends with two appendices, one describing currently available CA hardware and software, and the other listing web pages related to CAs. The bibliography is extensive, although far from complete.

Overall, the author's choice of topics is suboptimal: some are out of date, some are extraneous, some deserve a more critical examination, and some are conspicuous by their absence. Nonetheless, there is much useful material here, and we are not aware of anything better with a comparable scope. CA enthusiasts will want copies on their shelves.

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DISCREPANCY OF SIGNED MEASURES AND POLYNOMIAL APPROXIMATION
(Springer Monographs in Mathematics)

By Vladimir V. Andrievskii and Hans-Peter Blatt: 438 pp., £59.50 (US\$79.95), isbn 0-387-98652-9 (Springer, New York, 2002).

Bull. London Math. Soc. 35 (2003) © 2003 London Mathematical Society
DOI: 10.1112/S0024609302251917

This book is devoted to discrepancy estimates for the zeros of polynomials and for signed measures. Most of the topics have emerged in the last two or three decades, and in most cases the book contains far-reaching generalizations of results published earlier.

A somewhat detailed discussion of Chapter 2 will reveal the main subject and the most characteristic features that the book dwells on. It is about zero distribution of monic polynomials. Let E be a compact set on the plane, and let $P_n(z) = z^n + \dots$ be a polynomial of degree n with leading coefficient 1. It is well known that the