ON A PROBLEM OF SIDON IN ADDITIVE NUMBER THEORY
AND ON SOME RELATED PROBLEMS

ADDENDUM

P. Erdős.

In a note in this Journal [16 (1941), 212–215], Turan and I proved, among other results, the following: Let $a_1 < a_2 < ... < a_x < n$ be a sequence of positive integers such that the sums $a_i + a_j$ are all different. Then $x < n^4 + O(n^4)$. On the other hand, there exist such sequences with $x > n^4(2^{-1} - \epsilon)$, for any $\epsilon > 0$.

Recently I noticed that J. Singer, in his paper "A theorem in finite projective geometry and some applications to number theory" [Trans. Amer. Math. Soc., 43 (1938), 377–385], proves, among other results, that, if $m$ is a power of a prime, then there exist $m+1$ numbers $a_1 < a_2 < ... < a_{m+1} < m^2 + m + 1$ such that the differences $a_i - a_j$ are congruent, mod $(m^2 + m + 1)$, to the integers 1, 2, ..., $m^2 + m$. Clearly the sums $a_i + a_j$ are all different, and since the quotient of two successive primes tends to 1, Singer's construction gives, for any large $n$, a set with $x > n^4(1 - \epsilon)$, for any $\epsilon > 0$. Singer's method is quite different from ours. His result shows that the above upper bound for $x$ is best possible, except perhaps for the error term $O(n^4)$.

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NOTE ON $H_2$ SUMMABILITY OF FOURIER SERIES

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1. Let $s_n(t)$ denote a partial sum of the Fourier series of an integrable function $f(t)$, periodic with period $2\pi$, and let $\phi(t) = \frac{1}{2} \{f(x+t) + f(x-t) - 2f(x)\}$. Recently I proved the following result of Hardy and Littlewood†:

If $\int_0^1 |\phi(u)| \left[1 + \log^+ |\phi(u)|\right] du = o(t)$ as $t \to 0$, then the Fourier series of $f(t)$ is summable $H_2$ to sum $s$ for $t = x$, i.e.

$$\sum_{r=0}^n |s_r(x) - s|^2 = o(n).$$

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