Assessment of Social Preference in Automotive Market using Generalized Multinomial Logistic Regression

by

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Abstract

Individual auto market share is always one of the major concerns of any auto manufacturing company. It indicates a lot of things about the company such as profitability, competitiveness, short term and long term development and so on. The focus of this paper is to construct a quantitative model that can precisely formulate the social welfare function of the auto market by relating the auto market share with the utilities of the significant vehicle-purchasing criteria (e.g. reliability, safety, etc.) that concern vehicle buyers. Social welfare function is defined as the additive form of the utility of each criterion considered, it's a good estimation of the customer preferences. The assessment methods used in this research include random utility theory and B-spline fitted logistic regression model. G-test is applied to select the criteria that is significant to the vehicle market social welfare, pseudo R-squareds are used as the model goodness-of-fit measures and Kendall rank correlation coefficient and Matthews correlation coefficient are applied to validate the assessment model. A case study using the U.S. auto market and vehicles related data collected in years of 2013 and 2014 are conducted to illustrate the assessment process of the social welfare function, and the data from 2015 are used to validate the assessment model.

Keywords: Market share prediction, Social welfare function, Random utility theory, B-spline fitted logistic regression, G-test, Pseudo R-squared, Kendall rank correlation coefficient, Matthews correlation coefficient.

Chapter 1. Introduction

The past decades have witnessed a steady decline in market share for the Big Three – GM, Ford, and Chrysler – which lost their dominance from 86% in 1970 to 45% in 2014 (Automotive-News (2014)). Meanwhile, other brands have expanded lineups and marketing. During the years 2002 – 2014 shown in Figure 1.1, GM slipped from 28.61% to 17.75%, Ford from 20.20% to 14.95%, Chrysler from 13.09% to 12.65%m, while Toyota, Nissan, and Hyundai all gained more than 4%. Train and Winston (2007) criticized the non-ideal performance of American brands for price, fuel economy, power, etc. Klier (2009) ascribed the losses of the Big Three to the emergence of government regulation for vehicle safety and emissions, together with the entry of foreign auto part and vehicle manufacturers since mid-1950s. Similarly, Consumer Report and J.D. Power Report gave the highest ratings to Asian and European vehicles for low-cost, high-quality and low fuel consumption (Auto-News (2014)). For example, Toyota Camry is the best seller of the passenger cars in North America in past few years, according to our data (shown in Figure 3.1), it has a dominance advantage over other vehicles on maintain and repair fee and comfort rating. Meanwhile, its advantages on price, acceleration ability, and yearly depreciation rate are also pretty obvious. Besides, Camry performs very well on fuel consumption and wheel base, which will be proved to be a representative of safety.

In this paper, we investigated the key attributes of well-sold vehicle brands, and assessed the social preference in the U.S. automotive market. In economics, a consumer's preference is characterized by an individual utility function (e.g. Akerlof and Kranton (2000); Akerlof and

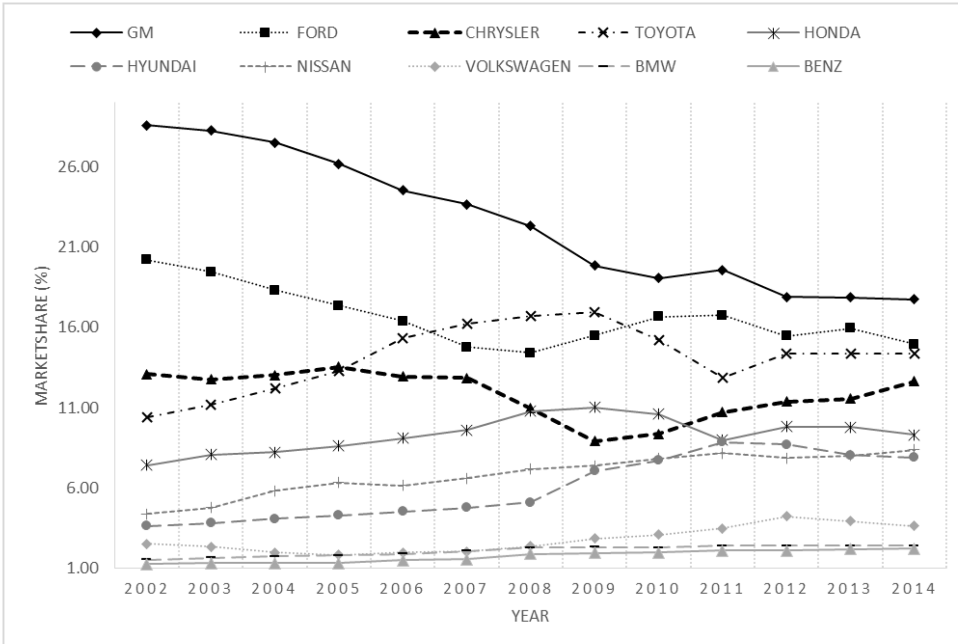


Figure 1.1 Historical Market Share for Top 10 Brands from 2002 to 2014

Kranton (2002); Johansson-Stenman and Martinsson (2006)). A social welfare function, aggregating each consumers' individual choices, represents a public preference (Arrow (1950); Goodman and Markowitz (1952); Kaneko and Nakamura (1979); Mitchell et al. (2015)). Indeed, the market share is a qualitative and quantitative outcome of the social preference. On the other hand, a social welfare function, if correctly elicited, provides an instructive explanation of the trend of market share changes.

Our research has three contributions. First, we specify the connection of the market share with the social preference. A statistical analysis based on the multi-attribute utility theory (MAUT) to assesses the effect of cost, vehicle performance, reliability, after-sales service and safety, and quantifies the relative importance among jointed attributes. Next, we develop a generalized multinomial logit regression method combining with the B-spline interpolation. As the results, this method generates a smooth utility curve for each key attribute, which virtualizes the level of sensitivity and significance of this attribute. Besides, based on the social welfare function in two consecutive years, we could predict the trend of the consumer preference on each vehicle. It means a lot for auto manufacturers, like get a better understanding of why they and their competitors sell more/less than last year, how could they improve their market share with less money, what attributes are consumers looking for etc.

This paper is organized as follows. First, we will review the literatures that are related to our research in the next chapter, including ideas, methods, and their contributions and drawbacks. Then, we give a description of the problem that we intended to solve and specify the attributes that will be considered in our research in Chapter 3. Next, we come up with a social welfare function, and based on random utility theory and B-spline fitted logistic regression model, together with the real market share to estimate the parameters raised in the B-spline model in Chapter 4. In this chapter, we also introduce some methods that used to assess and validate our model, like maximum likelihood estimation, G-test, pseudo R-squared, Kendall rank correlation coefficient and Matthews correlation coefficient. After that, we analyze the results of a real case based on the data we collected in terms of the passenger car sold in North America in 2013 and 2014 in Chapter 5. Based on the parameters estimated in our model, the data collected for 2015 are used to validate our model by applying the Kendall rank correlation coefficient and Matthews correlation coefficient statistics. Last, discussion of the results and contributions as well as the further researches and applications are introduced in the last chapter.

Chapter 2. Literature Review

Various studies have discussed about the changes of the market share of the “Big Three” in North America. Klier (2009) discussed how the Detroit automakers lost their dominance of the U.S. auto market. By analyzing the decline taking place in three distinct phases, the author concluded that the presence of foreign automakers, the price of gasoline, and the emergence of light trucks played major roles in this transition. Train and Winston (2007) found that the U.S. industry’s loss in share during the past decade could be explained almost entirely by relative changes in the most basic attributes of new vehicles, namely, price, size, power, operating cost, transmission type, reliability, and body type. Actually, all these changes influenced the consumers’ purchasing behaviors, which directly decided the market share. In addition, Aghdaie and Yousefi (2011) argued that the most important decision criteria for vehicle purchasers, include the technical performance, economic aspect, after sale services as well as its safety.

On the other hand, Agrawal and Schorling (1997) stated that in the traditional econometric modeling area, one technique which has emerged quite robust is the multinomial logit model (MNL) for the multi-choice problems. The MNL model has been shown to be more appropriate for modeling consumer’s probability of choice as a mix of continuous and discrete predictor variables compared to its rivals such as log liner, multiple regression, multiple discriminant models (Green et al. (1977); Gensch and Recker (1979); Malhotra (1984); Maddala (1986)). One widely recognized advantage of MNL is its ability to provide closed form solution for the choice probabilities in a competitive setting where marketing activities of all players are taken into consideration. The choice probability can be aggregated to yield estimates of brand shares for a particular marketing mix environment. The MNL model has been applied to many other researches areas related to the market share (Neagu and Hoerl (2005); Kleijnen et al. (2004); Barone et al. (2007); Sallis and Deo Sharma (2009); and Kirkos et al. (2010)).

One research that need to be specifically stressed here is the research on vehicle choice behavior and the declining market share of U.S. automakers conducted by Train and Winston (2007). In their research, they developed a consumer-level model of vehicle choice to shed light on the erosion of the US automobile manufacturers' market share during the past decades. The MNL was used to estimate the probability of an individual consumer to buy a particular vehicle. However, including this research, all the mentioned and other traditional researches using MNLs just modeled a linear combination of each predictor variable, which means the assessment is appropriate only if the contribution of the predictor variable is linear within the relevant range (Winkler and Murphy (1970)). Unfortunately, this is often not the case. Instead, the contribution of each predictor variable is often regarded as nonlinear (Berry (1994); Berry et al. (1995), Walker and Ben-Akiva (2002)), where linear contribution is only a special case of the nonlinear.

In addition, all the research efforts so far were based only on the individual customer behavior. In our research, we are trying to study the customer behavior as a whole in the perspective of social choice instead of individual choice. For social choice, the model given by Lu and Boutilier (2011) aimed computing the outcomes of different commonly studied voting rules. The model proposed by Soufiani et al. (2013) aimed at computing the MAP (Maximum a-posteriori) of GRUMs (general random utility model) that was developed from principles in Bayesian experimental design. In comparison, the model we proposed aims at computing the MLE (Maximum Likelihood Estimator) to obtain the social preference curve for each attribute.

A SWF is defined as a measure of group preferences (Arrow (1950); Goodman and Markowitz (1952); Kaneko and Nakamura (1979); Mitchell et al. (2015)). It holds the same property as utility: the higher the social welfare is, the more likely that the consumer will choose the corresponding products. Sanayei et al. (2008) provided an integrated approach of weighted utility additive method for rating and choosing the best supplier. Thus, we can define SWF as an additive form of all the utility function for each attribute.

In terms of how to simulate the utility function for each variable, Karande and Chakraborty (2015) provided a weighted utility additive method, an extension of utility additive approach, that was based on ordinal regression and consisted of a piecewise linear additive decision model from a preference structure using linear programming. However, this piecewise linear fitted utility function is not differentiable, which is a base requirement for social welfare function, especially when we want to look into the marginal social welfare in the future. A second order differentiable curve fitting method was proposed by Park and Lee (2007) by using a new approach of B-spline curve fitting to a set of ordered points.

Based on pros and cons of the research in the above reviewed literature, we intended to construct a general social welfare function for passenger cars, which is an additive form of the B-spline fitted utility function of each criterion we selected. In order to estimate the parameters defined in our B-spline fitted model, in our study we followed what Conitzer and Sandholm (2012), Procaccia et al. (2012), Roos et al. (2011), Xia and Conitzer (2011), Xia et al. (2010), and Conitzer et al. (2009) have done in their researches which was applying the Maximum Likelihood Estimation (MLE). In addition, the likelihood ratio test is applied for testing the significance of the overall model with selected criteria. (Hosmer Jr et al. (2013); Fienberg (2007)), which is known as G-test. Via G-test, only the criteria have significant influence on the likelihood ratio will be selected into the final model.

In traditional linear regression model, the coefficient of determination, R^2, is a statistical measure of how well the regression line approximates the real data points (Rao (2009)). However, for a generalized regression model besides linear model, it is not possible to compute a single R^2 statistic that functions as the regular R^2 in the linear model. Usually, pseudo R-squareds are used to measure the coefficient of determination. Among all the pseudo R-squareds, Cox and Snell's R^2 (Cox and Snell (1989)), Nagelkerke's R^2 (Nagelkerke (1991)), and McFadden's R^2 (McFadden (1973)) are the most commonly used as measures of coefficient of determination. In our research, all these three pseudo R-squareds are discussed and applied to test the significance of our model.

In terms of the trend of the market share changes, it is a summarize of the consumer preferences regarding to different attributes. According to our model, social welfare is a measure of consumer preferences, we can use the social welfare of two consecutive years to predict the market share change direction. In traditional pattern recognition with binary classification, precision and recall are used to evaluate the measurement. Recall (called Sensitivity in Psychology) reflects how many of the relevant cases your classification rule picks up, while Precision (called Confidence in Data Mining) donates the proportion of predicted positive cases that are correctly real positive (Matthews (1975)). However, both of them focus only on the positive predictions and neither of them captures any information about the negative cases. Although there is no perfect way of describing the confusion matrix by a single number, the Matthews correlation coefficient (MCC) is generally regarded as being one of the best such measures (Powers (2011)). Therefore, Matthews correlation coefficient is applied as another measurement of our model validations.

Considering the positive relationship between market share and social welfare, as long as we getting the social welfare and estimated market share (EMS), we could get the ranks for both market share changed and EMS changed for each vehicle. The Kendall rank correlation coefficient, known as Kendall's tau, is first introduced by Kendall (1948) to evaluate the degree of similarity between two sets of ranks given the same set of objective. It is used to measure the ordinal association between two observed quantities. Intuitively, the Kendall's tau will high when the observations have similar rank between two variables, and low when the observations have a dissimilar rank between two variables. Thus, we could use Kendall's tau to validate the goodness of our model regarding to the assessment of social welfare.

Chapter 3. Problem Description

The automobile market share reflects the social preference of vehicle choices, which represents customer attitudes toward capital expenditure, vehicle performance, safety consideration, reliability reputation, and after-sales service. In economics, the social preference is characterized by a social welfare function, which ranks social states as less desirable, more desirable, or indifferent for every possible pair of social states. In our research. our primary motivation is to formulate a social welfare function that can rank the consumers' preference for all the passenger vehicles in the U.S. auto market. Furthermore, as long as the social welfare function is obtained, the simulation of the market share change tendency could be fulfilled by analyzing the consumers' preference in two consecutive years.

3.1 Attributes Definition

In order to construct one precise and appropriate model, the attributes selection is of great importance. The attributes considered in the model should have certain significant influence on the decision of consumer buying behavior, like capital expenditure, vehicle performance, safety consideration and so on. As our model is based on the social level instead of individual consumer level, all the attributes of individual consumer are not considered in our research. Taking the previous research results (Aghdaie and Yousefi (2011); Vrkljan and Anaby (2011); Train and Winston (2007)) and data availability into consideration, we defined 11 attributes that covering cost, performance, reliability, consumer service level and safety, which will be discussed in the following paragraphs

3.1.1 Cost

Sale price, estimated insurance fees in five years, fuel economy (MPG: miles travelled per gallon), yearly depreciation rate (DR), and estimated maintain and repair fees in five years are all have the property of cost. Sales price is very intuitively and the primary consideration of most consumers because a large part of their cost is due to how much they paying for purchasing. Insurance fees is the estimated amount a consumer paying for insurance in the first 5 years after purchasing. Fuel economy not only have the property of cost but also can be viewed as a measure of performance. At some occasions, it is refereed as a measure of how much the consumer would pay to travel a specific distance when talking about cost. However, on the other hand, it is also a measure of the performance regarding to the relationship between the distance traveled and the amount of fuel consumed by the vehicle to some degree. Assumed that a vehicle is still under good condition after 10 years of using with a mileage of 110 thousand miles, DR is defined as yearly depreciation rate based on the sales price and trade-in price. DR has the property of cost, performance and reliability. Usually, when talking about cost and performance, it indicates the potential remaining value of a vehicle in the future and how the vehicle is performed, the lower the depreciation rate is, the higher the resale value and better performance will be. Actually, it can also be considered as one of the indications of the reliability reputation of a vehicle to some degree, only if your vehicle brand and model is reliable reputed, the depreciation rate of your vehicle could remain a low value. That is why the second hand vehicles from Asian and Europe could be sold at a better price than American vehicles when the vehicle condition is the same. Maintain and repair fees is a measure of the anticipated cost on maintain and repair in the first five years after purchasing. It belongs to cost, reliability and consumer service level at the same time, because if the manufacturer thought their products are reliable enough, they would like to offer more warranty for their products after selling, thus a better experience of consumer service and less expense on maintain and repair are very likely to be happened.

3.1.2 Performance

Besides fuel economy and DR, acceleration ability, comfort rating and storage are part of performance as well. Acceleration ability is measured by the time a vehicle consumed from 0 to 60 miles, it represents the horsepower of a vehicle, an attribute which almost all consumer will consider about. Comfort rating is a 5-scale rate of the driver and passenger experience on comfort, it is a measure of the vehicle design regarding to space and materials. Storage is the space of the trunk in cubic feet, it is related to the size of the vehicle sometimes, it measures the capability of carrying luggage of a vehicle.

3.1.3 Reliability

Reliability only have two components in our research, DR and maintain and repair fees. Both of them could be at a relatively low level if the corresponding vehicle is reliable enough.

3.1.4 Consumer Service Level

Besides maintain and repair fees, dealership is another measure of consumer service level. Dealership is represented by the number of dealers in North America, it directly influences the consumer satisfaction and consumer experience. When you have more dealers, it means your consumers are much easier and more convenient to get serviced, furthermore more time and money would like to be saved, thus a better service level is achieved.

3.1.5 Safety

Wheelbase and weight are measuring the safety of a vehicle. In terms of wheelbase, Subramanian (2006), a staff from National Highway Traffic Safety Administration (NHTSA), examined the occupant fatality rates by vehicle type and size and found that in all fatal crashes, when broken down by size, compact scars have the highest occupant fatality rate while the "large vans" category has the lowest occupant fatality rate. The standard he used to classify the size of the passenger cars is wheelbase. On the other hand, based on the physics of car crashes results, Insurance Institute for Highway Safety (IIHS) argued that most of the very small cars generally can't protect people in crashes as how bigger and heavier models do (IIHS (2009)). Thus, in our research, wheelbase and weight are used as indications of the safety of a passenger car.

All these attributes and categories could be summarized in a table as shown in Table 3.1. As long as the attributes are selected, constructing an appropriate model that could connect these attributes and consumer purchasing behavior is a desiderate task at present. Train and Winston (2007) in their research used the logit function to estimate the probability of an individual consumer to buy a particular vehicle. Together with the social welfare function developed for the U.S. auto market, in this paper NML is used to estimate the probability of the whole society buying a particular vehicle. This estimated probability is defined as the estimated market share (EMS) of that particular vehicle in this paper. With the collected real market share data for each vehicle brand and model, we then applied maximum likelihood to estimate the parameters in the social welfare function we defined.

3.2 Data Collection

Based on our extensive literature review, we identify the 11 attributes that were considered to be significant factors for vehicle purchasing in other researches which has been discussed above. Along with the years of market share data (2002 - 2014), we collected the 12 attributes relevant data for 2013, 2014 and 2015 via different kinds of sources. These data are mainly collected from following websites, www.consumerreports.org, www.kbb.com, www.goodcarbadcar.net,

Table 3.1 Summary of Attributes Properties

Attributes	Cost	Performance	Reliability	Consumer Service Level	Safety
Price	Y				
Insurance Fees	Y				
Fuel Economy	Y	Y			
Yearly Depreciation Rate	Y	Y	Y		
Maintain & Repair Fees	Y		Y	Y	
Acceleration Ability		Y			
Comfort Rating		Y			
Storage		Y			
Dealership				Y	
Wheelbase					Y
Weight					Y

www.nada.com, www.fueleconomy.gov and www.edmunds.com.

3.3 Attributes Selection and Analysis

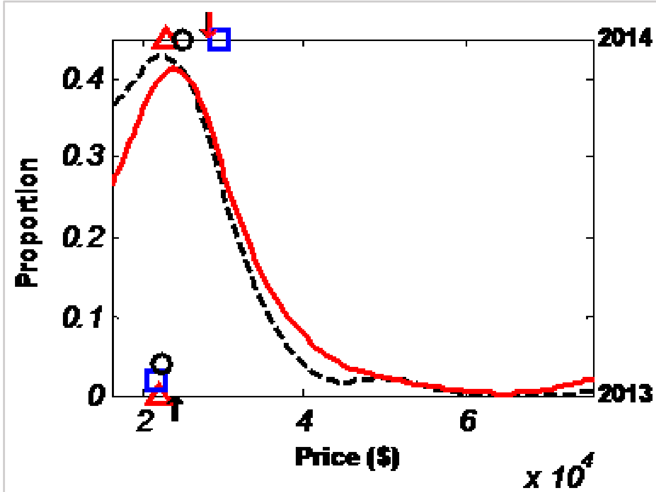
Previous research and experience showed that a lot of criteria could influence vehicle sales and further the market shares in the auto market. However, in our model, just part of them will be selected. In this research, we first come up with a generalized multinomial logistic regression model to assess the social welfare function of each vehicle by utilizing the knowledge of B-spline, discrete choice model and maximum likelihood, based on this model, G-test is applied as the model of goodness-of-fit to select the significant attributes. The criteria proved to be significantly important by the G-test are wheelbase, dealership, acceleration ability, comfort rating, fuel economy, sale price, yearly depreciation rate, and maintenance and repair fees.

It is obvious that sales price, fuel economy, acceleration ability, comfort rating, and number of dealerships are the core roles in influencing the cost, performance and consumer service level, as what has been mentioned in previous researches (Aghdaie and Yousefi (2011); Vrkljan and Anaby (2011); Train and Winston (2007)). The rest three criteria, wheelbase, DR and maintenance and repair fees are also of the great importance in influencing consumers' purchasing behaviors, although they have seldom been mentioned in previous research. Wheelbase is the representative of safety, DR and maintain and repair fees have the property of cost and reliability at the same time. Besides, DR is also a measure of performance.

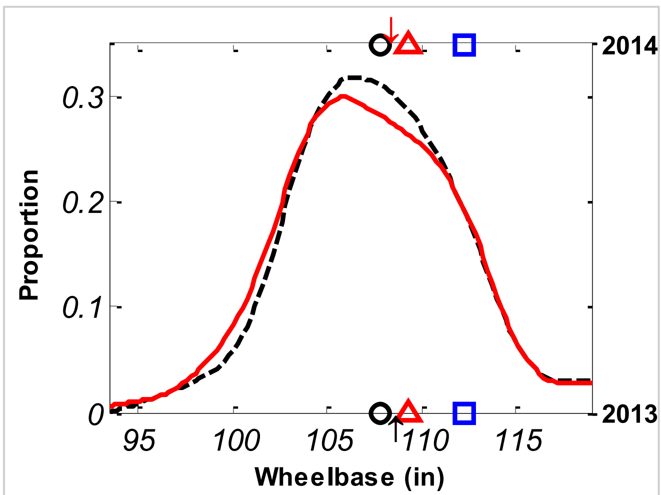
Figure 3.1 shows how the proportions are distributed with these important criteria that are incorporated in our model, and how the three sample vehicles (Ford Fusion, Chevrolet Malibu, and Toyota Camry) are located in the criteria span together with the attribute mean level in 2013 and 2014. With regards to the detailed attributes proportion distribution, the detailed analysis is discussed below:

Figure 3.1 (a) indicates that the prices of most vehicles are located between 18,000 and 40,000 dollars and all the three sample vehicles are located in this range. However, around 5% of the consumers show higher interest for vehicles whose price range from 40,000 dollars to 60,000 dollars than those whose price lower than 40,000 dollars. Actually, most of the vehicles whose price located in this range are luxury passenger cars, like Audi A6, Lexus GS, Lincoln MKS, Cadillac CTS and so on. Thus the bulge between 40,000 and 60,000 dollars could be explained by the fact that price is not the primary consideration for consumers who are intended to buy a luxury car with price less than 60,000 dollars.

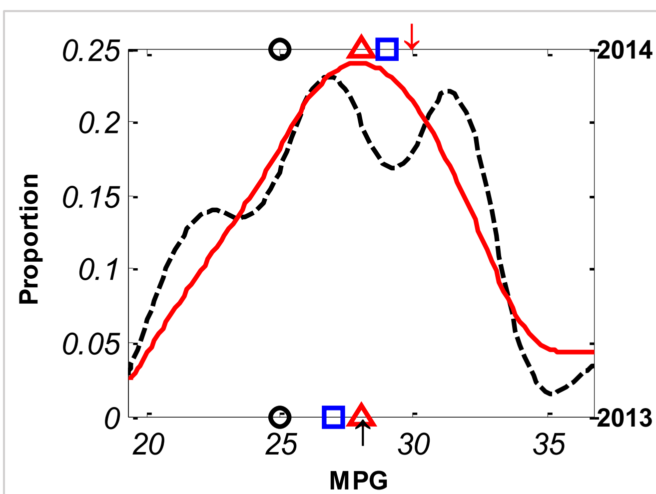
Wheelbase as a measure of safety, Figure 3.1 (b) shows that the proportion distribution of wheelbase is centralized between 100 inches and 115 inches. Over 95% of the vehicles sold including the three sample cars are compact or larger cars (wheelbase greater than 100 inches), there are only less than 5% of the vehicles sold are subcompact cars, which has been proved to be the riskiest group by Subramanian (2006) when broken down by size. This phenomenon also



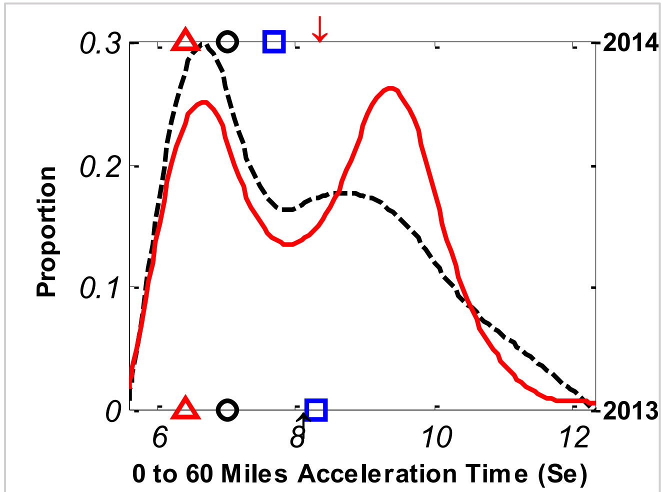
(a) Price



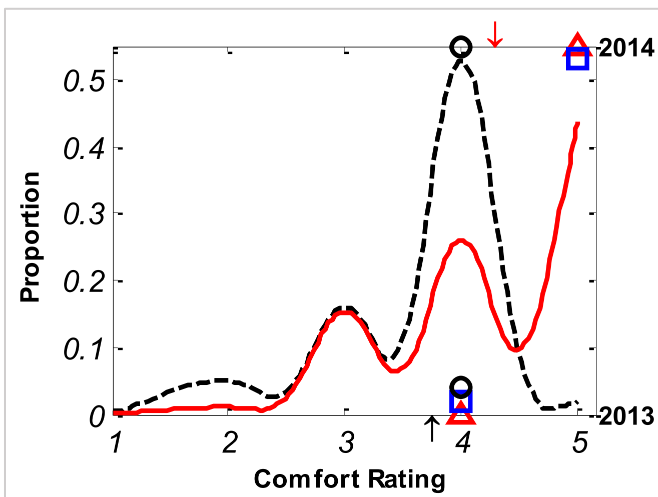
(b) Vehicle Size



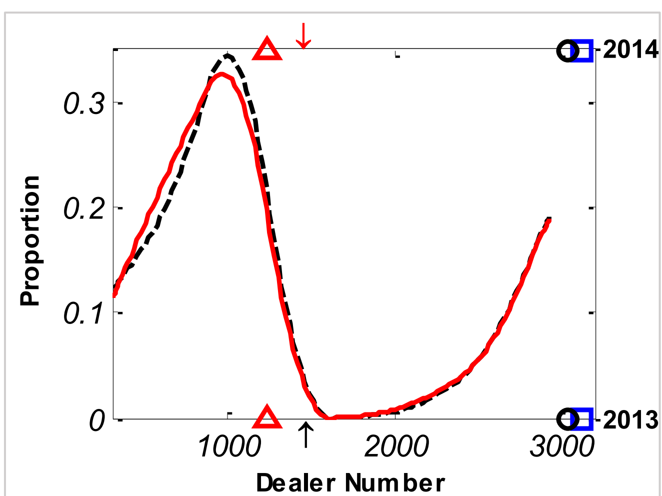
(c) Fuel consumption



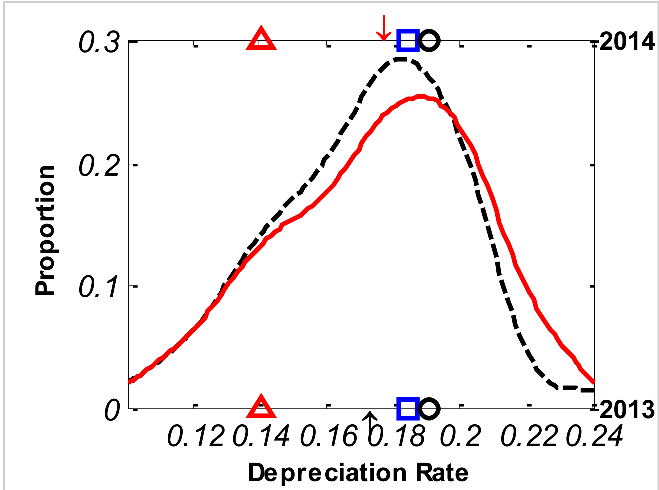
(d) Acceleration



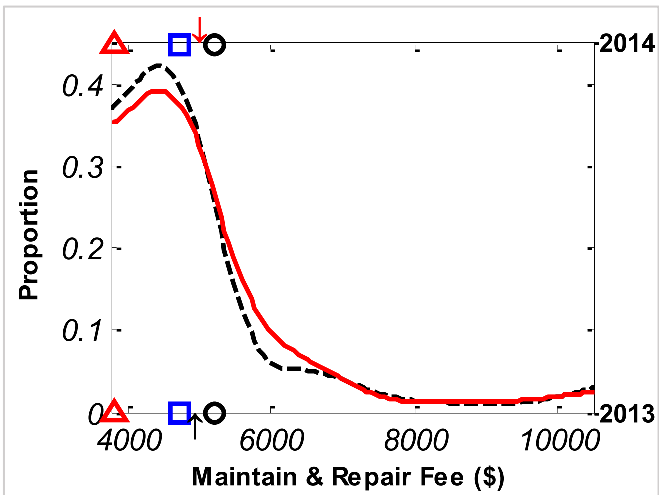
(e) Comfort



(f) Dealership



(g) Yearly Depreciation Rate



(h) Maintain and Repair Fee

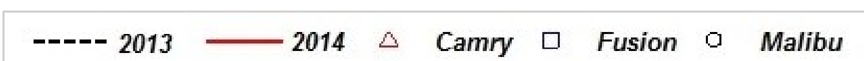


Figure 3.1 Proportion Distribution for 2013 and 2014 and Samples Comparison

implies that consumers are more cautious when choosing subcompact vehicles comparing to choosing some larger size vehicles.

Based on Figure 3.1 (c), compared with 2013, the overall MPG is increased in 2014, there are obvious proportion increases for range from 23 to 31 and over 34. The increase between 23 and 31 might due to the high gasoline price in 2013, when design for the new vehicle for 2014, engineers would pay more attention to the fuel consumption part and higher MPG vehicles are very likely to be designed. However, the increasing for those over 34 is mainly attributed by the mature of hybrid technology, because most of the vehicles who have such a high MPG are Hybrid vehicles.

The proportion distribution induced by acceleration time is shown in Figure 3.1 (d, compared to 2013, more vehicles could accelerate to 60 miles between 9 and 10 seconds and less vehicles between 6 and 9 seconds in 2014. Actually, the changes of acceleration ability and MPG are related. This is because higher MPG usually are based on the sacrificing of horsepower and acceleration time, this explains why acceleration time increased when more vehicles sold due to better performance in fuel economy.

Figure 3.1 (e) shows that there were more vehicles located at higher comfort rating (i.e. 4 and 5) and a lot of vehicles located at 4 in 2013 increased its rating to 5 in 2014. This indicates that consumers and manufacturers are paying more attention to comfort experiences.

In terms of number of dealerships, Figure 3.1 (f) shows that most vehicle sold have around 1000 dealerships but some vehicle sold have dealerships around 3000. The large difference is due to differences of the nationality of the vehicle brand, domestic brands with no doubt have a relatively larger number of dealers, like Ford, Chevrolet. However, for foreign brands, like Toyota, Honda, they usually have a dealership around 1000.

Figure 3.1 (g) shows how that there are slightly more vehicles with DR below 20% in 2013. This could be explained by the improvement of the average income of potential buyers in 2014. When the economy is improved, consumers will prefer a new vehicle to a used one. Less demand for used vehicles will further harm the resale price of used vehicles, which will directly increase the DR of a vehicle. An interesting phenomenon is that the vehicles with DR less than 0.15 are mainly from Japan, this is in accordance with our common sense that Japanese cars are more reliable and second-hand vehicles could be sold at a better price.

As shown in Figure 3.1 (h), over 85% of the vehicles sold including the three sample vehicles have a maintain and repair fees less than 6,000 dollars.

Among the three samples, Toyota Camry is the best seller of the passenger cars in 2013 and 2014. From the perspective of cost, Camry has a lower price than average, its maintenance and repair fee is even the lowest of all. In terms of performance, its acceleration ability and comfort rating are much better than the average, its fuel consumption is also around the mean level when a lot of hybrid vehicles are considered. As to safety, wheelbase is referred as an important criterion to represent safety factors, Camry is still performing better than average level. With regard to reliability, Camry holds a pretty low yearly depreciation rate which is much lower than most of the passenger vehicles. As Toyota is a Japanese brand, it has a relatively fewer dealer in America, but it still very close to the average numbers. All in all, though Toyota Camry somehow has some disadvantages in consumer service part, it performs very well in cost, performance, safety and reliability perspectives, it inevitably turned out that Toyota Camry is the best seller in both 2013 and 2014.

On the other hand, as for Fort Fusion and Chevrolet Malibu, another two vehicles we selected as samples to demonstrate our model, based on their performance in these criteria, it is not hard to infer that which is the best seller and which is the worst seller of the three. For cost, both of them are higher than Camry, Malibu performs better than Fusion in price but worse in maintain and repair fee. In terms of performance, Camry still performs the best of three, Malibu is better than Fusion in acceleration ability however Fusion does a better job in comfort rating and fuel consumption part. In safety part, Fusion is the best of the three and Malibu performs the worst. In the perspective of yearly depreciation rate, Fusion does better than Malibu but not as well as Camry. Of course, both Fusion and Malibu are better in dealer numbers because they domestic brand, but Fusion is even better than Malibu. According to these comparison, we could come the conclusion that Fusion would definitely sell better than Malibu but not as well as Camry in 2013 and 2014, and the reality turned out to be the same.

Chapter 4. Methodology

4.1 Glossary

$\boldsymbol{I} = \{1,2, \dots, I\}$ the index vector of attributes

$\boldsymbol{M} = \{1,2, \dots, M\}$ the index vector of year

$\boldsymbol{J_m} = \{1_m, 2_m \dots, J_m\}$ the index vector of alternative vehicles in year m

$x_i^{j,m}$ the i^{th} attribute of the j^{th} vehicle in year m

$\boldsymbol{X^{j,m}} = \{x_1^{j,m}, x_2^{j,m}, \dots, x_I^{j,m}\}$ the vector of all the attributes of the j^{th} vehicle in year m

$u_i(x_i^{j,m})$ the utility of the i^{th} attribute of the j^{th} vehicle in year m

$U^{j,m}(\boldsymbol{X^{j,m}})$ the adaptive utility of the j^{th} vehicle in year m

$S^{j,m}(\boldsymbol{X^{j,m}})$ the social welfare function of the j^{th} vehicle in year m

a_i the scale parameter of the i^{th} attribute

β_i the weight of the i^{th} attribute

$S^{j,m}$ the social welfare of the j^{th} vehicle in year m

$\epsilon^{j,m}$ unobserved component of $S^{j,m}$

$U^{rest,m}$ the estimation of the social welfare for all the rest vehicles in year m that are not included in the J_m vehicle

$S^{rest,m}$ the social welfare of the rest vehicles

$\epsilon^{rest,m}$ unobserved component of $S^{rest,m}$

$P^{l,m}$ the probability of choosing vehicle l in year m

$\boldsymbol{C_i} = \{c_{i,1}, c_{i,2}, \dots, c_{i,n}\}$ the index vector of the control points in terms of the i^{th} attribute

$R^{j,m}$ the market share of the j^{th} vehicle in year m

$R^{rest,m}$ the market share of the rest vehicles in year m

4.2 Introduction

This section introduces all the methodologies applied in our model and it is organized as follows. We firstly state the definition of social welfare function (SWF) we adopt in this study. Then we introduce how B-spline is used to construct the utility function of each attribute and to define the parameters in the utility functions. After that we define the relationship between SWF and real market share. In addition, this section also provides the solution to work out the parameters defined in B-spline. Last we introduce the method used to test the goodness of fit for the SWF model and provide two methods to validate the SWF model.

4.3 Social Welfare Function (SWF)

Social welfare is defined as a measure of group preferences (Arrow (1950); Goodman and Markowitz (1952); Kaneko and Nakamura (1979); Mitchell et al. (2015)). A SWF can be constructed to measure the preferences of consumers in North America market for different types of vehicles. In addition, an integrated approach of weighted utility additive method was applied to rate and choose the best supplier by Sanayei et al. (2008). Thus, we can define SWF as an additive form of all the social welfare induced by each attribute.

In year m, J_m is the number of vehicles to be considered, the SWF of the j^{th} vehicle in year m can be defined as

$$S^{j,m}\,(\boldsymbol{X^{j,m}}) = U^{j,m}\,(\boldsymbol{X^{j,m}}) + \epsilon^{j,m} \tag{1}$$

where $\epsilon^{j,m}$, a random variable, is an error terms, which includes the impact of all the unobserved attributes on the social welfare of $S^{j,m}$. As $U^{j,m}$ is the additive utility of each attributes, thus

$$S^{j,m}\left(\boldsymbol{X^{j,m}}\right) = \sum_{i\in \boldsymbol{I}} a_i\, \beta_i\, u_i\left(x_i^{j,m}\right) + \epsilon^{j,m} \tag{2}$$

where $a_i > 1$, $\beta_i \in [0,1]$, $\sum_{i\in I} \beta_i = 1$, and $u_i\left(x_i^{j,m}\right) \in [0,1]$. As both a_i and β_i are used to differentiate the importance of each attribute, in order to simplify our model, we assume that all the a_i are equal to each other. Thus, (1) can be written as

$$S^{j,m}\left(\boldsymbol{X^{j,m}}\right) = a \sum_{i\in \boldsymbol{I}} \beta_i\, u_i\left(x_i^{j,m}\right) + \epsilon^{j,m} \tag{3}$$

Due to the availability of data collection, we cannot consider all the passenger vehicles in each year. We use $S^{rest,m} = U^{rest,m} + \epsilon^{rest,m}$ to represent the social welfare of the rest vehicles that do not included in the J_m vehicles.

Taking all the vehicles in year m into consideration, vehicle l will be chosen if and only if $\forall j \neq l$, $S^{l,m} > S^{j,m}$ and $S^{l,m} > S^{rest,m}$. The general expression of the probability of choosing vehicle l in year m is then

$$P^{l,m} = P(S^{l,m} > S^{1,m}, \dots, S^{l,m} > S^{J,m}, S^{l,m} > S^{rest,m}) \tag{4}$$

To simplify matters more, researchers often use the following assumption for the distribution of the error terms: error terms are assumed to be independently and identically distributed (IID) following the double exponential (Gumbel Type II extreme value) distribution (DDWiki (2010)). Based on the theory of MNL, the logit probability of purchasing vehicle l in year m is given by

$$P^{l,m} = \frac{e^{U^{l,m}}}{\sum_j e^{U^{j,m}} + e^{U^{rest,m}}} = \frac{e^{a \sum_{i\in I} \beta_i u_i\left(x_i^{l,m}\right)}}{\sum_j e^{a \sum_{i\in I} \beta_i u_i\left(x_i^{j,m}\right)} + e^{U^{rest,m}}} \tag{5}$$

According to (5), the ratio of the probability of choosing any two vehicles is

$$Ratio = \frac{P^{vehicle1,m}}{P^{vehicel2,m}} = \frac{e^{a \sum_{i\in I} \beta_i u_i\left(x^{vehicle1,m}\right)}}{e^{a \sum_{i\in I} \beta_i u_i\left(x^{vehicle2,m}\right)}} \tag{6}$$

and the range of the ratio is (e^{-a}, e^{a}). If we do not introduce the scale parameter a in (3), in other words, $a = 1$, the range of the ratio will be $(e^{-1}, e^{1}) = (0.37, 2.72)$. In reality, the range of the ratio is much larger than $(0.37, 2.72)$, theoretically, the range of the ration should be $(-\infty, +\infty)$. Thus, it is reasonable and necessary to include a scale parameter $a > 1$ in our model.

4.4 B-spline

Based on (3), a better estimation of a, β_i and $u_i(x_i^{j,m})$ are critical if a more accurate SWF is desired. In order to give better estimations of scale parameter, individual social welfare and its weight of each attribute, an adequate model is required to approximate the real individual social welfare. However, in terms of how to simulate the individual social welfare for each variable, it is not an easy job. The traditional simple linear regression model is not enough anymore. Karande and Chakraborty (2015) provided a weighted utility additive method, an extension of utility additive approach, based on ordinal regression and it consists of building a piecewise linear additive decision model from a preference structure using linear programming.

However, the piecewise linear fitted function is not differentiable. It misses some properties of the individual social welfare, like the marginal social welfare for each attribute. A second order differentiable curve fitting method was proposed by Park and Lee (2007) by using a new approach of B-spline curve fitting to a set of ordered points. Taking the non-decreasing (or non-increasing) and differentiable properties of social welfare into consideration, a B-spline curve fitting model could be a better estimator.

Let's take $u_i(x_i^{j,m})$, the utility function of the i^{th} attribute of the j^{th} vehicle in year m, as an example to illustrate this model. Suppose we have n control points for the i^{th} attribute, $\boldsymbol{C_i} = \{c_{i,1}, c_{i,2}, \dots, c_{i,n}\}$. Considering the property of individual social welfare, the set of control points must be non-decreasing or non-increasing. For some attributes, like MPG, comfort rating, wheelbase and number of dealerships, the bigger the measurement is, the higher the individual social welfare is. The control points for these attributes should be non-decreasing. On the other hand, other attributes like price, acceleration time, depreciation rate and maintain and repair fee, the control points should be non-increasing. In addition, considering the normalization, the maximum control point for each attribute, $c_{i,max} = Max\{c_{i,1}, c_{i,2}, \dots, c_{i,n}\}$, must be equal to 1. With a degree of d, suppose we have $n + d + 1$ non-decreasing knots, $\boldsymbol{X_i} = \{x_{i,1}, x_{i,2}, \dots, x_{i,n+d+1}\}$, for the i^{th} attribute, base on the theory of B-spline, the utility of $x_i^{j,m}$ is given by

$$u_i\left(x_i^{j,m}\right) = \sum_{k=1}^{n} c_{i,k} B_{k,d}(x_i^{j,m}) \tag{7}$$

where $B_{k,d}(x_i^{j,m})$ is given by the recurrence relation

$$B_{k,d}\left(x_i^{j,m}\right) = \frac{x_i^{j,m} - x_{i,k}}{x_{i,k+d} - x_{i,k}} B_{k,d-1}\left(x_i^{j,m}\right) + \frac{x_{i,k+1+d} - x_i^{j,m}}{x_{i,k+1+d} - x_{i,k+1}} B_{k+1,d-1}\left(x_i^{j,m}\right) \tag{8}$$

and the function $B_{k,0}\left(x_i^{j,m}\right)$ is given by

$$B_{k,0}\left(x_i^{j,m}\right) = \begin{cases} 1, & x_{i,k} \le x_i^{j,m} \le x_{i,k+1} \\ 0, & otherwise \end{cases} \tag{9}$$

According to (8) and (9), once we know the $n + d + 1$ non-decreasing knots $\boldsymbol{X_i}$, the only variable in (7) is $c_{i,k}$. That is to say, we could estimate the SWF of year m, as long as we find the control points $\boldsymbol{C_i}$ and the corresponding weight parameters β_i for all $\boldsymbol{I}$. Thus, (5) can be simplified as

$$P^{l,m} = \frac{e^{\sum_{i \in I} \sum_{k=1}^{n} \alpha_{i,k} B_{k,d}\left(x_i^{l,m}\right)}}{\sum_j e^{\sum_{i \in I} \sum_{k=1}^{n} \alpha_{i,k} B_{k,d}\left(x_i^{j,m}\right)} + e^{U^{rest,m}}} \tag{10}$$

where $\alpha_{i,k} = a\beta_i c_{i,k}$ for all $\boldsymbol{I}$.

Of course, the probability of purchasing any vehicle that is not included in the vehicles in year m can be calculated by

$$P^{rest,m} = \frac{e^{U^{rest,m}}}{\sum_j e^{\sum_{i\in I}\sum_{k=1}^{n}\alpha_{i,k}B_{k,d}\left(x_i^{j,m}\right)} + e^{U^{rest,m}}} \tag{11}$$

4.5 Model Assessment

4.5.1 Parameters Estimation

To estimate model parameters ($\alpha_{i,k}$ and $U^{rest,m}$) that would help the predictions of the consumers preference that best match observed data (real market share), we collected data of real market share that is donated by $R^{j,m}$ for all $\boldsymbol{J_m}$ and all $\boldsymbol{M}$ and $R^{rest,m}$, the total market share of the rest vehicles in year m. Based on the maximum log likelihood theory (Hosmer Jr et al. (2013)), the parameters, $\alpha_{i,k}$ and $U^{rest,m}$, are therefore given by

$$\hat{\alpha}_{i,k}\,, \hat{U}^{rest,m} = argmax(\sum_j R^{j,m} logP^{j,m} + R^{rest,m} logP^{rest,m}) \tag{12}$$

According to $\hat{\alpha}_{i,k}$ obtained from (12) , the social welfare for each vehicle in each year could be estimated by (10).

4.5.2 G-Test

Likelihood ratio test was used by Hosmer Jr et al. (2013) and Fienberg (2007) to test the overall significance of the parameters for the independent variables in their models. The test was based on the statistic "G", and the null hypothesis of their test was that the coefficients for the covariates in the model were equal to zero. The G statistic is given by

$$G = -2\ln[\frac{\hat{L}(M_{-variable})}{\hat{L}(M_{variable})}] \tag{13}$$

where$\hat{L}(M_{-variable})$ stands for estimated likelihood without the variable and $\hat{L}(M_{variable})$ stands for estimated likelihood with the variable. The distribution of “G” is a chi-square with q degree of freedom, where q is the number of variables in the logistic regression equation.

In our model, for each attribute, there are more than one variables related to it. Actually, the number of variables of each attribute is equal to the number of control points related to the attribute. Thus, we cannot directly eliminate one variable from the model each time. Instead, we should eliminate all the variables related to the attribute that we are interested in. Therefore, the test statistic "G" is modified to

$$G = -2\ln[\frac{\hat{L}(M_{-attribute})}{\hat{L}(M_{attribute})}] \tag{14}$$

where $\hat{L}(M_{-attribute})$ stands for estimated likelihood without the attribute related coefficients and $\hat{L}(M_{attribute})$ stands for estimated likelihood with the attribute related coefficients. Here, the null hypothesis is that all the coefficients for the attribute in the model are equal to zero. If the test statistic "G" is significant, it indicates that the attribute is helpful in measuring the SWF. Otherwise, we can ignore this attribute without influencing our model accuracy too much.

4.5.3 Pseudo R-squareds

Cox and Snell's R^2 is a comparison of the log likelihood for the full model with the log likelihood for a baseline model (Cox and Snell (1989)). The ratio of the likelihood reflects the improvement of the full model over the baseline model. The Cox and Snell's R^2 is given by formula,

$$R^2 = 1 - \left[\frac{\hat{L}(M_{Intercept})}{\hat{L}(M_{Full})}\right]^{2/N} \tag{15}$$

where $\hat{L}(M_{intercept})$ is the estimated likelihood for the model without predictors, $\hat{L}(M_{Full})$ is the estimated likelihood for the model with predictors and N is the number of observations in the model. However, with categorical outcomes, it has a theoretical maximum value of less than 1, even for a "perfect" model with $R^2 = 1 - \hat{L}\left(M_{Intercept}\right)^{2/N}$.

Nagelkerke's R^2 is an adjusted version of the Cox & Snell R^2 that adjusts the scale of the statistic to cover the full range from 0 to 1 (Nagelkerke (1991)). It is given by the formula,

$$R^2 = \frac{1-\left[\frac{\hat{L}(M_{Intercept})}{\hat{L}(M_{Full})}\right]^{2/N}}{1-\hat{L}(M_{Intercept})^{2/N}} \tag{16}$$

McFadden's R^2 is another version, the likelihood of the intercept model is treated as a total sum of squares and the likelihood of the full model is treated as the sum of squared errors (McFadden (1973)). Thus, formula for McFadden's R^2 is very similar to traditional R^2,

$$R^2 = 1 - \frac{\ln \hat{L}(M_{Full})}{\ln \hat{L}(M_{Intercept})} \tag{17}$$

For both Cox and Snell's R^2 and Nagelkerke's R^2, a value close to 1 is expected to demonstrate the strength of association. However, for McFadden's R^2, this value tends to be smaller than traditional R^2 and values between 0.2 and 0.4 are enough to be considered as highly satisfactory.

4.5.4 Matthews Correlation Coefficient (MCC)

MCC is introduced by Matthews (1975), it is calculated based on the confusion matrix using formula

$$MCC = \frac{TP*TN-FP*FN}{\sqrt{(TP+FP)(TP+FN)(TN+FP)(TN+FN)}} \tag{18}$$

where TP is the number of true positives, TN the number of true negatives, FP the number of false positives and FN the number of false negatives. As a correlation coefficient, the MCC is the geometric mean of the regression coefficients of the problem and its dual.

4.5.5 Kendall Rank Correlation Coefficient

Assumed that $(a_1, b_1), (a_2, b_2), \dots, (a_n, b_n)$ are a set of observations joint from two rules, $\boldsymbol{a}$ and $\boldsymbol{b}$. $\forall i \neq j$, if both $a_i > a_j$ and $b_i > b_j$, or if both $a_i < a_j$ and $b_i < b_j$, the pair of observations (a_i, b_i) and $\left(a_j, b_{j2}\right)$ is defined as concordant. If $a_i > a_j$ and $b_i < b_j$, or if $a_i < a_j$ and $b_i > b_j$, the pair is defined as discordant. Otherwise, the pair is neither concordant nor discordant (Kendall (1948)). The Kendall's tau is defined as

$$\tau = \frac{n_c - n_d}{n(n-1)/2} \tag{19}$$

where n_c stands for the number of concordant pairs and n_d stands for number of discordant pairs. The range of the coefficient is between -1 and 1, where 1 means the two ranks are the same and -1 means one rank is the reverse of the other one. If $\boldsymbol{a}$ and $\boldsymbol{b}$ are independent, the coefficient would be approximately 0.

Significant test can also be applied to this data sets to test whether the value of τ is obtained by chance, it is given by the formula

$$Z = \frac{3(n_c - n_d)}{\sqrt{n(n-1)(2n+5)/2}} \tag{20}$$

Chapter 5. Results of Case Study

In this section we address the results of our model, including the G-test results and social welfare function we obtained. In order to make simplify our calculation, we set the number of control points $n = 5$, and degree $d = 2$. The results of G-test decide the attributes that will be considered in our final model. Based on the selected attributes, we construct a SWF for each vehicle and discuss the social welfare plot of each attribute. In addition, the three sample vehicles, Ford Fusion, Toyota Camry and Chevrolet Malibu, will be used to discuss the changes of their market share from 2013 to 2014 based on the social welfare we obtained. Furthermore, the data of 2015 is used to validate our model by applying MCC and Kendall's tau statistics.

5.1 Model Assessment

The $-2\log(likelihood)$ for the constant only model obtain by fitting the constant only model was 1172.459; and the $-2\log(likelihood)$ for the overall model was 912.989. Thus the value of the likelihood ratio test is

$$G = 1172.459 - 912.989 = 259.470$$

And the p-value for the test is $P[\chi^2(55) > 259.470] \cong 0.000$, which is highly significant at $\alpha = 0.1$ level. Thus we can conclude that at least one attribute should be included in our model. The likelihood ratio tests for all attributes and for each attribute are given in Table 5.1. From Table 5.1 we note that the G-tests relate to attributes of price, fuel consumption, wheelbase, acceleration, dealership, depreciation rate and maintain and repair fee are statistically significant. While the G-tests relate to attributes of weight, storage and insurance fee are not statistically significant at

Table 5.1 G-test Results

Model	-2log(likelihood)	G	q	P-value
Model with all attributes (full model)	912.989			
Model with constant only	1172.459	259.470	55	<0.001
Model without price	961.890	48.901	5	<0.001
Model without fuel consumption	1029.612	116.623	5	<0.001
Model without weight	917.173	4.184	5	0.523
Model without wheelbase	968.275	55.286	5	<0.001
Model without acceleration	956.973	43.984	5	<0.001
Model without comfort	923.179	10.190	5	0.070
Model without storage	918.240	5.251	5	0.386
Model without dealership	928.220	15.231	5	0.009
Model without depreciation rate	931.616	18.627	5	0.002
Model without insurance fee	914.078	1.089	5	0.955
Model without maintenance and repair fee	929.672	16.683	5	0.005

$\alpha = 0.1$ level. In addition, the p-value of G statistic for comfort is 0.070, which is also significant at $\alpha = 0.1$ level. Thus, there are eight attributes that will be included in our final social welfare function model. They are price, fuel consumption, wheelbase, acceleration, dealership, depreciation rate, comfort and maintain and repair fee.

Include all these eight attributes into our model, we could get another log likelihood, $-2\log(likelihood) = 919.475$, which is the maximized likelihood of our final model. Thus, the results of the pseudo R-squareds could be calculated, which are given in Table 5.2. As we mentioned before, for Cox and Snell R^2 and Nagelkerke R^2, a value close to 1 is better. On the other hand, for McFadden R^2, a value between 0.2 and 0.4 are large enough to be regarded as highly satisfied. Actually, we can find out that all the pseudo R-squareds are large enough to demonstrate that our model is a good measurement of the social welfare.

Table 5.2 Results for Pseudo R-squared

Cox and Snell	0.979
Nagelkerke	0.980
McFadden	0.216

Table 5.3 Results for $\alpha_{i,k}$

Attributes	$\alpha_{i,1}$	$\alpha_{i,2}$	$\alpha_{i,3}$	$\alpha_{i,4}$	$\alpha_{i,5}$
Price	1.0821	1.0821	1.2939	2.2696	2.2696
Fuel consumption	0.4813	1.2190	1.6504	1.6504	1.6504
Wheelbase	1.6710	1.7812	1.7812	1.7812	1.7812
Acceleration	0.6449	0.9440	1.1335	1.3221	1.5279
Comfort	0	0	0.5084	0.6693	0.7376
Dealership	0.3064	0.5891	0.5891	0.5891	0.5891
Depreciation rate	0.2640	0.4092	0.4092	0.4092	1.6122
Maintenance and repair fee	0	0	0	0.1229	0.1229

Table 5.4 Weight and Control Points

Scale Parameter $A = 10.2909$						
Attributes	β_i	$c_{i,1}$	$c_{i,2}$	$c_{i,3}$	$c_{i,4}$	$c_{i,5}$
Price	0.2205	0.4768	0.4768	0.5701	1.0000	1.0000
Fuel consumption	0.1604	0.2916	0.7386	1.0000	1.0000	1.0000
Wheelbase	0.1731	0.9381	1.0000	1.0000	1.0000	1.0000
Acceleration	0.1485	0.4221	0.6178	0.7419	0.8653	1.0000
Comfort	0.0717	0.0000	0.0000	0.6893	0.9074	1.0000
Dealership	0.0572	0.5201	1.0000	1.0000	1.0000	1.0000
Depreciation rate	0.1567	0.1638	0.2538	0.2538	0.2538	1.0000
Maintenance and repair fee	0.0119	0.0000	0.0000	0.0000	1.0000	1.0000

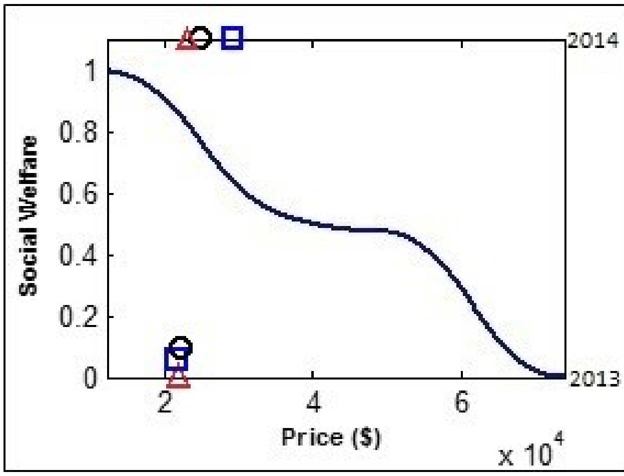
5.2 Utility Plots

According to the attributes selected in Model Assessment, we could get the results of the estimator $\alpha_{i,k}$ for all the attributes, which is given in Table 5.3. Meanwhile, the total social welfare of the vehicles not included in our model in 2013 and 2014 are 9.6601 and 8.4951 respectively. Based on the estimator $\alpha_{i,k}$ given in Table 5.3, the scale parameter, weight and control points for each attribute could be worked out, which are given in Table 5.4. We could easily learn that the biggest concern of a consumer when purchasing a vehicle is the price of the vehicle, because weight of price is the largest. The next three attributes the consumers will focus on are vehicle size, fuel consumption and acceleration. Besides price, safety, fuel economy and acceleration ability are the next three major concerns. The rest four attributes included in our model are average depreciation rate, comfort rating, dealership and maintain and repair fee, from most important to least important. With the significance of our model, we believe that this is always how most consumers are considering when purchasing a new vehicle. In other words, our model does explain the common sense of our real life.

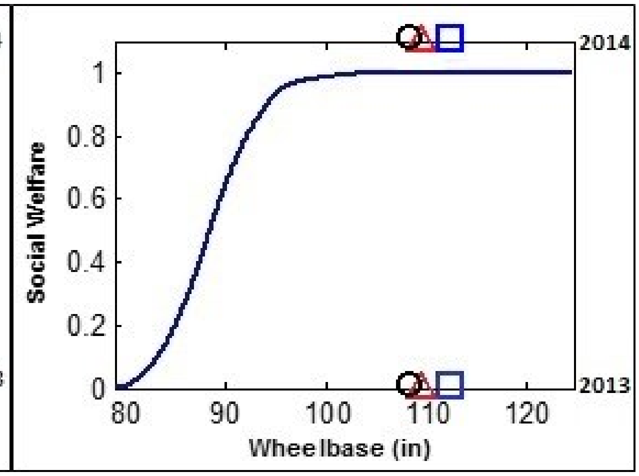
The social welfare of each attribute could be worked out by applying the scale parameter, weights and control points to the formula given in (7). The social welfare plots for each attribute are given in Figure 5.1.

As shown in Figure 5.1(a), the social welfare of price is a decreasing function, the lower the price is, the higher the social welfare is contributed. When the price is lower than 20,000 dollars, the social welfare can remain a relatively high level. However, the contribution of price to social welfare is nearly zero when the vehicle price is higher than 70,000 dollars. In addition, the social welfare of price decreases dramatically when the price is increasing from 20,000 to 40,000 dollars and increasing from 55,000 to 70,000 dollars, which indicate that the vehicles located in these two ranges can improve their social welfare dramatically by decreasing their sale price slightly. However, when price increases from 40,000 dollars to 55,000 dollars, the social welfare of price is relatively stable. This verifies our deduction that price is not the primary consideration when a consumer is considering buying a luxury passenger car with price less than 55,000 dollars, like Audi A6, Lincoln MKS etc. But if the price of the vehicle is higher than 55,000 dollars, like Audi A8, BMW 7-Series, Lexus LS and so on, price is still a big concern.

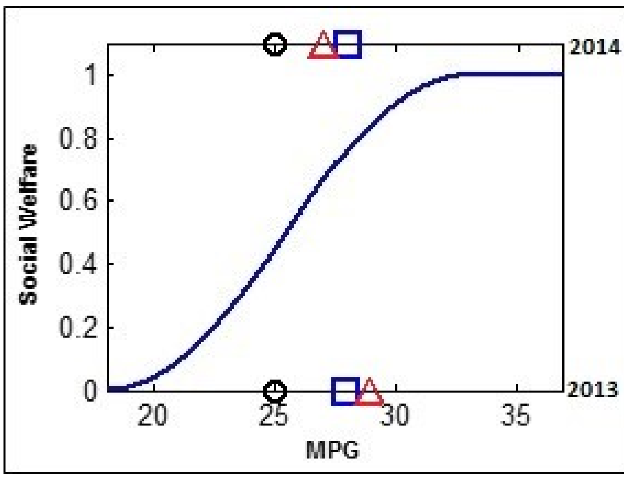
According to Figure 5.1(b), the relationship between the social welfare induced by wheelbase and wheelbase is positive. As we mentioned before, the wheelbase is a measure of safety, and it is reasonable to assume that everybody wants a vehicle with higher safety measure.



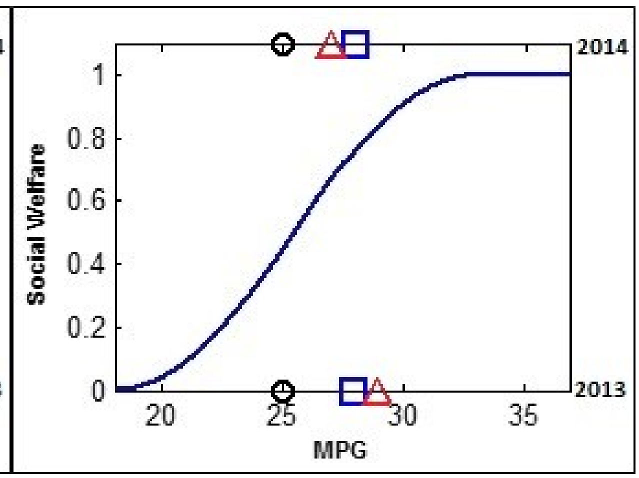
(a) Price



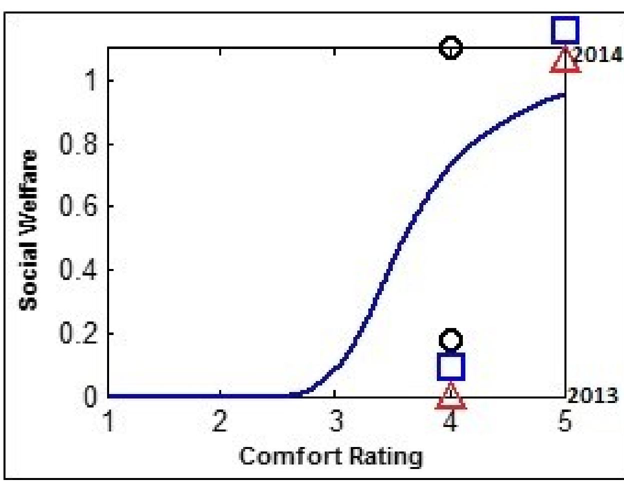
(b) Vehicle size



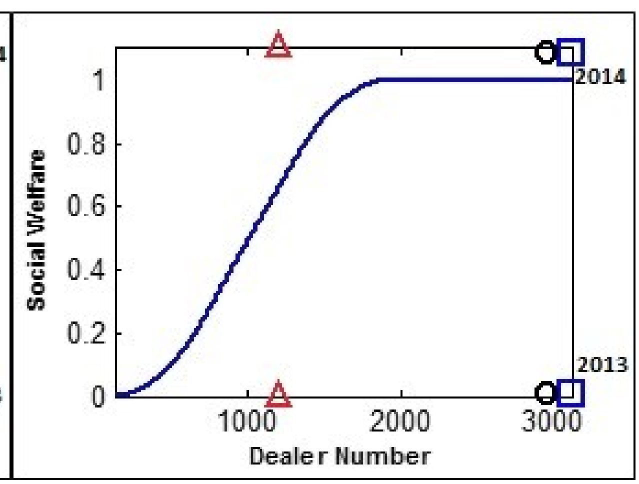
(c) Fuel Consumption



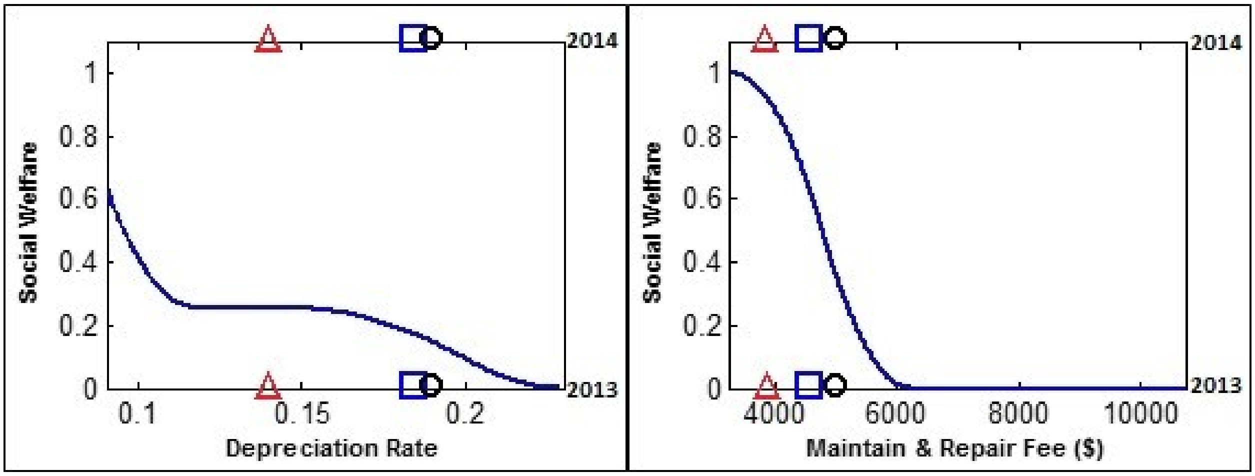
(d) Acceleration



(e) Comfort



(f) Dealership



(g) Average depreciation rate (h) Maintain and repair fee

Figure 5.1 Social Welfare for Attributes

Thus, the relationship between them should be positive, which is shown in the plot. In the plot, the most sensitive area is when the wheelbase is less than 100 inches, which represents the vehicles that are smaller than compact vehicles. In other words, the changes from subcompact to compact can bring a huge increase in social welfare. Another thing worth to mention is that when the vehicles are large enough, the social welfare comes from different vehicle sizes are not much different.

Fuel consumption plays a critical role when consumers make purchasing decisions. Figure 5.1(c) is the utility function of fuel economy (MPG). The influence of fuel consumption on social welfare is positive, and nearly constant when MPG changes from 20 to 31. However, when the MPG is large enough (greater than 32), the social welfare keeps the same. This is because vehicles located in this area usually are hybrid vehicles, if a consumer is considering buying a hybrid vehicle, he/she has paid enough attention to fuel consumption, his /her focus would be some other attributes, like price, acceleration time and so on, instead of fuel consumption. If the manufacturers want to increase the social welfare by changing MPG, the only way is to improve their MPG for these vehicles that have the MPG less than 32. For those vehicles whose MPG is greater than 32, the improvement on MPG could only contribute little on its social welfare.

Acceleration ability is the fourth consideration of vehicle purchasing behavior. The social welfare induced by acceleration time from 0 to 60 miles is given in Figure 5.1(d). Intuitively, greater acceleration time contributes less on its social welfare. When the acceleration time is longer than 12 seconds, it contributes nothing on its social. Thus, as long as the acceleration time is no longer than 12 seconds, any decrease in acceleration time would lead a dramatically improvement on its social welfare.

Social welfare for comfort rating is given by Figure 5.1(e). The plot shows that when the rating of comfort is less than 3, the contribution of comfort is zero. This indicates that it is crucial for those vehicles to improve their comfort ratings if the automakers want to improve their competitiveness. Besides, the contribution of comfort to the social welfare for the vehicles with rating 5 is nearly double of that of the vehicles with rating 4. Thus, it is worth considering improving the comfort rating of the vehicles from 4 to 5.

Figure 5.1(f) is the social welfare induced by the number of dealerships, which has a great relationship with the consumer satisfaction. When the number of dealerships increases from 0 to 1,800, the contribution to social welfare is increasing. However, after that, the utility keeps the same, which implies that when the number of dealerships is large enough, like domestic brands, the dealership is no longer the major concern for vehicle purchasing. Of course, those vehicles with dealership number less than 1,800 could still improve its social welfare by setting up more dealerships.

Figure 5.1(g) depicts the social welfare induced by DR, which is a measure of attrition of vehicle value. A vehicle with lower DR could be sold at a relatively higher price comparing to its purchasing price, and thus higher social welfare it brings. When the DR increases from 0.09 to 0.11, the social welfare induced by DR decreases dramatically. Improvement made on this range could make a big contribution to its social welfare. When the DR is between 0.11 and 0.15, where most Japanese vehicles located, the utility is relatively stable, this implies Japanese brands have built a quite good reputation on their reliabilities. When the DR increases from 0.15 to 0.23, where most American vehicles located, the social welfare it contributes changes from 0.3 to zero, which explains the big differences of reliability among different American vehicles. An interesting thing is that when the DR is moving to 0, the social welfare it brings is moving toward to infinite. In other word, if a vehicle has no depreciation, the social welfare it brings is infinite.

Figure 5.1(h) depicts the social welfare induced by maintenance and repair costs. As shown, only when the total cost is less than 6,000 dollars, the social welfare is greater than zero. The less the fee is, the more this attribute contributes to the social welfare. It is obvious that the only sensitive part is between 3,000 and 6,000 dollars. Thus improvement can be made in this range.

5.3 Social Welfare for Sample Cars

Based on the results shown in Table 5.4, the social welfare of any vehicle given in 2013 and 2014 can be calculated. The social welfare of the three sample vehicles we introduced before is given in Table 5.5. According to Table 5.5, within each year, the higher the social welfare is, the higher the real market share is. This rule is even true when the comparison is between the vehicles in different years. In addition, the estimation of the market share for all the three vehicles in both years are very close to their real market share. All these prove that we have constructed an efficient and appropriate model to assess the social welfare of these vehicles. Furthermore, the social welfare is

Table 5.5 Social Welfare Sample

Year			2013						2014					
Vehicle			Chevrolet Malibu		Ford Fusion		Toyota Camry		Chevrolet Malibu		Ford Fusion		Toyota Camry	
Social Welfare	Attribute	Weight	Value	SW	Value	SW	Value	SW	Value	SW	Value	SW	Value	SW
	Price	0.2205	*22,340*	0.8503	*21,970*	0.8608	*22,235*	0.8533	*25,065*	0.7622	*29,656*	0.6350	*22,970*	0.8315
	MPG	0.1604	*25*	0.4082	*27*	0.6211	*28*	0.7154	*25*	0.4082	*29*	0.7892	*28*	0.7154
	Wheelbase	0.1731	*107.8*	1.0000	*112.2*	1.0000	*109.3*	1.0000	*107.8*	1.0000	*112.2*	1.0000	*109.3*	1.0000
	Acceleration	0.1485	*7.00*	0.7747	*8.30*	0.6700	*6.40*	0.8230	*7.00*	0.7747	*7.7*	0.7185	*6.40*	0.8230
	Comfort	0.0717	*4*	0.7291	*4*	0.7291	*4*	0.7291	*4*	0.7291	*5*	0.9537	*5*	0.9537
	Dealership	0.0572	*3,035*	1.0000	*3,117*	1.0000	*1,234*	0.6938	*3,035*	1.0000	*3,117*	1.0000	*1,234*	0.6938
	ADR	0.1567	*0.1908*	0.1948	*0.1842*	0.2141	*0.1402*	0.2538	*0.1908*	0.1948	*0.1842*	0.2141	*0.1402*	0.2538
	M&R fee	0.0119	*5,190*	0.2566	*4,719*	0.5291	*3,778*	0.9405	*5,190*	0.2566	*4,719*	0.5291	*3,778*	0.9405
$U^{j,m}(\mathbf{X}^{\mathbf{j},\mathbf{m}}) = A\sum_{i\in\mathbf{I}}\beta_i u_i(x_i^{j,m})$			7.0408		7.3205		7.6272		6.8409		7.3401		7.7434	
$P^{l,m}$			0.0251		0.0384		0.0525		0.0239		0.0393		0.0535	
Real Market Share			0.0257		0.0378		0.0524		0.0238		550.0387		0.054	

a good estimator of the market share, which could be applied to lots of researches related to market in the future.

In 2013, the social welfare of Toyota Camry was the highest among the three sample vehicles. This was because most of the individual attribute social welfares of Camry were the highest except price and number of dealerships. Although the number of dealerships of Camry was the lowest, as dealership only weights 5.72%, it did not influence significantly on its total social welfare. Meanwhile, the social welfare due to price was very close to the highest of the three. Thus, it is not strange that the Camry could hold the 1st place. On the other hand, for Chevrolet Malibu, except the number of dealerships was better than the other two vehicles, the social welfares induced by most of its attributes were the lowest. So, it is reasonable to see that the sale of Malibu was the worst of the three vehicles.

In 2014, though the social welfare induced by fuel consumption (MPG) for Camry was no longer the highest any more, the social welfare from price ranked 1st, which counted most among the eight attributes. Besides, the social welfare from comfort rating increased, too. Thus, Camry could still hold the first place. However, for Malibu, the market share of it was still the lowest of the three. Although the social welfare of its price ranked 2nd, the social welfare due to comfort kept the same as the year before when the other two vehicles improved their social welfares in this field.

From 2013 to 2014, Fusion and Camry increased their social welfare to certain degrees while the social welfare of Malibu decreased a lot. Thus, it is not hard to explain why the market share of Malibu decreased from 2013 to 2014 when the other two increased. In detail, from 2013 to 2014, for Malibu, the social welfare from price decreased while others kept the same; for Fusion, though it lessened its social welfare from price as well, the social welfare due to wheelbase, acceleration and comfort increased much more; for Camry, the increased social welfare from comfort rating increased was much more than the decreased social welfare from price.

5.4 Model Validation

Besides 2013 and 2014, the date in 2015 are also collated. In order to validate our assessment model, two different validation methods are applied here to test whether our model could identify the change of consumer preferences from 2014 to 2015. First we use the Matthews correlation coefficient to validate whether the change direction of the EMS based on social welfare is concordant with the change direction of market share. After that, Kendall's tau is applied to validate whether our model could tell the order of the quantity of market share changed based on the order of the EMS changed. Finally, two pairs of sample vehicles are used to illustrate the influence of the weight for different attributes on the trend of market share change.

5.4.1 Matthews Correlation Coefficient

There are 85 passenger vehicles in 2015 being used as input of the SWF, according to the coefficients we got, the social welfare and EMS (estimated market share) of each vehicle in 2015 could be calculated. The EMS of these vehicles in 2014 can also be obtained from the previous data, thus the change of EMS for each vehicle and corresponding market share change could be generated and it is shown in Figure 5.2.

From Figure 5.2, we can see that our model could predict the market share change direction pretty well for most of the vehicles although most of time it is more likely to predict a bigger change than it actually does. Of course, we still have some vehicles located in the second quadrant and the fourth quadrant, which means our model either predicted a decreasing of market share when it

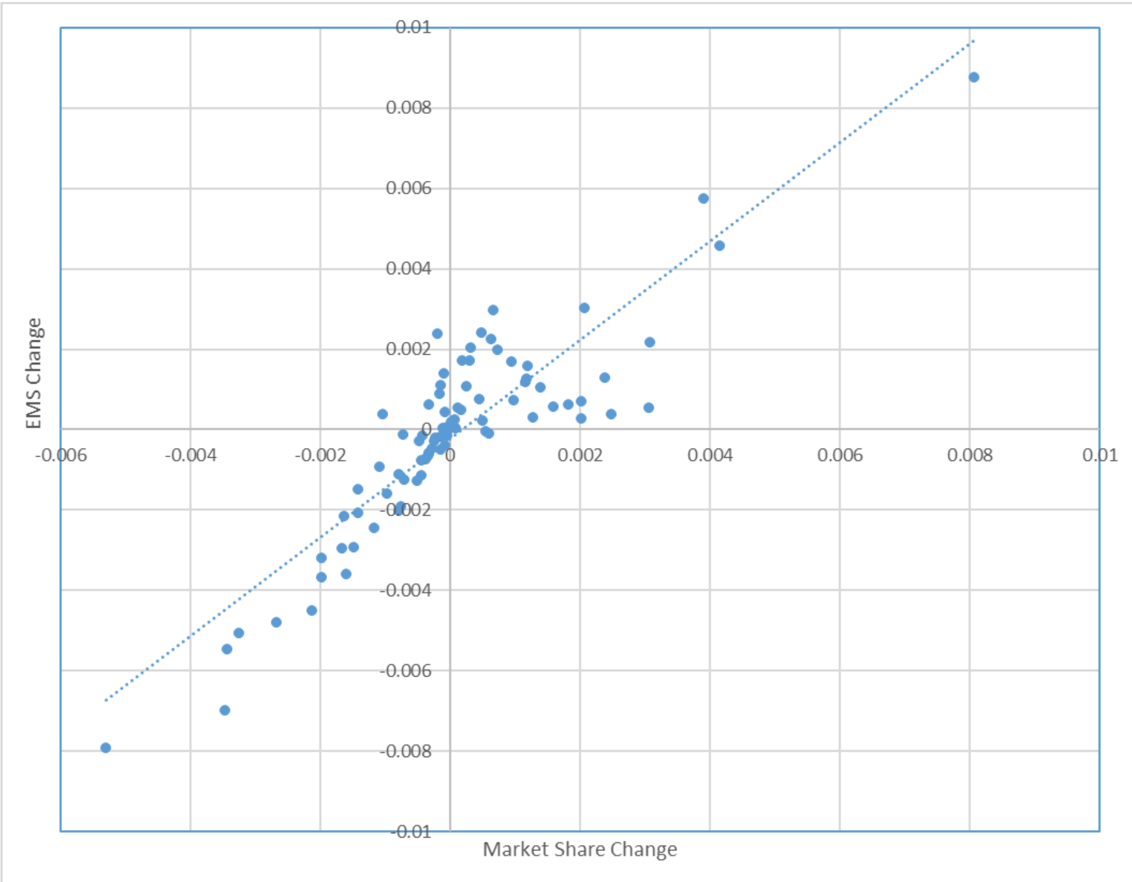


Figure 5.2 Comparison of Changes in Market Share with EMS

Table 5.6 Confusion Matrix for Validation Date Sets

		EMS Change Direction	
		+	-
Market Share Change Direction	+	35	2
	-	9	39

actually increased or predicted an increasing trend when it actually decreased. According to our review, MMC is a good method to measure how well our model is performing regarding to the prediction of market share change direction. Based on the change direction of both market share and EMS for each vehicle, the confusion matrix of these points in Figure 5.2 could be summarized in Table 5.6. Based on this table, we could calculate the MCC using formula

$$MCC = \frac{TP * TN - FP * FN}{\sqrt{(TP + FP)(TP + FN)(TN + FP)(TN + FN)}}$$

$$= \frac{35 \times 39 - 9 \times 2}{\sqrt{(35 + 9)(35 + 2)(39 + 9)(39 + 2)}} = 0.753$$

In additional, the predict accuracy (ACC) of our model is

$$ACC = \frac{TP + TN}{TP + FP + FN + TN} = \frac{33 + 39}{35 + 9 + 2 + 39} = 0.871$$

A coefficient value of 0.753 means the predicted directions obtained from our model is highly correlated to the actual market share change directions. Besides, 0.871 accuracy indicates that our model identified over 87% of the market share change directions. Both of these two measures demonstrate that our model performs very well on predicting the change directions of market share. That is to say, in terms of consumer preferences, our model could tell whether consumers would prefer a specific vehicle more or less based on their social welfare in two consecutive years.

5.4.2 Kendall Rank Correlation Coefficient

Actually, besides comparing consumer preferences to a particular vehicle form last year to this year, we want to also identify consumer preferences regarding two different vehicles in two consecutive years. For example, if the real market share changes for both vehicle A and vehicle B are positive and increase for vehicle A is greater than vehicle B, we would like to see whether our model could detect the same changes. For these kinds of ordinary data, Kendall rank correlation coefficient is a good measurement to quantify the correlation between two observations. For any two pairs of our 85 vehicles, 3335 of these pairs are concordant and 235 of them are discordant, thus

$$\tau = \frac{n_c - n_d}{n(n-1)/2} = \frac{3335 - 235}{85(85-1)/2} \approx 0.868$$

This large value of τ indicates that our model is strongly agree with the reality on the evaluation of the orders of market share changes. The corresponding Z value is

$$Z = \frac{3(n_c - n_d)}{\sqrt{n(n-1)(2n+5)/2}} = \frac{3(3335 - 235)}{\sqrt{85(85-1)(2 \times 85 + 5)/2}} \approx 11.77$$

This value of $Z = 11.77$ is large enough to reject the null hypothesis even at the $\alpha = 0.001$ level, and therefore we could come to the conclusion that our model has shown a significant agreement with the reality regarding to the prediction of the consumer preferences changing order for all the vehicles.

5.4.3 Validation Cases

In traditional time series method, if we have multiple years of data, we might could predict the trend of the market share change, and this prediction usually has a lag compared to elastic. What if we only have the market share of one year? Traditional time series is not applicable any more. In this part, we select two pairs of sample vehicles to validate our model on the ability of predicting market share change trend, i.e. consumer preferences, even if we only have one-year data for market share. Both pairs of vehicles have a common feature, one vehicle sells better before 2014 and another sells better in 2015. That is to say, from 2014 to 2015, the market shares of the vehicles within each pair have exchanged their ranks. We want to use our model to demonstrate why this happened.

The first case is Toyota Corolla and Honda Accord, the related data is listed in Table 5.7. Actually, from 2013 to 2014, both vehicles have an increasing trend on their market share. However, from 2014 to 2015, Corolla continues increasing its market share while Accord has a decrease in its market share. If we compare the changes of the attributes for the two vehicles from

Table 5.7 Comparison Table for Toyota Corolla and Honda Accord

		Toyota Corolla			Honda Accord		
		2014	2015	Difference	2014	2015	Difference
Attributes	Weights	4.28%	4.67%	0.39%	4.89%	4.57%	-0.32%
Price	0.2205	16300	17625	1325	20675	22440	1765
Fuel consumption	0.1604	32	32	0	30	30	0
Wheelbase	0.1731	106.3	106.3	0	109.3	109.3	0
Acceleration	0.1485	9.9	9.9	0	6.3	7.7	1.4
Comfort	0.0717	5	5	0	4	5	1
Dealership	0.0572	1234	1234	0	1042	1042	0
Depreciation rate	0.1567	0.1288	0.1426	0.0138	0.1546	0.1546	0
Maintenance and repair fee	0.0119	3922	3836	-86	4348	4187	-161

2014 to 2015, we could identify that three of the eight attributes did not changes, which has no effect on the market share change. However, the rest attributes, like price, acceleration, comfort, depreciation rate, and maintain and repair fee, changed for at least one of the two vehicles. For price, both of them increased but Accord increased more. Besides, the price of Accord is located at higher marginal utility position in price utility plot shown in Figure 5.1(a), which means Accord would have a larger loss on its social welfare than Corolla in terms of increasing price. For acceleration, it is obvious that Accord would lead a greater loss on its social welfare. Regarding to comfort and maintain and repair fee, Accord did better than Corolla. As for depreciation rate, because Corolla increased its rate within the 0 marginal utility range (can be found in Figure 5.1(g)), both of them had no effect on their social welfare. However, if you look at the weight of each attribute, you could figure out that although Accord did better than Corolla on comfort and maintain and repair fee, on other two attributes with higher weight, Accord did much worse, it inevitable lead a loss of market share and overtaken by Corolla. In fact, based on our prediction, Corolla would increase its market share by 0.43% and Accord would have a decrease of its market share by 0.06%. Though it is not accurate, it predicts the trend very well.

Table 5.8 Comparison Table for Hyundai Elantra and Chevrolet Cruze

		Hyundai Elantra			Chevrolet Cruze		
		2014	2015	Difference	2014	2015	Difference
Attributes	Weights	2.80%	3.11%	0.31%	3.44%	2.91%	-0.53%
Price	0.2205	14825	15950	1125	14975	16525	1550
Fuel consumption	0.1604	27	33	6	33	33	0
Wheelbase	0.1731	106.3	106.3	0	105.7	105.7	0
Acceleration	0.1485	9.8	9.8	0	9.7	9.7	0
Comfort	0.0717	3	4	1	4	4	0
Dealership	0.0572	825	825	0	3035	3035	0
Depreciation rate	0.1567	0.2045	0.2045	0	0.1908	0.1908	0
Maintenance and repair fee	0.0119	3753	3693	-60	4227	4289	62

Another case is Hyundai Elantra and Chevrolet Cruze. It is interesting that from 2013 to 2014, Elantra has a decreasing trend on its market share and Cruze shows an increasing trend on its market share, however, from 2014 to 2015, Elantra shows an increasing trend on its market share and Cruze has a decreasing trend on its market share. With traditional time series model, it cannot explain these kinds of variation precisely and timely. But our model performs very well on predicting these changes. Based on the related date given in Table 5.8, four of the eight attributes keep the same from 2014 to 2015 for both vehicles. For the other four attributes, Elantra changes all and Cruze changes two of them. It is obvious that, for fuel consumption, comfort, and maintain and repair fee, Elantra is getting better and Cruze is either getting worse or not change, which will definitely increase the social welfare of Elantra and decrease the social welfare of Cruze. On the other hand, both of them have a higher price in 2015 but Cruze changes more even they are changing with similar marginal utility. All these four attributes indicate that Cruze would lead a drop on its market share while Elantra would lead a jump. Actually, our model predicts 0.05% of market share increase for Elantra and 0.54% of market share decrease for Cruze. Both cases indicate that our model has the capability of predicting consumer preferences precisely and timely.

Chapter 6. Conclusion

This paper develops an automotive market social preference assessment method based on a generalized multinomial logistic regression model using multi-attributes data of multiple vehicles in multiple years. The concept of social preference is defined as social welfare using the additive form of each individual attribute. The social welfare of each vehicle can be further used to estimate its market share. The individual attribute social welfare is assumed to be a second order differentiable concave curve. The attributes that included in our final model are selected by applying the G-test to the significance of each attribute. In addition, the scale parameter of the SWF, together with the weight of each attribute, are given by our model.

Finally, there are eight attributes proved to be important by our model when the purchasing behavior is regarding as a social behavior. They are price, wheelbase, fuel consumption, average depreciation rate, acceleration, comfort rating, dealership, and maintenance and repair costs, from the most significant one to the least significant one. Among them, price and maintenance and repair costs are concerning about the cost the consumer will pay. Acceleration, comfort rating and fuel consumption belong to the performance of the vehicle, which give great influence on the driving experiences. Wheelbase and average depreciation rate are representing the safety and reliability measurement of a vehicle. The dealership is a measurement of the consumer service level, which indicates a strong association with the consumer satisfaction. In other words, cost, performance, safety and reliability, and consumer service level, are usually the consumers' four major concerns when they are making the purchasing decisions. Thus, our model performs well with the reality and common purchasing experiences.

The social welfare plot for each attribute is given in Figure 5.1. Based on these figures and their corresponding weights, automakers could figure out what the most sensitive attribute is when a specific vehicle is chosen. After knowing the unit cost of changing each attribute, the automaker could find a way to increase its social welfare most by a fixed budget. According to the validation analysis, it proved that our model could not only predict the consumer preferences on particular vehicle from year to year, i.e. the market share change direction, but also identify how much the consumer like or dislike a particular vehicle, i.e. the rank of the changes on market share for each vehicle. Besides, the trend line in Figure 5.2 indicates that our model is real sensitive to the changes on attributes from year to year, which is a great advantage over traditional time series methods on evaluating the trend of the changes, especially when the measurement is fluctuating over time.

Furthermore, this generalized multinomial logistic regression model could be applied in many other fields other than automotive market to identify the consumer preferences on various of goods.

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