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Chapter 7 — Good Teaching of Calculus I

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As indicated in Chapter 2, the analysis of student survey data revealed that student responses to questions about instructor characteristics factored into three clusters: Good Teaching, Technology, and Ambitious Pedagogy. Of these three, only Good Teaching had a positive effect on the change in students' attitudes towards mathematics (a composite of three outcomes: mathematics confidence, enjoyment, and persistence).

In this chapter, we further analyze the data collected in the study to understand better what the construct Good Teaching means. We seek to answer the question: What are the features of Good Teaching that are revealed through student responses, interviews with faculty and administrators, and classroom observations in the institutions participating in the CSPCC study? The chapter is organized into four sections. We start with a brief description of how Good Teaching has been conceptualized in the literature. We next describe the sources of data and the analysis we performed to answer our question. Following this, we present our major finding—namely, that Good Teaching in this study has three components: Classroom Interactions that Acknowledge Students, Encouraging and Available Faculty, and Fair Assessments. After describing these components, we conclude with suggestions for institutions, departments, and faculty about using this information to infuse changes in their practice.

Good Teaching in the Literature

There is a strong body of research dedicated to the identification of features of teaching that are important for various aspects of students' experiences in college. For example, in a now classic study, Chickering and Gamson (1991) identified the following seven principles of good practice in undergraduate education:

1. Student-faculty contact
2. Cooperation among students
3. Active learning
4. Prompt feedback
5. Time on task
6. High expectations, and
7. Respect for diverse talents and ways of learning

Chickering and Gamson (1991) identified these principles via surveys of large numbers of students in various types of post-secondary institutions and a variety of disciplines. Their work was seminal because the operationalization of these principles allowed further research that investigated the impact of these practices on students' college experiences. More current work has provided evidence that high expectations, specifically the use of active learning (e.g., via inquiry-based learning), result in positive affective and cognitive gains among undergraduate students (Kuh, 2008; Kuh, Kinzie, Buckley, Bridges, & Hayek, 2007). Likewise, having high aspirations for student learning, setting clear expectations for student performance, and standards for holding students accountable have been highlighted as key elements that faculty need to consider in their pedagogy (see, e.g., Hassel & Laurey, 2005; Tagg, 2003). Rendon (1994) has advocated for the use of validation activities (e.g., calling students by name, working one-on-one, praising students, encouraging students to see themselves as capable of learning, and providing encouragement and support) as important for "transformational changes in students, accompanied by an increased interest and confidence in their capacity to learn" (as cited in Kuh et al., 2007, p. 67). Notice the recurring themes across these works: active engagement and high expectations.

Other researchers indicate that knowledgeable and enthusiastic instructors who also encourage students to express their views through discussion and invite them to interact with others inside and outside of class help build a positive learning classroom atmosphere that is conducive to learning (Angelo & Cross, 1993; Pascarella & Terenzini, 2005), which in turn results in better dispositions towards learning. Providing timely feedback that is both supportive and corrective, and that includes individualized instruction as needed, has been shown to make a difference in students' learning (Angelo & Cross, 1993; McKeachie & Svinicki, 2006). Finally, and perhaps more importantly, using challenging, novel, meaningful, and stimulating activities and assignments influences students' growth and satisfaction (e.g., Strauss & Volkwein, 2002), , which is conducive to positive attitudes towards learning.

While this literature has not differentiated these principles by disciplinary orientation the mathematics education literature adds other features to these ideas of good teaching. For example, the K-12 mathematics education literature on high-quality teaching has emphasized the importance of giving students tasks of high cognitive demand (Henningsen & Stein, 1997). Such tasks require students to engage with mathematical content in ways that go beyond using known facts in standard procedural ways, and instead pose questions for which the students have to generate new connections and find the required knowledge (Stein, Grover, & Henningsen, 1996). Students of teachers who consistently use tasks at the higher level of cognitive demand show significant gains in standardized tests (Silver & Stein, 1996), and teachers who are recognized as exemplary teachers are more likely to use high-cognitive demand tasks in their teaching and in their assessments than teachers who are not exemplary (Silver & Mesa, 2011; Silver, Mesa, Morris, Star, & Benken, 2009). The literature in undergraduate mathematics education has documented that inquiry-based approaches to the learning of mathematics has a positive impact on affective gains, especially for women and minority students (Laursen, Hassi, Kogan, & Weston, 2014; Rasmussen & Kwon, 2007; Stephan & Rasmussen, 2002).

Thus there are features in this literature that point to characteristics of the instructor, to characteristics of the engagement of students, and to characteristics of the tasks that student are asked to work on. This literature informed the design of the surveys and of the interviews we used to collect data in the CSPCC study. As we will see, the analysis of the data we collected partially supports some of the ideas found in the literature. We present next a brief note on the methods of analysis for this chapter.

Methods and Limitations

We had a variety of sources at our disposal: student, instructor, and coordinator surveys collected during the first phase of the study; interviews with faculty, administrators, and staff; focus groups with students; and observations of Calculus I teaching at the 18 institutions selected in the second phase of the study. Using the full data set from the surveys (N = 3,448[1]) we conducted a factor analysis using the 22 survey items identified by Sadler and Sonnert as defining Good Teaching (see Chapter 2). Three factors emerged: Classroom Interactions that Acknowledge Students, Encouraging and Available Faculty, and Fair Assessments (see Table 1).

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[1] The file used to perform the analysis includes data from students from two pilot institutions that were later included as part of the study. See Hsu, Mesa, and The Calculus Case Collective (2014).

Table 1: Mean, standard deviation, and reliability of each of the Good Teaching factors.

Factor (Survey items included)	N	Mean[a] (SD)	Cronbach α
Classroom Interactions that Acknowledge Students (e.g., presented more than one method for solving problems; asked questions to determine if I understood what was being discussed; listened carefully to my questions and comments; allowed time for me to understand difficult ideas; helped me become a better problem solver; provided explanations that were understandable; discussed applications of calculus; frequently asked for questions; prepared extra material to help students understand calculus concepts or procedures; (-)made students feel nervous during class)	3,448	4.48 (.954)	0.918
Encouraging and Available Instructor (e.g., encouraged students to enroll in Calculus II; (-) discouraged me from wanting to continue taking Calculus; acted as if I was capable of understanding the key ideas of calculus; made me feel comfortable in asking questions during class; encouraged students to seek help during office hours; was available to make appointments outside of office hours, if needed; showed how to work specific problems; made class interesting)	3,448	4.78 (.766)	0.788

Fair Assessments (e.g., assignments completed outside of class were challenging but doable; my Calculus I exams were a good assessment of what I learned; my exams were graded fairly; my homework was graded fairly)	3,439	4.56 (.921)	0.714

Note: a. Average of the items and average across all available data. Items measured on a scale from 1-6. Negative items, marked with (-), were rescaled.

Although the survey data analysis is revealing, a main limitation is that the data are self-reported—as are the data used in the studies conducted in the higher education literature reviewed at the beginning of the chapter. In our case we also had low response rates in several of the institutions and over representation of the PhD institutions in the survey data. These sampling issues make it difficult to rely solely on the survey data, which is why we sought to corroborate some of these trends with the information gathered during the in-depth site visits to the selected institutions. Thus, we augmented the factor analysis with concrete examples, seeking to understand in more detail how participants expressed each of these features of Good Teaching. A potential issue with the case study data is that we visited institutions about two years after the survey data were collected. The students to whom we spoke were a different than those who responded to the surveys. In some cases, we also talked with a different group of faculty and administrators. We believe, however, that this is a strength of the design, as it helps us understand the stability of the responses over time.

We drew from data collected during Phase Two of the CSPCC study to illustrate these factors. We began with analyzing institutional documents produced by each team that synthesized major key features of each of the institutions visited (henceforth: ‘Facts and Features documents’). We then analyzed the student focus groups looking for confirming and disconfirming evidence for these three features, using at least one focus group transcript from each of the institutions visited (*n* = 20) attending mainly to questions about their instructors, their class interaction, and their assessments. Finally, we analyzed 72 reflective memos that drew on evidence from various record-keeping logs used by observers during our classroom observations in the project (see White & Mesa, 2012). We summarized the trends in responses to 13 interview questions[2] that spoke directly about classroom interactions and encouraging and available instructors.[3]

Good Teaching in the CSPCC Data

The 22 items in the Good Teaching factor clustered into three main factors, which we call Classroom Interactions that Acknowledge Students, Encouraging and Available Instructor, and Fair Assessments (Table 1), because of the types of items that made up each factor. We discuss them next, listing first the survey items that clustered in each and then providing corroborating data from institutional documents, student focus groups, and classroom observations.

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[2] There were 916 responses to the 13 questions. Of these, 71 (8%) were not included in the analysis because they were empty, ambiguous, or non-interpretable. For more details refer to Sümer and Mesa (2014). The full analysis included 1,252 comments.

[3] There were no questions regarding assessment in the observation protocol and we explicitly requested to observe lessons in which no examinations were to be administered. A copy of the questionnaire is available through MAA (see www.maa.org/cspcc).

Classroom Interactions that Acknowledge Students

The survey items that clustered in this factor refer to specific actions that occur inside the classroom and that describe how teachers and students relate to each other. This factor mainly includes things teachers do to encourage student participation: they present more than one method for solving problems and help students improve in their problem-solving skills; they ask questions to gauge students' understanding, but they also listen to questions, comments, and they invite students' questions and comments. These instructors also prepare extra explanations and additional materials, and talk about applications of Calculus I.

Out of the 12 Facts and Features documents that described classroom interaction and instruction, lecture was the predominant form of interaction in nine of them, although in eight cases, the lecture was combined with other forms of interaction: small group work, individual work and student presentations. The variety in the interaction is important because lecture tends to be characterized in the literature as a form of instruction that discourages classroom interaction. We note that 14 out of 16 institutional reports that addressed classroom size described it as small, with 30 to 35 students. Only two institutions had lectures of over 200 students with small recitation sections of 30 students. Small class sizes are more conducive to having interaction in the classroom, which may explain these observations.

Data from the student focus groups also suggest that lecture with many questions was the norm in these courses. Students of all teachers indicated that their instructors lectured during some portion of the lesson and they labeled the lectures as "interactive." In interactive lessons students were allowed and encouraged to ask questions and their teachers responded to their questions thoroughly, making sure they understood the answers, as expressed by a student in a bachelor's granting institution:

> It's more interactive though. It's not just blah, blah, blah; I mean when I came to college for math I expected that math teachers would be really boring and in this case, I mean like I said in this case, [my professor] isn't boring, but at the same time like we don't have some crazy, frizzy-haired, mathematician or anything. (BA1[4])

In nearly half of the institutions (seven out of 18), there were instructors who included more interactive forms of engagement, sending students to the board, doing interdisciplinary projects, or working in groups during class:

> My teacher will lecture for like a couple minutes and the stuff he lectures seems really vague to me… and then he'll hand out a worksheet for everyone to do and if [we] have a question [we] can ask it. (TY2)

Students recognized that having smaller classes made it easier to get to know their teachers, and that taking classes in large lecture halls limited the amount of interaction. Yet even in these large classes, their teachers made sure students participated:

> I think for the most part, it's hard to be interactive with a massive lecture hall, but he took the time to wake you up or to make sure that you would understand things.

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[4] We refer to the type of institution (MA, BA, TY) and the internal identification of the institution (a number between 0 and 4). We refer to the PhD-granting universities as LPU (Large Public University) 1 or 2, LPrU (Large Private University), PTU (Public Technical University), or PTI (Private Technical Institute).

And he would prompt you with questions if you needed it. So I guess there was a degree of interaction between the professor and the students. (LPU2)

We also corroborated these impressions in our classroom observations. The observers reported that the atmosphere was such that it encouraged students to ask questions. Indeed, in very few classes did the observers say that students did not ask questions (seven out of 65[5]). They mentioned that the level of student questioning seemed adequate, with students asking for clarification about procedures or requesting justifications for the work. Pacing is also germane to making it possible for students to feel at ease in a lesson. Observers indicated that the pace of the class seemed very reasonable (53 out of 69); and that this pace was such that students could participate and seemed comfortable following the presentation.

The analysis further suggests that the most popular interaction form in the classes was an exchange known as IRE/F, which describes a three-turn pattern of speech: Initiation, in which the instructor asks a question; Response, in which a student provides a short answer to the question, and Evaluation/Feedback in which the teacher judges the answer as correct or incorrect or gives more information to the student (Cazden, 1986; Hicks, 1995-1996). These are short exchanges that may occur at a fast pace in any given lesson. In addition, observers noted use of questions during lecture meant to elicit more information from the students. IRE/F questions and other questions in the lecture were reported in 43 of 65 observations. Sharing in pairs or quiet

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[5] There were 74 observation documents, but not all the observers answered all questions. The reader will notice the fluctuation of the available data in the reporting.

exchanges among students was observed in 16 observations, whereas group work or student presentations were infrequently mentioned (only four times each).

Although students may participate via IRE/F interactions, in general they were not observed making contributions to content delivery, although such contributions did occasionally occur, such as when students presented their own solutions to problems or provided detailed answers to questions (eight observations each). In only three cases the observers noted that students actually guided the content of the class. In 13 cases observers said that students did not make any contribution to content delivery. These figures confirm that lecture, guided by the instructor, is a primary form of interaction in the classrooms observed.

These observations indicate that, when teaching, the majority of the observed teachers created an environment that encouraged students' participation via questions, with interesting examples, and opportunities to correct mistakes. Students of those teachers indicated that the lessons observed were fairly typical. Thus, although lecturing dominated the content delivery, there was interaction between the instructor and the students; this interaction was mainly directed by the teacher and involved few students each time. Students participated both by asking questions and responding to questions posed by the instructor. Group work or student presentations on problems that were not from the homework were not very commonly observed.

Encouraging and Available Instructor

The survey items that clustered around this factor (see Table 1) refer specifically to the perception students had that their instructors encouraged them to continue taking Calculus II, and invited them to seek help during office hours, making themselves available outside of office hours as needed. This factor also includes perceptions that the instructors acted as if students were capable of understanding the material, made them feel comfortable in asking questions during class, and showed how to work specific problems. This factor captures the perception that teachers care about students' learning, that they believe in students' abilities, and that they see students as capable of taking more Calculus courses.

The analysis of the Facts and Features documents provided more details about how this factor was perceived by participants. Out of 15 Facts and Features documents that explicitly mentioned instructors, seven characterized them as caring, available, flexible, and seeking to maintain high expectations. We also note that, for the most part, faculty assigned to teach Calculus I *wanted* to teach the course or were rigorously trained (in the case of teaching assistants or visiting professors) to engage in the specific pedagogy that was supported by their institution.

Eleven out of 12 Facts and Features documents that mentioned faculty status indicated that Calculus I was taught by full-time or tenure-track faculty only, with the majority being at baccalaureate and two-year institutions. The 2010 CBMS statistical report indicates that the percentage of all sections of mainstream Calculus I taught by full-time faculty was 53% for four-year institutions and 90% for two-year institutions (Blair, Kirkman, & Maxwell, 2013, p. 18). It appears that in the selected institutions the proportion of Calculus I sections taught by full-time faculty was possibly above the 2010 figure. Because full-time faculty are more likely to be on campus than adjunct faculty, this availability might be playing a role in the importance of this aspect of Good Teaching. However, further analysis would need to be performed to understand better the interaction between faculty status and the characteristics of the instructors described in this factor.

Students in focus groups eloquently described how their instructors were encouraging and available. In all the institutions we visited, students said that the Calculus I faculty made them feel that they were capable of working through difficulties, and sometimes went out of their way to assist them when they needed help. For example one student at a bachelor's granting institution said:

> Well I never wanted to go to the board and I went up one time and I actually knew what I was doing, so I felt better that I knew the material that we were learning. He expects us all to understand it and I think he knows that we all can do it. (BA4)

Likewise, students described instructors' availability, and patience dealing with them:

> In my class mostly he's on top of things, so… I'm kind of (…) it takes me a little longer to get it, but he understands and he's really understanding: if you need help… outside of class… he'll make time for that. (LPrU)

Observers described a positive atmosphere in the classroom in which teachers were encouraging, asking for more questions, making sure the whole class was on the same page or pausing to wait for a response. The majority of the observers (51 out of 65) described the observed lessons as "interesting" because of the teacher (energetic, friendly, justified steps in a novel way). They also described the students as spellbound by the instructors, and indicated that the content was original or included challenging problems. Although tangential to being available, these observations do suggest that in most cases the observed faculty behaved in ways that made the students feel at ease, which might be interpreted by the students as encouraging and trusting in their capacity to do challenging work.

Fair Assessments

This feature of Good Teaching refers to students' rating of the challenge of the assignments completed outside of class and how doable they were. In addition it speaks about students' perceptions that their Calculus I exams were a good assessment of what they learned and that both exams and homework were graded fairly. We called this factor simply Fair Assessments.

The Facts and Features reports suggested that faculty had more latitude in assigning and designing homework and quizzes than in designing exams. Indeed, in 11 of 18 institutions the final exams were common or had questions that were required for all students taking Calculus I. However all faculty were consulted on the contents of the common exams.

The student focus group data corroborated that students felt their exams were fair. By "fair" students meant that they were rarely surprised in the exams. That is, the instructors would give them as much information as possible to make sure they were ready and prepared. This information came in the form of homework (the homework was usually described as harder than the examples given in the lecture) and quizzes; students also relied on exams done in previous years and attended review sessions. Students knew what to expect:

> The exams are like the homework problems, there are no surprises. A lot of what I do it's… there's past exams online, so if you go through past exams and stuff it kind of helps you because you're prepared for like the types of story problems. Like they're not going to be exactly the same, but you're prepared for what kind of material you're going to be tested on. (LPU1)

This did not mean that the exams or assessments (including homework) were easy. Quite the contrary, students found them challenging. In all institutions, students indicated that although the exams were not surprising, the faculty changed them or contextualized them in ways that they had to show they understood the material. Memorization alone was not useful:

> Like in the last test we had to put the limit definition in our own words and we couldn't use like formal definitions. If we tried to put the formal definition she wouldn't accept it. (TY3)

Another way in which instructors increased the challenge of the exams was by making them cumulative:

> You can't just memorize. My teacher mixes and matches the problems, the solutions, you have to think … all the way back to like say chapter 1. (MA3)

In general we note that faculty in selected institutions had an important say in the assessment of the courses, both in terms of homework and quizzes but also in terms of designing exams or making contributions when they were common.

Implications

Good Teaching is a construct identified by Sadler and Sonnert (see Chapter 2) that contributed in our identification of institutions that had positive changes in students' attitudes toward math (a composite of three of our outcomes: mathematics confidence, enjoyment, and persistence). In this chapter we have delved deeper into understanding the components of Good Teaching using the data we collected from students, faculty, and administrators, corroborating some of those with research literature. Our analysis reduced 22 survey items defining Good Teaching into three factors: Classroom Interaction that Acknowledge Students, Encouraging Instructors, and Fair Assessments. These factors are under the control of the instructors. They can provide concrete and actionable elements that can be used by instructors and departments to improve practice or to evaluate it.

The definition of Good Teaching described in this chapter, as coming from the voices of students, administrators, and instructors, combines a mixture of three elements, all related to what the instructor does in the classroom, with students, and with the mathematics. Good Teaching involves, first, a teacher who is encouraging of students' efforts to learn and who is available to answer their questions and to support them in their learning; second, a class environment that fosters interactions geared at eliciting students' participation as a mean to promote mastery and understanding of the material; and third, tests and assignments that are perceived as fair. These features are not novel. They have been documented in the literature on good practice in teaching, and they are present in the institutions we selected for our case studies. We propose the following five practices related to Good Teaching of Calculus I:

1. Positive atmosphere in which the instructors encourage students to ask questions.
2. Positive attitude towards students' mistakes.
3. Reasonable pacing of the lecture to ensure all students are on the same page, with time for individual, pair, or group work.
4. High standards and clear expectation that all students can meet.
5. Availability to answer student questions and respond to their needs.

In this list it is worth noting that Practices 1 to 4 add to what the literature says about good practices in teaching, because they are centered on the actual calculus lessons we observed. Practice 5 is not novel (it was already identified by Chickering & Gamson, 1991). As stated earlier, these five practices are under direct control of the instructor. Even though most of the instructors in our study delivered lectures, they were nonetheless able to have lectures that were interactive, with many exchanges between students and instructors, and were carried at a reasonable pace that allowed students to understand the material. The exchanges were quick, in the form of short questions and answers, and involved more than a handful of students. This work that faculty do in classrooms corroborates the crucial importance of teachers in creating an environment in which students can learn Calculus I through interaction with the material.

In the selected institutions we visited, most of the administrators explicitly indicated that the reason their calculus program was successful was because of their instructors, as opposed to other features that they could mention, like the use of technology, or the small class size. Our analysis has underscored the importance of getting the best instructors into the Calculus I classroom. If student interest in taking more mathematics hinges on the quality of the experiences in the classroom, the everyday contact with a knowledgeable, caring, and supportive teacher, then departments should ensure that this is the case. For Calculus I, the teacher may be the student's most important resource. As such, investing in faculty development and in hiring high quality instructors would be of paramount importance.

In addition to hiring and supporting instructors, this study suggests that departments and institutions may need to consider class size and assigning full-time faculty to the teaching of calculus. Small class size enables faculty to create a classroom environment in which students interact and their interactions are valued, thus facilitating the exchanges that help students engage with the material. As a bonus, smaller classes allow faculty to get to know students personally and encourage them to stick with their mathematics regime, as the personal connection can create personal accountability. Assigning full-time faculty can also foster such relationships because they are more likely to be available for students than part-time faculty.

In all, our take home point from the analysis of Good Teaching is that the instructor and what she or he does in the classroom matters for students. It is an old time truth that continues to be corroborated today.

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