Online Appendix to:

A Characteristics Approach to Optimal Taxation: Line Drawing and Tax-Driven Product Innovation

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Nested Constant Elasticity of Substitution Preferences

In this appendix we provide a specific example, where each consumer has a utility function of the nested constant elasticity of substitution (CES) form, as follows:

$$u(x,c_1,c_2;\omega)=\left[a_1(\omega)\,c_1^{\frac{\sigma-1}{\sigma}}+\left(a_0x^{\frac{\varepsilon-1}{\varepsilon}}+a_2(\omega)\,c_2^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{\sigma-1}{\sigma}\right)}\right]^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

In expression (1), σ is the elasticity of substitution between characteristic one and both characteristic two and the untaxed numeraire x, and ε is the elasticity of substitution between characteristic two and the untaxed good. It is assumed that $a_2(\omega)$ is increasing in ω and that $a_1(\omega)$ is decreasing in ω, which implies that, at any set of prices, the higher is ω the greater is the consumer's implicit demand for c_2 and the lower is the consumer's implicit demand for c_1.

It is informative to consider a special case in which, at pre-tax prices, each consumer spends the same share of their income on the characteristics good (and thus also on the untaxed good). Consumers differ in the type of characteristics good they prefer, but not in their share of income spent on the characteristics good; as $a_2(\omega)$ rises, $a_1(\omega)$ declines by enough to keep expenditure at pre-tax prices on the characteristics good constant. When the goods above and below $\bar{c}_2$ differ in their substitutability with the untaxed numeraire, it is optimal with preferences given by (1) to introduce a notch. (For analytical convenience, we maintain the assumption that the social marginal utility of income is constant across consumers, but our qualitative conclusions are unaffected by this assumption.)

Proposition 4 (CES Preferences): *If consumer preferences are given by (1), the expenditure share on the untaxed good does not vary across consumers in the absence of taxes, the social marginal utility of income is constant across consumers, and the revenue requirement is sufficiently small, then a notch is optimal for* $\sigma \neq \varepsilon$.

Proof:

Consumers have constant elasticity of substitution (CES) preferences, with a type-ω consumer having utility given by

$$u(x, c_1, c_2; \omega) = \left[a_1(\omega) c_1^{\frac{\sigma-1}{\sigma}} + \left(a_0 x^{\frac{\varepsilon-1}{\varepsilon}} + a_2(\omega) c_2^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{\sigma-1}{\sigma}\right)} \right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where σ is the elasticity of substitution between characteristic one and both characteristic two and the untaxed good x, and ε is the elasticity of substitution between characteristic two and the untaxed good. The expenditure function for a type-ω consumer is

$$e(p,u) = \frac{u\left[a_0^{\varepsilon} p_0^{1-\varepsilon} + a_1^{\sigma} p_1^{1-\sigma} \left(a_0^{\varepsilon} p_0^{1-\varepsilon} + a_2^{\varepsilon} p_2^{1-\varepsilon} \right)^{\left(\frac{\sigma-\varepsilon}{1-\varepsilon}\right)} + a_2^{\varepsilon} p_2^{1-\varepsilon} \right]}{\left[a_1^{\sigma} p_1^{1-\sigma} \left(a_0^{\varepsilon} p_0^{1-\varepsilon} + a_2^{\varepsilon} p_2^{1-\varepsilon} \right)^{\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\varepsilon-\sigma}{1-\varepsilon}\right)} + \left(a_0^{\varepsilon} p_0^{1-\varepsilon} + a_2^{\varepsilon} p_2^{1-\varepsilon} \right)^{\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\varepsilon}{1-\varepsilon}\right)} \right]^{\left(\frac{\sigma}{\sigma-1}\right)}}, \tag{3}$$

and the associated Hicksian compensated demand functions are

$$x(p,u) = \frac{u a_0^{\varepsilon} p_0^{-\varepsilon}}{\left[a_1^{\sigma} p_1^{1-\sigma} \left(a_0^{\varepsilon} p_0^{1-\varepsilon} + a_2^{\varepsilon} p_2^{1-\varepsilon} \right)^{\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\varepsilon-\sigma}{1-\varepsilon}\right)} + \left(a_0^{\varepsilon} p_0^{1-\varepsilon} + a_2^{\varepsilon} p_2^{1-\varepsilon} \right)^{\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\varepsilon}{1-\varepsilon}\right)} \right]^{\left(\frac{\sigma}{\sigma-1}\right)}}, \tag{4}$$

$$c_1(p,u) = \frac{u a_1^{\sigma} p_1^{-\sigma} \left(a_0^{\varepsilon} p_0^{1-\varepsilon} + a_2^{\varepsilon} p_2^{1-\varepsilon} \right)^{\left(\frac{\sigma-\varepsilon}{1-\varepsilon}\right)}}{\left[a_1^{\sigma} p_1^{1-\sigma} \left(a_0^{\varepsilon} p_0^{1-\varepsilon} + a_2^{\varepsilon} p_2^{1-\varepsilon} \right)^{\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\varepsilon-\sigma}{1-\varepsilon}\right)} + \left(a_0^{\varepsilon} p_0^{1-\varepsilon} + a_2^{\varepsilon} p_2^{1-\varepsilon} \right)^{\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\varepsilon}{1-\varepsilon}\right)} \right]^{\left(\frac{\sigma}{\sigma-1}\right)}}, \tag{5}$$

and

$$c_2(p,u) = \frac{u a_2^{\varepsilon} p_2^{-\varepsilon}}{\left[a_1^{\sigma} p_1^{1-\sigma}\left(a_0^{\varepsilon} p_0^{1-\varepsilon} + a_2^{\varepsilon} p_2^{1-\varepsilon}\right)^{\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\varepsilon-\sigma}{1-\varepsilon}\right)} + \left(a_0^{\varepsilon} p_0^{1-\varepsilon} + a_2^{\varepsilon} p_2^{1-\varepsilon}\right)^{\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\varepsilon}{1-\varepsilon}\right)}\right]^{\left(\frac{\sigma}{\sigma-1}\right)}}. \tag{6}$$

By differentiation of (5) and (6), the compensated cross-price demand elasticity for each characteristic with respect to a change in the price of the untaxed good can be shown to be

$$\varepsilon_{10}^{c,\omega} = \frac{\partial c_1^{\omega}(p,u)}{\partial p_0}\frac{p_0}{c_1^{\omega}} = \sigma s_0^{\omega}, \tag{7}$$

and

$$\varepsilon_{20}^{c,\omega} = \frac{\partial c_2^{\omega}(p,u)}{\partial p_0}\frac{p_0}{c_2^{\omega}} = \sigma s_0^{\omega} + (\varepsilon-\sigma)\left(\frac{s_0^{\omega}}{s_0^{\omega}+s_2^{\omega}}\right), \tag{8}$$

where $s_0^{\omega} \equiv (p_0 x^{\omega})/Y$ is the expenditure share for the untaxed good (for a type-ω consumer), and $s_j^{\omega} \equiv \left(p_j c_j^{\omega}\right)/Y$ is the expenditure share on characteristic j. Hence, recalling that we have defined $\theta_1^{\omega} = s_1^{\omega}/(s_1^{\omega}+s_2^{\omega})$ to be the expenditure share of the first characteristic in the composite good purchased by the consumer, and similarly $\theta_2^{\omega} = s_2^{\omega}/(s_1^{\omega}+s_2^{\omega})$ for the second characteristic, by combination of (7) and (8) it can be shown that

$$\theta_1^{\omega}\varepsilon_{10}^{c,\omega} + \theta_2^{\omega}\varepsilon_{20}^{c,\omega} = \sigma s_0^{\omega} + (\varepsilon-\sigma)\left(\frac{s_0^{\omega}}{1-s_0^{\omega}}\right)\left(\frac{s_2^{\omega}}{s_0^{\omega}+s_2^{\omega}}\right). \tag{9}$$

Next, suppose $\tau_1 = \tau_2$, and that the social marginal utility of income, γ, is constant across consumers. Then (A. 36) and (A. 37) in the main text take the form

$$\frac{1}{F(\bar{\omega})}\int_{\omega^{min}}^{\bar{\omega}} Y(1-s_0^{\omega})\left[-\left(1-\frac{\gamma}{\lambda}\right) + \left(\frac{\tau_1}{1+\tau_1}\right)\left(\sigma s_0^{\omega} + (\varepsilon-\sigma)\left(\frac{s_0^{\omega}}{1-s_0^{\omega}}\right)\left(\frac{s_2^{\omega}}{s_0^{\omega}+s_2^{\omega}}\right)\right)\right] dF(\omega) = 0 \tag{10}$$

and

$$\frac{1}{1-F(\bar{\omega})}\int_{\bar{\omega}}^{\omega^{max}} Y(1-s_0^{\omega})\left[-\left(1-\frac{\gamma}{\lambda}\right) + \left(\frac{\tau_2}{1+\tau_2}\right)\left(\sigma s_0^{\omega} + (\varepsilon-\sigma)\left(\frac{s_0^{\omega}}{1-s_0^{\omega}}\right)\left(\frac{s_2^{\omega}}{s_0^{\omega}+s_2^{\omega}}\right)\right)\right] dF(\omega) = 0. \tag{11}$$

Because s_2^{ω} is increasing in ω, and for a sufficiently small revenue requirement s_0^{ω} is almost constant across consumers, $\sigma s_0^{\omega} + (\varepsilon-\sigma)\left(s_0^{\omega}/1-s_0^{\omega}\right)\left(s_2^{\omega}/s_0^{\omega}+s_2^{\omega}\right)$ is monotonically increasing or decreasing in ω if $\sigma \neq \varepsilon$. For any location of the line that has at least some consumers purchasing above and below $\bar{c}_2$, (10) and (11) cannot be simultaneously satisfied by $\tau_1 = \tau_2$. Hence, $\tau_1 \neq \tau_2$ and a tax-price notch is optimal. □

If $\sigma > \varepsilon$, in which case goods above $\bar{c}_2$ (which feature the second characteristic prominently) are less substitutable with the numeraire than are goods below $\bar{c}_2$ (which feature the first characteristic prominently), then $\tau_2 > \tau_1$, and vice versa. The social planner is willing to tolerate at least some distortion from bunching behavior to mitigate the distortion caused by substitution from taxed characteristics to the untaxed good.