MEASURING RECOGNITION OF THE PROFESSIONAL OBLIGATIONS OF MATHEMATICS TEACHING: THE PROB SURVEYS

Patricio Herbst[1]
University of Michigan
pgherbst@umich.edu

Inah Ko
University of Michigan
inahko@umich.edu

This paper shows validation data of an instrument that estimates high school teachers' recognition of four obligations of the mathematics teaching profession. Measures of internal consistency show three instruments reliably measure three of the four obligations. Factor analyses support a 3-factor model for the disciplinary obligation and 2-factor models for each of the individual, interpersonal, and institutional obligations. We inspected correlations between scores on those obligations and other individual teacher measures, including experience teaching and teachers' beliefs (measured with the survey by Stipek et al., 2001), and found very low correlations that suggest recognition of obligations and beliefs are different constructs.

Keywords: Instructional activities and practices, measurement, teacher beliefs, research methods, high school education

The Study of Mathematics Teaching: Background and Theoretical Framework

The last twenty years have seen increased attention in mathematics education research to the work of teaching and learning mathematics in classrooms. The work of teaching is seen as a phenomenon of interest to describe and explain in its own right, not the least because it can help us understand what it takes to improve the opportunity to learn for all children that teachers construct in classrooms. The research reported here contributes to theoretical and methodological progress understanding what the work of mathematics teaching is.

Educational researchers with an interest in teaching had in the 60s and 70s framed their work according to one of two paradigms. The *presage product* paradigm saw teaching as the expression of teacher characteristics and the *process product* paradigm instead saw teaching as the enactment of behaviors (Shulman, 1986). As researchers deepened their focus on what teachers do in classrooms, paradigms for research have grown more sophisticated, expanding interest in the individual teacher to their cognition and knowledge, as well as expanding interest on classroom behaviors to the meanings transacted through those behaviors. But the two somewhat different orientations have persisted. On the one hand some researchers have been interested in understanding the individual characteristics of teachers that play a role in the work

[1] This work was done with the support of NSF grant DRL- 0918425 to Patricio Herbst. All opinions are those of the authors and do not necessarily represent the views of the Foundation. This paper is an extended version of a Brief Research Report presented at the 2017 Annual Meeting of PME-NA, Indianapolis, IN. The survey items studied here were developed by the GRIP Lab at Michigan, under the direction of Patricio Herbst; the authors acknowledge the contribution of GRIP members who were involved in item development, cognitive pretesting, and peer validation, including Wendy Aaron, Justin Dimmel, Ander Erickson, Kristi Hanby, and Karl Kosko. Individuals or groups interested in using the PROB surveys may contact the first author.

they do in the classroom. In the PME community this orientation has taken the form of numerous studies of teachers' beliefs and teacher knowledge (e.g., Even, 2009; Gates, 2001; Leatham, 2004). On the other hand some researchers have been interested in understanding how teachers interact with students. In the PME community this has been taken the form of studies of classroom discourse, norms, and patterns of interaction (e.g., Cobb, 1998; Yackel. 2001). These two approaches have complemented each other, often drawing data from classroom observations, but seeing it alternatively as projection of an individual teacher's goals, beliefs, and orientations (Schoenfeld, 2010) or as adaptations of the teacher to the context of his or her interactions with the students and the content (Voigt, 1985).

Absent from consideration thus far has been attention to how the institutionalized settings of school frame both what it means to be a mathematics teacher and what it is required to do in mathematics teaching. Quite often our community has looked the interactions between students, teacher, and content without paying considering the environments where those interactions take place (Cohen, Raudenbush, & Ball, 2003). Yet those environments, schools and educational systems, are important, not the least because they warrant the encounter among teacher, students, and content. We contend that it is necessary to understand how those environments frame the work of teaching, particularly how they create expectations that frame the position of mathematics teacher.

Herbst and Chazan (2012) proposed the notion of professional obligations to identify those environmental expectations. They contend that the institutional position of the mathematics teacher, at least in US public schools, makes mathematics teachers accountable to stakeholders that look at mathematics teaching from four different perspectives, which Chazan, Herbst, & Clark (2016) call Knowledge, Client, Society, and Organization. From the Knowledge perspective, mathematics teachers are obligated to the discipline of mathematics--to communicate mathematically correct knowledge and engage students in mathematical practice. From the Client perspective, mathematics teachers are obligated to the individual student--to tend to their cognitive, emotional, physical, and other needs. From the Society perspective, mathematics teachers are obligated to the interpersonal collective of their class--to promote social values such as fairness and respect. From the Organization perspective, mathematics teachers are obligated to institutional policies and practices of the district, school, and department.

These obligations are hypotheses; confirmation involves, among other things, looking at how much mathematics teachers themselves recognize being under those obligations in contrast with other people that might not be so obligated. As a contribution to move toward this confirmation, we have undertaken to develop an instrument that can measure the extent to which teachers recognize each of the four obligations. Our interest in this paper is to describe the properties of a set of survey instruments (the PROB surveys) that purport to measure recognition of those obligations. If teachers' recognition of each of those obligations could be measured, that could be another variable that could help explain what teachers do with students in the classroom. We contend that recognition of obligations is a different construct than beliefs; we suggest also that professional obligations could help explain classroom instruction as more than response to interactional context. In what follows we describe the instruments and provide validation data, then we show data from correlating the responses to PROB with responses to the beliefs questionnaire designed by Stipek et al. (2001) as well as with other individual variables such as years of experience teaching.

The PROB Surveys

There are four PROB surveys, PROB-MATH, PROB-INDV, PROB-INTP, and PROB-INST, designed to measure recognition of the obligations to the discipline, the individual student, the interpersonal collective of the class, and the institutions of schooling respectively (see also Herbst, Dimmel, Erickson, Ko, & Kosko, 2014). The items of all four surveys (e.g., PROB-MATH and PROB-IND have 18 items each; PROB-INST has 20 and PROB-INTP has 29) have the same format. Each item asks participants to consider a statements that avowedly describes mathematics teaching (e.g., "Mathematics teachers take time to discuss school policies") and then asks participants to "Rate the degree to which mathematics teachers are expected, as professional educators, to act in the manner that this statement describes" using a 4-point Likert-type of scale that ranges from (1 = Teachers are **never** expected to act in this manner to 4 = Teachers are **always** expected to act in this manner). These items can be answered by teachers of mathematics at different levels as well as by non-teachers, all of them being asked to indicate their stances toward statements that say what a teacher of mathematics is purportedly expected to do. We developed the survey through several iterations that included brainstorming, item writing, internal and external vetting, piloting with teachers, and examining the collected pilot data using classical test theory (Crocker & Algina, 1986). Examples of the statements included in the items are provided in Figure 1. These questionnaires were developed using iterative procedures, relying on cognitive pretesting to make sure that they elicited considerations congruent to what we were aiming for: The rating prompt in particular was designed so as to produce the kind of thinking we were after, not whether the participant thought the actions described were good to do but whether the participant thought mathematics teachers were expected by others to act in the way described.

Obligation	Statement
Disciplinary	Mathematics teachers describe new procedures to students in ways that a mathematician would endorse.
Individual	Mathematics teachers are responsive to individual student's emotions.
Interpersonal	Mathematics teachers ensure that they address questions from all students.
Institutional	Mathematics teachers assign grades that represent how much of the curriculum students have learned

Figure 1. Sample items for each of the PROB surveys

Method

From March 2015 to January 2016, the PROB surveys were administered to a national-distributed representative sample of 497 U.S. high school mathematics teachers who were located across 47 states. Participants responded to test items as well as to survey questionnaires asking about their educational background and teaching experience. All questionnaires were administered through the Lesson*Sketch* online platform and participants completed the instrument by logging into this website. The majority of participants were Caucasian (83%) and female (59%). These figures are consistent with nationally representative data obtained from the NCES database. On average, participants had been teaching mathematics for 14.1 years (SD = 8.7, min = 1, max = 40), and had taken 14 college-level mathematics courses (SD = 7.25, min = 2, max = 40).

The analysis we present below looked at the internal consistency of the surveys and dimensionality of the constructs we attempted to measure. To save space we condense specific description of methods with their application into the analysis of the data.

Analysis

Reliability as Internal Consistency

To evaluate the reliability of the PROB surveys we examined all the items in each survey using indices from classical test theory (item-test and item-rest correlation) to make sure they would contribute to a consistent measure of the recognition constructs. To evaluate internal consistency of retained items we used both Cronbach's Alpha and the mean inter-item correlation. A commonly accepted rule for describing internal consistency using Cronbach's Alpha considers values over .7 as acceptable and over .8 as good (Kline, 2005) though a large number of items can artificially inflate the value of alpha (Cortina, 1993). The mean inter-item correlation is another way to assess internal consistency; in examining this statistic we looked for our set of items to meet a benchmark of between .15 and .25 suggested by Clark and Watson's (1995) for a broad and higher-order construct. The PROB surveys had the measures of internal consistency shown in Table 1. As can be noted, the disciplinary, interpersonal, and individual surveys had good internal consistency, but the institutional survey had only acceptable Cronbach Alpha and a low average inter-item correlation.

Table 1. Internal consistency of PROB surveys

Obligation	Number of items	Average inter-item correlation	Cronbach's Alpha
Disciplinary	18	0.273	0.8711
Institutional	20	0.1117	0.7154
Interpersonal	29	0.2226	0.8925
Individual	18	0.3082	0.8891

Dimensionality validation

Cronbach's Alpha is a good measure of internal consistency if it is possible to assume that items are unidimensional, that they are all equally good to measure the construct, and that their errors are uncorrelated. Because the possibility existed that one or more of those assumptions were not met, we examined the factorial structure of disciplinary, individual, interpersonal, and institutional scales using factor analysis. To determine the appropriate number of dimensions needed to explain the relationships among items in a given survey we first run exploratory factor analysis (EFA) for disciplinary, individual, and interpersonal surveys using Mplus (Muthen & Muthen, 2012) with 248 participants (half of whole sample of 497 participants who completed all items). We set up the software to extract, a range from 1 to 4 factors using WLSMV estimator, which is optimal for categorical variables (Muthén, DuToit, & Spisic, 1997) with the oblique rotation of GEOMIN. When more than a single factor was feasible, we used the likelihood ratio test (LRT) to check whether a model with N factors was significantly better than one with N-1 factors or one with N+1 factors. At that point, based on factor loadings for each item, item content, and pairwise polychoric correlations we decided, in some cases, to exclude poorly performing items. After deciding on the number of factors and the retained items, we ran a confirmatory factor analysis (CFA) with 249 participants (the other half of the sample). For institutional survey, a two-factor CFA model with the whole sample of 497 was conducted based on a hypothesized factor structure. In all CFAs, the WLSMV estimator, which is optimal for categorical variables with a small sample size was used to test the factor model. To find the best model we considered the Root Mean Square Error of Approximation (RMSEA) looking for a value of RMSEA less than 0.6, the Tucker-Lewis index (TLI) and the Comparative Fit Index (CFI), in both of these looking for values greater than 0.95 (Hu & Bentler, 1999). Factor means were set to 0 and factor variances were set to 1. The specific factor models tested are described in detail below.

A three-factor model, where 7 items have the same estimated loadings (discrimination) and two pairs of items are correlated due to the same wording, fits our PROB-DISC data well (RMSEA=0.064; CFI=0.955; TLI=0.949 with all standardized factor loadings greater than 0.5. Three suggested factors are interpretable in regards to the item statements (see Figure 2).

F1: Obligation to the discipline insofar as member of a community contributing to increase and extend appreciation of knowledge outside of the classroom (9 items)
F2: Obligation to the discipline insofar as responsible for its correct representation in classroom interaction (5 items)
F3: Obligation to the discipline insofar as responsible for its correct representation in study resources (3 items)

Figure 2. Factors of the disciplinary obligation

Using similar procedures we determined that the items in the PROB-INDV survey could inform a two-factor model of recognition of the individual obligation (RMSEA=0.048; CFI=0.975; TLI=0.971). The two individual factors we found are defined in Figure 3a. The items in the PROB-INTP survey and PROB-INST were also best accounted for by a two-factor model (PROB-INTP: RMSEA=0.051; CFI=0.954; TLI=0.949; PROB-INST: RMSEA=0.027; CFI=0.963; TLI=0.954) which are defined in Figure 3b (PROB-INTP) and in Figure 3c (PROB-INST).

The results above show a mostly positive outcome of the PROB surveys. It is of interest to investigate how these measures relate to other constructs being used in research on teaching, particularly other measures of teacher characteristics. Years of experience teaching showed significant positive correlation with the all three PROB-DISC factors though no significant correlations with either of the others.

Our participants had also taken the survey by Stipek, Givvin, Salmon, & MacGyvers (2001), which measures 7 different aspects of teachers' beliefs. We were interested in correlations between factor scores in the obligations (estimated from the validated factor models by Mplus) and mean scores in the seven factors of Stipek's questionnaire. They are shown in Table 2 and 3.

F1: Obligation to the academic needs of individual students (12 items)
F2: Obligation to the socio-emotional needs of individual students (6 items)

Figure 3a. Factors of the individual obligation

F1: Obligation to support social interaction among small groups of students (16 items)
F2: Obligation to support social interaction in the whole class (8 items)

Figure 3b. Factors of the interpersonal obligation

F1: Obligation to support policies for school-wide events and activities (4 items)
F2: Obligation to support school policies that concern classroom activities (9 items)

Figure 3c. Factors of the institutional obligation

Some of those correlations are significantly different from 0 and that suggests a need for further inquiry. For example the factor 3 of the disciplinary obligation that indicates obligation to the correct representation of mathematics in classroom interaction has a small correlation with the entity view of mathematical ability (i.e., the notion that there mathematical ability is fixed, there are some math people and others who are not math people). Likewise the disciplinary factors 3 and 2 (that indicates obligation to correct representation of mathematics in study materials such as textbooks) correlate significantly with the belief that mathematics learning aims at correct answers. The individual obligation's factor 1 (obligation to support academic learning needs of individual students) is significantly correlated with teachers' confidence teaching mathematics. And both factors of the interpersonal obligation, as well as factor 2 of the institutional obligation, are correlated with the belief that mathematics is enjoyable. Many of these significant correlations can be made sense of post-hoc. Yet, the most important finding is not that some significant correlations exist, but that all of these correlations are uniformly low. This suggests that recognition of these obligations not only is consistently defined but also does not measure the same thing as this measure of beliefs.

Table 2. Correlations between obligation factors and four belief factors (* indicates significantly different than 0 at the 0.05 level)

	PROB-INTP		PROB-INST		PROB-INDV		PROB-DISC		
	F1	F2	F1	F2	F1	F2	F1	F2	F3
Entity (high) vs. Incremental (low) view of intellectual ability	-0.06	-0.03	0.03	-0.02	-0.06	-0.02	0.01	0.05	0.13*
Correct answers (high) vs. understanding (low) as primary goal of learning.	-0.06	-0.02	0.00	0.00	-0.05	-0.03	0.06	0.11*	0.12*
Confidence in teaching math. A higher score associated with higher confidence.	0.09*	0.10*	-0.08	0.06	0.10*	0.04	0.06	0.08	.05
Teacher control versus some child autonomy in classroom lessons. A higher score indicates belief in teacher control.	-0.09	-0.08	0.01	-0.01	-0.09	-0.07	0.03	0.09*	0.14*

Table 3. Correlations between obligation factors and three belief factors (* indicates significantly different than 0 at the 0.05 level)

Extrinsic versus intrinsic motivation. A higher score associated with extrinsic motivation	0.02	0.05	0.05	0.08	-0.02	0.00	0.10*	0.13*	0.17*
Enjoyment of Math. A higher score associated with more enjoyment of math	0.10*	0.12*	0.04	0.16*	0.13*	0.05	0.03	0.03	.01
Math as a set of operations versus a tool for thought. A higher score tells us the answer is more associated with math as a set of operations	0.04	0.13*	0.00	0.06	0.09	0.08	0.18*	0.14*	0.16*

Conclusion

We warranted this instrument development on the need to understand how the environments in which mathematics instruction is deployed might establish expectations on the professional position of the teacher. These environments might be described in reference to cultural, economic, historical, institutional, and political characteristics and these considerations might help choose populations whose recognition of obligations might contrast with those of US high school mathematics teachers: Do teachers in other countries recognize these obligations at the same level as American teachers do? Questions like that one can help us understand the professional position of mathematics teachers in America. This understanding may in turn help explain instructional actions as more than expression of individual resources or response to interactional context. Along with Chazan, Herbst, and Clark (2016), we suggest instead that inasmuch as individuals who teach take on a position that is framed by these expectations, those expectations are likely to shape what individual resources are able to bring of themselves to instruction, and how how teachers respond to the interactional demands of the work of instruction.

References

Chazan, D., Herbst, P., and Clark, L. (2016). Research on the Teaching of Mathematics: A Call to Theorize the Role of Society and Schooling in Mathematics. In D. Gitomer and C. Bell (Eds.), Handbook of research on teaching (5th ed., pp. 1039-1097). Washington, DC: AERA.

Cohen, D., Raudenbush, S., & Ball, D. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis, 25*(2), 1–24.

Cobb, P. (1998). Analyzing the mathematical learning of the classroom community: The case of statistical data analysis. In A. Olivier& K. Newstead (Eds.) Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education. Vol. 1 (pp. 33-48).Stellenbosch, South Africa:University of Stellenbosch

Cortina, J.M. (1993). "What is coefficient alpha? An examination of theory and applications". *Journal of Applied Psychology*. 78: 98–104.

Crocker, L., & Algina, J. (1986). *Introduction to classical and modern test theory*. Orlando, FL: Holt.

Clark, L. A., & Watson, D. (1995). Constructing validity: Basic issues in scale development. *Psychological Assessment*, 7(3), 309-319.

Gates, P. (2001). Mathematics teacher belief systems: Exploring the social foundations. In M. Van den Heuvel-Panhuizen (Ed.), Proceedings of the 25th PME International Conference, 3, 17-24.

Herbst, P., & Chazan, D. (2012). On the instructional triangle and sources of justification for actions in mathematics teaching. *ZDM-The International Journal of Mathematics Education, 44*(5), 601–612.

Herbst, P., Dimmel, J., Erickson, A., Ko, I., & Kosko, K. (2014). Mathematics teachers' recognition of an obligation to the discipline and its role in the justification of instructional actions. In P. Liljedahl, C. Nicol, S. Oesterle, & D. Allen (Eds.), *Proceedings of the 2014 annual meeting of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 273-280). Vancouver, Canada: Simon Fraser University.

Leatham, K. R. (2004). Viewing Teachers' Beliefs as sensible Systems. North American Chapter of the International Group for the Psychology of Mathematics Education October 2004 Toronto, Ontario, Canada, (pp.944-950).

Muthén, B., du Toit, S. H., & Spisic, D. (1997). Robust inference using weighted least squares and quadratic estimating equations in latent variable modeling with categorical and continuous outcomes. Unpublished technical report.

Kline, R. B. (2005). *Principles and practice of structural equation modeling*. New York: Guilford.

Schoenfeld, A. (2010). *How we think*: *A theory of goal-oriented decision making and its educational applications.* New York, NY: Routledge.

Shulman, L. S. (1986). Paradigms and research programs in research on teaching: A contemporary perspective. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 3–36). New York, NY: Macmillan.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*(2), 213–226.

Voigt, J. (1985). Patterns and routines in classroom interaction. *Recherches en Didactique des Mathématiques, 6*(1), 69–118.

Yackel, E. (2001). Explanation, justification and argumentation in mathematics classrooms. In M. van den Heuvel-Panhuizen (Ed.), Proceedings of the 25th conference of the International Group for the Psychology of Mathematics Education (Vol. 1, pp. 9-24). Utrecht, The Netherlands: PME