TEACHERS' RECOGNITION OF THE DIAGRAMMATIC REGISTER AND ITS RELATIONSHIP WITH THEIR MATHEMATICAL KNOWLEDGE FOR TEACHING

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We examine responses from a national sample of high school mathematics teachers to a questionnaire, which had been developed to study teachers' recognition of a system of hypothesized norms that stipulate that geometry proof problems are to be posed using a diagrammatic register. We report on the psychometric properties of the questionnaire, as well as the relationship between these mathematics teachers' mathematical knowledge for teaching geometry (MKT-G) and their stances on breaching those norms. Although Herbst et al. (2013) hypothesized that the system consisted of five distinguishable sub-norms, the factor structure of the questionnaire suggested that two of those norms might not truly be distinguishable. We also found a positive and significant relationship between teachers' MKT-G and their stances on breaching two of the determined components of the system of norms.

Keywords: Geometry and Geometrical and Spatial Thinking, High School Education, Reasoning and Proof, Measurement

Behavior in social situations involves participants negotiating their way around norms-- ways of behaving in a given social situation that are routine and tacitly expected by those familiar with that situation. Norms are unremarkable when complied with, but elicit comment when breached (Garfinkel, 1963). Our group has been working on bringing these ideas to scale by using visual representations of instructional situations in the context of online questionnaires. This paper reports on a piece of such work, following earlier investigations of the diagrammatic register in geometry proofs (Herbst, Kosko, & Dimmel, 2013).

The present study tests the psychometric properties of Herbst et al.'s (2013) DRN (a.k.a., N3) instrument. We conjectured that teachers with higher MKT-Geometry scores (see Herbst & Kosko, 2014) might be more likely to strategically breach a norm of the diagrammatic register. We examined covariation between the DRN data and data collected with the MKT-G instrument. This work is significant as any efforts to improve mathematical practices in classrooms must come to terms with the norms that undergird ordinary practice (Cobb, Zhao, & Dean, 2009) and because the results suggest that increasing mathematical knowledge for teaching geometry might provide resources for teachers to depart from the norms of ordinary practice.

Theoretical Framework

Our work builds theory to describe the work of teaching. We assume that describing the work of teaching mathematics requires attention to the specificity of the subject matter transactions between students and teacher. To operationalize such specificity we model the transactions between particular items of subject matter knowledge and the tasks in which students can lay claim to them. *Instructional situations* (e.g., doing proofs, solving equations; Herbst, 2006) are modeled by articulating sets of norms that describe what teacher and students are expected to do in those tasks and what knowledge and skills the accomplishment of the task counts toward. The diagrammatic register norm states that proof problems are presented in a diagrammatic register. Herbst et al. (2013) decomposed it into five sub-norms: (SN1) Properties: The statement of the problem does not make explicit properties of betweenness, intersection, separation, collinearity, or concurrency, which are left for the diagram to communicate; (SN2) Diagram: The teacher provides a diagram for students to use while doing the proof; (SN3) Labels: The teacher assigns a proof problem with an accompanying diagram where all the points needed in the proof are labeled (but not necessarily all points); (SN4) Statement: The proof problem is stated using symbols and labels for elements of a diagram; (SN5) Accuracy: When a teacher provides a diagram accompanying a proof problem, the diagram is accurate.

These norms describe the defaults for setting up problems in the situation of doing proofs, they do not necessarily describe what would be optimal for student learning. Arguably, it would enrich the mathematical experience of students if they could be expected to do themselves what ordinarily would be done by the teacher. Various questions can be asked that contribute to this theory: To what extent do teachers consider adherence to the various aspects of this norm to be appropriate (in comparison with alternatives that might also be compelling)? And: To what extent is mathematical knowledge for teaching geometry related to a teacher's disposition to deviate from these norms?

Methods

Data

As described by Herbst et al. (2013), the DRN instrument (a.k.a., the N3 instrument) contains 30 items that target the 5 sub-constructs of the diagrammatic register norm. Each item asks participants to compare the appropriateness (using a 6-point Likert-like closed response format) of two possible ways of setting up a proof problem: one that we conjecture to be normative and another one that breaches one of the sub-norms (but is otherwise normative). Eight items were designed to measure SN1, five to measure SN2, five to measure SN3, five to measure SN4, and seven to measure SN5. The MKT-G instrument has been described at length by Herbst and Kosko (2014).

The DRN and MKT instruments, as well as a background survey that included a question about participants' years of experience teaching geometry, were administered as part of a larger study to a nationally-distributed sample of high school mathematics teachers, using the Lesson*Sketch* online platform. Participants were randomly sampled from more than 10,000 public secondary schools in the United States, identified using the NCES 2012-2013 School Universe data set. The effective sample of those who completed the background survey, the DRN, and the MKT-G instruments was 300 high school mathematics teachers. The minimum number of years of experience teaching high school geometry that any teacher in the sample had was 1, the maximum was 35, the mean was 6.75, and the standard deviation was 5.96.

Measures

Diagrammatic register norm endorsement. To determine whether the items that are in the DRN instrument measured five distinguishable sub-norms, we conducted an exploratory factor analysis (EFA), after recoding certain items so that high values in the scale of each item indicated departures from the norm. We split the sample into two random sub-samples, then conducted an EFA using the first and a CFA using the second (for more information on this approach, see Duffy, et al., 2012). Once we had determined the factor structure of the items, we created factor scores by taking the mean of each participant's rating of the items that loaded onto each factor. We also created a DRN total score by taking the mean of a participant's ratings of all

30 items. Finally, we calculated alpha scores and average inter-item correlations (IIC) to determine the internal consistency of the set of items in each factor.

Mathematical knowledge for teaching geometry (MKT-G) scores. We used a two-parameter Item Response Theory (IRT) model to create MKT-G scores, after removing four items that the Item Characteristic Curves (ICC) suggested would not discriminate well between individuals. The minimum MKT IRT score was -2.20, the maximum was 2.17, the mean was -0.0000043, and the standard deviation was 0.902.

Analysis

An Ordinary Least Squares (OLS) regression model was then created, in which the DRN total score was regressed on the MKT IRT score and participants' years of experience teaching geometry. After finding that the relationship between the MKT-G score and DRN total score was statistically significant, even when controlling for years of experience teaching geometry, we decided to create four other similar stepwise OLS regression models, each of which used one of the DRN factor scores, rather than DRN total score, as the outcome variable. This was done in order to understand whether participants' stance on breaching the DRN was dependent on which of the sub-norms was breached.

Results

In terms of the EFA, we considered both the criterion of retaining factors with eigenvalues larger than 1 (Kaiser, 1960) and the criterion of retaining components above the point of inflection on a scree plot (Cattell, 1966), as well as factor loadings and fit statistics. Together, these suggested that four factors undergird the DRN instrument. The CFA confirmed that structure to the extent that no standardized item loading was less than 0.3 in the CFA and the fit statistics were reasonable - RMSEA: 0.066, CFI=0.853, TLI=0.841, SRMR=0.091, just short of the typical cut-points of RMSEA<=0.05, CFI>=0.95, TLI>=0.95 and SRMR<=0.6 (Hu and Bentler, 1999). According to that model, the items that were designed to target SN1, SN3, and SN5 loaded onto three factors, in the way that they were expected to. However, the items designed to target SN2 and SN4 loaded onto the same factor. We will hereafter refer to those factors as S1:PRO, S3:LAB, S5:ACC, and S2S4:DNS, respectively.

The means of the DRN total score, SI:PRO, S2S2:DNS, S3: LAB, and S5:ACC scores (described earlier) were 2.86, 3.32, 2.56, 3.38, and 2.42, respectively. The standard deviations of those scores, in the same order, were 0.54, 0.94, 0.89, 0.56, and 0.78. Cronbach's alpha for the DRN total score and each of the factor scores ranged from 0.6909 and 0.8522, and their average inter-item correlations (IIC) ranged from 0.1723 to 0.4344, both of which suggest that the entire set of items as well as the set of items that loaded onto each factor had good internal consistency (Clark & Watson, 1995).

The main take-away from the regression models is that there is a significant, positive association between teachers' comfort with breaches of the DRN and their MKT-G, which seemed to be due to the also significant and positive association between their MKT-G and comfort with breaches of the S2S4:DNS and S3:LAB components of that norm, independent of their years of experience teaching geometry. When the DRN total score was regressed on MKT and years of experience teaching geometry, the MKT regression coefficient and associated standard error were 0.14 and 0.04. When the S2S4:DNS score was used instead they were 0.36 and 0.06. When S3:LAB score was used, they were 0.08 and 0.04.

Discussion

A main finding of this study is the discovery of the somewhat unexpected, but nonetheless comprehensible, factor structure of the DRN instrument. Another is the discovery of relationship between teachers' endorsement of the diagrammatic register norm (as well as two of its subnorms) and their level of MKT. Upon reflection, we imagine that SN2 and SN4 items loaded onto the same factor because all of our proof problems that complied with SN4 (the expectation that the proof problem will be stated using symbols and labels for elements of a diagram) included a diagram and, therefore, complied with SN2 (the expectation that the teacher will provide a diagram for students to use while doing the proof).

In terms of MKT being a significant predictor of S2S4:DNS and S3:LAB, but not S1:PRO or S5:ACC, we would argue that understanding this result also requires careful consideration of each of the 5 subnorms. For example, we would expect that the Accuracy subnorm (SN5) was fairly easy to recognize, regardless of one's MKT, as there is a sense among teachers that even accurate diagrams can be misleading, and so providing inaccurate diagrams could make something that is to be regarded with suspicion even more problematic. SN1 (properties) deals with subtle issues of positioning. On the other hand, SN2 and SN4 are arguably more directly related to the kind of mathematical knowledge teachers readily have.

Acknowledgments

The research reported in this article had the support of the National Science Foundation, through grant DRL-0918425 to P. Herbst. All opinions are those of the authors and don't necessarily reflect the views of the Foundation.

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