How Do Mathematics Teachers Manage Students' Responses In-The-Moment?

by

Annick O.T. Rougée

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Doctoral Committee:

Professor Deborah Loewenberg Ball, Chair
Professor Hyman Bass
Professor Kevin F. Miller
Assistant Professor Matthew S. Ronfeldt

Annick O.T. Rougée

arougee@umich.edu

ORCD id: 0000-0002-3141-3698

DEDICATION

To my parents, Marc and Christiane

Merci pour tout!

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When I was younger, I used to read abnormal-psych books for fun and subject my poor little sister to couch sessions so I could play psychologist. In high school and as an undergraduate, I toyed with the idea of going into medicine but instead pursued physics and later in graduate school, materials engineering. From there, I realized that my hobbies over the past eight year—including teaching physics labs, leading math homework review sessions and working with elementary students in Phoenix—were more than hobbies; they were an indication of a deeper calling to teaching. The long, winding path that brought me to teaching and eventually to Michigan and the work I engaged in for my dissertation is built on many interests and many experiences. The research that I was fortunate to engage in for this dissertation feels like it has brought all of these various interests and experiences—in psychology, physiology, engineering (signal processing of EDA data!), and teaching—together. More than any other work I have done before this, it more comprehensively captures my interests and the type of scholar I want to become.

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LIST OF ABBREVIATIONS

PCK pedagogical content knowledge

MKT mathematical knowledge for teaching

PST pre-service teacher

CCK common content knowledge

KCT knowledge of content and teaching

IRE initiate, response, and evaluate protocol

GTE general teaching efficacy

SRS student response sequence

SAM Self-Assessment-Manikin

TR teacher response

TRC Teacher Response Coding scheme

MU mathematical understanding

MP mathematical point

ABSTRACT

A ubiquitous and complex challenge for mathematics teachers is managing students' responses in-the-moment during whole-class instruction. In-the-moment, managing students' responses involves a variety of internal resources—including specialized content knowledge for teaching, productive beliefs about teaching and learning mathematics, and the ability to self-regulate ones' own emotional state and anxiety—in addition to skill in perceiving and interpreting important aspects of students' answers. Beyond these specialized internal resources and skills which make this complex work, managing students responses becomes more salient to research when one considers that the public nature of this interaction means that how a teacher handles these situations can have important implications for the learning of *all* students in the class.

In this three-paper dissertation, I explore the complexities of managing students' responses in-the-moment. In the first paper, I explore a special case of managing students' responses when a student response is perceived by a teacher as incorrect. Here, I outline the important consequences of managing students' responses for students and explore why this might be difficult and challenging work for teachers. For the second and third papers, I used an interactive-video based teaching simulation to first explore *how* teachers responded to students (in paper 2) and then to consider what factors (such as various internal resources a teacher might have) might be related to various features teachers' responses (paper 3).

In paper 2, I describe the ways in which teachers responded to apparently correct and incorrect student answers. In general regardless of whether the student answer

is apparently correct or incorrect, teacher responses tend to go back to the student who provided the answer. However, the ways in which the same student was asked to think about particular mathematics depended on whether the student answer was apparently correct or incorrect. When a student answer was apparently incorrect, teacher responses predominantly asked questions to get the student to correct or fix their response. In contrast, when a student answer was apparently correct, the majority of teacher responses would ask the student to elaborate on some aspect of his/her response. A small subset of teachers in this sample did not exhibit these same, clear evaluative patterns and I discuss trends in their responses that could inform ways in which teacher education supports teachers in responding to students.

In paper 3, I quantitatively investigate how teachers responses to students relate to teachers' individual characteristics, including their anxiety, teaching experience, beliefs and mathematical knowledge for teaching, as well as their self-reported emotional reaction to student responses. The analyses showed that teachers' self-reported emotional reactions, with the exception of sense of control, had almost no significant correlations to any aspects of their responses to students. Additionally, teachers' anxiety was generally negatively correlated with aspects of teacher responses while years of experience was positively correlated with features of teachers' responses. Findings indicate that exploring teachers' anxiety is a promising avenue for understanding teaching performance and that important methodological considerations arise when attempting to assess an action-related competence such as managing students' responses.

CHAPTER I

Introduction

Motivation for Dissertation Study

During my three years as a field instructor for secondary mathematics teachers at the University of Michigan I was fortunate to work closely with both novice and experienced teachers discussing and observing lots of mathematics teaching. It was during those experiences that I began to notice that many of the preservice teachers I worked with had very distinct patterns in the way they responded to students. Specifically, these preservice teachers clearly validated correct answers (e.g., "Great", "Nice job", "Exactly") and negatively evaluated seemingly incorrect answers (e.g., "Not quite", "Anybody else?", "Interesting") in ways that were not necessarily productive for student learning. In the post-observation meetings where the preservice teacher and I would debrief their instruction we would discuss these patterns and their potential consequences for students. The preservice teachers were always eager to improve and we would brainstorm alternative ways in which they could manage responses from students. Typically though, despite their genuine desire to change and their newfound knowledge of alternative strategies, the next time I would observe them I would see the same evaluative patterns; nothing seemed to change. This apparent paradox—between their desire and knowledge, and their performance—perplexed and troubled me until I came across Feldon's (2007) article on automaticity in teaching.

In this article, Feldon (2007) describes dual-process models of cognition (which stipulate that human cognition is comprised of both automatic and conscious processes) in relation to empirical findings in teaching. This dual-process lens resonated with my own experiences teaching and observing teaching in a way that prior work on teacher decision-making had not and it prompted me to seriously reconsider my conceptualizing of what it takes to teach, let alone teach well. In particular, it offered me both a more generous way to view teaching and suggested other disciplines to scour for answers in my reconceptualization of teaching.

First, instead of assuming that teaching is an entirely conscious endeavor, applying a dual-process lens, which means considering how unconscious processes play a part in complex human endeavors, humbled me to be more generous in my observations of others' teaching. Instead of wondering if my preservice teachers were being disingenuous in their desire to improve, I could step back and appreciate that they were instead doing what humans have necessarily evolved to do: in life, and in complex performance situations in particular, we automate as much as possible the ways in which we operate. As they were standing in front of the classroom, trying to navigate the intellectual and emotional needs of 30 or more students in addition to their own jitters, these preservice teachers might not have been able to consciously focus on every single response they provided to students. They had to automate some of their work and it was not unreasonable for them to perhaps default to the evaluative response patterns they had likely experienced in their own schooling. Rather than placing the onus for change squarely on the individual teacher, for me this new dual-process lens forces me, as well as other teacher educators, researchers and teacher education in general, to carefully reconsider how to train teachers given this reality of human cognition.

Second, Feldon's article spurred me to look into dual-process theories and, more broadly, into the disciplines of psychology and psychophysiology to begin to explore what other fields had learned about in-the-moment decision-making that might help inform my reconceptualization of teaching. Though not an exhaustive look by any means, these forays into other fields shaped my thinking about teaching and scholarship in teacher education more broadly. In particular, it encouraged me to explore the ways in which affect—a dimension of the work of teaching that is often neglected in teacher education programs—could impact teaching performance and to find ways to argue for why the affective dimension of teaching warrants more attention. Further, it pushed me to consider other methodologies, including incorporating different measures (such as electrodermal activity or sweat, and self-assessed emotional reactions), and to design a teaching simulation that would allow me to explore how teachers respond to students and to begin exploring the various factors (including knowledge, anxiety and beliefs) that might explain why they responded as they did. It also led me into literature from Europe, and Germany in particular (e.g., the Max Planck Institute for Human Development), that informed a great deal of the work presented here and reminded me that scholarship and progress in education are international, not just national, pursuits.

The Dissertation Study

This dissertation study explores how mathematics teachers manage students responses in-the-moment. In addition to this introductionwhich provides some personal background relevant to this workthe dissertation includes three scholarly articles. The first article explores a special case of managing students responses, namely when a student provides a mathematical response that is apparently incorrect. In this conceptual piece, I use a vignette to illustrate what I mean by managing students apparently incorrect responses and then describe four ways in which this is consequential for students. I then explain why this might be difficult work for the teacher and explore what it might entail to manage students apparently incorrect responses. In the second paper I explore how teachers respond to three different student responses: one apparently correct response and two apparently incorrect responses. In the third paper, I explore whether there are relationships between teachers' emotional reactions to student answers and the characteristics of their responses, as well as whether teachers' internal resources (such as their knowledge, beliefs and anxiety) might have associations with characteristics of their responses.

CHAPTER II

Managing Students' "Apparently" Incorrect Mathematical Responses: A Conceptual Framework

Managing Students' "Apparently" Incorrect Mathematical Responses: A Conceptual Framework

What do I mean by managing "apparently" incorrect student responses?

Every teacher has experienced the challenge of navigating a student's incorrect response during whole-class instruction. This situation can become particularly dicey for the teacher and students when a student response seems incorrect but might not actually be wrong; in other words, when a student response is "apparently" incorrect. In this paper I explore how teachers manage students' apparently incorrect responses and discuss the challenges and complexities of this ubiquitous and high-stakes situation.

To begin my exploration, I present a vignette from a high school, accelerated geometry class to illustrate an "apparently" incorrect student response. The vignette serves two purposes. First, it provides a starting point to develop a shared understanding of "apparently incorrect". Second, the vignette includes additional information (e.g., some of the teacher's thoughts, feelings and actions) that impacts how teachers manage these situations. The complex and varied nature of this information—which includes the many visible actions as well as the invisible factors that contribute to how the classroom interaction in the vignette unfolds—is why I have chosen the verb "manage" (as opposed to "respond" or "decide", for example) to describe the work of teaching in this situation. After analyzing the interaction in the vignette, I discuss four potential implications of how teachers manage apparently incorrect responses for students.

Next, I turn my attention to the teacher and describe four ways in which managing these responses can be difficult. Here I present a model to illustrate the complexity of this in-the-moment decision-making. I then use this model to detail the different cognitive, affective and motivational-volitional resources as well as the situation-specific skills that managing students' apparently incorrect responses might entail. Finally, I conclude by discussing possible implications for teacher education.

Vignette.

In the following vignette, imagine you are observing a high school, accelerated geometry class during a lesson on the perimeter and area ratios of similar figures. On this occasion, you have access to both the visible actions that occur and the invisible factors that might be at play. Specifically, you can see what the students and teacher say and do, including some of the teacher's facial expressions (offset from the rest of the vignette below by brackets) and you have insight into some of the invisible factors at play, including the teacher's thoughts and feelings (denoted by italics).

As the lesson begins you can tell the teacher is a little anxious. Although she is strong in her mathematical content knowledge (i.e., she knows how to easily calculate the perimeters and areas of these figures), she is nervous about how students might take up this content and starts the lesson slowly. She begins by reading aloud some notes on the board about the relationship between the ratios of perimeters and areas of similar figures that students jot down as she speaks. She then asks students to work individually on the following problem (Figure 3.1):

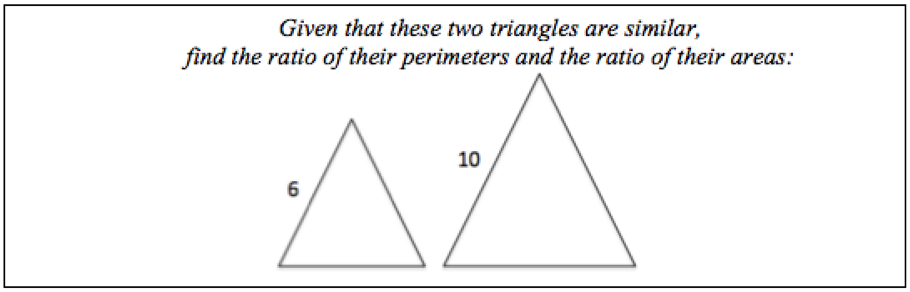


Figure 2.1: Problem given to students.

As students work, the teacher circulates and notices that most students are able to quickly set-up the perimeter ratio (6:10 or 3:5) and area ratio (9:25) correctly. She feels relieved. After a few minutes, she calls on a student who provides both ratios correctly. She thinks, Okay, it seems like they get this so far, so how can I push them a little moremaybe I can go for something more general. Yeah, to think about why this works, to consider how perimeter is one-dimensional and area is two-dimensional...So she asks the class, "So, now I want you guys to take a second and think about why this relationship makes sense. Why do we go from 3 to 5, to 9 to 25? [hhmmmaybe that's not clear, I should reword that] So we're squaring it, why does this make sense? Think about it, talk to each other and when your group has an answer, signal and I'll keep an eye out"

As students start to discuss this question with their group members, the teacher stands at the board looking around the room. Sometimes, Jean and his group members take a few minutes to get started so she keeps an eye on them until she sees they are working together. She mulls over her ideas, preparing herself for when she brings students back together to talk. Okay, perimeter is one-dimensional and area is two-dimensional, area is the square of length, so squaring it makes sense since you're going from one-dimensional to two-dimensional. She feels good. This was a good question to ask, it will push them to think beyond the procedure and to connect this to other work they've done. After about two minutes, it seems to her that most groups have had a chance to talk and she confirms this by quickly checking in with a group of students. They confirm that they are ready to answer, so she calls the class back together and says, "Okay, raise your hand if you would like to share your thoughts about my question?"

She waits several seconds and a few hands go up. She notices that Jean has his hand up and thinks Great! He's got an answer. Jean hardly ever volunteers so she's excited that he seems to have something to say. "Yes, Jean?" Jean responds, "Okay, if you think of it like boxes [she furrows her brow, What is he talking about, there's nothing 3-D going on here?!] and you have a box that's 3 by 3 and a box that's 5 by 5, then there's 9 boxes in the box that's 3 by 3 [she nods, okay, that's true] and 25 boxes in the box that's 5 by 5 [she nods, yup, 5 times 5 is 25 but her spirit sinksuurggghhh, he's not answering my question, he's not generalizing, where's the one-dimensional and two-dimensional stuffhe doesn't get it. She clenches her jaw, her heart starts to race and her palms begin to sweat a little...how do I respond to this???!!!]. So the 9 to 25 rather than 3 to 5 cause it's...yeah"

Then, without hesitation, she replies, "Okay, that's an interesting answer [I don't want you to feel bad Jean, thanks for trying] but it doesn't necessarily answer why. When we talk about units squared we're talking about area, it's 2-dimensional and the perimeter is 1-dimensional, that's why the ratio of the areas is the square of the ratio of the perimeters because area is 2-dimensional." She scans the room and deems that the students' facial expressions convey that they understand her explanation but she misses Jean's perplexed expression. Okay, she thinks, let's move on to a more complicated problem. "Alright then, let's go ahead and try a few harder problems. Everyone grab a book and turn to page 56 please."

A Situation in Teaching: Apparently Incorrect Responses.

The vignette above illustrates a frequent, often important and complex situation in mathematics classrooms: a moment during whole-class instruction when a student (e.g., Jean) provides an apparently incorrect response. To understand the importance and potential complexity of this situation first necessitates common ground about what is meant by "apparently incorrect". To establish this common ground, I take a closer look at Jean's response and the teacher's subsequent reply.

Several aspects of Jean's answer provide evidence of insightful mathematical understanding. In particular, in his response there is evidence that Jean has not only a clear understanding of how to compute areas but also a conceptual understanding of the meaning of area. This is evident in his description of the number of unit squares (9 and 25) that comprise the areas of the two squares (3 by 3 and 5 by 5). Specifically, he is building on the tacit assumption made in school mathematics that the reference unit of length is the unit interval [0,1] on the number line and hence, the unit of area measure is the unit square. To measure the area of some plane region (such as a 3 x 3 square), Jean is providing the number of unit squares that fit, without overlap, within that region (using what he knows about computing the area of a square to get "9 boxes"). He then contrasts this unit of measure of area with that of the unit of measure for length ("9 to 25 rather than 3 to 5"). The connections he makes between length and area throughout his example and his contrasting of the area and perimeter ratio at the end of his response indicate he is indeed, correctly answering the teacher's question. However, though Jean's answer is correct, there are two surface features of his response that might make it more challenging to follow.

First, Jean has made a lexical selection error, specifically in his choice of the words "box" and "boxes." Jean uses the word "box" not in reference to a 3-dimensional object but rather to refer to the 2-dimensional squares in his example (one with side length 3, the other with side length 5). Similarly, he uses the word "boxes" rather that "unit squares" to describe the total number of unit squares that comprise the area of each larger square ("9 boxes" and "25 boxes"). Second, he does not provide a general justification for why the ratio of the areas of similar figures is the square of the ratio of their perimeters. Instead, he provides a concrete example using squares (3 x 3 and 5 x 5) and unit squares ("9 to 25 rather than 3 to 5") to explain his reasoning. Despite this minor issue in language and lack of generalization, Jean's explanation clearly indicates understanding and sense making. In other words, though his response is not quite correct, it is also not completely incorrect either. Essentially, responses from students can be thought of as occurring on a continuum of correctness (see Figure 3.1).

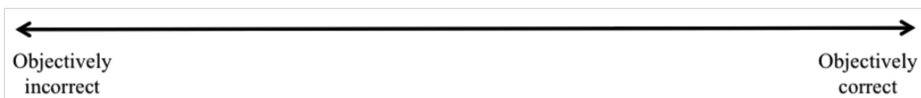


Figure 2.2: Continuum of correctness

At the far right of the continuum are responses that can be objectively deemed correct. These responses are likely to contain underlying mathematical ideas and reasoning that are procedurally and conceptually sound; and, further, these ideas are presented articulately, with precise mathematical language. These correct responses, far to the right, are likely to be objectively evaluated as correct. On the far left of the continuum, are responses that can be objectively deemed incorrect. In these types of responses, the underlying mathematical procedures and concepts are troublesome and deeply, logically flawed. Additionally, the presentation of the ideas is problematic due to ambiguous pronouns, incorrect terminology, or confusing uses of mathematical language. At this end of the continuum, responses are likely to be objectively evaluated as incorrect. Most student responses, like Jean's, tend to fall somewhere in-between the two ends of this continuum and hence, require some interpretation and sense-making on the part of the teacher, as we will see next.

Once Jean has responded, the teacher needs to manage Jean's response. In the vignette, she manages this situation by responding as follows: Okay, that's an interesting answer but it doesn't necessarily answer why. When we talk about units squared we're talking about area, it's 2-dimensional and the perimeter is 1-dimensional, that's why the ratio of the areas is the square of the ratio of the perimeters because area is 2-dimensional.

Though the teacher begins her response by acknowledging Jean's answer is "interesting" she quickly proceeds to how his answer is wrong ("but it doesn't necessarily answer why"). She then continues by providing what she likely considered the correct answer, one that uses the terms "1-dimensional" and "2-dimensional" (terms she had in mind before Jean responded and that she was likely fishing for with her original question). Hence, though she begins with what could have been a positive evaluation of Jean's response, what follows clearly indicates that she has interpreted Jean's answer to be wrong. To the teacher in the vignette, Jean's response is more towards the left-end of the continuum, closer to incorrect. It is "apparently" incorrect.

Again, although there are times when a student's response may be objectively incorrect (e.g., "2 plus 2 is 5"), the vignette above illustrates that it is not always an objective act to manage students' responses. As I described above, students' answers typical fall on a continuum between objectively incorrect and objectively correct. Hence, a teacher must hear and make sense of (consciously or unconsciously) what a student has said, a process that occurs through the filter of a teacher's experiences, knowledge, emotional state, and so forth. In other words, teachers manage students' response through a subjective lens and hence, a judgment of correctness cannot always be objective. As in the vignette above, seen through her filter, the teacher perceives Jean's answer as wrong and her response indicates that she is managing an apparently incorrect (to her) response.

Section summary: Managing Students' Apparently Incorrect Responses.

With the vignette and this exploration of Jean's and the teacher's responses, I hope that I have illustrated for the reader what I mean by "apparently" incorrect. Specifically, my aim is to have established that students say lots of things (many of which fall on a continuum, somewhere between objectively correct and objectively incorrect) and that making sense of a student response is not a completely objective act. Additionally, I hope to have clarified the deliberate choice of the verb, "manage", to capture both the visible actions a teacher takes in response to a student (i.e., what she says or does, or doesn't do) as well as the invisible factors that contribute to these visible actions (i.e., the affective reaction, the cognitive resources and mental processes at play). With this shared context to build from, I turn next to considering the following question: Does it matter how teachers manage these moments?

Why Does it Matter?

The ways in which a teacher manages students' apparently incorrect responses during whole-class instruction has important implications for the learning of not only the student who has responded but also the rest of the class. Since the interaction occurs during whole-class instruction it is public and hence, a teacher's subsequent actions impact not only the student who provided the response but also the rest of the students in the class who are observing this interaction take place. This interaction can have important consequences for students' through the four important messages it can send, implicitly or explicitly, about: (1) what mathematics students should learn, (2) what mathematics is and hence what it means to do mathematics, (3) the role of errors in mathematics learning and in learning more broadly, and (4) who can learn mathematics.

The consequences of these messages are best illustrated in the case when, like in the vignette above, a teacher negatively evaluates the student's response. This type of interaction pattern, where the teacher initiates a question, gets a response from a student and then evaluates that response (IRE) (Mehan, 1979) is predominant in U.S. classrooms (Franke et al., 2007). Hence, the case in the vignette is likely representative of a common occurrence in US classrooms and the potential consequences I discuss below have salience and urgency for students currently experiencing this in mathematics classrooms.

Why does it matter: What mathematics students should learn.

A very immediate impact of the teachers' actions is on what mathematics students will learn. On a moment-to-moment level, how a teacher manages students' responses indicates what mathematics is correct or incorrect and hence, conveys to students what mathematics they should be learning. For example, in the vignette above, students experiencing this interaction between the teacher and Jean might take away the idea that using concrete examples in the process of abstract mathematical generalization is ill advised. Polya (1945), Lakatos (1976) and many other prominent mathematicians are likely to cringe at this idea that concrete examples (i.e., inductive reasoning) have no part in abstract mathematical generalizations. Another way to make sense of this problematic message is to think of what happens in the opposite occurrence, when a student response is apparently correct. A classic illustration of this occurrence is Erlwanger's (1973) interview with Benny, a 12-year old student making "better than average" progress through a behavioral, initiate, response, and evaluate protocol (IRE) structured mathematics program. Though Benny's answers on the automated program seem to indicate he understands how to add and multiply fractions and decimals, upon closer inspection it is clear he does not. For example, in solving 0.7 times 0.5 Benny provides the correct answer of 0.35 but explains the position of the decimal as follows, "because there's two points, one in bothin front of each number; so you have to add both of the numbers left1 and 1 is 2; so there has to be two numbers left for the decimal" (Erlwanger, 1973, p.8). Though Benny's answers are apparently correct, they conceal deep misunderstandings and have actually reinforced some incorrect mathematical thinking. In other words, how a teacher manages students' apparently incorrect (or correct) responses signals to students what mathematics they should learn and has the dangerous potential, as in the case of Jean and Benny, to lead to mathematical misconceptions and misunderstandings.

Why does it matter: What mathematics is and what it means to do mathematics.

Beyond the moment-to-moment interactions when a teacher predominantly engages in an IRE discourse pattern in a mathematics classroom, she can send implicit or explicit messages at a higher level about what mathematics is and hence what it means to do mathematics. Specifically, this pattern of interaction places the emphasis on getting the right answer, rather than developing understanding and exploring the methods or strategies used to produce an answer (Franke et al., 2007, p.229). This focus on the "right" answer privileges procedural competence rather than meaningful, conceptual mathematical understanding. It sends messages to students about what the discipline of mathematics is—a set of rules to be memorized rather than a connected discipline that can be reasoned through (H. Ball D. L. Bass, 2000)—and hence what it means to do mathematics.

For example, rather than viewing mathematics as a connected web of interrelated concepts, Benny understands mathematics as a set of rules, " invented 'by a man or someone who was very smart' " (Erlwanger, 1973, p.x). For Benny, rather than thinking of doing mathematics as a sense-making activity, he describes learning mathematics as a "wild goose chase" (p.53) in which he is searching for the right, "magic" answers (p. 54). In the vignette, Jean may similarly feel frustrated and disillusioned with mathematics when his answer is so quickly dismissed as incorrect. He too may start to wonder if mathematics is a disconnected set of rules and facts, a body of knowledge that is to be transferred into his mind, rather than an activity of making sense and discovery. He might start to believe that doing mathematics is just about getting the right answer.

Why does it matter: The role of errors.

Another important consequence of this interaction is what it tells students about the role of errors in mathematics learning and in learning more broadly. It could be that, at this point, after reading about Benny and Jean's thoughts, some readers may wonder about what role the student plays in all of this and, in particular how students' own personalities and idiosyncrasies play a role in making meaning of these interactions. Indeed, there has been quite a bit of work over the past decade about individuals' differing mindsets and how these shape the ways in which they make different meanings of the same situations (e.g., the growth versus fixed mindset work of Dweck (e.g., Dweck, 2008)). Specifically, a student's orientation towards their learning goals (mastery for one's own competence versus performance to demonstrate high-competence or avoid showing low-competence) and their academic self-concept (i.e., their beliefs about their domain specific abilities) are highly correlated to their behaviors in response to making mistakes. However, recent work challenges the idea that students' adaptive or maladaptive motivational and behavioral patterns around errors and failure can be solely explained by their individual attributes (Steuer et al., 2013).

Instead, this recent body of research demonstrates that, above and beyond students' individual attributes, a classroom's "error climate" significantly contributes to understanding student's behaviors and motivation around errors (Steuer et al., 2013). The multi-faceted construct of a classroom's "error climate" includes the ways in which teachers might set-up classrom norms (e.g., making the classroom a safe-place to make mistakes and take risks) that support a positive error climate as well as how a teacher navigates students' errors in-the-moment. A teacher's in-the-moment navigation of errors includes a teacher's tolerance of those errors, the ways in which errors are tied to summative assessment in early learning phases, support following an error, and a teacher's disapproving reactions to errors, all of which have important implications for the error climate of a classroom (Steuer et al., 2013). This perceived error climate is predictive of students' "behavioral and cognitive engagement in academic work (in terms of the quantity and self-regulation of their effort)" (p. 207). In other words, while students' individual affective and behaviorial orientations towards errors are important, the perceived error climate (positive or negative) of a classroom also significantly impacts how students react to errors.

This error climate can likely reinforce students' mindsets around the meaning of errors in mathematics—as "springboards for inquiry" (Borasi, 1994) or as demonstrations of incompetence—and the ways in which they cope with errors and failure in general. Reinforcing negative perceptions around errors can reinforce maladaptive coping mechanisms such as avoiding challenges or taking risks and hiding mistakes in addition to increasing students' negative affect towards a particular subject or to learning and failure (Steuer et al., 2013; Tulis, 2013; Rybowiak et al., 1999; Dweck, 2008).

Why does it matter: Who can learn mathematics.

A final message that can be sent to students through the manner in which a teacher manages apparently incorrect responses is about who can learn mathematics or who cannot (or should not). One way to understand how this might be the case is to re-read the vignette and replace the student "Jean" with "Shareese"—a Black, female student. In this version of the vignette, with the student's gender and race now fore grounded, it is likely that the teacher's quick, corrective response feels heavier and more consequential than when the student was simply "Jean". This qualitative difference is likely due to the fact that this micro-level interaction speaks to macro-level educational inequities across race and gender (to name a few). At the macro-level there is a well-documented absence of people of color, and females in particular, in STEM fields and a so-called mathematics "achievement gap" between Black and White students that continues to widen (e.g., Lubienski, 2002; Flores, 2007). As researchers have turned their focus on understanding the underlying mechanism that give rise to the "achievement gap", they have begun to reframe it as an "opportunity gap" (Flores, 2007). This change in naming the problem more accurately reflects the underlying mechanisms at the school and classroom level (such as student-teacher interactions) that seem to be emerging as likely explanations for the differences in the mathematical performance of Black and White students.

Specifically, research is accumulating evidence of the alarming disparities in the quality of mathematics education that students of color receive in the United States (e.g., Ladson-Billings, 1997; Lubienski, 2002; Flores, 2007). At the school level, Black, LatinX, and low-income students are more likely to be placed in remedial mathematics courses (already placing them at a disadvantage to their Anglo and Asian peers) and have inexperienced and under-qualified mathematics teachers (Flores, 2007). At the classroom-level, these students are more likely to have teachers with deficit mindsets who believe that students' lack of achievement is explained by "student characteristics such as differences in motivational levels, work ethic, and family support" (Bol & Berry, 2005, p.32). These systemic issues then translate into lower-expectations for students and problematic mathematics instruction.

In mathematics classrooms these issues manifest themselves through instruction that is focused on building basic computational skills and learning procedures rather than engaging in higher-level, cognitively demanding tasks and assessments (e.g., Ladson-Billings, 1997; Means & Knapp, 1991). At the level of student-teacher interactions, which are likely a mechanism through which the macro-levels disparities are perpetuated (Battey & Leyva, 2016), less research exists. However, the research that does exist can illuminate how something as small as how a teacher manages different students' apparently incorrect responses could send messages to students about who can (or should) do mathematics.

In education research more broadly, scholars have found that teacher-student relationships are important for both students' academic and psychological development but that these relationships are strained and perceived as more conflictual between teachers and Black students (e.g., Jerome et al., 2009). In mathematics education, Battey & Leyva (2013) examined teacher-student relationships through five dimensions to begin to unpack how these interactions might differ. They found that of these five dimensions, "acknowledging student contribution", which examined the ways in which teachers acknowledged student mathematical thinking by "valuing/devaluing, or praising/disparaging" student contributions (p. 982), was the only dimension (in that sample) that was statistically significant in predicting students' mathematics achievement on the California Standards Test (p.984). Additionally, though not quite statistically significant in the small sample, they did find some notable differences in the ways in which teachers acknowledge student contributions. Specifically, they found that teachers acknowledged student contributions in a way that advantaged Black girls and disadvantaged Black boys. In other words, Battey & Leyva (2013) found that how teachers acknowledge student contributions impacted students' mathematics achievement and was not consistent across students' gender. Even if these problematic interactions are unconscious and hence not deliberate or intentional on the part of the teacher but rather due to teacher's implicit racial bias, they can still cause harm to students (e.g., Van den Bergh et al, 2010). Essentially, the recent work of Battey Leyva (2013, 2016) points to the importance of how teachers' manage students' responses and how this could be a mechanism for "conveying messages about who is mathematically able, whose mathematical contributions are valid, and whose cultural and linguistic practices are legitimized in mathematics classrooms" (Battey & Leyva, 2013).

Section summary: Why does it matter?

How teachers manage students' apparently incorrect responses is consequential for students. Implicitly or explicitly it sends four critical messages to them. First, it signals to students what mathematics they should learn and, when done poorly, it can lead to mathematical misunderstandings and misconceptions. Second, it conveys messages about the nature of mathematics which in turn shape students' understanding of what it means to do mathematics—as a pursuit for the elusive right answer through correct procedures or as a creative, problem-solving endeavor for conceptual understanding. Third, it indicates to students what role errors play in mathematics learning and in learning more broadly. This can either reinforce students' fear of making mistakes and maladaptive coping mechanisms for failure or liberate them to see mistakes as opportunities for learning and growth that are worth the risks. Finally, it can send messages to students about who can (or should) learn mathematics through how it positions or favors students of different genders, races or class. Despite these significant implications for students, it is still common for teachers' in US classrooms to engage in evaluative patterns in response to all types of student answers (Franke et al ref). However, examining why it might be difficult for teachers to manage students' apparently incorrect responses sheds light on why this discourse pattern might persist.

Why might managing students' apparently incorrect responses be difficult?

In this section, I argue that managing students' apparently incorrect responses is difficult for four main reasons: (1) Teaching, and specifically managing apparently incorrect student responses, can be "unnatural" work. (2) Teaching, and specifically managing apparently incorrect student responses, can involve managing dilemmas. (3) Teaching, and specifically managing apparently incorrect student responses, can be anxiety provoking. (4) Making decisions in-the-moment can be difficult and cannot always be completely conscious.

Difficulty 1: Teaching is "unnatural" work.

The work of professional classroom teaching requires teachers to act in ways that are vastly different from those in their day-to-day life (D. L. Ball & Forzani, 2009) and this is particularly evident when a teacher needs to respond to a student. In that moment, to be able to 'hear' a child, a teacher must first be intently focused on really understanding what the student has said. In our everyday interactions, when we are in conversation with others, rather than listening to truly understand what another person is saying, our minds are usually busy crafting a response.

Even when we are genuinely interested in hearing another person, we can be looking to understand what they are saying in terms of how it provides value for us and not necessarily the other person (i.e., "what can I learn here", "how can respond so I look good, genuine, etc."). Teachers, on the other hand, are likely listening through the lens of students' learning and looking for ways to provide learning opportunities and value for students, not themselves. In other words, the teacher must suspend the urge to craft a response or seek value only for themselves and instead, cultivate an intense, "unnatural orientation towards others" (D. L. Ball & Forzani, 2009, p.499).

Additionally, in our day-to-day lives, we tend to spend time with people who are like us in some way, people with whom we have some shared perspective or identity. Hence, we can, usually correctly, make assumptions about what someone else means (e.g., "I totally hear what you are saying", "I get it"). In contrast, teaching requires that teachers be vigilant about not assuming a shared perspective and take care to not project their own understanding and thinking onto what students' have said.

This vigilance is closely tied to the work of "listening across divides" of age, race, culture, religion, language, and gender between the students and teacher (D. L. Ball, 1997). As D. L. Ball (1997) puts it, "how children think, talk, and represent their ideas is shaped by their varied identities and experiences. Trying to hear children challenges teachers while trying to listen across a gulf of human experience and meaning" (p. 787). In the vignette for example, the teacher seems to have difficulty hearing across the divide and listening without her own understanding clouding what she hears. She seems to understand the relationship between the ratios in terms of one-dimensional and two-dimensional objects and hence cannot hear past the language ("box", "boxes") and concrete example that Jean uses in his response. The teacher therefore assumes Jean does not understand the concept because he does not understand it in the way she does or use language she would use. In order to manage a student response productively, a teacher must first hear and understand in an "unnatural" way what a student has said. Once a teacher hears a student response, if she perceives it to be incorrect, she must then continue to engage in unnatural behaviors.

In day-to-day interactions, when a child or individual says something apparently incorrect, adults usually make efforts to quickly aid the individual in getting to the right answer. There is a strong urge to "smooth" things over and minimize the potential embarrassment of the situation. In contrast, when a student provides an apparently incorrect response a teacher will likely need to linger in that moment. She might ask purposefully probing questions that provide more information or intentionally cause cognitive disequilibrium and discomfort, as the work of learning often necessitates this uncomfortable struggle (D. L. Ball & Forzani, 2009). Teachers must suppress the natural urge to swoop in and swiftly 'fix' errors, to do something for someone else, and instead facilitate meaningful learning through productive struggle.

Managing students' apparently incorrect responses requires teachers to act in unnatural ways. They must be unnaturally focused on the words, understanding and needs of others and have to suspend natural assumptions of common identity and meaning. Additionally, if a student says something that is apparently incorrect, they need to unnaturally linger in the discomfort of the moment in order to facilitate students' learning.

Difficulty 2: Teaching can involve managing dilemmas.

The complexity of teaching in general and managing student responses in particular, often requires a teacher to navigate sometimes-competing teaching obligations (Buchman, 1986) in addition to their own goals and desires. In particular, as a teacher of mathematics in schools, a teacher has professional obligations to at least four main stakeholders: the discipline of mathematics (to represent and teach it in authentic and valid ways), the individual students (and their idiosyncratic feelings, thoughts, and needs), the class as whole (in particular, making sure the shared resources of time and space are used in "socially and culturally appropriate ways" by all, Herbst & Chazan (2012, p.610), and the institution (e.g., the school, state and federal education mandates, professional organizations, etc.) (Herbst & Chazan, 2012). In addition to these obligations or commitments (Lampert, 1985), teachers also have a variety of short-term goals (e.g. " responding at a specific moment to a particular student in an appropriate way", p. 15) and long-term goals (e.g. "helping students to develop over the course of a year", p. 15) for what they intend to achieve, mathematically or affectively (Schoenfeld, 2011). These goals are likely shaped by and in turn influence the ways in which a teacher interprets her various obligations to different stakeholders. One particular goal—the mathematical learning and understanding of children—can also manifest itself as the teacher's powerful desire for children to 'get' the right answer . D. L. Ball (1997) explains, "Teachers want students to understand... They care about their students. After investing time and effort in a particular student, a teacher wants to hear right answers, sensible reasons, creative ideasThe desire to have been effective, to have helped, is strong " (p. 800).

What can become difficult for teachers is that these various obligations, goals and desires together rarely present a single, clear, "right" course of action at any given moment. Frequently, teachers are faced with "a forced choice between equally undesirable alternatives" which creates seemingly impossible dilemmas to problems that "might be defined as unsolvable" but that a teacher must resolve nonetheless (Lampert, 1985, p.179). These dilemmas are not always simply resolved by cutting edge theories of mathematics teaching and learning since they are not simple matters of logic and knowledge. They need to be resolved in complicated contexts and involve real cognitive and affective consequences to other human beings and to the teacher. When a teacher responds to a student's apparently incorrect answer they are not only shaping students, they are also shaping their own identity as a teacher and, hence, dilemmas can further be "a conflict of identity" (p. 182). To illustrate how these various obligations, goals and desires might come into play let us revisit the vignette but this time consider how Jean's apparently incorrect response might give rise to dilemmas for the teacher.

When Jean provides his response the teacher is in a situation where she must navigate obligations to all four stakeholders, and her own goals and desires simultaneously. First, she is obligated by the discipline of mathematics to help students learn to speak in mathematically precise ways by using terms such as "one-dimensional" and "two-dimensional" rather than "box" and "boxes". Second, the teacher has a sense of obligation towards Jean's individual emotional and intellectual needs. Third, the teacher is likely highly aware, since the interaction occurs during whole-class instruction, of her interpersonal obligation to the class. Some teachers fear public, incorrect answers because they worry about the handful of students that might be paying attention only in that moment and then leave the class believing that perimeters and areas are "box" and "boxes" . Further, teachers are also responsible for the use of time in class and cannot spend inordinate amounts of it on Jean's response. Fourth, the teacher has institutional obligations that, in particular through the deluge of standardize testing, pressure her to make sure that students have a correct understanding of the content and, in some cases, that she keep a particular pace in her class (which creates time pressures around how long, if time is spent at all, to perhaps spend on apparently incorrect answers). Related to these obligations, are the teacher's own goals and desires. In this case, the teacher likely has a goal of developing students' understanding of these ideas as well as their use of mathematical terms. The teacher also has a strong desire for Jean to participate and to get the right answer. When he volunteers, she get's excited and she is looking forward to an answer to her 'good question.' Then there are likely her own desires to be a good and competent teacher, and the clearest proof of her success as a teacher will be that her students are able to produce the right answer (D. L. Ball, 1997).

All of these various obligations, goals and desires do not necessarily align to suggest one, correct course of action for the teacher. On the one hand, her individual obligation to Jean and her desire for him to participate and learn, might suggest one course of action where she follows-up with probing questions (e.g., "what do you mean by "boxes"?"). On the other, her obligations to the class as a whole and the discipline in addition to her desire for students to understand might suggest that a better course of action is for her to provide a correct, accurate response with the right terminology. These obligations and desires might also suggest a third option: asking a different student to voice their understanding of Jean's response by rephrasing what was said. Unlike choose-your-own-adventure books where there are usually only two possible courses of action, in teaching many instructional moments present three or more plausible choices for the teacher and these choices are not likely to satisfy every obligation, desire or goal at play in that moment. In some cases, a teacher might feel that these obligations, goals and desires are pulling her towards different, irreconcilable courses of action. Regardless, she likely does not have the luxury of time to contemplate each path of action to completion before needing to respond. She must make a decision and resolve the dilemma.

Difficulty 3:Teaching can be anxiety provoking.

As I mentioned earlier, I am exploring managing students' apparently incorrect responses in the context of whole-class instruction. This particular activity structure, regardless of the type of instruction occurring in that structure (i.e., lecture or discussion), means that the teacher's interaction with a student is public and this public-nature has important implications for both the students and the teacher. As I mentioned earlier, for students this public interaction has implications for what they learn mathematically, and about what it means to do mathematics, what it means to make mistakes, and who can learn mathematics. For the teacher there are also important implications due to the public nature of this interaction. Consider for a moment that in psychology experiments one of the easiest ways to "stress out" a participant is to engage them in a public speaking task as well as a public mental mathematics task (Kirschbaum et al., 1993). I contend that this experience, in many ways, is similar to the experience of the mathematics teacher, standing at the front of the room, responding to students. Like a participant in a psychology experiment, a teacher is speaking to an audience and performing cognitively demanding work (harder, I would argue, in most cases than simple mental computations). It is therefore plausible to imagine that, like in the psychology experiment, a teacher managing an apparently incorrect student response is likely to experience emotional arousal, quite possibly in the form of anxiety. A cause for concern is that this anxiety has important consequences for the teacher's cognitive abilities and performance in that moment through two potential mechanisms: occupying mental space and their physiological reaction.

Several researchers have argued that anxiety, such as math anxiety, needlessly occupies mental space and hence, negatively impacts performance (e.g., Beilock & Willingham, 2014; Wine, 1971). In other words, they hypothesize that during a math test, for example, an anxious math test taker inadvertently divides their attention between "self-relevant" (being "self-deprecating" and "self- preoccupied" during the task) and "task-relevant" (actually working on the task at hand) variables (Wine, 1971). This attention to both the worries and the task at hand reduces the capacity of working memory (Beilock & Willingham, 2014). Working memory is the short term, capacity-limited part of short-term memory which holds information temporarily such that it can be used for immediate, conscious perceptual and linguistic processing and hence, guides reasoning, decision-making and behavior ("https://en.wikipedia.org/wiki/Working_memory", accessed April 2017). Thus, preoccupying working memory with worry reduces its capacity to solve the task at hand. Recent work in neuroscience has confirmed this hypothesis.

When looking at the neural activity in the brain of children with high and low math anxiety as they worked on math problems, researchers found differences in the children's brain activity. Math anxious children, as compared to their peers, had more brain activity in their amygdala (which is used for processing negative emotions) and lower activity in the dorsolateral prefrontal cortex and the posterior parietal lobe (brain regions known to support working memory and numerical processing) (Christina B Yung, Saras S Wu, Vinod Menon, "neuro-developmental basis of math anxiety" psychological Science 23, 201, 492—501). In other words, worrying about a task activates the processing of negative emotions and decreases brain activity in areas that could be used to problem-solve.

This is exactly one of things that happens to the teacher is the vignette. In the vignette, we can see her worry as she listens to Jean's response: "What is he talking about, there's nothing 3-D going on here?!uurggghhh, he's not answering my question...he doesn't get it....how do I respond to this???!!!..." She worries about making sense of what Jean is saying, about Jean not understanding and about how to respond, all of which needlessly occupies her working memory. I hypothesize that consciously and unconsciously, teachers are likely worrying about many things—looking knowledgeable, making sure all students are paying attention, keeping an eye on how much time is left, etc. all the obligations, goals and desires creating dilemmas—simultaneously. These unavoidable concerns can, at times, give rises to anxiety and worries that then needlessly occupy mental space, hindering a teacher's cognitive capacity to fully listen and hear what a student has said and to respond productively. In addition to these worries, another challenge can be presented through the teacher's physiological response in that moment. Threat versus challenge: The physiological response.

When a teacher is at the front of the room and a student has just responded, the teacher is in a situation that requires an active performance—including cognitive, emotional and behavioral responses—on their part. The way in which the teacher perceives this situation (also known as appraisal of the situation) has importance consequences for their physiological response and their performance (e.g., Blascovich & Mendes, 2000; Jamieson et al., 2010). When an individual feels that they have "insufficient resources to meet the situational demands" they experience a threat response (Blascovich & Mendes, 2000, p.60). In contrast, if they feel that they have "sufficient or nearly-sufficient resources" they experience a challenge response. These different appraisals of a situation in light of one's resources have distinct physiological responses that impede or support performance. Though both responses activate the sympathetic-adrenal-medullary axis , in a challenge state the adrenal medulla releases higher levels of catecholamine neurotransmitters (such as adrenaline) that are associated with better task performance (e.g., Jamieson et al., 2010) and the body's cardiovascular response "mimics cardiovascular performance during aerobic exercise and represents the efficient mobilization of energy for coping" (Blascovich & Mendes, 2000, p.66). In other words, when an individual appraises a situation as a challenge, their body kicks into gear a physical response that better mobilize them to perform. In contrast, if a situation is appraised as a threat, an individual's body is preparing for physical harm so it constricts blood flow to minimize blood loss, ramps up inflammation and mobilizes immune cells needed to heal after the inevitable harm is inflicted. This reaction then impedes cognitive performance and, over the long-term, can increase the risk of cardio-vascular disease (McGonigal, 2015, pp.110-111).

When a teacher is at the font of the room and a student has provided an apparently incorrect response, a teacher might appraise the situation (consciously or not) as either a threat or a challenge. To illustrate these differences, let us imagine two different ways this moment might go. In the first, the teacher hears the student response and feels like she doesn't have the experience, knowledge or resources to handle it. This is the first time she has taught this content so she is unsure of what mathematics underlie the student thinking and where to go with what the student has provided. She feels underprepared and is not confident in her ability to manage this moment productively. Her body, sensing this perception of threat, responds accordingly. Her heart pounds, her palms sweat, her blood vessels constrict and reduce the blood flow and oxygenation to her brain. Her body is ready for harm and, as a consequence, provides her with less support to handle this moment. Now imagine instead that the teacher hears the student response and gets excited. She has wanted this particular misconception to come out so she can dig into it with her students. This is exactly what she had been reflecting on a few days ago as she planned out this lesson, anticipating various mathematical complexities and issues students might have and planning a variety of ways in which she might respond. She is confident in her knowledge and in her ability to manage this moment. In turn, her body perceives her challenge appraisal of the situation and mobilizes to support her. Her heart beats faster, her blood vessels dilate, there is a rush of blood and oxygenation to her brain, and adrenaline courses through her. Her body provides her with the physical resources to face this challenge.

Difficulty 4: Making decisions in-the-moment can be complex.

I argue that a fourth difficultly in managing students' apparently incorrect responses is the complexity of in-the-moment decision-making. Understanding into this difficulty first necessitates some common ground be established regarding what "decision making" might mean in general and clarifying how it is being operationalized here. Decision-making—loosely conceptualized as why and/or how people do what they do—has long been a focus of considerable interest by philosophers and researchers alike. This fascination has led to a prolific and diverse body of work on the subject across a variety of disciplines including education (e.g., Blömeke, Gustafsson, & Shavelson, 2015; Schoenfeld, 2011; Shavelson & Stern, 1981), psychology (e.g., work on behavioral decision making such as Kahneman (2011) and any of the work by Kahneman Tversky, or G. Gigerenzer as well as naturalistic decision making, e.g., Klein (1999)or Lipshitz et al. (2001)), social psychology (e.g., Chaiken & Trope, 1999), business (e.g. Buchanan O'Connell,2006) and neuroscience (e.g., the work of Antonio and Hanna Damasio on emotions and decision-making). While an exhaustive review of this vast, rich literature is impossible here, I provide an overview of the facets of decision-making relevant to understanding it in the context of managing students' apparently incorrect responses.

At a glance over this broad field one might notice four interrelated but slightly different facets of the work on "decision-making": 1. A focus on the outcome (decision) and/or the processes (the decision-making processes, e.g. weighing pros versus cons or using a rule of thumb) and/or the post-hoc rationalization of decisions (i.e., what people do versus the processes that occurs versus how they explain why they did what they did after the fact). 2. A focus on the attributes of individuals that influence the decision or decision-making process. 3. A focus on the characteristics of a situation that influence the decision or decision-making process. 4. A focus on the ways in which decision-making might evolve or be optimized over time.

All of these various foci have implications for how one might define, conceptualize and study any decision-making phenomenon, including how teachers manage students' apparently incorrect responses. For example, a focus on only the outcome or decision means looking closely at how teachers respond (what they say and do). In addition though, one might want to also consider the invisible decision-making processes that are occurring (e.g., it is a snap judgment, a heuristic or routine, or is there some mental deliberation the teacher is going through?). One might also explore how teachers rationalize or explain their decisions post-hoc (e.g., Herbst Chazan's practical rationality). Further, one might be interested in understanding what individual characteristics, such as particular types of knowledge and skills, might influence teachers decisions (e.g., Schoenfeld's work on goals, resources and orientations in teacher decision-making). There is also a need to consider the characteristics of a situation that influence the decision or decision-making process. For example, if one were interested in studying how implicit bias might impact how teachers' manage students' apparently incorrect responses this would need to occur in a context where differences in race are present. One might also be interested in how managing students' apparently incorrect responses differs between content areas (e.g. Algebra versus Calculus), student age groups (e.g., elementary, middle, or high-school or post-secondary students), or school settings (e.g., public versus private, or urban versus suburban). Finally, one might be interested in how teachers' manage students' apparently incorrect responses moment-to-moment or looking across time to see patterns in the ways this might evolve for novices or be improved with particular interventions such as professional development on children's thinking.

In order to allow for these different facets or foci, at a high-level I conceive of how teachers manage students' apparently incorrect responses as a type of professional competence and therefore, draw on Blömeke, Gustafsson, & Shavelson (**?**)odel of competence. In examining the research on competence, Blömeke, Gustafsson, & Shavelson (2015) pointed to a dichotomy between performance, on the one hand, captured through "behavioral assessment in real-life situations" and the "dispositions underlying such behavior," (p.5) on the other, which are usually "analytically divided into several cognitive and affective-motivational traits (or resources), each to be measured reliably and validly" typically with paper-and-pencil assessments (p. 2). To bridge this dichotomy, they propose a model of competence as a continuum that takes into account dispositions along with the situation-specific skills (i.e., processes such as the "perception and interpretation of a specific job situation together with decision-making", p. 8) that likely mediate between disposition and performance. "Thus, instead of insisting on an unproductive dichotomy view of competence, in particular knowledge or performance, competence should be regarded as a process, a continuum with many steps in between" (p. 8, italics added, see Figure 4.1 below).

Taking this model as a starting point, the competency of managing students' apparently incorrect responses can be conceived of as encompassing a variety of "dispositions" (e.g., cognitive, affective, motivational, volitional) and "situation-specific skills" (e.g., perception, interpretation, and decision making) that led to an observable behavior (e.g., a response to a student). Based on my focus here on in-the-moment decision-making, I revised their model as follows:

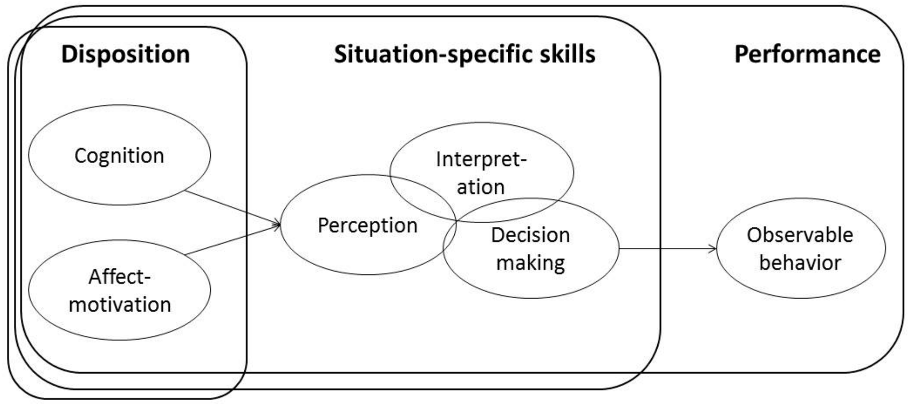


Figure 2.3: Model of competence as a continuum (Blömeke, Gustafsson, & Shavelson, 2015, p.9).

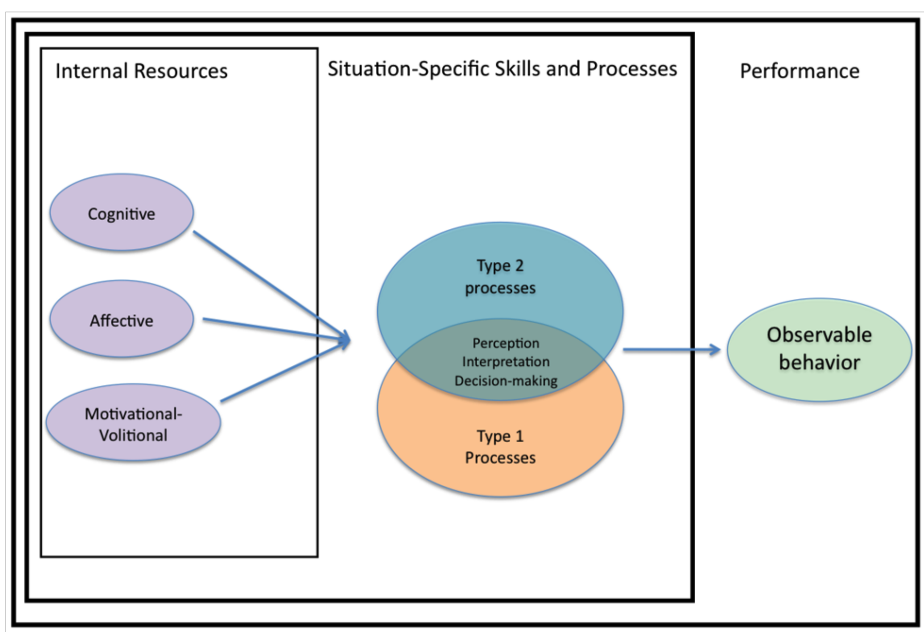


Figure 2.4: Revised model of competence for managing students responses.

In this model I have made four important changes. First, instead of using the label “dispositions”, I have used “internal resources” to indicate what an individual might have available to them in the moment. Dispositions can be thought of as a type of average of these internal resources since a disposition is a tendency over time rather than at a unique moment in time. Second, within this category I have chosen to differentiate between cognitive, affective and motivational-volitional resources. In general, there is some agreement that cognitive resources are “knowledge” or empirical facts as contrasted with beliefs and emotions. Here, I use affective resources to encompass teachers’ beliefs about the subject matter and about teaching and learning mathematics. In other words, affective resources are beliefs about things that are outside of themselves. In contrast, beliefs related to their abilities (inside themselves) such as self-efficacy, I categorize as motivational-volitional resources. To illustrate the difference in the way I have defined affective versus motivational-volitional resources, consider the difference between a belief in mathematics teaching and learning as encompassing students engaged in hands-on activity versus a belief in my ability to actually implement this type of instruction. As I will discuss in greater detail in the next section, managing students’ apparently incorrect responses necessitates several specialized cognitive, affective and motivational-volitional resources. A lack in any one of these resources is likely to impact performance and hence, a difficulty here is in having and/or accessing all of these resources in the moment of performance.

A third change I have made is in labeling the mediating step between resources and performance as “situation-specific skills and processes”. This change in label is directly related to the fourth and final change I have made to the model: layering Type 1 and Type 2 processes onto the “perception, interpretation, and decision making” skills. These processes, Type 1 and Type 2, come from research on dual-process theories of cognition. These theories essentially stipulate that human beings have (at least) two systems for processing information: Type 1 or the autonomous set of systems (in some work referred to as system 1) which can be faster, automatic and unconscious (including: "modular, habitual, and automated forms of processing" (Evans & Stanovich, 2013, p.225)), and Type 2 processes (in some work referred to as system 2) which can be slower, deliberate and conscious (Evans & Stanovich, 2013; Kahneman, 2011). The important distinction between these is that "Type 2 processing is distinguished from autonomous Type 1 processing by its nature—involving cognitive decoupling and hypothetical thinking—and by its strong loading on the working memory resources that this requires" (Evans & Stanovich, 2013, p.227). I have chosen to layer this into my model for several reasons.

First, the research on expertise, including expertise in teaching, suggests that one of the characteristics that distinguished experts from more novice members of a profession is their development of and reliance on heuristics and routines. A teacher might have developed a set of heuristics or routines to handle particular, common types of apparently incorrect student response. For example, an expert teacher might have developed something as simple as a wait-time or revoicing routine after particular types of responses that she knows will leave space for other students to jump-in or for the same student to reflect on their own response. Or an expert teacher might have a routine follow-up task or question to use when students exhibit a specific mathematical misconception. It is likely that in these familiar moments, a teacher might perceive, interpret and decide almost simultaneously and instantaneously on a course of action. These quick decisions are not only a sign of expertise when they occur at opportune moments but they are also a necessity in teaching because of the typically rapid-fire pace of events. The difficulty arises in having professional judgment and knowing (intuitively) when a quick decision can or should be made and heuristics utilized versus when the situation requires a more conscious deliberation before acting.

Second, the inclusion of Type 1 and Type 2 processes also affords room to more accurately and generously consider how implicit bias might appear in a teacher's practice in general and in how a teacher manages students' apparently incorrect responses in particular. Implicit bias is hypothesized to occur through an unconscious, associative mechanism in which a particular attribute of an object or situation automatically triggers a specific association (Greenwald & Banaji, 1995). These "so-called evaluative associations are created via evaluative conditioning or the repetition of particularly negative (or positive) information in the media, personal experiences, and so forth" (Van den Bergh et al., 2010). One does not need to look very far to see examples in day-to-day media that might bias individuals (e.g., watching the news or network shows and noticing how many instances there are of Black vs White males being portrayed as criminals). Regardless of their source, these evaluative associations are automatically triggered and hence are not under conscious control. In teaching, it is likely that racial implicit bias shapes the interactions between teachers and students (Battey & Leyva, 2016). It is therefore reasonable to assume this bias will impact the ways in which teachers manage students' apparently incorrect responses and it would unsurprising, for example, to find that teachers who have implicit bias more frequently negatively-evaluate apparently incorrect responses from students of color. Recognizing this as an implicit, unconscious, and likely Type 1 process has implications for not only a more realistic and generous view of teaching but, importantly, for suggesting alternative interventions. In other words, how teacher education might go about addressing a lack of situation-specific skills (e.g. noticing) is likely to be different than how it addresses an unconscious, situation-specific process, such as implicit bias.

What might be entailed in managing students' apparently incorrect responses?

The four difficulties of managing student's apparently incorrect mathematical responses discussed above—that it can be "unnatural" work, that it can involve managing likely un-solvable dilemmas, that it can be anxiety provoking, and that making decisions in-the-moment is complex—might make it seem impossible to fathom how a teacher would be able to consistently do this work competently. I propose, however, that having named these difficulties and in particular, having conceptualized managing students' apparently incorrect responses as a type of professional competence, provide a starting point for envisioning what it might entail for teachers to manage these responses. In this section, I review research that provides insights into the internal resources and situation specific skills that might be entailed in managing students' apparently incorrect responses. It should be noted that even though these various components are listed separately here for the purposes of discussion, in reality these components are closely interrelated and integrated. After exploring the relevant cognitive, affective, and motivational-volitional resources I discuss perception, interpretation and decision-making through the lens of the "noticing" literature. Throughout, I discuss when relevant how a particular resource or skill addresses one or more of the four difficulties identified above.

Figure 2.5

resources.

Cognitive resources.

"Cognitive resources" is a term intended to broadly capture "knowledge" (i.e., " Having a basis in or reducible to empirical factual knowledge" or cognition as "the mental processes of perception, memory, judgment and reasoning as contrasted with

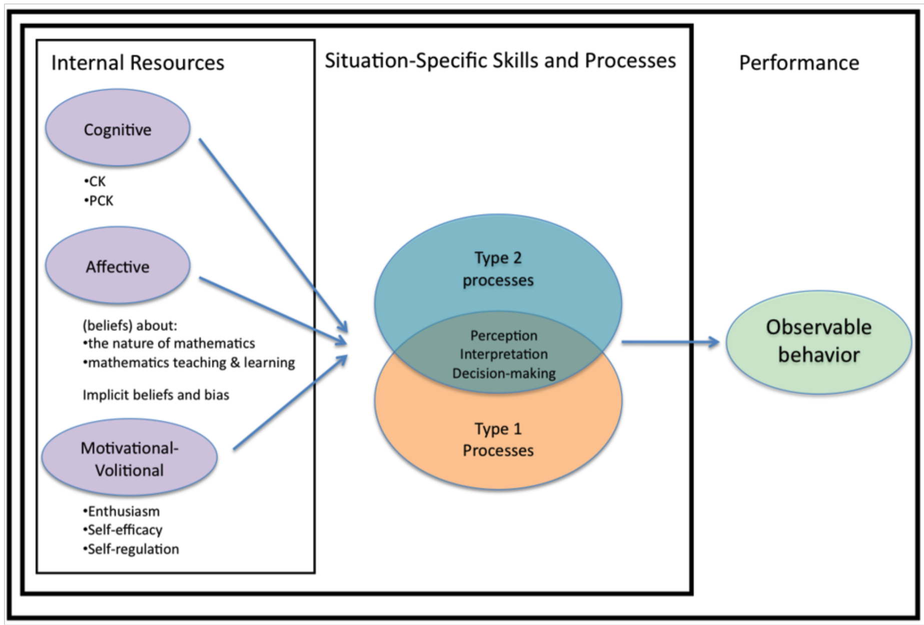


Figure 2.5: Model of managing competence with details on relevant internal resources.

emotional and volitional processes" -http://www.dictionary.com/browse/cognitive). It is generally agreed that teachers, like other professionals, needs particular types of specialized knowledge and work in mathematics education, spurred in part by Shulman (1987)'s research, is in progress to develop this knowledge base.

In articulating what a professional knowledge base for teaching might encompass Shulman (1987) proposed seven likely categories: content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational aims. It is likely that all seven of these various forms of knowledge shape teachers' competence and actions in important ways but I focus here on a sub-set. Specifically, the two forms of knowledge that are likely key to managing students' apparently incorrect responses are: content knowledge and pedagogical content knowledge.

Content knowledge (CK) and pedagogical content knowledge (PCK).

In mathematics teaching the first category, content knowledge, has been hypothesized to include common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (D. L. Ball et al., 2008). Essentially content knowledge broadly encompasses knowing mathematics content deeply such as knowing how to do mathematical procedures, use mathematical concepts, and understanding why particular algorithms or procedures work. This type of knowledge, specifically common content knowledge, is "the mathematical knowledge and skill used in settings other than teaching" (D. L. Ball et al., 2008, p.200). CCK is what a teacher utilizes to recognize when a student might have made a mistake. In addition, "sizing up the nature of an error, especially an unfamiliar error, typically requires nimbleness in thinking about numbers, attention to patterns, and flexible thinking about meaning in ways that are distinctive of specialized content knowledge (SCK)" (p. 401). These particular ways of knowing content, however, are not enough. Teachers also need pedagogical content knowledge (PCK) that is "most likely to distinguish the understanding of the content specialist from the pedagogue" (p.8 Shulman, 1987). In other words, teachers need a special type of mathematical knowledge for mathematics teaching that distinguishes them from a generalist or "pedagogue". Others have gone further to demonstrate that PCK also differentiates mathematics teaches from mathematicians or other math-intensive professions.

PCK has been theorized to include knowledge of content and students (KCS), knowledge of content and curriculum, and knowledge of content and teaching (KCT) (D. L. Ball et al., 2008). In order to first decipher a students' apparently incorrect response teachers need KCS or, "familiarity with common errors and [knowledge to decipher]which of several errors students are most likely to make" (p. 401). Further, KCS is a type of knowledge that supports teachers in being able "to hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language" (p. 401). Once a teacher has heard a student and identified a potential misconception, KCT can support a teacher in knowing how to respond to a student. KCT "combines knowing about teaching and knowing about mathematics" (p.401) which enables teachers to make instructional decisions about which student contributions to pursue and which to ignore or save for a later timewhen to pause for more clarification, when to use a student's remark to make a mathematical point, and when to ask a new question or pose a new task to further students' learning (p. 401).

Closely related to D. L. Ball et al. (2008)'s articulations of the specialized mathematical knowledge for teaching that teachers need are the types of knowledge some researchers speculate are important for "professional error competence" (Seifried & Wuttke, 2010, p.147), "error analysis" Peng & Luo (2009) or "diagnostic competence" (e.g., Schwarz et al., 2008; Hoth et al., 2016). Diagnostic competence refers to "the ability and the readiness of an assessing person [e.g., a teacher] to assess or analyse [sic] people or their performances according to predefined categories and terms or conceptions" (Schwarz et al., 2008, p.779). In addition to having strong, relevant content knowledge, this competence requires that teachers have the following types of PCK:

(1) Knowledge of possible error types: At first, teachers have to actually recognize the specific logical flaws and false assumptions made by students. To be able to do this, teachers need domain-specific knowledge about possible learner errors.

(2) Available strategies of action/teachers reaction: After having recognised [sic] the error, teachers must treat it adequately. For this they have to know about various alternatives of action (e.g. about giving adequate feedback or when it is better to ignore errors). (Seifried & Wuttke, 2010, p.150).

The first type of knowledge is essentially a sub-set of what D. L. Ball et al. (2008) described as knowledge of content and students (KCS) and what Peng & Luo

(2009) describe as a teacher's knowledge of the existence of the error, the underlying "rationality of the mathematical error" and the evaluation of the error (p. 23). The second type of knowledge (about available strategies of actions/ teacher reactions), is essentially equivalent to D. L. Ball et al. (2008)'s articulation of knowledge of content and teaching (KCT) or what Peng & Luo (2009) described as the teaching strategy presented to "eliminate [or remediate] the mathematical error" (p. 23).

Essentially, in order to manage students' apparently incorrect responses teachers likely need some form of all seven of Shulman (1987)'s types of knowledge but key to this competence are content knowledge (specifically CCK and SCK) and pedagogical content knowledge (KCS and KCT). These forms of specialized knowledge can support a teacher's situation-specific skills (as will be explored later) to diagnose a possible error and respond productively.

Affective resources.

Like "cognitive resources", "affective resources" is used to broadly capture the resources related to a person's emotions such as their "mood, feelings or attitudes" (Google search definition). Here, I will focus on the types of teacher attitudes or beliefs specific to teaching and learning mathematics that might support teachers in managing students' apparently incorrect responses. In this section, I do not include beliefs such as self-efficacy and have chosen to discuss those as motivational-volitional resources. In other words, this section focuses on beliefs teachers hold about the relevant ideas that are "out there" (e.g., their beliefs about constructivist versus behavioral instruction) rather than how those ideas relate to themselves (e.g., their own abilities to actually enact constructivist teaching). Obviously, these two categories are highly interrelated and interconnected and are only presented distinctly here for ease of presentation and discussion.

Beliefs about the nature of mathematics.

At a macro-level, what a teacher believes (implicitly or explicitly) about teaching and learning in general and specifically about mathematics will influence their behavior (e.g., Pajares, 1992). With regard to mathematics teaching, teacher's beliefs about the nature of mathematics influence their beliefs about teaching and learning mathematics, or as Pajares (1992) described it, "Teachers often teach the content of a course according to the values held of the content itself"(p. 308-309). In particular, three distinct views of the nature of mathematics have been described: an instrumentalist view ("mathematics is an accumulation of facts, rules and skills to be used in the pursuance of some external end. Thus mathematics is a set of unrelated but utilitarian rules and facts") (Skemp, 1978?) , a Platonist view ("mathematics as a static but unified body of certain knowledge. Mathematics is discovered, not created") and a problem-solving view ("mathematics as a dynamic, continually expanding field of human creation and invention, a cultural product. Mathematics is a process of enquiry and coming to know, not a finished product, for its results remain open to revision") (Ernst, 1988, p. 1-2). These different views about the nature of mathematics in turn shape teachers' beliefs about what it means to learn and teach mathematics.

Beliefs about mathematics learning and teaching.

In particular, teachers with more instrumentalist views of mathematics tend to believe mathematics learning looks like students being able to correctly carry out mathematical procedures and that in mathematics teaching, they should maintain the locus of control of instruction (Thompson, 1984). This then unfolds through instruction that is more structured and teacher centered with fewer digressions to address student difficulties and instructional patterns where teachers demonstrate procedures and then give students time to reproduce or practice these procedures

(Thompson, 1984; Stigler & Hiebert, 1999). These types of beliefs about mathematics instruction, sometimes described as "traditional" or "teacher-centered" ("transmission view", Perry et al. 1999), stand in contrast to the beliefs about mathematics learning and teaching of teachers with a problem-solving view of the nature of mathematics.

Believing that the discipline of mathematics is a dynamic, evolving field and that mathematics is a creative, problem-solving endeavor, tends to be accompanied with particular beliefs about what it looks like to learn and hence to teach mathematics. Specifically, teachers with these beliefs about the nature of mathematics also believe that learning mathematics means developing deep conceptual understanding and flexibility in integrating "knowledge of facts, concepts, and procedures so as to find solutions to a variety of related mathematical tasks" (Thompson, 1984, p.120). Consequently, mathematics instruction then involves releasing some intellectual control to students and having them actively do and reason about mathematics with their classmates (a perspective that learning is a social activity and hence a socio-constructivist view of learning) (Perry et al 1999: child-centerdness view). These types of beliefs about mathematics learning and teaching (sometimes referred to as "inquiry-oriented") do not necessarily mean that teachers do not believe in developing students' procedural and computational fluency but rather that they place value on instructional activities that engage students in reasoning, creativity, problem-solving, and constructing knowledge (Stipek et al., 2001). Teachers with these kinds of beliefs about mathematics learning and teaching may be less likely to simply correct students who have provided an apparently incorrect response. It is possible that because of their beliefs about the nature of mathematics, they might be more comfortable with the messy process of problem solving which inevitably involves struggle and making errors.

A related sub-set of these beliefs about teaching and learning mathematics, has to do with teacher's beliefs about the role of errors in learning and their related beliefs about ability. Most recently, Stipek et al. (2001) suggested that there might be a relationship between teachers' beliefs about students' mathematical abilities as more fixed than malleable (more fixed versus growth mindset, Dweck (2008)) and their beliefs about what mathematics learning might look like. In particular, they proposed that teachers who believe in a more traditional view of mathematics teaching and learning would be more likely to believe that students' abilities are fixed since both views focus on correct performance, in which there is little room for errors. Hence, it follows that for teachers to manage students' apparently incorrect responses productively they would need to believe that students' abilities are malleable and that errors are a productive part of learning, that mistakes are "springboards for inquiry" (Borasi, 1994). In particular, Borasi (1994) proposed that an orientation towards more progressive, inquiry-based instruction, "calls for highlighting ambiguity and uncertainty in the mathematical content studied so as to generate genuine conflict or doubt and, consequently, the need to pursue inquiry" (p. 168).

In summary, the following beliefs are most likely to support teachers in productively managing students' apparently incorrect responses:

(1) Beliefs about the nature of mathematics: as a "dynamic, continually expanding field of human creation and invention, a cultural product" (Ernst, 1988).

(2) Beliefs about mathematics learning: as stemming from abilities that are malleable rather than fixed; as an active and social process in which students construct deep conceptual understanding and flexibility in integrating "knowledge of facts, concepts, and procedures so as to find solutions to a variety of related mathematical tasks" (Thompson, 1984, p.120). This type of active, social and inquiry-based learning therefore will and must involve making and reasoning from errors (Borasi, 1994).

(3) Beliefs about teaching mathematics: as "providing necessary support to students' own search for understanding by creating a rich learning environment that

can stimulate students' inquiries and by organizing the mathematics classroom as a community of learners engaged in the creation of mathematical knowledge" (Borasi, 1992, p2-3 as cited in Borasi (1994)). This means that teachers believe that their role is more like that of a facilitator and guide than as a holder and transmitter of knowledge.

Implicit and Explicit Bias.

Another important affective resource for teachers is a lack, rather than the presence of, particular beliefs about students. Specifically, teachers are likely to be more effective in instruction and in managing all students' apparently incorrect responses when they do not hold negative biases about students. Though in some parts of the US blatant racism and discrimination still exist, overall explicit racism seems to have diminished; unfortunately, implicit racist attitudes have not (Pearson et al., 2009). These implicit attitudes are the unconscious, "introspectively unidentified (or inaccurately identified) traces of past experience that mediate favorable or unfavorable feeling, thought or action towards social objects" (Greenwald & Banaji, 1995, p.8). These implicit racial biases can lead to "aversive racism", which occurs when an individual has conscious, explicit, non-prejudiced views and considers the suggestion they might be prejudice as 'aversive' but, simultaneous, has a typically-unconscious, implicit, 'aversion' to Blacks in the form of fear, anxiety or other negative feelings (Pearson et al., 2009, p.317).

Most teachers, at least as part of their professional identity, must embrace teaching and learning for all students, a push for equitable education that is frequently reflected in statements for many national organizations including the National Council of Teachers of Mathematics (NCTM) (e.g., their position statement on Access and Equity in Mathematics Education released in April, 2014). Teachers must therefore publically and explicitly espouse non-prejudiced views of teaching and overtly provide equitable instruction. However, as I mentioned earlier, because of the complex and fraught nature of the media and the paradoxical, conflicting history of the US ("an 'American dilemma', as first described in 1944 by Swedish economist Gunnar Myrdal", from Pearson et al. (2009, p.314)), it is unlikely that all teachers do not hold some form of implicit racial bias. The danger of this aversive racism and these implicit biases is that these, "unconscious negative feelings and beliefs get expressed in subtle, indirect, and often rationalizable ways (Dovidio Gaertner, 2004; Gaertner Dovidio, 1986; Nail, Harton, Decker, 2003)" (Pearson et al., 2009, p.317).

In teaching, this rationalization is evident in the ways some secondary mathematics teachers explain students' lack of achievement as resulting from student characteristics, specifically "differences in students' motivational levels, work ethic, and family or parent support" (Bol & Berry, 2005, p.40). Though in their study Bol & Berry (2005) did not explore the role implicit bias played in shaping teachers' rationalizations, based on the work of Dovidio and Gaertner it is likely that implicit biases underlie these explanations and deficit mindsets. These rationalizations are dangerous for equitable education since they shift the blame for the achievement gap onto students and away from the teacher and the instructional quality these students receive. As Battey & Leyva (2013) demonstrated it is possible that these disparities in achievement are actually perpetuated and produced at the level of instruction, through the interactions of teachers with students. Hence, implicit bias is likely to rear its ugly head when a teacher manages students' apparently incorrect responses, especially if the teacher has to navigate multiple obligations under pressures of time in a complex environment (as they regularly must). Acknowledging that this affective factor exists and is a hindrance rather than a resource while also recognizing that it stems from different, unconscious mental processes is a first step towards actually realizing more equitable instruction.

Motivational-Volitional Resources.

In addition to the above cognitive and affective resources, teachers also need particular motivational-volitional resources to managing students' apparently incorrect responses. Here, I use motivational-volitional resources to group together resources having to do with an individual's desire or willingness to do something (in some disciplines motivation is distinguished from volition as follows: motivation is "the desire to do something" and volition is sometimes equated to the willpower). The notion that motivational-volitional resources might play a role in how teachers' manage students' apparently incorrect responses comes from psychological motivation theories that propose that motivation precedes behavior. As Kunter et al. (2008) summarize it, motivation, "is assumed to provide the energy, direction and quality of goal-directed behaviour (Ford, 1992; Pintrich, 2003b; Schutz, Hong, Cross, Osbon, 2006), and it is thought that differences in people's goals, emotions, beliefs and values predict differential behaviour and engagement (Eccles Wigfield, 2002; Ford, 1992; Linnenbrink, 2006; Pintrich, 2003b; Schutz et al., 2006)" (Kunter et al., 2008, p.470). In other words, teacher's motivational orientations are likely to impact their instructional behaviors. In considering which motivational-volitional resources are most likely to play a role in how teachers manage students' apparently incorrect response, I propose three: (1) enthusiasm for mathematics teaching, (2) self-efficacy in mathematics teaching and (3) the ability to self-regulate.

In the last decade, researchers have started to examine how teacher's enthusiasm, which had previously been considered to be an aspect of effective teaching but was left under-specified, is linked to their instruction (Kunter et al., 2008). Teacher's enthusiasm reflects, "the degree of enjoyment, excitement and pleasure that teachers typically experience in their professional activities" (Kunter et al., 2008, p.470). In mathematics teaching, teachers' enthusiasm for teaching mathematics (which is distinct from their enthusiasm for mathematics) has been linked to higher perceived instructional quality by students (Kunter et al., 2008). In Kunter et al. (2008), the study perceived instructional quality including three dimensions: monitoring ("the degree to which the teacher notices disruptive or inattentive student behaviour [sic]"), cognitive challenge ("the degree of cognitive challenge they [the students] experience in mathematics instruction"), and "the social support for students provided by their teacher" (p. 473). Though none of these dimensions has a one-to-one correspondence to the management of students' apparently incorrect responses, there is likely some overlap between the ways in which students' feel cognitively challenged and social support and how teachers' manage their responses. In other words, it is likely that students' perceptions of these two dimensions are in part reflective of how teachers' manage students' responses. Hence, continuing to work backwards from there, it is reasonable to speculate that teachers who are enthusiastic about mathematics teaching might also manage students' apparently incorrect responses in more productive ways. Put another way, an enthusiasm for mathematics teaching is likely to support teachers in managing students' apparently incorrect responses.

A related motivational-volition resource is a teacher's self-efficacy. Self-efficacy refers to one's belief in one's ability to succeed at a given task or in a specific situation (context specific) (Bandura, 1977) or "self-perception of competence rather than actual level of competence" (Tschannen-Moran et al., 1998). Teacher's self efficacy has been defined as " "the extent to which the teacher believes he or she has the capacity to affect student performance" (Berman, McLaughlin, Bass, Pauly, Zellman, 1977, p. 137), or as "teachers' belief or conviction that they can influence how well students learn, even those who may be difficult or unmotivated" (Guskey Passaro, 1994, p. 4) (as summarized in Tschannen-Moran et al. (1998, p.202)). Teaching self-efficacy has been defined as being context (e.g., working with particular types of students for example) and subject-specific specific (e.g., teaching Algebra I versus Geometry) and, even potentially teaching task specific (i.e., "teachers' sense of efficacy is not necessarily uniform across the many different types of tasks teachers are asked to perform" (Tschannen-Moran et al. (1998, p.219), summarizing Bandura (1977)). There are also distinctions that have been made between general teaching efficacy (GTE) (which goes beyond an individual's perceived capabilities but rather are about the capabilities of teachers in general or teaching more broadly) and personal teaching efficacy (which is more specific and individual that GTE) (Tschannen-Moran et al., 1998). Teaching self-efficacy has been linked to a variety of teacher characteristics ("teachers' classroom behaviors, their openness to new ideas, and their attitudes toward teaching") as well as impacting "student achievement, attitude, and affective growth" Tschannen-Moran et al. (1998, p.215). More recently, teacher self-efficacy has been found to be positive related to students' perception of instructional quality (as captured on three dimensions: classroom management, cognitive activation and individual learning support) (Holzberger et al., 2013). Interestingly, in the same longitudinal study, Holzberger et al. (2013) also found a reverse effect of instructional quality on self-efficacy over time, meaning that self-efficacy not only impacts instructional quality but that, in turn, instructional quality over time shapes a teacher's self-efficacy.

Again, though the dimensions of instructional quality are not a one-to-one correspondence with managing students' apparently incorrect responses, there is enough empirical evidence to suggest that teachers may feel more or less efficacious in this professional competence. It is also possible that, as suggested by prior work, this sense of efficacy is context and content dependent. In other words, a teacher might feel a strong sense of self-efficacy when managing students' apparently incorrect responses in Geometry but not Algebra or when managing students' apparently incorrect responses during small-group work versus whole-class instruction. The empirical evidence also suggests that this sense of self-efficacy is likely to support teachers in their performance. Hence, a teacher with a high sense of self-efficacy in managing students' apparently incorrect responses (in a given context and content) is likely to do so more productively (in that given context and content).

A final motivational-volitional resource that I propose can support teachers in managing students' responses is adaptive self-regulation around emotional labor that arises because of the demands of the "role" teachers need to play (Buchman, 1986). Teaching is a "role" that requires teachers to act in particular ways that align with their professional obligations, obligations that apply "regardless of personal opinions, likes or dislikes" (Buchman, 1986, p. 531). This can be particularly difficult when, as I mentioned earlier, a teacher needs to act in ways that are "unnatural" (D. L. Ball & Forzani, 2009), as can occur when teachers manage students' apparently incorrect responses. Though hearing a students' apparently incorrect response might evoke an emotional reaction (perhaps disappointment, or an emotional instinct to protect a child from embarrassment or even joy that a particular mathematical idea is now at play), a teacher must typically suppress and control these emotions and feelings to maintain composure and stay in their role. In the case where a teacher has appraised the situation as a threat rather than a challenge (another difficultly that can arise in this situation), they are likely experiencing negative emotions (e.g., anxiety, fear, uncertainty) that they must similarly be careful to manage appropriately. This "management of feeling to create a publicly observable facial and bodily display" is known as "emotional labor" (Hochshild,1983 as cited in James & Robin (2003)) and has implications for employee's mental well-being and job performance that have been extensively explored in organizational behavior research. The concept has only recently made its way into research on teaching as more scholars note the importance of teacher's mental and emotional well-being not only on teacher retention and burnout (e.g., Näring et al., 2006; Cheung et al., 2011) but also on instructional quality (Klusmann et al., 2008).

Since emotional labor can lead to emotional exhaustion and burn out (Näring et al., 2006) teachers need ways to deal with the emotional demands of teaching. In occupational research, it has been shown that employees, including teachers, with high engagement and high resilience tend to far better on several measures including reporting "less physical and psychological strain, had lower absence rates, and had lower means on the three burnout symptoms"(Schaarschmidt et al. (1999) as cited in Klusmann et al. (2008)). Engagement refers to, "the willingness to invest energy and effort in one's job" (Klusmann et al., 2008, p.704) and "resilience reflects the individual's reaction to work-related demands and describes the ability to deal with failure and to maintain a healthy distance with one's work (Schaarschmidt et al., 1999). It includes emotional distancing, a low tendency to give up after failure, active coping, and mental stability" (Klusmann et al., 2008, p.704). Though these constructs are typically studied at the macro-level of an individuals' overall engagement and resilience, I hypothesize that the ability to effectively self-regulate one's emotional state in-the-moment, at the micro-level, is likely reflective of these macro-level measures of an individual's engagement and resilience. Essentially, I am proposing that these macro-level constructs reflect the sum of the micro-level, moment-to-moment interactions a teacher experiences.

Therefore, I argue that teachers with high levels of engagement and resilience will fare better in self-regulating the emotional demands of managing student responses in-the moment and in responding productively. Again, though the interactions at the level of managing students' apparently incorrect responses have not been studied, there has been research that explores how these motivational-volitional resources impact instructional quality at the macro-level. Specifically, teachers with high engagement and high resilience (as compared to those with low engagement and high resilience, low engagement and low resilience, or high engagement and low resilience) have been shown to provide more social support, cognitive activation and teach at a lower interactional tempo as perceived by students (even after controlling for context-schools tracks) (Klusmann et al., 2008, p.711). Included in the dimension of social support is a sub-scale of relevance for managing students' apparently incorrect responses, namely, "the students' perceptions of the teachers' patience with students' mistakes ("Our teacher is patient if someone makes a mistake in the lesson"; 3 items) (Klusmann et al., 2008, p.711). Additionally, interactional tempo (captured through items such as "Our teacher does not leave us much time to think when asking questions" (Klusmann et al., 2008, p.711) is likely related to forms of general pedagogical knowledge that support teachers in managing students' apparently incorrect responses.

Situation-specific skills: "noticing."

In addition to the internal resources needed to manage students' apparently incorrect responses, teachers need situation-specific skills to enact this competency. In mathematics teaching, there has been a significant amount of work on these skills through the lens of teachers professional noticing (e.g., M. Sherin et al., 2011). Teacher's professional noticing builds on Goodwin's "professional vision": the specialized, "socially situated" and "historically constituted" ways of seeing and understanding events particular to a profession (p. 606). In the context of teaching, though there is some diversity in how this notion has been taken-up, most authors who discuss teacher noticing include two main processes: "attending to particular events in an instructional setting" and interpreting or "making sense of events in an instructional setting" (M. Sherin et al., 2011, p.5). These two processes are equivalent to what Chi (2011) described in experts as the processes of, "(1) perceiving particular events in an instructional setting; (2) interpreting the perceived activities in the classroom" (as summarized in Santagata & Yeh (2016, p.154)).

With regard to managing students' apparently incorrect responses these skills are related to the ways in which Jacobs et al. (2010) adapted these two processes for the specific work of "professional noticing of children's mathematical thinking" (p.

169). In their conceptualization of particular professional noticing, the process of "attending to particular events in an instructional setting" is narrowed to attending to "the mathematical details in children's strategies" (p. 172). In other words, to manage students' apparently incorrect responses teacher first need to see the mathematical intricacies and details within a students' responses. In addition to attending to the mathematics in a students' response, a teacher will also need to "interpret" the student thinking. This means going beyond what the student has provided to making-sense of what the child's response indicates about their understanding.

Decision-making.

Though some conceptualizations of noticing including decision-making and I have kept it as a situation-specific process as per Blömeke, Gustafsson, & Shavelson (2015)'s model, I hypothesize that there is not one consistently useful way in which teacher make decisions when managing students' apparently incorrect responses. Again, since this is a situation—hence context and content—dependent process, decision-making will at times occur unconsciously and out of habit, relying on some heuristic, routine or rule-of-thumb. It is quite likely that expert teachers have particular routines (e.g., repeating student contributions, asking other students to rephrase or to weigh in on an idea) that are productive in managing this situation. At other times, something about the student's response or the context might make a teacher pause and deliberate more consciously how to manage this situation. Knowing when to rely on a routine and when to pause and deliberate is a form of professional judgment, key to this competency, that is built on all the resources I previously described.

Conclusion and Implications

When a student has provided a response that seems wrong but might not actually be wrong, an apparently incorrect responses, how a teacher manages this response in that moment has important implications for students' mathematical learning and learning more broadly. This micro-level interaction can shape students' perceptions about mathematics, about failure and about who can and should do mathematics. Seen in this light, managing students' apparently incorrect becomes an issue of equity and quality mathematics instruction for all students. Despite the consequential nature of these interactions, they are not frequently managed productively and instead, non-discriminate patterns of negative and positive evaluation are typical in classrooms. The prevalence of these patterns, however, is less surprising when one considers the difficulty and level of complexity entailed in managing students' apparently incorrect responses.

In particular, teaching in general and managing student's apparently incorrect mathematical responses in particular can be "unnatural" work, requiring teachers to act in ways coherent with their role as teachers but in that stand in juxtaposition to their day-to-day adult ways of being. Additionally, this situation can involve navigating un-solvable dilemmas and can be anxiety provoking. Further, when one considers managing students' apparently incorrect responses as a professional competency and considers what it might entail, the complexities of in-the-moment decision making come to light.

In particular, in order to manage students' apparently incorrect responses teachers need a vast array of knowledge, and affective and motivational-volitional resources, as well as skill in noticing relevant situation-specific details. In addition to general pedagogical knowledge, teachers need specialized forms of professional knowledge including common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and students (KCS), knowledge of content and curriculum, and knowledge of content and teaching (KCT). They also need productive beliefs about the nature of mathematics as a problem-solving endeavor and about mathematics learning and teaching. Additionally, in order to really be able to enact equitable instruction, teacher need to be free of explicit and implicit biases that might overtly or subtly impact their interactions with students. Teacher must also have enthusiasm for teaching mathematics, a strong sense of self-efficacy and high-engagement and resilience in order to navigate the sometimes emotional challenges of managing students' responses.

All the these various internal resources can then support the teacher's situation specific skills and processes when they manage students' responses in the moment. Within a specific situation teachers must notice—perceive and interpret—relevant features of instruction and deploy professional judgment. Ideally, this professional judgment affords teachers the ability to know when to engage in habitual behaviors and employ a rule of thumb or when to pause and deliberate an appropriate course of action as called for by the specific student response and context. Having a clear conceptualization of managing students' apparently incorrect responses and the complexity it entails, I turn now to considering relevant future research and implications for teacher education.

Future Research.

As with any professional competence, there are questions about whether the internal resources I have highlighted here are in fact those that are indeed key to managing students' apparently incorrect responses. It is possible that other resources I did not mention here could play a part in this competence (e.g., some sub-sets of general pedagogical knowledge). Additionally, when considering these resources that are questions to be explored about, "how precisely the different resources are cobbled together, what this interplay depends on and how the resources can be built up" (Blömeke, Gustafsson, & Shavelson, 2015, p.7). There is also work that remains to understand how this competency is enacted, how it develops over time and what teacher education content and pedagogies might facilitate learning of and improvement in managing students' responses. Implications for Teacher Education.

With regard to teacher education, this paper both echoes old wisdom and offers newer suggestions. It is not new to suggest that teacher education carefully reconsider the type of knowledge it imparts on secondary mathematics teachers (e.g., Shulman, 1986). Indeed, there has been more of a push for joint efforts between mathematics and education departments for coherent instruction focus on learning mathematics for teaching and a growing body of research on specialized forms of MKT for different mathematical content. There is also accumulating evidence that situation-specific skills such as noticing are important for teaching and can be developed in teacher education through a variety of video-based pedagogies (e.g., M. G. Sherin & van Es, 2009; McDuffie et al., 2014; Stockero, 2014). Additionally, there are more efforts to incorporate pedagogies of enactment (Grossman, Hammerness, & McDonald, 2009) in mathematics teacher education that can afford opportunities for teachers to develop competence using their knowledge in action.

What might be less common in current conversations about teacher education is attention to the other resources that impact how teachers' manage students' apparently incorrect responses. In particular, there is little attention to teachers' implicit biases and no systematic means in teacher education to identify and work on these problematic beliefs. Additionally, though there is attention to teachers' beliefs about the nature of mathematics and its learning and teaching, there is little to no attention to developing teachers' self-efficacy and adaptive self-regulation practices. Teacher education seems to take for granted that teachers come to the profession already having developed the beliefs and practices they need to have high enjoyment and resilience.

Just as teacher educators might sometimes need to remind teachers that their role calls for them to attend not only to teaching mathematics, but to teaching children mathematics—and all the social-emotional, interpersonal work this entails—teacher education might also need to be reminded that it too is dealing with human beings. A more holistic approach to teacher education that attends to the complex, multi-faceted, humanistic nature of teaching and addresses key resources in several areas—cognitive, affective and motivational-volitional—might make for more competent, engaged and resilient teachers. An outcome that is not only beneficial for these teachers but also to the countless students that will come through their classrooms for years to come.

CHAPTER III

How Do Secondary Mathematics Teachers Respond to Student Thinking in Early Algebra?

Introduction

There is general agreement in mathematics education of the importance of teachers being responsive to students' thinking. For example, the National Council of Teachers of Mathematics highlights the importance of teachers eliciting and probing students' thinking to craft instruction that connects to and builds on student ideas (2000). Being responsive to student ideas reflects "the extent to which teachers 'take up' students' thinking and focus on student ideas in their moment-to-moment interactions" (Pierson, 2008, p.40). This type of responsive instruction has been linked with both rich, learning environments and improved student achievement (e.g., Pierson, 2008; Fennema et al., 1993).

Despite widespread agreement about its importance, research into how teachers respond to students is still ongoing. Research related to understanding how teachers respond spans a variety of different streams of work including research that examines the work of teachers pre-, post- and during- instruction. Though the field benefits from studies that examine the ways in which teachers plan for and reflect on instruction (e.g., Stein et al., 2008) these actions differ from those of responding in-the-moment to student-thinking. To support teachers in the enactment of responsive teaching requires a better understanding of the variety of ways teachers respond in-the-moment to different types of student thinking.

Within the work that explores how teachers respond in-the-moment there is an impressive range in the mathematical context and some variety in the focus of analysis. First, the mathematical content of studies varies greatly from whole-number operations in early elementary mathematics (e.g., Jacobs et al., 2010), to topics in Algebra (responding to student work on linear equations e.g., AMTE, Lesseig. et al.), to Geometry concepts such as ratio and proportion (J.-W. Son, 2013) and reflective symmetry (J. Son & Sinclair, 2010), to proofs of the Pythagorean Theorem (D. Zazkis & Zazkis, 2016) and quadratic equations Hu et al. (n.d.). Additionally, the analysis of teacher responses varies as well including exploring the content and pedagogical content knowledge that teachers used in interpreting and responding to students (e.g., J.-W. Son, 2013; D. Zazkis & Zazkis, 2016), or the pedagogical nature of the response (e.g., working with versus redirecting students' thinking; "show and tell" versus "give-ask", (J.-W. Son, 2013, p.56).

Though the mathematical content and analysis vary within this body of research, most studies obtain teacher responses in a similar manner. The predominant approach is to ask teachers to script or write their response to student work (e.g., J. Son & Sinclair, 2010; Crespo et al., 2011; R. Zazkis & Kontorovich, 2016; R. Zazkis et al., 2012). A slight variation of this method is to ask teachers to depict—using a comic, image based program—how they might respond to students (e.g., Rouge & Hersbt, in press; Pelton et al., n.d.; Chen, 2012; de Araujo et al., 2015).

Looking across this body of work, three important gaps emerge. First, few studies examine how teachers respond to students work in early algebraic content. Algebra is often described as the "gate-keeper" to higher-level mathematics and a variety of recent efforts, including introducing algebraic concepts in elementary grades and pushing more formal algebra into middle school mathematics, speak to the need to introduce students sooner than high school to Algebraic concepts. The transition from arithmetic thinking to algebraic thinking presents many adjustments for students including a shift of focus to "relations and not merely on the calculation of a numerical answer", "on operations as well as their inverses, and on the related idea of doing / undoing", "on both representing and solving a problem rather than on merely solving it" and "a focus on both numbers and letters, rather than on numbers alone" (Kieran, 2004, pp.140-141). This push for earlier introductions to algebraic content and the challenges this presents for students, means that students are likely to produce a range of correct and incorrect thinking as they reason through this content. In order to support teachers as they manage these student answers, research is first needed to explore how teachers might currently be navigating student responses in this mathematical domain.

A second gap emerges when looking at the type of student responses used as prompts. Most studies present teachers with either incorrect or correct student thinking. In studies focusing on exploring how PSTs respond to students, prompts tend to include problematic or incorrect student work since these are typically more challenging for novice teachers to respond to. Specifically, in addition to the content knowledge needed to identify an error a teacher also needs specialized knowledge of content and students D. L. Ball et al. (2008) to craft a response. Other studies include correct student work with rich and varied solution methods to explore how teachers might attend to, interpret and decide how to respond to students (Jacobs et al., 2010). Though Jacobs et al. (2010) provided teachers with one student response that includes a small computational error, even though the student work demonstrates a strong grasp and flexibility with numbers, the focus of their analysis was not on how teachers might have responded differently to this student. In order to better understand the ways in which teachers respond it would be important to explore the ways in which they respond to a variety of student responses, including comparing and contrasting responses to perceived (or what I will refer to as "apparently") correct and incorrect student responses.

A third important gap arises when examining the methods used, specifically the ways in which teacher responses are obtained. As I mentioned above, the predominant means of obtaining information about how a teacher would respond to a student is to ask the teacher to write or script their response (e.g., Jacobs et al., 2010; R. Zazkis et al., 2012). Though this provides interesting information about what teachers report they might do and is likely related to how they might actually respond to students, the fidelity between this setting and a real classroom setting is still low. Therefore, caution should be used in extending the results obtained in these settings to actual classroom performance. Further, very few studies ask teachers to respond verbally to student thinking (e.g., Van Zoest, Stockero, etc. 2017 in press PME-NA proposal) and even fewer do so under conditions that replicate some of the time-pressures of actual classrooms (e.g., Knievel et al., 2015). The current study aims to address these three gaps by reporting on the ways in which teachers responded to a variety of correct and incorrect student thinking in early algebra during a teaching simulation with more fidelity to classroom conditions. In particular, I asked:

1. How do teachers respond to apparently correct student responses?
2. How do teachers respond to apparently incorrect student responses?
3. How do teachers' responses to these different types of student responses vary?

Conceptual Framework

There are two general concepts that undergird the work in this study: (1) that student responses are usually not simply right or wrong, and (2) that there are some overarching, guiding principles about what features of teachers' responses that make them more or less productive for student learning.

Types of Student Thinking

One of the challenges that teachers face when responding to students is that student responses are rarely objectively correct or incorrect. Instead, responses from students can be thought of as occurring on a continuum of correctness (see Figure 3.1).

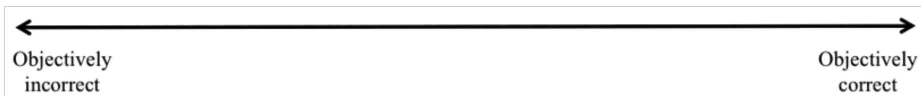


Figure 3.1: Continuum of correctness

At the far right of the continuum are responses that can be objectively deemed correct. These responses are likely to contain underlying mathematical ideas and reasoning that are procedurally and conceptually sound. Further, these ideas are presented articulately, with precise mathematical language. These correct responses, far to the right, are likely to be objectively evaluated as correct. On the far left of the continuum, are responses that can be objectively deemed incorrect. In these types of responses, the underlying mathematical procedures and concepts are troublesome and deeply, logically flawed. Additionally, the presentation of the ideas is problematic due to ambiguous pronouns, incorrect terminology, or confusing uses of mathematical language. At this end of the continuum, responses are likely to be objectively evaluated as incorrect.

Though I have chosen to represent this continuum as two-dimensional there are a variety of ways in which a student response could vary (as I alluded to in my descriptions above). For example, a student answer might have a simple computational error and/or a more fundamental, logical error. Further, as I mentioned above, the way in which the answer is presented (through imprecise or inaccurate language) might superficially make the answer appear incorrect (as in the case of Jean and his "boxes" in Paper 1). In some cases, issues might arise when a student provides an incomplete answer that perhaps misses a logical step in a proof for example, or provides a correct answer with an incomplete justification for the answer. Perhaps, as is the case in the apparently incorrect responses used in this study, students are quite capable at doing the mathematics and are simply solving a problem that is different than the one the teacher intended. Essentially, one might imagine that between the two extremes of "objectively correct" and "objectively incorrect" lie a variety of other dimensions representing the multitudes of ways in which student answers might vary. Most student responses tend to fall somewhere in-between the two ends of this continuum and hence, will require some interpretation and sense-making by the teacher. However, there are some broad principles that guide how teachers could respond to students.

Responding to Student Thinking

Over the last three decades much research has focused on describing teacher responses to student thinking. These efforts seem to fall into three broader strands of work with each strand foregrounding one of three components core to effective mathematical discussions: engaging the class, focusing on the mathematics, and responsiveness to students. One example in the first strand is the general "talk moves" (e.g., revoicing, asking students to restate others' thinking) proposed by Chapin et al. (2009) that attend to students' positioning relative to each other and to the content to create collaborative discourse in classrooms. This work highlights the central goal of discourse to more broadly involve students, not just the teacher, in discussing content. A second strand of work on teacher responses foregrounds the mathematics. An example of this strand of work is the research of Conner et al. (2014) that describes teacher responses that support collective argumentation. This work highlights the importance that mathematics should play in teacher responses. A third strand focuses predominantly on characterizing teachers' responses with respect to whose ideas and what reasoning are the focus of a teacher's response (e.g., Pierson, 2008). This strand of work foregrounds responsiveness to student thinking and highlights the importance of teacher responses picking up the ideas that students put forth. Together these lines of work suggest that important aspects of teachers' responses include: (1) who they engage in the work, (2) what mathematics is considered and in what ways, and (3) the extent to which teacher responses are 'responsive' to student thinking.

Research Methods

Participants

Secondary mathematics in-service and pre-service teachers within and around a large midwestern university town were recruited in person and by email as part of a larger study on managing students' mathematics responses. Recruitment efforts targeted secondary mathematics teachers at middle and high schools, as well as public, private and charter schools. All participants volunteered and were compensated for their participation in this study. Hence, the final sample of 24 participants who completed all data collection (paper instruments and teaching simulation) for this study is a sample of convenience (it should also be noted that the author had previously worked with 10 of the 24 participants in the author's previous work as a university field-supervisor).

Within this sample of 24 secondary mathematics teachers, one participant self-identified as Black and 23 participants self-identified as White. Additionally, there were six participants who self-identified as male and 18 who self-identified as female. The sample included five preservice teachers, three of which were in their final year of a three-semester undergraduate secondary teaching certification program, one who was in the first semester of this same undergraduate program and one who was in a one-year masters teaching certification program. The remainder of the sample is comprised of 19 inservice teachers with a range of 2 to 38 years of teaching experience and a median of 10 years of experience. The 24 participants ranged in age from 19 to 64 years of age, with a median age of 35.5 years.

Of the 24 teachers, one was recently (within the last 5 years) retired, 10 were currently in middle school mathematics classrooms and 13 were in high school mathematics classrooms. One participant reported having taken no undergraduate or graduate mathematics courses while the rest of the participants all reported having taken three or more undergraduate or graduate mathematics courses. With respect to methods of mathematics teaching courses, seven participants reported having taken one such course, nine reported having taken two courses, and eight reported having taken three or more mathematics methods courses.

Teaching Simulation.

As part of a larger study exploring how mathematics teachers manage students' responses, I designed, piloted and used an interactive-video based teaching simulation. The teaching simulation takes place in a lab setting where participants sit at a desk and go through the teaching simulation on a laptop computer (see Figure 3.2 for a picture of the set-up). The experimenter, myself in this case, sits next to the participant and monitors their progression through the teaching simulation, intervening with additional information and feedback as indicated in a protocol script. In terms of the mechanics, the teaching simulation is essentially a collection of slides that participants click through with a computer mouse and videos of student responses that they can view only once.

Figure 3.2: Teaching simulation set-up in lab space.

Teaching simulation: Overview.

Here I present an overview of the teaching simulation and details of the student responses that are the basis of the analysis in this paper. Figure 3.3 below shows an overview of the contents of the simulation. To ensure consistency of the information presented and questions posed to participants, I used a simulation protocol script and checklist.

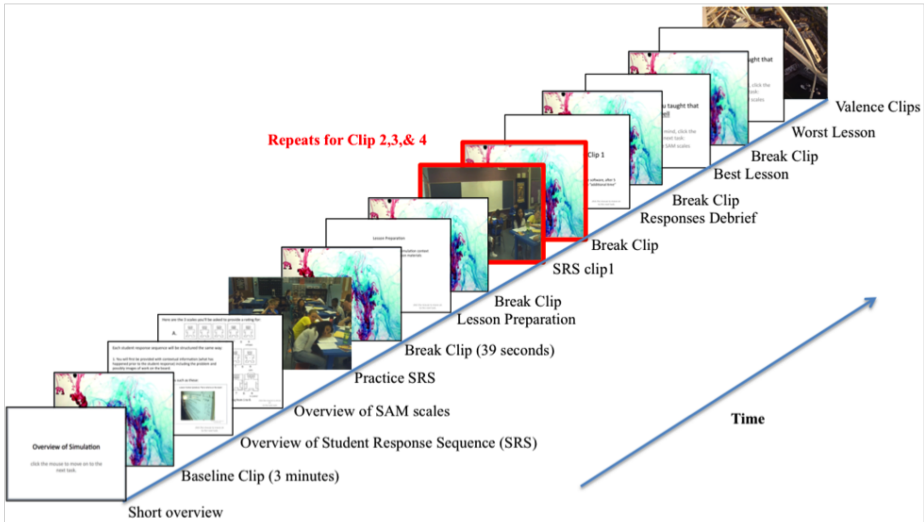


Figure 3.3: Overview of teaching simulation.

The teaching simulation begins with a short, three slide overview that presents participants with general information about the context of the simulation (responding to students) and what they will be ask to do (read, think-aloud, respond) in addition to explaining why baseline and break clips are built into the simulation (to provide rest periods for the physiological data that was also collected during the teaching simulation but not used in this study). After viewing the baseline clip, participants go through a series of slides that explain the structure and content of the Student Response Sequences (SRSs) that comprise the core of the teaching simulation. The teaching simulation includes four SRSs (three of which are analyzed in this paper).

Each student response sequence (SRS) begins with some instructional context including information about what problem students are working on and images of relevant board work. After the instructional context, participants see a slide with a question that they are to imagine they have posed to students. During the SRS participants can go through the context and question slides at their own pace. When they are ready to move past the question slide and click the mouse, a video of the student responding to the question immediately begins to play. The videos of the student responses used in the SRSs come from an actual 6^{th}-grade mathematics class and are taken from a first-person perspective (using Tobii eye-tracking glasses) from the front of the classroom. This makes the viewing of the video seem more like what one might see as the teacher in the class. These videos also include subtitles on the bottom left in case, for some reason, participants have difficulty hearing the student despite the good sound quality. Additionally, during a practice clip I confirm with the participant that the volume is adequate and adjust it for them as needed.

Once the student is done talking, the simulation automatically transitions to a slide prompting the participant to self-assess their emotional reactions to the student response using a paper-and-pencil version of the Self-Assessment-Manikin Self-Assessment-Manikin (SAM) (Lang, Bradley, xx). Participants have a total of 15 seconds to fill these out. The 15 seconds consist of the prompt slide (which is displayed for five seconds) and a ten-second, countdown video. After the ten-second countdown video ends, the simulation automatically transitions to a slide prompting participants to respond to the student. Participants are given a maximum of 90 seconds (a minute and a half) to respond. In earlier piloting of the simulation, this was found to be more than enough time especially given that I am asking participants to provide only their initial or first response to the student. The organization of the SRS and the time constraints imposed for responding are similar to some of the design considerations Lindmeier (2011) and Knievel et al. (2015) advocate to explore action-related competence (which is related to the ability to perform under more realistic conditions).

After participants have gone through these detailed instructions about the SRSs and we discuss the SAM scales, they are given a practice student response sequence. This provides them with an opportunity to get a sense for the flow and speed of the various components of the SRS, to ask any final questions, and for me to redirect them if they talk to me rather than speaking to the student when they respond. After this practice sequence, participants are given the context for the rest of the teaching simulation and SRS clips. Namely, participants are asked to imagine that they are substituting for a colleague's 6^{th}-grade mathematics class towards the end of the school year during a lesson on using fact-families to find unknowns. They are then given the lesson materials (Connect Mathematics Program lesson 4.4 "Finding the Unknown Value: Solving Equations I" version 4, 2011) to review for about 25-minutes, after which they go through the four SRSs.

Teaching simulation: Student response sequences.

Figures 3.4 and 3.5 provides a summary of the instructional contexts, the teacher questions and student responses for Clips 1, 2 and 4 that are the focus of this paper. Clip 3 was not used in this analysis since participants varied in their perceptions of whether it was correct or incorrect. This made it less useful in characterizing teachers' responses to apparently correct and incorrect student answers.

In the first SRS (Clip 1) the student notices that numbers in the facts of the fact families for multiplication and division are "rotating around in a circle." Though there is some imprecision in the student's language, she is noticing something visually that is indicative of underlying mathematical properties (e.g., the commutative property of multiplication, that division is the inverse operation to multiplication, and that when a number is divided by one factor the result is the remaining factor). Hence, the student response in Clip 1 is likely to be perceived as correct by teachers.

In SRS Clip 2 the student provides their answer to the following problem:

> On the Ocean Bike Tour test run, Sidney stopped the van at a gas station that advertised 25 cents per gallon off on Tuesdays. 1. Write the function that shows how to calculate the Tuesday discount price per gallon d from the price on other days p. (Connected Math Curriculum)

The correct answer to this problem is that d (the discounted price per gallon) is

SRS	Instructional Context	Teacher Question	Student Response
Clip 1	This takes place during the launch of the lesson on fact families to solve equations. The teacher has elicited two examples of fact families from students, including the following which is on the board: 3*4 =12 4*3 = 12 $\frac{12}{3} = 4$ $\frac{12}{4} = 3$	Tell me something else about fact families, other than giving me an example	So like for the multiplication it kinda rotates. So like the 4 goes where the 3 was and then the 3 ...like...but then…when it goes …uhhhmm...multiplication to division it's like, it, the 12 goes where the 4 was and the 4 goes to the 3 and then the 3 goes to the answer so like they kinda just rotates around in a circle
Clip 2	After students have had a chance to work on parts A and B of the lesson. The participant is told that none of the students have had a problem with part A, so they have decided to start the class discussion with part B. Part B is as "On the Ocean Bike Tour test run, Sidney stopped the van at a gas station that advertised 25 cents per gallon off on Tuesdays. 1. Write the function that shows how to calculate the Tuesday discount price per gallon *d* from the price on other days *p*"	Tell me your answer for the first part of B, what are your variables and what equation did your group come up with?	So when d is the discounted price, p is normal price, and g is gallons…g is gallons, uhmm we said d equals p minus point 25g and what that means is that 25 is…you get 25 cents off for every gallon so p is normal price and then you would minus the 25 times the gallons to get the difference

Figure 3.4: Summary of SRSs Clips 1, and 2: instructional context, teacher question and student response.

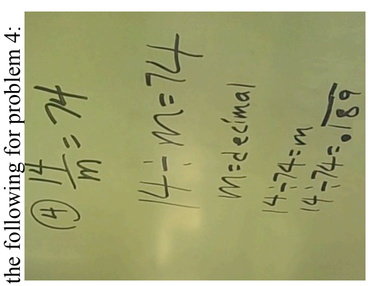
SRS	Instructional Context	Teacher Question	Student Response
Clip 4	Students have worked on problems 1-4 of part D and volunteers have put their answers on the board. Including the following for problem 4:  After going over problems 1-3 with students, the teacher goes over problem 4 and asks the students if they have any questions.	Yes, what's your question about number 4?	The answer is one thousand thirty six because one thousand thirty-six divided by fourteen …

Figure 3.5: Summary of SRSs Clip 4: instructional context, teacher question and student response.

equal to p (the normal price per gallon) minus 0.25 (dollars per gallon). The student's solution can be summarized as follows:

> d is the discounted price, p is normal price, and g is gallons,
>
> therefore $d = p$ -0.25*g, which means you get 25 cents off for every gallon

There is some ambiguity on the exact definition of the student's variables, though it can be inferred that the student means these to be total prices (rather than price per gallon) because of the way the student explains what their equation represents. Given their definition of the variables, the student solution is actually correct. However, the student response does not answer the question posed in the problem. The core issue stems from how the variables d and p are being defined. If they represent total price, then one indeed needs to know the number of gallons (g) in order to calculate the total discount price from the total normal price. If, as the problem intends, the prices are unit prices (price per gallon) then there is no need to specify the number of gallons (g) in order to calculate the total discount price per gallon from the total normal price per gallon.

In Clip 4, the student is providing an alternative response to a problem 4 that is already on the board. The board work is as follows:

> $14 \div m = 74$
>
> $m =$ decimal
>
> $14 \div m = 74$
>
> $14 \div 74 = 0.189$

The student proposes that, "the answer is one thousand thirty six because one thousand thirty-six divided by fourteen..." [equals 74]. Again, like in Clip 2, the student answer is not completely incorrect. Based on the end of the student's response, it appears that the student is actually solving a different problem, one in which m is divided by 14 rather than 14 divided by m. The core issue becomes understanding why the student solved $m \div 14 = 74$ rather than the original problem $14 \div m = 74$.

Coding of teacher responses.

In this study, I am interested in characterizing teachers' initial responses to apparently correct and incorrect student answers in early algebra.The choice to examine a teacher's initial response to a student is, in part, a consequence of the design of the teaching simulation (the student in the simulation cannot respond to the teacher). Additionally, though a teacher's initial response is only a thin slice of their instruction and is not necessarily representative of their overall practice, it can be argued that the ways in which a teacher starts an interaction with a student idea can immediately either open or close the conversation that follows. If this initial response shuts down or narrows the focus of conversation immediately, the interaction that follows has less potential to engage students in meaningful mathematical conversation. On the other hand, an initial response that is more open-ended (though it might not necessarily stay that way as the conversation unfolds) provides greater potential for the interaction to be responsive to student thinking and to engage the class in conversation. In other words, though a teacher's initial response provides only a glimpse into how they might manage student thinking, it is a crucial first-move for the teacher and important insight can be gained by examining the way in which teachers choose to begin their interaction around student thinking.

In the section that follows, I describe the two analysis I conducted to describe teachers' responses to students' early algebraic thinking. The first analysis I conducted used a coding scheme to describe various characteristics of teachers' responses. These characteristics are informed by the features of teachers' responses that make them more or less productive for student learning (identified early in the conceptual framework). This first analysis provides a way to describe trends in features of teacher responses within each SRS clip and across apparently correct and incorrect SRS clips. This broader level of analysis however, necessitates loosing some of the interesting, context specific details of teacher responses. In order to see some of the more SRS context specific nuances in teachers' responses, I also conducted a thematic analysis. This afforded me an opportunity to explore how teachers responded in the three different student response contexts and complements the first analysis by painting a richer picture of what teachers said.

Analysis 1: identifying the initial teacher response and qualitative coding.

After transcribing the 72 teacher responses (24 teachers and three student clips), I identified the initial teacher responses. I define a teacher's initial response as their first full turn of talk that starts when the teacher begins to speak and ends either when the teacher stops talking or there are indications that they would let students take a turn of talk. In some cases, teachers responded with more than just their initial response. In these cases, I looked for indications within their full response that they would have let a student take a turn and I also reviewed the video of their teaching simulation experience for other indicators that their initial turn of talk was over (e.g., looking for wait times of more than a few seconds). For example in response to SRS Clip 2 about finding the discounted price per gallon, participant id07 responded with,

> Okay Paul, so can you read the problem out loud to me again, cause I want to talk about let's start with the variables that you chose. Okay, you had uhhmm g for gallons, d for discounted uhhhmmmm price, so I want to talk about the variables first, does anybody else have different ideas for the variables, what variables we maybe can use from the problem now that's he's read it out loud again...uhh. it says discounted...so, I want people to raise their hand if they have an idea of what operation discounted could

mean...

There is evidence that the teacher's initial response is to ask the student to read the problem out loud. After the student has read the problem out-loud, the teacher would then follow-up by asking other students in the class about the variables. Based on the lexical clues and the pause in speech and the lexical clues that indicate the student has read the problem, I defined this teacher's initial response as follows:

> Okay Paul, so can you read the problem out loud to me again, cause I want to talk about let's start with the variables that you chose. okay, you had uhhmm g for gallons, d for discounted uhhhmmmm price, so I want to talk about the variables first

Once I had identified and catalogued the initial teacher responses I qualitatively coded the response using the Teacher Response Coding scheme (TRC) (Peterson, Van Zoest, Rougée, Freeburn, Stockero, Leatham, in press). The TRC was developed to provide a descriptive way to capture multiple features of teachers' responses that have been identified in the literature as important for effective mathematics discussion. Specifically, it contains four categories: *actor* (which identifies who is publicly being asked to consider the student thinking), *move* (which captures what the actor is doing or being asked to do), *recognition* (the extent to which the student might recognize their response in the teacher's response through the take-up of the student's language— *student actions* subcategory—and main ideas—*student ideas* subcategory), and *mathematics* (the alignment between the mathematics towards which the teacher response seems to be headed and the mathematics underlying the student's idea). These categories and codes, including additions that I made to the coding scheme, are described in greater detail below. The TRC can be applied at various grain sizes including the entire teacher response or at the level of each different move (distinguished by changes in inferable instructional intent) within a teacher response. For example, the teacher response "Nice job Johnny, what do others think?"

can be coded either holistically (the entire teacher response) or can be decomposed into two moves ("Nice job Jonny" and "what do others think?") that can then each be coded separately. In this study, the unit of analysis was the entire, initial teacher response. Hence, each teacher response received an *actor, move, recognition- student actions, recognition- student ideas,* and *mathematics* code.

To establish some validity in the coding 19 out of the 72 (26% of the data) responses were coded by two other researchers. Both researchers had been previously involved (as was I) with the development of the coding scheme and had prior experience applying the TRC codes to teacher responses from classroom videos and interview scenarios. Each researcher independently coded 10 teacher responses (chosen at random from across Clip 1, Clip 2 and Clip 4) for all categories of the TRC. All but one of the teacher responses assigned to the external researchers were unique, hence a final count of 19 rather than 20 teacher responses. After the researchers and I had independently coded the assigned teacher responses, I met with each one individually to discuss our coding. During these meetings, the researcher and I would discuss the coding that differed and discuss the instance untill agreement was reached. Hence, after these meetings I had 100% agreement with the researchers for all TRC codes for the 19 teacher responses. After these meeting, I re-coded the remaining 53 teacher responses based on the agreements and understanding reached through those discussions.

TRC: Actor

The coding category actor captures who is publicly being engaged or asked to engage in the intellectual work. The actor of a teacher response or move can be coded as: *teacher, same student(s), other student(s),* and *whole class.*

TRC: Recognition

The code *recognition* captures the extent to which the student might recognize his/her response in the teacher's response through two sub-categories of codes: (1) through the take-up of the student's language (*student actions*) and (2) through the take-up of the student's ideas (*student ideas*). Student actions can be an important or a superficial way for teachers to pick up on what students have said. On the one hand, using the student's language, work or gestures (their actions) might reasonably increase the probability that the teacher's response is close to the student's ideas. On the other hand, it is possible that a teacher response picks-up the student's actions without attending to the student's underlying main idea. In addition to attending to the degree to which a teachers' response uses a student's actions (coded: *explicit, implicit,* or *not*), I also captured when teachers attempted to use a students' language explicitly but did so incorrectly. For example, saying "you got 1076" rather than 1036 in response to Clip 4. Hence, I added the code *explicit-incorrect* to this sub-category.

The coding sub-category *student ideas*, in contrast to *student actions*, is a more meaningful way for teachers to attend to what a student has said. The codes in this category capture the extent to which the main idea in the teacher's response is related to the student's main idea. Codes in this category include: *core, peripheral, other, cannot infer* and *not applicable*.

TRC: Moves

The coding category *moves* captures what the actor is being asked to publicly do. For example, the actor might be asked to *justify, clarify* or *repeat* some particular aspect of the mathematics being discussed. A detailed list of the various *moves*codes and their definitions can be found Peterson, Van Zoest, Rouge, Freeburn, Stockero, and Leatham (in press).

TRC: Mathematics

This category captures the alignment between the mathematics towards which the teacher response seems to be headed (the mathematical understanding (MU) of the teacher response) and its relation to the mathematical point (MP) underlying the student's idea. Van Zoest et al (2016- theorizing MP) define a mathematical understanding as “a concise statement of a non-subjective truth about mathematics” and the mathematical point of a student’s response as “the mathematical understanding that (1) students could gain from considering a particular instance of student thinking and (2) is most closely related to the SM [student mathematics] of the thinking” (p. 326). The degree to which the MU of a teacher’s response aligns with the MP of the student response can be coded as: core, peripheral, other, cannot infer (CNI), and non-mathematical. Additionally, the tag “-incorrect” can be added to the code other when the teacher response (TR) seems to be going towards incorrect mathematics (I extend this tag to use with the code peripheral). When more than one mathematical point underlies the student’s idea a tag, “-MP1”, “-MP2” and so forth, can be added to any of the main codes to further specify what mathematics a teacher response could be headed towards (details about these various codes can be found in the next paper of this dissertation).

Analysis 2: identifying themes in teacher responses.

Though the TRC provides a means to describe various characteristics of teachers’ responses in a way that allows for comparisons across responses to different student answers, I also analyzed the teacher responses thematically. Essentially, for each set of 24 teacher responses I went about grouping the responses from most to least productive to better understand the various ways in which teachers might respond. For each group of 24 responses I went through roughly the same process. In a first pass, I first created two groups from the 24 responses: a less and more productive group. This initial choice was informed in part by whether the teacher did all of the intellectual work or not, whether or not there were issues in the language or wording of the response that might make it confusing for students, and the general sense for whether the mathematics seemed related or not to what the student had proposed. I then went into each group to look for similarities between responses in the type of work teachers might be asking students to engage in (e.g., were students being asked to check their work or put their answer on the board). This resulted in clusters of teacher responses with similar themes that are described in the results that follow.

Results

In the sections that follow I explore the answers the following:

1. How do teachers respond to apparently correct student responses?

2. How do teachers respond to apparently incorrect student responses?

3. How do teachers' responses to these different types of student responses vary?

How do teachers respond to an apparently correct student answer?

In response to the student who, correctly, notices a relationship between the numbers in the multiplication and division fact of the fact family most teachers responded by going back to the *same student* (13 out of 24 responses). Nine teacher responses engaged only the *teacher* and two responses engaged the *whole class*. Additionally, a little over half (14 out of 24) used the student's language *explicitly* and 10 used it *implicitly*. Most teacher responses were close to the student idea with half (12 out of 24) coded as *core* to the student ideas. The remaining 13 responses were *peripheral* (5 responses), *CNI* (1 response), *other* (3 responses), or *NA* (3 responses). Though there was some distribution in the type of move (see Table 3.4, the most frequently occurring move was an *elaborate*. With regard to the *mathematics*, most of the responses to Clip 1 were *cannot infer* or *CNI*, meaning they were vague but not imprecise or confusing (see Table 3.5 for the distribution).

Thematic grouping of teacher responses to Clip 1

In qualitatively grouping the 24 teacher responses the following seven categories emerged: (1) “Come up and show us”, (2) “why is it doing that?”, (3) “what do you mean?”, (4) focusing on peripheral/other mathematics, (5) “that’s good, can you tell me more...?”, (6) “that’s good, what else?”, and (7) only the teacher engages in the intellectual work.

(1) “Come up and show us” There were four teacher responses (from id06, id08, id07, and id18) that, with only slight variations, essentially all asked the student to come to the board and “...show a little bit what you’re talking about with how you’re noticing the numbers move around in these fact families”—id08. These responses all focused on the student idea and asked the student to either repeat, clarify or elaborate on the idea of the numbers rotating in a circle by coming up to the board. In two of these responses teacher also provided a rationale for their request (“...cause I could...can see a couple of people in class not really understanding your way of thinking about this.”—id18 and “...so that others can see what you’re thinking.”—id06).

(2) “Why is it doing that...” Another group of six (id21, 11, 19, 01,16, and 17) teacher responses focused on some version of probing the underlying mathematical operations and preservation of equality, with questions in the vain of “why it rotates like that”—id19. Of these, two responses asked student(s) to think about “anything that’s happening between those two numbers that might cause them to rotate?”—id 21 or “why is it doing that?”—id 11. Two other responses posed what could be described as a two-part question, where the first question, like these first two responses asked, “why it rotates in a circle” —id01. The responses then continued with another, related question. In one case, the participant asked, “could we write them differently where they wouldn’t rotate?”—id19. In the other, the participant asked a similar but more ambiguously worded question, “can you do it without doing the whole circle or do you need to write it down every time?”—id01. Both of these follow-up questions

seem to be attempting to support (or funnel?) students towards the preservation of equality implicit in the fact families. A fifth response in this category was longer than the other but also more deliberately and clearly focusing students on investigating "...are all three numbers interchangeable, uhhmmm...can can I switch the 12 and the 4 and the 3 and put them in any spot and still always have a true answer?.."—id17. The final response in this group seemed to also intend to probe the underlying "why" but used wording that was unclear, "are you sure that the 12 is moving around in the circle...errrr...or what do you think might be going on with that, that 3 and that 4?"—id16.

(3) "what do you mean?" Two teacher responses, after some verbal processing, settled on asking the student to clarify or elaborate on what they meant. In one case, the participant started with "what do you mean by it goes around in a circle" but then veered into reiterating that the conversation was about facts of multiplication and division, asking "what's the connection between multiplication and division, what's the connection between addition and subtraction and going around in circles", before finally settling back on "what do you mean by it going around in circles?"—id02. The other participant also seemed to flounder to find the question they wanted to ask the student. This participant started by honing in on the word rotate, "when you use the word rotate, that's an interesting word", then veered into wanting the student to connect this to "some other words that you've heard in your math classes uhhh that kind of mean the same thing as rotate..." before finally articulating the question "I'm interested in like when you use the word rotate, like it goes in a circle [gestures], uhmm, what do you, could you say some more about that?"—id23.

(4) Focusing on peripheral/other mathematics Two teachers responses (id09, 24), after validating the student responses, focused on a piece of mathematics that was rather far from the student thinking. As mentioned in the discussion above on the mathematics of the moves, at the heart of the student's response is the mathematical

idea that there is some pattern (hence the “rotating”, and “circle”) related to the relationships between the mathematical operations within facts of the fact family. The two responses in this group appeared to deviate significantly from this mathematical terrain. In one response the participant seemed to focus on what they perceived to be the student’s imprecision in language, “you said that the 3 becomes the answer, how do we know if there’s an answer here if we’re not solving for anything?”—id09. Interestingly, the student is not incorrect when they describe the number that remains by itself in the fact as “the answer”. The answer to the problem “12 divided by 4” is in fact 3.

The other response in this category was much longer and hence, an instance where the participant seemed to be verbally processing the student thinking. During this processing, the participant seemed to hone in on the underlying mathematical operations in the fact families, “...so when you’re saying something like the 3 can kinda rotate around and move to the other side of the equation you’re noticing that uhmm, that you can move the numbers around with mathematical operations...”—id24. Then the response veers into, “if we go from 12 equals 3 times four that we have up here to something like 12 divided by 3 equals 4, then what mathematical operation have we done to both sides of that equation?”—id24. Here, the response has taken a turn towards mathematical territory (inverse operations) that is related but likely mathematically inappropriate for this lesson and grade level (this lesson is essentially the start to introducing the concept of inverse operation but in a less formal way).

(5) “That’s good... what more?” Two teacher responses, after validating the student response (“awesome”—id12 and “that’s a very good observation”—id22) asked the student to follow up with more. In both cases, however, the request for more information was vague and unclear: “what more can you tell me when you see that they’re rotating, is there something else that you could tell me about?”—12 and “is there anything else you could think of that uhh this could relate to or why this is

important?”—”—idid22. There is nothing in the questions that supports student in responding to them. Though, on the one hand, one could argue they are open and hence leave space for student thinking, on the other hand, a student is unlikely to have any idea what to think about and say in response to this.

(6) “That’s good... what else?” In this group of responses, after validating the student’s response, the teacher went on to collect input from other students. The three teacher responses in this group all start out similarly by validating the student responses: “that’s a great observation”—id13, “that was really good”—id05, and “that’s a really good observation”—id03. Two responses (id13, and id05) went on to repeat, with varying degrees of accuracy, part of what the student had said (e.g., “...You noticed that each family you can reverse the position so the 3 and the 4 uhh...make 12, as does the 4 and the 3. Then you go back to, to make 12 and multiply...”—id 05). All three responses ended by asking for input from other students: “...does anybody else have a suggestion on that? how do you see it Vanessa sitting in the front row?”—id05, “...do we have any other , other insights into what’s going on here?”—id 13 and “...but is there anything else you guys noticed about it?”—id03)

(7) Only the teacher engages in the intellectual work There were five teacher responses that basically just made a statement about the student response and never re-engaged the same student or any other students in the class in the intellectual work.

Version 1- “Awesome job there” One of these teacher-centric responses essentially made a succinct statement validating the student response: “That’s really interesting how you think like that, putting it in a circular motion, that’s really a cool idea”—id 25.

Version 2-“That’s good and here are the connections I see”

The other four versions of this teacher-centric response validated the student response and made, to varying degrees, connections between what the student had said and other ideas. In one of these, it seemed that the teacher was verbally processing what the student had said as she was speaking, almost as if she was doing a think-aloud: "I think you're right on target. You know you're going to use all the three numbers all the time and that if you're using two of the numbers then the other number is the answer, you said it kind of goes around in a circle, kinda like that triangle idea. So if you switch the 3 and the 4 yes the answer is still the third number and nice job, I think you've got a good handle on it"—id 10. On the one hand, as this teacher is verbally processing, she is making her thinking and hence, the ideas in the student response, more visible to the rest of the class. However, it seems that her verbalization is more for her benefit rather than to intentionally broadcast the student's idea. This seems clear when she references "kinda like that triangle idea." The connection between fact families and "that triangle idea" is a connection this teacher made during the lesson-planning segment as she was making sense of the content. It was never a part of the instructional context and hence this reference would be something the hypothetical students in the simulation would have no obvious reason to understand.

A second response in this group made a connection between the student's response and inverse operations for both multiplication and division, and addition and subtraction: "...So when you're looking at rotating, you could give those words those operations uhhh, they're inverse operations so that's why they can operate like that, ... so if we've talking about multiplication and division those, those fact families are together, right? They can kind of [gesturing a circle] undo and redo each other. Which the same is true about addition and subtraction and they rotate like that, that's good, good explanation"—id15. In this response, the teacher again seems to be verbally processing and making connections as she is speaking. Though there are moments where it sound like she could have re-engaged students ("right?") she does not pause and hence these are rhetorical questions rather than questions intended to engage students with the ideas.

The third response in this group similarly made a validation of the student thinking but then followed it up with a word of caution to students: "now we just have to be cautious with that when we start getting into negative numbers and uhhmm when we start to see more operations"—id26. It seems as though this teacher made some kind of connection between the pattern the student saw and exceptions to it but her articulation of this connection to negative numbers and "more operations" is left ambiguous in her response.

The final response again started with a validation, "Okay, you just made an interesting observation"—id20, then repeated the student response in a way that seemed to help the teacher (rather than the imaginary class of students) process the student thinking. The response then made a large mathematical leap from fact families to interpreting division problems:

> so you're noticing that these are kind of rotating but where those are, those specific positions, that's actually an important part of division problems, right? So, there's specific terms where uhhmm, where those numbers are located and what that means with the context of a problem. So, we're dividing for example into four groups and seeing how many are in each group or dividing into three groups and seeing how many are in each group. So these are, those are different ways of interpreting those division problems

This particular participant had talked about partative and measurement models of division during her lesson planning, which might explain the leap to division in her response.

How do teachers respond to an apparently incorrect student answer?

Though both Clip 2 and Clip 4 had apparently incorrect student answers, they presented different mathematical challenges for the teacher. In particular, with regard to the Clip 2 in which the student uses total pries versus unit prices, the student's answer provides information both about *what* the student is doing incorrectly and *why* (they are using different variables that represent total amounts rather than unit prices). Hence, the student answer provides sufficient information to suggest a particular course of action for the teacher (namely addressing the definition of the variables somehow). In contrast, the student answer in Clip 4 provides insight into *what* the student is doing incorrectly (solving a different problem) but not *why* the student is doing this. Hence, it is less clear how a teacher might proceed. Given these important differences, I explore each clip separately.

Coding of teacher responses to Clip 2

In response to the student who incorrectly uses total prices to determine their equation for discount price, most teachers responded by going back to the *same student* (14 out of 24 responses). Five teacher responses engaged only the *teacher* and five responses engaged the *whole class*. Additionally, over half (16 out of 24) used the student's language *explicitly*, 2 used it incorrectly, 3 did not use it at all and 3 used it *implicitly*. Again, most teacher responses were close to the student idea with a little less than half (11) coded as *core* to the student ideas and the remaining 13 responses were coded as *peripheral* to the student idea. Though there was some distribution in the type of move (see Table 3.4, the most frequently occurring move was a *correct*. With regard to the *mathematics*, there was a wide variety in the responses to Clip 2. Most responses to Clip 2 were either *CNI* (6 out of 24), *CNI-Core* (5 out of 24) or *peripheral* (5 out of 24) for *mathematics* (see Table 3.5 for the distribution).

Thematic grouping of teacher responses to Clip 2

In qualitatively grouping the 24 teacher responses the following four categories emerged: (1) focusing on the variables, (2) variations on 'g', (3) monitoring first, and (4) "did you test that?".

(1) Focusing on the variables

A total of seven teacher responses (id 07, 18, 06, 02, 08, 23, 17) focused on the number or definition of the variables that the student had chosen but chose to address this issue in very different ways. One response, after providing a condensed version of the student's equation ("...d equals p minus point-two-five g"), opened the conversation about the variables to the class, "anybody want to comment on these variables?"—id08. One response included a short think-aloud, "I'm interested in uhh about G, the variable G uhh so what I'm wondering is when you think about what D stands for and what P stands for, could you..." then asked the students to revisit the definition of the variables, "let's talk again about like what those variables actually stand for"—id23. This response seemed to convey, "I'm interested in your variables, let's talk about what those stand for" though it is not quite clear how a student could respond to this since they have already provided the definition for their variables. This teacher response does not provide any guidance as to how the student or class might go about revisiting the variable definitions.

Another teacher response focused on probing the student for justification of their equation and variable choice by asking a rapid-fire series of questions, "why are you subtracting twenty five from the regular price?... how could you find out the number of gallons that a customer is going to buy and the cost, how could I find out the cost on Monday versus the cost on Tuesday? which is more economical buying gas on Monday or Tuesday? and why would this be economical? why'd you choose these variables for...current price and the discounted price?"—id02. Though this TR might be confusing for students because it asks so many, slightly different questions, it does settle on a core issue around the definition of the variables, "why'd you choose these variables for uhhhmmm current price and the discounted price?"

Two teacher responses (id07 and id18) asked the student to read the problem aloud again because in one case, "...I want to talk about the variables first..."—id07 and in the other, "let's take a look back at the question again and see if we can...think about the 3 variables that you had..."—id18. A final response in this group focused on the number of variables but probed the student with a series of two confusing, yes-no questions: "Is there a way that you could write this without using all those variables? Is there something that you know that you can put instead of all those letters?"—id06. It is clear that the participant wants the student to focus on the number of variables but the teacher response provides students with no clear support in how to revisit the number of variables. "Is there something you know that you can put instead of all those letters?" sounds like it is fishing for something but provides little guidance for students. Hence, it is unlikely that students will have any idea how to respond to this question. In the final response in this category, after re-reading the beginning of the problem aloud, the teacher tells students suggests that, "let's look at the situation when we're only thinking about one gallon at a time cause I think that might even make a nicer problem for us. So I think we should investigate that."—id 17.

(2) Variations on 'g'

A majority (10 of the 24 teacher responses) focused on the variable G (for gallons) that the student proposed and, as in the group above that focused on the variables more broadly, the responses addressed this issue in several different ways. Within this group, two teacher responses focused on the "multiplying...at the end" in the student's response and seeking an elaboration from the student about this. The responses (after an incorrect repetition of the student's variables in the case of one response, "so it sounds like you are...looking at price per gallon"—id 13) basically asked the student to say more about "the multiplying by G at the end there"—id21. Another response focused on the multiplication but used it in a different way than these two responses. In this response, the teacher requested that the student come up, "and just look at the problem a little bit clearer" because, "I don't quite understand...where the multiplication is coming from which is what I think I saw on the board"—id 12. In other words, the confusion generated by the student's use of multiplication in the equation was used as the rationale for the teacher's request to have the student review the original problem statement.

Another set of responses focused on the inclusion of gallons in conjunction with considering unit price. Within this set of eight responses (id 01, 09, 03, 11, 25, 10, 24,15) there were variations in the ways in which responses made use of the connection between the number of gallons and unit price. One response put it succinctly, "we're looking for the price per gallon, so what's the g actually going to tell you?"—id01. One response funneled the issue to an either-or-question but was a imprecise, asking the asking, "is gallons going to be a variable in our equation or are we just talking about price ?"—id09. A final variation on teacher responses that focused on unit price did so by having students consider whether or not unit price will change if only one gallon is purchased. After validating the student's response, one teacher response asked "but then what if I just had the price is two dollars and fifty cents, is it going to matter how many gallons I buy, if I buy more than one gallon is the price going to be different than two dollars and fifty cents?"—id03. In another teacher response, after repeating the student's equation the participant asked, "well does price change at all depending upon the number of gallons?" but then immediately opened the conversation up to input from other students, "anyone else have a different response here?"—id 11.

A sub-set of four teachers responses that focused on the variable G explicitly corrected the student response. In one case, the teacher response was short and to the point, “we probably wouldn’t need to use a G would we, because we know the price is twenty-five cents and we know that we’re referring to gallons”—id25 then ended by validating the student’s use of the other two variables (d and p). Another corrected the student’s use of g, “I don’t think you have the understanding of a variable ’g’ in here. They never mention ’g’ to be how many gallons”, explaining that the prices are “per gallon”—id10. The response then goes on to validate the student’s use of subtraction and then finally provide the correct answer, “it’s like the price minus the point two five is your discounted price or the original price, I should say can be found by adding twenty-five to the discount”—id10. The last two responses in this subset were both much longer because it appeared that the teacher was verbalizing their own thinking (almost like a think aloud).

In one response, the participant pointed out that the student (whom they named “Billy”) was “actually making the equation kinda complicated” because “it’s kind of a complicated situation” and “he’s so mathematically capable”—id24. The response then focused on streamlining the number of variables, “...we’re going to try to simplify what’s going on here uhhmm to limit the number of things that we have, variables that we have to deal with...” by using just, “...the ones that were given to us in B which were d is the discounted price per gallon and p is the regular price per gallon”. Which lead to dismissing the use of the variable g, “So really the number of gallons that we’re buying is not a factor in this question all were’ looking at is just the unit prices per gallon, so umm, so g is not a variable it’s just d and p...”. After dismissing the student’s answer as incorrect, this teacher response then asked the class for a different answer, “...uhmm did anybody else get a different equation for uhh for part B number one that they’d like to share?”

In the other response, the participant again seemed to be verbalizing their thinking and, as they are talking, they seem to be changing their mind about where they want to go with students. In the beginning of this response, the participant immediately cautioned the student that in this problem “we’re talking about one gallon no matter what” and then quickly compliment the student on their use of the other two variables (d and p). The response then goes on to, “take away this gallons...because that’s the total price when I put in a whole bunch of gallons, right?... we’re not looking for the total price but just for one...one gallon.” Eventually, the participant explains, “on Tuesday it happens to be discount day so you have your D which is great and what do you have to do to that D, what’s, what’s the actual price, I’m gonna back up, what’s the price and you have to take-away that twenty-five cents, right, to get the discounted price”—id 15. Even after essentially giving students the answer ($d = d0.25$), the response continues and finally tasks students with, “I’m going to leave it open there cause I’m hoping you can come back with an equation, how do we relate the discount, the d, with the price and the twenty-five cents. which what are you taking away from what?”

A final pair of teacher responses (id 16,22) that focused on the variable g did so without attention to the way in which the student had defined the variables p and d. In one, after acknowledging how students’ demonstrated “a good intuition” by including g, the participant asked an either-or-question (“is the question asking for ...the total price of a certain number of gallons of gas or is it just asking us to look at the price per gallon?”—id16) but continued with, “and what one small change would we need to make to your groups equation to better fit the problem situation?”. This final question seems to imply that the student’s equation would be correct if they simply remove the variable g. In the other response, after validating the student’s work the teacher asked two questions in a row, “is it necessary to have g there as a variable in order to calculate the price per gallon? if we were just looking for just one gallon of gas what the price would be?”—id22. Though the first question hits on the key issue at play, the second question seems to guide students towards simply substituting “one” for the number of gallons in their equation to get the discounted price, which neglects the ways in which d and p are currently defined in the student's equation.

(3) Monitoring first.

In another group of responses (3 total), the participant decided to monitor the student thinking in the class by asking students, "...raise your hand if you also used three variables?..."—id20 or "how many people got the same answer?"—id 05 or "how does everybody else feel about that response?"—id26. In theory, this is a type of formative assessment that could allow the teacher to gather more information about what other students are thinking and to use this to inform the teacher's next steps. Unfortunately, participant id 26 initially repeated the student response incorrectly ("...so you set-it up as Tuesday price is equal to regular day price minus point two five...") before asking other students to chime in.

(4) "Did you test that?"

One participant asked the student if his group had tested their equation by substituting "any possible number of gallons to see if that equation makes sense?"—id19. It is possible though that this participant is assuming that the students used unit prices (price per gallon for d and p) rather than total prices. If this were indeed the cases, then students would eventually get negative prices for any purchase over a certain amount of gallons (e.g., $d = 2.76 - 0.25 * 12$ =2.763= -0.24). Though this line of questioning by the teacher could work if this was indeed the mistake that students had made, it is not as helpful given that the student has defined the prices as total prices. Therefore, this teacher responses is unlikely to help students hone in on the definition of their variables.

Coding of teacher responses to Clip 4

In response to the student who incorrectly solves the problem $m/\ 14 = 74$ most teachers, again, responded by going back to the *same student* (13 out of 24 responses).

Of the remaining 11 responses, 10 teacher responses engaged only the *teacher* and one response engaged the *whole class.* Additionally, a little less than half (10 out of 24) of the teacher responses used the student's language *explicitly* 2 used it incorrectly, 10 did not use it at all and 2 used it *implicitly.* Again, most teacher responses were close to the student idea with a little less than half (11) coded as *core* to the student ideas. The remaining 13 responses were *peripheral* (5 responses), *other* (3 responses), or *NA* (5 responses). Though there was some distribution in the type of move (see Table 3.4, the most frequently occurring move was a *correct.* Most responses to Clip 4 were either *CNI* (6 out of 24), or *Other* (5 out of 24) (see Table 3.5 for the distribution).

Thematic grouping of teacher responses to Clip 4

In qualitatively grouping the 24 teacher responses the following four categories emerged: (1) exploring the student's response (2) "does your answer make sense?" (3) "Youre solving the wrong problem", (4) correcting the student and (5) taking it literally.

(1) Exploring the student's response.

A group of eight TR followed up on the student thinking in ways that had the most potential for productive conversation about the student response. In one TR, after making an inference about how the student got their answer of 1036 ("I heard you say that...you have multiplied together the fourteen and the seventy-four, and that when you did that multiplication you got ten-thirty-six"—id08), the teacher asked others to comment, "does anybody want to either support or challenge this idea?" This provides space for other students to potentially question the student about whether he is solving the same problem or using a fact that does not belong in this fact family. Two TR probed the student for how they came to their answer. In one, the TR probed the student for information about "what was the equation that you set-up to get that answer?"—id09. This TR does not make an inference about how the student got their answer of 1036 but does begin to focus the conversation on potential relevant mathematical content. Similarly, id20 asked the student, "so how did you get that answer?" One response asked the student to go back to what they had done and the beginning of the lesson and think about how to "use our fact families to find our solution for m"—id22.

Two teacher responses focused on mathematical operations. In one, the TR seems to make an inference about how the student obtained the answer of 1036 and asks, "why would you multiply to find this answer?"—id02. In the other TR, the participant seems to focus on the end of what he student has said (the incomplete statement about "1036 divided by 14...") and asks, "how did you know that you had to use division the way that you used division?"—id 23. In one TR, the participant essentially asked the student to come up to the board and represent their work next to the other answer on the board so that "then we can look at the two of them side by side and figure out which one is uhhmm the is the better way to go"—id 24. A final TR in this group asked the student, "now expand upon that for just a second?"—id19. On the one hand, this TR provides room for the student to say more but, in the other, it is not clear what the student should be saying more about. Hence, it leaves the mathematical terrain of the conversation particularly nebulous.

(2) Does your answer make sense?

A group of six TRs essentially focused on whether the student answer made sense. One response focused on the idea that m should be a decimal and five of these TRs focused on having the student plug their answer back into the original problem. These TRs did varying degrees of work to connect the problem the student seemed to be doing and the original problem on the board. This connection seems important for student understanding because tin the student response, the student has essentially already plugged their answer back in ("because one thousand thirty-six divided by fourteen..." [equals 74]).

In one TR, the participant explicitly pointed the student back to the problem on the board, "...let's think about putting it in the problem...which position on this problem that's on the board does that number fit into"—id18, to focus the student's elaboration on "...why that [1036] is a correct answer?"—id18. Another TR inferred the student had multiplied 14 and 74 to get 1036 and then simply stated, "but if we put one thousand thirty-six in for m. check that we got fourteen divided by one thousand thirty-six and what do you get when you do that? is it seventy-four?"—id01. Although the TR does not explicitly explain how the original problem is connected to the one the student seems to have solved, the TR is at least very explicit about what the 1036 is being plugged back into. Similarly, participant id13, after some verbal processing, asked the student "so that, that value you're substituting ...we have fourteen divided by that value if we, if we plug that into our calculators or if we write that down on the board as a fraction, what does that look like? fourteen divided by that number you said ...". Though the participant could not recall the student's exact answer they were clear about the student using their answer in the problem "fourteen divided by that value." Additionally, the participant seemed to hypothesize that the student's issue arose from the notation in the original problem and that representing the problem differently (vertically as a fraction rather than horizontally) would also clarify things for the student.

Two other TRs essentially also asked the student to "plug it back into the equation to see if that gave you a true statement?"—id 05 or "let's plug that in uhhh and see how the arithmetic pans out"—id 21 but neither of these responses specific what equation the 1036 was being plugged back into. The one response that focused on making sense of the size of m given the original problem asked the student, "If I'm taking fourteen and I'm dividing it, uhmmm into pieces and I end up with seventy four of them, would it make sense for our divisor to be a whole number or a decimal?"—id 12. Though the response is clear about what problem the teacher is talking about, it does nothing to connect it with the problem the student seemed to be solving.

(3) You're solving the wrong problem.

Three TRs pretty explicit told the student, "but it, it doesn't ask what divided by fourteen is seventy-four, the question is fourteen divided by what is seventy-four"—id06. In one case, participant id06 this was their whole response. The other two TRs continued solving the original problem by explaining "so when you solve for m in this case, you're gonna take fourteen and divide by 74 to get to your answer of m"—id26. Participant id10 went as far as the final solution, "to get 'm' which was the decimal point one eight nine". Like participant id13, participant id26 seemed to hypothesize that the notation had caused the problem, so she also wrote "it out a different way that might help you see it a little bit better."

(4) Correcting the student

A group of six TRs corrected, to varying degrees and with different mathematical foci, the student's response. One participant seemed very taken with the work on the board for problem 4, in which the student volunteer had noted that "m = decimal" and focused their response to the student on that idea. In this response there was a lot of verbal processing, but the participant did state the problem again, "fourteen divided by something is equal to a bigger number" and suggested the class consider an easier version of the student's response by stating, "you're going to say fourteen divided by a bigger number, one thousand something is equal to seventy-four, so let's try just a smaller number like fourteen divided by...by twenty-eight, what would that be..." to see if that "makes sense with the context of this problem"—id07. Unfortunately, the TR seems to consider neither the student's answer of 1036, instead opting for a smaller number, or the fact that the student seemed to have solved a different problem.

Another set of three TRs that were also rather wordy touched both on the idea that m is a decimal and using fact families to solve the problem. One TR quickly stated that 1036 was wrong and that mental math would confirm this (14 divided by 1036 does not equal 74) then the response goes on to talk about how division problems do not need to have the "the largest number and divide it by a smaller number"—id 17. The TR then mentions the work on the board that noticed that m was a decimal and goes on to talk about how, earlier, the class had talked about how the numbers in the fact families can only go in certain positions. Another participant walked through the facts of the fact family ($14/m = 74$ so $14 = 74 * m$, then we can divide out the m and get $14/74 = m$) and asking rhetorically, "and fourteen divided by seventy-four is that going to be bigger than one or smaller than one?"—id11. The TR then comments on how the student's thought to multiply (the 14 and 74) would have been "the right thing to do if it were m divided by fourteen" but that in the facts for this fact family they needed to multiply 74 and m.

A third response of this sub-type, immediately cautioned the student, "so be careful," then reiterated the original problem "fourteen divided by M equals seventy-four" and pointed out that the answer has to be a decimal. The response then went on to putting another fact family on the board ($35 = 5*7$, $35/5 = 7$, etc.) to highlight that you can get one factor by diving by the other factor and connecting this to the student response in the first clip that mentioned the numbers "can rotate."

Another set of TRs were much more concise in their corrections. One TR essentially explained that the participant had made the same mistake and that rewriting it horizontally helped them see how to solve it correctly. The TR then walks through the algebraic steps, "we had to multiply the m on both sides to get fourteen equals seventy-four m and then divide the seventy-four and that's how we got point one eight nine repeating", to solve the problem. The other concise TR assumed the student had made an error with order of operations in his calculator and cautioned the student to "be careful" because "order is very important...so when we put it into our calculators we need to make sure we're doing fourteen divided by seventy-four"—id 16. This participant also seemed to believe that rewriting the problem horizontally would support students in seeing the original problem correctly and that from there, "...so that when we put it in our calculators ...so make sure that that we're doing fourteen divided by seventy-four, not seventy-four divided by fourteen, okay? auhmmm , alright."

(5) Taking it literally One TR seemed to take the teacher prompt for this student response ("Yes, what's your question about number 4?") very literally and essentially told the student to hold his answer because "first we just want to know if you have any questions about how to do part four..."—id25.

How do teachers' responses to these different types of student responses vary?

Tables 3.1, 3.2, 3.3, 3.4 and 3.5 display the distributions for various codes across the five categories in the TRC: *actor, student actions, student ideas, move* and *mathematics.*

In general, participants seemed inclined to either go back to the same student or be the main actor to publicly consider the student thinking. There was the more variation in the actor in teacher responses to Clip 2, where the student provided the answer of $d = p - 0.25 * g$. In that case, though the majority of the participants (14 out of 24) went back to the same student, the rest were evenly split between publicly inviting the whole class to consider the student thinking and being the only ones to publicly consider the student response.

Table 3.1: *Distribution of actor codes in Clips 1, 2, and 4.*

	Same Student	Whole Class/ Other student	Teacher
Clip 1	13	2	9
Clip 2	14	5	5
Clip 4	13	1	10
Total	40	8	24

With regard to being responsive to student thinking, as captured by the student actions and student ideas TRC categories, overall participants' responses frequently explicitly used the student's language and were core to the student's idea. In response to Clips 1 and 2 (both of which were had longer student answer and in which the mathematical ideas were more intricately tied to the language the student used), the majority of participants explicitly used the students' words (14 out of 24 for Clip 1, using "rotate" and/or "circle" and 16 out of 24 in Clip 2, using "g" and "number of gallons"). When responding to the student's response of 1036 in Clip 4 (a student answer that was much shorter than the student answers from Clips 1 and 2), participants' were almost split between explicitly using the student's language (repeating the answer of 1036, or attempting to repeat it but doing so incorrectly) and not using it at all.

Table 3.2: *Distribution of student actions codes in Clips 1, 2, and 4.*

	Explicit	Implicit	Not	Explicit-Incorrect
Clip 1	14	10	0	0
Clip 2	16	3	3	2
Clip 4	10	2	10	2
Total	40	15	13	4

The teacher responses across all three student response clips tended to be fairly responsive to students. In other words, the student's within all three Clips would be likely to recognize their idea (a code of *core*) or a closely related idea (a code of *peripheral*) as being made the object of consideration in the teacher responses. This is evident from the 17 teacher responses in Clip 1, 24 teacher responses in Clip 2 and 16 teacher responses in Clip 4 that were coded as either *core* or *peripheral* (see Table 3.2). Participants' responses seemed to more easily make the student response (or a closely related idea) the main object of consideration in Clip 2, in which a little less than half of the teacher responses were core to the student ideas and slightly more than half were peripheral to it. Teacher responses to the other clips had more variation, with about one eighth TRs (3 out of 24) to both clip 1 and clip 4 veering towards other ideas and about a few (3 in Clip 1 and 5 in Clip 4) not pursuing the student idea (a code of *NA*).

Table 3.3: *Distribution of student ideas codes in Clips 1, 2, and 4.*

	Core	Peripheral	CNI	Other	NA
Clip 1	12	5	1	3	3
Clip 2	11	13	0	0	0
Clip 4	11	5	0	3	5
Total	34	23	1	6	8

Teacher responses across the three clips employed a variety of different moves as can be seen in Table 3.4. In response to the mostly correct student response of Clip 1, a little over a third of participants (9 out of 24 TRs) tended to engage the actor in an elaboration of some aspect of the student idea. In contrast, in response to the mostly incorrect student responses of Clips 2 and 4, a third of participants (8 out of 24 TRs) tended to engage the actor in a correction of some aspect of the student idea.

Table 3.4: *Distribution of moves codes in Clips 1, 2, and 4.*

	Clip 1	Clip 2	Clip 4	Total
Justify	4	2	1	7
Allow	0	1	0	1
Elaborate	9	2	5	16
Collect	0	1	0	1
Connect	0	3	0	3
Clarify	1	0	0	1
Monitor	0	0	4	4
Repeat	2	1	1	4
Evaluate	1	0	1	2
Literal	0	6	2	8
Validate	1	0	0	1
Adapt	3	0	0	3
Correct	0	8	8	16
Dismiss	3	0	2	5

As can be seen in Table 3.5 there was some interesting variation in the mathematics code across the various student responses. When responding to the mostly correct student response of Clip 1, teacher responses seemed to leave more room for students' to explore the mathematical terrain (as suggested by the 10 *CNI* and 5 *CNI-core* coded responses). In the clips that were mostly incorrect, participants had different trends. In response to the student using the incorrect variables (total price versus price per gallon, in Clip 2), a little more than a third of the participants seemed to provided responses that touched on (4 out of 24, *Core*) or had the potential to touch on (5 out of 24, *CNI-core*) the core mathematical issue about the way in which the student had defined their variables and the implications of defining these as total versus unit price.

Table 3.5: *Distribution of mathematics codes in Clips 1, 2, and 4.*

	Clip 1	Clip 2	Clip 4	Total
CNI-core	5	5	1	11
CNI	10	6	6	22
Core-MP1	0	0	0	0
Core	0	4	4	8
Peripheral	1	5	3	9
Peripheral-beyond	2	0	1	3
CNI-imprecise	4	4	3	11
Peripheral-incorrect	0	0	0	0
Other	1	0	5	6
Other-incorrect	1	0	0	1
Non-math	0	0	1	1

In contrast, when responding to Clip 4 (where the student suggests an answer of 1036 for m) participants seemed to have more difficulty picking up, on the fly, the core mathematical issue to pursue from the student's response as indicated by the 8 TRs that veered into peripheral, peripheral-beyond or other mathematics. Put another way, these 8 responses veered into mathematical terrain that drifted away from the core mathematical point underlying the student response. That core mathematical point can be summarized as follows: While fact families switch around location of three values in inverse operations, a/b=c is not equal to b/a=c unless c=1.

Patterns within teachers

Another facet of understanding how teachers responded to different types of students emerges from looking at patterns across the student responses within teachers. As can be seen in Table 3.6, half of the teachers in this sample provided responses with the same actor for their responses to all three student responses (9 had the same student as the actor for all three responses and 3 had the teacher as the actor for all three responses).

Table 3.6: *Patterns in actor code within teachers*

Pattern in Actor Code	Number of teachers
All 3 same student	9
All 3 teacher	3
2 Teachers/ 1 Same Student/Whole Class	5
1 Teacher/ 2 Same Student/Whole Class	6
2 Whole Class/ 1 Same Student	1

As can be seen in the TRC results in Table 3.2, the majority of teacher responses tended to explicit use the student actions. Looking at these instances within teachers, as can be seen in Table 3.7, teachers were predominantly explicit in the majority of their responses to students. Though few were explicit in their responses to all three student answers (only 4 out of 24), a little less than half of the teachers used the students' exact language in two of their three responses (13 out of 24).

Table 3.7: *Patterns in actor code within teachers*

Patterns in Explicit Student Actions Code	Number of teachers
Explicit in all 3	4
Explicit in 2	13
Explicit in 1	6
Explicit in none	1

In looking across the TRC coding and within teachers, another interesting pattern that emerged was the way in which different teachers seemed to treat the student responses.

Table 3.8: *Patterns across student response types]*

Pattern in responses to "correct" versus "incorrect" student responses	Number of teachers
Group 1: Evidence that Clip 1 is correct and Clips 2 and 4 are incorrect	12
Group 2: Evidence that responses 1, 2, and 4 were treated similarly	7
Group 3: Less clarity but more evaluative than not	3
Group 4: Miscellaneous	2

As can be seen in Table 3.8 half of the teachers (id 01, 03, 05, 06, 10, 12, 13, 15, 16, 17, 22 and 23) (Group 1) in this sample responded to Clips 2 and 4 in a way that differed significantly from how they had responded to Clip 1. Namely, these teachers responded in a way that generally indicated that the student answers in Clips 2 and 4 were incorrect. In four of these 12 responses, teachers used a correct move overall in the response to both Clip 2 and Clip 4. Another frequent combination of moves within this subset of teachers was a correct in response to one clip and a literal move in response to the other (three teachers did this). The remaining five teachers had various combinations of moves across their responses to the two clips including correct, and literal, as well as monitor (which, for these five teachers, was always in response to clip 4), connect, justify and dismiss. There were no distinct patterns that appeared with respect to their choices of actor, student actions, student ideas or mathematics.

Three teachers (id 07, 11, and 24; Group 3 in Table 3.8) provided responses that were also more evaluative than not, but made this distinction with a little less clarity. Two of these teachers clearly evaluated the student response in Clip 4 as incorrect (with a dismiss and a correct) but were a bit more ambiguous in their treatment of the student response in Clip 2. In one case the teacher asked back-to back questions, first indicating there might be an issue ("well does price change at all depending upon the number of gallons?") but then deciding to collect responses from the class ("...anyone else have a different response here?"—id 11). In the other, the teacher asked the student from Clip 2 to "read the problem out loud to me again" so they could, "talk about the variables first" id 07. The third teacher more evidently corrected the student response from Clip 2 (by essentially providing the right answer) but was not as obvious in their evaluation of the correctness of the student response from Clip 4. In response to Clip 4, the teacher asked the student to come up and write their answer on the board so it could be discussed without obvious indications they thought it was wrong.

There was another group of seven teachers that provided responses to the students that did not as obviously seem to indicate that students were correct or incorrect. This group of six teachers (and 21 responses) had a number of similarities. With the exception of one teacher in this group, the teachers engaged the *same student* or the *whole class* in all three of their responses. The only exception was a teacher whose response to Clip 1 engaged the *teacher* but whose responses to Clips 2 and 4 engaged the *whole class* and the *same student*. This group of teachers also seemed to be more responsive to students. Their responses more frequently used the students' exact language (all seven had *explicit student actions* in two of their three responses) and they were predominantly either *core* or *peripheral* to the students' ideas. Another trend across this group of teachers was the predominance of *CNI* codes for *mathematics*. Six of the seven teachers gave responses that were coded with one of the three variations of the *CNI* code for at least two of their responses, with *CNI* and *CNI-Core* occurring most often (13 out of the 18 instances of a CNI code in this sub-set of six teachers). This indicates that, in general, the responses from this group of teachers tended to give less away or be less leading and, hence, leave more room for students to do the intellectual work.

When contrasting the *move* choices of these seven teachers' responses with the 17 teachers with clearly evaluative responses, one distinction arises in the choice of *move* for Clip 4. In particular, the majority of teacher responses in Group 1 used potentially lower-level cognitive moves such as *correct, literal, dismiss* and *monitor*. In contrast, a majority of the seven teacher in Group 2 used more cognitively demanding moves such as *justify* and *elaborate* in response to Clip 4.

Within Group 4 ("miscellaneous") one teacher response (id26) misremembered the students' answer in Clip 2. This response repeated the correct answer rather than the student's answer, and hence, it is unclear how this teacher would have actually responded if she had correctly recalled what the student had said. The other teacher in this "miscellaneous" group (id09) provided responses that were generally imprecise, vague and confusing making it difficult to determine what, exactly, the teacher responses were conveying to students. Therefore, it was not possible to determine whether or not these two teachers had normative, evaluative response patterns to student answers. This meant they could not be placed into Groups 1, 2 or 3.

Discussion and Conclusion

Though current mathematics reforms and research efforts point to the importance of how teachers respond to students, work in the area in still ongoing. In particular, when looking across the research that does exists three gaps emerge: (1) a lack of studies examining how teachers respond to students around algebraic content, (2) a lack of studies comparing and contrasting teacher responses to correct and incorrect student responses and (3) a need to explore how teachers respond in more realistic ways under more authentic conditions. The current study aims to address these three gaps by reporting on the ways in which teachers responded to a variety of correct and incorrect student thinking in early algebra, in a teaching simulation with more fidelity to classroom conditions. In particular, I asked:

1. How do teachers respond to apparently correct student responses?
2. How do teachers respond to apparently incorrect student responses?
3. How do teachers' responses to these different types of student responses vary?

In examining how teachers responded to correct student responses, teachers in this study responded in a variety of ways. Namely, seven different types of repose categories emerged: (1) "Come up and show us", (2) "why is it doing that?", (3) "what do you mean?", (4) focusing on peripheral/other mathematics, (5) "that's good, can you tell me more...?", (6) "that's good, what else?" and (7) only the teacher engages in the intellectual work. As hinted by the category names, the majority of teacher responses took-up the correct student thinking. Overall, teacher responses were responsive to the student thinking and went back to the same student asking for an elaboration without giving away all the mathematical ideas to be explored.

In responding to the student who used total prices and provided an equation that included the variable g for gallons (a response that is correct given how the student defines their variables but does not answer the original problem) teacher responses fell into four categories: (1) focusing on the variables, (2) variations on 'g', (3) monitoring first, and (4) "did you test that?". Though teacher responses were responsive to the student thinking and went back to the same student without giving away all the mathematical ideas to be explored, they tended to use moves that funneled or guided the student to a correct answer. One interesting issue that arose in the teacher response to this student was a problem with precision in language around units. Even as teacher responses seemed to be honing in on the mathematical issue of defining variables and/or units precisely, many still used incorrect units (e.g., referring to price when they meant price per gallon).

In response to the student who found that m is 1036 and seems to have solved $m/14 = 74$ rather than $14/m = 74$, teacher responses fell into the following four categories: (1) exploring the student's response, (2) "does your answer make sense?", (3) "You're solving the wrong problem", (4) correcting the student and (5) taking it literally. With the exception of the first category, most teacher responses sought to correct, through various means, the student's answer. This was also what emerged in looking at the coding distributions of the Teacher Response Coding scheme, where again though most teacher responses were responsive to the student thinking and re-engaged the same student, they tended to use moves that funneled or guided the student to a correct answer.

Additionally, of the three student answers used in this simulation, the student answer in Clip 4 seemed to be the most difficult for teachers to mathematically interpret in-the-moment. Several teacher responses seemed to misinterpret the underlying mathematical issue in the student thinking and hence veered into mathematics that did not seem closely related to the mathematical point underlying the student thinking. Another interesting issue that arose in these responses was the inference that teachers' responses seem to make. These ranged in the amount of inference: from a small amount (such as completing the students though, " 1036 divided by 14" equals 74), to making a little bigger of a leap (assuming the student got 1036 by multiplying 14 and 74), to making large inferences about why the student made a mistake (not plugging it into their calculator correctly, misusing fact families, not reading the original problem correctly).

When looking at all 72 teacher responses across all three clips and within teachers a two interesting patterns emerge that seem to speak to some normative behaviors in teacher responses, regardless of the type of student response. First, as can be seen in Table 3.1 teachers seemed to be inclined to re-engage the same student in their response (40 out of 72, a little over half, teacher responses in this sample were coded as same student). When exploring the choice of actors within teachers, (Table 3.6) a similar conclusion emerges. Namely, more than half (15 out of 24) of the teachers had a pattern of predominately going back to the same student (9 had the same student as the actor for all three responses and 6 had the same student as the actor for two of the three responses). Second, teachers seem to be inclined to repeat part or all of a student's contribution verbatim in their response. As can be seen in Table 3.2, 40 teacher responses explicitly included the student's actions, a predominant pattern that also emerged within teachers. 17 of the 24 teachers used explicitly used the student's actions in two of their three responses (Table 3.7).

Of note in this sample is that not all teachers exhibited clear evaluative patterns in their responses to students (Table 3.8). Indeed, though the majority of teachers in this sample responded in ways that made it clear that the first student answer was correct and that the other two were incorrect, this was not the case for seven of the teachers in this sample. These seven teachers tended to provide responses that predominantly engaged the same student or the whole class in all three of their responses and were highly responsive to the student thinking, especially with regard to using, verbatim, the students' language. They also tended to provide responses that were clear but that did not completely give away the underlying mathematical point that was being explored. Hence, they invited students to engage in the intellectual work in ways that were both clear and vague.

In conclusion, this study provides some implications for teacher education with regard to considering the ways in which teachers respond to student thinking. First, it provides a methodological alternative that can provide teachers with an opportunity to respond under more realistic time constraints in a context with more fidelity to actual classroom instruction. Second, it points to some possible norms of teacher responses—about repeating student thinking and going back to the same student—that can both provide a starting point for novices and also a point of discussion to help teachers broaden their practice with an eye towards discerning when they can involve more students versus when there is not enough information and they need to go back to the same student. Finally, in contrast to much of the deficit rhetoric about teachers, there is evidence that not all teachers default to simply evaluating student responses. In other words, there is some hope that by understanding what might support this different group of teachers and describing the characteristics of their responses, that teacher educators might gain insight into how to better support teachers in responding to student thinking.

CHAPTER IV

Managing Students' Responses: Examining Individual Resources That Might Influence the Outcome

Introduction

There is general agreement in mathematics education of the importance of teachers' being responsive to students' thinking. For example, the National Council of Teachers of Mathematics highlights the importance of teachers eliciting and probing students' thinking to build instruction that connects to and builds on student ideas (2000). Being responsive to student ideas reflects "the extent to which teachers 'take up' students' thinking and focus on student ideas in their moment-to-moment interactions" (Pierson, 2008, p.40). This type of responsive instruction has been linked with both rich, learning environments and improved student achievement (e.g., Pierson, 2008; Fennema et al., 1993).

Despite widespread agreement about its importance, research into what enables teachers to successfully enact responsive instruction is still ongoing. Some might suggest that skill at eliciting student thinking precedes a teacher's ability to respond. It does, undoubtedly, seem logical that in order to have student thinking to respond to, a teacher would first need to be skilled at "eliciting" that student thinking; in other words, she would need to be skilled in "the set of teaching actions that serve the function of drawing out students' mathematical ideas" (Lobato et al., 2005, p.111). Others might propose that what teachers need is broader than skill at eliciting and suggest that teachers need to have positive affect towards and beliefs about mathematics, and mathematics teaching and learning. There is in fact evidence that teachers' self-efficacy in teaching mathematics and their social-constructivist beliefs about mathematics, and mathematics teaching and learning can positively impact their instruction (e.g., Kunter et al., 2008; Thompson, 1984, 1992; Stipek et al., 2001).

Still others might suggest that being responsive to student thinking depends more on a teacher's specialized pedagogical content knowledge (PCK) (Shulman, 1986, 1987). Indeed professional development around children's mathematics thinking, a facet of PCK, has been shown to improve teachers' instruction including the ways in which student thinking is made central to and built upon during instruction (e.g., Carpenter et al., 1989; Fennema et al., 1993). Further, a version of specialized knowledge for teaching that brings together PCK and content knowledge, mathematical knowledge for teaching (MKT) (D. L. Ball et al., 2008), has been linked to overall gains in students' mathematics achievement (Hill et al., 2005) and positively correlated with responding to students appropriately—as captured by "the degree to which teacher can correctly interpret students' mathematical utterances and address student misunderstanding" (Hill et al., 2008, p.437). The result of Hill et al.'s (2008) study regarding appropriately responding to students also suggests that responding to students involves more than MKT.

Specifically, it seems to indicate that responding to student thinking also depends on a teacher's ability to attend to and interpret student thinking; in other words, a teacher's skill in noticing students' mathematical thinking (Jacobs et al., 2010). This particular form of professional noticing encompasses three interrelated, component skills: "attending to children's strategies", "interpreting children's mathematics understandings", and "deciding how to respond" (Jacobs et al., 2010). All three of these skills seem to be supported by professional development that focuses on children's mathematical understandings, and ways to elicit and respond to those understandings (Jacobs et al., 2010). These findings echo the results of prior research on the effects of professional development focused on children's mathematical understandings. Specifically, they reaffirm the importance of teachers' specialized pedagogical content knowledge for teaching and the need for a variety of skills that support responding to students.

Although the work of Jacobs et al. (2010) starts to bring together skill and knowledge in considering how teachers respond to students, it does not yet provide a complete understanding of what is entailed in responding productively to students' thinking. In this paper, I propose that a more comprehensive approach might be to consider how teachers manage students' responses as a type of professional competence—which would encompass knowledge, skills, and beliefs as well as affective and motivational factors—and explore what this might entail with regard to the in-the-moment enactment of this competence. In particular, I contend (as is argued in more detail in the following section) that in addition to teachers' knowledge, beliefs, skills and affective resources, teachers' emotional reaction to a student's answer might also play a role in shaping teachers' responses. Hence, in this paper, I report on a study that asked what might explain how teachers respond to students' ideas. In particular, I asked:

1. Are participants' self-assessed emotional reactions to the student responses related to characteristics of their responses?

2. Are participants' individual characteristics (predictor variables) related to characteristics of their responses to students?

Theoretical Framework

Conceptualizing "Managing Student Responses" as a Competence

As the research and knowledge base on teaching mathematics has continued to grow, more attention is being given to the ways in which various factors might come-together in a more holistic conception of what it might take to teach well. Specifically, recent work (most prolifically from the Max Planck Institute for Human Development's COACTIV studies in Germany) has proposed that conceptualizing teaching as a competence might serve to bridge various lines of research on teacher quality (e.g., Kunter et al., 2013). Broadly, teaching competence can be thought of as the various "skills, knowledge, attitudes, and motivational variables that form the basis for mastery of specific situations (see Epstein & Hundert, 2002; Kane, 1992; Klieme, Hartig, & Rauch, 2008)" (Kunter et al., 2013, p.807) (see also Weinert et al., 1990). In teaching this would likely entail skills such as professional noticing (e.g., M. G. Sherin & van Es, 2009; Jacobs et al., 2010), specialized pedagogical content knowledge (Shulman, 1987), productive beliefs about teaching and learning (e.g., Thompson, 1992), self-efficacy in teaching (Tschannen-Moran et al., 1998) and productive self-regulation mechanisms (e.g., Klusmann et al., 2008). Indeed, evidence is accumulating that these various facets shape the quality of teachers' instruction that then subsequently impacts students' learning (e.g., Kunter et al., 2013). It is likely that this higher-level conception of teaching as a competence can be conceptualized as comprising of a set of smaller-grained sized competencies such as competency in planning instruction, in leading whole-class discussions, providing oral and written feedback, etc. (e.g., TeachingWorks high-leverage practices might be re-conceptualized as competencies). One competency that I contend is ubiquitous to teaching in general and mathematics teaching in particular, is managing students' responses.

Regardless of the activity structure of instruction (one-on-one, small group, or whole class), core to the interactions between teachers and students is the back-and-forth about mathematical ideas and in particular about students' mathematical ideas. This mathematical discourse around student ideas reflects the "moment-to-moment decisions and interactions [that] are the life of the learning environment" and "influences student learning" (Pierson, 2008, p.14). This indisputable fact about mathematics discourse means that a core part of teaching becomes managing students' responses. Managing students' responses is best described as the in-the-moment form of what others have described as teachers' "diagnostic competence" (Weinert et al., 1990; Hoth et al., 2016).

Diagnostic competence.

In describing a more holistic model of teaching that considered how classroom context and teacher expertise might impact student achievement, Weinert et al. (1990) proposed diagnostic competence as one of four key variables defining teacher expertise. In medicine, the field from which the term likely stems, diagnostic competence refers to one's ability to determine, through the use of diagnostic tests, what is wrong with a patient and subsequently decide on an appropriate course of treatment (Hoth et al., 2016, p.43). In mathematics teaching, diagnostic competence can be thought of similarly and can apply to a range of teaching situations using a variety of diagnostic assessments (e.g., pre and post tests, formative or summative assessments, etc.). One particularly important version of diagnostic competence is situation-based diagnostic competence (Hoth et al., 2016) and is closely related to what I referred to earlier as "managing student responses."

This type of diagnostic competence, as described by Hoth et al. (2016) builds on Blömeke, Gustafsson, & Shavelson (2015) model of competence as a continuum. In response to a dichotomy in the literature on "competence" as focused on internal resources or dispositions (e.g., various forms of knowledge) on the one hand and performance on the other, Blömeke, Gustafsson, & Shavelson (2015) proposed a model that brings these two perspectives together. They proposed that competence involves various cognitive and affective-motivational dispositions and the situation specific skills (namely perceiving, interpreting and deciding) that mediate between these dispositions and performance (see Figure 4.1).

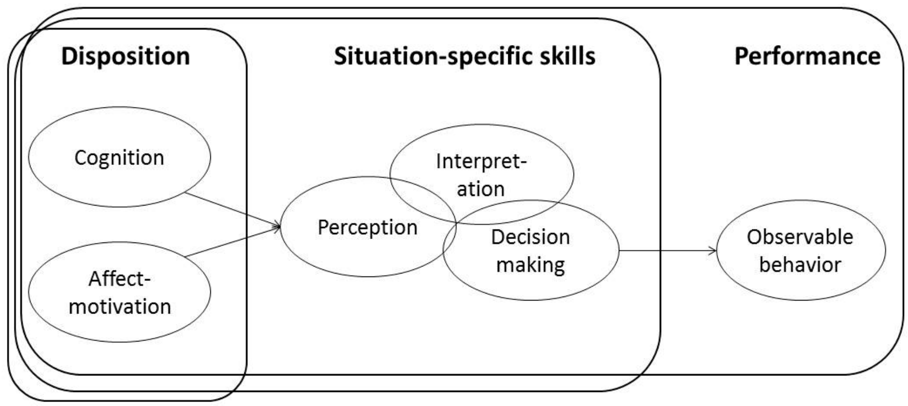


Figure 4.1: Model of competence as a continuum (Blömeke, Gustafsson, & Shavelson, 2015, p.9).

Hence, situation-based diagnostic competence likely necessitates particular cognitive resources and affective-motivational resources in addition to skill at perceiving, interpreting and making decisions about student thinking.

As mentioned above, with regard to mathematics education, there is general agreement that two core cognitive resources for mathematics teachers are likely to be content knowledge and pedagogical content knowledge. Indeed, empirical evidence suggests that mathematical knowledge for teaching, which encompasses these two different forms of knowledge, supports how teachers interpret and respond to student thinking (Hill et al., 2008). Others would add that in-the-moment, situation-based diagnostic competence will be shaped by, "general pedagogical knowledge" (p. 42). In their study, Hoth et al. (2016) found that differences in these forms of knowledge, especially in content versus general pedagogical knowledge, had important implications for what teachers' perceived and interpreted in a video-based assessment. In particular, they found that teachers with lower mathematics content knowledge tended to focus on behavioral aspects of the video while teachers with "above-average mathematics content knowledge" tended to focus on aspects of the video related to students' "understanding and learning" (p. 51).

In addition to these cognitive resources, this form of diagnostic competence also likely involves particular affective-motivational factors, typically conceptualized as "epistemological beliefs about mathematics and about mathematical knowledge acquisition" (p. 42). Indeed, researchers have found that teachers can hold varying beliefs about the nature of mathematics that seem to influence their beliefs about teaching and learning mathematics (e.g., Thompson, 1992). Within the body of research on beliefs and instructional practice, few have looked specifically at how teachers respond to students. One notable exception is Bray's (2011) case study that examined the influence of both beliefs and knowledge on how teacher's responded to errors during whole-class discussions. In particular, Bray (2011) found one teacher's intentional practice of making student errors a focus of instruction for the purposes of learning were supported by both the teacher's content and pedagogical content knowledge in addition to her beliefs, specifically her belief that "students' flawed solutions...[are] partially correct solutions with underlying logic" (p. 25). This study, like prior research that examines how beliefs are linked to instructional practice, highlights that beliefs about teaching and learning mathematics are likely to influence instructional practices, including how teachers' manage students' responses, but that the complexity of teaching means that beliefs cannot be studied in isolation. In addition to beliefs one needs to consider other internal and external factors including teachers' knowledge, contextual factors at the classroom, school, district and national level, and other motivational-affective resources (such as self-awareness and self-reflection) (Buehl & Beck, 2015, p.74).

With regard to managing students' responses, this attention to the external factors is captured in part by considering the situation-specific skills that teachers "need to perceive relevant aspects [of an instructional situation] and interpret them in order to decide about reasonable ways to act" (p. 44). These skills build on the literature that explores expert teachers' "noticing" (Kaiser et al., 2015). Though there are variations in the ways researchers conceptualize "noticing", most include teachers' attending to or perceiving relevant aspects of an instructional situation and making-sense of or "interpreting" these aspects (e.g., M. Sherin et al., 2011). In the competence literature, these situation-specific skills are usually articulated as the "PID-model" which includes: "(a) Perceiving particular events in an instructional setting, (b) Interpreting the perceived activities in the classroom and (c) Decision-making, either as anticipating a response to students' activities or as proposing alternative instructional strategies" (Kaiser et al., 2015, p.374). Similar to Jacobs et al. (2010)'s articulation of "professional noticing of children's mathematical thinking" the PID model includes perceiving, interpreting and decision-making.

There is quite a bit of research that has explored mathematics teacher's perception, interpretation and/or decision-making (see Stahnke et al., 2016, for a review). Across the studies that included teachers' decision-making there are three important trends worth noting. First, there seems to be some consistency in the findings that knowledge plays an important part in teachers' decisions about how to respond to students. For example, J. Son & Sinclair (2010) found that in a scenario-based task on reflective symmetry, though many pre-service teachers were able to identify the scenario-student's error on a conceptual level, the interventions pre-service teacher (PST)s proposed were predominantly focused on addressing procedures. In other words, the knowledge needed to diagnose a student's error or misunderstanding (e.g., common content knowledge (CCK)) differs from the knowledge needed to decide an appropriate response (i.e., knowledge of content and teaching (KCT)) (D. L. Ball et al., 2008). Additionally, the ways in which teachers propose responding to students seems to be greatly shaped by their areas of strength. Specifically, Hoth et al. (2016) found that in their proposed responses to students, teachers with greater content knowledge focused on "content specific characteristics" while those with greater general pedagogical knowledge focused on "pedagogical aspects such as the classroom management or methodological decisions" (p. 52).

A second important trend builds on the first by considering the impact of beliefs in addition to knowledge on decision-making. For example, in exploring changes during teacher induction, Blömeke, Hoth, et al. (2015) found that teachers with higher levels of knowledge (including mathematical content, general pedagogical and mathematical pedagogical content knowledge) and productive beliefs (including mathematics as dynamic, constructivist beliefs about learning and ability as malleable) scored higher on both their mathematical-PID and pedagogical-PID in a video based assessment.

A third important trend to note in these studies is in the methods used to ascertain teachers' decision-making. Specifically, these studies used either static (written scenario) or dynamic (classroom video clips) stimuli with prompts to describe and explain (usually in writing) how the teacher might follow-up with the students. Under these conditions, it is likely that what is being collected provides insight into teachers' "reflective competence"—or the "abilities required to master subject-specific pre- and post-instructional tasks"—rather than their "action-related competence" (Knievel et al., 2015, p.313). Action-related competence refers to the component of professional competence needed for tasks that occur during instruction and "are determined by the spontaneous, immediate, interactive, complex, and concurrent demands of teaching mathematics" (Knievel et al., 2015, p.313). Though action-related competence, like reflective competence, requires particular types of knowledge and skills, it occurs under pressure. Hence, "there is no time for reflective application of the knowledge and deep elaboration processes in instruction" in other words, "conscious decision-making possibilities are limited. Hence, for instructional situations, the rapid accessibility of subject-specific—perhaps implicit—knowledge gains importance" (p. 314).

Managing student responses is a type of situation-based, diagnostic competence that has both a reflective and an action-related component. This means it entails particular types of knowledge (e.g., mathematical content knowledge, mathematical pedagogical content knowledge and general pedagogical knowledge) as well as beliefs about the nature of mathematics, and the teaching and learning of mathematics. Additionally, it involves skills in perceiving, interpreting and making decisions based on student thinking that are inextricably linked to teachers' knowledge. Most of the research that has explored competency related to managing student thinking has used methods that focused on exploring the reflective component of this competency. Though it is likely that information about reflective competence provides some insight into the action-based competence, there is a dearth of research on the action-based competency of managing students' responses.

A focus on this action-based component necessitates attention to additional factors—beyond knowledge, beliefs and PID skills—that can play a role in in-the-moment decision-making. In particular, I propose that both a teachers' emotional reaction to a student response and their anxiety will be important factors when exploring action-based competency. The case for exploring these two additional factors comes from considering work outside educational research on decision-making. Essentially, the case for considering both of these factors comes from research on the role of emotions or affect in decision-making and in particular, the bi-directional connection between the brain and the body.

The case for the first factor—considering a teacher's emotional reactions to a student response—comes from considering the "somatic marker hypothesis" in neuroscience (Bechara et al., 2000). Through studies of the decision-making of patients with various cognitive impairments (due to disease or injury), neuroscientist Antonio Damasio (1994) demonstrated that emotions and feelings play an important part in the decision making process. Specifically, the somatic marker hypothesis, "proposes that a defect in emotion and feeling plays an important role in impaired decision making" and "specifies a number of structures and operations required for the normal operation of decision making" (Bechara et al., 2000, p.295). Essentially, when presented with a complex situation for which some factual aspects have been previously experienced and categorized, those factual aspects trigger a variety of pertinent processes and information, including emotions and facts, consciously and/or unconsciously, that impact decision-making. "Emotion, feeling, and biological regulation all play a role in human reason" (Damasio, 1994/2005, p.xvii).

The role of emotions in decision-making, resonates with how D. L. Ball (1997) described teaching as, "one part intellect, [and] three parts emotion" (p. 800). D. L. Ball (1997) describes, at a broad level, essentially how teachers' emotions can impact their actions:

> Teachers are disappointed when they confront their students' confusions, missing pieces, distorted understandings. They care about their students. After investing time and effort in a particular student, a teacher wants to hear right answers, sensible reasons, creative ideas. Teachers ask leading questions, fill in where students leave space, and hear more than what is being said because they so hope for student learning

In other words, D. L. Ball (1997) is describing in teaching what (Damasio, 1994/2005) has postulated in decision-making more generally. Emotions impact our decisions and our actions, and teaching is not immune to this reality of human decision-making. I hypothesize that the emotions triggered by a student's responses will impact how a teacher responds to a student. In particular, if a teacher perceives a student's response to be incorrect and this triggers disappointment or unhappiness, I hypothesize that a teacher will be more likely to engage in the response behaviors D. L. Ball (1997) described.

Additionally, I propose that an examination of the action-related facet of the competence of managing students' responses necessitates consideration of teachers' anxiety. This hypothesis is in part supported by, again, appealing to the theory that emotions play a part in decision-making but also by looking closely at the ways in which anxiety can impact performance. Specifically, anxiety can impact performance through two mechanisms: (1) active worry and (2) the body's physiological reaction. In studies of students' with math testing anxiety, research has shown that individuals dilute their mental capacity to solve the problem at hand by worrying about it (Beilock & Willingham, 2014). Instead of allocating all of their working memory to solving a task, these individuals are preoccupying part of their working memory with concerns about their performance.

A second mechanism through which anxiety can impact performance is through the body's physiological reaction to a situation that has been appraised as a threat. When faced with a situation that requires an active performance—including cognitive, emotional and behavioral responses—on their part (as is the case for managing students' responses), an individual might experience a threat or a challenge depending on whether they perceive they have the capacity or not to meet the demands of the situation (e.g., Blascovich & Mendes, 2000; Jamieson et al., 2010). When an individual feels they do not have the capacity to meet the situation's demands, they feel threatened and their body responds accordingly. Specifically, an individual's body will prepare for physical harm and constricts blood flow to minimize blood loss, ramps up inflammation, and mobilizes immune cells needed to heal after the inevitable harm is inflicted. This reaction then impedes cognitive performance and, over the long-term, can increase the risk of cardio-vascular disease (McGonigal, 2015, pp.110-111). Hence, if a teacher has high levels of anxiety in general and with respect to tasks of teaching, they are likely to feel anxious about responding to students in-the-moment. This anxiety is likely to then negatively impact their physiological reaction and, in-turn, their action-related competency.

This paper explored the action-related component of the competency of managing student responses. It adds to the conceptualization of action-related competence by considering the ways in which teachers' emotional reactions to student responses and teachers' anxiety might play a role in performance.

Research Methods

Participants

Secondary mathematics in-service and pre-service teachers within and around a large midwestern university town were recruited in person and by email as part of a larger study on managing students' mathematics responses. Recruitment efforts targeted secondary mathematics teachers at middle and high schools, as well as public, private and charter schools. All participants volunteered and were compensated for their participation in this study. Hence, the final sample of 24 participants who completed all data collection (paper instruments and teaching simulation) for this study is a sample of convenience (it should also be noted that the author had previously worked with 10 of the 24 participants in the author's previous work as a university field-supervisor).

Within this sample of 24 secondary mathematics teachers, one participant self-identified as Black and 23 participants self-identified as White. Additionally, there were six participants who self-identified as male and 18 who self-identified as female. The sample included five preservice teachers, three of which were in their final year of a three-semester undergraduate secondary teaching certification program, one who was in the first semester of this same undergraduate program and one who was in a one-year masters teaching certification program. The remainder of the sample comprised 19 inservice teachers with a range of 2 to 38 years of teaching experience and a median of 10 years of experience. The 24 participants ranged in age from 19 to 64 years of age, with a median age of 35.5 years.

Of the 24 teachers, one was recently (within the last 5 years) retired, 10 were currently in middle school mathematics classrooms and 13 in high school mathematics classrooms. One participant reported having taken no undergraduate or graduate mathematics courses while the rest of the participants all reported having taken three or more undergraduate or graduate mathematics courses. With respect to methods of mathematics teaching courses, seven participants reported having taken one such course, nine reported having taken two courses, and eight reported having taken three or more of these math methods courses.

Data Collection

Measures of Internal Resources

In order to investigate the hypothesis that how teachers respond to students is related to their anxiety, mathematical knowledge for teaching, and beliefs about teaching and learning mathematics, I used four different paper instruments to capture these factors. Participants completed these instruments in person, in about one to two hours, several days or months prior to completing the teaching simulation.

Anxiety: STAI and Teaching anxiety. I used two paper and pencil instruments to assess participants' level of anxiety: the State-Trait Anxiety Inventory (STAI) for adults and a teaching anxiety instrument adapted from Parons's (1973) Teaching Anxiety Scale. I chose these two instrument to explore if and how general anxiety and context-specific anxiety, namely teaching anxiety, might have similar or difference associations with teacher's responses.

The STAI was developed by Spielberger, Gorsuch, Lushene, Vagg, and Jacobs (1983) and is used in clinical settings to aid in the diagnosis of anxiety disorders (`http://www.apa.org/pi/about/publications/caregivers/practice-settings/`

assessment/tools/trait-state.aspx). Reliability (Spielberger et al., 1983) and construct validity of the instrument have been established (Spielberger, 1989). The STAI contains a set of 40 items, 20 that assess state anxiety (anxiety felt at that moment, right now as the person is filling out the survey) and 20 assessing trait (or general) anxiety. On the state anxiety portion of the STAI, participants indicate the degree to which 20 different statements (e.g." I feel calm","I am jittery", etc.) seem to describe how they feel at that moment (1="not at all", 2= somewhat", 3="moderately so", and 4 ="very much so"). On the trait anxiety portion of the STAI, participants indicate the frequency (1="almost never", 2= sometimes", 3="often", and 4 ="almost always") with which they generally experience 20 different statements (e.g."I feel like a failure","I am happy","I get in a state of tension or turmoil as I think over my recent concerns and interests", etc.). Both the individual state and trait anxiety score range from 20 (low) to 80 (high).

The teaching anxiety instrument I used pulled from three different sources for the 26 items in the final instrument. The majority of the items (17 of the 26) in this instrument came from Parsons's (1973) Teaching Anxiety Scale and were used with only slight modifications in language (e.g., changing"I would feel..." to"I feel..."). Additionally, I used two items translated and adapted from Schwarzer & Jerusalem (1999) that measure teachers' appraisal of their profession (in my questionnaire items 21 "I am confident that I am up for the demands of teaching mathematics" and 22"I feel that teaching is an interesting job because it challenges me in new ways"). I also created three questions specifically about student responses and students' struggling with content ("I feel capable of handling unanticipated, incorrect student responses","I feel relieved when students do not struggle with the mathematical content they are learning", and"I feel capable of handling unanticipated, correct student responses"). For these 22 items (17 from Parsons (1973), 2 from Schwarzer & Jerusalem (1999) and 3 that I created), participants were asked to indicate how frequently they experienced the statement by circling a response from 1 to 6 (1= always, 2=usually, 3=often, 4=sometimes, 5=seldom, 6= never).

I also created a set of four questions that probed how frequently teachers experienced different emotions (excited, worried, calm, tense, upset, relaxed) during different instruction formats (one-on-one, small groups, whole-class instruction) and when teaching new content. For each emotion, participants again circled a response form 1 to 6 (1= always, 2=usually, 3=often, 4=sometimes, 5=seldom, 6= never) indicating how frequently they experienced that emotion in context given in the question statement. This resulted in a final instrument with 26 different statements and 46 different items. Items were reverse coded such that a higher score indicated experiencing more teaching anxiety. Hence, the minimum score on this instrument is 46 and the maximum is 276. Together, these 46 items for the 26 participants who took this instrument have a Cronbach's alpha of 0.959 and an average inter-item correlation of 0.351; indicating acceptable internal consistency of this collection of items. Hence, all items were used as is and each participant was assigned a teaching anxiety score based on their scored responses to the 46 items (a simple sum of the score for individual items, where each item is scored from 1 to 6 based on the answer circled).

Mathematical Knowledge for Teaching Algebra. In the teaching simulation I designed for this study, participants are asked to imagine they are substituting for a 6^{th} grade mathematics teacher during a lesson on using fact families to solve equations with one unknown. This mathematical content lies in early Algebra so I used a selection of Mathematical Knowledge for Teaching (MKT) items around this content. Since there are no instruments intended to assess MKT in this specific domain of Algebra, I pulled together a set of 12 items from three different MKT instruments: (1) the Learning Mathematics for Teaching Project (LMT) instrument on Middle School Patterns, Functions, and Algebra Content Knowledge (University of Michigan), (2) the Content Knowledge for Teaching: Algebra 1 Assessment (Phelps, Gitmore, 2012), and (3) the Teacher Education Study in Mathematics (TEDS-M): released items from the Future Teacher Mathematics Content Knowledge (MCT) and Mathematics Pedagogical Content Knowledge (MPCK) Secondary (2011). Using the scoring rubrics provided by each instrument developer, each item was scored accordingly. Though for the full LMT instrument weights for each item are suggested, these were not used in the final scoring since only a sub-set of the LMT items were used. In other words, using the weights determined on the full set of items on a sub-set of items is not appropriate. A simple sum score was used as a proxy for participant's MKT. Participants could obtain a score between 0 and 30 on the set of MKT items I used.

Beliefs about teaching and learning mathematics. In order to assess participants' beliefs about teaching and learning mathematics, I used the survey instrument developed by MacGyvers, Stipek, Salmon Bogard (1993) and used in used in Stipek et al. (2001). After an initial review of the literature, used in Stipek et al. (2001) created a total of 57 items covering seven different beliefs domains (e.g."Math as a set of operations versus a tool for thought","Correct answers versus understanding as primary goal"). For each of these items, participants are asked to indicate the degree (1="strongly agree" to 6 = "strongly disagree") to which they agree with the statement. Since the original instrument was only used with a small sample (24 teachers), I ran some reliability statistics on the data I obtained with the 26 teachers who completed this instrument to narrow the original 57 items to an appropriate, statistically reliable subset.

This resulted in a final list of 32 items from the original survey that grouped into two categories similar to those used in used in Stipek et al. (2001). Beliefs category 1 comprises 26 items and includes teacher's beliefs about"Math as a tool", "Extrinsic versus intrinsic motivation", "Teacher Control", and"Correct Answers." In Beliefs category 1, scores range from 26 to 156 and a lower score indicates more productive beliefs about mathematics learning and teaching. Beliefs category 1 had a Cronbach's alpha of 0.861 and an average inter-item correlation of 0.188. Beliefs category 2 included 6 items on participants' enjoyment of and confidence in mathematics. In this category scores can range from 6 to 36 and a higher score indicates higher confidence in mathematics teaching and enjoyment of math. For this sample, Beliefs category 2 has a Cronbach's alpha of 0.740 and an average inter-item correlation of 0.378.

Teaching Simulation.

As part of a larger study exploring how mathematics teachers manage students' responses, I designed, piloted and used an interactive-video based teaching simulation. The teaching simulation takes place in a lab setting where participants sit at a desk and go through the teaching simulation on a laptop computer (see Figure 4.2 for a picture of the set-up). The experimenter (myself in this case) sits next to the participant and monitors their progression through the teaching simulation, intervening with additional information and feedback as indicated in a protocol script. In terms of the mechanics, the teaching simulation is essentially a collection of slides that participants click through with a computer mouse and videos of student responses that they can view only once.

Figure 4.2: Teaching simulation set-up in lab space.

Teaching simulation: Overview. Here I present an overview of the teaching simulation and details of the student responses that are the basis of the analysis in this paper. Figure 4.3 below shows an overview of the contents of the simulation. To ensure consistency in the information presented and questions posed to participants, I used a simulation protocol script and checklist.

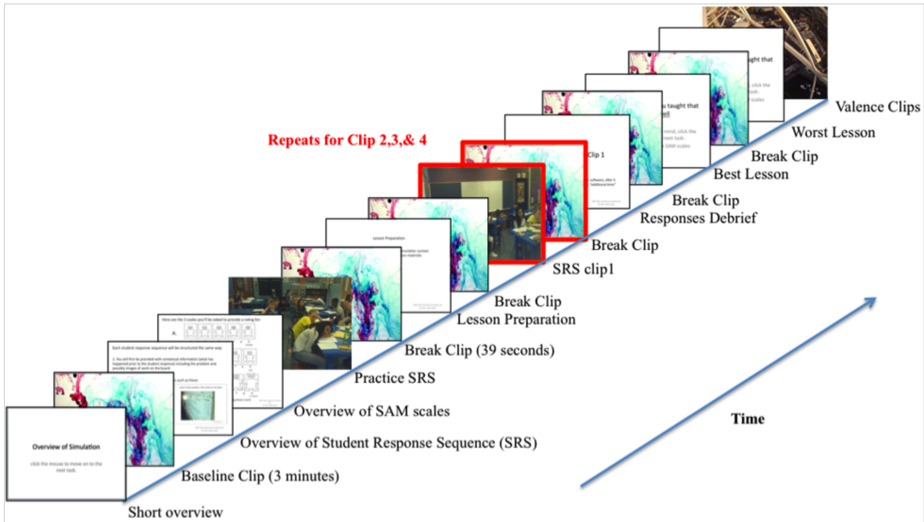


Figure 4.3: Overview of teaching simulation.

With regard to the content, the teaching simulation begins with a short, three slide overview that presents participants with general information about the context of the simulation (responding to students) and what they will be ask to do (read, think-aloud, respond) in addition to explaining why baseline and break clips are built into the simulation (to provide rest periods for the physiological data that was also collected during the teaching simulation but reviewed elsewhere). After viewing the baseline clip, participants go through a series of slides that explain the structure and content of the Student Response Sequences (SRSs) that comprise the core of the teaching simulation. The teaching simulation includes four SRSs (three of which are analyzed in this paper).

Each SRS begins with some instructional context including information about what problem students are working on and images of relevant board work. After the instructional context, participants see a slide with a question that they are to imagine they have posed to students. During the SRS participants can go through the context and question slides at their own pace. When they are ready to move past the question slide and click the mouse, a video of the student responding to the question immediately begins to play. The videos of the student responses used in the

SRSs come from an actual 6th-grade mathematics class and are taken from a first-person perspective (using Tobii eye-tracking glasses) from the front of the classroom. This makes the viewing of the video seem more like what one might see as the teacher of the class. They also include subtitles on the bottom left of the video in case, for some reason, participants have difficulty hearing the student, although the sound quality is good. During a practice clip, I also confirm with the participant that the volume is adequate and adjust it for them as needed.

Once the student is done talking, the simulation automatically transitions to a slide prompting them to self-assess their emotional reactions to the student response using a paper-and-pencil version of the Self-Assessment-Manikin (SAM) (Bradley & Lang, 1994). Participants have a total of 15 seconds to fill these out. The 15 seconds consists of the prompt slide (which is displayed for five seconds) and a ten-second, countdown video. After the ten-second, countdown video ends, the simulation automatically transitions to a slide prompting participants to respond to the student. Participants are given a maximum of 90 seconds (a minute and a half) to respond. In earlier piloting of the simulation, this was found to be more than enough time since I am asking participants to provide only their initial response to the student. The organization of the SRS and the time constraints imposed for responding are similar to some of the design considerations Lindmeier (2011) and Knievel, Lindmeier, Heinze (2015) advocate to explore action-related competence.

After participants have gone through these detailed instructions about the SRSs and we discuss the SAM scales, they are given a practice student response sequence. This provides them with an opportunity to get a sense for the flow and speed of the various components of the SRS, to ask any final questions and for me to redirect them if they talk to me rather than speaking to the student when they respond. After a quick break, participants are told that they are being asked to imagine they are substituting for a colleague's 6th-grade mathematics class towards the end of the school year during a lesson on using fact-families to find unknowns. They are then given the lesson materials to review (Connect Mathematics Program lesson 4.4 "Finding the Unknown Value: Solving Equations I" version 4, 2011) after which they go through the four SRSs.

Teaching simulation: Student response sequences. Figures 4.4 and 4.5 provides a summary of the instructional contexts, the teacher questions and student responses for Clips 1, 2 and 4 that are the focus of this paper.

In SRS Clip 1 the student notices that numbers in the facts of the fact families for multiplication and division are "rotating around in a circle." Though there is some imprecision in the student's language, she is noticing something visually that is indicative of underlying mathematical properties (e.g., the commutative property of multiplication, that 3*4=4*3, that division is the inverse operation to multiplication, that when a number is divided by one factor the result is the remaining factor, so that 12/3 =4 and 12/4 = 3). Hence, her response is mostly correct.

In SRS Clip 2 the student provides their answer to the following problem:

> On the Ocean Bike Tour test run, Sidney stopped the van at a gas station that advertised 25 cents per gallon off on Tuesdays. 1. Write the function that shows how to calculate the Tuesday discount price per gallon d from the price on other days p. (Connected Math Curriculum)

The correct answer to this problem is that d (the discounted price per gallon) is equal to p (the normal price per gallon) minus 0.25 (dollars per gallon). The student's solution can be summarized as follows:

> d is the discounted price, p is normal price, and g is gallons,
> therefore $d = p$ -0.25*g, which means you get 25 cents off for every gallon

There is some ambiguity on the exact definition of the student's variables though it can be inferred that they mean these to be total prices (rather than price per gallon)

SRS	Instructional Context	Teacher Question	Student Response
Clip 1	This takes place during the launch of the lesson on fact families to solve equations. The teacher has elicited two examples of fact families from students, including the following which is on the board: 3*4 =12 4*3 = 12 $\frac{12}{3} = 4$ $\frac{12}{4} = 3$	Tell me something else about fact families, other than giving me an example	So like for the multiplication it kinda rotates. So like the 4 goes where the 3 was and then the 3 ...like...but then…when it goes …uhhhmm...multiplication to division it's like, it, the 12 goes where the 4 was and the 4 goes to the 3 and then the 3 goes to the answer so like they kinda just rotates around in a circle
Clip 2	After students have had a chance to work on parts A and B of the lesson. The participant is told that none of the students have had a problem with part A, so they have decided to start the class discussion with part B. Part B is as "On the Ocean Bike Tour test run, Sidney stopped the van at a gas station that advertised 25 cents per gallon off on Tuesdays. 1. Write the function that shows how to calculate the Tuesday discount price per gallon *d* from the price on other days *p*"	Tell me your answer for the first part of B, what are your variables and what equation did your group come up with?	So when d is the discounted price, p is normal price, and g is gallons…g is gallons, uhmm we said d equals p minus point 25g and what that means is that 25 is…you get 25 cents off for every gallon so p is normal price and then you would minus the 25 times the gallons to get the difference

Figure 4.4: Summary of SRSs Clips 1, and 2: instructional context, teacher question and student response.

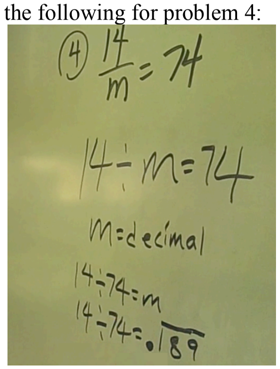
SRS	Instructional Context	Teacher Question	Student Response
Clip 4	Students have worked on problems 1-4 of part D and volunteers have put their answers on the board. Including the following for problem 4:  After going over problems 1-3 with students, the teacher goes over problem 4 and asks the students if they have any questions.	Yes, what's your question about number 4?	The answer is one thousand thirty six because one thousand thirty-six divided by fourteen …

Figure 4.5: Summary of SRSs Clip 4: instructional context, teacher question and student response.

because of the way the student explains what their equation represents. Given their definition of the variables, the student solution is actually correct. However, the student response does not answer the question posed in the problem. The core issue stems from how the variables d and p are being defined. If they represent total price then one does, indeed, need to know the number of gallons (g) in order to calculate the total discount price from the total normal price. If, as the problem intends, the prices are unit prices (price per gallon) then there is no need for the number of gallons (g) to calculate the total discount price from the total normal price. In Clip 4 the student is providing an alternative response to a problem 4 that is already on the board. The board work is as follows:

$14 \div m = 74$

$m = \text{decimal}$

$14 \div m = 74$

$14 \div 74 = 0.189$

The student proposes that, “the answer is one thousand thirty six because one thousand thirty-six divided by fourteen...” [equals 74]. Again, like in Clip 2, the student answer is not completely incorrect. Based on the end of the student’s response, it appears that the student is actually solving a different problem, one in which m is divided by 14 rather than 14 divided by m. The core issue becomes understanding what the student did to get from the original problem $14 \div m = 74$ to the problem they are solving $m \div 14 = 74$.

Teaching simulation: Self-assessment manikin. To capture participants’ self-assessed emotional reaction to the student response, immediately after the student response clip ended a screen prompted participants to circle the SAM scales (see Figure 4.6 below). During the simulation overview, when I discussed the SAM scales with participants, participants were provided paper versions of this screen (one paper for each of the six times they were asked to do this during the simulation). In order to capture participants' reactions as authentically as possible, participants were given a total of 15 seconds to circle their ratings for the three scales. This short amount of time is enough to complete the task but provides little time to overanalyze or over-think their emotional response. Hence, these ratings are likely to be more representative of participants' authentic immediate reactions to the student response clip.

The self-assessment manikin (Lang, 1980) provides a quick and easy method to capture participants' assessment of their emotional state along three dimensions: valence or pleasure (happy to unhappy), arousal (excited to calm) and dominance or agency (out-of-control to in-control). Though the original instrument does not include any text, language was added to provide an anchoring for each of the extremes of the three scales.

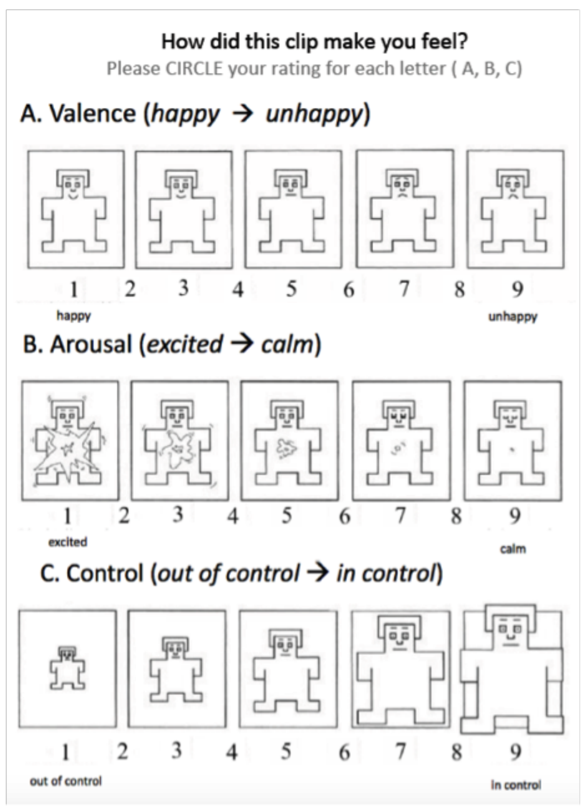


Figure 4.6: Screen prompting participants to fill out the SAM scales.

Data Analysis

Scoring of measures of internal resources and SAM.

The four paper-and-pencil instruments discussed above (STAI, Teaching Anxiety, MKT and Beliefs) were scored as described in the previous section but I summarize here those processes. In the STAI instrument each item received a score of 1 to 4, where some items are reverse coded, and the final score is a simple sum of the scores for individual items. For both state and trait anxiety final scores can range from 20 (low) to 80 (high). In the teaching anxiety instrument all items were answered on the same 6-point Likert scale and again, after appropriate items were reverse coded, a simple sum of individual item scores was computed. Scores on the teaching anxiety instrument can range from 46 to a maximum is 276. For the MKT items, these were scored based on the answer keys provided by the instrument developers. Since weights for items were not available, a simple binary system (1-point for each right answer and 0-points for each wrong answer) was used and, again a sum of individual item scores was used. For this instrument scores can range from 0 to 30.

The final beliefs instrument contained two categories of items. These categories were defined based on the factor analysis of Stipek et al. (2001) and the individual items within each category were determine through iterative decisions aimed at achieving acceptable internal consistency. Beliefs category 1 is comprised of a final list of 26 items on teacher's beliefs about "Math as a tool", "Extrinsic versus intrinsic motivation", "Teacher Control", and "Correct Answers." Beliefs category 2 included six items on participants' enjoyment of and confidence in mathematics, where higher agreement meant more enjoyment and confidence. All items on the beliefs instrument (and hence for beliefs category 1 and 2) were answered on the same 6-point Likert scale and a simple sum was used to calculate totals for Beliefs category 1 and category 2. In Beliefs category 1, scores range from 26 to 156 and a lower score possibly indicates more productive beliefs about mathematics learning and teaching. In Beliefs category 2 scores can range from 6 to 36 and a higher score indicates higher confidence in mathematics teaching and enjoyment of math.

With the SAM scale participant's score for each of the three dimensions was recorded for each student response clip. In order to explore relationships that might exist across clips an average was computed for each of the three dimensions. For example, average valence score for a participant was obtained as follows:
(Clip1 valence + Clip2 valence + Clip3 valence) / 3.

Coding and quantification of teacher responses.

In this study, I am interested in exploring the relationships between a teacher's initial response to student and their emotional reaction to that student answer, as well as various internal resources the teacher might have at her disposal. The choice to examine a teacher's initial response to a student is, in part, a consequence of the design of the teaching simulation (the student in the simulation cannot respond to the teacher). Additionally, though a teacher's initial response is only a thin slice of their instruction and is not necessarily representative of their overall practice, it can be argued that the ways in which a teacher starts an interaction with a student idea can immediately either open or close the conversation that follows. If this initial response shuts down or narrows the focus of conversation immediately, the interaction that follows has less potential to engage students in meaningful mathematical conversation. On the other hand, an initial response that is more open-ended (though it might not necessarily stay that way as the conversation unfolds) provides greater potential for the interaction to be responsive to student thinking and to engage the class in conversation. In other words, though a teacher's initial response provides only a glimpse into how they might manage student thinking, it is a crucial first-move for the teacher and important insight can be gained by examining the way in which teachers chose to begin their interaction around student thinking.

Step 1: identifying the initial teacher response and qualitative coding. After transcribing the 72 teacher responses (24 teachers and three student clips) , I identified the initial teacher responses. I define a teacher's initial response as their first, full turn of talk that starts when the teacher begins to speak and ends either when the teacher stops talking or there are indications that they would let students take a turn of talk. In some cases, teachers responded with more than just their initial response so I looked for indications in the video for the end of their initial response (e.g., looking for wait times of more than a few seconds) or lexical clues in their full response. For example in response to SRS Clip 2 about finding the discounted price per gallon, participant id07 responded with,

> Okay Paul, so can you read the problem out loud to me again, cause I want to talk about let's start with the variables that you chose. okay, you had uhhmm g for gallons, d for discounted uhhhmmmm price, so I want to talk about the variables first, does anybody else have different ideas for the variables, what variables we maybe can use from the problem now that's he's read it out loud again...uhh. it says discounted...so, I want people to raise their hand if they have an idea of what operation discounted could mean

It is clear that the teacher's initial response is to ask the student read the problem out loud, after which the teacher would follow-up by asking other students in the class about the variables. Using the pause in speech and the lexical clues that indicate the student has read the problem, I defined this teacher's initial response as follows:

> Okay Paul, so can you read the problem out loud to me again, cause I want to talk about let's start with the variables that you chose. okay, you had uhhmm g for gallons, d for discounted uhhhmmmm price, so I want to talk about the variables first

Once I had identified and catalogued the initial teacher responses I qualitatively coded the response using the Teacher Response Coding scheme (TRC) (Peterson, Van Zoest, Rouge, Freeburn, Stockero, Leatham, 2017 accepted PME proposal). The TRC was developed to provide a descriptive way to capture multiple features of teachers' responses that have been identified in the literature as important for effective mathematics discussion. Specifically, it contains 4 coding categories: *actor* (identifying who is publically being asked to consider the student thinking), *move* (which captures "what the actor is doing or being asked to do with respect to the instance of student thinking"), *recognition* (the extent to which the student might recognize their response in the teacher's response through the take-up of the student's language and main ideas), and *mathematics* (the alignment between the mathematics towards which the teacher response seems to be headed and its relation to the mathematics central to the student's idea). These categories and codes, including additions that I made to the coding scheme, are described in greater detail below in the section that details how these codes were quantified (as well as being described in the previous Dissertation Paper). In this study, the unit of analysis was the entire, initial teacher response. Hence, each teacher response received an *actor, move, recognition- student actions, recognition- student ideas,* and *mathematics* code.

To establish some validity in the coding 19 out of the 72 (26% of the data) responses were coded by two other researchers. Both researchers had been previously involved (as was I) with the development of the coding scheme and had prior experience applying the TRC codes to teacher responses from classroom videos and interview scenarios. Each researcher independently coded 10 teacher responses (chosen at random from across Clip 1, Clip 2 and Clip 4) for all categories of the TRC. All but one of the teacher responses assigned to the external researchers were unique, hence a final count of 19 rather than 20 teacher responses. After the researchers and I had independently coded the assigned teacher responses, I met with each one individually to discuss our coding. During these meetings, the researcher and I would discuss the coding that differed and discuss the instance untill agreement was reached. Hence, after these meetings I had 100% agreement with the researchers for all TRC codes for the 19 teacher responses. After these meeting, I re-coded the remaining 53 teacher responses based on the agreements and understanding reached through those discussions.

Step 2: quantifying the qualitative coding. In this study, I am interested in exploring the relationship between the characteristics of a teacher's response and their emotional reactions to a student answer, as well as to their various internal resources. Though there are several ways in which one could go about exploring these relationships, I chose to quantify the qualitative TRC coding and look for correlations between the quantified TRC scores and the various factors of interest (e.g. SAM scale valence rating, MKT score, etc.). There are a few reasons (conceptual and practical) for this choice.

Though one option could have been to simply create box plots in which I compared, for example, the MKT score distribution for each of the three actor coding categories (*same student, teacher, whole class*) this would provide answers for a series of different research questions (such as "Are there differences in the means or distributions of MKT scores between teachers who provide responses where the *same student* is the actor versus teachers with responses coded *whole class* versus teachers with responses coded *teacher*?"). Additionally, since I am interested in many variables (eight internal resources measures, three emotional scales, across three clips) looking across correlation tables versus a large numbers of box plots provides a more accurate and efficient way to look for trends in relationships. Further, (and I elaborate on this in what follows) there are arguments that can be made, based on the literature, that (under particular classroom conditions) certain codes represent more productive choices (ones that have more potential to led to student learning) than other codes.

For example, given the same student response in the same instructional context it might be possible to make a case for why it would be better for a teacher response to go back to the *same student* rather than open the conversation to the *whole class* at that moment. In what follows, I explain the values I assigned to each code and why those values were chosen (by both considering the other codes within a coding category and thinking across all five categories in relation to one another).

In addition, this quantification provides a mean to assign an aggregate, overall score to each teacher response so that trends across all three student response clips can be explored. Since the TRC was originally intended to provide a way to describe, rather than evaluate, teacher responses, I made some minor additions to the codes (described below) to capture the nuances in teachers' responses that made them more or less productive.

To check that the final results were not simply a product of this quantification process and idiosyncratic to this model but rather indicative of existing relationships, I also explored the results for a different model, which used a binary coding process (e.g., for the *actor* category instead of 5 points for *same student*, 3 point for *whole class* and 0 points for *teacher*, the *same student* and *whole class* were assigned a 1 and *teacher* was assigned a 0). Overall, the results for both of these different models were almost identical (see Appendix A for a comparison of the coefficients and p-values and Appendix B discussion of the rare differences). In the only case where an important difference did occur (for Clip 2 word count and clip 2 *moves* scores), when I explored this relationship further it seemed to be the result of outliers in the data rather than the quantification process.

In order to make judgments about the value of different codes within each category and sub-category of the TRC, I drew from the ideas that informed the TRC (about responsiveness, teacher moves, and productive discourse in general) and used the details of the context and content of the student's response to make decisions about what might be more or less productive for a given student response. Additionally, I sorted the responses to each of the three SRS from what had the potential to be the most productive to the least productive teacher responses. This qualitative ranking served two purposes. First, it provided me with a sense for which characteristics' of teacher responses might be weighed more in quantifying the teacher responses. Second, comparing this qualitative ranking with the quantitative ranking served as a way to verify that the quantification of the teacher responses accurately captured and weighed the underlying characteristics of the responses appropriately.

In assigning numerical values for each code, I considered both the code's value in comparison to other codes in the same category (e.g. an explicit code for student actions is more productive, hence scores higher than a not code for student actions) and how the category of codes might be more or less productive in comparison with other categories of codes (e.g., who the actor is in a TR seemed to influence the productivity of a TR more than the extend to which the TR picks up the student actions). Below, I discuss for each coding category the ways in which numerical values were assigned to codes.

Actor

To quantify the actor codes, the first question I considered was whether or not the student had provided enough information within their response for a teacher to reasonable open the student idea up to the class. If the student had not provided enough information, it would be more productive for the teacher to go back to the same student and follow-up the student's idea in some way. Overall, I assigned numerical codes for the actor based on the underlying assumption that getting the student or students to engage in the intellectual work is generally more productive for student learning than if the teacher is the only actor engaging in the intellectual work.

In general, in all three clips (Clip1, 2, and 4) there was a level of ambiguity in the students' responses that made it viable (though not crucial) for the teacher to go back to the same student but did not preclude the response from being considering in some capacity by other students in the class. For example, in Clip 1, the student uses the terms "rotate" and "in a circle" which are not mathematically precise and a little ambiguous. Though she grounds this in concrete examples ("...the multiplication...so like the 4 goes where the 3 was and then the 3" and "when it goes ...multiplication to division...the 12 goes where the 4 was and the 4 goes to the 3 and then the 3 goes to the answer"), even these concrete examples are a little unclear. Hence, it is viable that going back to the same student for some kind of information can be more productive than either having the teacher be the only actor considering the student ideas (which likely requires the teacher to make some important inferences about what the student says) or engaging other students just yet in the conversation about the student response.

Similarly, in Clip 2 the student provides a response that is also somewhat ambiguous but contains enough information that some reasonable inferences might be made. In their response, the student has provided a definition of their variables as well as a complete equation (d= p- 0.25g), including providing a brief explanation about what the equation means ("you get 25 cents off for every gallon so p is normal price and then you would minus the 25 times the gallons to get the difference"). Though the student's explanation seems to indicate they have defined the variables as total prices rather than unit prices, there is ambiguity in the initial definition they provide of their variables ("discounted price" and "normal price"). In this case, it is reasonable to go back to the same student for more information about the variables though, as in Clip 1, there is enough information available to ask other students to engage in some way with the student thinking. In Clip 4, like in Clip 2, the student provides more clarity than in Clip 1 but leaves some ambiguity because they do not complete their final thought ("because one thousand thirty-six divided by fourteen..."). Additionally, it is not clear why the student seems to be solving the problem m/14 =

74 rather than 14/m = 74. Again, it is reasonable to go back to the same student for more information or to engage other students with some aspect of the student thinking. Hence, for teacher responses to Clip 1, 2 and 4 the actor codes were scored as followed:

Same Student = 5, *Other student/Whole class*= 3, *Teacher* = 0.

Recognition: Student actions and student ideas.

The code recognition captures the extent to which the student might recognize their response in the teacher's response through two sub-categories of codes: (1) through the take-up of the student's language (student actions) and (2) through the take-up of the student's ideas (student ideas).

Student actions can be an important or superficial way for teachers to pick up on what students have said. On the one hand, using the student's language, work or gestures (their actions) might reasonably increase the probability that the teacher's response is closer to the student's ideas. On the other hand, it is possible that a teacher response picks-up the student's actions without attending to the student's underlying idea. Hence, both of these categories are important for gauging the responsiveness of a teacher's response.

In the TRC the codes for this category include: *explicit, implicit, not.* In general, a student is more likely to recognize that their idea is the focus of conversation of a teacher explicitly uses the student's language. Therefore, from most to least productive the codes for *student actions* are *explicit, implicit, not.*

Additionally, in clip 2 and clip 4, I wanted to capture instances where teacher responses attempted to explicitly use the student's actions but did so incorrectly (e.g., saying "you got 1076" rather than 1036 in response to Clip 4). Hence, I added the code explicit-incorrect and considered this to be problematic for a teacher response to do since it can possibly evoke confusion about what is being discussed.

After deciding on a general order of productiveness within this category (from most to least:*explicit, implicit, not, explicit-incorrect*, I considered where this category might stand with respect to the other four TRC categories. As I mentioned before, *student actions*, as opposed to *student ideas* could be a superficial way for teachers to pick up a student's idea. Hence, the final scores in this category are, in general, worth less than in *student ideas.* This category is also less important for student learning than.*actor, moves* or *mathematics.*

Further, I determined that attention to *student actions* should carry the same weight in all three clips since, in all three clips, it is ideal for a teacher to pick up a student's language correctly. The instructional contexts across the clips do not seem to suggest than in one case, versus another, it would be more prodcuive for a teacher to *not* use a student's words, for example. Hence, for teacher responses to Clips 1, 2 and 4 the *student action* codes were scored as followed: *explicit* = 2, *implicit* = 1, *not* = 0, *explicit-incorrect* = -1.

The sub-category *student ideas* is a more meaningful way for teachers to attend to what a student has said. The codes in this category capture the extent to which a student would recognize their main idea as the one being taken-up in the teacher's response and include the codes: *core*, *peripheral*, *cannot infer*, *other*, and *not applicable.* Again, across all three student clips it is more productive for teachers to pick up on the student's main idea and hence, the order in which I have just represented these codes, is the order in which I ranked them within this category.

When comparing this category with the other four TRC categories, I had (as mentioned above) pointed out that *student ideas* rather than *student actions* is a more important means to take-up a student's answer. With respect to the other categories, the choice of *move* and *mathematics* are both more important than *student ideas.* For example, a teacher response that picks up a student's main idea but engages students in low-level cognitive work (such as answer a simple, factual question) or steers them into unrelated mathematical content is more problematic and hence less productive than a teacher that, for example, is a little vague (and is coded *cannot infer* for *student ideas*) but engages students in higher-level intellectual work (such as justifying) in the relevant mathematical terrain. With regard to *student ideas* in comparison with the *actor* category, I consider these two categories to be related in their conceptual purposes (to engage students) and hence consider these on par with each other. This resulted in the following quantification of the *student ideas* codes for teacher responses to Clip 1, 2 and 4, the *student ideas* codes were scored as followed: *core* = 5, *peripheral* = 3, *cannot infer* = 0, *other* = -2 and *not applicable* = -4.

Moves

After having coded the moves across all three clips, the final list included 14 moves. These were ranked and scored based on which moves have the most potential to engage the actor in more cognitively demanding work. For example, asking a student to *justify* some mathematical idea is more intellectually demanding work than simply asking them to *repeat* their answer. This, as I mentioned above, is more crucial than *student ideas* or *student actions*.

With regard to this category versus the *actor* category, I would contend that it is more productive for the teacher response to only engage the *teacher* in *justifying* some mathematical idea than to engage the *whole class* in simply *validating* the student answer. However, one might suggest that regardless of teh *move*, a teacher response is most productive when it engages an *actor* other than the *teacher*. Additionally, the productivity of an individual *move* however, is highly contingent upon the *mathematical* ideas that are the focus of the move. Hence, though the *move* is an important factor in determining whether one teacher response is more or less productive than another, I did not want these scores to mask issues that might be present with regard to who was being asked to engage in the work (the *actor* codes) and the type of mathematics that is being pursued (the *mathematics* code. Thus, the highest score in this category is much higher than in the *student actions* and *student ideas* categories

but not much higher than in the *actor* or *mathematics* categories. The final scoring was as follows:
justify, or allow = 6, *elaborate, collect, or connect* = 5, *clarify or monitor* = 4, *repeat* = 3, *evaluate* = 2, *literal* = 1, *validate* = 0, *adapt* = -2, *correct* = -4, and *dismiss* = -4.

Mathematics

This category captures the alignment between the mathematics towards which the teacher response seems to be headed and its relation to the mathematics point (MP) underlying the students idea (see Leatham et al., 2015 for additional details about the mathematical point as it is being used in this context). The original TRC has the following codes in this category: core, peripheral, other, cannot infer (CNI), and non-mathematical. Additionally, the tag "-incorrect" can be added to the code other when the TR seems to be going towards incorrect mathematics (I extend this tag to use with the code peripheral). When more than one mathematical point underlies the students idea the tag "-MP1", "-MP2" and so forth can be added to any of the main codes to further specify what mathematics a teacher response could be headed towards. In qualitatively sorting the TRs in this sample I noticed a few nuances that the coding scheme did not seem to capture for me so I added 5 codes in this category: *CNI-core, CNI-Imprecise, peripheral-beyond,* and *peripheral-incorrect.* Below, provide more details about the 11 codes that emerged in this data set. *When an MU cannot be articulated: "Cannot infer" code variations.* In order to determine the alignment between the mathematics in the TR and the mathematical point underlying the students idea, the TRC necessitates that one articulate, to some extent, the mathematics in the TR (also know as the mathematical understanding or MU underlying the teachers response). When the TR is mathematical but too vague to be able to reasonably articulate the MU (which often happens), then mathematics is coded as cannot infer or CNI. In coding the TR in this sample, I noticed that there appeared to be three variations within TR that were coded CNI: TR that were vague and open but clear, TR that were confusing or imprecise and TR that were vague but contained indications that the TR had potential to the mathematics central to the students idea.

TR that were simply vague or open (e.g., "Okay, so we just had a different...uhhmmm possible solution. So let's talk about that. So you said that the answer was one thousand thirty-six, so how did you get that answer?"- id20) were coded CNI. In some TRs the MU could not be articulated because the TR was confusing and/or used imprecise language. This often occurred when the TR used pronouns such as "it" or "that" in critical places where precision and specificity were crucial to understanding what idea was being considered. Since, even when using contextual information, the meaning of the pronouns could not be sufficiently deciphered the MU of the responses could not be accurately inferred. These types of responses were coded as CNI-Imprecise.

In other cases, the TR was vague but contained some clues that seemed to indicate that the mathematical terrain was being narrowed towards the mathematical point underlying the students idea. For example, in response to the final SRS, participant id09 responded, "So, since we're talking about setting up equations and expressions, if you think that the answer is a hundred and thirty-six what was the equation that you set-up to get that answer?" Though it is unclear exactly what mathematical understanding the teacher is going towards, there is some indication that this response is honing in on the core issue presented by the student response. Namely, that the student seems to be solving a different problem. These TR were coded CNI-core to indicate that, even though the response was vague and a precise MU could not be articulated, the responses contained evidence that the TR could be headed towards the mathematics central to the students idea.

When an MU can be articulated. When an MU can be reasonably articulated, it can then be compared to the mathematical point underlying the students idea.

The alignment can then be described as: core, peripheral, or other. The code core is assigned when the MU in the TRs is clearly going to words the mathematical point underlying the students idea (this typically occurs when the TR is very explicit about the mathematics). In student responses that could have contained more than one mathematical point (MP), I specified which MP the TR was core to (e.g., with the code core-MP1). When the MU is still related but not core to the mathematical point underlying the students idea, it is coded peripheral. An example of this is the TR from participant id02 to Clip 2 that hones in on the connection between the operations but extends it to addition and subtraction. So though this TR is related to what the student has said, it is broader and hence veering into peripheral mathematics. TRs that were coded as peripheral-incorrect were mathematically related to what the student has said but incorrect. An example of this would be an MU that has students focus on just correcting the inclusion of g in response to Clip 2.

Though this will superficially fix the issue of the extra variable in the students equation and is hence, peripherally related to what the student has said, it incorrectly addresses the mathematical issue at stake. The core mathematical issue is about the definition of the variables (total prices versus unit prices) and the direct implications these definitions have for the equation created. In some cases, TRs started peripheral to the MP but veered into mathematical content that was likely beyond the level of 6th graders (e.g., talking about inverse operations or solving equations with two-steps). Essentially, these responses were related to the MP underlying the students idea but were mathematically beyond what is likely appropriate for that grade level. Hence, I characterized these types of responses as peripheral-beyond and added this code to the TRC.

When the MU in the TR did not seem to be related in anyway to what the mathematical point underlying the students idea, the TR mathematics was coded as other. If the MU was not related and also appeared to be incorrect, the TR was coded other-incorrect. Finally, if the MU in a TR is not mathematical (e.g., they are attending to classroom management or behavioral issues) then the TR is coded as non-mathematical.

In considering how to assign numerical values to these 11 codes, I first grouped the codes into two broad categories: codes that seem to indicate that not all the intellectual work has been done in the teacher response and codes that seem to indicate that most of the intellectual work has been done in the teacher response. The former group included the codes CNI-Core and CNI, and the latter group included the remaining 9 codes (Core- MP, Core, Peripheral, Peripheral- beyond, Peripheral-Incorrect, CNI-Imprecise, Other, Other-incorrect, Non-Mathematical). Next, I looked at within category and ranked the codes from those with the most potential to be productive for student learning to those with the least potential to be productive.

Recall that the codes CNI and CNI-core apply when a teacher response is mathematical but vague. The main difference between these codes is that teacher responses with some indication that the mathematics could be headed into ideas that are core to the underlying MP of the students response are coded CNI-Core. Hence, I ranked CNI-Core as more productive than CNI. The second broad category contained the remaining 9 codes that all indicate that most of the intellectual work has been done in the teacher response. As a first pass in ranking these, I decided that a teacher response that receives one of these codes would be more productive if the MU is closer to the underlying MP of the students response. This led to ranking the Core-MP and Core codes above the three peripheral codes (Peripheral, Peripheral- beyond, Peripheral-Incorrect) that were ranked above the Other, Other-incorrect, and non-mathematical codes. The only remaining code to rank in this group was CNI-imprecise, which is used when TR are mathematical but vague in a way that is unclear or confusing because of imprecision in the language in the TR.

I reasoned that teacher responses that received this code had the potential to be more productive than TRs coded as Peripheral-beyond, that veer into correct mathematical terrain that is beyond and hence likely inappropriate for 6th graders. In qualitatively contrasting TRs coded as Peripheral versus CNI-imprecise I concluded that TRs with either code did not have significantly different potentials to be productive for student learning and assigned them the same value. I chose to assign positive values to the codes that seem to indicate not all the intellectual work has been done in the initial teacher response and negative values to the 9 codes in the other grouping since these, overall, are likely less productive for student learning. The final ranking and scoring of all 11 mathematics codes is as follows:

CNI-Core = 4, *CNI* = 2, *Core-MP1* = -1, *Core* = -2, *Peripheral* = -3, *CNI-Imprecise* = -3, *Peripheral-beyond* = -4, *Peripheral-incorrect* = -5, *Other* = -6, *Other-incorrect* = -7, and *Non-math* = -8.

Problematic inaccuracy in language.

In line with the issues in teachers' language that I seen in teachers' responses to Clip 4, I noticed a related issue regarding precision in language in Clip 2. Specifically, even though the core mathematical issue at stake in the student's responses in Clip 2 is the definition of the variables as total prices versus price per gallon, there were many teacher responses that were imprecise or incorrect in the units they used. For example, even as their responses seemed to be pointing out that the student's units were incorrect, many of the teachers' responses themselves were fraught with imprecise language around units (issues are italicized), "...but then what if I just had the price is two dollars and fifty *cents*, is that going to matter how many gallons I buy..."—id03. In responding to Clip 2 precision with units is crucially important. Hence, when a teacher response used incorrect units (as in the TR of participant id03), I assigned a penalty of 2 points to the overall TRC score (i.e., every teacher response to Clip 2 received either a 0 or -2 for language imprecision).

Word count Another aspect of teacher responses that I chose to explore was word count. In general, I hypothesized that it is likely that teachers who do less speaking are more likely to be leaving space for students to do the intellectual work. Though this suggests that some of the five TRC categories might be correlated with word count, it is potentially capturing a different aspect of teachers responses and therefore worth investigating separately. When individuals are nervous or anxious, it is likely that they speak more (and faster) though this effect might be mitigated by various internal resources they might have or their emotional state. Therefore, I looked to see how teachers' emotional reactions and internal resources might also be associated with word count.

Results

As discussed in the introduction, there is significant research on the characteristics of skilled and competent mathematics teachers. Skilled mathematics teachers have a litany of internal resources including specialized mathematical knowledge for teaching, productive beliefs about teaching and learning mathematics, and productive motivational and affective traits such as self-regulation and self-efficacy. Though these various lines of research have provided crucial insights about the resources of skilled mathematics teachers, it is typical that these various resources are explored in isolation from each other and in settings that are typically inauthentic and therefore lose fidelity to real classroom instruction. This study takes a step towards addressing both a need to look at various teacher characteristics, including their affective reactions in-the-moment, in conjunction as they relate to the competency of managing students' responses and by using methods that maintain some fidelity to the constraints of actual classroom instruction. In this section I discuss the results to the two research questions explored in this paper:

1. Are participants' self-assessed emotional reactions to the student responses related to characteristics of their responses?

2. Are participants' individual characteristics (predictor variables) related to characteristics of their responses to students?

Before delving into each of these research questions, I first provide a brief description of how teachers responded overall and across each individual clip to provide some context for the results that follow. After these brief descriptions (more details can be found in Paper 2), I explore each research question in turn and begin the results to those questions by presenting some descriptive information about the predictor variables explored here, namely the SAM scores used to capture teachers' emotional reaction and the various measures of their internal resources (e.g., state anxiety, MKT, etc.). I then turn my attention to answering the two research questions through a multi-level exploration of the results. By multi-level, I mean that I start my exploration to each research question by looking at the level of the average teacher responses, across all three clips. I then explore whether there are distinctions between relationships at the level of apparently correct and incorrect by looking at each of the three student response clips. Finally, I look within each of the three student response clips to unpack the nuances.

In general, I found very few relationships that were significant at the $p < 0.05$ level, likely to due to the small sample size (24) of this study. Since this study is underpowered, in this paper I report on correlations with values greater than or equal to 0.250 as being "weak" associations, despite being non-significant.

Brief Description: How did teachers respond?

In this sample, the average aggregate TRC score across all three student responses ranged from -2.67 to 19.33, with a mean of 8. As can be seen in Table 4.1, below, this average aggregate score is, unsurprisingly, positively and strongly correlated with the average score for each of the five, individual TRC categories, with the exception of average *student actions* score ($p = 0.302$). The non-significant correlation between average aggregate TRC score and average *student actions* score suggests that there is not a relationship between these two variables or that the average aggregate TRC score does not reflect the *student actions* scores. Overall, however, the remaining correlations suggest that the average aggregate TRC score is reasonably reflective of the underlying characteristics of the teacher's responses (as captured by the TRC). Which suggests it is meaningful to say that a higher average aggregate TRC score has higher scoring characteristics for each of the five individual TRC categories.

Using the mode for each category as well as the distribution of codes within each clip (from Appendix C), the most frequently occurring average teacher response had the following characteristics:

- An actor score of 5 (indicating the *same student* was the actor in the response),
- A move score of 2 (which, considering the distributions of moves across the 3 clips, can't be described meaningfully but rather indicates that even if teachers had lower scoring moves in some of their responses, they had higher ones in other responses),
- A student actions score of 2 (indicating is *explicitly* used a student's language)
- A student ideas score of 4.33 (which does not correspond meaningfully to one *code* in this category but rather, suggests that teachers responses were likely to be coded *core* for two of the three responses and *implicit* for one of the three)
- A mathematics score of -3 (which indicates that either all three teachers responses were *CNI-Imprecise* or that the teacher responses varied in the *mathematics* code, with a prevalence for negative codes, meaning the teacher responses had more of tendency to veer away from the underlying mathematical point of the student response).

With regard to average word count, as indicated in Table 4.1 there is an overall negative trend between average word count and all but one (student actions) of the average TRC category scores. This suggests that as average word count increases

Table 4.1: *Average aggregate TRC, average word count, and average TRC categories: Pearson's correlation coefficients and Descriptive Statistics (N=24).*

	Average Actor Score	Average Move Score	Average Student Actions Score	Average Student Ideas Score	Average Mathematics Score
Avg. aggregate TRC Score	0.782^{**}	0.931^{**}	0.22	0.764^{**}	0.765^{**}
Avg. Word Count	-0.583^{**}	-0.410^{*}	0.116	-0.283	-0.3
M	3.111	1.639	1.292	2.708	-0.75
SD	1.822	2.338	0.576	1.786	2.032
Mode	5	2	2	4.333	-3

$^{*}p < 0.05$, $^{**}p < 0.01$

there is an associated decrease in average actor score (-0.583^{**}) and move score (-0.410^{*}), and possibly a slight decrease in average student ideas score (-0.283, p-value =0.181) and average mathematics score (-0.300, p-value = 0.154). Word count has little to no association with student actions (0.116, p-value = 0.591). Additionally, the average word count across all three responses for all 24 teachers varied from a minimum of 20 words to a maximum of about 204, with a mean of about 74 words a standard deviation of about 51 words and a mode of about 36 words.

CLIP 1: Response characteristics

In Clip 1, the apparently correct student responses about fact family numbers "rotating" in a"circle", the Clip1 TRC aggregate is positively and significantly correlated with all but one (student actions) of the TRC categories describing the teacher response. This is not unexpected since the Clip 1 aggregate TRC is a sum of these five individual characteristics.

Using the modes in Table 4.2 and code distributions in Appendix C,a typical teacher response to Clip 1 can be described as follows:

- An *actor* score of 5 (indicating the *same student* was the actor in the response),

Table 4.2: *Clip 1 aggregate TRC, word count, and individual TRC categories: Pearson's correlation coefficients and Descriptive Statistics (N=24).*

	Actor	Move	Student Actions	Student Ideas	Mathematics
Clip 1 TRC aggregate	0.808^{**}	0.882^{**}	0.185	0.920^{**}	0.524^{**}
Clip 1 Word Count	-0.453^{*}	-0.279	0.214	-0.283	-0.279
M	2.96	2.63	1.58	2.38	0.04
SD	2.4	3.62	0.5	3.42	3.48
Mode	5	5	2	5	2

$^{*}p < 0.05,^{**}p < 0.01$

- A *move* score of 5 (an *elaborate* move)
- A *student actions* score of 2 (indicating it *explicitly* used a student's language)
- A *student ideas* score of 5 (indicating the teacher response was *core* to the student ideas)
- A *mathematics score* of 2 (a *CNI* code meaning the response was vague but not imprecise and without meaningful hints within the response to make a strong or confident inference about the mathematics the teacher response was headed towards).

In looking across word count, though actor is the only TRC category that is significantly correlated with Clip 1 word count, there is a trend, with the exception of student actions, of negative correlation coefficients in this row (all of which have p-values below 0.188). This suggests that, although not statistically significant, there is a weak, negative association between word count and *move*, *student ideas* and *mathematics* as well as *actor* (meaning that as word count increases all four of these TRC category scores decrease). Additionally, responses to Clip 1 had an average word count of about 59 words with a standard deviation of 40 words and a mode of 43 words. Teacher responses ranged in length from 18 words to 155 words.

CLIP 2: Response characteristics

In Clip 2, the apparently incorrect student responses about total discount price being "d= p-0.25*g," the Clip 2 TRC aggregate is, unsurprisingly, positively and significantly correlated with all but one (*student actions*) of the TRC categories describing the teacher response. Unlike in Clip 1, the correlation coefficient between Clip 2 aggregate TRC score and student actions is negative. A look at the data reveals that this is likely a result of 16 of the 24 teachers providing responses that *explicitly* used the used the students' words, hence scoring a two, but that these 16 teachers had a large range in their overall scores, ranging from -1 to 21, hence pulling the trend line down to the right (therefore producing a negative correlation coefficient).

Table 4.3: *Clip 2 aggregate TRC, word count, and individual TRC categories: Pearson's correlation coefficients and Descriptive Statistics (N=24).*

	Actor	Move	Student Actions	Student Ideas	Mathematics
Clip 2 TRC aggregate	0.642^{**}	0.879^{**}	-0.146	0.453^{*}	0.750^{**}
Clip 2 Word Count	-0.635^{**}	-0.467^{*}	0.063	-0.006	-0.287
M	3.54	1.38	1.38	3.92	-0.13
SD	2.02	3.61	1.01	1.02	2.95
Mode	5	-3	2	3	-3

$^{*}p < 0.05$, $^{**}p < 0.01$

Using the modes in Table 4.3 and code distributions in Appendix C,a typical teacher response to Clip 2 can be described as follows:

- An *actor* score of 5 (indicating the *same student* was the actor in the response, like in Clip 1),
- A *move* score of -3 (a *correct* move, unlike the predominant *elaborate* move in Clip 1)
- A *student actions* score of 2 (indicating it *explicitly* used a student's language, similar to Clip 1)

- A *student ideas* score of 3 (indicating the teacher response was *peripheral* to the student ideas, unlike Clip 1 where *student ideas* was, on average, coded as *core*)
- A *mathematics score* of -3 (which, given the distributions in Table C.5 of Appendix C cannot really be meaningfully interpreted to describe the mathematics of a typical response to Clip 2).

With regard to word count, *actor* and *move* are the only TRC categories that are significantly correlated with Clip 2 word count. Additionally, although not significant, there is a weak, negative association between mathematics and word count (p-value = 0.174). Student ideas and student actions appear to have little-to-no association with Clip 2 word count (unlike in Clip 1 where *student ideas* had a larger coefficient and smaller p-value, though it was also not statistically significant). Responses to Clip 2 had an average word count of about 77 words with a standard deviation of 64 words and a mode of 33 words. Teacher responses ranged in length from 11 words to 257 words.

CLIP 4: Response characteristics

In Clip 4, again the aggregate TRC score is positively and significantly correlated with almost all the individual TRC categories (*student actions* and Clip 4 TRC aggregate has a coefficient of 0.338 with $p = 0.106$).

Using the modes in Table 4.4and code distributions in Appendix C, a typical teacher response to Clip 4 can be described as follows:

- An *actor* score of 5 (indicating the *same student* was the actor in the response, like Clips 1 and 2),
- A *move* score of -3 (a *correct* move, like Clip 2 but differing from the predominant *elaborate* move in Clip 1)
- A *student actions* score of 2 (indicating it *explicitly* used a student's language,

Table 4.4: *Clip 4 aggregate TRC, word count, and individual TRC categories: Pearson's correlation coefficients and Descriptive Statistics (N=24).*

	Actor	Move	Student Actions	Student Ideas	Mathematics
Clip 4 TRC aggregate	0.809^{**}	0.895^{**}	0.338	0.846^{**}	0.875^{**}
Clip 4 Word Count	-0.574^{**}	-0.586^{**}	0.139	-0.464^{*}	-0.381
M	2.83	0.92	0.92	1.83	-2.17
SD	2.48	3.75	1.06	3.8	3.34
Mode	5	-3	2	5	-3

$^{*}p < 0.05, ^{**}p < 0.01$

like Clips 1 and 2)

- A *student ideas* score of 5 (indicating the teacher response was *core* to the student ideas, like in Clip 1 but unlike Clip 2 where the average teacher response was coded as *peripheral* to the student ideas)
- A mathematics score of -3 (which, given the distributions in Table C.5 of Appendix C, cannot quite be meaningfully interpreted to describe the mathematics of a typical response to Clip 4 though it was likely either *CNI* or *Other*).

Clip 4 word count is negatively and significantly correlated with most of the TRC categories scores (mathematics is almost significantly correlated with a p-value of 0.066). Again, the exception to this negative coefficient trend is the correlation between word count and student actions score in Clip 4 in which there appears to be little-to-no relationship. Responses to Clip 4 had an average word count of about 86 words with a standard deviation of 71 words and a mode of 33 words. Teacher responses ranged in length from 8 words to 284 words. Given these general descriptions of the teacher responses overall and for each of the three clips, I now turn my attention to the results of the two research questions.

Research Question 1: Are participants' emotional reactions to student answers related to their response?

As mentioned in the theoretical framework of this paper, recent theories about human behavior have begun to demonstrate the crucial role that emotions play in decision-making (Damasio, 1994). Teaching as a human decision-making endeavor is not immune to this fact and has actually been described as"one part intellect, [and] three parts emotion" (Ball,1997, p. 800). Hence, I hypothesized that how a teacher feels, specifically their emotional reaction to a student's response, might have some relationship to the response the teacher provides to the student. I therefore asked the following research question: Do participants' self-assessed emotional reactions to the student responses help us understand their responses?

To answer this question, I first provide some descriptive information about the measures of teachers' emotional reactions and then explore the association between these emotional reactions and the teachers' responses to students. I explore these associations at varying levels, starting with the teachers' average response score then looking at the responses to the three different student clips and finally, at the level of the teacher response characteristics.

SAM scale descriptives

Recall that during the teaching simulation, immediately after watching a video of the student response, participants were asked to rate (within 15 seconds of watching the video) their emotional reaction to the student response. Participants self-assessed their emotional reactions using the SAM instrument that has the following three dimensions:

- Valence (where 1 indicates feeling happy and 9 is unhappy, hence an increase in this score indicates feeling unhappier)
- Arousal (where 1 indicates feeling excited and 9 is calm, hence an increase in

this score indicates feeling calmer)

- Control (where 1 indicates feeling out of control and 9 is in-control, hence an increase in this score indicates feeling more in-control of the situation)

The SAM instrument is based on the assumption that emotional response can be assessed along these three, distinct dimensions and hence, it is expected that there would be little to no correlation between participants ratings on these scales within an SRS Clip (see Appendix D for confirmation of this). Participants' self-reported emotional reactions to Clips 1, 2, and 4, and on average are shown in Table 4.5 below.

Table 4.5: *SAM scale means and standard deviations for each clip and averages across all three clips.*

	Valence	Arousal	Control
Clip 1	3.708	4.792	6.375
	(1.756)	(1.978)	(1.952)
Clip 2	3.5	4.833	6.208
	(1.745)	(1.949)	(1.841)
Clip 4	4.208	5.083	6.75
	(1.719)	(2.225)	(1.726)
Average	3.8	4.9	6.4
	(1.154)	(1.553)	(1.222)

In reaction to clip 1, participants reported feeling slightly happy (valence of 3.708), neither excited or calm (arousal of 4.792) and slightly in-control (control of 6.375) [1]. In reaction to Clip 2 participants also reported feeling somewhat happy (valence of 3.5), neither excited or calm (arousal of 4.883) and slightly in-control (control of 6.208) after listening to the student response in Clip 2. Finally, in reaction to clip 4 participants still reported feeling slightly happy but less so than in Clips 1 or 2 (valence of 4.208), neither very excited or very calm (arousal of 5.083) and somewhat in-control (control of 6.750). On average (across all three student responses), participants self-reported feeling slightly happy (valence of 3.8), neither very calm or very excited (arousal of 4.9), and slightly in-control (control of 6.4). On average, participants reported

[1] please see Appendix E for an explanation of the language choices relative to the SAM scales.

feeling happiest while listening to Clip1 (valence of 3.708) and the least content but still positive when listening to Clip 4 (valence of 4.208). Additionally, participants generally reported feeling neither very aroused nor very calm, in other words neutral (closer to 5) when listening to the student responses. Across all responses, they also reported feeling slightly in-control (control of 6).

Emotional Reactions and Average Teacher Response Characteristics

At a broad level, I examined whether there were associations between a participants' average response score across the three student responses or average word count and their average SAM scores. As can be seen in Table 4.6 below there were no statistically significant associations between a participant's average response score (average TRC score) and any of the average SAM dimensions ratings (all p-values were greater than 0.5). This suggests that there is not a relationship between participant's average self-reported emotional reactions across all three student answer clips and their average aggregate TRC score (the potential productivity of their response). There is one statistically significant association between average word count and average SAM control. This negative correlation indicates that as participant's average self-assessed SAM control score (meaning they reported feeling more in-control) increased there was an associated decrease in the average number of words in their response (see Figure F.2 in Appendix F for plots of these data). Average valence and arousal have no association with average word count.

Table 4.6: *Average SAM scales and average aggregateTRC score and average word count: Pearson's correlations coefficients. (N= 24).*

	Average Valence	Average Arousal	Average Control
Average aggregate TRC Score	0.033	0.052	0.131
Average Word Count	0.152	−0.017	-0.549^{**}

$^{*}p < 0.05, ^{**}p < 0.01$

At the level of average TRC characteristics, there are also no statistically significant correlations as can be seen in Table 4.7. Only the negative correlation between average control and average student actions score (−0.391) is close to significant with a p-value of 0.059. This suggests that an increase in average self-reported control might be associated with a decrease in average *student actions* score. Otherwise, it appears that average emotional reactions (as captured by the SAM scales) to the student answers have no association with any of the average characteristics of a teacher's response (which is suggested by the close to zero correlation coefficients and high p-values, all of which were larger than 0.250 for the remaining 14 coefficients in the table).

Table 4.7: *Average SAM scales and average individual TRC category scores: Pearson's correlations coefficients. (N= 24).*

	Average Valence	Average Arousal	Average Control
Average Actor Score	0.057	0.002	0.244
Average Move Score	−0.029	0.035	0.219
Average Student Actions Score	−0.151	−0.156	−0.391
Average Student Ideas Score	−0.087	0.097	0.046
Average Mathematics Score	0.211	0.088	0.029

Emotional Reactions and Individual Clip Teacher Responses Characteristics

After looking at average aggregate teacher response scores and participants' emotional reaction, I examined individual associations within each of the three student responses.

As can be seen in Table 4.8 there were no statistically significant correlations between a participant's aggregate Clip 1 TRC score and any of the average SAM dimensions ratings (all p-values were greater than 0.3). In Clip 1 the TRC score ranged from -8 to 21, with a mean of 9.542 and standard deviation of 10.236. The

Table 4.8: *Clip 1 SAM scales, aggregate TRC score and word count: Pearson's correlations coefficients. (N= 24).*

	Clip 1 SAM Valence	Clip 1 SAM Arousal	Clip 1 SAM Control
Clip 1 TRC aggregate	−0.187	0.075	0.146
Clip 1 Word Count	0.294	−0.165	−0.448*

$^{*}p < 0.05, ^{**}p < 0.01$

word count in responses to Clip 1 varied from 18 to 155 words, with a median of about 49 words. There was only one statistically significant correlation between Clip 1 word count and Clip 1 SAM control. This negative correlation indicates that as the number of words in a teacher's response to Clip 1 increased there was an associated decrease in their self-assessed SAM control score (meaning they reported feeling more out-of-control in response to Clip 1). The only other correlation of note, although not significant, is the weak, positive association between Clip 1 word count and Clip1 valence (0.294, p-value = 0.164). This suggests that an increase in self-reported valence in response to the student answer in Clip1 (indicating feeling unhappier) might be associated with an increase in word count (talking more in their response). Otherwise, the remaining four correlation coefficients (and their p-values) suggest that there is no relationship between valence and Clip 1 aggregate TRC score, arousal and aggregate TRC score or word count, and control and aggregate TRC score.

For Clip 2, there were similar results to those described in Clip 1. As can be seen in Table 4.9, in response to Clip 2 there were no statistically significant correlations between a participant's aggregate Clip 2 TRC score and any of the average SAM dimensions ratings (all p-values were greater than 0.1). The correlation coefficient between Clip 2 control and aggregate TRC though not significant, did meet my criteria for a"weak" relationship (0.2889, p-value = 0.171). This suggests a non-significant, weak, positive association between control and aggregate TRC score (meaning that an increase in self-reported control in response to the student answer in Clip2 (indicating feeling more in-control) might be weakly associated with a slight increase in aggregate TRC score (a potentially more productive response). With regard to word count, there were no statistically significant correlations between a participant's response word count and any of the average SAM dimensions ratings in Clip 2 (all p-values were greater than 0.1). Though, again, the correlation between word count and control (-0.329, p-value $= 0.116$) is almost significant at the 0.100-level. Like in Clip 1, this suggests that an increase in control might be associated with a slight decrease in word count.

Table 4.9: *Clip 2 SAM scales, aggregate TRC score and word count: Pearson's correlations coefficients. (N= 24).*

	Clip 2 SAM Valence	Clip 2 SAM Arousal	Clip 2 SAM Control
Clip 2 TRC aggregate	0.217	0.09	0.289
Clip 2 Word Count	−0.133	−0.026	−0.329

Table 4.10, shows the results of the Pearson's correlation across Clip 4 aggregate TRC score , word count and SAM scales.

Table 4.10: *Clip 4 SAM scales, aggregate TRC score and word count: Pearson's correlations coefficients. (N= 24).*

	Clip 4 SAM Valence	Clip 4 SAM Arousal	Clip 4 SAM Control
Clip 4 TRC aggregate	0.12	0.015	−0.24
Clip 4 Word Count	0.219	0.109	−0.137

The aggregate TRC scores for SRS Clip 4 had the largest range of all three clips, with scores going from -16 to 21, and the lowest mean, with an average of 2.583 with a standard deviation of 10.669. The word count in responses to Clip 4 varied from 8 to 284 words, with a median of about 62 words (the highest median of all three clips). As can be seen from Table 4.10 there were no statistically significant correlations between any of the variables (all p-values > 0.2). The six coefficients and their associated p-values suggest that there is no relationship between any of the three emotional dimensions and either aggregate TRC score or word count for Clip 4.

Emotional Reactions and Characteristics of Individual TRC Categories

After looking at the level of the aggregate TRC score for each clip, I explored whether there might be associations between specific characteristics of a teacher's response and their emotional reaction to each of the three different student responses.

Table 4.11: *Clip 1 SAM scales and individual TRC categories scores: Pearson's correlations coefficients. (N= 24).*

	Actor score	Move score	Student Actions Score	Student Ideas Score	Mathematics Score
Clip 1 SAM Valence	−0.024	−0.209	−0.045	−0.263	−0.048
Clip 1 SAM Arousal	0.099	0.098	−0.135	0.076	−0.005
Clip 1 SAM Control	0.078	0.076	0.122	0.089	0.189

As can be seen from Table 4.11 there were no statistically significant correlations between any of the three emotional dimensions of the SAM instrument and any of the five different TRC categories for Clip 1(all p-values > 0.2). Only *student ideas* and valence are weakly but not statistically significantly associated (−0.263, p-value = 0.214). This relationship would suggest that an increase in valence (feeling unhappier) is weakly associated with a slight decrease in *student ideas* score (meaning that the student might be less likely to recognize their idea in the teacher response). The remaining 14 coefficients and their p-values suggest there are no associations between emotional reactions to Clip 1 and characteristics of a teacher's response to Clip 1.

With regard to Clip 2, again there were no statistically significant correlations between any of the three emotional dimensions of the SAM instrument and any of the five different TRC categories for Clip 2 (all p-values ¿ 0.1) (see Table 4.12). Here there are two correlations that might be meaningful with more data. First, valence and *actor* could be related (0. 265, p-value = 0.211), suggesting an increase in valence (feeling unhappier) might be associated with an increase in *actor* score (meaning the teacher response engages the *same student* or *whole class* rather than the *teacher*). Second, there might be a positive relationship between control and *moves* (0.335, p-value = 0.110). This would suggest that an increase in sense of control might be associated with an increase in *move* score,(e.g., perhaps asking the *actor* to *jusitfy* rather than to *correct*). Otherwise, the remaining 13 correlation coefficients and their p-values suggest that there is no relationship between these remaining combinations of emotional reactions and Clip 2 teacher responses characteristics.

Table 4.12: *Clip 2 SAM scales and individual TRC categories scores: Pearson's correlations coefficients. (N= 24).*

	Actor score	Move score	Student Actions Score	Student Ideas Score	Mathematics Score
Clip 2 SAM Valence	0.265	0.197	−0.061	−0.024	0.122
Clip 2 SAM Arousal	−0.109	0.139	−0.077	0.212	0.072
Clip 2 SAM Control	0.144	0.335	−0.067	0.126	0.157

In Clip 4, again there were no statistically significant correlations between any of the three emotional dimensions of the SAM instrument and any of the five different TRC categories (all p-values > 0.2 with one exception). The only correlation that was close to significant was the one between mathematics score and Clip 4 SAM control (−0.377, $p = 0.070$), indicating that a modest increase in Clip 4 SAM control is associated with a decrease in mathematics score (meaning the teacher responses is possibly veering away from the underlying mathematical point of the student's response). Additionally, there is a weak, negative association (a correlation coefficient of −0.250) between Clip 4 SAM control and *student actions*. Otherwise, there appears to be no relationship between emotional reaction to the student answer in Clip 4 and the Clip 4 teacher response characteristics.

Table 4.13: *Clip 4 SAM scales and individual TRC categories scores: Pearson's correlations coefficients. (N= 24).*

	Actor score	Move score	Student Actions Score	Student Ideas Score	Mathematics Score
Clip 4 SAM Valence	−0.043	−0.017	0.058	0.205	0.226
Clip 4 SAM Arousal	0.026	−0.004	0.003	−0.034	0.078
Clip 4 SAM Control	−0.183	−0.117	−0.25	−0.113	−0.377

Research Question 2: Are participants' internal resources related to their response to students?

Considering managing student's mathematical responses as a competency means considering the various internal resources that might impact a teacher's performance. This study set out to explore the possible association between characteristics of a teacher's response to a student and various internal resources a teacher might have, including their: state, trait and teaching anxiety, as well as beliefs, MKT, years of teaching experience and general life experience (age). Recall that state, trait and teaching anxiety, as well as beliefs and MKT measures were captured through paper-and-pencil instruments, administer in a quiet space days to month before the participant completed the teaching simulation. Hence, these various score (with the exception of the MKT instrument) are reflective of their general self-assessment on these measures. The teaching anxiety, beliefs and MKT instruments are context-specific in the sense that the items in those instruments are about teaching situations and task (e.g.," I find it easy to admit to the class that I don't know the answer to a question a student asks"). Table 4.14 below includes descriptive statistics for these eight measures (additionally, see histograms in Appendix G).

In addition to the general distributions of these variables for participants in this

Table 4.14: *Descriptive statistics for internal resources variables.*

Variables	Minimum	Maximum	Mean	Standard Deviation
State Anxiety	20	43	29.71	6.31
Trait Anxiety	21	50	33.42	8.31
Teaching Anxiety	61	171	102	28.24
MKT raw score	14	30	24.75	3.8
Beliefs Category 1	50	90	71.58	11.12
Beliefs Category 2	26	36	32.33	2.79
Years Teaching	0	38	11.67	11.7
Age	19	64	38.29	14.05

sample, I also explored associations between these variables as can be seen in Table 4.15 below.

Table 4.15: *Internal resources variables:Pearson's correlation coefficients. (N= 24)*

Variables	1	2	3	4	5	6	7
1. State anxiety	–						
2. Trait anxiety	0.639^{**}	–					
3. Teaching anxiety	0.668^{**}	0.771^{**}	–				
4. MKT score	-0.021	-0.127	0.131	–			
5. Beliefs category 2	-0.517^{**}	-0.480^{*}	-0.531^{**}	-0.033	–		
6. Beliefs category 1	0.643^{**}	0.569^{**}	0.592^{**}	-0.068	-0.676^{**}	–	
7. Years teaching	-0.165	-0.295	-0.337	-0.096	-0.091	0.029	–
8. Age	-0.296	-0.39	-0.486^{*}	-0.233	0.164	-0.192	0.851^{**}

$^{*}p < 0.05, ^{**}p < 0.01$

In this sample state, trait, and teaching anxiety are all positively and statistically-significantly correlated, as suggested by the Pearson's correlation coefficients of 0.639^{**}, 0.668^{**} and 0.771^{**}. Essentially, an individual with self-reported higher state anxiety is also likely to self-report higher anxiety in general (trait anxiety) and higher anxiety around tasks of teaching (teaching anxiety). Additionally, state, trait and teaching anxiety were all weakly negatively and significantly correlated with Beliefs category 2, with correlations of -0.517^{**}, -0.480^{**}, and -0.531^{**} respectively. This indicates that an individual with higher state, trait or teaching anxiety was likely to report feeling less enjoyment of and confidence in mathematics (as captured by the paper and pencil beliefs instrument). State, trait and teaching anxiety were all also positively and significantly correlated with Beliefs category 1 (with correlations of 0.643^{**}, 0.569^{**} and 0.592^{**} respectively). This indicates that an individual with higher state, trait or teaching anxiety is likely to report less productive beliefs about teaching and learning mathematics (higher Beliefs category 2 scores).

Of the three different types of anxiety measured, only teaching anxiety was significantly correlated with age (-0.486^{**}). However, the trend across correlation coefficients for all three anxiety measures and age are negative. In other words, in this sample, older participants reported less teaching anxiety and might also report less state and trait anxiety. Though again not statistically significant, there was a similar trend in correlation coefficients for all three anxiety measures and years of teaching. This is not entirely surprising given that all three anxiety measures are highly correlated with each other and that age and years of teaching were highly correlated with each other (0.851^{**}). The correlation between years of teaching and age simply indicates that older participants were more likely to report more years of teaching experience. The overall trends of negative anxiety measure correlation coefficients suggests that as participants get older and gain more teaching experience, they are likely to report feeling less state, trait and teaching anxiety.

The only other statistically significant correlation in this sample is between Beliefs categories 1 and 2 (-0.676^{**}). This negative correlation between the two beliefs categories suggests that an individual who reports generally less-productive beliefs about teaching and learning mathematics (a higher Beliefs category 1 score) is also likely to report feeling less confidence in and enjoyment of mathematics (a lower Beliefs category 2 score). The remaining correlation coefficients for Beliefs categories

1 and 2 were all non-significant and mostly close to zero. Suggesting that there possibly no association between these beliefs (as measured) and MKT (coefficients -0.033 and -0.068 for category 1 and 2 respectively), or years of teaching experience (coefficients -0.091 and 0.029 for beliefs category 1 and 2 respectively) or with age (coefficients -0.192 and 0.164, and both p-values > 0.367, for beliefs category 1 and 2 respectively).

Though I had anticipated that MKT might be correlated with other predictors, as can be seen in Table 4.15, the raw MKT scores in this sample were not statistically-significantly correlated with any other variables. The correlation coefficients and their associated p-values suggest that there is no relationship between this measure of MKT and any of the other seven internal resources.

Internal resources variables and overall TRC Scores, word count

At a broad level, I examined whether there were associations between a participants' average response score across the three student responses or average word count and their scores of the internal resources measures. As can be seen in Table 4.16 below, there were no statistically significant correlations between a participant's average response score (average TRC score) and any of the internal resources measures or the average word count of a participant's response and any of the internal resources measures (all p-values were greater than 0.05).

Table 4.16: *Average aggregate TRC score and word count, and internal resources variables:Pearson's correlation coefficients. (N= 24)*

	State Anxiety	Trait Anxiety	Teaching Anxiety	raw MKT score	Beliefs Category 1	Beliefs Category 2	Years Teaching	Age
Average TRC score	−0.257	−0.236	−0.012	0.018	−0.079	−0.153	0.404	0.235
Average Word Count	−0.222	−0.029	−0.129	0.333	−0.073	0.159	−0.13	−0.085

There are three correlations that were close to significant and/or meet my criteria for a weak association (coefficient absolute value greater than or equal to 0.250). First, the negative, non-significant correlation coefficient between state anxiety and average aggregate TRC score (−0.257, p-value =0.226) suggests that an increase in state anxiety might be weakly associated with a slight decrease in average aggregate TRC score (i.e., a decrease in the potential productivity of teacher responses). Second, the correlation coefficient between years of teaching experience and average aggregate TRC score was significant at the 0.100-level (p = 0.051). The positive correlation coefficient (0.404) suggests that an increase in years of teaching experience might be associated with an increase in average aggregate TRC score. Third, the only other correlation that was close to being significant at 0.100-level(p = 0.112) was the positive coefficient between raw MKT score and average word count (0.333). This suggests that an increase in raw MKT score could be associated with an increase in a teacher's response word count. Otherwise, there seems to be no relationship between most of the internal resources measures and aggregate TRC score or average word count.

Though at the average aggregate TRC score level there were no statistically significant correlations, there are some statistically significant correlations between internal resources measures and the average mathematics score (as can seen in Table 4.17).

Specifically, an increase in state anxiety or trait anxiety is associated with a decrease in average *mathematics* score (-0.430^* and -0.438^*respectively), while an increase in years of teaching is associated with an increase in average *mathematics* score (0.412*). Additionally, though not statistically significant, the correlation coefficients and their associated p-values between average *mathematics* score and age (0.288, p-value = 0.173), and between average *mathematics* score and beliefs category 1 (−0.270, p-value = 0.201) suggest weak associations between these variables. It seems that the average *mathematics* score has the most potential associations with

Table 4.17: *Average individual TRC scores and internal resources variables: Pearson's correlation coefficients. (N= 24)*

	State Anxiety	Trait Anxiety	Teaching Anxiety	raw MKT score	Beliefs Category 1	Beliefs Category 2	Years Teaching	Age
Average Actor Score	-0.081	-0.164	-0.01	0.015	-0.029	-0.156	0.155	-0.075
Average Move Score	-0.176	-0.147	0.009	-0.089	-0.013	-0.207	0.367	0.181
Average Student Actions Score	-0.143	0.016	-0.122	0.061	-0.107	0.063	0.006	0.111
Average Student Ideas Score	-0.106	-0.024	0.136	-0.02	0.096	-0.32	0.391	0.353
Average Mathematics Score	-0.430^{*}	-0.438^{*}	-0.125	0.147	-0.27	0.14	0.412^{*}	0.288

$^{*}p < 0.05$, $^{**}p < 0.01$

the various internal resources (five out of the eight internal resources).

Years teaching and age are significantly associated (at the 0.100-level) with average *student ideas* score (0.391, p-value = 0.059 and 0.353, p-value= 0.091). While beliefs category 2 (enjoyment of and confidence in mathematics) has a non-significant, weak, negative association with average *student ideas* score (−0.320, p-value = 0.128). These suggest that an increase in years teaching or age might be associated with a slight increase in average *student ideas* score (i.e., a student would be likely to recognize their ideas as being "taken-up" by the teacher response). On the other hand, an increase in beliefs category 2 is weakly associated with a slight decrease in average *student ideas* score (i.e., a student would be likely to recognize their ideas as being "taken-up" by the teacher response). A final correlation of note that is significant at the 0.100-level is between years teaching and average *move* score (0.397, p-value = 0.077). This relationship suggests that an increase in years teaching might be associated with an increase in average *move* score (meaning the teacher response uses a more cognitively demanding move). Otherwise, the remaining 31 correlation coefficients and their accompanying p-values suggest there are no relationships between the remaining internal resources and average individual TRC characteristics.

Internal resources variables and Individual Clip Teacher Responses Characteristics

After looking at overall teacher response scores and participants' internal resources measures, I examined associations in each of the three student responses.

As can be seen in Table 4.18, there were no statistically significant correlations between a participant's Clip 1 aggregate TRC score and any of the internal resources measures or participant's Clip 1 response word count and any of the internal resources measures (all p-values were greater than 0.05). The only correlation that was statistically significant at the 0.100-level was between state anxiety and Clip1 TRC

Table 4.18: *Clip 1 aggregate TRC score and word count, and internal resources variables:Pearson's correlation coefficients. (N= 24)*

	State Anxiety	Trait Anxiety	Teaching Anxiety	raw MKT score	Beliefs Category 1	Beliefs Category 2	Years Teaching	Age
Clip 1 TRC aggregate	−0.404	−0.308	−0.234	0.001	0.071	−0.063	0.279	0.316
Clip 1 Word Count	−0.243	−0.117	−0.184	0.301	−0.03	−0.215	0.005	−0.071

aggregate score (p = 0.051) which suggests that an increase in state anxiety might be associated with a decrease in Clip1 TRC aggregate score.

There are a few other, potential trends between Clip1 TRC aggregate score and three other internal resources. Specifically, there is a non-significant, weak, negative association between trait anxiety and Clip1 TRC aggregate score (−0.308, p-value = 0.143). Additionally, both years teaching and age have non-significant but positive, weak associations with Clip1 TRC aggregate score (0.279, p-value = 0.186 and 0.316, p-value = 0.133). This suggests that as years teaching or age increase there is a weakly associated slight increase in the overall score of a teacher's response to Clip 1.

When examining the correlations between the internal resources and Clip 1 word count it appears there is a weak, positive association between MKT score and word count (0.301, p-value = 0.153). This suggests that as MKT score increases there is a weakly associated slight increase in words in teacher responses to the apparently correct student answer of Clip 1. Otherwise, none of the remaining seven internal resources variables seem to have any relationship with Clip1 word count.

In looking at the teacher responses to Clip 2, there are again no statistically significant correlations between a participant's Clip 2 aggregate TRC score and any of the internal resources measures or participant's Clip 2 response word count and any of the internal resources measures (all p-values were greater than 0.05). The correlation coefficient between years of teaching experience and Clip 2 aggregate TRC score was significant at the 0.100-level (with a p-value of 0.058). This suggests that an increase in years of teaching experience is associated with an increase in Clip 2 aggregate TRC score. Additionally, like in Clip 1, there is a weak, negative association between Clip 2 aggregate TRC score and state anxiety (−0.262, p-value = 0.216). In looking at the correlation coefficients and their associated p-values, it appears there are no relationships between Clip 2 word count and any of the eight internal resources measures.

Table 4.19: *Clip 2 aggregate TRC score and word count, and internal resources variables:Pearson's correlation coefficients. (N= 24)*

	State Anxiety	Trait Anxiety	Teaching Anxiety	raw MKT score	Beliefs Category 1	Beliefs Category 2	Years Teaching	Age
Clip 2 TRC aggregate	−0.262	−0.238	0.034	0.042	−0.099	−0.106	0.392	0.216
Clip 2 Word Count	−0.203	−0.02	−0.169	0.208	0.008	0.117	−0.192	−0.127

Finally, as can be seen below in Table 4.20, there were also no statistically significant correlations between a participant's Clip 4 aggregate TRC score and any of the internal resources measures or participant's Clip 4 response word count and any of the internal resources measures (with only one exception all p-values were greater than 0.100).

Only the positive correlation coefficient between raw MKT score and Clip 4 word count was significant at the 0.100-level with a p-value of 0.094. Additionally, word count in Clip 4 has a weak, positive association with beliefs category 2 (0.251, p-value= 0.236) but a weak, negative association with Clip 4 aggregate TRC score (−0.256, p-value = 0.228). It is possible that an increase in beliefs category 2 (indicating more enjoyment of and confidence in mathematics) could be weakly associated with a slight increase in word count and a slight decrease in aggregate TRC score for teacher responses to Clip 4 (essentially meaning slightly longer and less productive teacher responses).

Table 4.20: *Clip 4 aggregate TRC score and word count, and internal resources variables:Pearson's correlation coefficients. (N= 24)*

	State Anxiety	Trait Anxiety	Teaching Anxiety	raw MKT score	Beliefs Category 1	Beliefs Category 2	Years Teaching	Age
Clip 4 TRC aggregate	0.07	0.01	0.161	0.005	−0.02	−0.256	0.207	−0.003
Clip 4 Word Count	−0.152	0.023	−0.018	0.35	−0.04	0.251	−0.106	−0.027

Internal resources variables and Individual Clip Teacher Responses Characteristics

In this section, I explore for each clip in-turn how different characteristics of a teacher's response were associated with their internal resources.

Internal resources variables and Clip 1 Teacher Responses Characteristics With regard to individual TRC characteristics of teacher responses to Clip 1 and internal resources measures, there was only one statistically significant correlation (-0.486^*) (as can be seen in Table 4.21). This correlation suggests that as trait anxiety increases there is an associated decrease in the TRC mathematics score. Other that this, trait anxiety has no relationship with any of the other charactersitics of teacher responses to Clip1.

Though not significant at the 0.05-level, there were a few correlations that were significant around the 0.100-level and some general trends in the table worth mentioning. One trend that stands-out is the negative correlations between state anxiety and four of the five TRC categories. In particular, an increase in state anxiety is not-significantly but weakly associated with a decrease in *move* (−0.277, p-value = 0.190). State anxiety also has significant (at the 0.100-lelve) associations with *student actions* (−0.368, p-value = 0.077), *student ideas* (−0.377, p-value = 0.069), and

Table 4.21: *Clip 1 individual TRC categories scores and internal resources variables:Pearson's correlation coefficients. (N= 24)*

CLIP 1	State Anxiety	Trait Anxiety	Teaching Anxiety	raw MKT score	Beliefs Category 1	Beliefs Category 2	Years Teaching	Age
Actor	−0.187	−0.138	−0.085	0.051	−0.02	−0.121	0.146	0.036
Move	−0.277	−0.081	−0.205	−0.029	0.048	−0.004	0.132	0.145
Student Actions	−0.368	−0.227	−0.349	−0.216	−0.126	−0.082	0.034	0.079
Student Ideas	−0.377	−0.207	−0.226	−0.059	−0.048	−0.018	0.211	0.339
Mathematics	−0.341	−0.486*	−0.141	0.086	−0.155	0.325	0.367	0.404

$^{*}p < 0.05,^{**} p < 0.01$

mathematics (−0.341, p-value = 0.102) scores. Teaching anxiety also appears to have a significant (at the 0.100-level), negative association with *student actions* (−0.349, p-value = 0.095).

A second trend of note is the significant (at the 0.100-level), positive relationships between Clip 1 *mathematics* score and beliefs category 2 (0.325, p-value = 0.121), years teaching (0.367, p-value = 0.077), and age (0.404, p-value = 0.050). These suggest that an increase in years teaching, age or beliefs category 2 are associated with a slight increase in Clip 1 *mathematics* score (meaning the teacher responses are likely vague but not imprecise or closer to the mathematics underlying the student response). Finally, age is also significantly (at the 0.100-level) and positively associated with *student ideas* (0.339 p-value = 0.105).

Internal resources variables and Clip 2 Teacher Responses Characteristics

With regard to individual TRC characteristics of teacher responses to Clip 2 and internal resources measures, there were four statistically significant correlation and a few other correlations worth mentioning (see Table 4.22 below). Both state and trait anxiety were significantly and negative correlated with *mathematics* scores (-0.559^{**}

and -0.506^{**}) and teaching anxiety is also significantly at the 0.100-level, negatively correlated with the *mathematics* score for Clip 2 (-0.358, $p = 0.086$). This suggests that an increase in state, trait or teaching anxiety is associated with a decrease in *mathematics* score in teacher responses to Clip 2. Additionally, *mathematics* score for Clip 2 is significantly (at the 0.100-level) and negatively associated with beliefs category 1 (beliefs about teaching and learning mathematics) (-0.316, p-value = 0.133) and not-significant, positively and weakly associated with years of teaching (0.293, p-value = 0.164). These correlations suggest that an increase in beliefs category 1 (which likely indicates potentially less-productive beliefs about teaching and learning mathematics) is associated with a slight decrease in *mathematics* score while an increase in years of teaching experience is weakly associated with a slight increase in *mathematics* score for teacher responses to Clip 2.

Two other statistically significant correlations for Clip 2 occurred between *student ideas* scores and years of teaching experience (0.450^{**}), and *student ideas* and age (0.442^{*}). These correlations suggest that an increase in years of teaching or age is associated with an increase in *student ideas* score (meaning a teacher response that is possibly *core* rather than *peripheral* to the student idea).

There are three other trends worth noting with respect to Clip 2 teacher response characteristics and internal resources. First, teaching anxiety is significantly (at the 0.100-level) and negatively associated with *student actions* (-0.343, p-value = 0.100). Teaching anxiety is also weakly, positively associated with both *actor* (0.275, p-value = 0.193) and *moves* (0.263, p-value = 0.214) scores. This would mean that an increase in teaching anxiety is weakly associated with teacher responses that *implicitly* or do *not* use the student's language but that engage either the *same student* or *whole class* in a more challenging *move*. A second trend to notice occurs across years of teaching experience. As mentioned earlier, this is positively associated with *student ideas* and weakly, positively associated with *mathematics* score. Additionally, years of teaching experience is positively and significantly (at the 0.100-level) associated with *moves* at the (0.346, p-value = 0.098). Age is significantly (at the 0.100-level) and positively associated with *student actions* (0.352, p-value = 0.091) while there is a non-significant, weak, positive association between beliefs category 2 (enjoyment of mathematics) and *student actions* (0.277, p-value = 0.191). In other words, an increase in either age or enjoyment of mathematics is weakly associated with a slight increase in *student actions* scores (meaning a teacher response to Clip 2 that likely uses the student language *explicitly*). The remaining correlations and p-values suggest that there are no other relationships between the internal resources measures and characteristics of teacher responses to Clip 2.

Table 4.22: *Clip 2 individual TRC categories scores and internal resources variables:Pearson's correlation coefficients. (N= 24)*

CLIP 2	State Anxiety	Trait Anxiety	Teaching Anxiety	raw MKT score	Beliefs Category 1	Beliefs Category 2	Years Teaching	Age
Actor	0.136	0.027	0.275	0.058	0.159	−0.264	0.013	−0.246
Move	−0.033	−0.024	0.263	−0.044	0.005	−0.224	0.346	0.195
Student Actions	−0.213	−0.133	−0.343	−0.031	−0.128	0.277	0.139	0.352
Student Ideas	−0.119	−0.006	0.136	−0.006	0.028	−0.082	0.450*	0.442*
Mathematics	−0.559**	−0.506*	−0.358	0.125	−0.316	0.137	0.293	0.167

$^{*}p < 0.05, ^{**}p < 0.01$

Internal resources variables and Clip 4 Teacher Responses Characteristics

With regard to individual TRC characteristics of teacher responses to Clip 4 and internal resources measures, there was only one statistically significant correlation (−0.413*) (as can be seen in Table 4.23). This correlation suggests that as beliefs category 2 (enjoyment of mathematics) increases there is an associated decrease in the

TRC *student ideas* score in the teacher response to Clip 4. The remaining correlations in that column suggest that beliefs category 2 has no association with the remaining four TRC categories. With the exception of three correlations that I will mention in a moment, the rest of the correlations and their accompanying p-values suggest that there are no relationships between state anxiety, MKT, beliefs category 1, years of teaching or age and any of the five TRC categories for Clip 4 responses.

There is a non-significant, weak, positive association between teaching anxiety and *student actions* (0.295, p-value = 0.162) and a significant (at the 0.100-level), positive association between teaching anxiety and *student ideas* (0.359, p-value = 0.085). In other words, an increase in teaching anxiety is associated with an increase in *student actions* and weakly associated with a slight increase in *student ideas* scores (indicating a more responsive teacher response) in Clip 4. Finally, though not significant, there is a weak, positive association between *student actions* and trait anxiety (0.261, p-value = 0.218).

Table 4.23: *Clip 4 individual TRC categories scores and internal resources variables:Pearson's correlation coefficients. (N= 24)*

	State Anxiety	Trait Anxiety	Teaching Anxiety	raw MKT score	Beliefs Category 1	Beliefs Category 2	Years Teaching	Age
Actor	−0.109	−0.25	−0.165	−0.065	−0.175	−0.01	0.19	0
Move	−0.03	−0.173	−0.038	−0.096	−0.075	−0.167	0.226	0.01
Student Actions	0.146	0.261	0.295	0.232	0.008	−0.122	−0.139	−0.194
Student Ideas	0.223	0.155	0.359	0.027	0.171	−0.413*	0.242	0.073
Mathematics	0.066	0.154	0.236	0.068	−0.053	−0.203	0.11	−0.044

$^{*}p < 0.05$, $^{**}p < 0.01$

Discussion

Attention to students' mathematical learning has meant attention to the interactions of students and teachers around mathematical content. In particular, there is evidence that whether teachers do or do not take-up students' thinking has important implications for students' learning. Exploring this interactional aspect of instructional quality means considering the capacities teachers need to be able to do this complicated, interactional work in the complex environment of the classroom. Multiple but typically parallel lines of research have produced evidence that teachers' instructional quality depends on teachers having specialized knowledge, particular beliefs, and various affective-motivational resources. With these various bodies of work to build on, research on teaching is now poised to bring these mostly independent lines of inquiry together and to develop a more comprehensive and holistic understanding of what is involved in teaching.

One promising perspective involves considering teaching as a form of professional competence in which multiple internal resources and situation-specific skills come together to produce particular performance outcomes. For a teaching competency such as managing students' mathematical responses, for example, teachers need mathematical content, general pedagogical and mathematical pedagogical content knowledge in addition to productive beliefs about the nature of mathematics and its teaching and learning. Further they need emotional-motivational resources that support them in productively regulating their emotions and motivation both in-the-moment and over the long-term. These internal resources in-turn support and are supported by teachers' skills at noticing: perceiving and interpreting important and relevant aspects of students' mathematical responses. All of these resources and skills culminate in some type of decision and then observable behavior.

A further nuance of the competence perspective that is consequential for, "the spontaneous, immediate, interactive, complex, and concurrent demands of teaching mathematics" is the distinction between reflective and action-related competency (Knievel et al., 2015, p.313). Specifically, recognizing that the ways in which teacher might plan for, reflect on or hypothesize about how they might respond to students involves a related but different competence than the competence they need to actually respond to students during instruction. A focus on this action-related competence necessitates building not only on the research within education but also utilizing findings from research outside of education that has explored individuals' decision-making in-the-moment. Most notable and relevant from this research are the findings on the role of emotions or affect in decision-making and additionally the bi-directional nature of the connection between the brain and the body.

Specifically, "emotion, feeling, and biological regulation all play a role in human reason" (Damasio, 1994/2005, p. xvii). One notable illustration of this is the way in which anxiety can impact human reason and performance. Experiencing anxiety about a task can manifest itself in worry, which occupies mental space in working memory, essentially diminishing the mental capacity one might otherwise have available to reason about the task. With regard to performance, when faced with a situation that requires an active performance—including cognitive, emotional and behavioral responses—the way in which an individual appraises the situation—as either a threat or a challenge depending on whether they perceive they have the capacity or not to meet the demands of the situation—will impact their physiological reaction which in-turn impacts their performance (e.g., Blascovich & Mendes, 2000; Jamieson et al., 2010).Essentially, emotions impact our decisions and our actions, and teaching, which is "one part intellect, [and] three parts emotion" (D. L. Ball, 1997, p.800), is not immune to this reality of human decision-making.

With this lens and a focus on the active-related component of managing students' responses, this paper explored two questions:

1. Are participants' self-assessed emotional reactions to the student responses re-

lated to characteristics of their responses?

2. Are participants' individual characteristics (predictor variables) related to characteristics of their responses to students?

By using a novel teaching simulation and various paper instruments, I collected data about teachers to explore how they responded to three different types of student responses: one response that is apparently correct in which a student notices that numbers in a multiplication and division facts family "rotate" in a "circle"; a second, apparently incorrect response in which a student provides an equation relating total discount price, normal price and number of gallons rather than using unit prices; and a third, apparently incorrect student response in which a student appears to be solving a different problem (m /14 = 74 instead of 14 / m = 74).

In what follows, I discuss the answers to these research questions using visual representations of the results. There are a few things that are helpful to know about these visuals in order to make sense of what they represent. First, the colors used indicate that there is no relationship (orange), a positive relationship (green text and green arrow) or a negative relationship (red text and red arrow) between the variables. These determinations about the relationships (none, positive or negative) are based on the value of the correlation coefficient and the p-value.

Orange indicates that the absolute value of the correlation coefficient is less than 0.250 (suggesting there is no statistically significant or non-significant and weak association between those variables). Red indicates that the correlation coefficient is less than or equal to -0.250 (suggesting a weak, negative relationship) and green indicates that the correlation coefficient is greater than or equal to 0.250 (suggesting a weak, positive relationship).

Within these visual representations of the results, significance, if it occurred, is indicated with the appropriate superscript-asterisk or letter as follows: ^{a} indicates $p < 0.100$, * indicates $p < 0.05$, **, and indicates $p < 0.01$.

Another thing to note when interpreting the results displayed is that each visual is organized in a similar manner. Specifically, the coefficients represented by the color-coded text ("Average", "Clip 1", "Clip 2" and "Clip 4") are to be read in the context of an increase in the predictor variable (emotional reaction or internal resource). For example, in Figure 4.7 the first visual representation shows the relationships between valence and aggregate TRC scores and the word count correlation when valence is increasing (as suggested by the blue arrow, right below the valence SAM image, that is going from left to right).

Finally, the visuals also summarize information about the average (across all three clips) teacher responses as well as the teacher responses to Clips 1, 2 and 4. This aspect of the visual has implications for the way that individual results are read. For example, in Figure 4.7 though the SAM scales shown represent the general scale (valence, arousal or control), when interpreting the individual results the SAM scale should be read in the context of that specific result. In other words, in Figure 4.7 all the results for "Clip1" should be taken in the context of the SAM scale representing participant's self-assessed valence in response to Clip 1, while all the results for "Clip2" should be taken in the context of the SAM scale representing participant's self-assessed valence in response to Clip 2, and so forth. Hence, when reading the result for valence and word count for "Clip 1", this result should be read as "an increase in valence in response to the student answer in Clip 1 was associated with an increase in the word count of the teacher responses to Clip 1."

In what follows I discuss the significant results and a few of the interesting trends that emerge (namely predominant trends or trends that are interesting in light of the apparently correct or incorrect nature of the student responses).

Are teachers' emotional reactions to student answers related to characteristics of their responses?

As can be seen in Figure 4.7 participants emotional reactions across all the Clips and in response to individual Clips generally have no relationship to their corresponding aggregate TRC scores. This is indicated by the predominance of orange text (the color used, as mentioned above, to indicate small correlation coefficients and large p-values, or, in other words, that there was no relationship between the variables). Essentially, participants self-report valence and arousal have no relationship to the aggregate TRC scores across all three clips and at the individual Clip level. With regard to participant's sense of control, this seems to have a positive association with teacher responses to Clip 2 (as indicated by the green text and arrow) suggesting that an increase in sense of control after hearing the student answer in Clip 2 might be associated with an increase in the overall productivity of the teacher response to Clip 2.

When looking across the emotional reactions and word count results there is again a predominance of orange text. The only notable exception to this trend are the red text and arrows for the control SAM scale. In this case, an increase in participant's self-reported sense of control after hearing the students answers in Clip 1 and 2 was associated with a decrease in the word count of their responses to those clips. When participants reported feeling more in control, they talked less. It is interesting to note that this results did not occur for Clip 4 in which the apparently incorrect student response does not provide information to participants about why the error is occurring. In contrast, in response to Clip 2 where the student answer provides both information about what the error is and why it is occurring (using total price versus unit price), an increase in sense of control was associated with shorter, more productive teacher responses.

When examining these relationship at the level of individual TRC characteristics

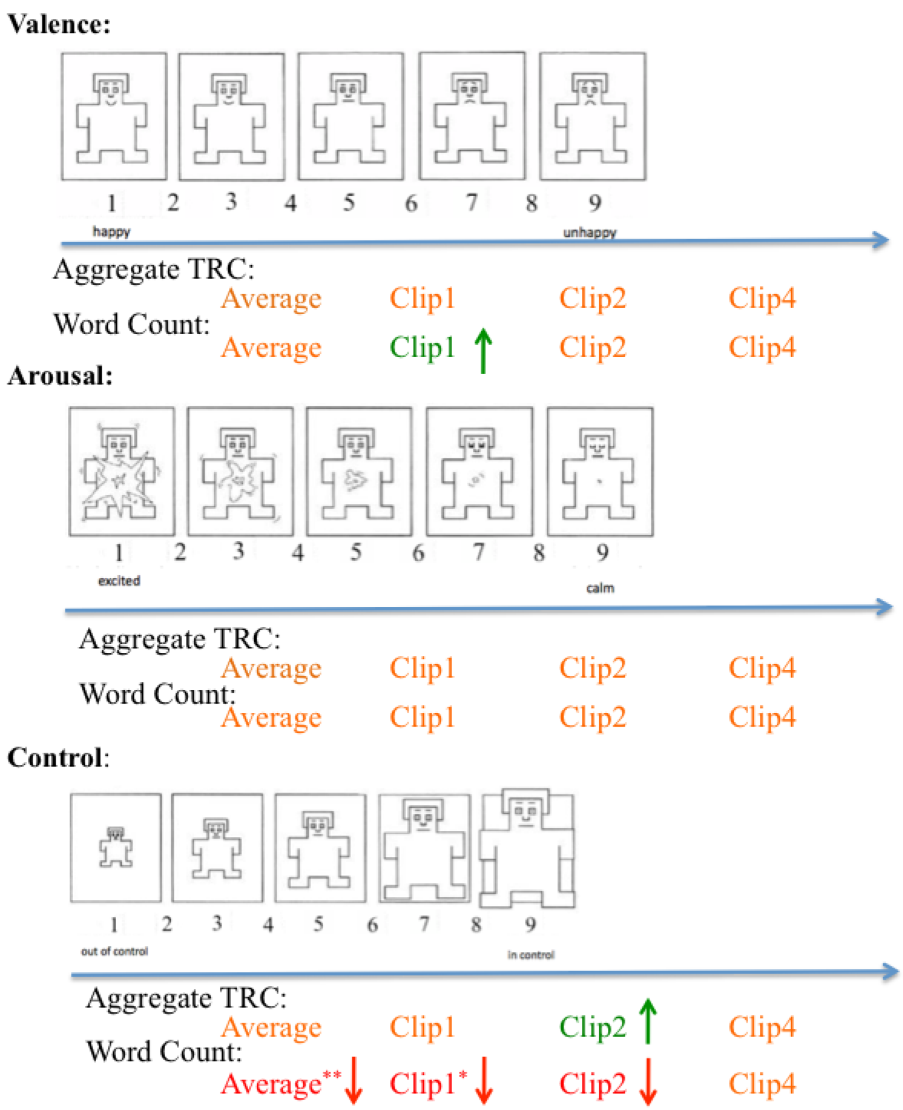


Figure 4.7: Summary Visual of SAM scores versus aggregate TRC and word count for the across-clip average, Clip 1, Clip 2 and Clip 4.

scores rather than aggregate TRC scores, the trends are essentially the same as those in Figure 4.7 (see Figures 4.8, 4.9, and 4.10 below). Specifically, with only two exceptions (as can be seen in Figure 4.8), valence has little to no association with any of the individual TRC categories for responses to Clips 1, 2, and 4 as well as for the average teacher response. Additionally, Figure 4.9 shows that there were no associations at all between participant's self assessed arousal and any of the individual TRC categories scores for responses to Clips 1, 2, and 4 as well as for the average teacher response. Though there are a few more potential relationships when looking at Figure 4.10, which summarizes the results across the control dimension, there is still overall little-to-no association between a participant's self-reported sense of control and any of the individual TRC categories for responses to Clips 1, 2, and 4 as well as for the average teacher response.

Despite this lack of relationships it is interesting to note that participant's sense of control, on average, might be associated with a slight decrease in average *student actions* scores potentially suggesting that teacher responses were less likely to use student's language or actions *explicitly*. Additionally, it is also interesting that in Clip 4, where the student answer provides insight into *what* the student is doing incorrectly (solving a different problem) but not *why* the student is doing this, there is a negative association between sense of control and *mathematics* score. It is possible that after hearing the student's answer teachers feel in control since they can likely identify the student's mistake. However, when they are crafting their response to the student, they struggled in addressing the underlying reason for the student's error since it was not evident from what the student had said. It could be that if participants were asked to self-assess their sense-of-control after responding to Clip 4 (rather than before) that some teachers might have felt less sense of control.

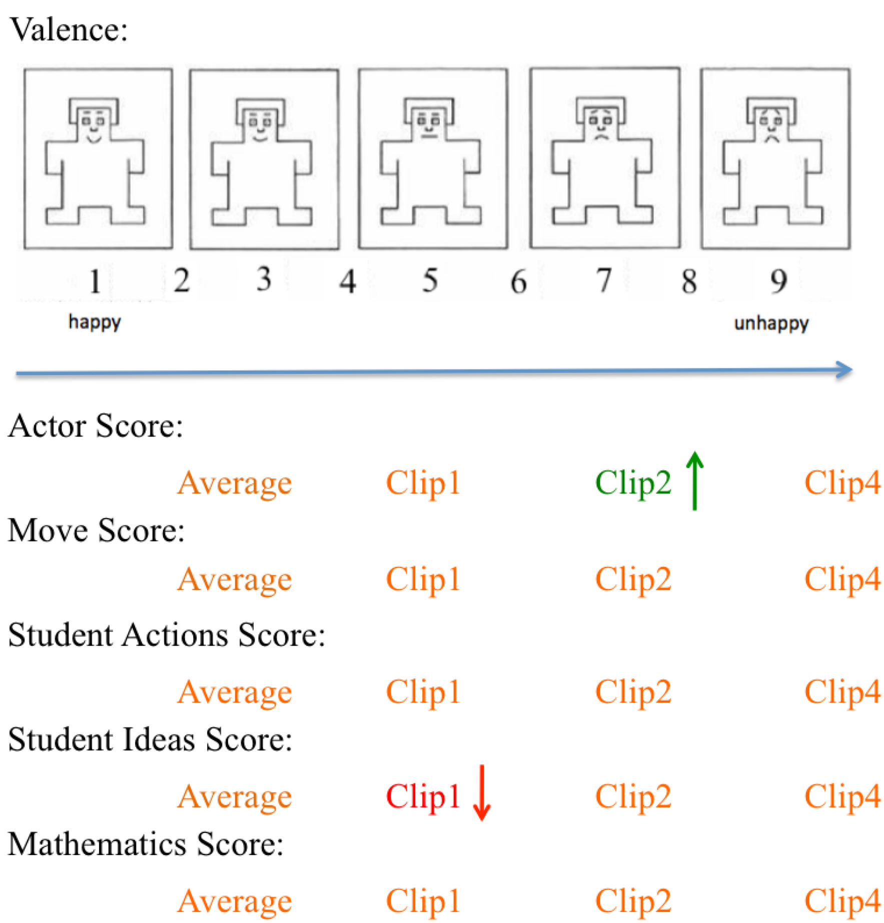
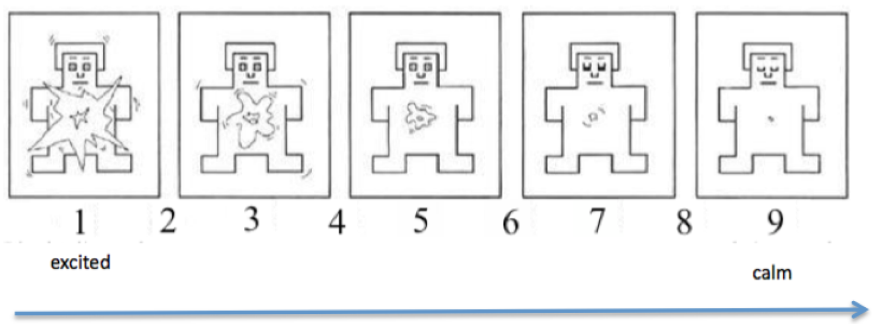


Figure 4.8: Summary Visual of average SAM scores versus aggregate TRC and word count for the across-clip average, Clip 1, Clip 2 and Clip 4.

Arousal:



Actor Score:

Average Clip1 Clip2 Clip4

Move Score:

Average Clip1 Clip2 Clip4

Student Actions Score:

Average Clip1 Clip2 Clip4

Student Ideas Score:

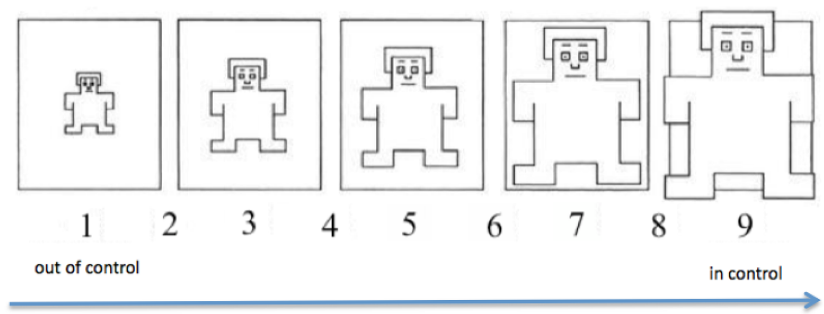
Average Clip1 Clip2 Clip4

Mathematics Score:

Average Clip1 Clip2 Clip4

Figure 4.9: Summary visual of arousal SAM scores versus aggregate TRC and word count for the across-clip average, Clip 1, Clip 2 and Clip 4.

Control:



Actor Score:

Average Clip1 Clip2 Clip4

Move Score:

Average Clip1 Clip2 ↑ Clip4

Student Actions Score:

Average ↓ Clip1 Clip2 Clip4 ↓

Student Ideas Score:

Average Clip1 Clip2 Clip4

Mathematics Score:

Average Clip1 Clip2 Clip4 ↓

Figure 4.10: Summary visual of control SAM scores versus aggregate TRC and word count for the across-clip average, Clip 1, Clip 2 and Clip 4.

Even though there are a few interesting relationships in teacher's self-assessed sense of control and the characteristics of their responses to student, overall however, there is a clear absence of relationships between teachers' emotional reactions to student answers and characteristics of their responses to those answers. In particular, out of the possible 84 associations (12 across the aggregate TRC scores, 12 across word count and 60 at the individual TRC characteristics level), only 10 (less than 12%) were non-zero.

Nonetheless, this lack of significance and results could be more indicative of methodological limitations rather than theoretical ones. In particular, though this aspect of the teaching simulation was designed with attention to some of the issues with self-reports—by limiting the time participants were given to fill out the self-report and by using a scale that uses images rather than just text—it could be that this was not enough to overcome other challenges with self-reports. Namely, even under the constraints of time, participants still likely engaged in some level of cognitive work to translate what they felt into a numerical rating on each of the three SAM scales. Some might also argue that asking participants "how do you feel?" could invoke higher-order cognitive processes that are difficult to subjectively capture (Nisbett Wilson, 1977). Essentially, by asking participants to be introspective it is possible that what is being captured is what participants' think they feel rather than what they actually feel.

Are teachers' internal resources related to characteristics of their responses to students?

Again, to summarize the trends in the associations between various internal resources and aggregate TRC scores and word counts, I have compiled the results into Figures 4.11, 4.15 and 4.19. I discuss these and the results at the individual TRC characteristics (also summarized visually) in what follows.

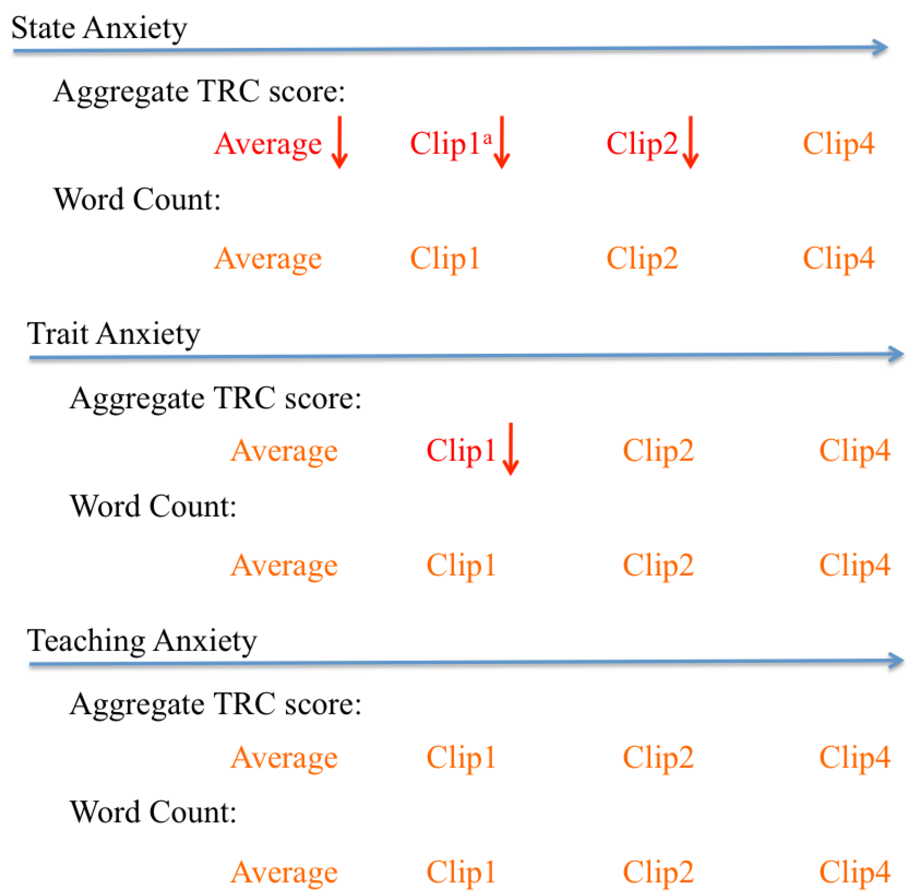


Figure 4.11: Summary visual of state, trait and teaching anxiety versus aggregate TRC and word count for the across-clip average, Clip 1, Clip 2 and Clip 4.

As can be seen in Figure 4.11 all three types of anxiety had no relationships to word count but state and trait anxiety had some negative associations with aggregate TRC scores. In looking more closely at the trends between the three different anxiety measures and the individual TRC scores (see Figures 4.12, 4.13, and 4.14) though the majority of the associations are non-existent (as indicated by the orange text) there are some interesting patterns that emerge in the associations that do exist.

First, the TRC characteristics scores of teacher responses to Clip 1 have negative associations (where they do occur) with all three anxiety measures. State anxiety in particular has negative associations with four of the five TRC characteristics for Clip 1. Recall that state anxiety was a measure of how anxious teachers felt while filling out the paper-and-pencil instruments. Essentially, those negative trends suggest that an increase in feeling more anxious while filling out the STAI instrument was associated with generally less productive (as measure by the TRC categories) responses to the apparently correct student answer. Though efforts were made to make participants feel at ease while both taking the pencil-and-paper instruments and when completing the teaching simulation, it could be that both of these situations, nonetheless, felt evaluative, high-stakes and thus anxiety-provoking for teachers. This state anxiety could then have negatively impacted teachers' responses to Clip 1 though it is unclear why it simultaneously had essentially no relationship with their responses to the apparently incorrect student answers of Clips 2 and 4.

A second interesting trend across these Figures is the mix of positive and negative associations seen for the TRC characteristics scores of teacher responses to Clip 2. Essentially, both an increase in state or trait anxiety is associated with a decrease in the *mathematics* scores for Clip 2. However, teaching anxiety has positive associations with Clip 2 *actor* and *moves* scores but negative (and close to significant) associations with Clip 2 *student actions* and *mathematics*. It is not evident why this would be the case though one possibility is that within the range of teaching tasks covered in the teaching anxiety instrument some might have supported teachers while responding in-the-moment and others might have hindered them.

A third trend emerges when looking at the Clip 4 teacher responses. Specifically, where there are associations they are positive. Both trait and teaching anxiety had positive associations with Clip 4*student actions* scores (meaning a teacher responses that was more likely to use the student's language *explicitly*). Additionally, teaching anxiety was positively and almost significantly associated with Clip 4 *student ideas*.

Of the three anxiety measures, teaching anxiety seems to show the most promise for providing insights into the relationship between anxiety and teacher responses. Future work will need to look more closely at this measure including both conceptual and instrument development. With regard to conceptual development, the current instrument covers a wide range of teaching tasks and it is possible that there teachers might have anxiety in some sub-domains of teaching work and not others. Therefore, developing a conceptual map of the categories of teaching work that are likely to cause teachers anxiety could better map the terrain of this broader anxiety. This conceptual development is also likely to inform instrument development, particularly the development of items for the various sub-categories of teaching work that are likely to be anxiety provoking.

Two additionally, methodological insights emerge from the anxiety data. On the one hand, these results provide evidence that anxiety can impact a teacher's performance and should be further explored in both research and teacher education. On the other hand, out of the 84 possible associations across all three anxiety measures (12 for the aggregate TRC scores, 12 for word count and 60 at the individual TRC characteristics level), only 21 (or 25%) were non-zero.

This suggests that perhaps other methods for collection information about anxiety might be worth exploring. In particular, as with the emotional reactions, using instruments and measures that do not require participants to self-assess and can be used in-the-moment rather than *a priori* during the teaching simulation might provide more information. A second methodological consideration is suggested in part by the mix of positive and negative associations that occurred across Clips 2 and 4. Specifically, in paper 1 I mentioned research has shown that how an individual interprets the anxiety—as either a threat or a challenge—experienced in a situation can impact there physiological reaction and their ability to perform. It would be worth exploring whether teachers experience students' apparently incorrect answers as threatening or challenging and if these perceptions lead to variable teacher performance when responding to students.

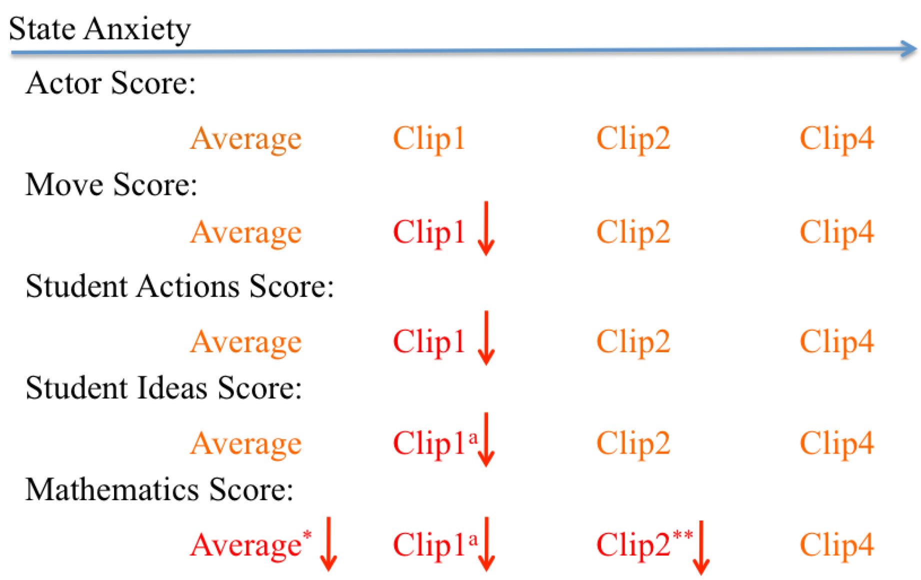


Figure 4.12: Summary visual of state anxiety versus individual TRC characteristics scores for the across-clip average, Clip 1, Clip 2 and Clip 4.

As can be seen in Figure 4.15 below there were no statistically significant associations between raw MKT score, beliefs category 1 and beliefs category 2 with aggregate TRC scores or word count. Though raw MKT score has a positive association with word count, suggesting that an increase in MKT is associated with lengthier teacher responses in Clips 1 and 4, raw MKT score had no relationships with aggregate TRC scores and no associations at the level of individual TRC characteristics (see Figure

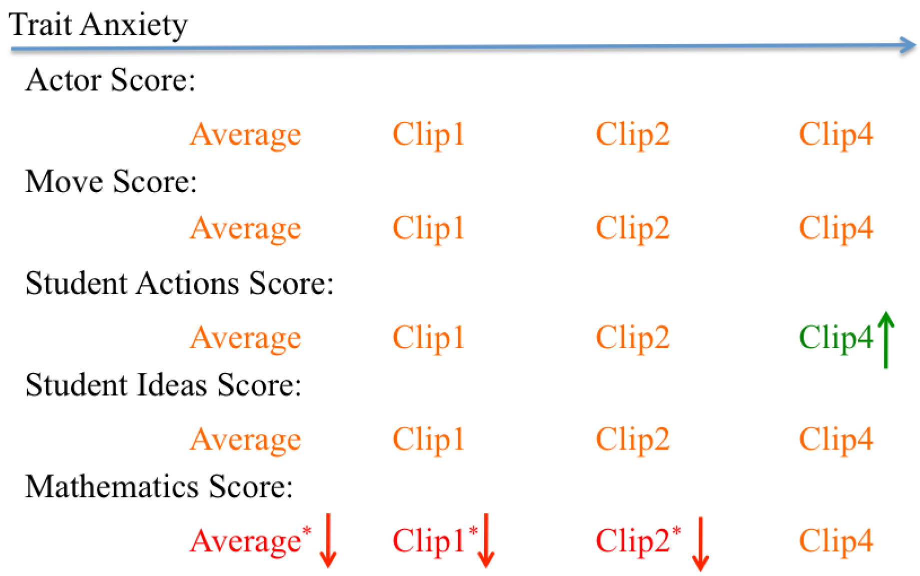


Figure 4.13: Summary visual of trait anxiety versus individual TRC characteristics scores for the across-clip average, Clip 1, Clip 2 and Clip 4.

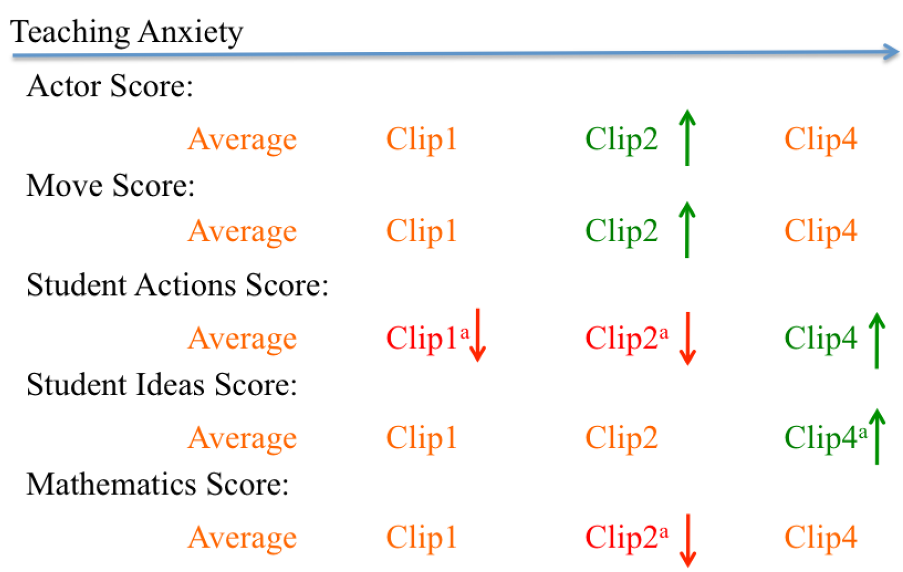


Figure 4.14: Summary visual of teaching anxiety versus individual TRC characteristics scores for the across-clip average, Clip 1, Clip 2 and Clip 4.

4.16). As I mention in the limitations below, one potential explanation for this lack of significance could be related to methodological considerations. However, the increase in word count with an increase in MKT score could be related to the instrument and "teacher lust." Namely, the MKT instrument has items that are mostly focused on common content knowledge and some specialized mathematical knowledge items. It could be that it is more of a proxy for specialized content knowledge rather than pedagogical content knowledge. Thus it is likely more of an indication of an individual's content strength. It is possible that these individuals, who are scored higher in MKT, experience "teacher lust" when they hear students answers and are excited about discussing the mathematics. Hence, their responses become lengthier.

Beliefs category 1 has no associations with word count or aggregate TRC scores and, even at the level of individual TRC characteristics (see Figure 4.17) has associations with only Clip 2 and average *mathematics* scores. It is possible that this lack of associations is a methodological issue again. Recall that this beliefs category was comprised of several sub-categories ("Math as a tool", "Extrinsic versus intrinsic motivation", "Teacher Control", and "Correct Answers") and therefore covers many different types of beliefs. It is possible that these broader beliefs are not ones that influence how teachers manage students' responses.

Beliefs category 2 also had very few associations of note. LIke raw MKT, an increase in this beliefs category (which indicates an increase in enjoyment of and confidence in mathematics) is associated with an increase in word count. I would hypothesizes that, similar to this phenomenon with the MKT score, it is possible that teachers who enjoy and are more confident in mathematics might also enjoy talking more about the mathematics. At the level of the individual TRC characteristics, this beliefs category is positively associated with aspects of teacher responses to Clips 1 and 2 but negatively associated with the *student ideas* score for teacher responses to Clip 4. It is possible, due to the nature of the apparently incorrect answer in Clip 4, that teachers who are more confident in mathematics might also feel more confident in inferring why the student made the mistake he did, even though his answer does not provide this information. Hence, teacher responses then start to veer further from the main *student idea*.

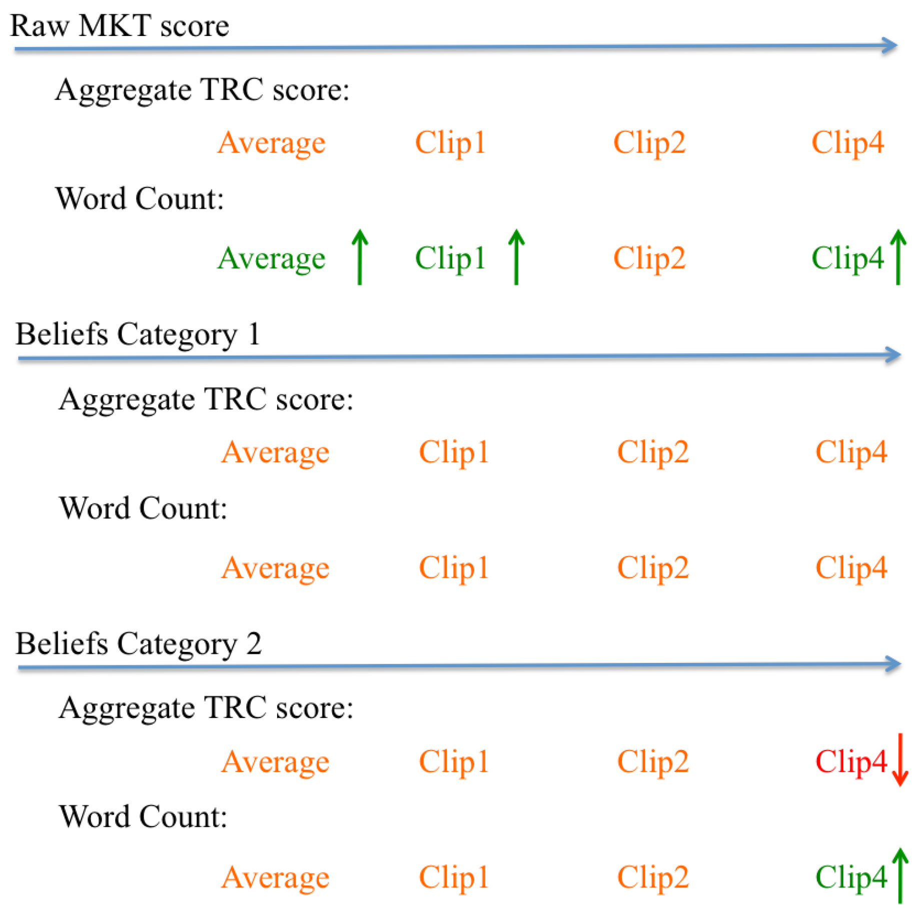


Figure 4.15: Summary visual of MKT, Beliefs category 1 and Beliefs category 2 versus aggregate TRC and word count for the across-clip average, Clip 1, Clip 2 and Clip 4.

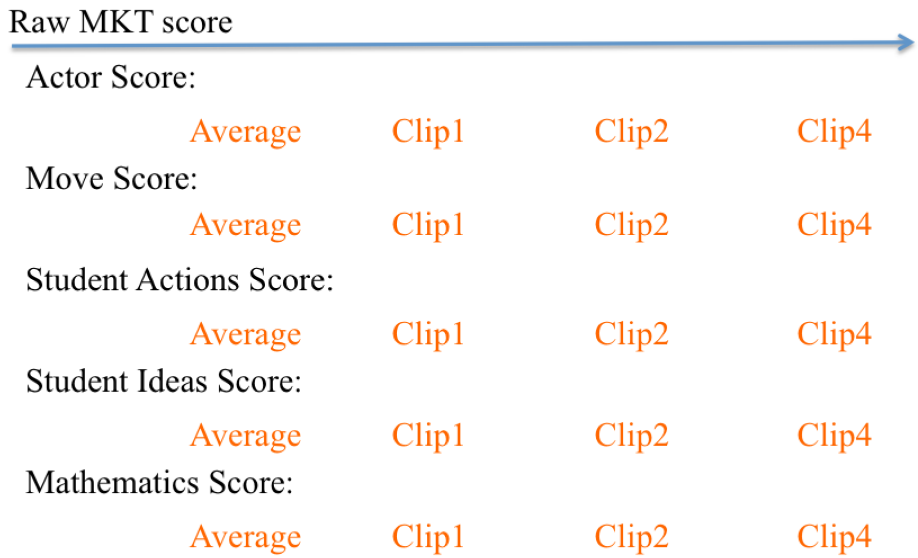


Figure 4.16: Summary visual of raw MKT score versus individual TRC characteristics scores for the across-clip average, Clip 1, Clip 2 and Clip 4.

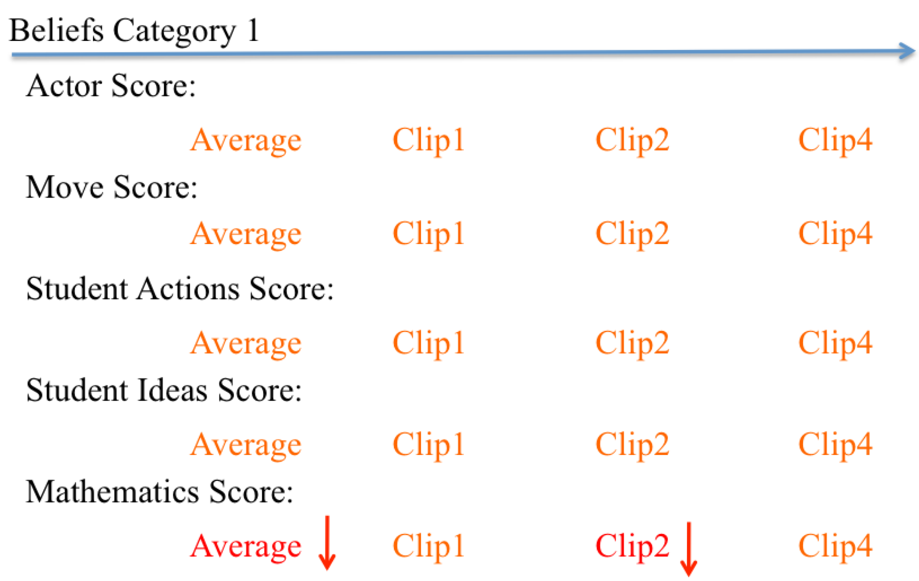


Figure 4.17: Summary visual of Beliefs category 1 versus individual TRC characteristics scores for the across-clip average, Clip 1, Clip 2 and Clip 4.

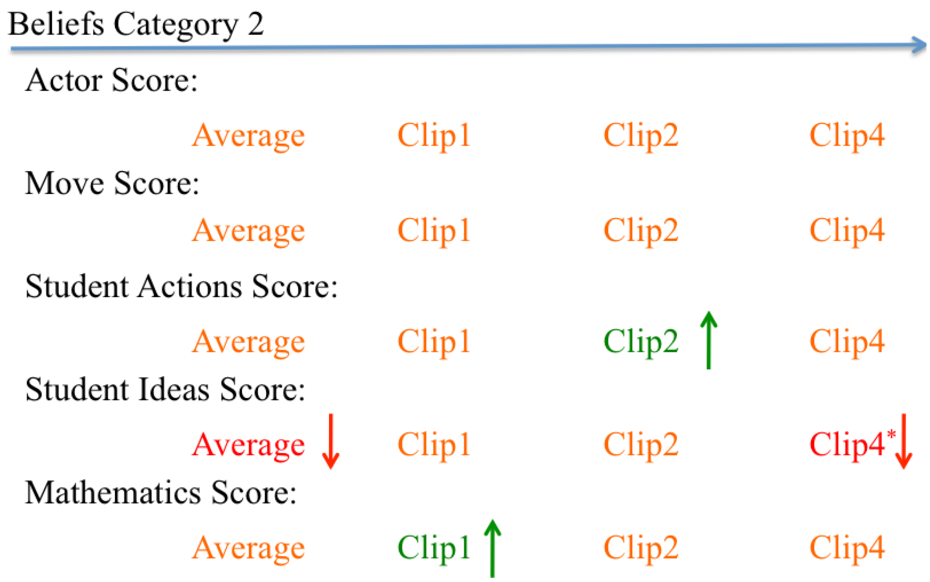


Figure 4.18: Summary visual of Beliefs category 2 versus individual TRC characteristics scores for the across-clip average, Clip 1, Clip 2 and Clip 4.

Finally, of the eight internal resources variables measures years teaching and age show the most association with various aspects of teachers' responses to Clips 1 and 2 (see Figures4.19, 4.20 and 4.21). Overall, years teaching has positive associations several aspects of teachers responses to Clip 2 and to the *mathematics* score of Clip 1 responses. In general, age is also positively associated with aspects of teacher responses to both Clips 1and 2. Out of a possible 56 associations (8 across the aggregate TRC scores, 8 across word count and 40 at the individual TRC characteristics level) there were 18 (about 32%) that were not zero.

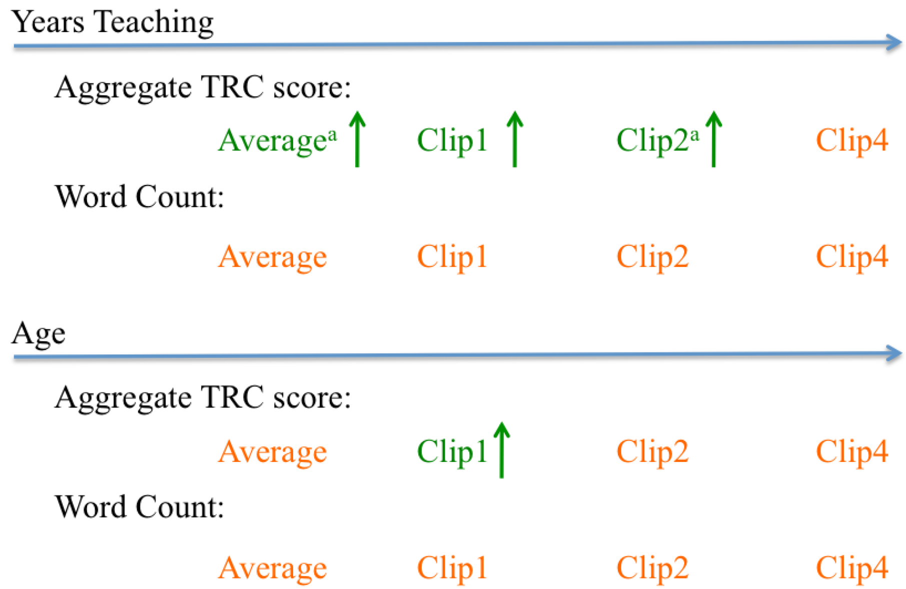


Figure 4.19: Summary visual of years teaching and age versus aggregate TRC and word count for the across-clip average, Clip 1, Clip 2 and Clip 4.

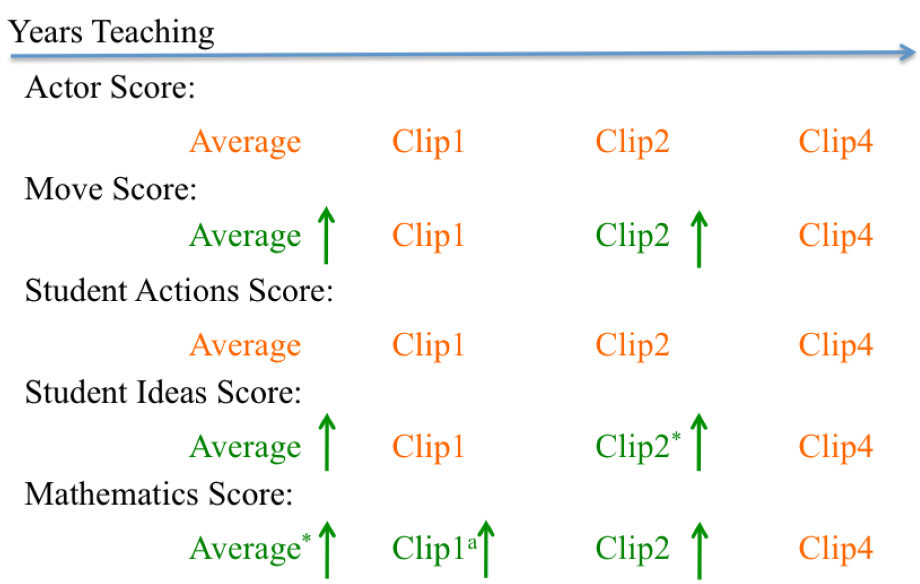


Figure 4.20: Summary visual of years teaching versus individual TRC characteristics scores for the across-clip average, Clip 1, Clip 2 and Clip 4.

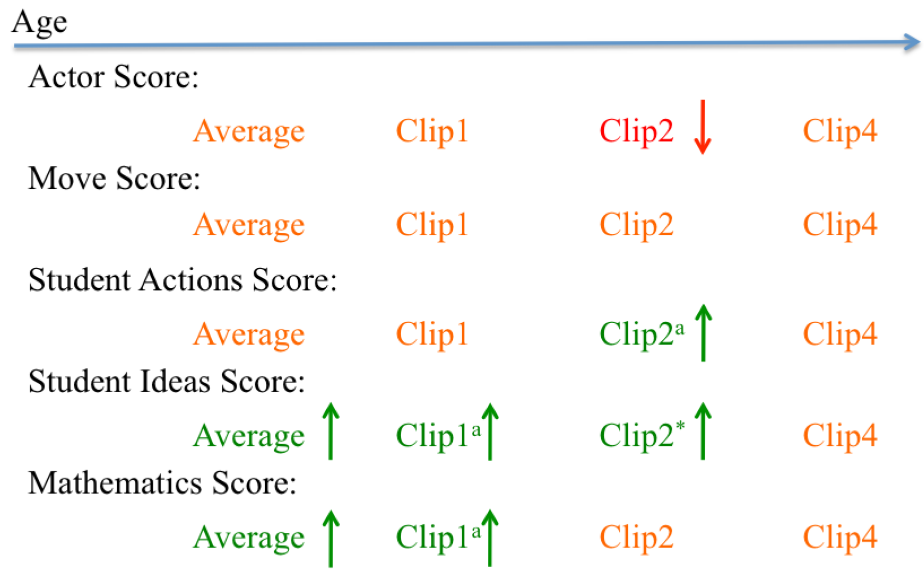


Figure 4.21: Summary visual of age versus individual TRC characteristics scores for the across-clip average, Clip 1, Clip 2 and Clip 4.

In general, the eight internal resources had the most associations with aspects of teacher responses to Clip 2. Out of a possible 56 associations (1 with aggregate TRC score, 1 with word count, and 5 with the individual TRC characteristics for each of the eight resources) there were 16 associations (or almost 29%) that were non-zero and almost evenly split between positive (nine) and negative (seven) associations. The number of associations with Clip 1 teacher responses was close (15 out of 56, or about 27%) with, again, an almost even split between positive (seven) and negative (eight) associations. In contrast, there were only seven associations of note for Clip 4 (or about 12.5%) and most of these were positive (five of the seven).

Though there were fewer associations across the emotional scales, there were again more associations between emotional reactions and aspects of teacher responses to Clip 2 (4 out of 21 possible, or about 19%). Clip 1 was again a close second with three associations (about 14%), while Clip 4 only had one association (or about 5%) across all possible associations with emotional reactions. In other words, the measures used in this study for emotional reactions and internal resources seemed to be the most helpful for understanding aspects of teacher responses to Clips 2 and 1, rather than Clip 4.

Though the lack of results suggest many refinements for the instruments, this pattern across the types of student responses also suggests that future designs of the teaching simulation that vary the student answers in strategic ways might provide different kinds of insights. In particular, it could be that different kinds of internal resources are more or less helpful in responding to particular types of student responses (such as the more incomplete apparently incorrect student answer in Clip 4).

Limitations

Reviewing these findings raises some limitations and several avenues for design improvements in future iterations. In particular, as I consider the various instruments used to measure the internal resources variables, I reflect on both what the instruments captured and how they captured this information.

As I mentioned in the description of the methods, there are no paper-and-pencil instruments that comprehensively assess the MKT in the specific mathematical domain covered in the teaching simulation. Hence, the items used were taken from the closest related content, specifically Algebra. It is therefore possible that the MKT captured by the paper-and-pencil assessment is significantly different than the MKT needed to respond to the student clips. Additionally, it could be that assessing MKT in this way is indicative of its use in reflective competence versus action-related competence. Indeed, this is the part of the argument that Knievel, Lindmeier and Heinze (2015) put forth in support for their video-vignette computer based assessment.

In this study, the beliefs instrument I used captured teachers beliefs about the nature of mathematics and the teaching and learning mathematics (beliefs category 1) as well as teachers' personal enjoyment and confidence in mathematics (beliefs category 2). Category 1 included beliefs about "Math as a tool", "Extrinsic versus intrinsic motivation", "Teacher Control", and "Correct Answers" which touch on some relevant facets of beliefs about mathematics and its learning and teaching. It is possible that the original instrument from which the items were pulled has some limitations including covering beliefs that may not be as crucial in shaping how teachers' respond to student thinking. With regard to beliefs category 2, which had no relationship to any of the teacher responses characteristics in any of the three student responses, it is likely that a more relevant measure would be of teachers' enjoyment and confidence in teaching mathematics rather than of just mathematics. This is a theoretical distinction that Kunter et al. (2008) are careful to point out and explain,

> given the clich of the highly knowledgeable, but pedagogically untalented, mathematics or science teacher who is fascinated by the subject of instruction, but would prefer not to have to interact with students, the topics-vs. activity-specific [enthusiasm] distinction seems particularly applicable to teaching (see also Shulman, 1987) (p. 470).

Hence, in future iterations, two important consideration will be: (1) which beliefs about mathematics teaching and learning are most relevant to assess with respect to managing students' responses and additionally (2) which beliefs about enjoyment and confidence in teaching mathematics should be assessed. A final consideration about the beliefs instruments speaks again to broader methodological debates. Specifically, there are likely inherent issues with asking teachers to provide self-reports about their beliefs (i.e., assessing beliefs through introspection). It is possible that beliefs might be better conceptualized as tacit and more authentically captured through breach experiments (in which beliefs come to light when they are violated or breached).

In considering the measures of anxiety, it is possible that the teaching anxiety questionnaire was not specific enough to the tasks of teaching relevant for managing students. Additionally, questions similar to those I posed with respect to assessing participants' emotional reactions arise when considering the measures of anxiety that I used; specifically, that perhaps assessing anxiety in-the-moment, through physiological measures rather than introspection could provide more accurate measures of teachers' anxiety.

Conclusions and Implications

This study sought to explore various factors that might help researchers better understand how teachers respond to students and in particular, to provide empirical evidence to warrant future investigations into the role of affect, particularly anxiety, in teaching. It also raised many methodological issues and questions that will need to be addressed as scholars explore the dynamic, action-related competencies of teaching. Of importance to teacher educators is the finding that anxiety does, indeed, play a role in shaping teachers' responses to students. In particular, it implies that teacher education may need to consider pedagogies of enactment that are likely to decreases teachers' anxiety while developing their self-efficacy and action-related competencies. Additionally, it suggests that teacher candidates might need support in developing emotional-motivational skills that are likely to impact not only their instructional quality but also their longevity in the profession.

CHAPTER V

Conclusion

In this dissertation, I explored how mathematics teachers manage students' responses in-the-moment during whole-class instruction. In my first paper I described a special case of this phenomenon when a student has provided an answer that the teacher perceives as incorrect, an "apparently incorrect" student response. After providing an illustrative vignette, I discussed four potential consequences for students' learning. Namely, that how a teacher manages students' apparently incorrect responses has consequences for: what mathematics students learn, what students learn about what it means to do mathematics, who can learn mathematics and what it means to make mistakes. Despite these consequences, research shows that in most US classrooms students' answers are still simply evaluated as right or wrong. In light of this reality, I unpacked the difficulties and complexities involved in managing students' responses to illustrate why something as seemingly simple as responding to students is actually incredibly difficult to do and therefore not often observe being skillfully enacted in classrooms. Specifically, I conceptualized managing students' response as a competency and described the various cognitive, affective and motivational factors that are likely to play an important part in a teacher's ability to do this work.

In my second paper, I looked at how teachers responded to both apparently correct and incorrect student answers in a teaching simulation early algebraic content. In this data I found trends in teacher responses that seemed to hold across both types of student answers as well as interesting differences across the two types. In general, regardless of whether a student answer was apparently correct or incorrect, teacher responses had a tendency to want to engage the same student in some kind of follow-up. Teacher responses across both types of student answers were also fairly responsive—as captured by the way teacher responses picked up a student's language and the extend to which a student might recognize their idea in what is being discussed. This result is encouraging since it seems to suggest that most teachers do make efforts to take-up what students are saying and engage at least the student who has spoken in conversation, rather defaulting to lecturing (or some other form of mathematical monologue).

What was less encouraging are the results about the teacher response characteristics that differed depending on whether the student answer was apparently incorrect or correct. Here I found that most teachers, even in this simulated instructional context, exhibited evaluative patterns around student responses. When a student answer was apparently incorrect, the majority of teacher responses employed moves clearly aimed at getting the student to correct their mistake. In contrast, when the student answer was apparently correct, teacher responses sought elaboration from the student on some aspect of their idea.

What is interesting is that there was a subset of the teachers in my sample of 24 that did not have these same clear patterns. In particular, when looking across their responses to the three different student answers there was not a clear pattern in the features of their responses to apparently correct versus apparently incorrect student answers. In particular, their responses to apparently incorrect student answers seemed to suspend judgment or evaluation and instead, sought, in general, to get more information from the student. Though the ultimate goal of instruction is for students to have correct knowledge, there is something of value to be gleaned from this subset of teachers. In particular, by suspending judgement in their responses to students, these teacher responses potentially provide space for students in the class to do the intellectual work and have more agency in developing mathematical knowledge and determining what is right and wrong. These teacher responses have more potential to be productive for student learning.

In consider the features of this subset of teacher responses and the other results of this paper, some points of leverage for teacher education emerge. In particular, most teachers are able to provide productive responses to student answers that are visibly correct but struggle in crafting responses to apparently incorrect answers. This indicates that it might be beneficial to spend time exploring incorrect student thinking within mathematics content and methods courses. Additionally, these explorations of student work could engage teachers (as other researchers have suggested) in looking carefully at the student thinking for both what a student does and does not understand. These claims of student understanding, however, should be supported by evidence from the student answer. Essentially, mathematics teachers could benefit from more opportunities to explore incorrect student thinking in ways that intentionally develop in them a more cautious disposition. By engaging with student work with a focus on both what students seem to understand and using evidence to support claims about these understandings, teacher could become more discerning in the inferences they make and recognize when and how to suspend judgment about the incorrectness of a student answer.

In addition to suggesting the development of particular dispositions towards student answers and thinking, the results of this paper also provide some recommendations for how teachers might respond to students. Specifically, there were particular moves (such as elaborate and justify) that teachers could be encouraged to utilize with apparently incorrect responses. In particular, the results indicate that teachers have little difficulty taking-up student's language and ideas in their responses but might need more support in developing a larger and more productive repertoire of moves to employ when a student answer is apparently incorrect. Teacher education efforts could again take-up this charge by providing opportunities for teachers to brainstorm alternative moves to incorrect student answers and then to practice utilizing these new moves in low-stakes but high-fidelity simulations of responding to student answers.

In my third paper, I turned my attention from the rich nuances and details of the teacher responses to exploring what factors might be related to features of these teachers' responses. Specifically, I explored how both teachers' emotional reactions to the student answers in the simulation and their various internal resources (e.g., knowledge, beliefs, anxiety) might be associated with aspects of their responses to students. In particular I explore how the word count, aggregate Teacher Responses Coding scheme (TRC) score and individual TRC category scores might be associated with the teachers' emotional reactions and internal resources.

Overall, there were few significant correlations in the results, which I hypothesize is in part due to methodological rather than conceptual limitations. When looking at the correlations that were significant as well as those with the potential to become significant in a much larger sample, a few notable patterns emerged. Specifically, teachers who reported feeling more in control after hearing the student answers in Clips 1 and 2 (the apparently correct and the first apparently incorrect answers), gave shorter responses. Additionally, one of the few potential associations between a teachers' sense-of-control and their responses was a negative relationship with the mathematics score for Clip 4 responses.

In essence, though a sense of control is likely to reflect a teacher's sense of self-efficacy and could, in theory, support a teacher in responding more productively, this results suggests that in some cases this initial reaction might actually be misleading. As I mentioned in the discussion of that paper, it is likely that upon hearing the student answer teachers can easily identify what the student did incorrectly and feel confident. However, this is likely a false sense of confidence since in their responses they had difficulty identifying *why* the student was making a mistake and seemed to struggle in determining what mathematics to follow-up on. Essentially, self-efficacy or confidence could be a double-edge sword and future work could explore the conditions that make it an affordance or a hinderance. Otherwise, it seems that either student answers do not evoke much emotional reactions from teachers and/or participants are not able to accurately self-assess their emotional state.

In exploring the potential relationships between features of teachers' responses and their internal resources again very few significant correlations occurred in this sample. Noteworthy nonetheless are the associations that did emerge when looking at state anxiety and teaching anxiety in particular. These results do suggest that anxiety can, like confidence, be a double-edge sword. With regard to anxiety, it would be worth exploring whether teachers perceive particular instructional situations as threatening or challenging since this might explain the mix of positive and negative associations I found.

These particular results can also be used to inform teacher education. The potentially detrimental effects of anxiety for both teachers (e.g., stress has been linked to an increased risk of heart disease in the general population and anxiety has been linked to burn-out in teachers) and students (e.g., through the possibly problematic ways this might impact the instruction they receive) might give teacher education programs that fully embrace the apprenticeship model pause. Though novices can gain much from these experiences (as suggested in part by the positive associations found in this study between years of teaching and features of teachers' responses), the preparation and support they receive to navigate the stress and complexity of real classroom teaching warrants closer attention. It could be that novices would be better prepared for these opportunities and their accompanying anxiety through carefully structured approximations of practice Grossman, Compton, et al. (2009).

In general, the work presented here—like much of the work in teacher education more broadly—has hopefully served to paint a more complete picture of the complexity and challenges of mathematics teaching in general and managing students' responses in particular. By continuing to expand the methodologies utilized in education research and considering teaching more holistically–including the affective components of the work that are often ignored—progress can be made more rapidly in understanding teaching and improving teacher education.

APPENDICES

APPENDIX A

Comparison of TRC models

In order to confirm that the final results were indicative of existing relationships between variables rather than simply a consequence of the quantification choices I made, I compared the results of two models. In one model, I used the quantification scheme described in the paper (e.g., for the actor category: 5 points for same student, 3 point for whole class and 0 points for teacher) and in a second model I reduced the codes to a low and a high (0/1) group. Below, I briefly discuss the binary grouping for each of the five TRC categories and provide a short rationale for the groupings within each category.

In the *actor* category *same student* and *whole class* were grouped together in the high or 1 group and *teacher* into the low or 0 group. In general, and in the specific context of the student response clips used in this study, student learning is more likely to happen if the teacher is not the only one doing the intellectual work. In the *recognition-student actions* category I defined the high group as the *explicit* code and the low group as *implicit, not,* and *explicit-incorrect*. In the *recognition-student ideas* category I defined the high group as the *core* code and the low group as *peripheral, CNI, Other* and *NA*. Students will be more likely to recognize their thinking in a teacher response if the teacher response uses the students exact words and stays core to the students idea. Conceptually, responsive teaching is more likely to be accomplished through teacher responses that would be coded *explicit* for *recognition-student actions* and *core* for *recognition-student ideas* (as opposed to the other codes in these categories). In the *moves* category *justify, allow, elaborate, collect* and *connect* were grouped together in the high or 1 group and *clarify, monitor, repeat, evaluate, literal, validate, adapt, correct* and *dismiss* into the low or 0 group. The five moves in the high category, as opposed to those in the low category, have greater potential to engage students in more cognitively demanding work. Finally, in the *mathematics* the codes *CNI-core* and *CNI* were grouped together in the high or 1 group and *Core-MP1, core, peripheral, peripheral-beyond, CNI-imprecise, peripheral-incorrect, other, other-incorrect*, and *non-math* into the low or 0 group. Teacher responses that are coded *CNI-core* or *CNI* do not give away so much of the mathematics that students have little intellectual work to do, are possibly closer to the mathematical point underlying the student thinking and do not have language issues (i.e., imprecision) that could be confusing for students.

In order to compare these models, I conducted Pearsons correlation coefficient analyses for the quantified codes model (model 1) and Chi-squared and Logit analyses for the binary model (model 2). I then compared the resulting coefficients' signs (positive and negative) and p-values. A detailed discussion of differences between the model can be found in Appendix B.

		Clip 1 Actor (model 1/ model 2)	Clip 1 Move (model 1/ model 2)	Clip 1 Student Actions (model 1/ model 2)	Clip 1 Student Ideas (model 1/ model 2)	Clip 1 Mathematics (model 1/ model 2)
Clip 1 Actor (model 1)	Pearson Correlation	-	-	-	-	-
	Sig. (2-tailed)	-	-	-	-	-
Clip 1 Actor (model 2)	Chi-squared, X^2	-	-	-	-	-
	p	-	-	-	-	-
	Symmetric Measure (nominal by nominal) Phi's	-	-	-	-	-
Clip 1 Moves (model 1)	Pearson Correlation	0.807^{**}	-	-	-	-
	Sig. (2-tailed)	0.000	-	-	-	-
Clip 1 Moves (model 2)	X^2	10.752^{**}	-	-	-	-
	p	0.001	-	-	-	-
	Symmetric Measure	0.669^{**}	-	-	-	-
Clip 1 Student Actions (model 1)	Pearson Correlation	0.129	0.316	-	-	-
	Sig. (2-tailed)	0.549	0.133	-	-	-
Clip 1 Student Actions (model 2)	Chi-squared, X^2	0.046	1.386	-	-	-
	p	0.831	0.239	-	-	-
	Symmetric Measure	0.044	0.240	-	-	-
Clip 1 Student Ideas (model 1)	Pearson Correlation	0.641^{**}	0.843^{**}	0.321	-	-
	Sig. (2-tailed)	0.001	0.000	0.126	-	-
Clip 1 Student Ideas (model 2)	Chi-squared, X^2	4.444^{*}	1.510	0.000	-	-
	p	0.035	0.219	1.000	-	-
	Symmetric Measure	0.430^{*}	0.251	0.000	-	-
Clip 1 Mathematics (model 1)	Pearson Correlation	0.187	0.112	-0.337	0.345	-
	Sig. (2-tailed)	0.381	0.604	0.108	0.099	-
Clip 1 Mathematics (model 2)	Chi-squared, X^2	5.344^{*}	0.120	3.311	10.971^{**}	-
	p	0.021	0.729	0.069	0.001	-
	Symmetric Measure	0.472^{*}	0.071	-0.371	0.676^{**}	-

Figure A.1: Comparison of Pearson's correlation coefficients from model 1 and Chi-squared results from model 2 for Clip 1 TRC coding.

		Clip 2 Actor (model 1/ model 2)	Clip 2 Move (model 1/ model 2)	Clip 2 Student Actions (model 1/ model 2)	Clip 2 Student Ideas (model 1/ model 2)	Clip 2 Mathematics (model 1/ model 2)
Clip 2 Actor (model 1)	Pearson Correlation	-	-	-	-	-
	Sig. (2-tailed)	-	-	-	-	-
Clip 2 Actor (model 2)	Chi-squared, X^2	-	-	-	-	-
	p	-	-	-	-	-
	Symmetric Measure (nominal by nominal) Phi's	-	-	-	-	-
Clip 2 Moves (model 1)	Pearson Correlation	0.519**	-	-	-	-
	Sig. (2-tailed)	0.009	-	-	-	-
Clip 2 Moves (model 2)	X^2	3.789	-	-	-	-
	p	0.052	-	-	-	-
	Symmetric Measure	0.397	-	-	-	-
Clip 2 Student Actions (model 1)	Pearson Correlation	-0.103	-0.349	-	-	-
	Sig. (2-tailed)	0.630	0.094	-	-	-
Clip 2 Student Actions (model 2)	Chi-squared, X^2	0.505	0.800	-	-	-
	p	0.477	0.371	-	-	-
	Symmetric Measure	-0.145	-0.183	-	-	-
Clip 2 Student Ideas (model 1)	Pearson Correlation	-0.083	0.494*	-0.179	-	-
	Sig. (2-tailed)	0.701	0.014	0.402	-	-
Clip 2 Student Ideas (model 2)	Chi-squared, X^2	0.087	5.919*	0.084	-	-
	p	0.769	0.015	0.772	-	-
	Symmetric Measure	0.060	0.497*	-0.059	-	-
Clip 2 Mathematics (model 1)	Pearson Correlation	0.267	0.453*	-0.129	0.242	-
	Sig. (2-tailed)	0.208	0.026	0.548	0.254	-
Clip 2 Mathematics (model 2)	Chi-squared, X^2	5.344*	2.517	1.343	0.621	-
	p	0.021	0.113	0.247	0.431	-
	Symmetric Measure	0.472*	0.324	-0.237	0.161	-

Figure A.2: Comparison of Pearson's correlation coefficients from model 1 and Chi-squared results from model 2 for Clip 2 TRC coding.

		Clip 4 Actor (model 1/ model 2)	Clip 4 Move (model 1/ model 2)	Clip 4 Student Actions (model 1/ model 2)	Clip 4 Student Ideas (model 1/ model 2)	Clip 4 Mathematics (model 1/ model 2)
Clip 4 Actor (model 1)	Pearson Correlation	-	-	-	-	-
	Sig. (2-tailed)	-	-	-	-	-
Clip 4 Actor (model 2)	Chi-squared, X^2	-	-	-	-	-
	p	-	-	-	-	-
	Symmetric Measure (nominal by nominal) Phi's	-	-	-	-	-
Clip 4 Moves (model 1)	Pearson Correlation	0.952**	-	-	-	-
	Sig. (2-tailed)	0.000	-	-	-	-
Clip 4 Moves (model 2)	X^2	5.714*	-	-	-	-
	p	0.017	-	-	-	-
	Symmetric Measure	0.488*	-	-	-	-
Clip 4 Student Actions (model 1)	Pearson Correlation	0.061	0.118	-	-	-
	Sig. (2-tailed)	0.778	0.581	-	-	-
Clip 4 Student Actions (model 2)	Chi-squared, X^2	0.020	0.229	-	-	-
	p	0.899	0.633	-	-	-
	Symmetric Measure	0.029	-0.098	-	-	-
Clip 4 Student Ideas (model 1)	Pearson Correlation	0.441*	0.591**	0.321	-	-
	Sig. (2-tailed)	0.031	0.002	0.127	-	-
Clip 4 Student Ideas (model 2)	Chi-squared, X^2	1.731	9.455**	1.386	-	-
	p	0.810	0.002	0.239	-	-
	Symmetric Measure	0.269	0.628**	0.240	-	-
Clip 4 Mathematics (model 1)	Pearson Correlation	0.542**	0.636**	0.339	0.775**	-
	Sig. (2-tailed)	0.006	0.001	0.105	0.000	-
Clip 4 Mathematics (model 2)	Chi-squared, X^2	7.059**	5.445**	3.601	6.331*	-
	p	0.008	0.020	0.058	0.012	-
	Symmetric Measure	0.542**	0.476**	0.387	0.514*	-

Figure A.3: Comparison of Pearson's correlation coefficients from model 1 and Chi-squared results from model 2 for Clip 4 TRC coding.

		Clip 1 Actor (model 1/ model 2)	Clip 1 Move (model 1/ model 2)	Clip 1 Student Actions (model 1/ model 2)	Clip 1 Student Ideas (model 1/ model 2)	Clip 1 Mathematics (model 1/ model 2)
Clip 1 SAM Valence (model 1)	Pearson Correlation	-0.024	-0.209	-0.045	-0.263	-0.048
	Sig. (2-tailed)	0.913	0.326	0.834	0.214	0.825
Clip 1 SAM Valence (model 2)	Logit coefficient (B)	-0.038	-0.126	-0.053	-0.263	0.005
	Sig. (2-tailed)	0.878	0.600	0.825	0.294	0.984
Clip 1 SAM Arousal (model 1)	Pearson Correlation	0.099	0.098	-0.135	0.076	-0.005
	Sig. (2-tailed)	0.646	0.649	0.531	0.723	0.982
Clip 1 SAM Arousal (model 2)	Logit coefficient (B)	0.152	-0.013	-0.143	-0.022	-0.096
	Sig. (2-tailed)	0.500	0.951	0.513	0.916	0.657
Clip 1 SAM Control (model 1)	Pearson Correlation	0.078	0.076	0.122	0.089	0.189
	Sig. (2-tailed)	0.719	0.724	0.571	0.680	0.375
Clip 1 SAM Control (model 2)	Logit coefficient (B)	0.067	0.098	0.129	0.162	0.129
	Sig. (2-tailed)	0.762	0.650	0.553	0.458	0.553

Figure A.4: Comparison of Pearson's correlation coefficients from model 1 and Logit coefficients results from model 2 for Clip 1 TRC coding and SAM scales.

		Clip 2 Actor (model 1/ model 2)	Clip 2 Move (model 1/ model 2)	Clip 2 Student Actions (model 1/ model 2)	Clip 2 Student Ideas (model 1/ model 2)	Clip 2 Mathematics (model 1/ model 2)
Clip 2 SAM Valence (model 1)	Pearson Correlation	0.265	0.197	-0.061	-0.024	0.122
	Sig. (2-tailed)	0.211	0.357	0.775	0.910	0.569
Clip 2 SAM Valence (model 2)	Logit coefficient (B)	0.233	0.091	-0.064	-0.029	0.029
	Sig. (2-tailed)	0.468	0.712	0.800	0.905	0.905
Clip 2 SAM Arousal (model 1)	Pearson Correlation	-0.109	0.139	-0.077	0.212	0.072
	Sig. (2-tailed)	0.614	0.517	0.720	0.320	0.739
Clip 2 SAM Arousal (model 2)	Logit coefficient (B)	0.242	0.024	-0.068	0.228	0.131
	Sig. (2-tailed)	0.413	0.912	0.762	0.307	0.545
Clip 2 SAM Control (model 1)	Pearson Correlation	0.144	0.335	-0.067	0.126	0.157
	Sig. (2-tailed)	0.503	0.110	0.756	0.558	0.464
Clip 2 SAM Control (model 2)	Logit coefficient (B)	0.292	0.488	-0.019	0.143	0.258
	Sig. (2-tailed)	0.274	0.121	0.936	0.541	0.295

Figure A.5: Comparison of Pearson's correlation coefficients from model 1 and Logit coefficients results from model 2 for Clip 2 TRC coding and SAM scales.

		Clip 4 Actor (model 1/ model 2)	Clip 4 Move (model 1/ model 2)	Clip 4 Student Actions (model 1/ model 2)	Clip 4 Student Ideas (model 1/ model 2)	Clip 4 Mathematics (model 1/ model 2)
Clip 4 SAM Valence (model 1)	Pearson Correlation	0.265	0.197	-0.061	-0.024	0.122
	Sig. (2-tailed)	0.211	0.357	0.775	0.910	0.569
Clip 4 SAM Valence (model 2)	Logit coefficient (B)	0.233	0.091	-0.064	-0.029	0.029
	Sig. (2-tailed)	0.468	0.712	0.800	0.905	0.905
Clip 4 SAM Arousal (model 1)	Pearson Correlation	-0.109	0.139	-0.077	0.212	0.072
	Sig. (2-tailed)	0.614	0.517	0.720	0.320	0.739
Clip 4 SAM Arousal (model 2)	Logit coefficient (B)	0.242	0.024	-0.068	0.228	0.131
	Sig. (2-tailed)	0.413	0.912	0.762	0.307	0.545
Clip 4 SAM Control (model 1)	Pearson Correlation	0.144	0.335	-0.067	0.126	0.157
	Sig. (2-tailed)	0.503	0.110	0.756	0.558	0.464
Clip 4 SAM Control (model 2)	Logit coefficient (B)	0.292	0.488	-0.019	0.143	0.258
	Sig. (2-tailed)	0.274	0.121	0.936	0.541	0.295

Figure A.6: Comparison of Pearson's correlation coefficients from model 1 and Logit coefficients results from model 2 for Clip 4 TRC coding and SAM scales.

		State Anxiety	Trait Anxiety	Teaching Anxiety	raw MKT score	Beliefs Category 1	Beliefs Category 2	Years Teaching	Age	Word Count
Clip 1 Actor (model 1)	Pearson Correlation	-0.187	-0.138	-0.085	0.051	-0.121	-0.020	0.146	0.036	**-0.453***
	Sig. (2-tailed)	0.382	0.519	0.695	0.812	0.574	0.925	0.495	0.866	**0.026**
Clip 1 Actor (model 2)	Logit coefficient (B)	-0.065	-0.028	-0.006	0.035	-0.048	-0.009	0.029	0.010	**-0.023**
	Sig. (2-tailed)	0.360	0.597	0.715	0.756	0.758	0.824	0.467	0.745	**0.070**
Clip 1 Move (model 1)	Pearson Correlation	-0.277	-0.081	-0.205	-0.029	-0.004	0.048	0.132	0.145	-0.279
	Sig. (2-tailed)	0.190	0.706	0.336	0.892	0.984	0.824	0.537	0.499	0.186
Clip 1 Move (model 2)	Logit coefficient (B)	-0.115	-0.061	-0.022	0.027	0.207	-0.009	0.002	-0.002	-0.007
	Sig. (2-tailed)	0.125	0.247	0.185	0.805	0.212	0.804	0.962	0.957	0.497
Clip 1 Student Actions (model 1)	Pearson Correlation	-0.368	-0.227	-0.349	-0.216	-0.082	-0.126	0.034	0.079	0.214
	Sig. (2-tailed)	0.077	0.286	0.095	0.311	0.702	0.559	0.873	0.712	0.316
Clip 1 Student Actions (model 2)	Logit coefficient (B)	-0.133	-0.058	-0.028	-0.130	-0.062	-0.024	0.006	0.012	0.012
	Sig. (2-tailed)	0.089	0.275	0.108	0.304	0.687	0.541	0.866	0.698	0.310
Clip 1 Student Ideas (model 1)	Pearson Correlation	-0.377	-0.207	-0.226	-0.059	-0.018	-0.048	0.211	**0.339**	-0.283
	Sig. (2-tailed)	0.069	0.331	0.288	0.783	0.933	0.823	0.323	**0.105**	0.180
Clip 1 Student Ideas (model 2)	Logit coefficient (B)	-0.183	-0.103	-0.016	0.024	0.139	-0.041	0.064	**0.070***	-0.011
	Sig. (2-tailed)	0.039	0.077	0.318	0.827	0.379	0.302	0.130	**0.049**	0.325
Clip 1 Mathematics (model 1)	Pearson Correlation	-0.341	-0.486*	-0.141	0.086	0.325	-0.155	0.367	0.404	-0.279
	Sig. (2-tailed)	0.102	0.016	0.510	0.689	0.121	0.471	0.077	0.050	0.187
Clip 1 Mathematics (model 2)	Logit coefficient (B)	-0.089	-0.152*	-0.007	0.043	0.199	-0.042	0.056	0.051	-0.014
	Sig. (2-tailed)	0.217	0.024	0.644	0.698	0.224	0.293	0.193	0.132	0.211

Figure A.7: Comparison of Pearson's correlation coefficients from model 1 and Logit coefficients results from model 2 for Clip 1 TRC coding and internal resources measure.

		State Anxiety	Trait Anxiety	Teaching Anxiety	raw MKT score	Beliefs Category 1	Beliefs Category 2	Years Teaching	Age	Word Count
Clip 2 Actor (model 1)	Pearson Correlation	0.136	0.027	0.275	0.058	-0.264	0.159	0.013	-0.246	-0.635**
	Sig. (2-tailed)	0.528	0.899	0.193	0.788	0.212	0.457	0.950	0.246	0.001
Clip 2 Actor (model 2)	Logit coefficient (B)	0.090	0.032	0.037	0.031	-0.276	0.023	0.011	-0.034	-0.037*
	Sig. (2-tailed)	0.319	0.619	0.153	0.814	0.262	0.616	0.815	0.360	0.036
Clip 2 Move (model 1)	Pearson Correlation	-0.033	-0.024	0.263	-0.044	-0.224	0.005	0.346	0.195	-0.467*
	Sig. (2-tailed)	0.878	0.910	0.214	0.840	0.292	0.981	0.098	0.361	0.021
Clip 2 Move (model 2)	Logit coefficient (B)	-0.055	-0.005	0.010	-0.022	-0.024	-0.023	0.072	0.049	-0.009
	Sig. (2-tailed)	0.442	0.928	0.535	0.843	0.877	0.557	0.080	0.135	0.306
Clip 2 Student Actions (model 1)	Pearson Correlation	-0.213	-0.133	-0.343	-0.031	0.277	-0.128	0.139	0.352	0.063
	Sig. (2-tailed)	0.317	0.536	0.100	0.886	0.191	0.550	0.516	0.091	0.772
Clip 2 Student Actions (model 2)	Logit coefficient (B)	-0.036	0.021	-0.006	-0.072	0.165	-0.005	-0.002	0.036	0.002
	Sig. (2-tailed)	0.609	0.697	0.696	0.564	0.304	0.895	0.950	0.289	0.786
Clip 2 Student Ideas (model 1)	Pearson Correlation	-0.119	-0.006	0.136	-0.006	-0.082	0.028	.450*	.442*	-0.006
	Sig. (2-tailed)	0.580	0.978	0.526	0.979	0.705	0.898	0.027	0.030	0.978
Clip 2 Student Ideas (model 2)	Logit coefficient (B)	-0.039	-0.001	0.010	-0.003	-0.060	0.005	0.095	0.073	0.000
	Sig. (2-tailed)	0.562	0.977	0.509	0.978	0.690	0.893	0.049	0.043	0.977
Clip 2 Mathematics (model 1)	Pearson Correlation	-0.559**	-0.506*	-0.358	0.125	0.137	-0.316	0.293	0.167	-0.287
	Sig. (2-tailed)	0.004	0.012	0.086	0.561	0.523	0.133	0.164	0.434	0.174
Clip 2 Mathematics (model 2)	Logit coefficient (B)	-0.174*	-0.117*	-0.025	0.021	0.076	-0.050	0.052	0.019	-0.019
	Sig. (2-tailed)	0.046	0.054	0.156	0.847	0.619	0.220	0.183	0.538	0.111

Figure A.8: Comparison of Pearson's correlation coefficients from model 1 and Logit coefficients results from model 2 for Clip 2 TRC coding and internal resources measure.

		State Anxiety	Trait Anxiety	Teaching Anxiety	raw MKT score	Beliefs Category 1	Beliefs Category 2	Years Teaching	Age	Word Count
Clip 4 Actor (model 1)	Pearson Correlation	-0.109	-0.250	-0.165	-0.065	-0.010	-0.175	0.190	0.000	-0.574**
	Sig. (2-tailed)	0.613	0.239	0.442	0.764	0.961	0.414	0.374	0.999	0.003
Clip 4 Actor (model 2)	Logit coefficient (B)	-0.040	-0.064	-0.011	-0.019	0.008	-0.041	0.034	0.002	-0.025*
	Sig. (2-tailed)	0.553	0.235	0.468	0.868	0.960	0.309	0.381	0.954	0.019
Clip 4 Move (model 1)	Pearson Correlation	-0.030	-0.173	-0.038	-0.096	-0.167	-0.075	0.226	0.010	-0.586**
	Sig. (2-tailed)	0.888	0.418	0.859	0.655	0.434	0.728	0.288	0.962	0.003
Clip 4 Move (model 2)	Logit coefficient (B)	-0.031	-0.005	0.003	-0.212	-0.292	0.007	0.015	-0.017	-0.043
	Sig. (2-tailed)	0.690	0.931	0.878	0.118	0.109	0.880	0.712	0.639	0.078
Clip 4 Student Actions (model 1)	Pearson Correlation	0.146	0.261	0.295	0.232	-0.122	0.008	-0.139	-0.194	0.139
	Sig. (2-tailed)	0.497	0.218	0.162	0.275	0.569	0.970	0.517	0.364	0.517
Clip 4 Student Actions (model 2)	Logit coefficient (B)	0.059	0.050	0.012	0.201	-0.030	-0.019	-0.001	-0.010	0.006
	Sig. (2-tailed)	0.393	0.344	0.433	0.155	0.627	0.627	0.981	0.743	0.314
Clip 4 Student Ideas (model 1)	Pearson Correlation	0.223	0.155	0.359	0.027	-0.413*	0.171	0.242	0.073	-0.464*
	Sig. (2-tailed)	0.295	0.469	0.085	0.900	0.045	0.423	0.255	0.733	0.022
Clip 4 Student Ideas (model 2)	Logit coefficient (B)	0.010	0.042	0.015	-0.064	-0.373*	0.018	0.003	-0.016	-0.014
	Sig. (2-tailed)	0.884	0.413	0.337	0.567	0.067	0.637	0.924	0.589	0.083
Clip 4 Mathematics (model 1)	Pearson Correlation	0.066	0.154	0.236	0.068	-0.203	-0.053	0.110	-0.044	-0.381
	Sig. (2-tailed)	0.761	0.471	0.268	0.751	0.341	0.804	0.610	0.837	0.066
Clip 4 Mathematics (model 2)	Logit coefficient (B)	-0.104	-0.037	-0.005	0.026	-0.009	-0.083	0.014	0.005	-0.011
	Sig. (2-tailed)	0.207	0.514	0.746	0.833	0.956	0.095	0.715	0.871	0.208

Figure A.9: Comparison of Pearson's correlation coefficients from model 1 and Logit coefficients results from model 2 for Clip 4 TRC coding and internal resources measure.

APPENDIX B

Discussion and plots of differences between TRC model 1 and 2

In comparing the coefficients and p-values of models 1 and 2 for various variables there are 13 differences that warranted further exploration. Again, differences were flagged when the sign of the coefficients and/or the significance level differed since, in general, the magnitudes of the coefficients cannot be necessarily compared in a meaningful way (especially in the case of Pearsons correlation versus Logit coefficients since the units are, in one case, related to a change in y for a one unit change in x and, in the other case, related to the log-odds for a change from group 0 to group 1).

Relationships within TRC categories

In comparing the Pearsons correlation coefficients of model 1 and Chi-squared results of model 2 in looking at the relationship within the five TRC categories for each clip, there were 6 differences between the models that warranted investigating.

In Clip1, the relationship between *mathematics* and *actor* was positive in both models but not significant in the quantitative model (model 1, Pearsons correlation coefficient p-value = 0.381) and significant in the binary model (model 2, Chi-squared coefficient p-value = 0.021). Similarly, the relationship between *mathematics* and *student ideas* was positive in both models but not significant in the quantitative model (model 1, p-value = 0.099) and significant in the binary model (model 2, p-value = 0.001). In looking at the relationship *move* and *student ideas* though again both coefficients were positive, in this case the coefficient was significant in model 1 (p-value = 0.000) and not significant in model 2 (p-value =0.219).

In Clip 2, the relationship between *mathematics* and *actor* was positive in both models but not significant in the quantitative model (model 1, Pearsons correlation coefficient p-value = 0.208) and significant in the binary model (model 2, Chi-squared coefficient p-value = 0.021). Similarly, the relationship between *mathematics* and *move* was positive in both models but significant in the quantitative model (model 1, p-value = 0.026) and not significant in the binary model (model 2, p-value = 0.113).

In Clip 4, the relationship between *actor* and *student ideas* was positive in both models but significant in the quantitative model (model 1, p-value = 0.031) and not significant in the binary model (model 2, p-value = 0.810).

Relationships between TRC categories and SAM scales

In this group of relationships only one difference stood out. In Clip 2, the relationship between *actor* and Clip 2 SAM Arousal was non-significant in both models but negative in model 1 and positive in model 2. As can be seen in Figure B.1 below, the plots of Clip 2 SAM arousal versus model 1 (quantitative model, top plot) and model 2 (binary model, bottom plot) provide some explanation for this result. In particular, it appears that teachers with Clip 2 responses that were coded *teacher* (0 in the quantitative model) or *same student* (5 in the quantitative model) actually seem to self-assess about the same level of arousal (the average for the 5 participants in the *teacher* group was 4.20 and the average for the 10 participants in the *same student* group was 4.14). In contrast, the participants with Clip 2 responses coded as *whole class* for the actor (a score of 3 in the quantitative model) reported higher arousal (the average for the 5 participants in this group was 7). Thus, the negative trend line in the model 2 plot reflects this slight negative difference in the means of arousal between the 0-score and 5-score groups (average of 4.20 versus 4.14) while in the model 2 plot, the 1-score group (which is a combination of the 3- and 5-score groups of model 1) reflects the higher arousal of teachers whose responses were coded *whole class*.

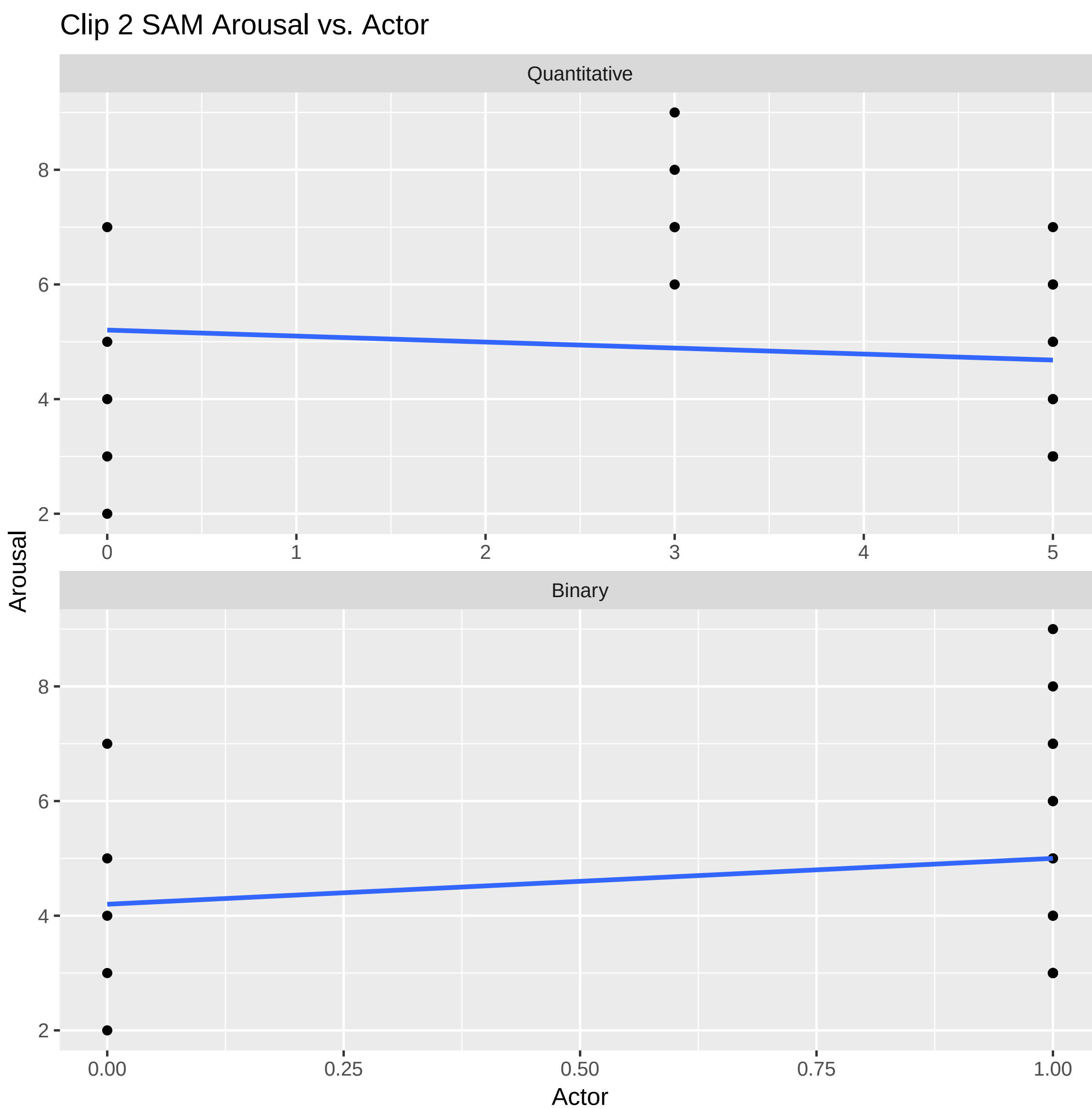


Figure B.1: Clip 2 SAM arousal versus *actor* for model 1 (quantitative model, top plot) and model 2 (binary model, bottom plot).

Relationships between TRC categories and internal resources

In this group of relationships six differences stood out, most of which involved word count as one of the two variables.

In Clip 1, the relationship between *actor* and Clip1 word count was negative in both models but significant in the quantitative model (model 1, Pearsons correlation coefficient p-value = 0.026) and not quite significant in the binary model (model 2, Logit coefficient p-value = 0.070). The plots of Clip 1 word count versus model 1 (quantitative model, top plot) and model 2 (binary model, bottom plot) in Figure B.2 below show this slight difference. The combination of the *whole class* group with the *same student* group results in a trend line that is not as steep in the binary model. This likely explains why this result is not quite significant in model 2 but is significant in model 1. The similarities in these results and plots provide reassurance that, in this case, the result of model 1 is meaningful and not simply a byproduct of the quantification process.

Additionally, the relationship between Clip 1 *student ideas* and age was positive in both models but not significant in the quantitative model (p-value = 0.105) and significant in the binary model (p-value = 0.049). A comparison of the plots (see Figure B.3 below) shows this slight difference between the two models. The similarities in these results and plots suggest that there might exist a positive association between Clip 1 *student ideas* and age.

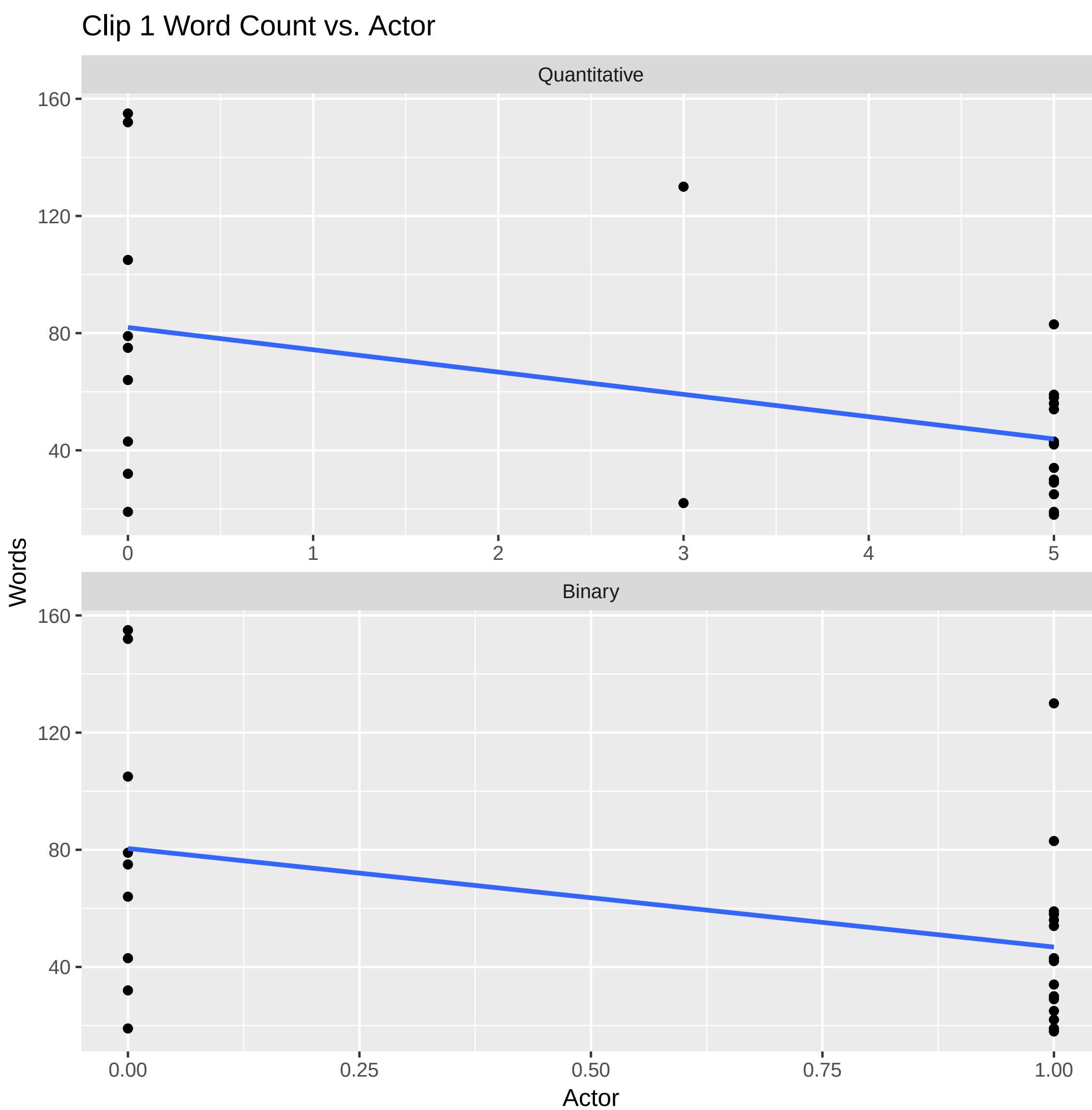


Figure B.2: Clip 1 word count versus *actor* for model 1 (quantitative model, top plot) and model 2 (binary model, bottom plot).

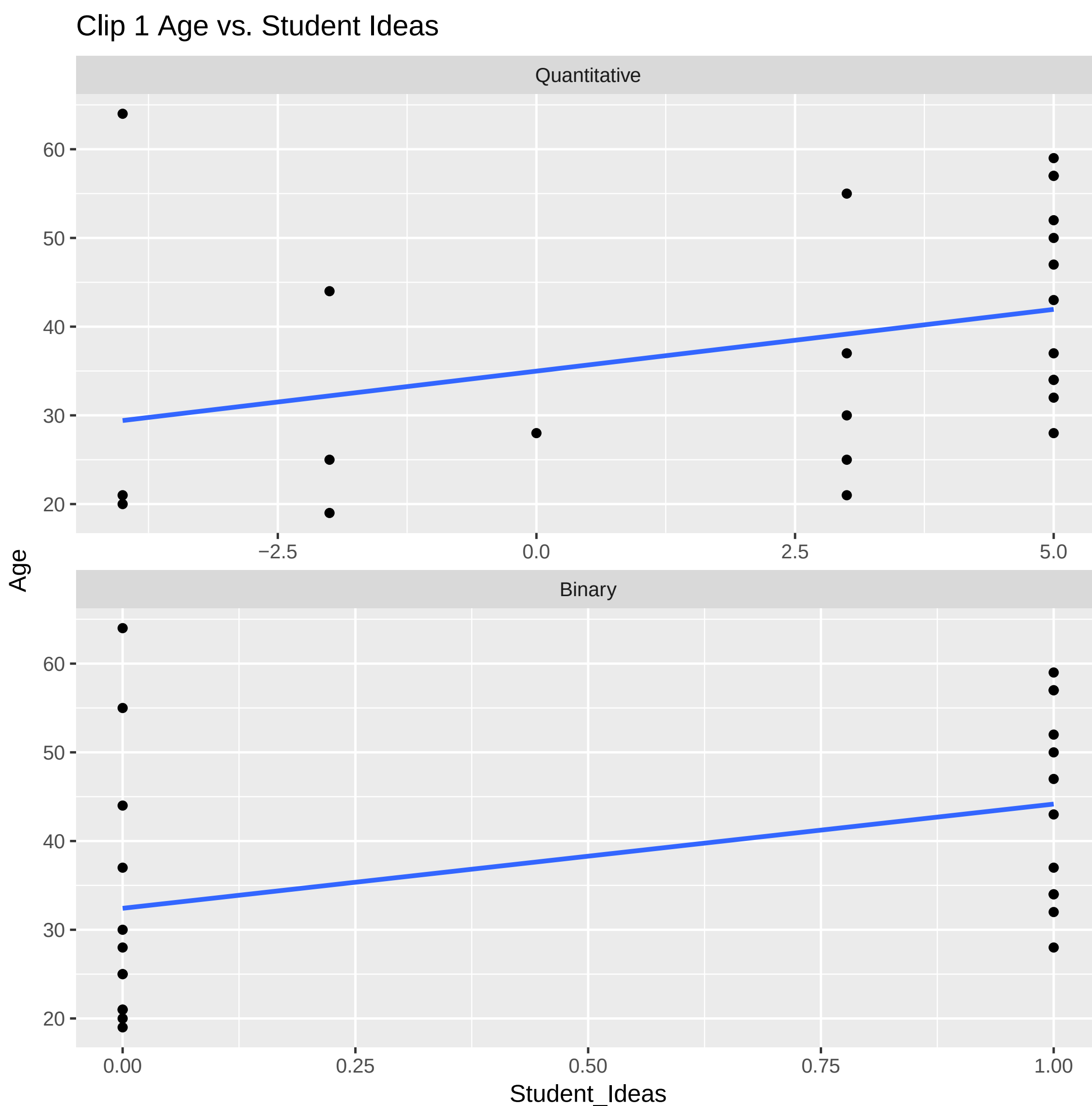


Figure B.3: Age versus Clip 1*student ideas* for model 1 (quantitative model, top plot) and model 2 (binary model, bottom plot).

In Clip 2 the relationship between *moves* and word count was negative in both models but significant in the quantitative model (p-value = 0.021) and not significant in the binary model (p-value = 0.306). As can be seen from the plots of the word count versus *moves* for models 1 and 2, it appears that in collapsing the codes (which meant grouping the -3, 1 and 3-point scoring moves into the low group for the binary model) lowered the average score of the low group and hence, flattened the trend line between the two extremes of the plot (see Figure B.4). The top plot does seem to indicate that a few outlier in the -3 *moves* group might be skewing the results and, indeed, after removing the three outliers (with word counts greater than 120 words) there seems to be no relationship between *moves* and word count (see Figure B.5).

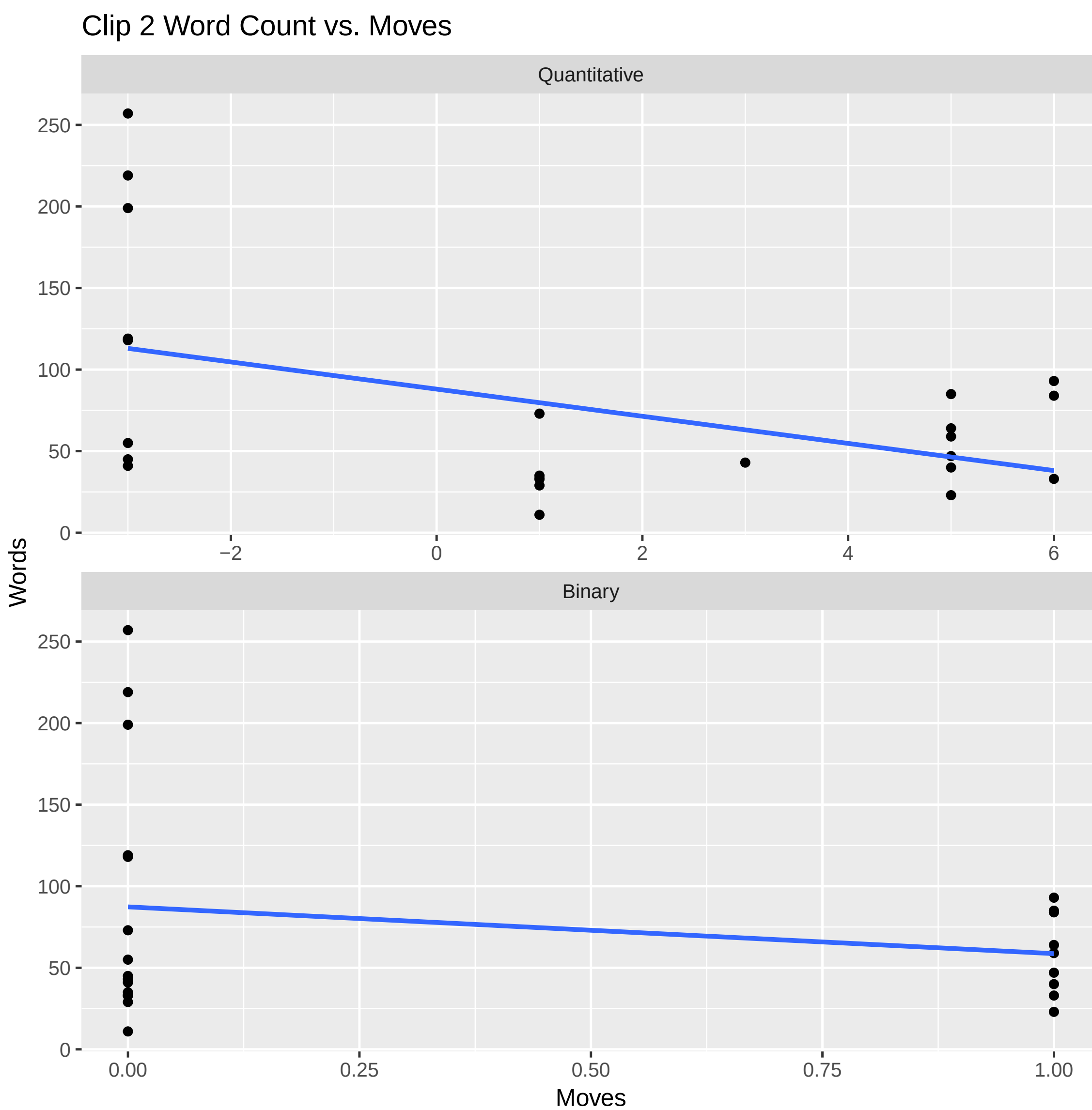


Figure B.4: Clip 2 word count versus *moves* for model 1 (quantitative model, top plot) and model 2 (binary model, bottom plot).

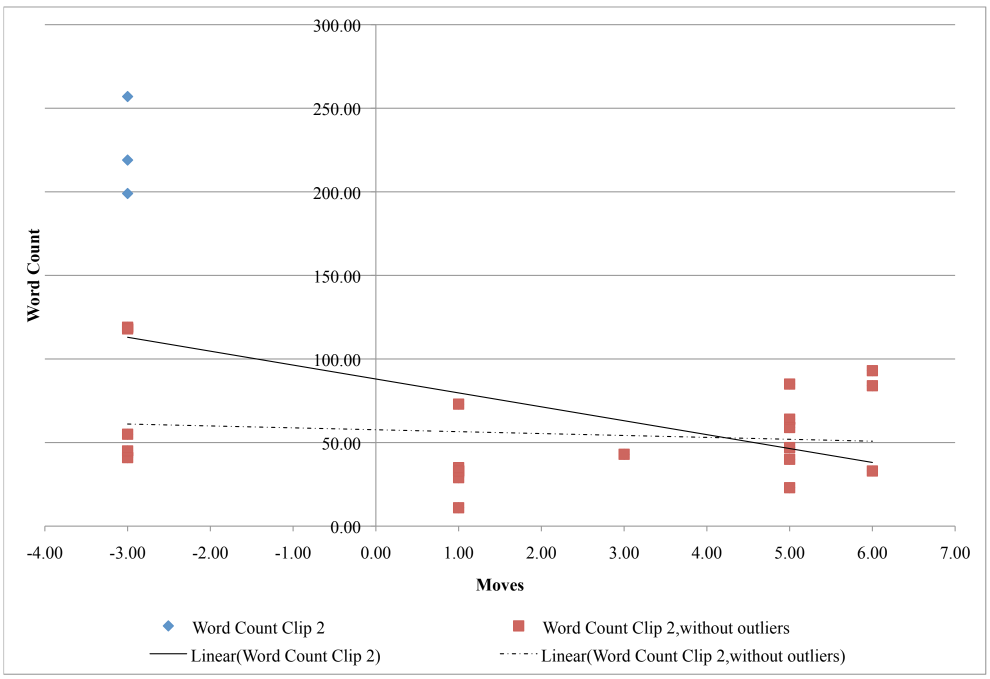


Figure B.5: Word count versus *moves* for model 1 with and without outlier points.

In Clip 4 the relationship between *moves* and word count was negative in both models but significant in the quantitative model (p-value = 0.003) and not quite significant in the binary model (p-value = 0.078). The plots of these models shows these slight differences (see Figure B.6) and, after removing the four outlier responses, a negative trend is still prominent in model 2 (B.7). These plots suggest that the result of model 1 is meaningful and not simply a byproduct of the quantification process or outliers.

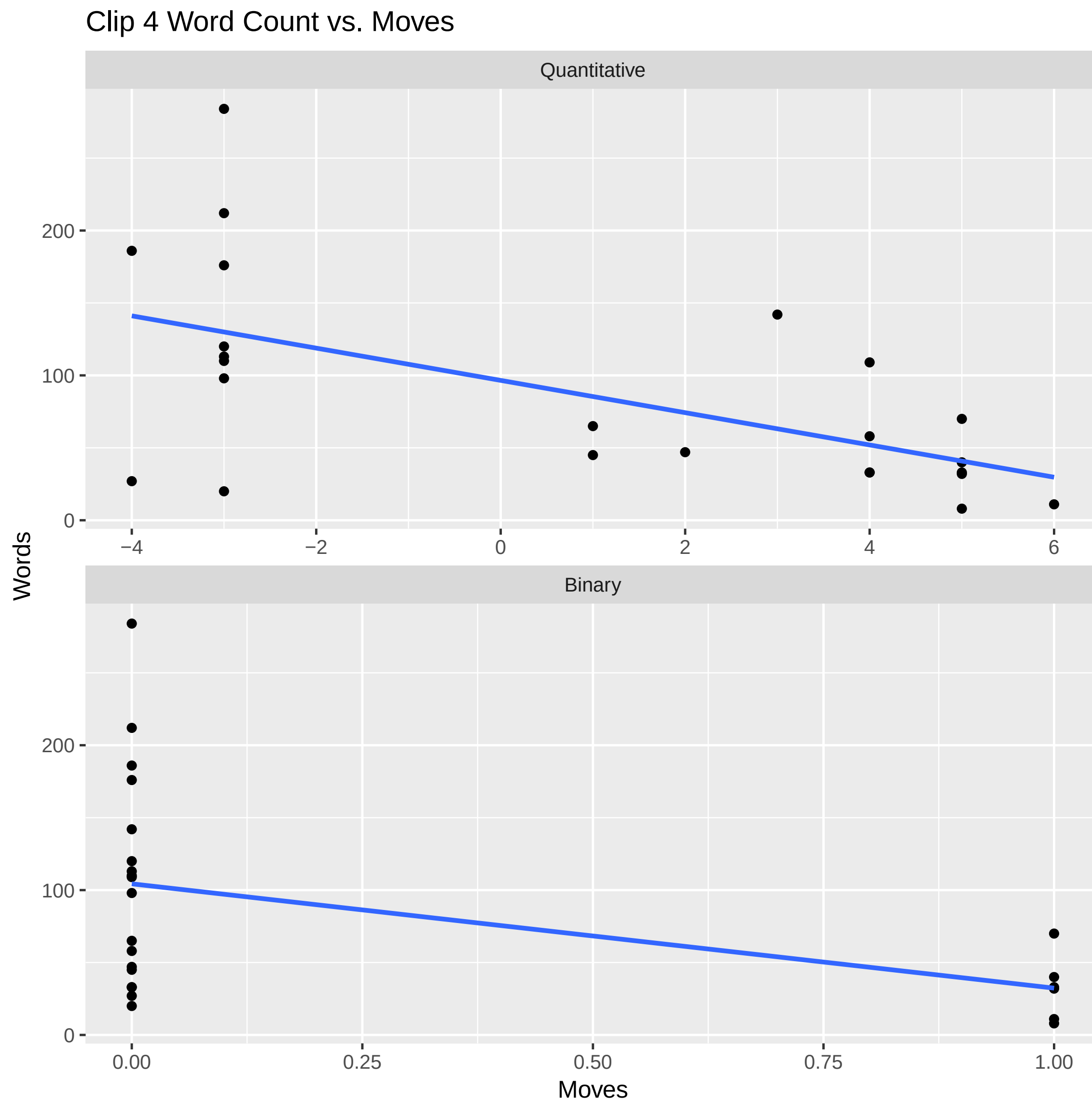


Figure B.6: Clip 4 word count versus *moves* for model 1 (quantitative model, top plot) and model 2 (binary model, bottom plot).

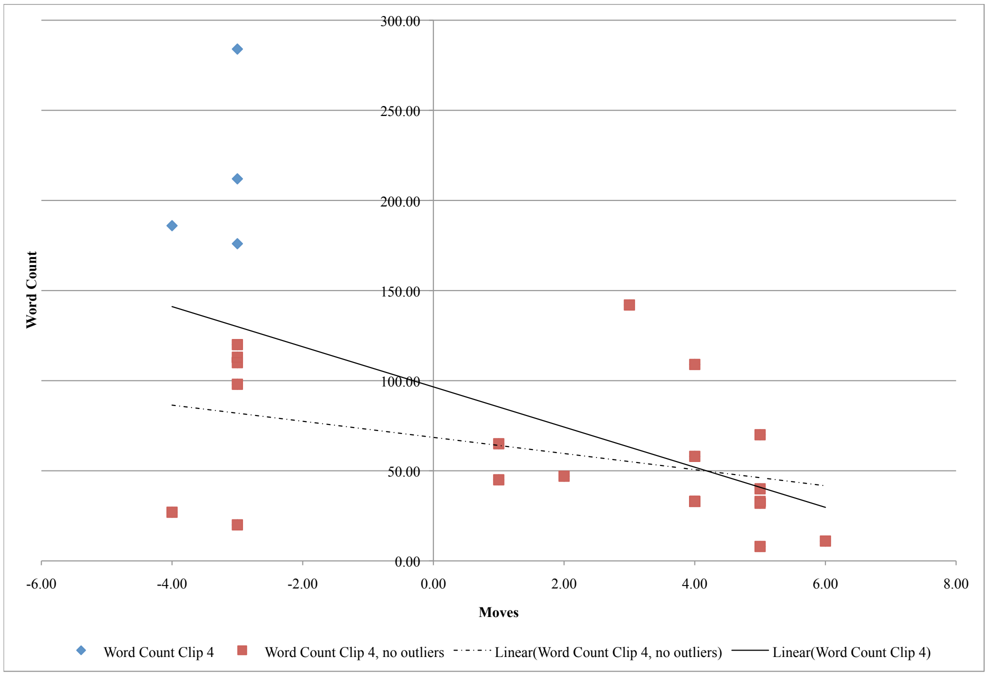


Figure B.7: Clip 4 word count versus *moves* for model 1 with and without outlier points.

Also, in Clip 4 the relationship between *student ideas*and word count was negative in both models but significant in the quantitative model (p-value = 0.022) and not quite significant in the binary model (p-value = 0.083). Again, the plots of this relationship in each model shows this slight different (see Figure B.8). In this case, outliers are not a factor in the quantitative model since these are spread across three of the four *student ideas*categories (see Figure B.9).These plots seem to confirm that the negative association between Clip 4 *student ideas*and word count is not idiosyncratic to the quantitative model or a only a result of outliers.

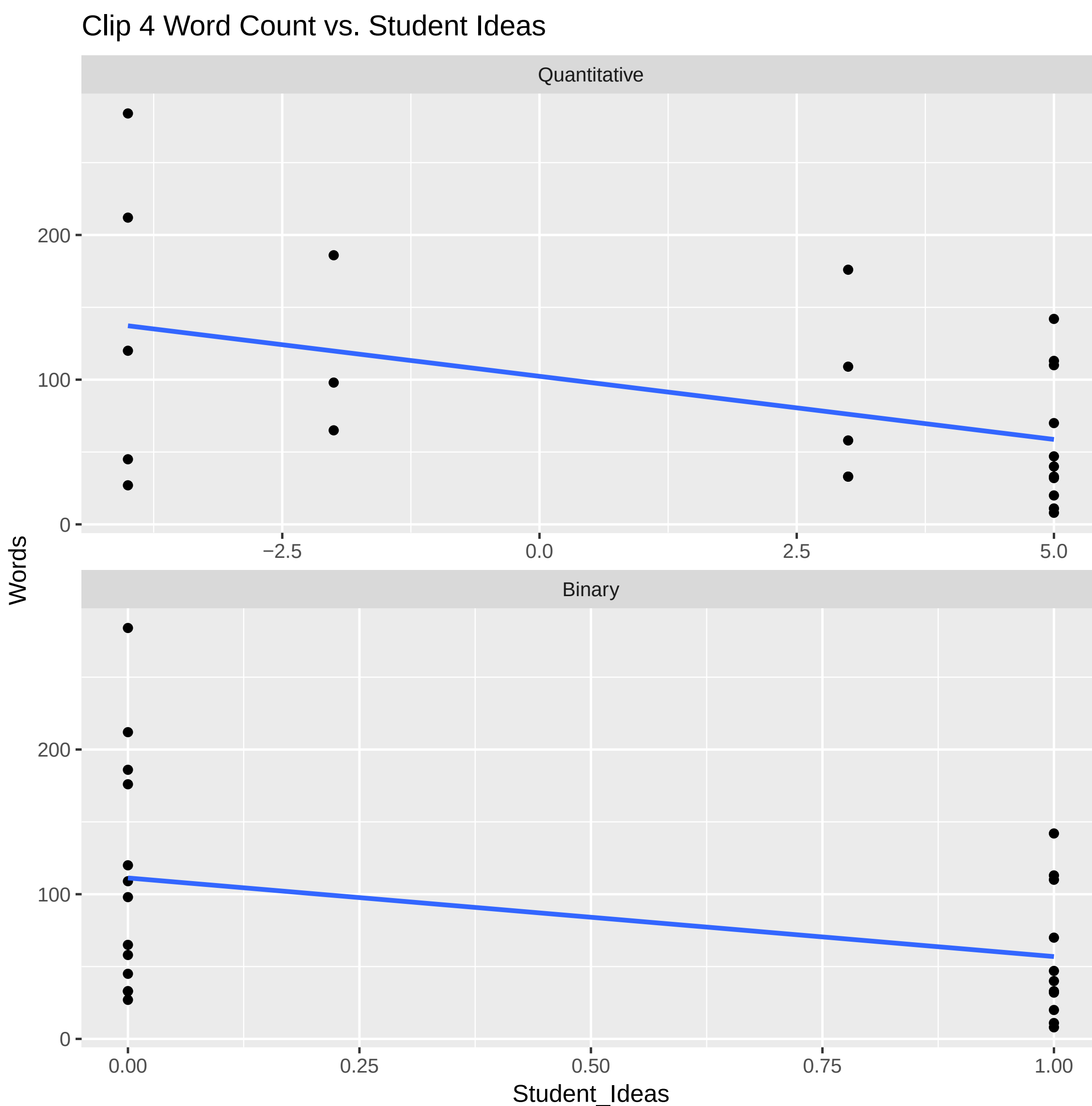


Figure B.8: Clip 4 word count versus *moves* for model 1 (quantitative model, top plot) and model 2 (binary model, bottom plot).

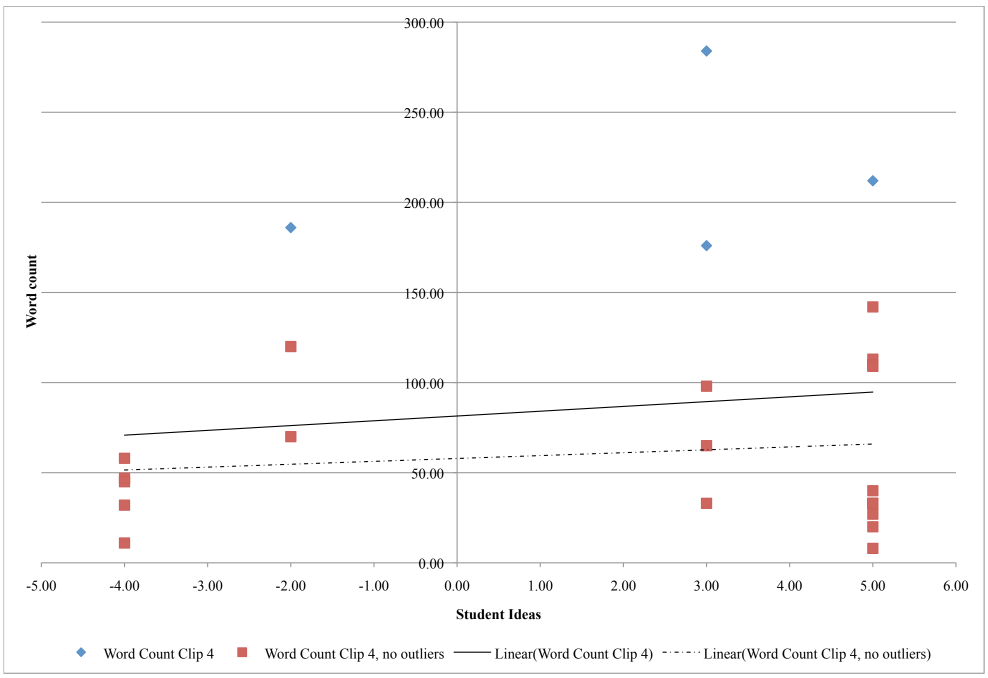


Figure B.9: Clip 4 word count versus *student ideas* for model 1 with and without outlier points.

Finally, in Clip 4 the relationship between *student ideas* and beliefs category 2 was negative in both models but significant in the quantitative model (p-value = 0.045) and not quite significant in the binary model (p-value = 0.067). Plots of beliefs category 2 versus *student ideas* in model 1 and in model 2 (see FigureB.10) indicate that a slight, negative association seems to exits, regardless of the model.

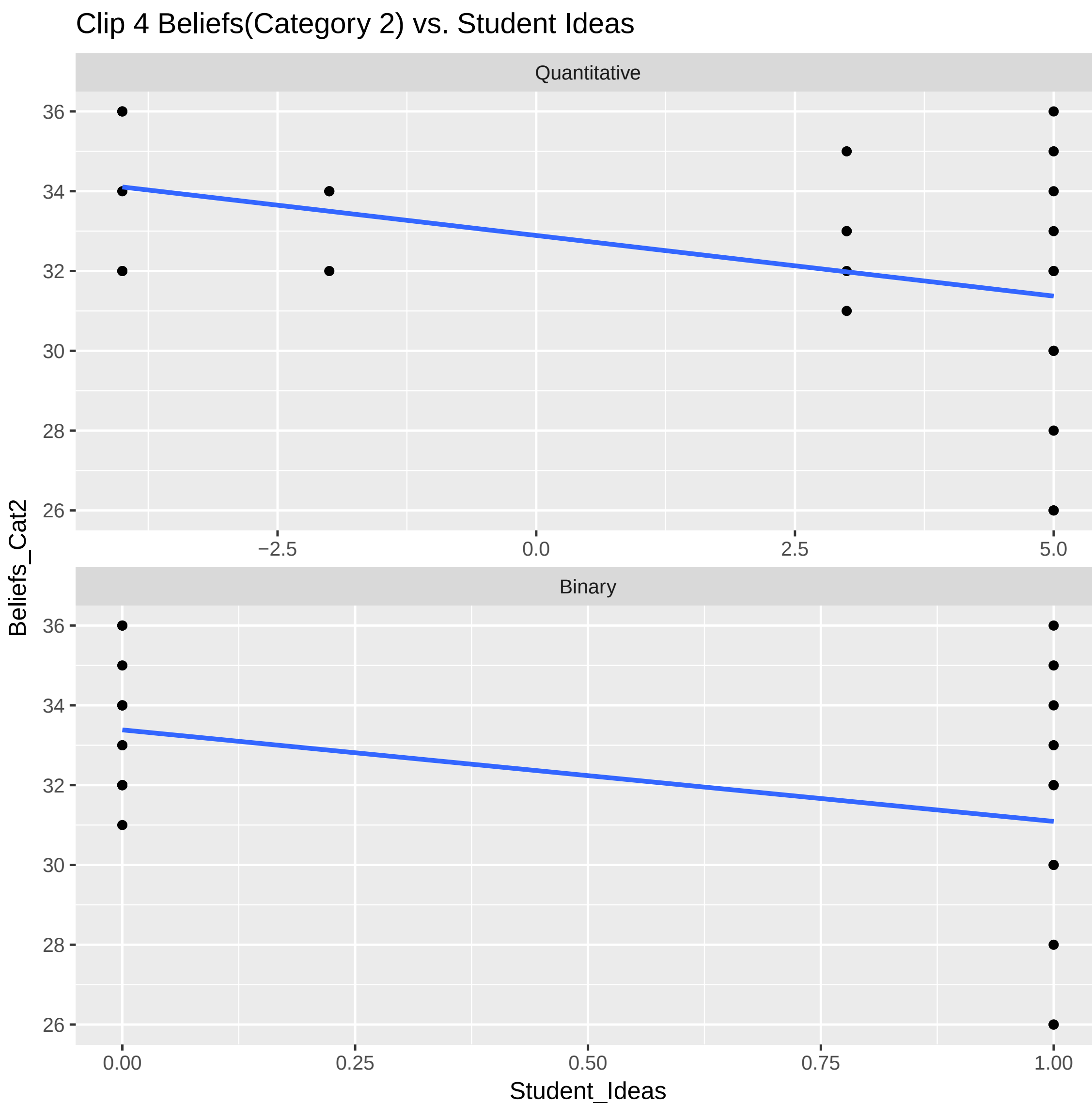


Figure B.10: Beliefs category 2 versus Clip 4 *student ideas* for model 1 (quantitative model, top plot) and model 2 (binary model, bottom plot).

APPENDIX C

TRC code distribution in clips

Table C.1: *Distribution of actor codes in Clips 1, 2, and 4.*

	Same Student	Whole Class/ Other student	Teacher
Clip 1	13	2	9
Clip 2	14	5	5
Clip 4	13	1	10
Total	40	8	24

Table C.2: *Distribution of student actions codes in Clips 1, 2, and 4.*

	Explicit	Implicit	Not	Explicit-Incorrect
Clip 1	14	10	0	0
Clip 2	16	3	3	2
Clip 4	10	2	10	2
Total	40	15	13	4

Table C.3: *Distribution of student ideas codes in Clips 1, 2, and 4.*

	Core	Peripheral	CNI	Other	NA
Clip 1	12	5	1	3	3
Clip 2	11	13	0	0	0
Clip 4	11	5	0	3	5
Total	34	23	1	6	8

Table C.4: *Distribution of moves codes in Clips 1, 2, and 4.*

	Clip 1	Clip 2	Clip 4	Total
Justify	4	2	1	7
Allow	0	1	0	1
Elaborate	9	2	5	16
Collect	0	1	0	1
Connect	0	3	0	3
Clarify	1	0	0	1
Monitor	0	0	4	4
Repeat	2	1	1	4
Evaluate	1	0	1	2
Literal	0	6	2	8
Validate	1	0	0	1
Adapt	3	0	0	3
Correct	0	8	8	16
Dismiss	3	0	2	5

Table C.5: *Distribution of mathematics codes in Clips 1, 2, and 4.*

	Clip 1	Clip 2	Clip 4	Total
CNI-core	5	5	1	11
CNI	10	6	6	22
Core-MP1	0	0	0	0
Core	0	4	4	8
Peripheral	1	5	3	9
Peripheral-beyond	2	0	1	3
CNI-imprecise	4	4	3	11
Peripheral-incorrect	0	0	0	0
Other	1	0	5	6
Other-incorrect	1	0	0	1
Non-math	0	0	1	1

APPENDIX D

SAM scale correlations

Tables D.1 , D.2, and D.3 show the SAM scale correlations for Clips 1, 2, and 4. As mentioned earlier, we would not expect there to be correlations between these scales within a clip as they are meant to measure conceptually different aspects of the construct emotion. The tables confirm that this is the case for these three clips.

Table D.1: *Clip 1 SAM scales: Pearson's correlations coefficients (N = 24).*

Variables	1	2	3
1. Clip1 SAM Valence	–		
2. Clip1 SAM Arousal	−0.244	–	
3. Clip1 SAM Control	−0.36	0.314	–
M	3.708	4.792	6.375
SD	1.756	1.978	1.952

Table D.2: *Clip 2 SAM scales: Pearson's correlations coefficients (N = 24).*

Variables	1	2	3
1. Clip 2 SAM Valence	–		
2. Clip 2 SAM Arousal	−0.128	–	
3. Clip 2 SAM Control	−0.196	0.18	–
M	3.5	4.833	6.208
SD	1.745	1.949	1.841

Table D.3: *Clip 4 SAM scales: Pearson's correlations coefficients (N = 24).*

Variables	1	2	3
1. Clip 4 SAM Valence	–		
2. Clip 4 SAM Arousal	0.086	–	
3. Clip 4 SAM Control	−0.319	0.096	–
M	4.208	5.083	6.75
SD	1.719	2.225	1.726

APPENDIX E

SAM scale language

The SAM scale is designed with visuals in order to avoid some of the potential problems inherent with asking participants to interpret text or words; however, in order to teach participants how to use the scale during the teaching simulation the extremes (1 and 9) and middle (5) were described to participants with both text and images. In order to described the results, I assigned language after-the-fact to the remaining SAM scale positions (2, 3, 4, 6, 7, and 8) and chose language intended to reflect each number's relative position between the neutral point (a 5) and the closest extreme (see Table E.1 below).

Table E.1: *SAM scales language.*

Rating	Valence Scale	Arousal Scale	Control Scale
1	very happy	very aroused/excited	very out-of-control
2	considerably happy	considerably happy	considerably out-of-control
3	somewhat happy	somewhat happy	somewhat out-of-control
4	slightly happy	slightly happy	slightly out-of-control
5	neither happy not unhappy (neutral)	neither excited not calm (neutral)	neither excited not calm (neutral)
6	slightly unhappy	slightly calm	slightly in-control
7	somewhat unhappy	somewhat calm	somewhat in-control
8	considerably unhappy	considerably calm	considerably in-control
9	very unhappy	very calm	very in-control

APPENDIX F

Plots of average SAM scales versus average aggregate TRC scores and average word count.

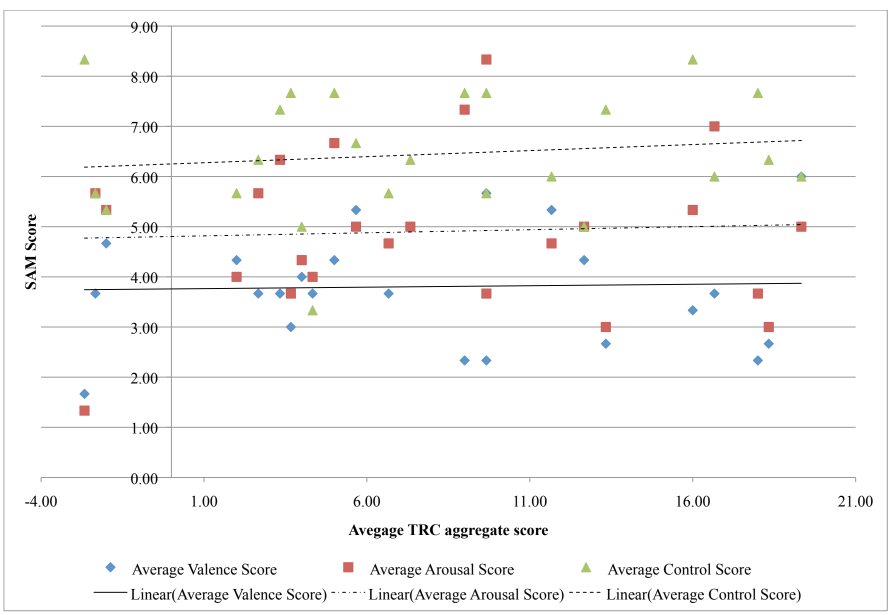


Figure F.1: Plot of average SAM scores versus average aggregate TRC scores.

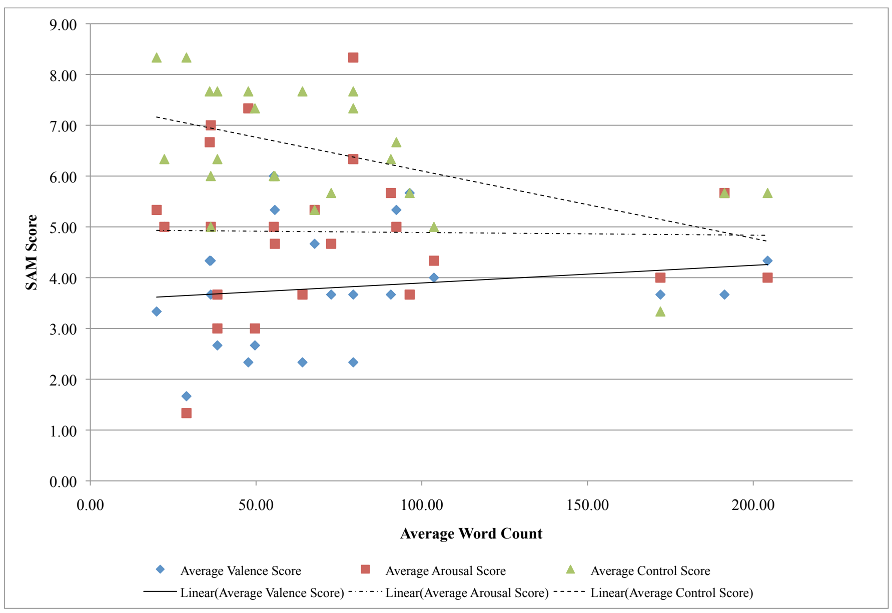


Figure F.2: Plot of average SAM scores versus average word count.

APPENDIX G

Histograms of Eight Internal Resources Measures

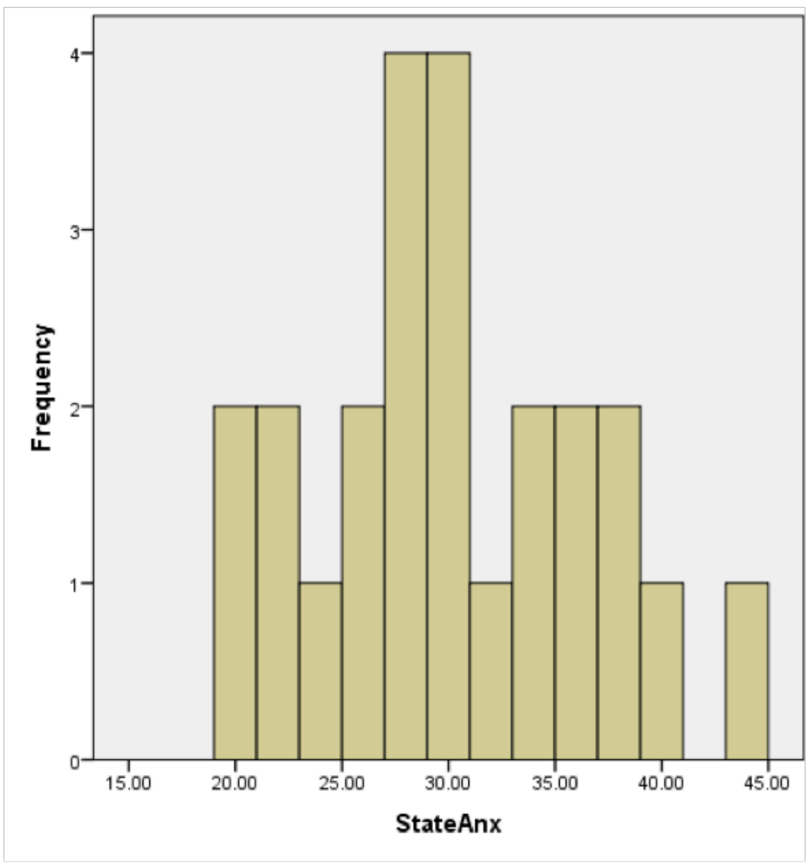


Figure G.1: Histogram of state anxiety score (N=24)

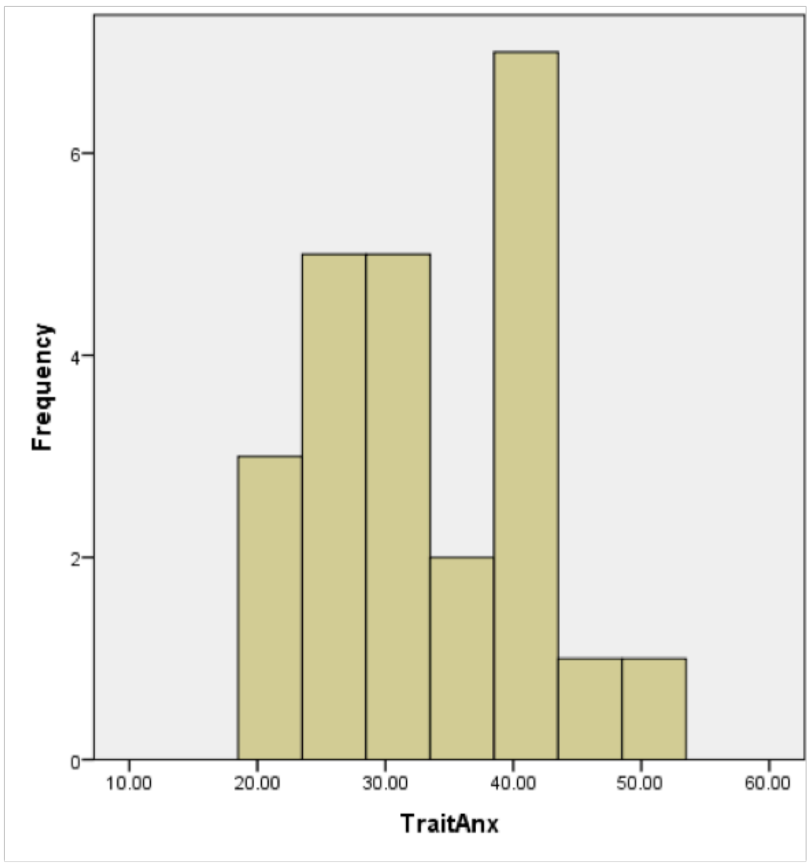


Figure G.2: Histogram of trait anxiety score (N=24)

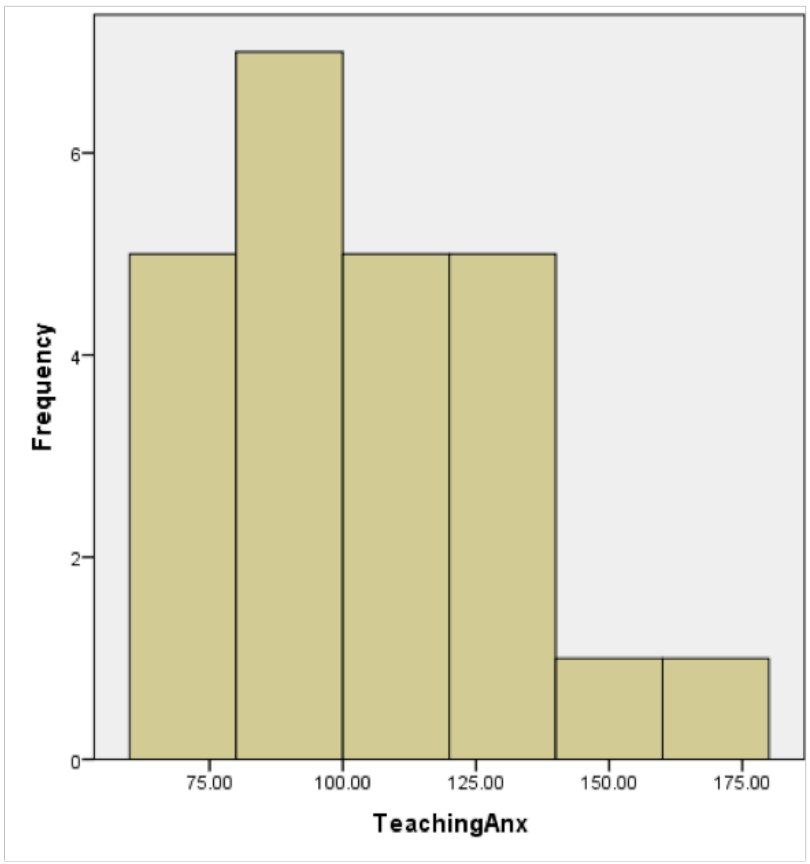


Figure G.3: Histogram of teaching anxiety score (N=24)

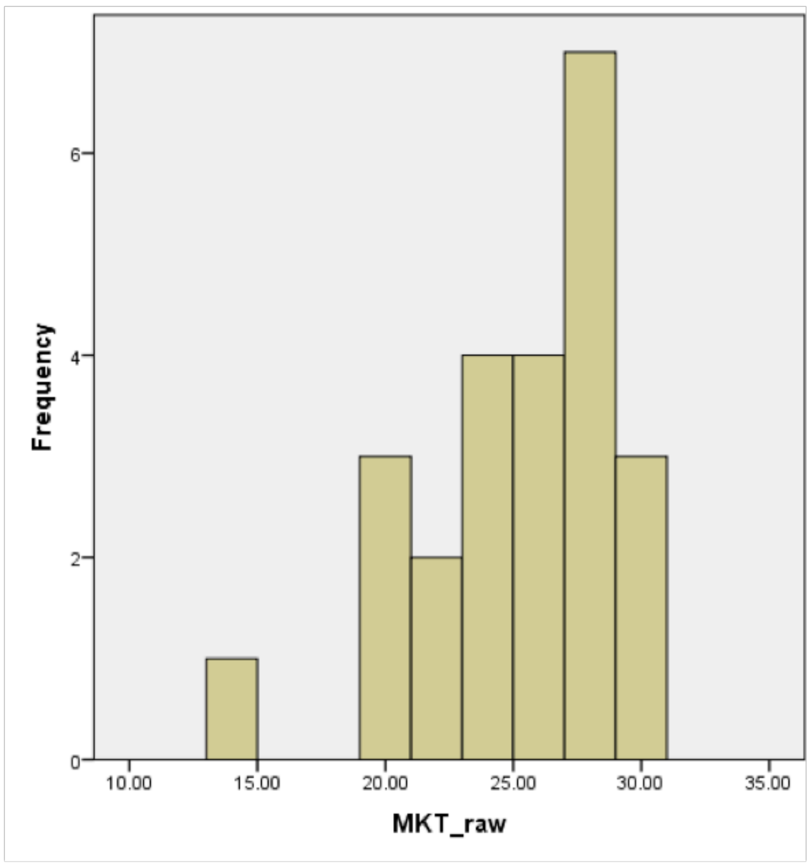


Figure G.4: Histogram of raw MKT score (N=24)

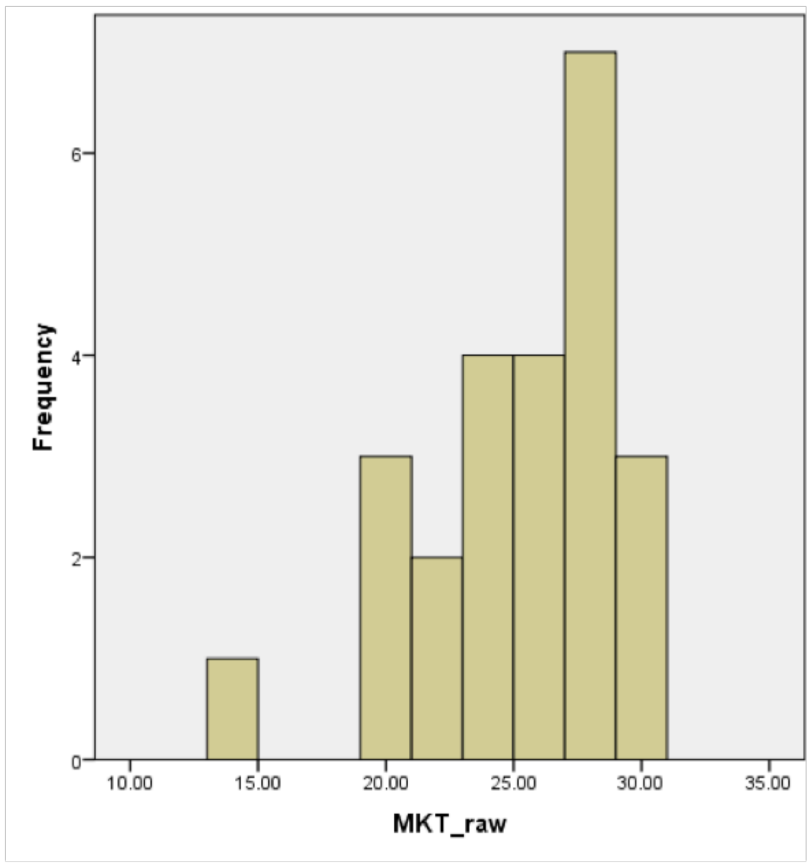


Figure G.5: Histogram of raw MKT score (N=24)

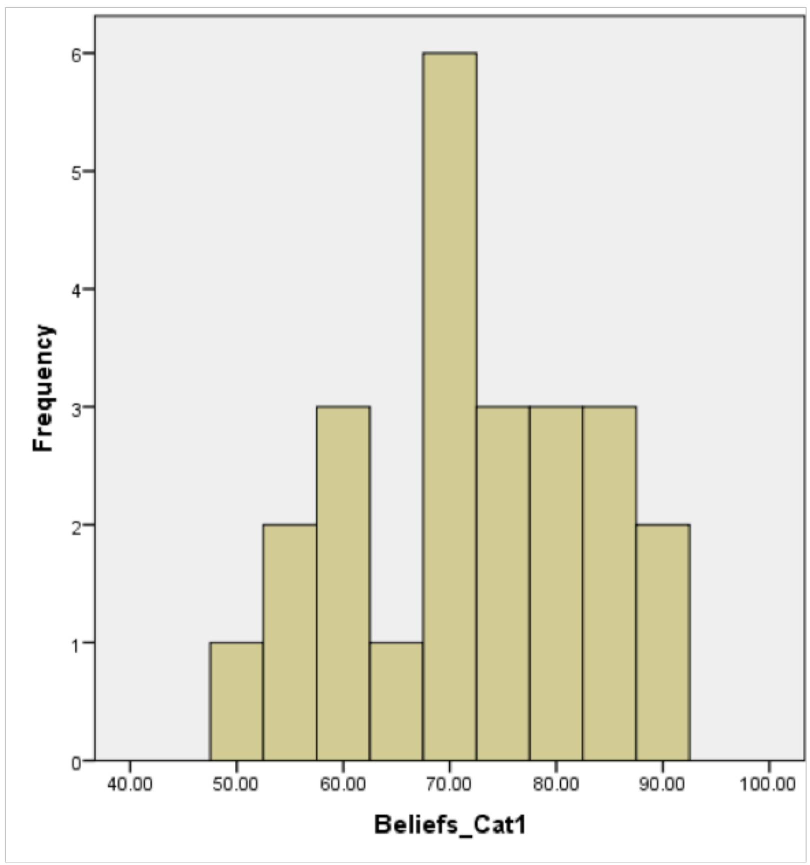


Figure G.6: Histogram of beliefs category 1 (N=24)

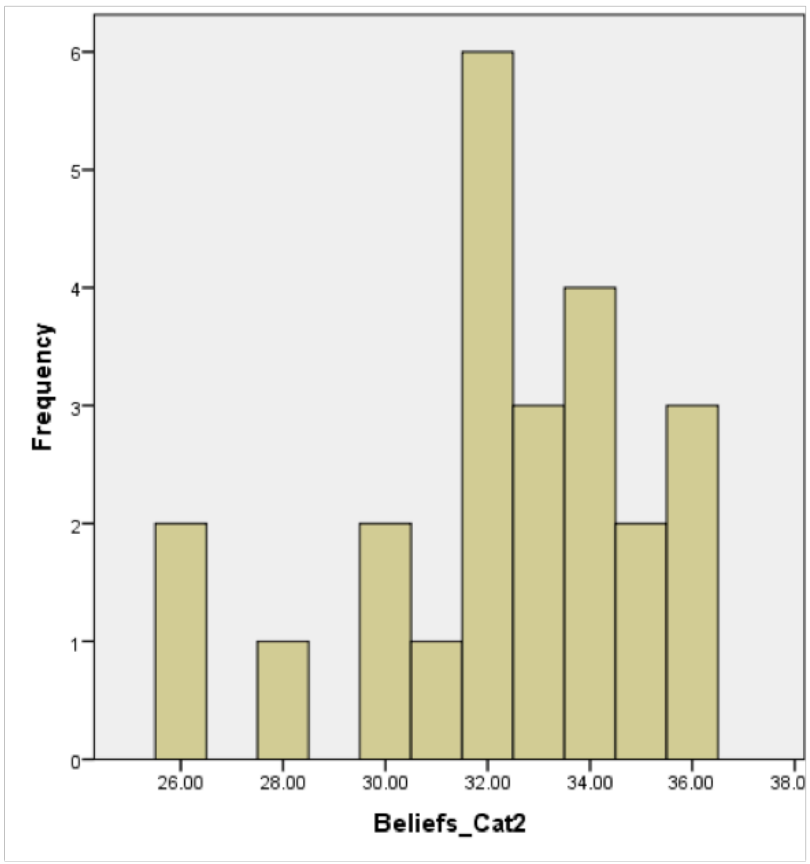


Figure G.7: Histogram of beliefs category 2 (N=24)

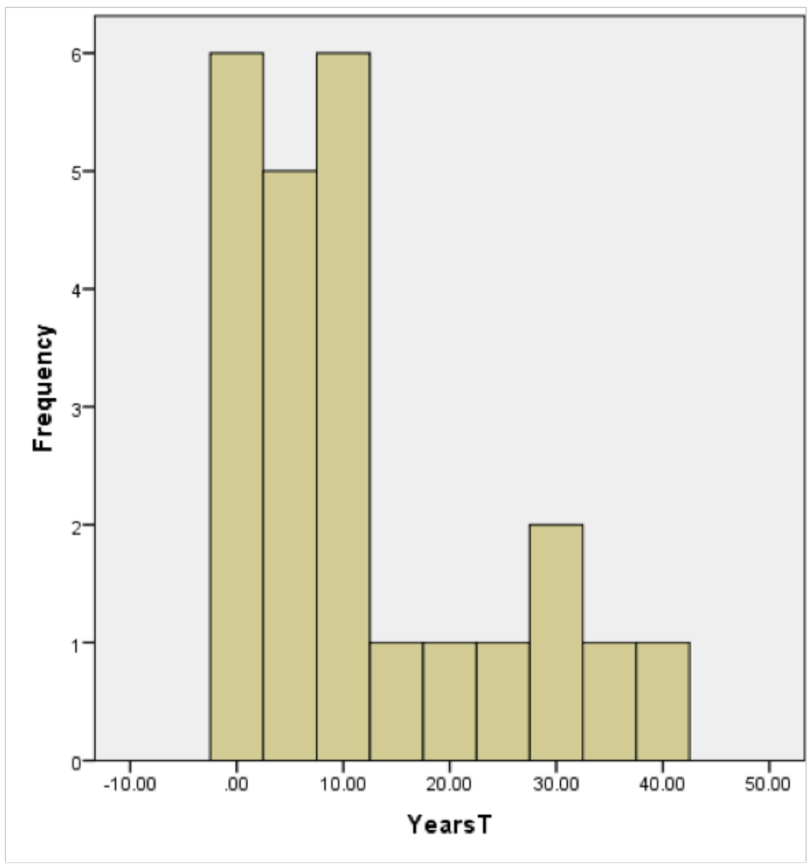


Figure G.8: Histogram of years of teaching experience (N=24)

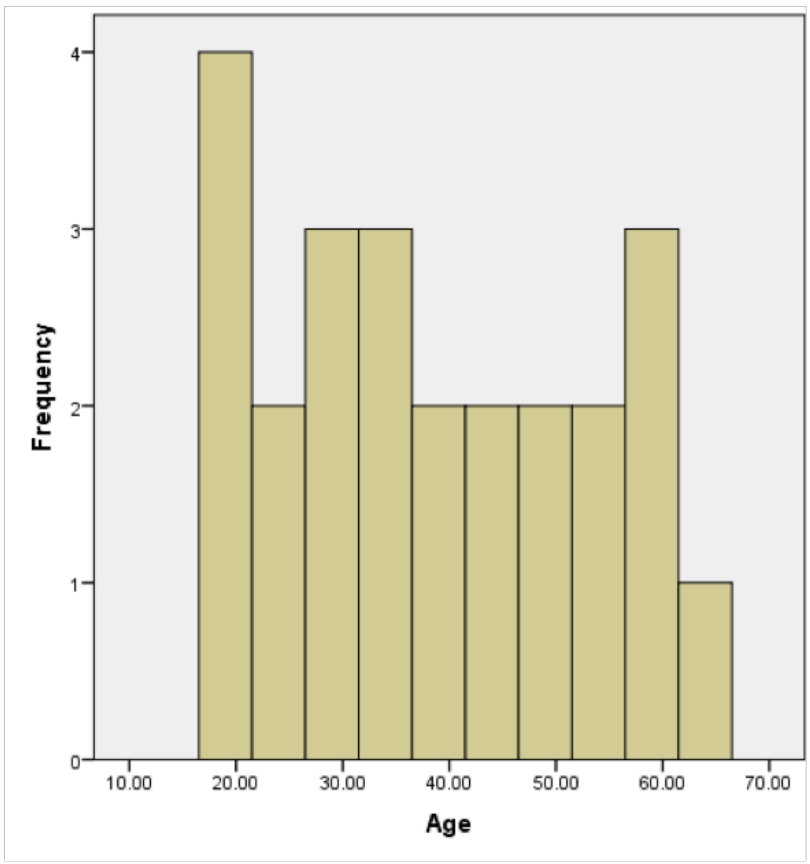


Figure G.9: Histogram of age (N=24)

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