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OPTIMUM AMBULANCE LOCATION IN SEMI-RURAL AREAS

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Abstract

As large medical centers become ever more capable, the existence of well equipped, well staffed, and rapid emergency ambulance service becomes increasingly important. This paper presents a method for determining the optimum location of ambulance stations to minimize the average response time to emergency calls. A new point-to-point driving time model is introduced, and a computer optimization algorithm is used to determine optimum locations. A constraint that the average response time to any point in the service area be less than some specified minimum is also considered. The method is applied to Washtenaw County, Michigan.

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Introduction

Emergency ambulance service is vital in any community, large or small. The availability of an ambulance, or even a few minutes difference in the time of its arrival, may make the difference between life and death for a patient. It is important, therefore, that adequate service be provided and that the facilities available be utilized so as to derive maximum public benefit from them. To achieve this, three basic subsystems of the emergency ambulance service must be considered; they are communication, transportation, and medical treatment. Fortunately, for most purposes these may be considered separately. In this report the second subsystem, transportation, is studied. In particular, the locations at which a given number of ambulances should be stationed to minimize the response time to a call is determined. By noting the results for varying numbers of ambulances one can determine the benefit to be derived from purchasing additional vehicles. These are important considerations, for in many emergency situations one of the most critical factors is how rapidly aid can arrive.

The technical problems which arise in the determination of ambulance locations are similar to those which occur in conjunction with many service- and business-location studies. Banks have considered location determination for branch offices, religious organizations for church sites, governmental agencies for a variety of services, etc. (See [1], [2], and [3] for a few examples.) The list could be very long, since the problem has been considered in many different contexts. In most cases, however, the work was primarily that of analysis. Only recently have people begun to utilize simulation and optimization techniques. Notable examples of this are the studies on fire station location currently being conducted by the Fels Institute at the University of Pennsylvania, and Dartmouth College, and the investigations into ambulance location by Savas [4], and by Gordon and Zelin [5].

Savas studied the allocation and location of ambulances for a hospital district in New York City. The primary results obtained there were: the number of ambulances should be sufficient to prevent the formation of significant queues, the ambulances should be dispersed throughout the service area, and large service areas with no district restrictions on ambulance travel were most efficient.

The study described in this report differs from that conducted in New York City in several ways. It was conducted for Washtenaw County, a semi-rural area in southeastern lower Michigan (see fig. 1), whose basic characteristics differ from those of a district or section of a large city. The population density is much lower. Consequently, a requirement that there be enough vehicles to prevent significant queues from forming is inadequate. With the smaller total population, the number of ambulances necessary to prevent this is quite small. The larger geographical area, on the other hand, poses an additional problem. Enough ambulances must be available to keep the driving time to various points in the county acceptably small. In semi-rural Washtenaw County this number is larger than that required to prevent significant queues.

Also, there is a basic difference in approach in this investigation. In the Savas study, the locations of the dispersed ambulances were determined through trial and error by a human operator using Monte Carlo techniques with a relatively straight-forward simulation model. In this effort, a more complex driving time model is used with explicit equations for the average response time, and the ambulance locations are determined by an iterative optimization algorithm. The purpose of the study is twofold: to obtain explicit usable results for Washtenaw County, and to investigate the feasibilities of the model and of the optimization procedures used for semi-rural areas.

This report is organized so the nonmathematically-oriented reader may study it without being encumbered with detailed equations. Chapter 2 discusses the problem formulation and the basic assumptions and approximations made; it also describes the data base used for the study. Chapter 3 presents the results. All of the detailed derivations of the equations are contained in the appendices.

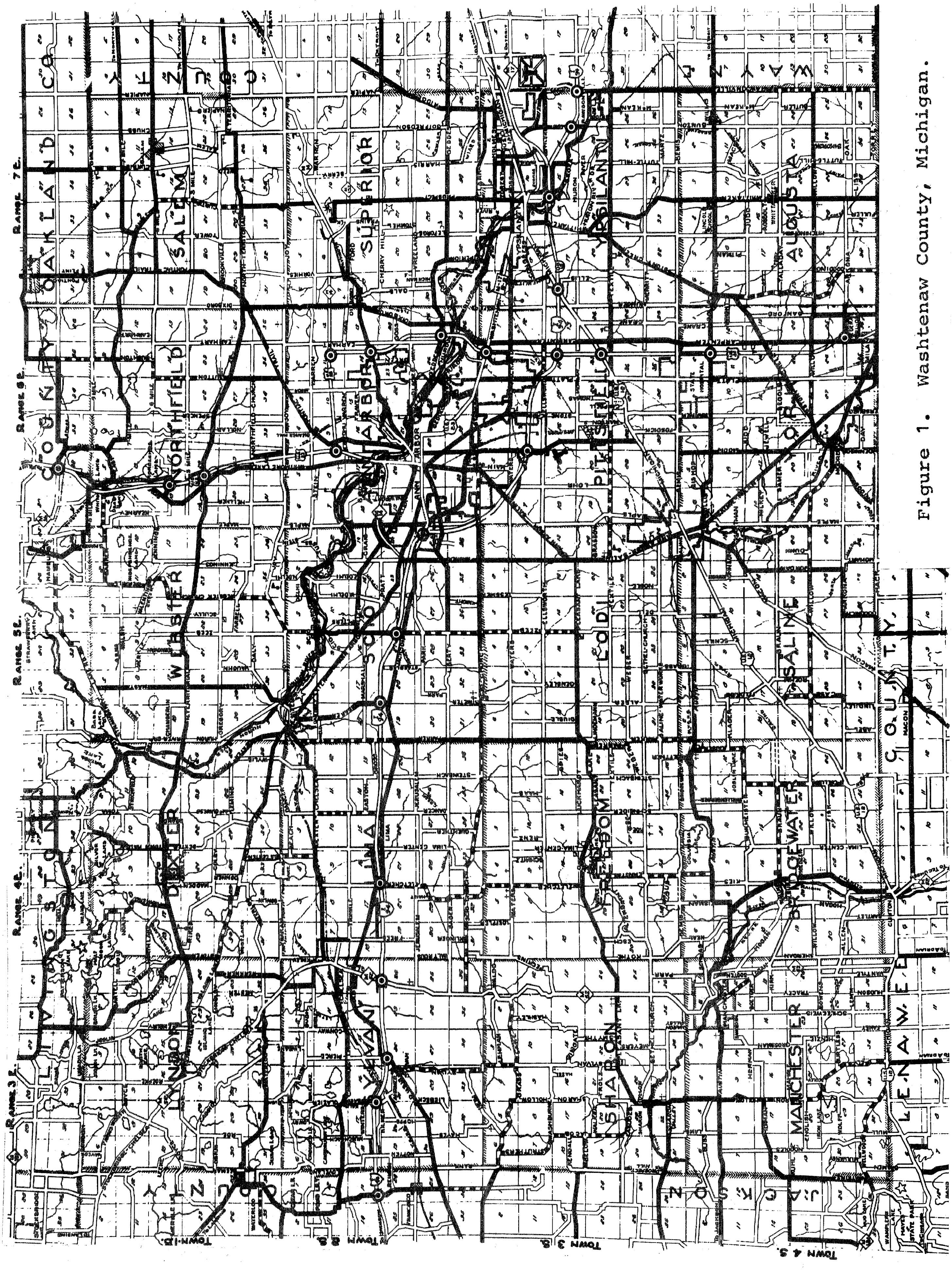

Figure 1. Washtenaw County, Michigan.

Chapter 2
Problem Formulation

Problem Statement and Assumptions

One of the important characteristics of an ambulance service is its ability to respond rapidly to emergency calls. There are many important factors which influence this; they include the number of ambulances assigned, the locations at which they are stationed, road conditions, time of day, day of week, etc. Over many of these factors the ambulance service has little or no control, e.g. time of call. Consequently, we examined two factors over which control can be exercized - the number of ambulances and their station locations - and treated all other factors as uncontrolled random inputs. As a means of evaluating the system performance, the response time - defined as the time difference between the receipt of a call requesting ambulance service and the arrival of an ambulance at the scene - is useful. However, it is not only the response to a single call that is important, but the overall performance of the system. Consequently, the average response time with respect to all emergency calls in the county was used. The problem may be stated very simply as follows:

Problem 1. Given that there are N ambulances to provide service to the county, determine the station locations for these ambulances which minimize the expected (or average) value of the response time.

The effect the number of ambulances has on the response time can be determined by solving problem 1 for varying values of N.

An expression for the average response time, denoted $\bar{T}_r$, is developed in Appendix A. Using the driving time model described in Appendix B and the minimization procedures in Appendix C, a set of ambulance locations was determined which minimized $\bar{T}_r$. In these developments, several simplifying assumptions and approximations were made. Reasonable operating procedure dictates that when a call is received, the nearest (in the sense of driving time) ambulance should be dispatched. It was assumed that if K ambulances are in service, the remaining, N-K, ambulances are optimally located. That is, we assumed that every time an ambulance goes into service, the remaining vehicles are instantaneously relocated in an optimal manner.* The fact that transition from one station location to another is not instantaneous would be a problem only if a vehicle received a call during a transition. Since the transition time is small, the probability of this occurring is also small. Even if this did occur, the ambulance would be on the way to the new station and the time difference in many cases would be small. Consequently it was felt that this assumption is justified.

A second important consideration is route selection. With known techniques it is not possible to specify the optimum route for the ambulance in a short enough time to allow the solution to problem 1 to be carried out. Therefore it was assumed that the driver would make a reasonable choice of route and that his route would be close to the optimal; it was further assumed that if a reasonable route were picked from the map it would be close (in the sense of driving time) to that actually selected by the driver. It was recognized that this introduces a margin for error. However, more accurate methods do not appear feasible at this time, and it was believed that if care is taken the errors can be held to acceptable limits. The details of the route selection from the map are discussed in Appendix C.

To simplify the calculation of $\bar{T}_r$, the county, which is 30 miles by 24 miles, was divided into squares one mile on a side. All calls within a one mile square were considered to come from a single representative point within that square and all distances from that square were measured relative to that point. This simplified the arithmetic because each location could then be represented by a pair of integer coordinates. If desired, smaller squares could be used; the procedures would be the same. The only difference is that greater computation time would be required.

*The idea of dynamically relocating the remaining vehicles is reasonable since the existing ambulance service utilizes a relocation scheme.

To achieve simplification it was assumed that the source of a call and the number of vehicles in service when it is received are statistically independent of all the variables in the system. In a strict sense this need not be true. For example, poor road conditions would lead to a greater number of highway accidents, and thus it would change somewhat the distribution on source of calls. However, the errors should be relatively small, and if desired, the problem could be segmented to achieve an even better approximation. One could solve the problem under different fixed conditions; e.g., 4-6 p.m. on weekdays when traffic is heavier, or times for which roads are covered with snow. For each of these a different solution may be obtained, and for each the assumption is valid. While this segmentation is theoretically possible, neither the data to make many segmented solutions meaningful nor the funds to support the necessary computer work were available. Consequently, no such segmentation was done.

Finally, it was assumed that there is always an ambulance available when a call is received. That is, a user would never need to wait for an ambulance to be released from a previous call. This is reasonable since only twice during a 12-month period were all of the ambulances in the present service simultaneously in use.

The response time, then, was evaluated under these assumptions and approximations using the procedures described in Appendices A and B. Using the method in Appendix C, a digital computer solved for the ambulance locations that minimize $\bar{T}_r$.

Although the average response time is a significant measure of the ambulance system performance, one might also want to specify that no response be longer than some predetermined maximum, T_m. In an absolute sense this cannot be guaranteed. However, one could include a constraint which would require that the average response time to any point in the county be less than T_m if at least r ambulances are available at the time a call is received. This latter condition on the number of ambulances available is necessary because if the number were too low there might be no set of locations from which one could reach any point within T_m minutes.

Thus, one can state:

Problem 2: Given that there are N ambulances to provide service to the county and that the average response time to any call is required to be less than T_m if at least r ambulances are available at the time the call is received, determine the ambulance station locations which minimize the expected (or average) value of the response time.

This problem will be called the constrained problem and problem 1 will be referred to as the unconstrained problem.

Data

The data used in this study were made available through the Washtenaw County Health Department and the Superior Ambulance Company. Superior maintains a thorough record on each call received. Among the pertinent information recorded are the location from which an ambulance left, the location to which it went, the time of the call, the time of departure, the time of arrival at the scene, the time the ambulance left the scene, and the time it arrived at the hospital (or other secondary destination). The times are recorded by a radio dispatcher using a time clock whose scale is in minutes.

The information obtained from these data include the density of emergency calls* in the county, an estimate of ambulance speeds on different types of roads, and the probability that a given number of ambulances will be in use when a call is received. Because response time is most important in emergency cases, only those calls were used in the first two computations. For the period October 1, 1967 to September 30, 1968 emergency calls totaled 1523. For the last computation all calls during the year were included.

Figure 2 shows the density of calls throughout the county. Each number in the figure is the number of emergency calls received

*A call was considered to be an emergency if both siren and lights were used by the ambulance.

0	0	0	0	0	0	0	0	1	0	0	7	1	0	0	0	1	0	8	9	2	0	1	0	1	1	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	2	0	0	0	1	0	0	5	2	1	0	0	0	0	1	1	0	0	0
0	0	0	0	0	0	3	1	0	1	0	4	1	1	1	0	0	0	1	1	0	0	0	0	0	1	0	1	0	0
		LYNDON						DEXTER						WEBSTER						NORTHFIELD						SALEM			
0	0	0	0	1	1	0	0	0	0	0	1	0	1	0	0	2	0	0	5	3	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	3	0	1	0	0	0	0	0	0	0	0	0	1	1	0	1	0	2	0
0	0	0	0	0	2	1	0	0	0	0	1	5	2	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	1	0	0	0	0	1	2	31	0	1	0	0	0	0	1	1	1	1	1	1	0	0	3	0	1
0	2	2	1	0	48	0	0	0	0	0	0	0	0	1	1	3	0	2	2	4	1	0	0	2	0	1	0	2	0
2	1	0	0	0	3	0	1	9	1	0	0	1	1	3	4	0	2	1	2	3	5	9	4	2	0	0	4	1	1
		SYLVAN						LIMA						SCIO						ANN ARBOR						SUPERIOR			
0	0	0	2	0	1	0	0	4	3	0	0	5	1	4	8	12	10	22	26	15	5	2	0	0	0	1	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	1	2	21	20	105	36	9	3	13	2	0	3	1	0	1
0	0	0	0	0	0	0	0	0	0	0	0	1	0	2	1	1	2	15	20	38	23	4	8	0	2	2	4	11	1
0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	3	2	3	4	13	17	32	17	8	22	21	22	26	16
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	3	10	9	9	2	3	36	107	37	30	6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	3	0	2	2	1	3	2	10	11	34	10	10	16
		SHARON						FREEDOM						LODI						PITTSFIELD						YPSILANTI			
0	0	0	0	0	0	0	1	3	0	2	2	0	0	0	0	0	1	1	0	1	0	3	5	0	2	3	1	0	11
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	0	0	2	2	0	1	0	4	0	0	1	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	0	0	0	1	1	0	2
0	0	0	0	4	0	1	0	0	0	2	2	0	0	0	1	0	41	0	1	1	0	5	0	1	0	1	0	2	1
0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	3	2	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	2	0	0	0	0	0	0	0	3	0	0
		MANCHESTER						BRIDGEWATER						SALINE						YORK						AUGUSTA			
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	2	0	0	0
0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	3	2	0	4	0	1	1	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	16	2	0	1	3	2	0	0

Figure 2. Density of emergency ambulance calls:
Oct. 1, 1967 through Sept. 30, 1968.

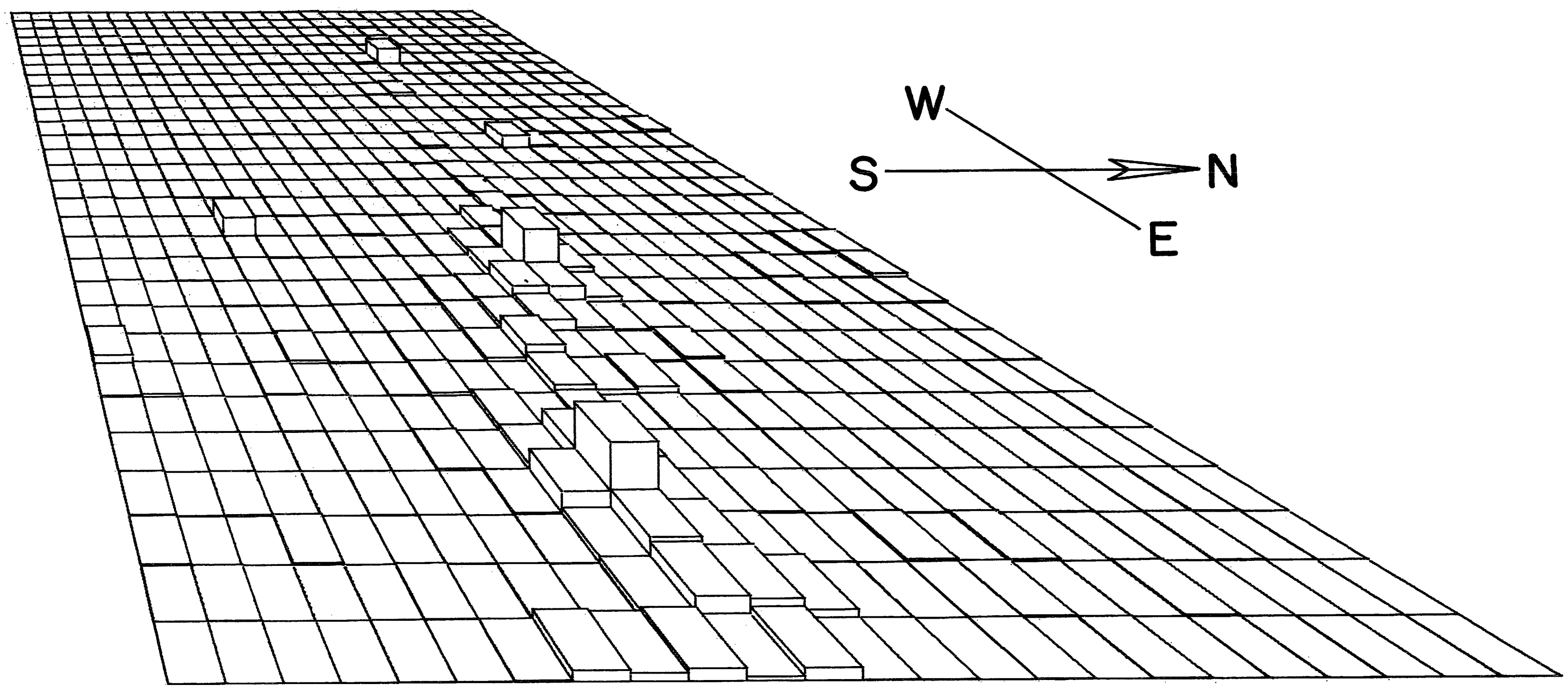

Figure 3. HSRI 3-D plot: density of emergency ambulance calls.

in the one-mile square indicated (compare with figure 1). As expected, there are high peaks in the major cities of the county and very few calls from the remote rural areas. While one can see some evidence of calls originating along the major expressways, this effect is certainly minimal. Figure 3 provides a perspective view of this density; the density pattern is viewed from the eastern direction.

The velocity coefficients were determined by selecting a sample of 273 cases, determining typical routes from county maps, and doing a least square fit of the data. The resultant velocities are shown in table I.

TABLE I

Average Ambulance Speed by Road Type

Type of Road	Average Speed
Expressway	73.2 mph
Other paved highway	58.1 mph
City streets	25.6 mph
Unpaved county roads	29.6 mph

To estimate the predictive capability of this choice a different set of 142 cases was used, and the predicted and measured times were compared. The average error in this was 10^{-4} minute. A similar test run was performed on the original 273-case sample, and an average error of 4.2 seconds was obtained.

The probability of K ambulances being in service when a call is received is computed using equation (A-5) and is shown in table II. From this it can be seen that the assumption that all N ambulances are not in service when a call is received is valid for $N \geq 4$.

TABLE II

Solutions to Equation (A-5) for Washtenaw County

K	p(K)
0	0.67031
1	0.26815
2	0.05362
3	0.00715
4	0.00071
5	0.00005

Chapter 3
Results

In order to provide a check on the validity of the entire model it was used to compute the average response for the ambulance system as it presently operates. This could not be done exactly, however, because a slightly variable relocation scheme is currently employed and the model assumes a fixed relocation scheme. What was done was to assume an average relocation scheme for present operation. The result was a predicted response time of 8.81 minutes. The measured average response time for emergency calls over a year's operation was 8.94 minutes.

The measured response time may itself be considered a random variable dependent upon the calls used in the sample. Using the standard deviation of the sample's sum together with its mean gives a better idea of the accuracy of the model. The standard deviation was estimated to be 0.15 minutes. Thus, it was seen that the model is a reasonable approximation of this system.

Table III shows the results of the unconstrained optimization runs for varying number of ambulances. The coordinates given are the x,y coordinates of the one-mile squares in which the ambulances would be located.

TABLE III

Solutions to Unconstrained Problem

Number of Ambulances	Ambulance Location	Average Response Time
1	(23,9)	--------
2	(19,14), (27,11)	--------
3	(9,17), (20,14), (27,11)	9.48 min
4	(9,17), (20,14), (27,11), (23,9)	8.64 min
5	(6,17), (13,18), (20,14), (27,11), (23,9)	8.03 min
6	(6,17), (13,18), (20,14), (27,11), (18,6), (22,13)	7.52 min

It should be pointed out that the times obtained in these runs will be approximated if the ambulances are placed at the representative points for each of the one-mile squares. Location of any ambulance at some other point in the square would result in a slightly different predicted response time. By looking at the response time for placing the vehicle at representative points in neighboring squares one can interpolate to estimate the response time for the location within a square.

It can be seen that for N=5 (which is the number of ambulances presently in use) an improvement of about 10% over present operation could be obtained by employing a different location scheme. The major differences in location involve moving some of the ambulances closer to the point of highest density of calls, and moving some of the outlying vehicles to points that would provide them with easy entry to the portions of the cross-country highway network they are likely to use. By purchasing a sixth ambulance an improvement of about an additional half minute could be obtained.

In order to determine how the average response times to individual locations varied throughout the county the optimal locations were used with the driving time model and an array of driving times to each of the one-mile squares was computed. This was done for N=4,5,6. The results are shown in figures 4, 5, and 6, in which the times are given in tenths of a minute and the ambulance locations are circled. In addition to giving information on how the response times vary, these results provide insight into which ambulance should be dispatched to which location. By looking at the ridges of peak response times between ambulance locations, one can divide the county into the regions to be covered by each vehicle.

It can be seen from figures 4, 5, and 6 that the response time to some of the remote corners of the county is nearly 34 minutes. Even though the population density in these areas is low, the maintenance of some minimum level of service may be desired. To try to reduce these long response times the constrained optimization procedure was used. A T_{max} of 20 minutes was chosen and it was decided that r should be four; with less than four vehicles available,

220	200	206	232	186	197	198	190	179	190	172	168	205	164	182	215	227	197	163	165	176	215	246	235	174	174	192	221	241	241
193	184	186	212	179	185	196	179	158	178	187	164	193	154	181	191	205	183	175	152	189	204	225	214	163	171	181	188	221	221
206	167	163	190	188	163	158	159	139	158	148	144	164	143	159	171	190	174	142	149	172	191	191	185	152	152	176	176	212	222
LYNDON						DEXTER						WEBSTER						NORTHFIELD						SALEM					
204	202	163	162	171	146	160	140	120	139	140	140	148	136	168	169	162	151	137	131	150	171	171	164	136	168	167	167	222	218
179	179	162	141	160	135	140	119	99	118	137	125	144	128	162	173	181	194	154	139	156	155	144	133	128	139	149	161	188	202
165	156	143	132	120	123	121	101	81	98	117	121	124	118	145	150	172	175	155	119	139	177	147	135	146	153	163	168	201	200
182	151	163	157	126	108	100	79	60	80	99	117	102	122	122	138	153	155	123	110	116	156	142	142	142	161	154	161	171	183
135	121	126	131	120	100	79	60	39	59	79	97	107	116	135	136	136	134	114	94	97	136	113	126	136	143	145	154	149	162
124	118	108	106	109	77	73	56	55	65	82	108	106	107	125	123	132	114	94	76	96	116	104	111	125	151	135	148	149	162
SYLVAN						LIMA						SCIO						ANN ARBOR						SUPERIOR					
132	126	108	92	80	77	71	60	70	77	75	84	99	98	108	111	113	94	74	55	75	96	98	109	125	128	115	123	154	154
128	116	122	122	90	90	83	87	87	115	107	99	117	145	132	114	94	74	54	35	55	75	89	93	100	105	95	105	122	111
147	154	146	146	102	112	133	115	120	126	130	109	125	135	125	123	101	91	74	55	75	89	89	90	101	95	75	95	115	103
207	185	151	141	123	115	130	135	135	155	161	143	125	135	125	113	102	92	85	75	96	116	93	84	88	75	54	75	95	103
247	196	172	147	132	130	152	157	161	178	187	136	153	174	162	138	115	107	89	85	93	106	86	73	74	54	34	54	68	87
250	218	196	173	141	141	153	178	179	197	149	138	164	183	142	129	118	105	117	116	97	89	69	72	64	74	54	55	61	76
SHARON						FREEDOM						LODI						PITTSFIELD						YPSILANTI					
252	228	218	201	183	151	194	216	197	182	172	162	172	152	141	129	118	130	125	107	92	72	49	62	86	79	74	95	95	75
234	213	213	204	183	182	218	224	218	214	191	191	199	203	180	161	137	126	138	102	76	66	60	78	78	94	95	115	104	89
239	239	245	223	193	208	222	255	238	257	227	206	209	210	192	165	143	120	99	88	98	90	71	100	111	122	106	124	135	111
274	261	247	239	207	228	252	237	228	206	194	180	186	174	160	150	139	116	131	125	117	99	86	100	105	123	121	140	140	130
273	253	252	242	230	235	235	230	230	219	225	208	219	206	168	153	153	138	139	120	113	94	88	102	104	121	115	123	148	136
281	274	274	262	255	250	260	260	271	271	245	212	196	183	170	156	159	161	175	143	149	105	99	123	130	146	144	147	163	157
MANCHESTER						BRIDGEWATER						SALINE						YORK						AUGUSTA					
306	269	269	285	275	260	273	271	262	247	237	212	181	184	207	194	162	177	187	160	147	121	110	134	148	166	166	170	186	186
295	331	326	318	296	271	277	283	272	251	228	198	211	206	209	184	162	174	169	152	138	146	120	131	145	147	155	178	178	180
339	325	319	298	291	284	282	262	269	226	214	225	249	224	218	188	184	184	167	159	168	153	153	131	161	161	182	180	201	212

Figure 4. Average time from receipt of call to arrival at scene: unconstrained optimal solution for four ambulances in Washtenaw County. (Times are given in tenths of a minute.)

191	171	176	202	155	166	169	161	158	151	133	129	166	125	143	176	194	196	162	164	175	215	246	235	174	174	191	220	241	241
163	154	156	182	148	154	167	151	136	146	149	125	151	115	142	153	172	168	174	151	188	203	224	213	163	170	180	188	220	220
177	138	134	160	157	132	128	135	124	121	110	105	126	104	121	133	153	150	141	148	171	191	191	185	151	151	175	175	212	222
		LYNDON						DEXTER						WEBSTER						NORTHFIELD						SALEM			
174	173	134	132	141	117	135	145	126	132	101	101	109	98	129	132	129	134	136	130	149	170	170	163	136	167	166	166	222	218
149	149	132	112	127	105	121	122	103	104	103	86	94	89	123	136	149	172	154	138	155	154	143	132	127	138	148	160	188	202
136	127	113	103	91	87	101	110	115	86	84	82	73	79	107	113	140	158	156	118	139	176	146	134	143	152	163	166	196	195
162	139	128	111	88	68	81	90	92	90	81	71	53	73	85	102	122	141	123	109	115	156	136	136	137	155	154	157	168	180
129	116	103	89	72	51	60	69	78	88	78	80	69	79	98	107	119	127	115	93	96	135	107	120	130	140	145	154	149	162
118	112	102	97	83	60	73	83	85	96	102	96	83	88	99	95	106	115	95	75	95	115	98	105	119	150	135	148	149	162
		SYLVAN						LIMA						SCIO						ANN ARBOR						SUPERIOR			
126	121	103	87	75	71	81	90	101	95	86	83	79	78	89	92	105	95	74	54	75	95	93	103	119	125	115	123	154	154
123	110	116	116	85	85	93	117	117	131	113	91	97	126	114	106	95	74	54	34	54	74	87	87	94	102	95	105	122	111
142	148	141	141	96	106	127	140	154	150	136	102	115	128	126	123	102	92	74	54	75	86	82	83	97	95	74	95	115	102
202	179	146	136	118	109	125	164	165	167	151	133	115	128	126	114	102	92	85	74	95	115	93	77	86	74	54	74	95	103
241	191	166	142	127	125	147	184	200	187	177	126	143	164	163	139	115	107	89	84	88	96	76	65	74	54	34	54	68	87
245	213	191	168	135	135	149	194	197	188	139	128	154	173	142	129	118	105	117	115	88	77	57	63	63	74	54	55	61	75
		SHARON						FREEDOM						LODI						PITTSFIELD						YPSILANTI			
247	223	213	196	177	146	191	211	191	172	162	152	162	152	141	129	118	130	117	97	77	57	36	55	76	76	74	95	95	75
229	208	208	199	177	177	215	218	217	204	181	181	189	201	178	159	135	124	130	90	65	54	48	67	71	87	95	115	104	89
234	234	240	218	188	203	215	253	240	247	221	201	204	202	180	157	136	112	86	76	85	78	59	88	107	118	104	123	135	111
269	256	242	234	202	223	249	228	218	195	181	168	173	161	148	137	126	103	119	114	106	88	75	89	95	116	116	139	139	130
268	248	247	237	225	231	233	224	217	207	212	195	206	194	155	141	141	125	127	108	101	83	76	90	93	112	112	122	147	136
276	269	269	257	250	245	255	257	267	260	233	199	183	170	157	143	147	148	163	132	138	94	87	111	119	140	143	146	163	157
		MANCHESTER						BRIDGEWATER						SALINE						YORK						AUGUSTA			
301	264	264	280	270	256	270	264	256	234	225	199	168	171	194	182	149	165	176	149	136	110	99	122	136	162	162	169	186	186
291	327	323	314	292	268	274	273	267	238	216	185	198	193	196	171	149	161	157	141	127	135	108	120	133	137	148	178	177	179
335	320	312	292	285	278	276	251	257	213	202	213	236	211	205	175	171	171	156	148	157	142	142	120	150	150	181	180	200	211

Figure 5. Average time from receipt of call to arrival at scene: unconstrained optimal solution for five ambulances in Washtenaw County. (Times are given in tenths of a minute.)

182	162	168	193	146	156	160	152	152	138	121	117	154	113	131	164	183	196	162	164	175	215	246	235	174	174	191	220	241	241
155	145	147	173	139	144	158	142	129	136	136	113	138	103	130	141	162	164	174	151	188	203	224	213	163	170	180	188	220	220
168	129	125	151	147	123	119	128	119	109	97	93	113	92	108	121	140	143	141	148	171	191	191	185	151	151	175	175	212	222
		LYNDON						DEXTER						WEBSTER						NORTHFIELD						SALEM			
165	164	125	123	131	108	127	148	129	130	88	88	97	85	117	119	118	129	136	130	149	170	170	163	136	167	166	166	214	210
140	140	124	103	117	96	116	124	106	100	92	74	77	77	110	124	138	164	154	138	155	154	143	132	127	138	148	160	180	195
127	118	104	94	82	76	96	114	127	83	73	70	57	67	94	101	130	152	156	118	138	161	146	134	143	152	163	167	196	195
157	136	117	97	77	56	75	94	104	94	75	57	37	57	73	89	112	136	123	109	115	141	136	136	136	158	153	157	169	180
128	115	96	77	57	37	55	74	92	99	79	75	56	67	86	97	113	124	115	92	96	121	110	120	130	140	144	152	149	162
117	112	102	95	76	55	74	93	95	106	109	92	76	81	91	86	97	115	95	74	95	100	98	104	119	150	135	148	149	162
		SYLVAN						LIMA						SCIO						ANN ARBOR						SUPERIOR			
126	120	102	86	74	71	85	101	112	102	90	83	73	72	83	86	102	95	74	54	74	80	87	96	113	119	115	123	154	154
122	110	115	115	84	84	97	128	128	137	115	89	91	119	107	103	95	74	54	34	54	60	78	79	88	96	95	105	122	111
141	147	140	140	95	105	127	150	166	158	138	100	111	125	126	123	102	92	74	54	60	48	62	73	90	95	74	95	115	102
201	178	145	135	117	109	124	174	176	172	148	130	112	125	126	114	102	92	85	74	80	71	80	67	77	74	54	74	95	103
241	190	166	141	126	124	146	194	214	190	174	122	140	161	163	133	99	107	89	85	93	80	76	68	74	54	34	54	68	87
244	212	190	167	135	135	148	198	202	185	135	125	146	168	119	106	95	91	117	115	95	89	75	76	68	74	54	55	61	75
		SHARON						FREEDOM						LODI						PITTSFIELD						YPSILANTI			
246	223	212	195	177	145	190	208	187	167	157	147	152	129	118	106	82	101	113	116	97	98	71	76	104	85	74	95	95	75
228	208	208	198	177	177	214	215	215	197	166	173	175	161	138	118	95	84	97	96	74	78	79	97	91	106	95	115	104	89
233	233	240	217	187	202	211	216	204	215	180	160	164	154	133	112	91	70	68	73	92	98	90	119	120	130	110	127	135	111
268	252	241	229	201	217	212	187	177	153	139	125	131	119	105	92	74	53	72	85	97	107	98	112	118	135	130	142	142	130
267	244	243	232	220	206	193	183	175	164	170	153	164	151	113	99	93	74	85	95	105	104	99	113	116	135	122	125	150	136
272	264	264	253	246	241	239	217	247	218	190	157	141	128	115	101	105	95	114	96	116	117	110	134	142	158	146	149	163	157
		MANCHESTER						BRIDGEWATER						SALINE						YORK						AUGUSTA			
296	260	260	275	266	245	243	223	215	192	182	157	126	129	152	140	107	114	132	125	122	115	122	145	159	173	173	172	186	186
286	306	293	282	260	237	240	231	226	196	174	143	156	151	154	129	107	119	141	127	113	121	131	143	157	160	169	178	181	182
313	279	271	251	244	237	235	209	215	171	159	170	194	169	163	133	129	129	142	134	143	128	137	143	173	173	185	180	203	214

Figure 6. Average time from receipt of call to arrival at scene: unconstrained optimal solution for six ambulances in Washtenaw County. (Times are given in tenths of a minute.)

it is not possible to provide reasonable service for the areas of high density and rapid service for the remote areas of the county. The results of this study are summarized in table IV, and figures 7, 8, and 9 show how the individual response times changed.

TABLE IV

Solutions to Constrained Problem

Number of Ambulances	Ambulance Locations	Average Response Time
1	(23,9)	--------
2	(19,14), (27,11)	--------
3	(9,17), (20,14), (27,11)	--------
4	(7,5), (9,17), (23,9), (20,16)	9.97 min
5	(7,5), (9,17), (23,9), (20,14), (27,12)	9.01 min
6	(7,5), (9,17), (23,17), (20,14), (23,9), (27,11)	8.24 min

Response times previously longer than 30 minutes were reduced to approximately 19 minutes. Only in a few locations was the constraint violated, and then only by a small amount; this was deemed acceptable. The main changes involved taking one ambulance away from a higher density area and placing it near Manchester, in the southwestern corner of the county (see fig. 1), and moving some of the vehicles covering Ann Arbor and Ypsilanti slightly north to provide more rapid response to the northeastern portion of the county.

The overall average response time, of course, increased with the incorporation of this constraint. However, it can be seen that if properly distributed, five ambulances could provide nearly the present average level of service and still furnish adequate coverage for the remote parts of the county. With an additional vehicle, remote service could be provided and overall performance improved.

220	200	206	232	186	197	198	190	179	190	172	168	205	164	182	201	202	172	138	139	150	190	229	226	165	165	183	212	231	231
193	184	186	212	179	185	196	179	158	178	187	164	193	154	181	177	180	157	149	127	164	179	207	204	154	162	172	179	210	210
206	167	163	190	188	163	158	159	139	158	148	144	164	143	159	157	167	149	117	124	147	174	174	176	142	142	166	166	203	213
LYNDON						DEXTER						WEBSTER						NORTHFIELD						SALEM					
204	202	163	162	171	146	160	140	120	139	140	140	148	136	162	155	136	125	112	106	124	153	153	154	127	159	157	157	203	199
179	179	162	141	160	135	140	119	99	118	137	125	144	128	162	161	162	166	129	114	131	131	135	124	119	130	140	151	169	184
165	156	143	132	120	123	121	101	81	98	117	121	124	118	145	147	155	146	126	93	114	148	137	125	143	144	154	158	196	196
182	151	163	157	126	108	100	79	60	80	99	117	102	122	122	137	140	126	103	85	94	127	142	142	142	147	143	161	167	179
135	121	126	131	120	100	79	60	39	59	79	97	107	116	130	132	120	105	85	66	72	107	99	126	136	143	133	142	139	151
124	118	108	106	109	77	73	56	55	65	82	108	106	107	125	117	104	85	65	46	67	87	102	111	125	151	135	141	139	151
SYLVAN						LIMA						SCIO						ANN ARBOR						SUPERIOR					
132	126	108	92	80	77	71	60	70	77	75	84	99	98	108	111	113	94	74	55	75	96	98	109	125	128	145	141	170	170
128	116	122	122	90	90	83	87	87	115	107	99	117	145	132	125	113	103	83	64	84	95	89	93	100	105	120	123	148	138
147	154	146	146	102	112	133	115	120	126	130	109	125	136	133	130	109	107	103	84	100	89	89	90	101	106	113	119	143	130
207	185	151	141	123	115	130	135	135	155	161	143	125	136	133	121	109	107	93	96	107	116	94	84	88	109	108	118	124	130
238	196	172	147	129	130	152	157	161	178	187	136	153	174	170	146	123	123	98	85	93	106	86	73	91	76	74	104	95	114
210	185	166	151	123	123	127	156	166	186	149	138	164	183	145	133	122	118	117	116	97	89	69	72	64	88	84	82	88	103
SHARON						FREEDOM						LODI						PITTSFIELD						YPSILANTI					
190	171	162	145	126	117	137	159	159	162	166	162	172	155	144	133	122	133	125	107	92	72	49	62	86	79	88	121	115	102
171	149	149	148	120	119	132	147	160	173	169	176	177	203	180	161	137	126	138	102	76	66	60	78	78	94	97	118	115	116
161	153	158	139	109	130	117	141	151	170	181	165	167	188	192	165	143	120	99	88	98	90	71	100	111	122	106	124	143	138
173	153	143	131	108	119	111	127	141	150	152	139	144	150	155	150	139	116	131	125	117	99	86	100	105	123	121	140	140	149
168	145	144	134	121	107	92	109	129	141	157	166	178	185	168	153	153	138	139	120	113	94	88	102	104	121	115	123	148	151
173	166	166	154	143	127	115	129	147	168	177	182	192	183	170	156	159	161	175	143	149	105	99	123	130	146	144	147	170	173
MANCHESTER						BRIDGEWATER						SALINE						YORK						AUGUSTA					
197	161	161	176	163	144	133	150	161	177	189	197	181	184	207	194	162	177	187	160	147	121	110	134	148	166	166	170	192	196
187	222	218	203	182	161	148	171	178	193	189	180	200	206	209	184	162	174	169	152	138	146	120	131	145	147	155	183	178	180
230	229	226	206	199	182	167	170	177	167	172	194	215	224	218	188	184	184	167	159	168	153	153	131	161	161	182	186	201	212

Figure 7. Average time from receipt of call to arrival at scene: constrained optimal solution for four ambulances in Washtenaw County. (Times are given in tenths of a minute.)

216	196	202	228	182	193	196	188	176	188	170	166	203	162	180	209	218	187	154	155	166	206	225	221	170	170	188	206	221	221
189	180	181	208	175	180	194	176	156	176	186	163	192	152	179	186	196	173	165	143	180	195	218	210	159	167	172	172	201	201
202	163	159	186	183	159	155	156	136	156	147	142	163	141	158	166	182	165	133	140	163	185	185	181	148	148	166	166	193	203
		LYNDON						DEXTER						WEBSTER						NORTHFIELD						SALEM			
200	198	159	158	167	142	156	136	116	136	138	138	146	135	164	164	153	141	128	122	140	164	164	160	132	164	157	157	198	195
174	174	158	137	156	130	136	115	95	115	136	124	142	127	160	169	175	185	146	129	147	146	140	129	124	135	145	151	164	179
161	152	139	128	116	119	116	95	75	95	115	119	122	116	144	149	167	166	146	109	130	166	143	131	142	149	154	142	183	183
178	147	159	153	122	104	95	75	55	75	95	115	101	122	122	138	149	146	116	101	108	146	136	136	137	142	125	143	149	162
130	117	122	127	116	95	75	55	34	55	75	95	106	116	134	136	132	126	105	83	88	126	102	120	130	125	116	124	120	133
119	114	104	102	105	73	69	51	51	61	79	107	105	107	125	122	124	105	85	65	85	105	98	105	119	129	116	120	120	133
		SYLVAN						LIMA						SCIO						ANN ARBOR						SUPERIOR			
128	122	104	88	76	73	67	56	67	74	73	83	99	98	109	112	115	95	74	54	75	95	93	103	119	118	110	104	134	134
124	112	118	118	86	86	79	83	83	113	105	97	116	145	133	118	101	84	64	44	64	81	87	87	94	91	85	86	114	111
143	149	142	142	97	108	129	111	116	123	128	108	125	136	128	126	105	97	84	64	83	86	82	83	96	84	73	84	109	108
203	180	147	137	119	111	126	131	131	153	161	142	125	136	128	116	105	97	88	81	98	115	95	77	78	71	57	74	90	106
231	192	168	143	124	126	148	153	156	176	186	135	152	174	165	141	118	112	92	84	88	96	76	65	95	76	62	85	96	116
193	170	152	140	113	113	115	147	159	181	148	137	163	182	143	130	119	109	117	115	88	77	57	63	63	92	79	86	92	107
		SHARON						FREEDOM						LODI						PITTSFIELD						YPSILANTI			
166	148	139	123	104	102	117	139	145	154	163	161	171	153	142	130	119	131	117	97	77	57	36	55	76	76	93	118	121	106
147	124	124	125	95	95	100	120	139	158	161	171	169	201	178	159	135	124	130	90	65	54	48	67	71	87	98	119	119	120
131	121	125	108	77	100	81	102	121	141	160	146	150	173	180	157	136	112	86	76	85	78	59	88	107	118	104	123	145	143
136	114	104	91	71	80	63	83	102	121	126	113	118	129	141	137	126	103	119	114	106	88	75	89	95	116	116	139	139	155
129	105	104	94	82	62	43	63	83	102	122	140	151	165	155	141	141	125	127	108	101	83	76	90	93	112	112	122	147	156
133	126	126	114	102	82	63	83	103	123	142	160	178	170	157	143	147	148	163	132	138	94	87	111	119	140	143	146	172	178
		MANCHESTER						BRIDGEWATER						SALINE						YORK						AUGUSTA			
158	122	122	137	122	102	83	103	122	142	161	179	168	171	194	182	149	165	176	149	136	110	99	122	136	162	162	169	194	200
148	184	180	163	143	122	103	124	141	162	164	161	184	193	196	171	149	161	157	141	127	135	108	120	133	137	148	185	177	179
192	192	190	169	162	143	123	128	134	135	145	171	191	211	205	175	171	171	156	148	157	142	142	120	150	150	181	187	200	211

Figure 8. Average time from receipt of call to arrival at scene: constrained optimal solution for five ambulances in Washtenaw County. (Times are given in tenths of a minute.)

216	195	201	228	181	192	195	188	176	188	170	166	203	162	180	203	204	173	140	141	152	190	190	196	140	140	157	181	195	195
188	179	181	208	174	180	193	176	156	176	186	162	191	152	179	180	182	159	151	129	166	181	174	179	129	136	145	146	174	174
201	163	159	185	183	159	155	156	135	156	147	142	163	141	158	160	170	151	119	126	149	156	150	151	117	117	136	136	166	176
LYNDON						DEXTER						WEBSTER						NORTHFIELD						SALEM					
199	198	159	157	167	142	156	135	115	135	138	138	146	135	164	158	139	127	114	108	126	136	129	129	102	133	127	127	167	163
174	174	157	137	155	130	135	115	95	115	135	123	142	126	160	164	166	172	132	115	133	120	108	98	93	104	114	121	133	148
161	152	138	128	116	119	115	95	74	95	115	119	122	116	144	150	164	174	149	100	120	116	94	100	110	118	124	120	157	157
178	147	159	153	122	103	95	74	54	74	95	115	101	122	122	139	152	154	122	91	99	96	77	92	104	101	103	120	127	139
130	117	121	126	115	95	74	54	34	54	74	95	106	116	135	137	136	134	110	87	79	75	53	72	90	102	93	102	99	111
119	113	103	102	104	72	68	51	50	61	79	107	105	107	126	124	132	113	93	73	82	84	61	72	86	110	97	101	99	111
SYLVAN						LIMA						SCIO						ANN ARBOR						SUPERIOR					
127	121	104	88	75	72	67	56	66	74	72	82	99	98	109	112	115	95	74	54	74	87	75	87	102	108	112	105	131	131
124	111	117	117	86	86	78	82	82	112	105	97	116	145	133	116	96	76	56	35	56	75	85	84	92	97	90	97	118	109
143	149	141	141	97	107	128	111	115	123	128	108	125	136	126	123	103	93	76	56	76	85	81	82	94	90	72	90	112	103
202	180	147	137	118	110	126	131	131	152	160	142	125	136	126	114	103	93	85	76	95	115	93	76	83	71	52	72	91	103
230	192	167	143	123	125	147	152	156	176	186	135	152	174	163	139	116	108	89	84	87	95	75	64	80	60	41	62	75	95
191	168	150	138	111	111	113	145	158	180	148	137	163	182	142	129	118	105	117	115	87	75	54	61	62	80	61	64	69	84
SHARON						FREEDOM						LODI						PITTSFIELD						YPSILANTI					
162	144	136	119	100	100	113	136	143	153	162	161	171	152	141	129	118	130	115	95	75	54	34	54	75	76	80	101	102	83
143	120	120	121	91	91	95	116	136	156	160	170	168	201	178	158	135	124	129	87	63	52	47	65	70	86	96	116	108	98
126	116	120	103	72	95	75	95	116	136	156	143	147	170	178	156	134	111	84	73	83	76	58	87	106	117	104	123	138	120
130	107	98	85	65	73	55	75	95	116	122	108	114	125	139	135	124	101	117	112	104	86	73	87	93	115	115	138	138	137
123	99	98	88	75	55	35	55	75	95	116	136	147	162	153	139	138	123	125	106	99	81	74	88	91	110	111	122	147	142
127	120	120	108	95	75	55	75	96	116	136	156	176	168	155	141	145	146	161	130	136	92	85	109	117	139	143	146	166	163
MANCHESTER						BRIDGEWATER						SALINE						YORK						AUGUSTA					
151	115	115	130	116	95	75	96	116	136	156	176	166	169	192	180	147	162	174	147	134	108	97	120	134	162	162	168	189	190
141	178	174	156	136	116	96	116	135	156	160	158	182	191	194	169	147	159	156	139	125	133	106	118	132	135	147	180	177	179
186	186	184	163	156	136	116	121	127	130	140	167	187	209	203	173	169	169	154	146	155	140	140	118	148	148	181	182	200	211

Figure 9. Average time from receipt of call to arrival at scene: constrained optimal solution for six ambulances in Washtenaw County. (Times are given in tenths of a minute.)

Chapter 4
Conclusions

This report has considered the application of modeling and simple optimization techniques to the problem of determining station locations for ambulances in a semi-rural environment. A statistical model for the average response time to emergency calls was developed, and a new model for the driving time between any two points in the county introduced. These were seen to work very well in modeling the behavior of the system. The major disadvantage of the approach is the amount of data that must be determined from maps. Relatively inexpensive student labor was used and this was not considered a problem.

The second feature of this study was the use of a simple optimization algorithm. This removed human participation in the selection of locations by one step: Instead of seeking a solution through trial and error using a simulation of the system, the operator merely selected different starting points for the optimization program so that different local minima were found. At some expense in computer time, this too could have been programmed on the computer. The computer time required for small numbers of ambulances was quite reasonable, being less than 80 seconds for N=2. However, the time required increased rapidly with N. For N=6 nearly 11 minutes were required. Fortunately, solutions were not needed for $N \geq 6$ and the total computation cost was reasonable.* The computation times were somewhat greater for the constrained case. As a result of this study and the existence of today's high-speed digital computers it is reasonable to consider the use of optimization algorithms for problems of this type.

This work was developed both to study the basic ideas involved and to apply them to ambulance operation in Washtenaw County, Michigan. The model and optimization procedure worked well for both the constrained and unconstrained cases. Examination of the resulting station locations, in terms of the county map, has shown that they are intuitively quite reasonable. In addition

*Approximately $400 was used for the final optimization runs. About $1700 additional was used for program development and data reduction.

to providing useful information on where to station ambulances, the study has provided insight on what the county would gain by purchasing a new ambulance. Implementation of these results has not yet taken place, but will be under consideration in the near future.

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Appendix A
Response Time

In this appendix an expression is developed for the expected value of the response time, which is defined as the time interval between the time a call is received and the time an ambulance arrives at the scene. It is assumed that when a call is received the nearest ambulance (in the sense of driving time) is dispatched. It is further assumed that if there are N ambulances assigned to provide service to the county of which K are in service, the remaining N-K are optimally located. That is, every time an ambulance goes into service, the remaining vehicles are instantaneously relocated in an optimal manner. Since the driving times for relocation are small relative to the average time between calls the assumption of instantaneous relocation, which greatly simplified the calculation, is reasonable.

From the above it is seen that one needs to determine not only the locations of N ambulances, but also the locations assuming 1, 2, ..., or N-1 ambulances are available. Thus, define x_j, $j=1,\ldots,N$ to be the set containing the j station locations to be used if only j ambulances are available. For convenience let $X_N=\{x_i\}_1^N$ denote the collection of all of these sets.

Let the driving time between any two points $z=(z_1,z_2)$ and $y=(y_1,y_2)$ be denoted by $\rho(z,y,\phi)$ where ϕ is a vector of all random variables which affect the driving time, e.g. weather condition, traffic condition, and time of day. Given that a call is received from a location y and that K ambulances are in service, the response time to that call will then be given by

$$\rho_2(X_N,y,K,\phi) = \min_{z\varepsilon x_{N-K}} \rho_1(z,y,\phi) \qquad \text{(A-1)}$$

To get the average response time we simply take the expected value of $\rho_2(X_N, y, K, \phi)$ with respect to the paramaters y, K, and ϕ. Denoting the average response time by $\bar{T}_r$ we obtain

$$\bar{T}_r = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho_2 \, (X_N, y, K, \phi) \; f(y, K, \phi) \; d\phi \; dK \; dy \tag{A-2}$$

where $\int dy$ and $\int d\phi$ denote the appropriate multiple integrals and $f(y,K,\phi)$ is the joint probability density function. It has been tacitly assumed that no more than N-1 ambulances are in service when a call is received (i.e., there is always an available ambulance).

To achieve some simplification of this, several assumptions will be made. First, the county (see fig.1) which is 30 miles by 24 miles, is divided into squares one mile on a side. All calls within a one-mile square will be considered to come from a single representative point within that square and all distances from that square will be measured relative to that point. Next, it will be assumed that the variables y, K, and ϕ are statistically independent. Then, noting that K takes only integer values between 0 and N-1, equation (A-2) can be written

$$\bar{T}_r = \sum_{K=0}^{N-1} \; \sum_{y_1=1}^{30} \; \sum_{y_2=1}^{24} \bar{\rho}_2 (X_N, y, K) \; p(K) \; p_1(y_1, y_2) \tag{A-3}$$

where $\bar{\rho}_2(X_N, y, K)$ is the expected value of $\rho_2(X_N, y, K, \phi)$ w.r.t. ϕ, $p(K)$ is the probability that K ambulances are in service, and $p_1(y_1, y_2)$ is the probability of a call occurring in the square with coordinates (y_1, y_2).

In this form, the problem of determining the location vector in the set X_N may be decoupled by writing equation (A-3) as

$$\bar{T}_r = \sum_{y_1=1}^{30} \sum_{y_2=1}^{24} \bar{\rho}_2(X_N, y, 0)\ p(0)\ p_1(y_1, y_2)$$
$$+$$
$$\sum_{y_1=1}^{30} \sum_{y_2=1}^{24} \sum_{K=1}^{N-1} \bar{\rho}_2(X_K, y, K)\ p(K)\ p_1(y_1, y_2) \qquad \text{(A-4)}$$

From equation (A-1) it is seen that the term on the left involves only the location set x_N, while the term on the right involves $x_1, \ldots, x_{N-1}$ but not x_N. Thus, one may successively solve for $x, x_2, \ldots, x_N$ using at each stage the left term in equation (A-4)

To get p(K), let there be Q calls in a year and let the average length of service of an ambulance be $\bar{T}_s$. Given that a call occurs at time t, p(K) may be approximated by the probability that exactly K calls occur during the previous $\bar{T}_s$ minutes. Since a call either does or does not fall within this time interval, binomial probabilities may be used, and

$$p(K) = \binom{Q-1}{K} \sigma^K (1-\sigma)^{Q-K-1} \qquad \text{(A-5)}$$

where $\sigma = \bar{T}_s/1$ yr. Table II (see Ch. 2) shows a few values of p(K) for Washtenaw County. It can be seen that the assumption that not all N ambulances are in service when a call is received is valid for $N \geq 4$.

Although the average response time is a significant measure of the ambulance system performance, it may also be desired that no response be longer than some predetermined maximum, T_m. In an absolute sense, this cannot be guaranteed. However, one can include a constraint which would require that the average response time to any point in the county be less than T_m if at least r ambulances are available at the time a call is received. This latter condition on the number of ambulances available is necessary because if the number is too low, there may be no set of locations from which one can reach any point within T_m minutes. This constraint is incorporated by using a penalty function and modifying the performance measure to:

$$\bar{T}_r = \sum_{y_1=1}^{30} \sum_{y_2=1}^{24} \sum_{K=0}^{N-r} \{\bar{\rho}_2(X_N,y,K) + \frac{1}{\varepsilon}\mathrm{Pos}\,[T_m - \bar{\rho}_2(X_N,y,K)]\}\, p(K)\; p_1(y_1,y_2)$$

$$+ \sum_{y_1=1}^{30} \sum_{y_2=1}^{24} \sum_{K=N-r+1}^{N-1} \bar{\rho}_2(X_N,y,K)\; p(K)\; p_1(y_1,y_2) \qquad \text{(A-6)}$$

where

$$\mathrm{Pos}\ (w) = \begin{cases} w \text{ if } w>0 \\ o \text{ if } w\leq 0 \end{cases}$$

and ε is an arbitrary small positive number.

Appendix B
Driving Time Model

From equation (A-1) it is seen that calculating the response time requires the ability to compute the point-to-point driving time between any pair of points. This can be written

$$\rho_1(z,y,\phi) = \int_\Gamma \gamma(w,\phi)\, dw + \gamma_0(\phi) \qquad \text{(B-1)}$$

where $\gamma(w,\phi)$ is the reciprocal velocity along the path Γ, and $\gamma_0(\phi)$ is the delay in starting, and Γ is the path chosen between z and y.

As an approximation, let all roads be divided into four categories (1) limited-access expressways, (2) paved county roads, (3) city streets, and (4) unpaved county roads. Then let $\Gamma_i = \{z: z\varepsilon\Gamma$ and z on a road of type i$\}$. In other words, Γ_i is that portion of Γ consisting of roads of type i. Thus $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4$ and the integral in equation (B-1) can be written

$$\rho_1(z,y,\phi) = \int_{\Gamma_1} \gamma(w,\phi)dw + \int_{\Gamma_2} \gamma(w,\phi)dw + \int_{\Gamma_3} \gamma(w,\phi)dw + \int_{\Gamma_4} \gamma(w,\phi)dw + \gamma_0(\phi)$$

But, $\int_{\Gamma_i} \gamma(w,\phi)dw$ is simply the average inverse velocity $\gamma_i(\phi)$ over Γ_i multiplied by the length of path Γ_i, d_i. Thus,

$$\rho_1(z,y,\phi) = \gamma_0(\phi) + \gamma_0(\phi)d_1 + \gamma_2(\phi)d_2 + \gamma_3(\phi)d_3 + \gamma_4(\phi)d_4 \qquad \text{(B-2)}$$

Letting $\bar{\gamma}_i = E[\gamma_i(\phi)]$ this can be written

$$\bar{\rho}_1(z,y) = \bar{\gamma}_0 + \bar{\gamma}_1 d_1 + \bar{\gamma}_2 d_2 + \bar{\gamma}_3 d_3 + \bar{\gamma}_4 d_4 \qquad \text{(B-3)}$$

where the expectation is carried out relative to the random variables ϕ.

In order to complete the driving time model, the path and, hence, the distances $d_1,\ldots,d_4$ must be specified. It is at this point that the key approximation in the model is made. A set of M (59 for Washtenaw County) ''major'' intersections were selected from the county map and an array of driving times between them computed using equation (B-3). Denoting this array by A, the driving time from intersection i to intersection j then is A(i,j). In this and subsequent time calculations, the distances d_i were obtained by using typical routes determined from area maps.

For each of the one-mile square subdivisions of the county, three surrounding intersections were selected. These may be viewed as possible entry points to the cross-country travel network. This information is stored in a second array B_o; $B_o(j,z_1,z_2)$ contains the index of the j-th (j= 1,2,3) intersection near the coordinate (z_1,z_2). Similarily a third array B_1 containing the driving time from (z_1,z_2) to intersection $B_o(j,z_1,z_2)$ may be generated.

$B_1(j,z_1,z_2)$ is the driving time from $B_O(j,z_1,z_2)$ to (z_1,z_2).

To determine an estimate of the driving time from $z=(z_1,z_2)$ to $y=(y_1,y_2)$ one can form

$$\alpha(z,y) = \min_{\substack{1\leq i\leq 3\\ 1\leq j\leq 3}} \{B_1(i,z_1,z_2)+ A[B_0(i,z_1,z_2),B_0(j,y_1,y_2)] + B_1(j,y_1,y_2)\} \tag{B-4}$$

In instances where z and y are close it will be faster for the ambulance to take a more direct route from z to y, which may be approximated by the sum of the coordinate distances. Since this route will generally only involve city streets or unpaved roads and since γ_3 and γ_4 do not differ widely, the inverse speed $\bar{\gamma}_4$ was applied to all such distances. Thus a second quantity

$$\beta\ (z,y) = (|z_1-y_1| + |z_2-y_2|)\cdot\bar{\gamma}_4 \tag{B-5}$$

is computed. Then the driving time from z to y is computed as

$$\bar{\rho}_1(z,y) = \min\ [\alpha(z,y),\beta(z,y)] \tag{B-6}$$

Finally, to obtain $\bar{\rho}_2(X_N, z, K)$ we form

$$\bar{\rho}_2(X_N, y, K) = \min_{z \varepsilon X_{N-K}} \bar{\rho}_1(z, y) \tag{B-7}$$

Appendix C
Optimization Algorithm

In order to minimize $\bar{T}_r$, the problem is decoupled as in equation (A-4) and iteratively solved for N=1,2,...,. Hence we assume that the solutions for $x_1,\ldots,x_{N-1}$ are known. The sum on the right of equation (A-4) depends only on $K\geq 1$, and hence only on $x_1,\ldots,x_{N-1}$. Thus, this term is known and the minimization may be carried out with respect to the left-hand sum

$$T_1(X_N) = p(o) \sum_{y_1=1}^{30} \sum_{y_2=1}^{24} \bar{\rho}_2(X_N, y, o)\; p_1(y_1, y_2) \qquad \text{(C-1)}$$

in which the only variable is x_N.

To minimize $T_1(x_N)$ a discrete version of steepest descents is used. For convenience, let x_N also denote an ordered vector of the elements of the set x_N. Beginning with an initial guess x_N^o, a sequence of location vectors is generated via the equation

$$x_N^{i+1} = x_N^i + \alpha^i \Delta T_1(x_N^i) \qquad \text{(C-2)}$$

where x_N^i is the i-th element in the sequence, $\Delta T_1(x_N^i)$ is a direction from x_N^i and α^i is chosen to minimize $T_1(x_N^i + \alpha\Delta T_1(x_N^i))$.

Due to the discretization of the county into one mile squares, several simplifications occur. First, only integer values need be considered for the entries in x_N^i. Thus a suitable descent direction may be found by forming the forward and backward differences $\Delta T_1^f(x_N^i)$ and $\Delta T_1^b(x_N^i)$ where j-th components are given by

$$\Delta T_{1j}^f(x_N^i) = T_1(x_N^i + e_j) - T_1(x_N^i)$$

$$\Delta T_{1j}^b(x_N^i) = T_1(x_N^i) - T_1(x_N^i - e_j)$$

where the e_j are the standard basis elements in E^{2N}. Then, one may choose $\Delta T_1(x_N^i)$ so that its j-th component is

$$\Delta T_{ij}(x_N^i) = \begin{cases} \Delta T_{1j}^f(x_N^i) & \text{if } 0 \le -\Delta T_{1j}^f(x_N^i) \ge T_{1j}^b(x_N^i) \\ \Delta T_{1j}^b(x_N^i) & \text{if } 0 \le T_{1j}^b(x_N^i) > -\Delta T_{1j}^f(x_N^i) \\ 0 & \text{otherwise} \end{cases} \qquad \text{(C-3)}$$

That is, each component of $\Delta T_1(x_N^i)$ is chosen to bring about the greatest decrease in T_1.

The sequence of $\{T_1(x_N^i)\}$ generated by the above procedure will clearly be nonincreasing. Since the entries of X_N may take on only integer values over a finite domain, there are only a finite number of possible values for $T_1(x_N^i)$. Hence, the iteration using equations (C-2) and (C-3) must eventually yield $x_N^{i+1}=x_N^i$. This is a natural stopping condition for the iteration. As a final check for a local minimum $T_1(x_N^i)$ is evaluated at all points in a neighborhood of x_N^i (this is reasonable since there are only a finite number of possibilities).

The major difficulty encountered in the implementation of the problem was the existence of local minima. These occur naturally at many points in the county. For example, it is reasonable to expect a local minimum to occur at most expressway entries, for if one moves the station a short distance away from the entry point, the driver would, for many calls, simply have to drive back to the expressway entry. This problem was overcome in the usual way by restarting the procedure from a number of different points.