Reduced Order and Prioritized Reference Governors for Limit Protection in Aircraft Gas Turbine Engines

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This paper considers the application of the reduced order and prioritized reference governors to protect aircraft gas turbine engines against constraint violation. The aircraft gas turbine engine control system must enforce various constraints that protect the engine from surge, over-speed, over-temperature and combustion blowout conditions, while satisfying actuator magnitude and rate limits. The reference governor is an add-on scheme augmented to a nominal engine controller, that modifies set-points that the nominal controller responds to if there is a danger of constraint violation. Prioritized reference governors enforce hard constraints and soft constraints in the order of their priority in cases when strictly enforcing all the constraints is infeasible. The reduced order reference governor theory facilitates the rigorous design of reference governors based on reduced order models so that the constraints are satisfied by the full order system. Simulation results based on a gas turbine engine model are reported that illustrate constraint handling using prioritized and reduced order reference governor techniques.

Nomenclature

WFE	Fuel flow
AJ	Variable area nozzle setting
IGV	Inlet guide vane setting
$OPR1$	Ratio of the HP compressor's outlet pressure to engine inlet pressure
$LPEMN$	LP compressors exit Mach number
NH	HP compressor's spool speed
x_i	State augmented in LQ tracking design
C_{thrust}	The row in C matrix that corresponds to thrust output
0_{ij}	Zero matrix with i rows and j columns
$r(t)$	Reference (set-point) command
$v(t)$	Reference governor output, i.e., the command actually applied to the system
H, h	Matrices defining inequality constraints
Q	Weight matrix in reference governor
$\tilde{O}_\infty$	Maximum output admissible set
$\tilde{v}(t)$	Augmented reference governor output
ϵ	Slack variable
Δ_1, Δ_2	Command constraints bounds
b_i	The ith component of the command constraint matrix
R_1, R_2	Weight matrices used by prioritized reference governor
λ_i	The ith diagonal component of the weight matrix
E_x	State error set used by the reduced order reference governor
E_y	Output error set used by the reduced order reference governor
ξ	Design parameter defined in output error set

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y_r	Reduced order system output

Subscripts

d	State space representation of discrete-time system
aug	System with augmented states
sd	Slow subsystem of discrete-time system after fast-slow decomposition
fd	Fast subsystem of discrete-time system after fast-slow decomposition

I.   Introduction

This paper considers the application of reference governors to aircraft engine control. The reference governors are add-on schemes that augment nominally designed controllers to enforce the constraints, see e.g., References[3,9] and citations therein. The reference governor is nominally inactive and minimally modifies the commands passed to the closed-loop system to ensure that the imposed state and control constraints are satisfied. This modification is performed based on the prediction into the future of the closed-loop system response.

In applying the reference governor to practical systems such as aircraft gas turbine engines, there are several challenges. These challenges include handling situations where satisfying all constraints at once is infeasible, and situations where the system model is high order thereby leading to a high computational complexity of the solution. Recent papers[5–7] have addressed these challenges. Prioritized reference governor design can enforce hard constraints and soft constraints in the order of their priority. Slight violation of the soft constraints is permitted (if all constraints cannot be strictly met) with less violation for higher priority soft constraints. This is accomplished by introducing and differentially weighting in the cost appropriately defined slack variables. Reduced order reference governor design facilitates rigorous enforcement of the constraints based on the low order slow subsystem models, while accounting for the contributions of both slow states, fast states and observer errors in the prediction. In this paper, these approaches are exemplified for limit protection in aircraft gas turbine engines.

The aircraft gas turbine engine control system must enforce a variety of constraints such as surge avoidance, over-speed and over-temperature limits, combustion lean blowout limit, actuator magnitude and rate limits, etc.[4,13] Furthermore, the aircraft gas turbine engine models can be high order[12] even though for control design their order is often reduced[4] by treating the dynamics of flows and pressures as being faster than the dynamics of the rotational speeds. In Reference[10] the application of the robust reference governor to aircraft gas turbine engine control based on low order models has been considered. In this paper we exemplify the designs enabled by recent advances in References[5,6] which include prioritized constraint handling and the treatment based on the high order model via the reduced order reference governor.

In principle, the reference governor is similar to the classical topping governor[4] used already in aircraft engine control applications. Its main difference with the topping governor is that it accounts for the model-based prediction of the engine dynamic response to determine command modifications necessary to enforce the full set of engine constraints. The reference governor is also a different strategy from Model Predictive Control (MPC) that has been applied to gas turbine engine control previously, see e.g., References.[2,11] While the reference governor is a variant of a predictive controller, the main advantages of the reference governor are in augmenting a nominal controller (which can be designed by a variety of linear and nonlinear control design techniques) rather than replacing it with a Model Predictive Controller, rigorously guaranteeing recursive feasibility under command changes, finite-time convergence properties for constant, nearly constant and slowly varying commands, and in simpler computations and software structure.

For the simulation case study in this paper, we use the linear engine model published by Skogestad and Postlethwaite.[12] The engine under consideration is a Rolls-Royce 2-spool reheated turbofan Spey engine. The model represents the engine dynamics at 87% of maximum thrust and sea level static conditions.

The paper is organized as follows. Details of the linear model and nominal control design are given in Section II. Constraints are discussed in Section III. In Section IV we review the prioritized and reduced order reference governor design procedures developed in References.[5–7] In Section V we consider several case studies where the use of prioritized reference governor and reduced order reference governor is illustrated. Concluding remarks are made in Section VI.

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II.   Aircraft Engine Model

Our case studies are based on the Spey engine model,[12]

$$\dot{x} = Ax + Bu, \tag{1}$$

$$y = Cx + Du. \tag{2}$$

The model has 18 states (15 engine states and 3 actuator states), and three control inputs,

$$u = \begin{bmatrix} WFE \\ AJ \\ IGV \end{bmatrix}.$$

where WFE is the fuel flow, AJ is the setting for the variable area nozzle, and IGV is the variable angle setting for inlet guide vanes. The model has three outputs,

$$y = \begin{bmatrix} OPR1 \\ LPEMN \\ NH \end{bmatrix}.$$

The engine thrust is correlated with the first output ($OPR1$), which is defined as the ratio of the HP compressor outlet pressure to engine inlet pressure. The second output is the LP compressor exit Mach number measurement ($LPEMN$) which is indicative of and used in enforcing the LP compressor surge margin constraint. The third output is HP compressor spool speed (NH); this output is used in defining the over-speed constraint. We do not consider other constraints in this paper as they are not represented in the model published in Reference.[12]

In the model (1)-(2), the thrust-related output ($OPR1$) is scaled so that one unit of it represents a 7.5% of maximum thrust. Similarly, the surge margin-related output $LPEMN$ is scaled so that one unit of it corresponds to 5% surge margin. Finally, the HP compressor spool speed output (NH) is scaled so that one unit of it corresponds to a 2.2% of maximum value. The inputs (WFE, AJ, IGV) are scaled so that one unit corresponds to 10% of each of their respective range.

The nominal controller is based on the LQ tracking design. Specifically, we augment an error integrator to the system with the state x_i and use the input WFE to control the output $OPR1$ to the set-point r_{OPR1}. Then the augmented system takes the form,

$$\dot{x}_{aug} = \tilde{A}x_{aug} + \tilde{B}_{WFE}u_{WFE} + \tilde{B}_{AJ,IGV}u_{AJ,IGV} + \begin{bmatrix} 0_{n1} \\ -r_{OPR1} \end{bmatrix}, \tag{3}$$

$$y_{thrust} = [C_{thrust}, 0]x_{aug} + D(1,:)u, \tag{4}$$

where

$$x_{aug} = \begin{bmatrix} x \\ x_i \end{bmatrix}, \tag{5}$$

$$\tilde{A} = \begin{bmatrix} A & 0_{n1} \\ C_{thrust} & 0 \end{bmatrix}, \ \tilde{B}_{WFE} = \begin{bmatrix} B(:,1) \\ D(1,1) \end{bmatrix}, \ \tilde{B}_{AJ,IGV} = \begin{bmatrix} B(:,2:3) \\ D(1,2:3) \end{bmatrix}, \ \tilde{u}_{AJ,IGV} = \begin{bmatrix} AJ \\ IGV \end{bmatrix}, \ C_{thrust} = C(1,:). \tag{6}$$

The fuel flow (WFE) nominal controller is of the form,

$$u_{WFE} = -K_1 x - K_2 x_i, \tag{7}$$

where the gains K_1 and K_2 are obtained on the basis of the linear quadratic theory applied to $\tilde{A}$, $\tilde{B}_{WFE}$ and weights Q and R defined by

$$Q = \begin{bmatrix} 0_{nn} & 0_{n1} \\ 0_{1n} & 1 \end{bmatrix}, \quad R = 1,$$

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and where n is the dimensionality of x.

With the LQ WFE controller, the closed loop system becomes,

$$\dot{x}_{aug} = (\tilde{A} - \tilde{B}_{WFE}[K_1, K_2])x_{aug} + \begin{bmatrix} 0_{n1} & B(:,2:3) \\ -1 & D(1,2:3) \end{bmatrix} \begin{bmatrix} r_{OPR1} \\ AJ \\ IGV \end{bmatrix}, \tag{8}$$

$$y = [C - D(:,1)K_1, -D(:,1)K_2]x_{aug} + [0_{31}, D(:,2:3)] \begin{bmatrix} r_{OPR1} \\ AJ \\ IGV \end{bmatrix}. \tag{9}$$

In the application of the reference governor, we replace the commanded

$$r(t) = \begin{bmatrix} r_{OPR1} \\ AJ \\ IGV \end{bmatrix},$$

by the reference governor output,

$$v(t) = \begin{bmatrix} v_{OPR1} \\ v_{AJ} \\ v_{IGV} \end{bmatrix}.$$

The modified set-points, $v(t)$, may deviate from $r(t)$ if the danger of constraint violation exists.

Converting the continuous-time closed-loop system model to discrete-time with the sampling period of 0.01 sec yields

$$x_{aug}(t+1) = A_d x_{aug}(t) + B_d v(t), \tag{10}$$

$$y(t) = C_d x_{aug}(t) + D_d v(t), \tag{11}$$

where A_d is a 19 by 19 matrix (15 engine states, 3 actuator states and 1 controller state), B_d is a 19 by 3 matrix (3 reference inputs), C_d is a 3 by 19 matrix (3 outputs), and D_d is a 3 by 1 zero matrix.

III. Constraints

We consider the following set of constraints. To ensure that an adequate surge margin is maintained, we impose a lower limit on $LPEMN$ of the form,

$$LPEMN_{min} \leq LPEMN. \tag{12}$$

To prevent over-speed conditions, we impose an upper limit on NH of the form,

$$NH \leq NH_{max}. \tag{13}$$

These constraints can be expressed in the following form:

$$y(t) \in Y, Y = \{y : Hy \leq h\}, \tag{14}$$

where

$$y = \begin{bmatrix} OPR1 \\ LPEMN \\ NH \end{bmatrix}, \quad H = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad h = \begin{bmatrix} -LPEMN_{min} \\ NH_{max} \end{bmatrix}.$$

Finally, we consider another constraint which is to avoid large deviations of the output of the reference governor, $v(t)$, from the original command, $r(t)$, i.e.,

$$-\Delta_1 \leq v(t) - r(t) \leq \Delta_2, \tag{15}$$

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where

$$r(t) = \begin{bmatrix} r_{OPR1} \\ AJ \\ IGV \end{bmatrix}, v(t) = \begin{bmatrix} v_{OPR1} \\ v_{AJ} \\ v_{IGV} \end{bmatrix}, \tag{16}$$

and the bounds Δ_1, Δ_2 are given.

We note that the constraint (15) is unusual and is not handled by the conventional reference governor theory. In applications, imposing this constraint can be useful to ensure that the reference governor does not create large modifications, however, with no information on future values of $r(t)$, feasibility, if this constraint is handled as a hard constraint, cannot be guaranteed. This constraint can, however, be treated more easily by a prioritized reference governor as we discuss next.

IV. Prioritized and reduced order reference governors

The reference governor schematics are shown in Figure 1. The reference governor modifies $OPR1$ as well as AJ and IGV settings.
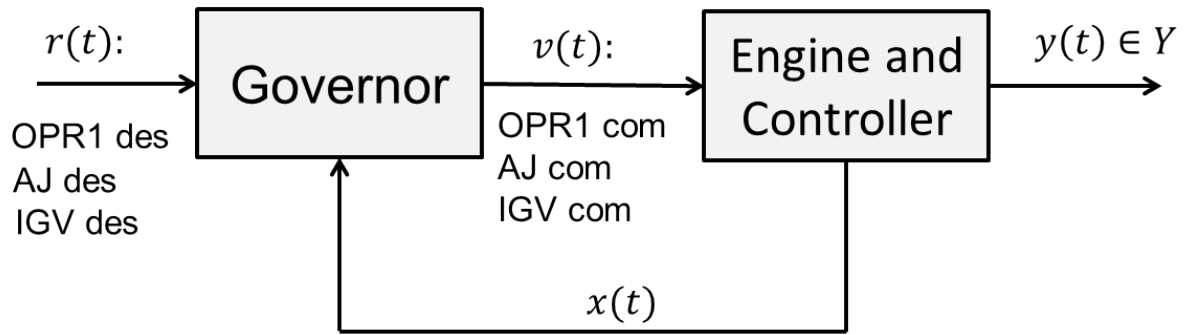

Figure 1: The system architecture. The commands with subscript *des* and $r(t)$ represent the original commands, the ones with subscript *com* and $v(t)$ represent modified commands with the reference governor. The reference governor enforces constraints $y(t) \in Y$ using state estimate, $x(t)$.

A. Reference governor

The conventional reference governor (based on command governor strategy[1]) is based on determining $v(t)$ via a solution of the following optimization problem:

$$minimize \ (v(t) - r(t))^T Q(v(t) - r(t)), \tag{17}$$

subject to

$$(v(t), x(t)) \in \tilde{O}_\infty,$$

where $Q = Q^T > 0$ and $\tilde{O}_\infty$ is the maximum output admissible set,[3] i.e., the set of all *constant* commands and closed-loop system states such that the ensuing response satisfies the imposed constraints. For the polyhedral-type constraints described in Section III, and under additional, reasonable assumptions, $\tilde{O}_\infty$ is a polytope defined by a finite number of linear inequalities.

B. Prioritized reference governor

In the Prioritized Reference Governor (PRG) approach,[5] some of the hard constraints are replaced by soft constraints. For instance, the hard output constraints in (14) can be replaced by soft constraints

$$Hy \le h + \epsilon_1, \tag{18}$$

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where $\epsilon_1 \geq 0$ is a vector of slack variables. A similar strategy can be pursued with respect to the constraint (15) by replacing it with a soft constraint

$$-\Delta_1 - \epsilon_2 \leq v(t) - r(t) \leq \Delta_2 + \epsilon_2, \tag{19}$$

where $\epsilon_2 \geq 0$ is a vector of slack variables. We define

$$\epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix}. \tag{20}$$

In the PRG design,[5] the closed loop system is extended with the slack variable vector as follows

$$x_{aug}(t+1) = A_d x_{aug}(t) + \begin{bmatrix} B_d & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ \epsilon(t) \end{bmatrix}, \tag{21}$$

$$y'(t) = \begin{bmatrix} y \\ \epsilon \end{bmatrix} = \begin{bmatrix} C_d \\ 0 \end{bmatrix} x_{aug}(t) + \begin{bmatrix} D_d & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} v(t) \\ \epsilon(t) \end{bmatrix}, \tag{22}$$

and the constraint set Y is replaced by Y' where

$$y' \in Y' = \{(y, \epsilon) : \epsilon = (\epsilon_1, \epsilon_2),\ Hy \leq h + \epsilon_1,\ \epsilon \geq 0\}. \tag{23}$$

With $\tilde{v}(t)$ defined as:

$$\tilde{v}(t) = \begin{bmatrix} v(t) \\ \epsilon(t) \end{bmatrix}, \tag{24}$$

and $\tilde{O}'_\infty$ (or its subset satisfying appropriate assumptions[3]) defined for the augmented system, the PRG governs the augmented signal based on the minimization of the cost,

$$minimize\ (r(t) - v(t))^T R_1 (r(t) - v(t)) + \epsilon^T R_2 \epsilon\ \text{w.r.t}\ v(t), \epsilon, \tag{25}$$

subject to the constraints,

$$(\tilde{v}(t), x(t)) \in \tilde{O}'_\infty,\ \ -\Delta_1 - \epsilon_2 \leq v(t) - r(t) \leq \Delta_2 + \epsilon_2,\ \tilde{v} = [v^\mathrm{T}, \epsilon^\mathrm{T}]^\mathrm{T},\ \epsilon = [\epsilon_1^\mathrm{T}, \epsilon_2^\mathrm{T}]^\mathrm{T}, \tag{26}$$

where $R_1 = R_1^\mathrm{T} > 0$, $R_2 = R_2^\mathrm{T} > 0$, are weighting matrices. This optimization problem is a quadratic programming problem with linear inequality constraints that can be easily handled by existing quadratic programming solvers.

The weight matrix R_2 can be defined to give different priorities to different constraints. For instance, if R_2 is diagonal, diagonal entries corresponding to higher priority constraints can be made larger.

To enforce some constraints as *hard* the corresponding elements of ϵ_1 or ϵ_2 can be constrained to zero. Alternatively, the corresponding constraints do not have to be relaxed by the slack variables from the start.[6]

To provide further insight, we note that based on the definition of the maximum output admissible set (and ignoring some technicalities), the minimization of the cost (25) is basically performed subject to constraints

$$-\Delta_1 - \epsilon_2 \leq v(t) - r(t) \leq \Delta_2 + \epsilon_2, \tag{27}$$

$$Hy(t+k|t, x(t), v(t)) \leq h + \epsilon_1, \forall k = 0, 1, ..., t^*, \tag{28}$$

$$\epsilon_1 \geq 0,\ \epsilon_2 \geq 0, \tag{29}$$

where $y(t+k|t, x(t), v(t))$ denotes the output prediction at the time instant $t+k$ given the state $x(t)$ at time t, command $v(t)$ (assumed constant over the prediction horizon, i.e., $v(t+k) = v(t)$, $k = 0, \cdots, t^*$), and t^* is the finite determination index of the maximum output admissible set.[3]

We note that the use of quadratic penalty functions on the slack variables has certain disadvantages, e.g., soft constraints can be slightly violated by the solution even if a feasible solution that strictly satisfies these constraints exists. If such violations are undesirable (i.e., soft constraints have to be strictly satisfied if possible), one can check for solutions of problem (25)-(26) with slack variables constrained to 0 and progressively relaxed, with highest priority constraints relaxed last. Another approach is to replace the quadratic penalty by linear (e.g., 1 or ∞ norm based) penalty with *sufficiently large* penalty weights.[8]

C.   Reduced order prioritized reference governor

For the development of a reduced order reference governor we consider a state transformation under which the closed-loop dynamic model decomposes into slow and fast subsystems[6] so that the governing closed-loop equations are given in discrete-time by,

$$
\begin{bmatrix} x_s(t+1) \\ x_f(t+1) \end{bmatrix} = \begin{bmatrix} A_{sd} & 0 \\ 0 & A_{fd} \end{bmatrix} \begin{bmatrix} x_s(t) \\ x_f(t) \end{bmatrix} + \begin{bmatrix} B_{sd} \\ B_{fd} \end{bmatrix} v(t), \tag{30}
$$

$$
y(t) = \begin{bmatrix} C_{sd} & C_{fd} \end{bmatrix} \begin{bmatrix} x_s(t) \\ x_f(t) \end{bmatrix} + D_d v(t), \tag{31}
$$

where $x_s(t) \in R^{n_s}$ and $x_f(t) \in R^{n_f}$ are the slow and fast state vectors. If it is assumed that the matrices A_{sd} and A_{fd} are Schur and the eigenvalues of A_{sd} are faster than those of A_{sf} .

The reduced order reference governor is designed based on the reduced order system model,

$$
x_s(t+1) = A_{sd} x_s(t) + B_{sd} v(t), \tag{32}
$$

$$
y_r(t) = C_{sd} x_s(t) + (C_{fd} \Gamma_f + D_d) v(t). \tag{33}
$$

where

$$
\Gamma_f = (I - A_{fd})^{-1} B_{fd}, \tag{34}
$$

and y_r is the system output that reflects the contributions of the slow states and steady-state contributions of fast states.

The reduced order reference governor is constructed to satisfy the tightened constraint on y_r and an additional constraint on $\Delta v(t) = v(t) - v(t-1)$ which ensures that command changes do not cause excessive deviations of fast states from their steady-state values:

$$
y_r(t+k|t) \in Y \sim E_y, \forall k \geq 0, \tag{35}
$$

$$
-A_{fd} \Gamma_f \Delta v(t) \in E_x \sim A_{fd} E_x, \tag{36}
$$

where the errors sets E_y and E_x are design parameters. The error set E_y is defined as

$$
E_y = \varepsilon Y, \tag{37}
$$

where $0 < \varepsilon < 1$ is a design parameter. The error set E_x is an A_{fd}-contractive set that can be computed as the maximum output admissible set corresponding to a scaled A_{fd} matrix and E_y as the output constraint set,[6]

$$
E_x = O_\infty(\frac{1}{q} A_{fd}, C_{fd}, E_y), \tag{38}
$$

where the contraction parameter is $0 \leq q < 1$.

For the development of the *prioritized reduced order reference governor,* we replace (30)-(31) with

$$
\begin{bmatrix} x_s(t+1) \\ x_f(t+1) \end{bmatrix} = \begin{bmatrix} A_{sd} & 0 \\ 0 & A_{fd} \end{bmatrix} \begin{bmatrix} x_s(t) \\ x_f(t) \end{bmatrix} + \begin{bmatrix} B_{sd} & 0 \\ B_{fd} & 0 \end{bmatrix} \tilde{v}(t), \tag{39}
$$

$$
y'(t) = \begin{bmatrix} C_{sd} & C_{fd} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_s(t) \\ x_f(t) \end{bmatrix} + \begin{bmatrix} D_d & 0 \\ 0 & I \end{bmatrix} \tilde{v}(t), \tag{40}
$$

where

$$
\tilde{v}(t) = \begin{bmatrix} v(t) \\ \epsilon(t) \end{bmatrix}, \tag{41}
$$

and $\epsilon = (\epsilon_1^{\mathrm{T}}, \epsilon_2^{\mathrm{T}})$ is a slack variable as introduced in Section B. The set Y is replaced by Y' defined in

$$
Y' = \{(y, \epsilon) : \ Hy \leq h + \epsilon_1, \ \epsilon \geq 0\},
$$

and the reduced order reference governor design is applied to this system (39)-(40) augmented with the slack variable ϵ and with the extended input (41).

V.   Simulation results

For the simulations, we choose $LPEMN_{min} = -1$, $NH_{max} = 5$ in (12)-(13), and a time-varying set-point command profile given as

$$r_{OPR1}(t) = 5(1 - e^{-t}), \quad r_{AJ}(t) = 3, \quad r_{IGV}(t) = 2, \quad t > 0. \tag{42}$$

The command constraints (19) are defined with

$$\Delta_1 = \Delta_2 = \begin{bmatrix} 1 \\ 1.5 \\ 1.5 \end{bmatrix}. \tag{43}$$

We first illustrate the operation of the full order PRG and its flexibility in handling different sets of soft and hard constraints. Three scenarios are considered.

In the first scenario, the surge constraint (12) and speed constraint (13) are treated as soft while the command constraint (19) is treated as hard. We further assign higher priority to the surge constraint than to speed constraint, so that the weight corresponding to the surge constraint is higher than that corresponding to the speed constraint. Then $\epsilon_2 = 0$ (as the command constraints are treated as hard) and the cost (25) is defined with

$$R_1 = diag([100, \ 10, \ 1000]), \quad R_2 = diag([100, 100, 0, 0, 0]). \tag{44}$$

Note that no weighting is applied to ϵ_2 since ϵ_2 is constrained to zero. This first scenario exemplifies a situation during aircraft emergency maneuvering where the control system accepts some level of risk[13] in enforcing the surge and speed constraints (as these are often set conservatively to begin with) provided it can ensure adequate thrust response.

In the second scenario we treat the surge constraint as hard and the speed constraint and command constraints as soft. We assign higher priority to speed constraint than to command constraints. Since there are three command constraints, the weight on the slack variable corresponding to $OPR1$ command is larger than weights corresponding to AJ and IGV commands. In this case, we constrain ϵ_1 to $\epsilon_1 = [0, \epsilon_{12}]$, where ϵ_{12} is a scalar and treat ϵ_2 as a three dimensional vector. The PRG weighting matrices are defined as

$$R_1 = diag([100, 10, 1000]), \quad R_2 = diag([0, 10000, 1000, 100, 100]). \tag{45}$$

Note that no penalty is applied to the first component of ϵ_1 since this component is constrained to be zero.

In the third scenario, the constraint and weight matrix are the same as in the second scenario. However, instead of applying an exponential profile (42) for OPR1, we investigate the response to a step change in $OPR1$ to the value 5 at 1 second.

The simulation results for these three scenarios are given in Figures 2-7. For the first simulation scenario, the speed constraint is violated, the surge constraint is violated less and the commands passed to the closed-loop system ride the constraint boundary. For the second simulation scenario, the surge constraint is strictly enforced, speed constraint is violated slightly while v_{OPR1} exceeds the bounds. For the third simulation scenario, the surge constraint is strictly enforced, speed constraint is violated slightly while v_{OPR1} exceeds the bounds and v_{AJ} is out of bounds for a short time to accommodate the sudden step change of $OPR1$.

Finally, we illustrate the operation of the reduced order PRG for the first scenario. The absolute values of the eigenvalues of the continuous-time closed-loop system model are

$$1.8763, 6.2272, 6.2272, 31.1960, 31.2500, 44.5930, 61.9103, 92.7397, 92.7397,$$

$$154.1014, 154.1014, 158.6060, 189.3077, 189.3077, 246.4429, 246.4429, 287.7015, 287.7015, 327.2601.$$

By considering eigenvalues with absolute values above 100 as fast we obtain a system decomposition into a slow subsystem with 9 states and a fast subsystem with 10 states. The reduced order PRG uses $q = 0.8$, $\varepsilon = 0.02$ (see (37)-(38) where these parameters are defined). The simulation results are given in Figures 8-9. Further model order reduction has been investigated. With the slow subsystem of 3rd order, the responses are not significantly different than the ones in Figures 8-9, with somewhat larger soft constraint violation, see Figures 10-11.
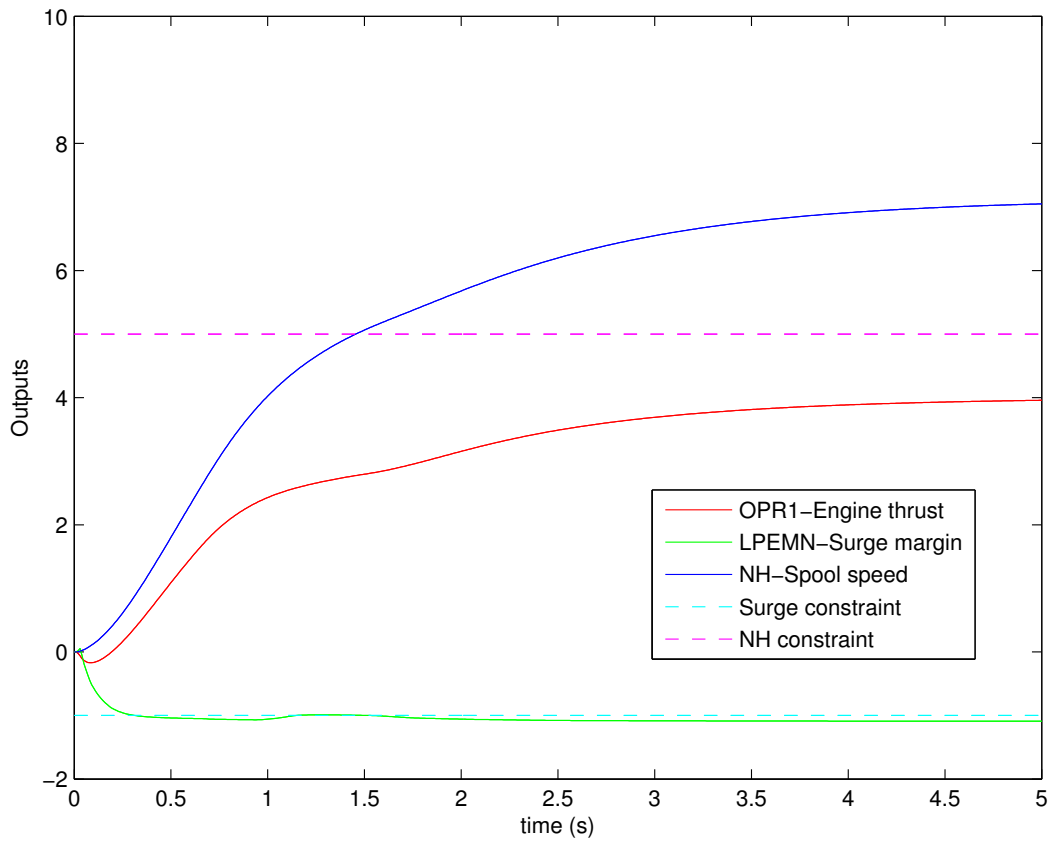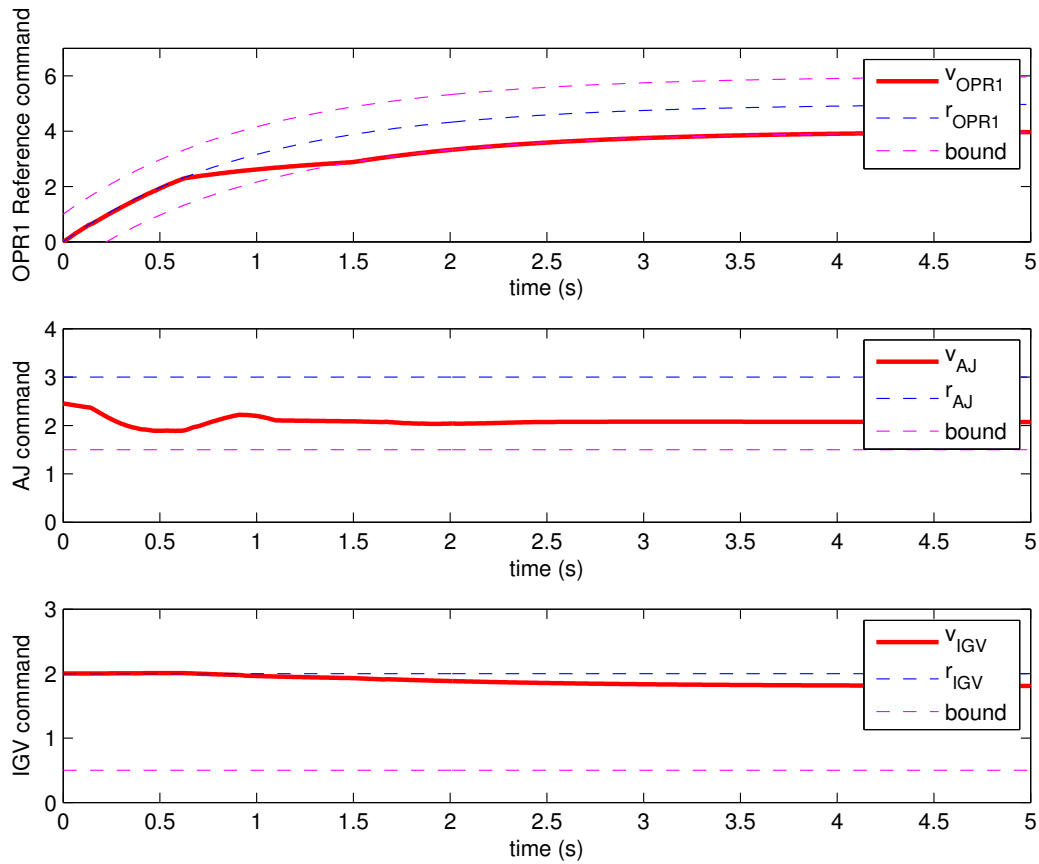
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Figure 2: Response with PRG in scenario 1.

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Figure 3: Commands (original and modified) with PRG in scenario 1.

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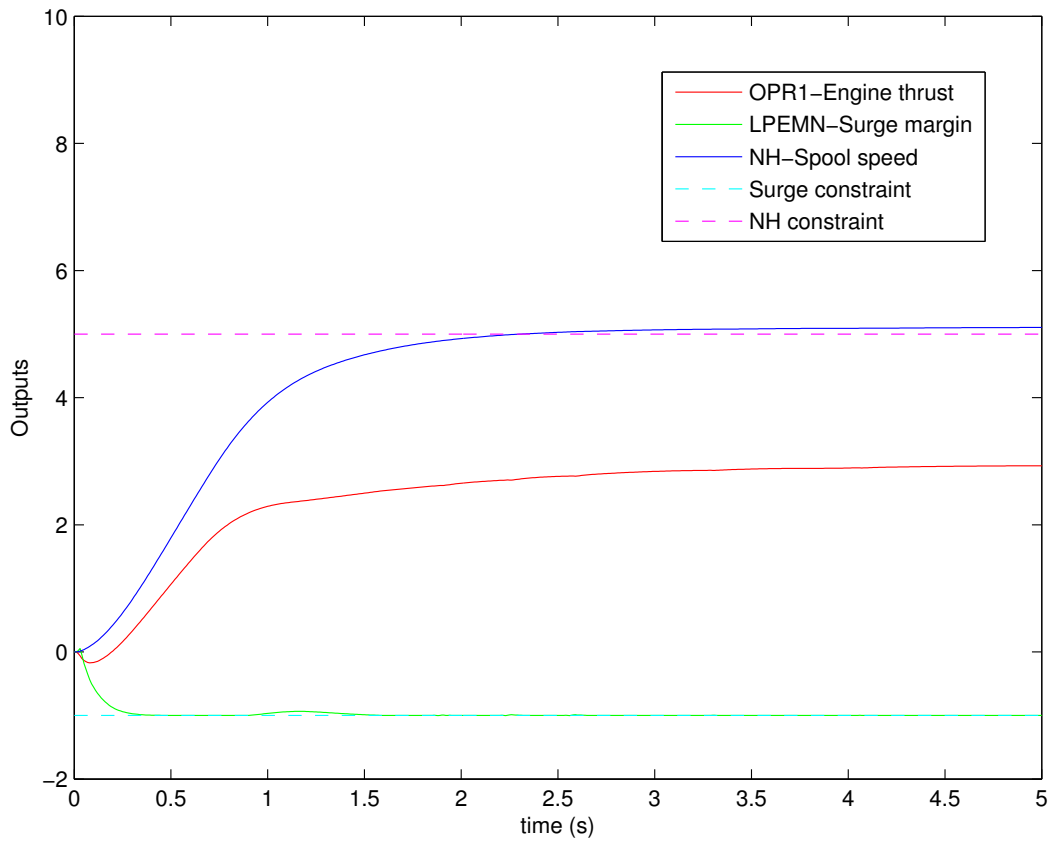
Figure 4: Response with PRG in scenario 2.

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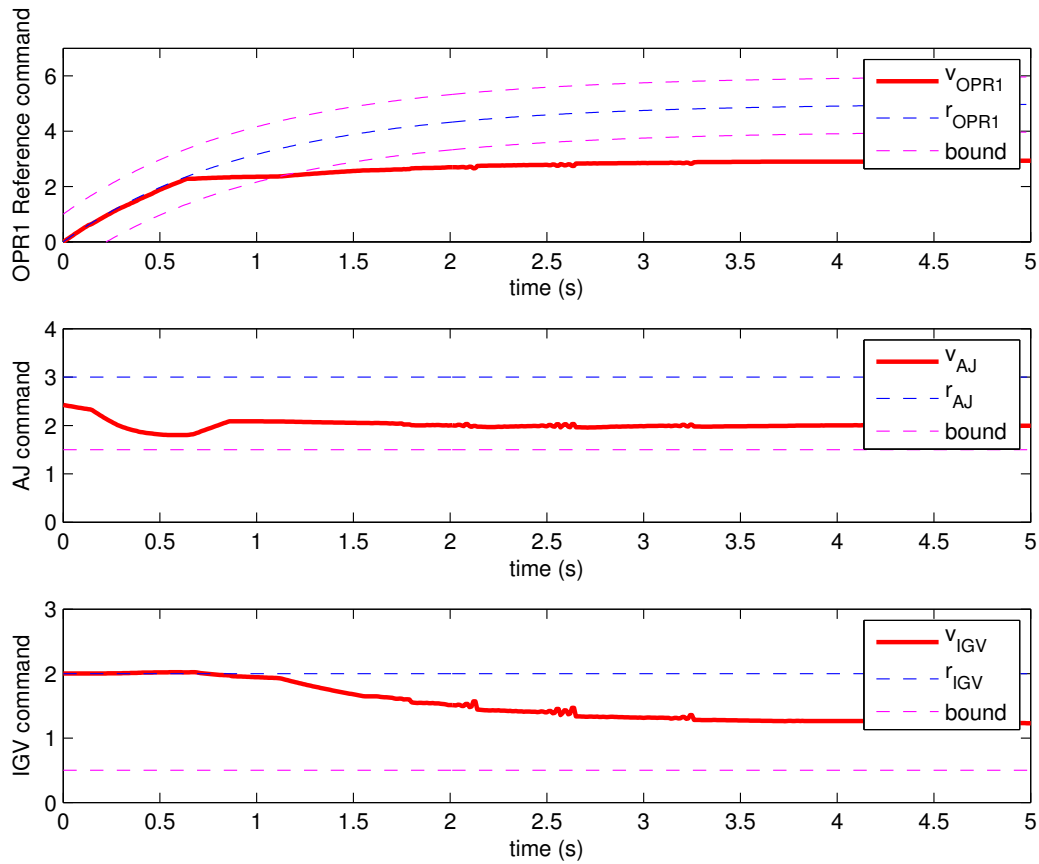
Figure 5: Commands (original and modified) with PRG in scenario 2.

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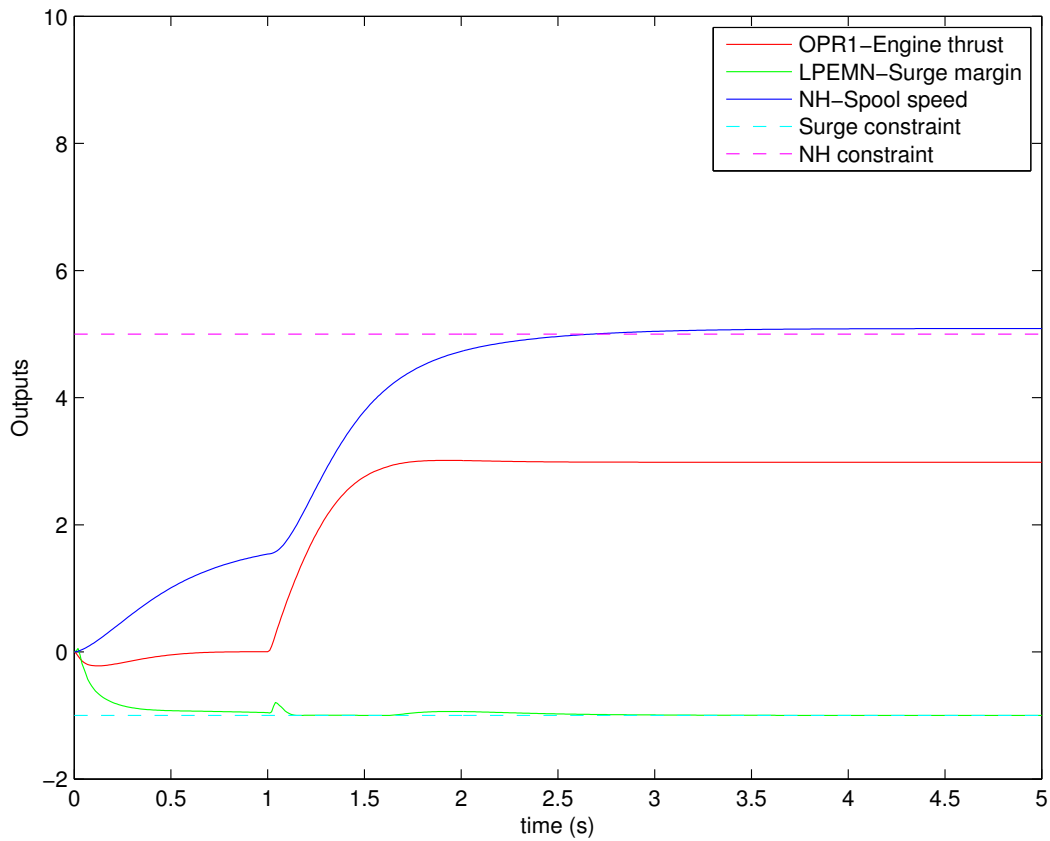
Figure 6: Response with PRG in scenario 3.

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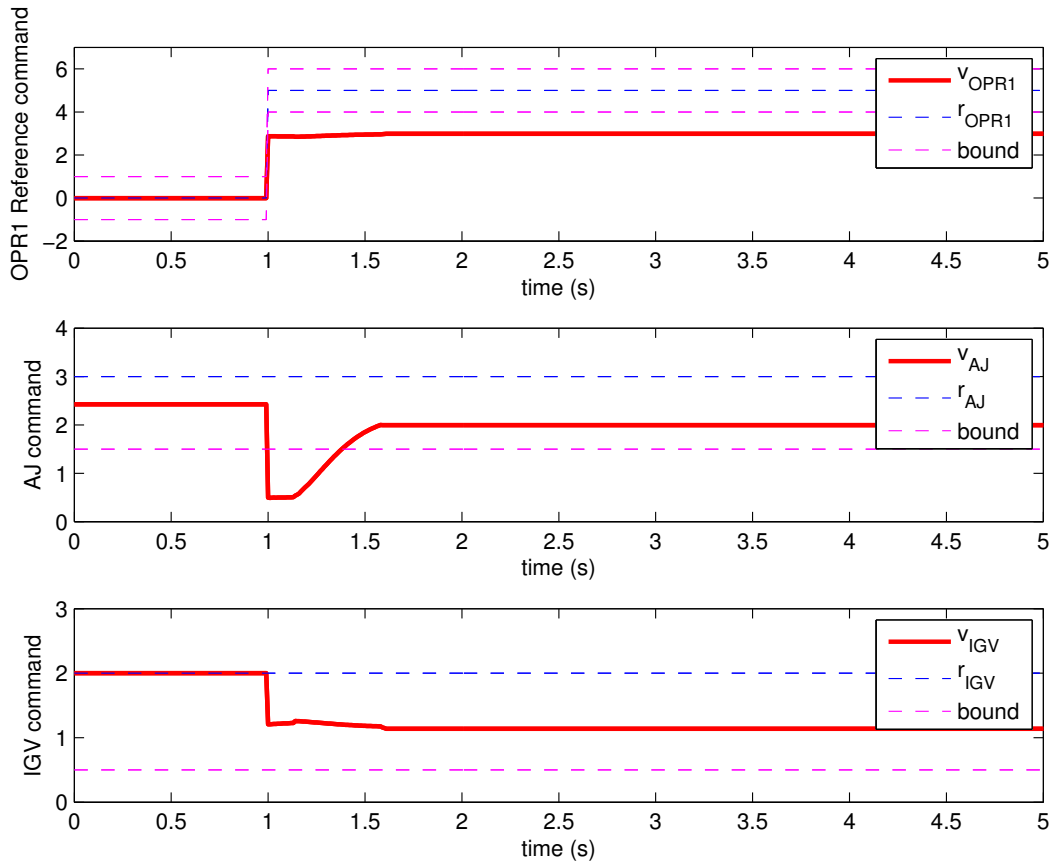
Figure 7: Commands (original and modified) with PRG in scenario 3.

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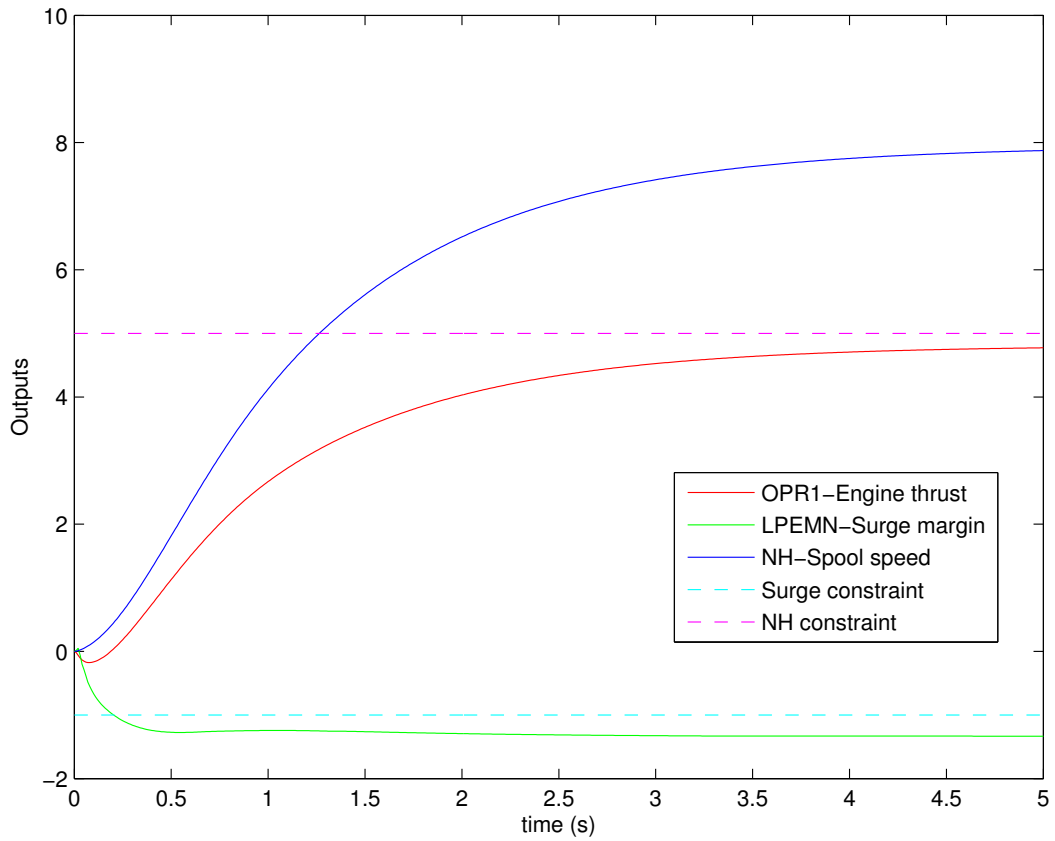
Figure 8: Responses with the reduced order PRG in scenario 1.

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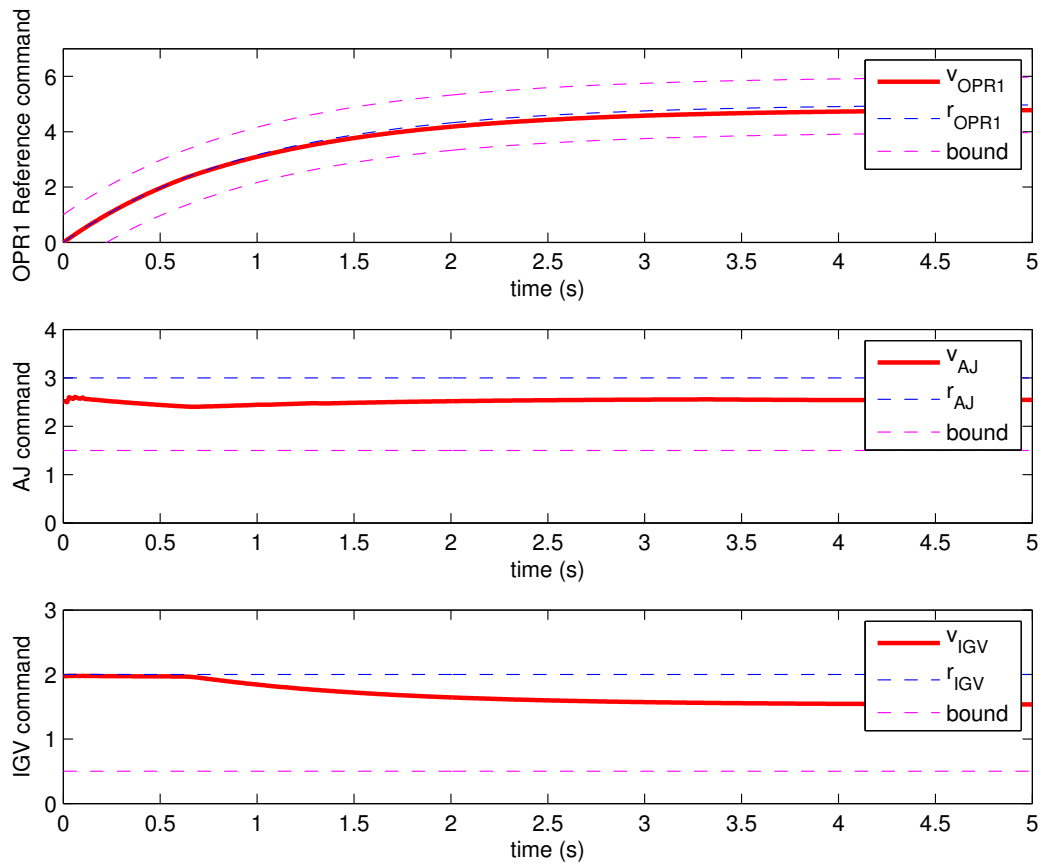
Figure 9: Commands (original and modified) with the reduced order PRG in scenario 1.

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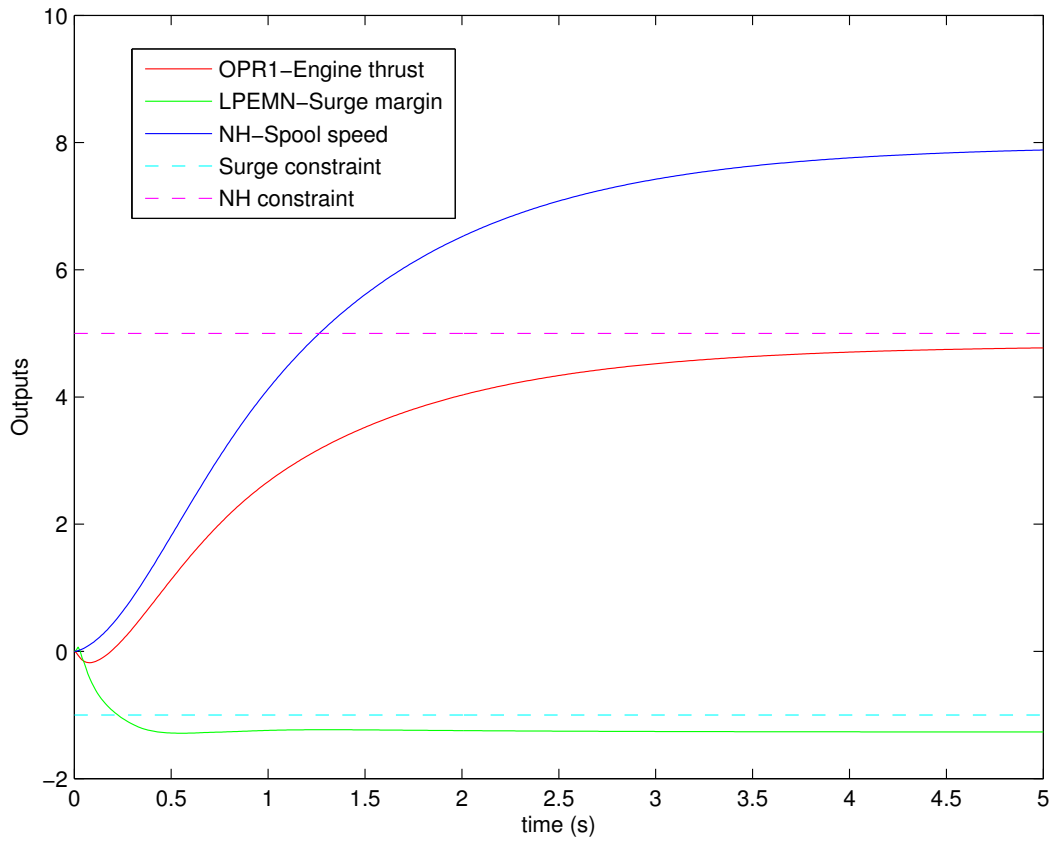
Figure 10: Responses with the reduced order PRG (subsystem of 3rd order) in scenario 1.

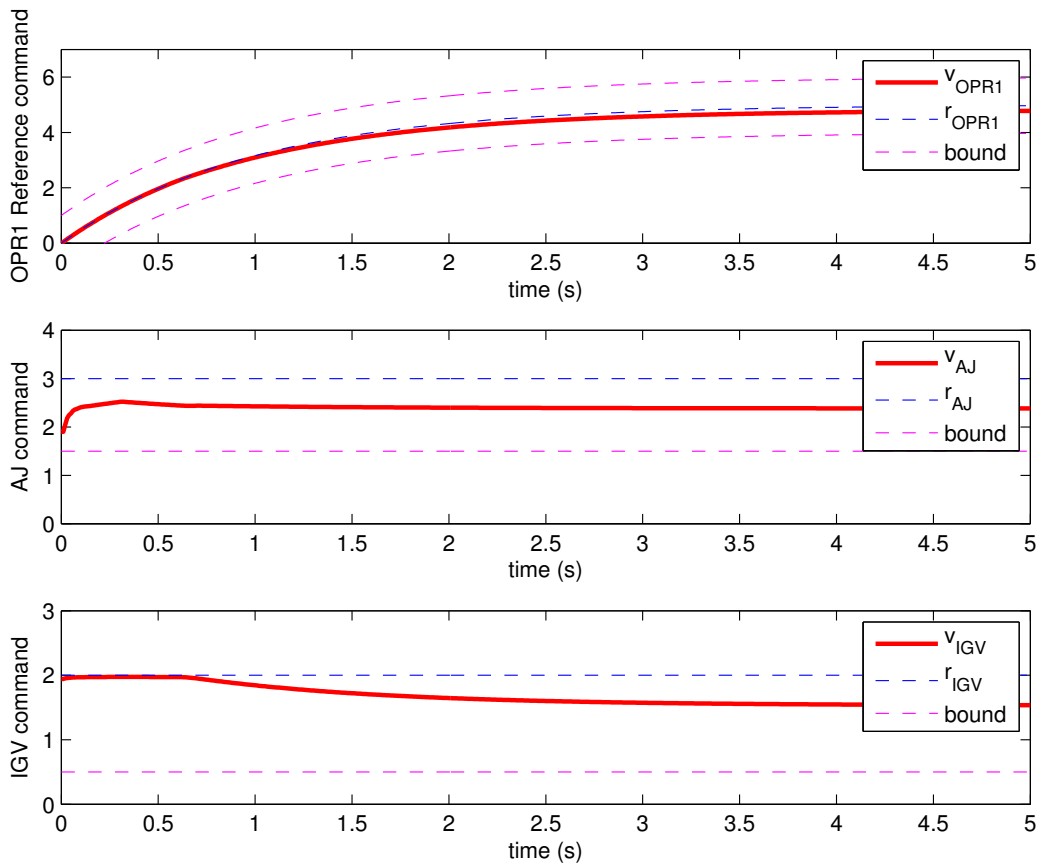American Institute of Aeronautics and Astronautics

Figure 11: Commands (original and modified) with the reduced order PRG (subsystem of 3rd order) in scenario 1.

VI. Concluding remarks

The paper illustrated the use of recently developed prioritized and reduced order reference governors for constraint handling in gas turbine engines. The prioritized reference governor addresses the situations when satisfying all the constraints at once becomes infeasible. It treats some of the constraints as soft (this selection of the soft constraints can be situation-dependent) and minimizes their violation reflective of the specified constraint priority. The reduced order reference governor rigorously enforces the constraints based on a low order model while accounting for the contributions of the omitted states. For the model used in this paper, the full order model had 19 states and we demonstrated that the design of a reduced order prioritized reference governor can be based on the 9th and 3rd order slow subsystem models. The full state was assumed to be available for implementation of the reference governor schemes in the simulations in this paper. While outside the scope of this paper, the observer estimation errors can also be rigorously handled.[6,7]

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