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Limit Protection in Gas Turbine Engines Based on Reference and Extended Command Governors *

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The paper considers the application of Reference Governors (RG), Command Governors (CG) and Extended Command Governors (ECG) to limit protection in gas turbine engines. The governors are add-on schemes to the nominal control design that operate by exploiting physical models of the constraints and predictively modifying the reference commands (set-points) to closed-loop systems as necessary to avoid constraint violation. It is shown that CG and ECG result in comparable speed of response, and both outperform the conventional RG. All three schemes have large constrained domains of attraction. The transient performance improvements in the CG and ECG cases are obtained at the expense of increased computing effort to solve a larger dimensional quadratic programming problem. Measured parametric uncertainties, unmeasured parametric uncertainties, time-varying disturbances and computational complexity reduction are addressed.

Nomenclature

CG	Command Governor
ECG	Extended Command Governor
RG	Reference Governor
W_f	Fueling rate
EPR	Engine Pressure Ratio
F_n	Engine thrust
$LPCSM$	Low Pressure Compressor (LPC) surge margin
$HPCSM$	High Pressure Compressor (HPC) surge margin
VBV	Variable Bleed Valve position
VSV	Variable Stator Vane position
QP	Quadratic Programming
$(\cdot)^+$	Value one step ahead
$(\cdot)^-$	Previous value
$(\cdot)_r$	Set-point
$(\cdot)_v$	Governed value (output of ECG/CG/RG)
x	Closed-loop state
$\bar{x}$	State of the auxiliary dynamic system
ρ	ECG output offset term
k^*, ϵ	Parameters in governor design
X	State of the augmented system
A_c, B_c, B_{cw}, C_c, D, D_{cw}	Closed loop system model matrices
A, B, B_w, C, D, D_w	Augmented system model matrices

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$\bigoplus$	Minkowski set sum
$\sim$	Pontryagin-Minkowski set difference
w	Disturbance
Y	Constraint set
Λ, λ	Matrices defining the constraint set Y
r	Vector of set-points
v	Vector of modified set-points
d	Output offset term
θ	Known constant disturbance
w_1	Unknown constant disturbance
w_2	Unknown time-varying disturbance
W_1	Set overbounding w_1
W_2	Set overbounding w_2
O_∞	Maximum constraint admissible set
P	Subset of maximum constraint admissible set
H, h	Matrices of inequalities defining O_∞ or P
R, Q	Weighting matrices in ECG design
Ω	Nonlinear control function of ECG/CG/RG
Z^+	Set of non-negative integers
I	Identity matrix

I. Introduction

Limit protection[6, 21] is an essential functionality of an aircraft gas turbine engine control system that handles various constraints such as surge avoidance, over-speed and over-temperature limits, combustion lean blowout limit, actuator magnitude and rate limits, etc. The conventional approach to handling constraints through fuel limiters, while reasonably well-understood, can be conservative and restrict unnecessarily engine thrust response.[16] Effective controllers for constrained systems can frequently benefit from exploiting prediction and optimization.

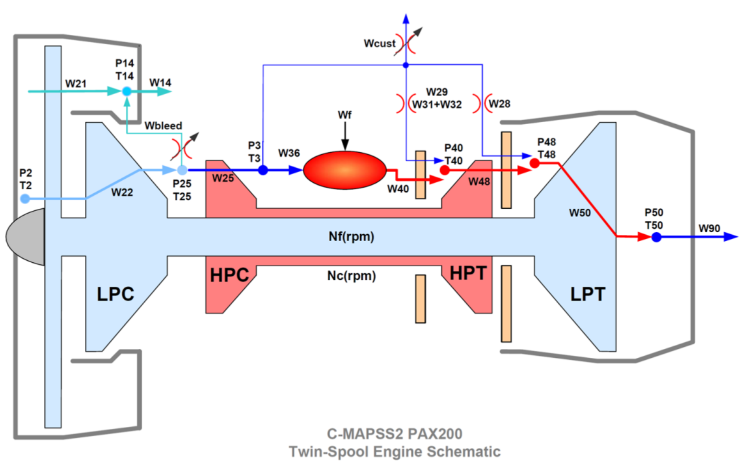

Figure 1. Schematic[17, 18] of a twin spool aircraft gas turbine engine with Fan, Low Pressure Compressor (LPC), Low Pressure Turbine (LPT), High Pressure Compressor (HPC), High Pressure Turbine (HPT) and Combustor.

We have been developing approaches to limit protection in gas turbine engines based on the application of Reference Governors (RG), Command Governors (CG) and Extended Command Governors (ECG), see Figure 2. These governors[12] are add-on schemes to the nominal control design and are introduced to protect the closed-loop system against violating pointwise-in-time state and control constraints. They operate by exploiting physical models of the constraints and predictively modifying the reference commands (set-points) to the closed-loop system when necessary in order to avoid constraint violation. These schemes remain inactive if there is no danger of constraint violation, and they become active only when the closed-loop system requires limit protection.

For gas turbine engine applications, these governors may also be viewed as algorithmic generalizations of the classical fuel topping governor.[6] The RG, CG and ECG are predictive nonlinear control schemes that differ in their respective prediction mechanism, with the ECG generally providing a larger constrained domain of attraction and better performance at the expense of the increase in the computing effort. Unlike solutions based on Model Predictive Control,[21,22] governors are add-on schemes and do not require replacing a nominal engine controller by a Model Predictive Controller. If designed appropriately, they also require less on-board computing power for implementation.

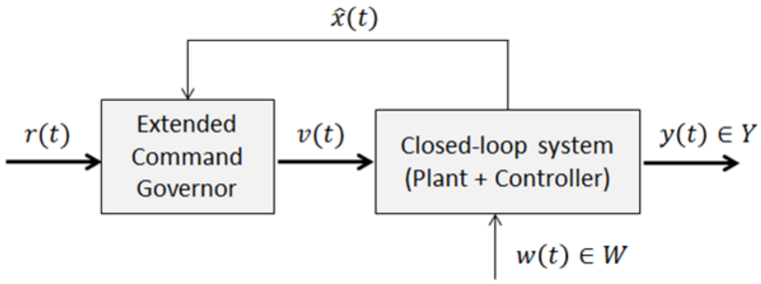

Figure 2. Reference governors, command governors and extended command governors are nonlinear control schemes that modify the original set-points, $r(t)$, to safe set-points, $v(t)$, as necessary to guarantee that the prescribed state and control constraints, expressed as $y(t) \in Y$, are enforced despite uncertainties/disturbances, $w(t) \in W$.

Our previous work on applications of governors to limit protection in gas turbine engines includes a robust RG that can enforce surge margin constraints and non-conservatively handle inlet distortions/disturbances.[11] Recent studies based on simulation models have been conducted demonstrating that constraints in gas turbine engines can be enforced using reduced order, prioritized and decentralized RG designs.[7,8,24] The research into the decentralized reference governor theory[7] has in fact been motivated by its potential use as an enabling technology for distributed control of aircraft engines.[2]

In this paper we consider the application of governors to enforce the LPC and HPC surge margin constraints, fuel limits, Variable Stator Vane (VSV) limits and Variable Bleed Valve (VBV) limits by modifying the Engine Pressure Ratio (EPR) set-point and commanded VSV and VBV position set-points. The ECG is compared with CG and RG in terms of transient response and constrained domain of attraction. It is shown that CG and ECG achieve comparable speed of response, and both outperform the conventional RG. All three schemes have large constrained domains of attraction. The transient performance improvements in the CG and ECG cases are obtained at the expense of increased computing effort. We also demonstrate the capability for robust constraint enforcement using governors in presence of unmeasured parametric uncertainties and time-varying disturbances.

The paper is organized as follows. We proceed by first discussing the ECG, which is the most general scheme, in Section II. We then specialize the developments to CG and RG cases in Section III. The simulation results and comparisons between the schemes are presented in Section IV. Conclusions are summarized in Section V.

II. Extended Command Governor

The Extended Command Governor (ECG) modifies reference commands (set-points) to a well-designed closed-loop system based on the predictions of the nominal *closed-loop* system response. This nominal closed-loop system consists of a plant and a controller, and is assumed to be asymptotically stable.

In this paper, a linear discrete-time model is used for prediction of the engine closed-loop response that has the following form,

$$\begin{aligned} x^+ &= A_c x + B_c v + B_{cw} w, \\ y &= C_c x + D_c v + D_{cw} w + d \in Y, \end{aligned} \tag{1}$$

where x denotes the current state of the closed-loop system, x^+ denotes the state one step ahead predicted by the model, v is the modified set-point by ECG, w is the unmeasured disturbance, y is the constrained output, and constraints are specified by the requirement that $y \in Y$ where Y is a given set.

The term d in (1) is an offset to the constrained output that remains constant in prediction. It is introduced for two reasons. Firstly, it can be used to compensate for the difference between the linear model prediction and actual output of the nonlinear system,[9,12,25] by setting it equal to

$$d(t) = -(y(t) + y_o) + y_{nonl}(t),$$

where y_o is the value of the output at the linearization point, $y(t)$ is the output of the linear model, and $y_{nonl}(t)$ is the actual output of the nonlinear system. Secondly, this term can be used to represent offsets to the limits and implement time-varying constraints.

In our application, we base the prediction model (1) on the linearization of CMAPSS-40k model[17,18] at the operating point corresponding to the altitude of 20000 ft, 0.5 Mach, and 60% PLA. To obtain a closed-loop model, we create an LQ-PI tracking loop that adjusts engine fuel rate, W_f, to control Engine Pressure Ratio (EPR) to a set-point, EPR_r. The design is performed by augmenting an integrator to the plant model and exploiting standard Linear Quadratic Regulator (LQR) theory.

Thus the state of the model (1) merges the integral state of the controller, x_{int}, and the plant model states, N_f (fan speed) and N_c (core speed),

$$x = (x_{int}, \Delta N_f, \Delta N_c)^{\mathrm{T}},$$

where here and subsequently $\Delta(\cdot)$ denotes the deviation of $(\cdot)$ from the nominal value at the linearization point. The update period of the closed-loop model (1) (and hence of the ECG) is $T_s = 0.1$ sec. The set-point, v, generated by ECG comprises the EPR set-point for the inner loop LQ-PI controller, EPR_v and the VSV and VBV commands,

$$v = (\Delta EPR_v, \Delta VSV, \Delta VBV)^{\mathrm{T}}.$$

Note that ECG directly commands VSV and VBV so that $\Delta VSV_v = \Delta VSV$, and $\Delta VBV_v = \Delta VBV$. The model output is

$$y = (\Delta EPR, \Delta LPCSM, \Delta HPCSM, \Delta F_n, \Delta W_f)^{\mathrm{T}},$$

where $LPCSM$ denotes the LPC Surge Margin, $HPCSM$ denotes the HPC Surge Margin, F_n denotes thrust, and W_f is the fuel flow generated by the nominal LQ-PI controller.

The components of the disturbance vector w in (1) reflect the fan health parameters of CMAPSS-40k model,[17,18] specifically, the fan efficiency modifier, the fan flow modifier and the fan pressure ratio modifier. Other health parameters of CMAPSS-40k model or the aggregate axial and circumferential inlet distortions parameters[11] can be treated similarly. The direct feed-through matrices D_c and D_{cw} in (1) are non-zero as the linearized model (1) is obtained after model order reduction and retaining only slower mechanical states.

The ECG determines v in (1) by exploiting (in prediction) an auxiliary dynamic system of the form,

$$\begin{aligned} \bar{x}^+ &= \bar{A}\bar{x}, \\ v &= \rho + \bar{C}\bar{x}, \end{aligned} \tag{2}$$

where $\bar{x}$ is the state of the auxiliary system, ρ is an output offset term and $\bar{A}$ is a Schur (asymptotically stable) matrix. Denoting the full state as

$$X = \begin{bmatrix} \bar{x} \\ x \end{bmatrix}, \tag{3}$$

the merged dynamics of (1)-(2) are represented by the following equations,

$$\begin{aligned} X^+ &= AX + B\rho + B_w w, \\ y &= CX + D\rho + D_w w + d \in Y, \end{aligned} \tag{4}$$

where

$$A = \begin{bmatrix} \bar{A} & 0 \\ B_c\bar{C} & A_c \end{bmatrix}, B = \begin{bmatrix} 0 \\ B_c \end{bmatrix},$$

$$B_w = \begin{bmatrix} 0 \\ B_{cw} \end{bmatrix}, C = \begin{bmatrix} D_c\bar{C} & C_c \end{bmatrix}, D = D_c, D_w = D_{cw}.$$

Note that a part of the state, X, specifically, $\bar{x}$ in (3), and the input ρ can be reset at each time instant to enforce constraints. Towards this end, we first consider the disturbance free case ($w(t) = 0$).

A. ECG in the disturbance free case

The constraint enforcement relies on a functional characterization of safe triples $(\rho, X(0), d)$ that lead to responses satisfying constraints. In the disturbance-free case (with $w(t) \equiv 0$) and based on the linear model (4), such triples satisfy the conditions,

$$\begin{aligned} CA^k X(0) + (C(I-A^k)(I-A)^{-1}B + D)\rho + d \in Y, \quad k = 0, \cdots, k^*, \\ (C(I-A)^{-1}B + D)\rho + d \in (1-\epsilon)Y, \end{aligned} \tag{5}$$

where k^* is sufficiently large (comparable to the settling time of the closed loop system) and where the last constraint with a small $\epsilon > 0$ is imposed to slightly tighten output constraints in steady-state.[3] Assuming that the set Y is polyhedral, i.e., described by a set of affine inequalities,

$$Y = \{y : \ \Lambda y \leq \lambda\}, \tag{6}$$

and $0 \in intY$, (5) reduces to another set of affine inequalities,

$$\begin{gathered} H_{x,k}x(0) + H_{p,k}\bar{x}(0) + H_{r,k}\rho + \Lambda d \leq \lambda, \\ H_\infty \rho + \Lambda d \leq (1-\epsilon)\lambda, \\ [H_{p,k}, H_{x,k}] = \Lambda C A^k, \ \ H_{r,k} = \Lambda(C(I-A^k)(I-A)^{-1}B + D), \ \ k = 0, \cdots, k^*, \\ H_\infty = \Lambda(C(I-A)^{-1}B + D). \end{gathered} \tag{7}$$

Let

$$O_\infty = \{(x(0), \bar{x}(0), \rho, d) : \text{ the system of inequalities (7) is satisfied}\}. \tag{8}$$

At the time instant $t = 0$, the ECG determines $\bar{x}(0)$ and ρ via the solution of the following Quadratic Programming (QP) problem,

$$(\rho - r(0))^{\mathrm{T}} R(\rho - r(0)) + \bar{x}(0)^{\mathrm{T}} Q \bar{x}(0) \to \min_{\rho, \bar{x}(0)}, \tag{9}$$

subject to

$$(x(0), \bar{x}(0), \rho, d) \in O_\infty, \tag{10}$$

where $r(0)$ is the actual set-point command at time $t = 0$, while $R = R^{\mathrm{T}} > 0$ and $Q = Q^{\mathrm{T}} > 0$ are weight matrices, with Q satisfying

$$\bar{A}^{\mathrm{T}} Q \bar{A} - Q < 0. \tag{11}$$

Through the solution of this problem, ρ is made as close as possible to $r(0)$ and $\bar{x}(0)$ is made as close as possible to zero. The solution to (9)-(10) can be written as a function, Ω, of the problem data ($r(0)$, $x(0)$, d),

$$\begin{bmatrix} \bar{x}(0) \\ \rho \end{bmatrix} = \Omega(r, x(0), d). \tag{12}$$

Based on the results in multi-parametric programming literature,[1] Ω is a continuous and piecewise affine function of its arguments.

At the time instant $t > 0$, $t \in Z^+$, the ECG action is defined similarly using the same function Ω applied to the current values of $r(t)$, $x(t)$ and $d(t)$,

$$\begin{bmatrix} \bar{x}(t) \\ \rho(t) \end{bmatrix} = \Omega(r(t), x(t), d(t)). \tag{13}$$

Once $\bar{x}(t)$ and $\rho(t)$ are determined, the ECG applies

$$v(t) = \bar{C}\bar{x}(t) + \rho(t), \tag{14}$$

to system (1) for all $t \geq 0$.

B. ECG with disturbances

With respect to the treatment of disturbance inputs, w, in (1), the usual assumption[10] is $w(t) \in W$ for all $t \in Z^+$, where W is a given compact set. For the application to the gas turbine engine control, we consider a modified assumption,

$$w(t) = \theta + w_1 + w_2(t), \tag{15}$$

where θ is a known parameter, w_1 is a constant unknown set-bounded parameter, and w_2 is a time-varying set-bounded disturbance. The bounding sets for w_1 and w_2 are W_1 and W_2, respectively, so that

$$w_1 \in W_1, w_2(t) \in W_2 \text{ for all } t \in Z^+. \tag{16}$$

The motivation for the disturbance model (15)-(16) is to be able to account for the estimation errors of the health parameters by an onboard estimator.[23] Specifically, θ is an estimate of changed health parameters of the engine that can be provided by an on-board estimator, W_1 over-bounds the error of such an estimator, and $w_2(t)$ represents a persistent time-varying uncertainty/disturbance.

With the disturbance model (15)-(16), the inequality conditions similar to (7) that delineate safe states and parameters can be given as

$$\begin{gathered}
H_{x,k}x(0) + H_{p,k}\bar{x}(0) + H_{r,k}\rho + \Lambda d \le h_k,\\
H_\infty \rho + \Lambda_\infty d \le h_\infty,\\
[H_{p,k}, H_{x,k}] = \Lambda_k C A^k,\ H_{r,k} = \Lambda_k(C(I-A^k)(I-A)^{-1}B + D),\\
h_k = \lambda_k - H_{w,k}\theta - \gamma_k,\\
h_\infty = (1-\epsilon)\lambda_\infty - H_{w,\infty}\theta - \gamma_\infty,\\
H_\infty = \Lambda_\infty(C(I-A)^{-1}B + D),\\
H_{w,k} = \Lambda_k(C(I-A^k)(I-A)^{-1}B_w + D_w),\\
H_{w,\infty} = \Lambda_\infty(C(I-A)^{-1}B_w + D_w),\\
\gamma_{k,j} = \max_l\{\Lambda_{k,j}[C(I-A^k)(I-A)^{-1}B_w + D_w]W_1^l\},\\
\gamma_{\infty,j} = \max_l\{\Lambda_{\infty,j}[C(I-A)^{-1}B_w + D_w]W_1^l\},
\end{gathered} \tag{17}$$

where $k = 0, \cdots, k^*$,

$$Y_k = \{y : \Lambda_k y \le \lambda_k\} = Y \sim D_w W_2 \sim CB_w W_2 \sim \cdots \sim CA^{k-1}B_w W_2,$$

$$Y_\infty = \{y : \Lambda_\infty y \le \lambda_\infty\} \subset Y_k \text{ for all } k \in Z^+ \text{ with } 0 \in int Y_\infty,$$

$\Lambda_{k,j}$ is the jth row of Λ_k, $\Lambda_{\infty,j}$ is the jth row of Λ_∞, $\gamma_{k,j}$ is the jth component of the vector γ_k, $\gamma_{\infty,j}$ is the jth component of the vector γ_∞, W_1^l denote the vertices of W_1, $l = 1, \cdots, n_l$, both W_1 and W_2 are assumed to be polyhedral, and $\sim$ denotes the Pontryagin-Minkowski set difference operation.[10] Note that Y_k and Y_∞ are computed off-line using linear programming techniques.[10] Let

$$O_\infty = \{(x(0), \bar{x}(0), \rho, d) : \text{ the system of inequalities (17) is satisfied}\}. \tag{18}$$

At the time instant $t = 0$, the ECG exploits the solution of a quadratic programming problem,

$$(\rho - r(0))^{\mathrm{T}} R(\rho - r(0)) + \bar{x}(0)^{\mathrm{T}} Q\bar{x}(0) \to \min_{\rho, \bar{x}(0)}, \tag{19}$$

subject to

$$(x(0), \bar{x}(0), \rho, d) \in O_\infty, \tag{20}$$

where $r(0)$ is the actual set-point at $t = 0$, while $R = R^{\mathrm{T}} > 0$ and $Q = Q^{\mathrm{T}} > 0$ are weight matrices, with Q satisfying (11). The solution of this problem has the form,

$$\begin{bmatrix} \bar{x}(0) \\ \rho \end{bmatrix} = \Omega(r(0), x(0), d, \theta, W_1), \tag{21}$$

where we make the dependence on θ and W_1 explicit. At the time instant $t > 0$, $t \in Z^+$, the ECG action is defined similarly, using the same function Ω applied to data at time t,

$$\begin{bmatrix} \bar{x}(t) \\ \rho(t) \end{bmatrix} = \Omega(r(t), x(t), d(t), \theta(t), W_1(t)), \tag{22}$$

where we make explicit the dependence of θ and W_1 on t. Note that to ensure recursive feasibility, conditions such as

$$\{\theta(t)\} \bigoplus W_1(t) \subset \{\theta(\tau)\} \subseteq W_1(\tau) \quad \text{for } \tau \leq t, \tag{23}$$

that imply that the estimation accuracy does not degrade with time may need to hold.[13]

Once $\bar{x}(t)$ and $\rho(t)$ are determined, the ECG applies

$$v(t) = \bar{C}\bar{x}(t) + \rho(t), \tag{24}$$

to system (1) for all $t \geq 0$.

C. Computational considerations

We note that the systems of inequalities (7) and (17) can be large, and the solution of (9)-(10) or (19)-(20) can require extensive on-board computing time and effort. To simplify the computations, we note that some of the constraints in (7) or (17) can be redundant; hence they can be eliminated upfront by solving a series of linear programming problems.

Furthermore, the conditions (10) or (20) can be replaced by the condition

$$(x(0), \bar{x}(0), \rho, d) \in P, \quad \text{where } P \subset O_\infty. \tag{25}$$

There is a large flexibility in choosing the subset P. In particular, it can be much simpler than O_∞. The choice cannot be completely arbitrary, and has to satisfy certain technical assumptions[3,4] to ensure desirable convergence properties. One approach to computing P is to eliminate almost redundant constraints from the O_∞ polyhedron defined by (7) (or, respectively by (17)), and apply a pull-in transformation.[3]

With the use of a general P, which does not satisfy appropriate positive-invariance properties, feasibility of the ECG optimization problem may occasionally be lost. In this case, (13) for $t > 0$ is modified to

$$\begin{bmatrix} \bar{x}(t) \\ \rho(t) \end{bmatrix} = \begin{cases} \Omega(r(t), x(t), d(t)) & \text{if feasible solution exist,} \\ \begin{bmatrix} \bar{A}\bar{x}(t-1) \\ \rho(t-1) \end{bmatrix} & \text{otherwise,} \end{cases} \tag{26}$$

and (22) is modified to

$$\begin{bmatrix} \bar{x}(t) \\ \rho(t) \end{bmatrix} = \begin{cases} \Omega(r(t), x(t), d(t), \theta(t), W_1(t)) & \text{if feasible solution exist,} \\ \begin{bmatrix} \bar{A}\bar{x}(t-1) \\ \rho(t-1) \end{bmatrix} & \text{otherwise.} \end{cases} \tag{27}$$

The implementation of ECG requires solving a QP problem. The availability of QP solvers suitable for onboard implementation has been steadily growing in recent years.[15] In fact, the implementation is now possible in fixed point ECUs.[20] Alternatively, explicit implementation using multi-parametric quadratic programming can be pursued,[1] where the solution is precomputed offline and stored as a piecewise affine map for the online use.

III. Reference and Command Governors

The conventional RG and CG may be viewed as particular variants of the ECG. Specifically, in the CG case, $v = \rho$, and the optimization problem (19)-(20) is replaced by

$$\begin{gathered} (v - r(0))^{\mathrm{T}} R (v - r(0)) \to \min_{v}, \\ \text{subject to (17) with } \bar{x}(0) = 0 \text{ and } \rho = v. \end{gathered} \tag{28}$$

The resulting solution is of the form $v = \Omega(r(0), x(0), d, \theta, W_1)$ and $v(t) = \Omega(r(t), x(t), d(t), \theta(t), W_1(t))$ for all $t \geq 0$.

In the RG case, (28) is replaced by

$$v = v^- + \beta(r(0) - v^-), \tag{29}$$

where β is a scalar determined as the solution of the following optimization problem,

$$\begin{gathered} \beta \to \max \\ \text{subject to } 0 \leq \beta \leq 1 \text{ and (17) with } \bar{x}(0) = 0 \text{ and } \rho = v = v^- + \beta(r - v^-). \end{gathered} \tag{30}$$

The resulting solution is of the form $v = \Omega(v^-, r(0), x(0), d, \theta, W_1)$ and $v(t) = \Omega(v^-, r(t), x(t), d(t), \theta(t), W_1(t))$ for all $t \geq 0$. At the time instant $t = 0$, v^- is determined using an initialization procedure. For instance, assuming that the system starts up in a constraint admissible steady-state, a simple choice $v^- = r(0)$ may be made. At the time instant $t > 0$, v^- is set to $v(t-1)$.

Computationally, the RG case is the simplest, as in it only a scalar parameter β is optimized. The solution is obtained analytically without requiring a quadratic programming solver. On the other hand, the RG can be conservative especially when the system has multiple command channels as it constrains them to "move together". In the case of multiple command channels, hybrid strategies where the original set-point, $r(t)$ is first mapped to a a steady-state constraint admissible set-point, $r_{ss}(t)$, through a solution of a simple QP, and the RG is applied to respond to r_{ss} and enforce constraints in transients can be developed.[11]

IV. Simulation results

To illustrate the constrained performance of ECG, CG and RG, several simulations based on the linearized engine model are presented. In all the simulations, $d = 0$. The constraints are defined as

$$LPCSM \geq 2.5, \quad HPCSM \geq 2.5, \quad VSV \geq -32, \quad VSV \leq 5, \quad VBV \geq 0, \quad VBV \leq 1, \quad W_f \geq 0.1.$$

The disturbance free case is treated first.

A. ECG responses without disturbance

We consider the response of ECG with the shift register[4] auxiliary dynamics (2) corresponding to the horizon $n_h = 2$, i.e., with

$$\bar{A} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \bar{C} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

The ECG was designed with

$$R = \begin{pmatrix} 10 & 0 & 0 \\ 0 & \frac{1}{10000} & 0 \\ 0 & 0 & \frac{1}{100} \end{pmatrix},$$

emphasizing EPR command tracking. The matrix Q satisfying (11) was computed, based on $\bar{A}^{\mathrm{T}} Q \bar{A} - Q = -0.01I$, where I is the identity matrix, as

$$Q = \begin{pmatrix} \frac{1}{50} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{50} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{50} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{100} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{100} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{100} \end{pmatrix}.$$

The responses to steps in EPR set-point, EPR_r, while the set-points for VSV and VBV positions, i.e., VSV_r and VBV_r, respectively, remain constant ($VSV_r = -8.6628$, $VBV_r = 0.1252$) are shown in Figures 3-6. The constraints are strictly enforced by ECG with responses riding the constraint boundary in several time intervals. The first two up and down steps in EPR_r are feasible in steady-state, with virtual set-points generated by ECG (EPR_v, VSV_v, VBV_v) converging to the actual set-points (EPR_r, VSV_r and VBV_r, respectively) in finite time. Note that VSV and VBV are active in transients and deviate from VSV_r and VBV_r to facilitate fast response of EPR and constraint handling. The final two up and down steps in EPR_r are infeasible in steady-state, and, in fact, EPR_v, VSV_v and VBV_v deviate from EPR_r, VSV_r and VBV_r in steady-state. Still, the general response characteristics of EPR and thrust F_n are preserved, and constraints are strictly enforced.

Other choices of the auxiliary dynamics (2) have been tried, including the shift register with the longer horizon (up to $n_h = 20$) and Laguerre's sequence generators.[9] The transient response improvements with these alternative choices have not been significant, suggesting that simple auxiliary dynamics are adequate in this problem.

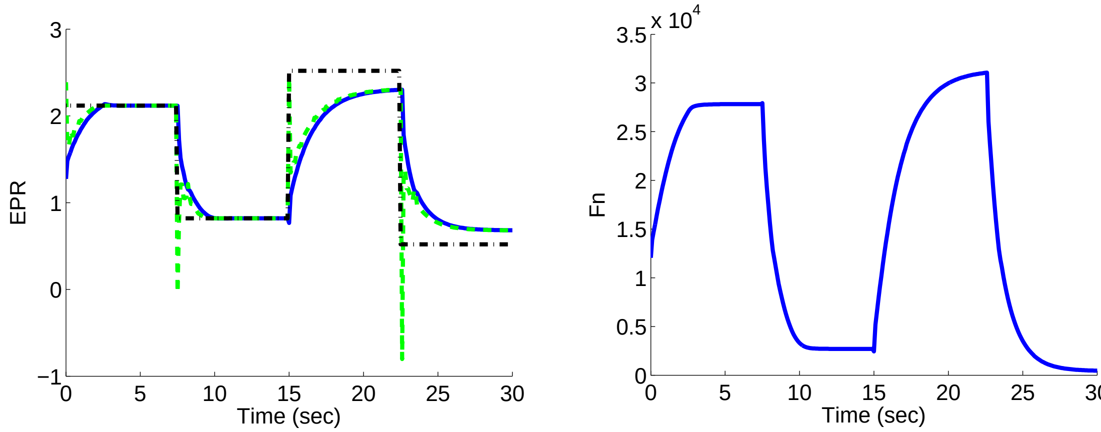

Figure 3. Left: The time history of EPR (solid line) when ECG is applied in the disturbance-free case with steps in EPR set-point, EPR_r, shown by dash-dotted line, and ECG generated virtual set-point EPR_v shown by the dashed line. Right: The time history of thrust F_n (solid) in response to steps in EPR set-point, EPR_r, with ECG.

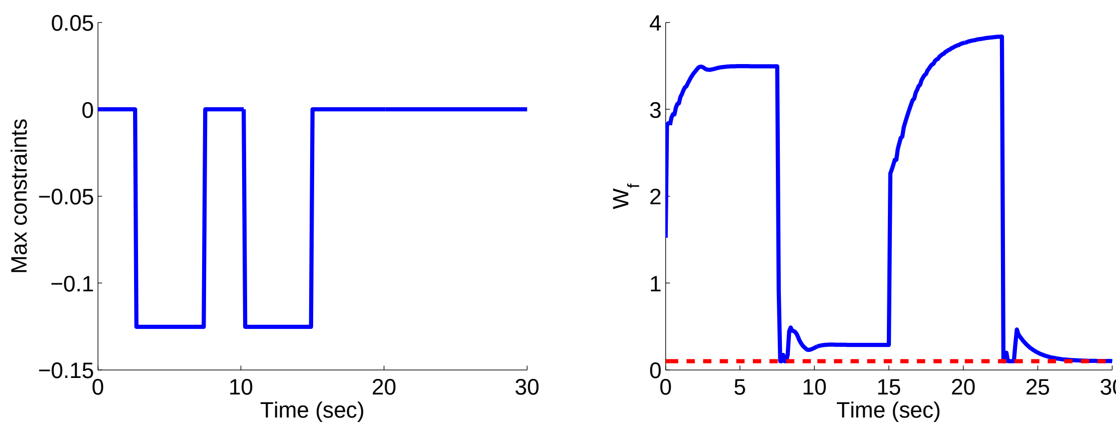

Figure 4. Left: The time history of the maximum of the constraints in response to steps in EPR set-point with ECG. Right: The time history of fuel rate W_f (solid) in response to steps in EPR set-point with ECG. The constraint is shown by the dashed line.

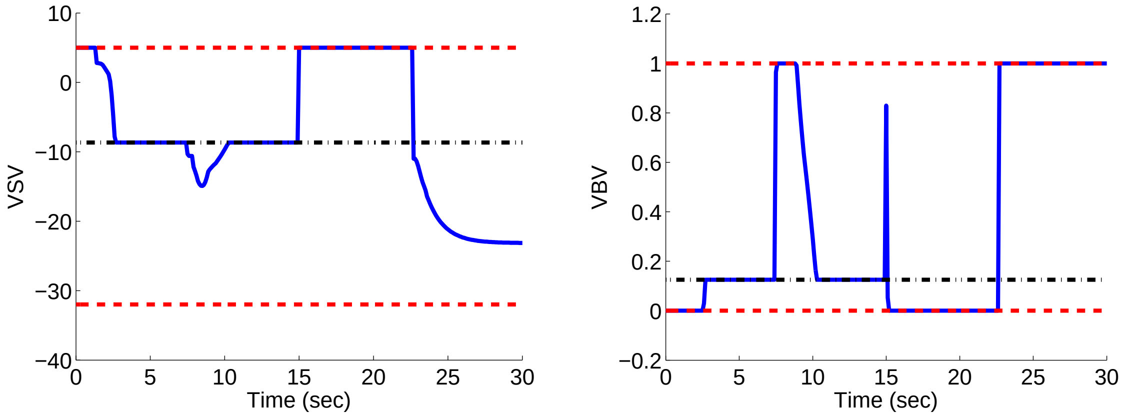

Figure 5. Left: The time history of VSV (solid) in response to steps in EPR set-point with ECG. The constant VSV set-point, VSV_r, is shown by the dash-dotted line. The constraints are shown by the dashed lines. Right: The time history of VBV (solid) in response to steps in EPR set-point with ECG. The constant VBV set-point VBV_r is shown by the dash-dotted line. The constraints are shown by the dashed lines.

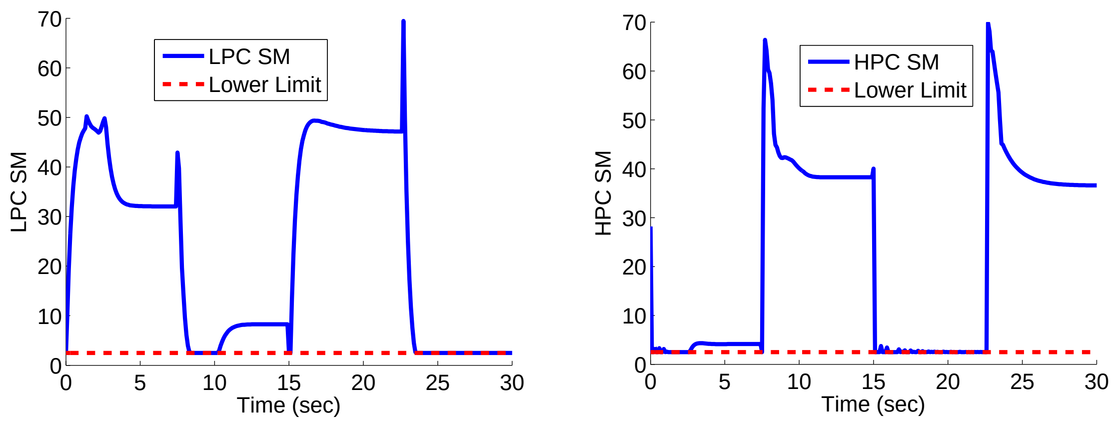

Figure 6. Left: The time history of $LPCSM$ (solid) in response to steps in EPR set-point, EPR_r, with ECG. The constraint is shown by the dashed lines. Right: The time history of $HPCSM$ (solid) in response to steps in EPR set-point, EPR_r, with ECG. The constraint is shown by the dashed lines.

B. Computational complexity reduction

The system of inequalities (7) with $k^* = 30$ contains 217 inequalities. The elimination of redundant inequalities yields 105 inequalities. For the pull-in factor of 1.2, the elimination of almost redundant inequalities[3] yields 37 inequalities, a reduction by a factor close to 3 with very little change in responses. See Figure 7.

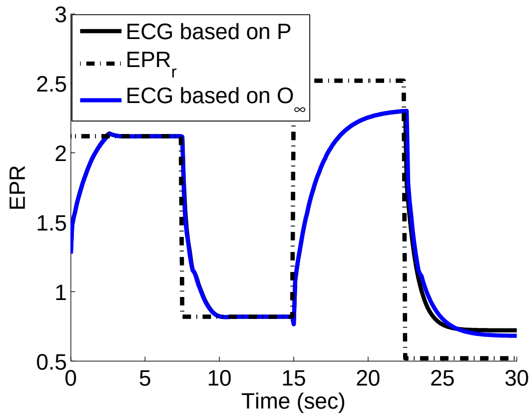

Figure 7. The time history of EPR with ECG when the implementation is based on O_∞ versus when it is based on P. The set-point, EPR_r, is shown by the dash-dotted line.

C. Comparison of RG, CG and ECG

Figures 8-9 compare the responses of EPR and thrust, F_n, during up and down steps in EPR set-point, EPR_r, when RG, CG and ECG are applied. The responses with ECG and CG are comparable, while the response with RG is slower. This is in part due to less flexible character of RG, in particular, inability to exploit VSV and VBV in handling constraints.

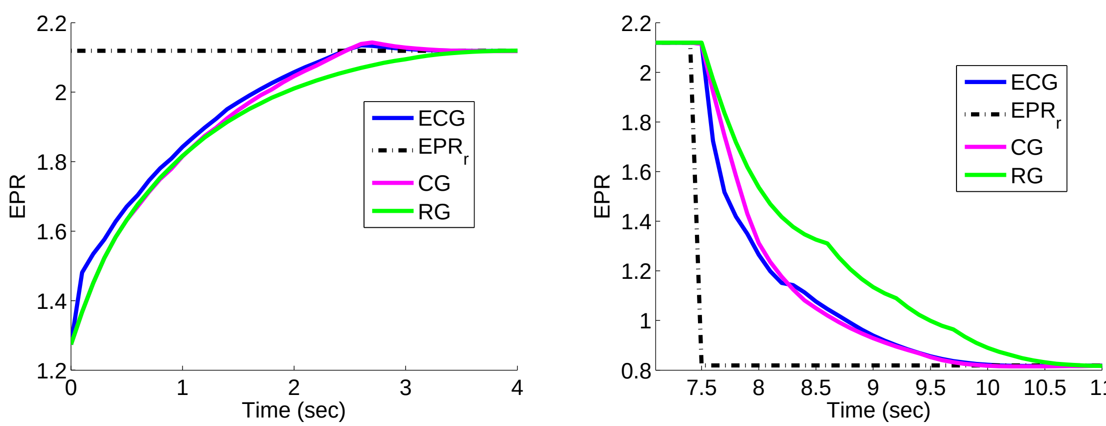

Figure 8. Left: The response of EPR to a commanded increase in EPR_r with RG, CG and ECG. Right: The response of EPR to a commanded decrease in EPR_r with RG, CG and ECG.

D. Comparison of constrained domains of attractions

Figure 10 compares the constrained domains of attraction of CG and ECG, obtained as projections of O_∞ on ΔN_c-ΔN_f plane, with the integral state, x_{int}, of the controller set to 0. These are sets of initial $\Delta N_c(0)$ and $\Delta N_f(0)$ values for which the respective schemes can guarantee subsequent constraint enforcement when $x_{int}(0) = 0$. The constrained domain of attraction is only slightly smaller in the CG case than in the ECG case. In the RG case, assuming that v^- at time 0 can be appropriately initialized, the constrained domain of attraction is the same as in the CG case.

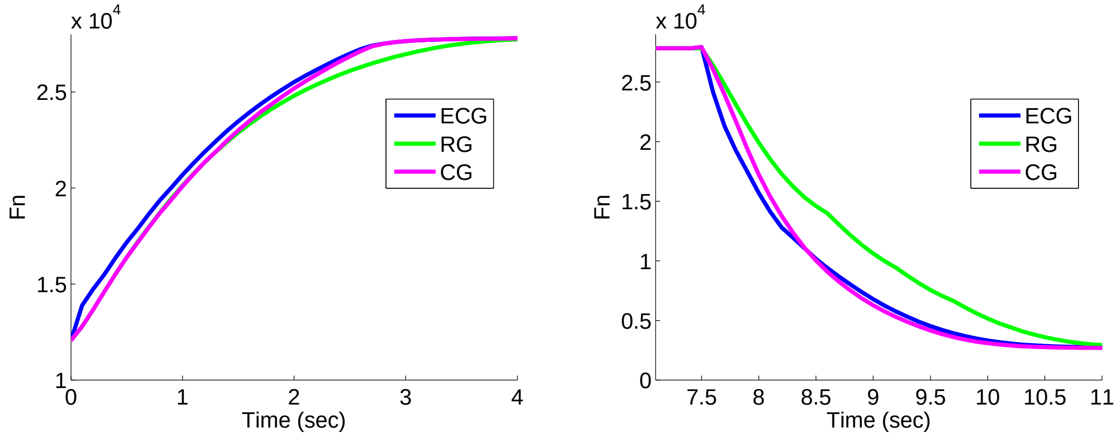

Figure 9. Left: The response of thrust, F_n, to a commanded increase in EPR_r with RG, CG and ECG. Right: The response of thrust, F_n, to a commanded decrease in EPR_r with RG, CG and ECG.

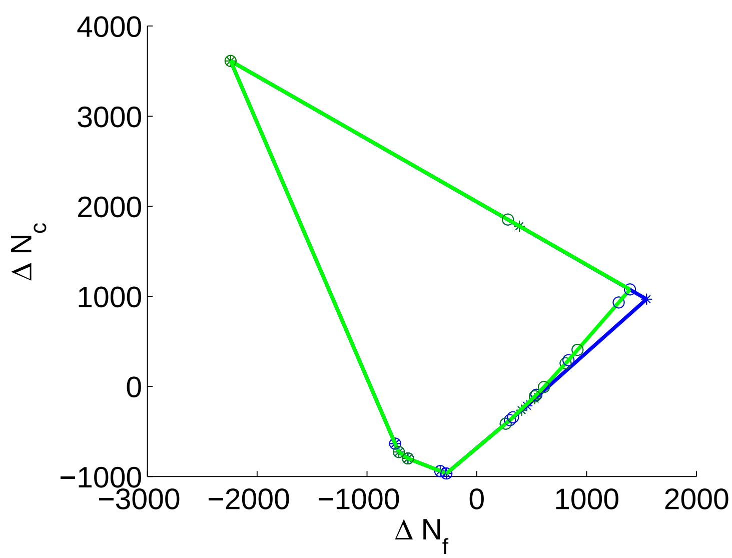

Figure 10. Projections of O_∞ on ΔN_c-ΔN_f plane in the RG/CG case and in the ECG case with $x_{int} = 0$.

These constrained domains of attraction can each grow significantly and the differences between them become more apparent if the initial integral state of the controller, $x_{int}(0)$, can be freely selected at the initial time, see Figure 11. These observations are based on the linearized model that may overestimate a large sized constrained domain of attraction. Effective strategies for constraint enforcement through a reset of controller integral states can be developed[14, 19] but are not pursued in this paper.

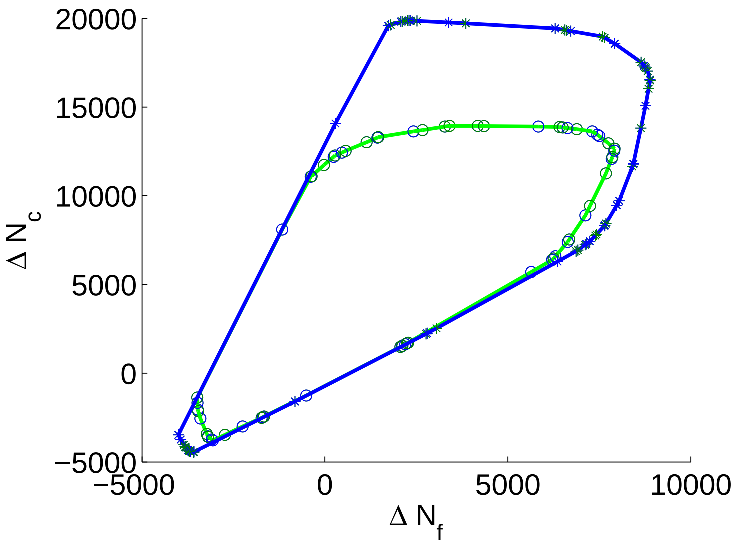

Figure 11. Projections of O_∞ on ΔN_c-ΔN_f plane in the RG/CG case and in the ECG case when the initial integrator state, x_{int}, is selectable.

E. Responses with disturbance

Suppose now the disturbances are modeled according to (15) with $\theta = 0$, $W_2 = 0.05 B_\infty$, where B_∞ is the unit cube in R^3, and where two cases with respect to w_1, $W_1(t)$ are considered: (i) $W_1(t) = 0.05 B_\infty$ for all $t \in Z^+$ and $w_1 = (0.05, -0.05, 0.05)^{\mathrm{T}}$ and (ii) $W_1(t) = 0.05 e^{-tT_s/5} B_\infty$ for all $t \in Z^+$ and $w_1 = (0, 0, 0)^{\mathrm{T}}$. The second case corresponds to a scenario when the uncertainty in the estimate of a constant parameter, θ, gradually reduces over time. In the simulations, the unknown disturbance component $w_2(t)$ is generated randomly within the prescribed limits.

The responses for these two cases are given in Figures 12-13. Note that the constraints are strictly enforced. In the case when $W_1(t)$ decreases with time, the EPR response is able to approach the set-points closer during the steps up in EPR_r and the achieved thrust levels are higher.

V. Conclusions

The developments in this paper indicate that Reference Governor (RG), Command Governor (CG) and Extended Command Governor (ECG) schemes can enforce a multitude of limits in as turbine engines by modifying the set-points for Engine Pressure Ratio (EPR), Variable Stator Vane (VSV) position and Variable Bleed Valve (VBV) position, without requiring a large dimensional auxiliary dynamics in the ECG case. No significant differences between constrained domains of attraction of these schemes have been found, assuming the integral state of the controller is not reset. Both CG and ECG exhibit comparable speed of transient response, and both are faster than RG. Thus CG represents an attractive option for implementation as it has lower computational complexity versus ECG. Based on the previous results,[7] a distributed implementation of CG for decentralized constraint enforcement is also feasible.

We have also shown that constraint handling can be merged with online estimation of uncertain parameters in presence of persistent time varying disturbances. With disturbances, the response becomes more conservative as the constraint violation must be avoided despite the worst possible action of the disturbance.

Finally, a significant potential for reducing computational complexity without compromising transient re-

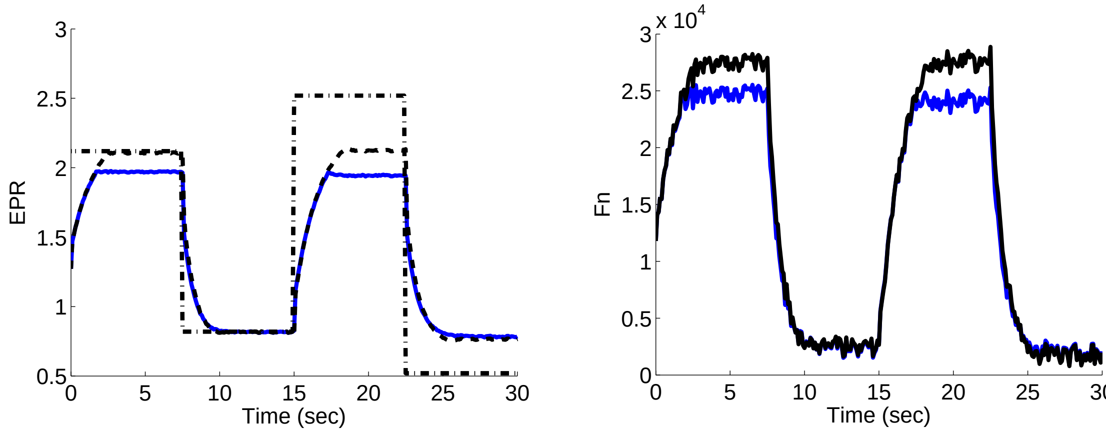

Figure 12. Left: The time history of EPR when ECG is applied in presence of disturbances with EPR_r shown by dash-dotted line, EPR response when $W_1(t)$ is constant shown by solid line, and EPR response when $W_1(t)$ shrinks with time shown by dashed line. Right: The time history of thrust, F_n, when ECG is applied in presence of disturbances with thrust response when $W_1(t)$ is constant shown by solid line, and thrust response when $W_1(t)$ shrinks with time shown by dashed line.

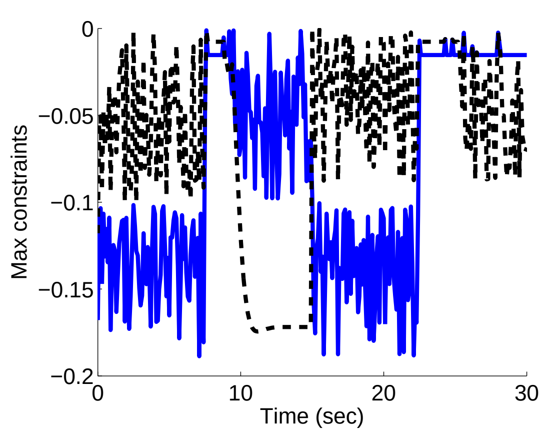

Figure 13. The time history of the maximum of the constraints when $W_1(t)$ is constant in time (solid line) and when $W_1(t)$ shrinks with time (dashed line).

sponse exist by eliminating almost redundant inequalities from the representation of the constraint admissible set used by the schemes, and applying a pull-in procedure.

The above conclusions have been reached based on the simulations of a linearized engine model. The procedure to handle nonlinearities through a constant in-prediction disturbance term has been discussed, and other approaches to implementation based on the nonlinear engine model exist[5] that are left as topics for future publications.

References

[1]Bemporad, A., Morari, M., Dua, V., and Pistikopoulos, E., The explicit solution of Model Predictive Control via multi-parametric quadratic programming, *Automatica,* vol. 38, no. 3, 2002.

[2]Distributed Engine Control Working Group, `http://www.decwg.org/`, Accessed June 28, 2014.

[3]Gilbert, E.G., and Kolmanovsky, I.V., "Fast reference governors for systems with state and control constraints and disturbance inputs," *International Journal of Robust and Nonlinear Control,* vol. 9, no. 15, pp. 1117-1141, 1999.

[4]Gilbert, E.G., and Ong, C.-J., "Constrained linear systems with hard constraints and disturbances: An extended command governor with large domain of attraction," *Automatica*, vol. 47, pp. 334-340, 2011.

[5]Gilbert, E.G., and Kolmanovsky, I.V., "Nonlinear tracking control in the presence of state and control constraints: a generalized reference governor," *Automatica*, vol. 38, no. 12, pp. 2063–2073, 2002.

[6]Jaw, L.C., and Mattingly, J.D., *Aircraft Engine Controls: Design, System Analysis, and Health Monitoring,* AIAA, 2010.

[7]Kalabic, U., and Kolmanovsky, I.V., "Decentralized constraint enforcement using reference governors," *Proceedings of 2013 IEEE Conference on Decision and Control,* Florence, Italy, pp. 6415–6421, 2013.

[8]Kalabic, U., Kolmanovsky, I.V. and Gilbert, E.G. "Reduced order extended command governor," *Automatica,* vol. 50, no. 5, pp. 1466–1472, 2014.

[9]Kalabic, U., Kolmanovsky, I., Buckland, J., and Gilbert, E., "Reference and extended command governors for control of turbocharged gasoline engines based on linear models," In *Proceedings of IEEE International Conference on Control Applications (CCA) / IEEE Multiconference on Systems and Control*, Denver, Colorado, pp. 319–325, 2011.

[10]Kolmanovsky, I.V., and Gilbert, E.G., "Theory and computations of disturbance invariant sets for discrete-time linear systems," *Mathematical Problems in Engineering,* vol. 4, pp. 317-367, 1998.

[11]Kolmanovsky, I.V., Jaw, L., Merrill, W., and Tran, H.-T., "Robust control and limit protection in aircraft gas turbine engines," *Proceedings of the 2012 IEEE Multi-Conference on Systems and Control,* Dubrovnik Palace Hotel, Dubrovnik, Croatia, October 3-5, 2012.

[12]Kolmanovsky, I.V., Garone, E., and Di Cairano, S., "Reference and command governors: A tutorial on their theory and automotive applications," *Proceedings of 2014 American Control Conference,* Portland, Oregon, pp. 226-241, 2014.

[13]Kolmanovsky, I.V., Gilbert, E.G., and Cook, J.A., "Reference governors for supplemental torque source control in turbocharged diesel engines," *Proceedings of the 1997 American Control Conference,* vol. 1, pp. 652–656, Albuquerque, New Mexico, 1997.

[14]Kolmanovsky, I.V., and Sun, J., "Parameter governors for discrete-time nonlinear systems with pointwise-in-time state and control constraints," *Automatica*, vol. 42, no. 5, pp. 841–848, 2006.

[15]Di Cairano, S., Brand, M., and Bortoff, S.A., "Projection-free parallel quadratic programming for linear model predictive control, " *International Journal of Control*, vol. 86, no. 8, 2013.

[16]May, R.D., and Garg, S., "Reducing conservatism in aircraft engine response using conditionally active min-max limit regulators," *Proceedings of ASME Turbo Expo 2012 GT2012*, Copenhagen, Denmark, Paper GT2012-70017, 2012.

[17]May, R.D., Csank, J., Lavelle, T.M., Litt, J.S., and Guo, T.-H., A high fidelity simulation of a generic commercial aircraft engine and controller, *Proceedings of 46th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit,* 25-28 July 2010, Nashville, TN, 2010, AIAA 2010-6630.

[18]May, R., Csank, J., Litt, J.S., and Guo, T.-H., Commercial modular aero-propulsion system simulation 40k (C-MAPSS40k) users guide, NASA/TM-2010-216831.

[19]McDonough, K., and Kolmanovsky, I.V., "Integrator resetting for enforcing constraints in aircraft flight control systems," *Proceedings of AIAA Scitech/AIAA Guidance Navigation and Control*, submitted, 2015.

[20]Patrinos, P., Guiggiani, A., Bemporad, A., "Fixed-point dual gradient projection for embedded model predictive control," *Proceedings of European Control Conference (ECC)*, pp. 3602–3607, 2013.

[21]Richter, H., *Advanced Control of Turbofan Engines,* Springer, 2011.

[22]Saluru, D., Yedavalli, R., and Belapurkar, R., "Active fault tolerant Model Predictive Control of a turbofan engine using C-MAPSS40k," *Proceedings of 5th Annual Dynamic Systems and Control Conference,* Paper DSCC2012-MOVIC2012-8730, 2012.

[23]Simon, D., and Simon, D. L., "Aircraft turbofan engine health estimation using constrained Kalman filtering," *Journal of Engineering for Gas Turbines and Power,* vol. 127, no. 2, pp. 323–328, 2005.

[24]Tian, Y., and Kolmanovsky, I.V., "Reduced order and prioritized reference governors for limit protection in aircraft gas turbine engines," *Proceedings of AIAA SciTech / AIAA Guidance, Navigation and Control Conference*, Baltimore, MD., AIAA 2014-1149, 2014.

[25]Vahidi, A., Kolmanovsky, I.V., and Stefanopoulou, A., "Constraint handling in a fuel cell system: A fast reference governor approach," *IEEE Transactions on Control Systems Technology,* vol. 15, no. 1, pp. 86-98, 2007.