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Finite Element Modeling of Longitudinal Metastructures for Passive Vibration Suppression

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The research presented here explores a finite element model of metastructure with distributed vibration absorbers experiencing unidirectional axial vibrations. This work builds off of previous work using a lumped mass model. The proposed model is compared to a baseline model where both models have equal mass. First, it is verified that the finite element formulation produces similar trends as the lumped mass model, specifically, that using vibration absorbers of linearly varying natural frequencies leads to a larger bandgap of reduced vibrations. Second, the two comparison models generated using the finite element formulation have different effective stiffnesses and the effects of this is examined. The results show that a small number of vibration absorbers, less than three, leads to a significant change in stiffness which in turn results in a larger dynamic response of the structure.

Nomenclature

A	=	state matrix for state space representation
A_i	=	cross sectional area of of rod for the *i*th absorber
B	=	input matrix for state space representation
C	=	output matrix for state space representation
D	=	finite element damping matrix
E	=	Young's modulus of the structure
$G(s)$	=	tip to tip transfer function of the system
H_2	=	norm of system
K	=	finite element stiffness matrix
k_i	=	stiffness of the *i*th absorber
ℓ_i	=	length of rod for the *i*th absorber
M	=	finite element mass matrix
m_i	=	mass of the *i*th absorber

I. Introduction

VIBRATION suppression is necessary throughout aerospace engineering. Unfortunately, adding damping to a structure is typically considered after the design of the structure has been completed and the damping is simply added on afterwards. This is called add-on damping and can add considerable mass to the structure, typically 20 – 30% of the host structure's mass. One example of add-on damping is using a tuned mass damper on a tall building to help reduce vibrations from earthquakes or windloading.[1] The new field of metastructures is aimed at integrating damping into the design of the structure by distributing the damping throughout the structure. This concept was originally discussed by Sun, et. al. They showed that a bandgap can be created over a specific frequency range.[2] In 2013, Baravelli and Ruzzene developed a chiral metamaterial for low-frequency applications. Their structure consists of an aluminum rectangular frame and a rubber internal lattice with steel mass inclusions distributed throughout. Through their design they were able to significantly reduce the damping in the structure.[3,4] These are promising results but they do not consider the mass which is added to the system. Building off of the previous studies, the work shown here attempts to achieve high damping without adding additional mass to the system, a concern in all aerospace structures.

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Previous work conducted by the authors has examined an analytical lumped mass model of a bar experiencing longitudinal vibrations.[5] Work performed by the author's colleagues tested a 3D printed metastructure and found that a bandgap can be created using these concepts.[6] The work here is an extension of previous longitudinal work but looks at using finite element methods instead of a lumped mass model to capture the behavior of the metastructure. The finite element model is necessary because it can capture the effects of the distributed mass and the stiffness of the point masses. Here we explore the effects on stiffness and damping for a structure while not increasing the mass. The finite element model presented here looks at vibrations solely in the axial direction exploiting a finite element model comprised of 2D bar elements. Future work will involve printing these metastructures using a 3D printer cabable of printing parts in a single print job composed of materials with different stiffnesses. The code developed allows for different density and stiffness properties to be used throughout the structure. The finite element code is created using MATLAB.

II. Design of the Metastructure

This design of the metastructure used in this paper is inspired by the lumped mass models show in Fig. 1 from previous work by the authors.[5] This representation of a bar looks at motion in the horizontal direction. This structure is composed of large masses connected via large springs which make up the host structure and set of smaller springs and masses, called vibration absorbers, such that the number of absorbers can vary. The natural frequency of the absorbers can be tuned by chosing parameters such that the ratio $\sqrt{k_i/m_i}$ matches the desired frequency. It has been determined that tuning the natural frequencies of these absorbers such that they span the fundamental natural frequency of the host structure produces a favorable dynamic response.[5] The results shown here, build off of the results from this lumped mass model transitions into a finite element model.

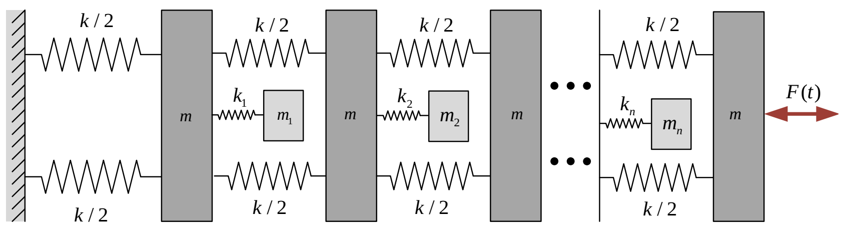

Figure 1. Lumped mass model with n vibration absorbers from previous work by the authors.

The finite element model is derived from the physical model shown in Fig. 2. This is a bar-shaped metastructure experience vibrations along the length of the rod. The model shown has five vibration absorbers distributed along the length of the bar. Simliar to the lumped mass model, the natural frequency of the absorbers can be tuned by looking at the the ratio $\sqrt{k_i/m_i}$ where $k_i = EA_i/\ell_i$. This structure is designed by setting the geometry of the the host structure and the masses of the absorbers, then the cross sectional area of the springs of the absorbers is varied such that the desired frequency is obtained. This metastructure will be compared to a baseline structure of equal mass and similar stiffness. The baseline structure has no absorbers and additional mass in the sections between the absorbers.

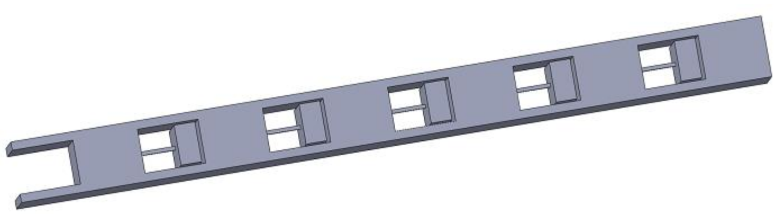

Figure 2. Metastructure with five absorbers located along the length of the beam.

III. Modeling Procedure

The modeling presented in this paper was performed using a finite element code developed by the authors using MATLAB. The structure is comprised of bar elements. Each section of the structure is discritised into elements. The size of the elements used is based on a mesh convergence study looking at the H_2 norm of the structure. The Fig. 3 below shows how the metastructure is reduced down to 2D bar elements. The vertical dashed lines are used to show nodes that are tied together. Since the absorber mass does not experience any loading, the stiffness of that mass is negiable and thus is accounted for using a point mass shown as the large black circle.

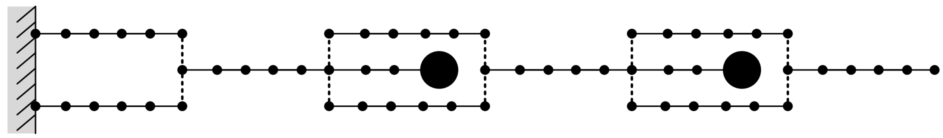

Figure 3. Finite element discritization of structure (dotted lines represent rigid elments).

Using finite element procedures, the mass and stiffness matricies of the structure were developed. Strucutral damping is used in this model, thus the damping matrix is calculated, $\mathbf{D} = \alpha\mathbf{K}$ where an apporiate α value is chosen. Next the system is put into state space using the following equations.

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}^{-1} \end{bmatrix}, \qquad \mathbf{C} = [\mathbf{I} \quad \mathbf{0}] \tag{1}$$

Once the system is in state space, the transfer function matrix $H(s)$ can be calculated, $H(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}.$ The specific transfer function of interest here, is where the input force is at the tip and the response is also at the tip which we will call $G(s)$. The performance measure of interest is the H_2 norm which is related to the area under the transfer function using Eq. (2).

$$H_2 = \|G(s)\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{tr}\big(G^\star(j\omega)G(j\omega)\big)\, d\omega \tag{2}$$

This value is calculated for both the metastructure and the baseline model. From these values, a percent decrease is then calculated. A higher percent decrease leads to a more favorable design. Additionly, the effective stiffness of each structure is also determined, by applying a tip force to the finite element model, measuring the displacement and finding the ratio of the two.

IV. Results

These results explore two different cases in depth, a structure with two absorbers and a structure with five absorbers and then looks at trends in behavior as the the number of absorbers increase. For the system with two absorbers, a diagram of the metastructure and the baseline structure drawn to scale are shown in Fig. 4. Both of these structures have the same mass. The baseline structure is longer to account for the additional mass in the sections between the slots for the absorbers. Because of this additional mass, the metastructure has a lower stiffness than the baseline structure by 32%. Both of these structures are tested in two different ways. First, the frequency response function is calculated as outlined in the design section of this paper. Second, the impulse response function is calculated based off of an impulse loading to the tip and the response of that tip. These results are shown below in Fig. 5.

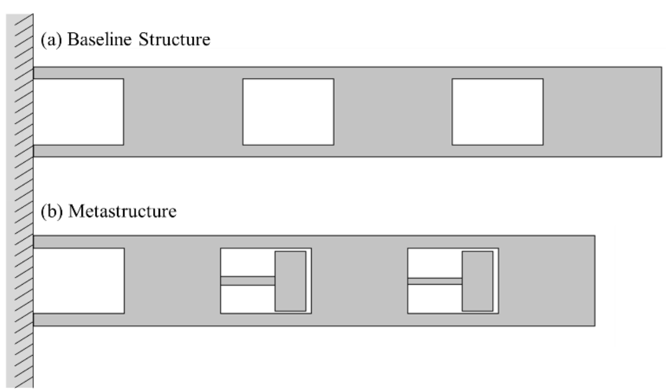

Figure 4. Scale drawing of (a) baseline structure and (b) metastructure with two vibration absorbers of equal mass.

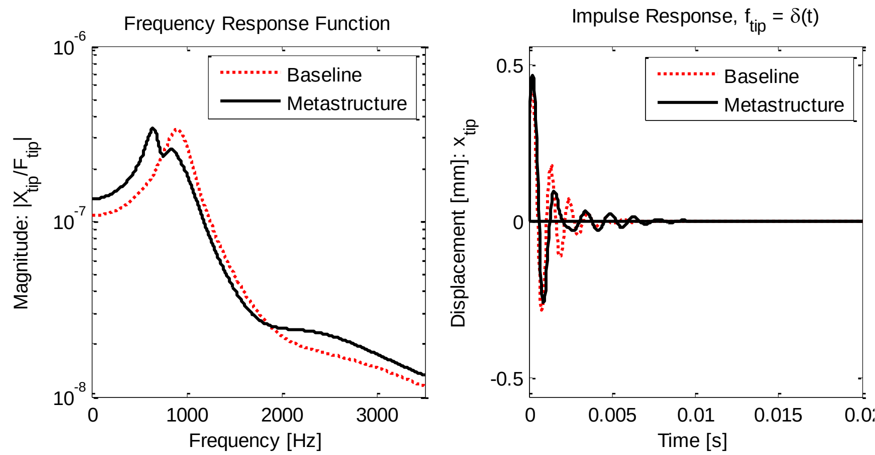

Figure 5. Frequency response function and impulse response for a system with two absorbers.

The difference in the values of the frequency response function for the static loading, at frequency of 0 Hz, is due to the difference in the stiffness values of the two structures. Since the metastructure is less stiff, it has a larger response from a static load than the baseline structure. This can been seen in both the frequency response function and the impulse response. The metastructure takes longer to die off when subjected to an impulse. Since the metastructure has a lower stiffness, the response is greater which is undesirable. The H_2 norm, which is related to the area under the frequency response function, is 4.1% greater for the metastructure than the baseline structure. This shows that the basestructure performs better in the vibration response and also has a higher stiffness.

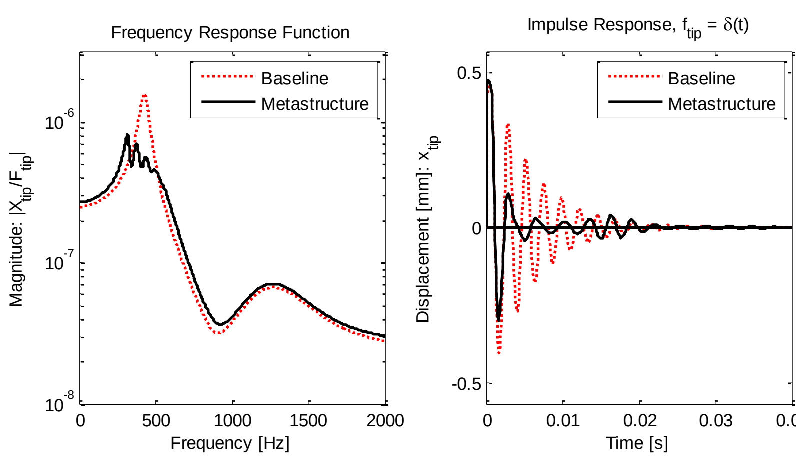

Figure 6. Frequency response function and impulse response for a system with five absorbers.

Next, we will examine a structure with five absorbers. The same results as shown in the previous case are shown in Fig. 6. Once again, the static values are different for the two structures; in this case the metastructure is 6% less stiff than the basline structure. Since this difference is much less than the stiffness discrepancy before, effects of the vibration absorbers can counter-act the negative effects of the lowered stiffness in the dynamic response. In the frequency response plot, the frequencies at which the individual abosrbers are tuned can be clearly seen as dips in the function. Overall, the metastructure has a 26.6% decrease in the H_2 norm compared to the baseline model. The increased performance can clearly been seen in the impulse response function. In this case, the metastructure has a more favorable response than the baseline model.

This leads to the question of how many absorbers are needed such that the negative effects of the decreased stiffness are counteracted. To explore this question, the plot in Fig. 7 was generated. In these simulations the number of absorbers was varied and the resulting H_2 norm values calculated and a percent decrease determined. A large decrease in the H_2 norm means better performance. Negative values indicate that the baseline model perfroms more favorably than the metastructure. Thus for the specific configuration tested here, the metastructure begins to outperform the baseline model once the number of absorbers reaches three.

V. Conclusion

The results of this paper show that this design of a metastructure can produce a favorable dynamic response. This was shown using a finite element formulation. The trends discovered used a lumped mass model in previous work of the authors were found to also hold for this finite element model. Namely, designing the absorbers such that the natural frequency of the absorbers linearly vary over the span of the peak for the fundamental natural frequency of the baseline mode can smooth out the fundamental natural frequency peak and result in a favorable dynamic response of the structure.

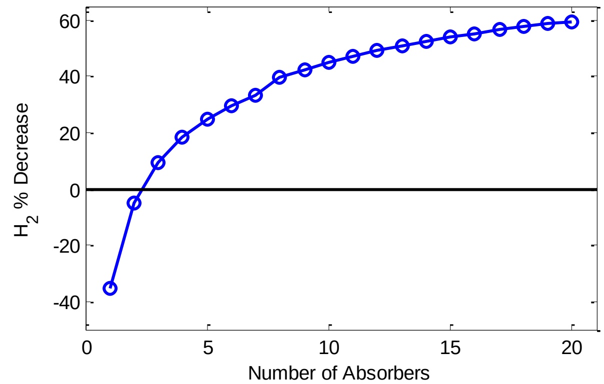

Figure 7. Plot of H_2 percent decrease for various numbers of absorbers.

Next, the paper shows that when you keep the mass constant from the baseline model and metastructure there is a tradeoff in stiffness of the structure. This reduction in the stiffness is more significant in the systems that have fewer number of absorbers. This reduction in stiffness is not beneficial in terms of performance of the structure but also the reduction in stiffness leads to an increase in the dynamic response of the structure. In order for a metastructure to have a favorable dynamics response but still have the same mass as the baseline structure, a larger number of vibration absorbers must be utilized.

Acknowledgments

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