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Engineering Notes

Aircraft Sensor Health Monitoring Based on Transmissibility Operators

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I. Introduction

SENSOR failure can have catastrophic consequences when the measurements are used for control. For example, failures of pitot probes for airspeed measurement have resulted in several major aircraft disasters [1]. Consequently, there is a pressing need to continually diagnose the health of sensors to ensure reliable control-system operation [2–5].

Various techniques can be used to detect a sensor failure. The most basic approach is to monitor the sensor signal and use characteristics of the data to detect faults [6–9]. However, this approach cannot distinguish between sensor faults and atypical system behavior due to anomalous disturbances. Alternatively, if the predicted response of a known control input differs significantly from the sensor data, then a sensor fault may be present. In this case, however, it may be difficult to distinguish sensor failure from the effects of disturbances, faulty actuators, and model error [1].

In the present note, we adopt a third approach, which is to use multiple sensors to detect discrepancies in the predicted output of one sensor based on the remaining sensors. The basic idea is to develop a model of the "transfer function" from one set of sensors to another set. Within the context of structural vibration, position-to-position, velocity-to-velocity, and acceleration-to-acceleration transfer functions are known as transmissibilities [10–14]. By using a transmissibility model, the residual between pairs or sets of sensors can be used to assess the health of a sensor. An advantage of the transmissibility approach is that there is no need to measure the excitation to the system, which can originate from either an actuator or ambient disturbances.

Examination of the equations relating one sensor to another sensor entails several issues that require special consideration. For example, because the transfer function between sensors does not arise as the forced response of a state space model, a sensor-to-sensor transfer function is not a transfer function in the usual sense. Therefore, we adopt the terminology pseudo-transfer function (PTF) and transmissibility operator to refer to a dynamic model relating sensor signals, which are called the pseudo-input and pseudo-output [15–17]. To use a PTF identified under healthy conditions for fault detection, it is necessary to show that the identified PTF is independent of both the initial condition and the input to the system. In fact, these properties can be shown to hold for single-input/single-output and multi-input/multi-output PTFs [18]. For the case of a pair of sensors and a single exogenous input, the numerator and denominator of the PTF are the numerators of the transfer functions between the input and the sensors. Generically, the poles of the plant play no role in the numerator and denominator of a PTF, whose roots are the zeros of the original transfer functions. These issues are discussed in [15] and applied to structural vibration in [16,17].

In the present note, we use PTFs for rate-gyro health monitoring in aircraft. The NASA generic transport model (GTM) [19,20] is used to simulate the fully nonlinear aircraft dynamics for data generation. In particular, we excite the aircraft by using the ailerons, elevator, and rudder, and we use rate-gyro measurements along with sideslip-angle measurements to construct a 1×3 transmissibility operator. We then use the transmissibility operator for health monitoring by computing the resulting one-step residual. The case of gyro drift is considered as an illustrative example.

II. Multi-Input/Multi-Output Transmissibility Operators

We consider the linear system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

$$x(0) = x_0 \tag{2}$$

$$y_i(t) = C_i x(t) + D_i u(t) \in \mathbb{R}^m \tag{3}$$

$$y_o(t) = C_o x(t) + D_o u(t) \in \mathbb{R}^{p-m} \tag{4}$$

where $u \in \mathbb{R}^m$, p is the number of measurements, m is the number of pseudo-inputs, and $p - m$ is the number of pseudo-outputs. The coefficient matrices have dimensions $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{m\times n}$, $C_o \in \mathbb{R}^{(p-m)\times n}$, $D_i \in \mathbb{R}^{m\times m}$, and $D_o \in \mathbb{R}^{(p-m)\times m}$.

The goal is to obtain a relation between y_i and y_o that is independent of both the initial condition x_0 and the input u. We consider a time-domain approach using the differentiation operator $\boldsymbol{p}$. Define

$$\Gamma_i(\boldsymbol{p}) \triangleq C_i \operatorname{adj}(\boldsymbol{p}I - A)B + D_i \delta(\boldsymbol{p}) \in \mathbb{R}^{m\times m}[\boldsymbol{p}] \tag{5}$$

$$\Gamma_o(\boldsymbol{p}) \triangleq C_o \operatorname{adj}(\boldsymbol{p}I - A)B + D_o \delta(\boldsymbol{p}) \in \mathbb{R}^{(p-m)\times m}[\boldsymbol{p}] \tag{6}$$

$$\delta(\boldsymbol{p}) \triangleq \det(\boldsymbol{p}I - A) \in \mathbb{R}[\boldsymbol{p}] \tag{7}$$

Then, the transmissibility operator from y_i to y_o is the operator [18]

$$\mathcal{T}(\boldsymbol{p}) = \Gamma_o(\boldsymbol{p})\Gamma_i^{-1}(\boldsymbol{p}) \tag{8}$$

Note that Eq. (8) is independent of both the initial condition x_0 and the input u. As in [18],

$$y_o(t) = \mathcal{T}(\boldsymbol{p}) y_i(t) \tag{9}$$

represents the differential equation

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$$\det \Gamma_i(\boldsymbol{p})y_o(t) = \Gamma_o(\boldsymbol{p})[\operatorname{adj}\Gamma_i(\boldsymbol{p})]y_i(t) \tag{10}$$

The transmissibility operator Eq. (8) is in continuous time. Henceforth, we assume that measurements of the output signals are obtained in discrete time, and we consider discrete-time transmissibility operators in the forward-shift operator $\boldsymbol{q}$ [21].

III. Noncausal Finite Impulse Response Approximation of Transmissibility Operators

Equation (8) shows that $\mathcal{T}(\boldsymbol{p})$ contains information about the zeros of the system and not the poles. Therefore, a nonminimum-phase zero in the pseudo-input channel of a transmissibility operator yields an unstable transmissibility operator. Moreover, if the pseudo-output channel of a transmissibility operator has more zeros than the pseudo-input channel, then the transmissibility operator is improper and thus noncausal. However, neither instability nor causality has the usual meaning associated with transfer functions. Nevertheless, to facilitate system identification, we consider a class of models that can approximate transmissibility operators that may be unstable, noncausal, and of unknown order. This class of models consists of noncausal finite impulse response (FIR) models based on a truncated Laurent expansion. The causal (backward-shift) part of the Laurent expansion is asymptotically stable because all of its poles are zero, whereas the noncausal (forward-shift) part of the Laurent expansion captures the unstable and noncausal components of the transmissibility operator [22].

Let $\mathcal{T}(\boldsymbol{q})$ be the discrete-time transmissibility operator whose pseudo-input is y_i and whose pseudo-output is y_o, that is,

$$y_o(k) = \mathcal{T}(\boldsymbol{q})y_i(k) \tag{11}$$

It follows from [22] that the truncated Laurent expansion

$$\mathcal{T}(\boldsymbol{q}, \theta_{r,d}) \triangleq \sum_{j=-d}^{r} H_j \boldsymbol{q}^{-j} \tag{12}$$

is a noncausal FIR approximation of $\mathcal{T}(\boldsymbol{q})$, where r and d are positive integers, $H_{-d}, \ldots, H_r \in \mathbb{R}^{(p-m)\times m}$ are coefficients of the Laurent expansion of the rational function $\mathcal{T}$ in an annulus that contains the unit circle, and

$$\theta_{r,d} \triangleq [H_{-d} \quad \ldots \quad H_r] \in \mathbb{R}^{(p-m)\times(r+d+1)m} \tag{13}$$

Using Eq. (12), the one-step predicted output is given by

$$y_o(k|\theta_{r,d}) \triangleq \mathcal{T}(\boldsymbol{q}, \theta_{r,d})y_i(k) = \sum_{j=-d}^{r} H_j y_i(k-j) \tag{14}$$

If $\mathcal{T}$ has a pole on the unit circle, then we define $\mathcal{T}_\alpha(z) \triangleq \mathcal{T}(\alpha z)$, where $0 < \alpha < 1$ and α is not the modulus of a pole of $\mathcal{T}$. Note that $\mathcal{T}_\alpha$ has no poles on the unit circle. Letting $H_{j,\alpha}$ denote the coefficients of the Laurent expansion of $\mathcal{T}_\alpha$ in an annulus containing the unit circle, it follows that $H_j = \alpha^j H_{j,\alpha}$ for all j.

IV. Identification of Transmissibility Operators Using Prediction Error Methods

To identify transmissibility operators that are possibly unstable, improper, and of unknown order, we use noncausal FIR models with prediction error methods (PEMs) [23].

For each choice of transmissibility coefficients

$$\bar{\theta}_{r,d} \triangleq [\bar{H}_{-d} \quad \cdots \quad \bar{H}_r] \in \mathbb{R}^{(p-m)\times(r+d+1)m} \tag{15}$$

it follows that

$$\mathcal{T}(\boldsymbol{q}, \bar{\theta}_{r,d}) = \sum_{j=-d}^{r} \bar{H}_j \boldsymbol{q}^{-j} \tag{16}$$

The residual of the transmissibility $\mathcal{T}(\boldsymbol{q}, \bar{\theta}_{r,d})$ at time k is defined to be the one-step prediction error

$$\begin{aligned} e(k|\bar{\theta}_{r,d}) &\triangleq y_o(k) - y_o(k|\bar{\theta}_{r,d}) \\ &= y_o(k) - \mathcal{T}(\boldsymbol{q}, \bar{\theta}_{r,d})y_i(k) \\ &= y_o(k) - \sum_{j=-d}^{r} \bar{H}_j y_i(k-j) \end{aligned} \tag{17}$$

The accuracy of $\bar{\theta}_{r,d}$ is measured by the performance metric

$$V(\bar{\theta}_{r,d}, \ell) \triangleq \frac{1}{\ell - d - r + 1} \sum_{k=r}^{\ell-d} \|e(k|\bar{\theta}_{r,d})\|_2^2 \tag{18}$$

where $\|\cdot\|_2$ is the Euclidean norm, and $\ell + 1$ is the number of data samples. Then, the PEM estimate $\hat{\theta}_{r,d,\ell}$ of $\theta_{r,d}$ is given by

$$\hat{\theta}_{r,d,\ell} \triangleq \underset{\bar{\theta}_{r,d}}{\arg\min}\ V(\bar{\theta}_{r,d}, \ell) \tag{19}$$

where

$$\hat{\theta}_{r,d,\ell} \triangleq [\hat{H}_{-d,\ell} \quad \cdots \quad \hat{H}_{r,\ell}] \in \mathbb{R}^{(p-m)\times(r+d+1)m} \tag{20}$$

It follows from Eq. (17) that the residual of the identified transmissibility $\mathcal{T}(\boldsymbol{q}, \hat{\theta}_{r,d,\ell})$ at time k is given by

$$\begin{aligned} e(k|\hat{\theta}_{r,d,\ell}) &= y_o(k) - y_o(k|\hat{\theta}_{r,d,\ell}) \\ &= y_o(k) - \mathcal{T}(\boldsymbol{q}, \hat{\theta}_{r,d,\ell})y_i(k) \\ &= y_o(k) - \sum_{j=-d}^{r} \hat{H}_{j,\ell} y_i(k-j) \end{aligned} \tag{21}$$

The identification data set used to obtain Eq. (19) is different from the validation data set used to compute Eq. (21).

V. Aircraft Example

To apply transmissibility operators to aircraft sensor health monitoring, we consider the NASA GTM model [19,20], which is a fully nonlinear model with aerodynamic lookup tables. GTM includes sensor models that can be modified to emulate sensor faults.

Let $\delta\beta$ denote the sideslip angle in degrees, and let $\omega \triangleq [\omega_x \quad \omega_y \quad \omega_z]^T$ be the angular velocity of the aircraft relative to the Earth resolved in the aircraft frame, where ω_x, ω_y, and ω_z are measured by rate gyros in degrees per second. Define $\mathcal{T}(\boldsymbol{q})$ to be the 1×3 transmissibility operator whose pseudo-input is $y_i \triangleq [\omega_x \quad \omega_y \quad \delta\beta]^T$ and whose pseudo-output is $y_o \triangleq \omega_z$, that is,

$$\omega_z(k) = \mathcal{T}(\boldsymbol{q}) \begin{bmatrix} \omega_x(k) \\ \omega_y(k) \\ \delta\beta(k) \end{bmatrix} \tag{22}$$

We set the sampling time $T_s = 0.01$ s, and we assume that sampled data are available for $t \in [0, 500]$ s, that is, $0 \le k \le 50{,}000$ steps. Let δa, δe, and δr denote the aileron, elevator, and rudder deflections, respectively. For all $0 \le k \le 50{,}000$ let $\delta a = \sin(\Omega k T_s)$ deg, $\delta e = \sin(\Omega k T_s + 45)$ deg, and $\delta r = \cos(\Omega k T_s)$ deg, where $\Omega = 30$ deg/s. Physically, the displacements of the ailerons, elevator, and rudder are sinusoidal with an amplitude of 1 deg and a period of 12 s. We consider the following initial GTM trim conditions: level flight, altitude = 8000.00 ft, equivalent airspeed = 89.18 kt, true airspeed = 100.58 kt, alpha = 3.00 deg, beta = 0 deg, gamma = 0 deg, roll = 0.066 deg, pitch = 3.00 deg, yaw = 45.00 deg, ground track = 45.00 deg, elevator = 2.70 deg, and throttle = 22.84%.

To emulate sensor noise, we add zero-mean white noise with a siganal-to-noise ratio (SNR) of 50 to all identification and validation measurements of ω_x, ω_y, ω_z, and $\delta\beta$. We use PEM with a noncausal FIR model with $r = 50$ and $d = 50$, along with identification data for $2500 \le k \le 20,000$ steps to obtain the identified transmissibility $\mathcal{T}(\boldsymbol{q}, \hat{\theta}_{r,d,\ell})$ of $\mathcal{T}(\boldsymbol{q})$. Figure 1 shows the Markov (impulse response) parameters of $\mathcal{T}(\boldsymbol{q}, \hat{\theta}_{r,d,\ell})$ from each pseudo-input ω_x, ω_y, and $\delta\beta$ to the pseudo-output ω_z. Data for $20,000 < k \le 50,000$ are used for validation. Figure 2 shows ω_z and its one-step prediction $\hat{\omega}_z \triangleq \mathcal{T}(\boldsymbol{q}, \hat{\theta}_{r,d,\ell})[\omega_x \quad \omega_y \quad \delta\beta]^T$ for $28,000 \le k \le 29,000$, that is, for $t \in [280, 290]$ s.

Next, we consider the case where a ramp-like drift with a slope of 0.05 deg$/s^2$ is added to measurements of either ω_x, ω_y, or ω_z starting at $t = 300$ s. Measurements of ω_x, ω_y, ω_z, and $\delta\beta$ are used with the identified transmissibility operator $\mathcal{T}(\boldsymbol{q}, \hat{\theta}_{r,d,\ell})$ to generate the residual using Eq. (21). For all $2500 \le k \le 50,000 - w - d$, where $w + 1$ is the data-window size, define

$$E(k|\hat{\theta}_{r,d,\ell}, w) \triangleq \sqrt{\sum_{j=k}^{w+k} e^2(j|\hat{\theta}_{r,d,\ell})} \tag{23}$$

Figure 3 shows $E(k|\hat{\theta}_{r,d,\ell}, w)$ for $w = 1000$ steps, where a ramp-like drift is added to measurements of either ω_x, ω_y, or ω_z. Figure 3 shows that the residual levels increase after $t = 300$ s, which indicates that, in all three cases, at least one of the sensors is faulty. However, we cannot conclude from Fig. 3 which sensor is faulty. Similar results can be shown for other types of faults, such as magnitude saturation, rate saturation, deadzone, and jam.

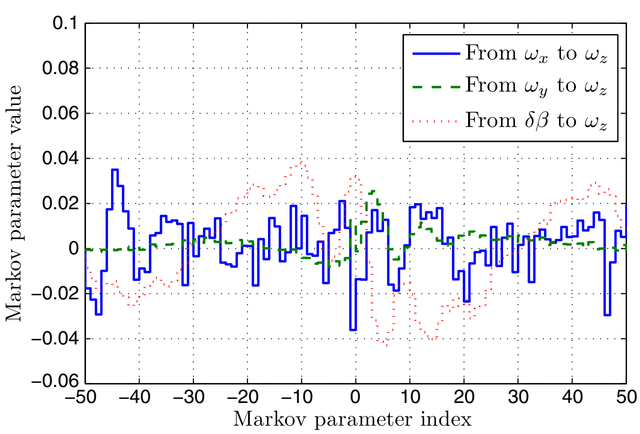

Fig. 1 Entries of the estimated Markov parameters $\hat{\theta}_{r,d,\ell}$ of $\mathcal{T}(q, \theta_{r,d})$ from each pseudo-input ω_x, ω_y, and $\delta\beta$ to the pseudo-output ω_z.

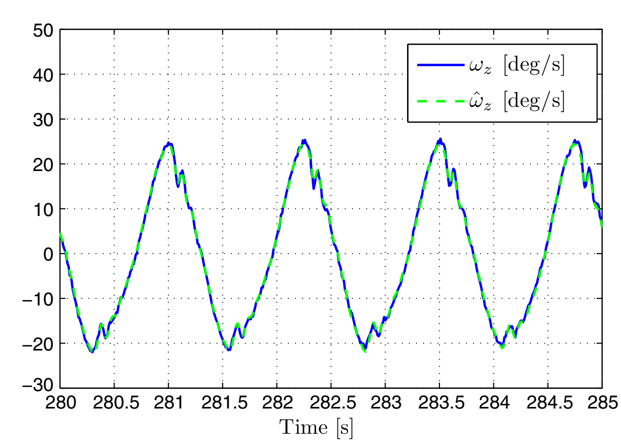

Fig. 2 Measurements of ω_z and the computed one-step prediction $\hat{\omega}_z$ under healthy sensor conditions with SNR of 50 for both the pseudo-inputs and the pseudo-output.

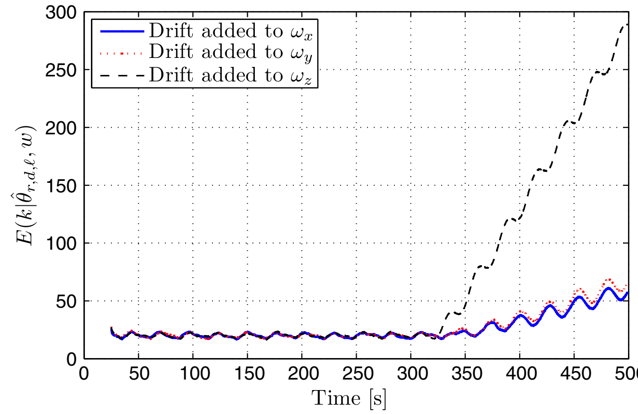

Fig. 3 $E(k|\hat{\theta}_{r,d,\ell}, w)$ for $w = 1000$ steps, where a ramp-like drift is added to measurements of either ω_x, ω_y, or ω_z.

VI. Conclusions

An estimate of the transmissibility operator between pairs or sets of sensors can be used to detect sensor faults in the presence of unknown external excitation. The ability to detect sensor faults by exploiting the presence of unknown external excitation is the key difference between this approach and techniques based on residual generation. In particular, the transmissibility operator is a relationship between pairs or sets of sensors that is independent of the time history of the external excitation.

Transmissibility-based fault detection depends on various assumptions. In particular, this approach assumes that the plant itself does not change between the identification and validation data sets and that the location of the external excitation does not change. By using the estimated transmissibility operator, the residual between pairs or sets of sensors can be used to detect a sensor failure. Moreover, the characteristic shape of the residual can be used to infer the type of sensor failure. However, this approach does not identify which sensor has failed. This problem is left for future research.

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