Adaptable Wave Propagation In Phononic Structures Via Origami Folding

by

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This work is dedicated to amma, nanna, anna and my better half who are a constant source of inspiration and to their sacrifices that made me who I’m today.
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ABSTRACT

Wave propagation inside a fluid-media with periodically distributed solid-inclusions (viz. phononic structures) exhibit bandgaps, which are frequency regions in transmission spectra where wave propagation is attenuated. Controlling acoustic bandgaps have many engineering applications such as in sound barriers, acoustic signal processing elements, infra-ultra sound imaging systems and acousto-optical/acousto-thermal devices; and incidentally, phononic structures that can control wave propagation in terms of band structure provide an excellent base for such applications. In spite of the diverse technical studies on the effect of geometric parameters of phononic structures on wave propagation, research on broadband acoustic bandgap adaptation that will have large impact on the abovementioned engineering applications is still outstanding.

One of the important geometric properties in ABG design is the lattice-type – five different 2D (two-dimensional) Bravais lattices (rectangle, square, hexagon, centered rectangle and oblique) are classified based on their symmetry properties. It is possible that by reconfiguring periodicity of phononic structures between distinct lattice-types, where each lattice-type exhibit fundamentally different dispersion relations, we may achieve broadband wave tuning.

In this spirit, to advance the state of the art, this thesis pioneers a new class of reconfigurable phononic structures that utilizes origami folding to reconfigure the periodicity of lattice and achieve significant phononic bandgap adaptation. The design of origami phononic structure is composed of cylindrical inclusions attached to the vertices of Miura origami sheet. It is found that origami folding, which can enable significant and precise topology reconfigurations in the lattice of inclusions between distinct Bravais lattice-types, can bring about drastic changes in waves propagating through phononic structure. Such versatile wave transmission control is demonstrated by extracting band structure of waves propagating in the plane perpendicular to the inclusions axis. Further, numerical and experimental simulations performed on finite origami structure demonstrate that wave propagation can be significantly changed in terms of bandgaps.
Major contributions, impacts and uniqueness of this research

- A reconfigurable origami architecture is proposed that can transforms the underlying periodicity between distinct symmetry Bravais lattices. Unlike previous theoretical studies that rely on symmetry reduction techniques to change bandgaps, here through origami folding, the symmetry of the lattice is shifted between different point groups that leads to drastic adaptation in bandgaps. To study the effect of such reconfiguration on wave propagation, a numerical model is developed that combines origami folding kinematics with wave propagation theory.

- We utilize the versatile wave tailoring capabilities of origami structures to advance unique wave control mechanisms such as (a) traffic noise mitigation (b) tunable waveguiding and (c) adaptive unidirectional transmission via achieving broadband tailoring in their frequency spectra. These unique adaptable wave control mechanisms are demonstrated via analytical, numerical and experimental investigations. The outcomes and findings of this research are useful to engineering applications such as traffic noise barriers and acoustic filters, especially under system uncertainties and variations.

- Being a one degree of freedom, the origami rigid-folding mechanism used for lattice reconfiguration, can achieve global topology transformation and the bandgap adaptation via minimal local actuation. Further, since the related folding principle is scale independent, origami can be fabricated at vastly different scales to target different frequency ranges. For example, due to the equivalence in wave equation of sound and light, the phononic crystal design proposed in this thesis can also be used to use to build photonic crystal and manipulate light wave propagation.
CHAPTER 1.
INTRODUCTION

1.1. Phononic structures and special characteristics

Periodic structures can be defined as structures with repeating material or geometric properties. These kinds of structures form the basis for many modern day technologies such as space structures, machines, transducers, communication devices etc. In the past decade, there has been special focus on engineered periodic structures with repetitive material properties viz. phononic structures, which can manipulate sound, light and elastic waves. Examples of phononic structures in 1D, 2D and 3D are shown in Fig. 1.1. The yellow continuous material is the host, while the red material represents inclusions. The impedance mismatch between the inclusions and the host inside the phononic structures leads to multiple reflections of waves, propagating inside this composite media. Such complex interference pattern leads to the formation of bandgaps — which are regions in transmission spectra where waves are attenuated.

Figure 1.1: Schematic of 1D, 2D and 3D Phononic structures [1]
Depending on the material properties and excitation scheme, the host media can support either longitudinal waves (in gaseous host media) or transverse waves / mixed longitudinal-transverse wave (in solid host media). Acoustic bandgaps (ABGs) generated in 2D phononic structures with longitudinal wave propagation will be the focus of this thesis.

Being able to control acoustic wave propagation has rich engineering applications [2–8] such as for acoustic signal processing, sound/vibration attenuation, sonar/medical imaging etc.; and incidentally, periodic structures with the ability to control sound waves, in terms of ABGs, form an excellent base for these applications (Fig. 1.2).

1.2. Literature review

Because of the wide spread applications potential, studies were performed on phononic structures to tailor their ABGs. While different properties of phononic structure can affect ABGs, viz. material or geometry [14,15], many of the studies in acoustics are focused on the later. This is because geometric features such as the spatial distribution of inclusions or shape/size of the inclusions that affect wave propagation can be modified easily in acoustic structures; as the host media that constitutes of gas or liquid, offers minimum resistance to the change in geometry and distribution of inclusions.

Figure 1.2: Application of controlling wave propagation (a) waveguide [9] (b) noise barriers [10] (c) earthquake proof structures [11] (d) ultrasonic transducers [12] and (e) under-water imaging systems [13]
1.2.1. Lattice symmetry

Among different geometric properties, lattice symmetry is the most important feature to tune ABGs; based on the space symmetry properties of the lattice there are five distinct 2D (two-dimensional) Bravais lattices, viz. rectangle, square, hexagon, centered rectangle and oblique. Each of these lattices exhibits distinct dispersion characteristics and each band structure contain degeneracies induced due to symmetry of the corresponding lattice [16–24]. More details about the co-relation between lattice symmetry and band structure will be provided in chapter 2.

For the purpose of understanding, here we discuss some basics of band structure. In Fig.1.3, we show the acoustic band structure of square and hexagonal lattice-types. A typical band structure provides information about eigen frequencies (vertical axis of Fig.1.3) along different wave directions (horizontal axis of Fig.1.3). Discontinuous regions in this band structure, where there are no eigen frequencies for all wave directions, are called complete ABGs (represented by horizontal shaded regions) and no wave propagation is possible for any direction of incidence.

Upon comparing the band structures of the two lattice symmetries in Fig.1.3, it can be noticed that (i) spectral locations where ABGs occur in square lattice-type (Fig.1.3a) are different from that of

![Figure 1.3: Effect of lattice symmetry on band structure [18] (a) square lattice (b) hexagonal lattice. Lattice distributions of different symmetries are provided in the insets.](image)

hexagonal lattice-type (Fig.1.3b) and (ii) inside band structure of a given lattice-type, band degeneracies are formed (where two different bands (modes) meet) that restrict the formation and widening of ABGs. This later feature of band structure is exploited in the past works to tune ABGs.

Since degeneracies are formed because of the symmetry in the lattice, they could be lifted by reducing its symmetry; and by doing so will lead to widening of existing bandgaps and formation of new ABGs. Some of the symmetry reduction mechanisms are reviewed in the following paragraphs.

**Addition of an extra inclusion**

By including additional inclusions in the unit-cell of the original lattice, spatial symmetry could be reduced. For example, [19] overlaid two square symmetry lattices – each with different inclusion size (refer Fig.1.4a), while in other studies [19,25] two different hexagon lattices are superimposed (refer Fig.1.4b). These techniques reduced the number of symmetry operations (which are composed of translational, rotation, inversion and mirror symmetries – all of these are collectively called *point-group symmetry*) of original lattice; for example, some of the translational properties are lost by adding second inclusion of different size into the unit-cell.
Changing the orientation of non-circular inclusions

In another category of studies [30–33] (Fig.1.4c), non-circular inclusions are re-orientated to reduce the symmetry of lattice; these techniques removed some of the mirror symmetry properties.

Changing the spatial distribution of inclusions

Finally, some of the more recent methods introduced topology transformation to reduce lattice symmetry [26–28,34]; for example, Yao, Xu and Rubio, studied the effect of relative distances between inclusions (Fig.1.4d-e), while Lee studied the effect of inclusion shape (Fig.1.4f) on acoustic band structure. In these techniques, some of the mirror symmetry operations are reduced.

Bandgap tailoring via symmetry reduction

All of the above symmetry reduction techniques are capable of tuning the band structure and the evolution of ABGs for some of the reduction techniques viz. for architectures given in Fig.1.4a, 1.4c and 1.4d are shown in Fig.1.5(a-c) respectively and are explained in the following paragraphs.

The bandgap evolution via the symmetry reduction technique in Fig.1.4a is given in Fig.1.5a. It can be observed that as the diameter of the extra inclusion is increased (represented along the horizontal axis), the lowest ABG width (represented by grey regions) slowly reduces and vanishes.

Figure 1.5: ABG evolution in symmetry reduction studies. Red curve represent the low frequency bandgaps while green lines represent bandgaps at high frequencies [19,30,34].
at diameter ratio of ~0.25 and then beyond a diameter ratio of ~0.4 it re-emerges and increases in width (follow the red line in Fig.1.5a to see the trend); further a high frequency ABG (refer green dashed-dot line in Fig.1.5a) is also formed, which increases in width with increase in diameter of extra inclusion.

In the case of symmetry reduction via rotating a square rod (Fig.1.4c), the lowest order ABG width (plotted along vertical axis and represented by red curves in Fig.1.5b) is increased with increase in rotation angle (represented along horizontal axis) and this effect is more prominent in lattices with higher filling fraction.

Finally, the results of topology transformation study (Fig.1.4d), are given in Fig.1.5c. In Fig.1.5c, each panel corresponds to band structure of different spatial distributions; where the spatial distribution of inclusions in unit-cell are shown in the inset. Further, we mark the low and high frequency ABGs by red solid and green dash-dot lines respectively.

**1.2.2. Filling fraction**

Apart from the lattice symmetry, another important geometric feature that affects bandgaps is the filling fraction. It is found that with increase in filling fraction of the lattice – via increasing the inclusion size or by reducing the lattice constant ($r$ or $a$ in inset of Fig.1.3) – the low frequency ABGs increase in width (represented by red lines) and some higher frequency ABGs (represented by green dash-dot lines) may come into existence and cease to exist, as shown in Fig.1.6; and this trend is similar for different lattice symmetries (square and hexagon lattice symmetries are represented by vertical and horizontal hashed regions) as shown in Fig.1.6.
1.3. Limitation of current research

In all of the above research, viz. either symmetry reduction or filling fraction studies, the bandgap tuning is associated with gradual change in low frequency ABG in conjunction with formation of new ABGs at higher frequency. Although these features are promising for broadband wave tuning, robust and practical wave adaptation and control that require shifting between ABGs of distinct spectral locations cannot be realized by these techniques, as discussed below:

- One could only reduce the number of symmetry operations of the lattice using these techniques but the overall point-group symmetry of the lattice cannot be changed. Therefore, the bandgaps features pertaining to the initial lattice symmetry (for example, the low frequency ABGs) will be present even after reducing lattice symmetry or modifying filling fraction, and they can only be continuously changed (as represented along green lines in Figs.1.5 & 1.6).
- The new high frequency ABG’s formation mechanism is due to the splitting of degenerate bands. Therefore, the spectral location of the newly created ABGs is also dictated by the lattice symmetry of the original lattice and cannot be created at any location as desired.
- Finally, the previous studies on ABG tuning via modifying geometric parameters are theoretical in nature and the studies to incorporate programmable and controllable geometry transformation mechanism are non-existent.

![Figure 1.6: Effect of filling fraction on ABGs of square and hexagon lattice symmetries [18.](image)](image)
In summary, the opportunities to selectively leverage the breadth of adaptable wave characteristics in an on-demand tunable manner would open new doors for engineering systems, responding to environmental variations in ways that maximize performance, enhance safety, and optimize operational effectiveness. Yet, such opportunities are not well explored and insufficiently elucidated.

1.4. Hypothesis, Research Goal and New Idea

While the effect of reducing symmetry in a given lattice is studied in detail, there has not been any concrete investigation on the effect of changing the entire point-group symmetry of the lattice. Since each of the five distinct 2D Bravais lattices-types have distinct point-group symmetries and correspondingly have different wave characteristics, being able to shift between them may lead to drastic bandgap adaptation and hence elevate the limitations of symmetry reduction studies and bandgap control of acoustic wave motion can be tremendously improved.

From reviewing the current state of the art and the above arguments, the thesis research goal is to (a) create an ideal reconfigurable structure with significant symmetry reduction and versatile topology transformation capable of shifting between different 2D Bravais lattice-types and exploit their distinct dispersion characteristics to achieve broadband ABG adaptation, (b) develop simple/efficient topology reconfiguration mechanisms that are practical to implement and scalable to target a wide spectra of applications, and (c) utilize the adaptive wave features to explore novel wave control mechanism.

To achieve our research goal, in this thesis we synthesize a new class of reconfigurable phononic structures viz. origami structures (OS) that can fold to achieve phenomenal bandgap adaption via topology reconfiguration. Our new idea here is to exploit the rigid origami folding to reconfigure the lattice topology of the underlying phononic structure between fundamentally different 2D Bravais lattice-types. Since the bandgap properties are strongly dependent upon the underlying lattice symmetry, the wave propagation characteristics can be drastically altered via origami folding.
Moreover, origami folding is scale independent and can be fabricated at vastly different length scales without losing reconfigurability; this feature can be utilized to target a wide spectra applications. Also, the rigid folding is a one degree-of-freedom mechanism that requires minimal local actuation for sophisticated and precise global topology transformation. All of the above features illustrate the potential of origami structure that can bring on-demand wave tailoring to a new level and advance the state of the art in tunable wave propagation. Towards this end, we utilize the versatile wave tailoring properties of origami phononic structures to explore unique wave control mechanisms such as (a) traffic noise mitigation (b) tunable waveguiding and (c) adaptive unidirectional transmission.
CHAPTER 2.
BACKGROUND

This chapter will serve as a detailed background for evaluating wave propagation in periodic structures. Through this chapter we seek to explain various aspects of acoustic band structures and help identify key geometric parameters of phononic structures that affect ABGs. First we introduce the equations describing acoustic wave motion followed by classification of periodic geometries. We then provide the mathematical framework required to cast the wave propagation problem of phononic structure and solve for band structures. Following mathematical formulation, parametric analysis will be done on different periodic geometries and we draw conclusions on the importance of lattice symmetry and filling fraction on bandgap formation.

2.1. Acoustic wave equation

In this thesis, our focus will be on 2D phononic structures with host media made of gas or liquid and inclusions made of solid material. Since gaseous/liquid (fluid) host cannot support shear deformations, only longitudinal wave propagation is allowed in this study. On the other hand, the periodically scattered solid inclusions should ideally support different polarizations of waves – i.e. both longitudinal and transverse; but due to the large contrast in the impedance values between solid inclusion and fluid host, most of the wave energy incident on the inclusion will be reflected back into the host. Hence, with little wave energy transfer into the solid inclusions, it is safe to assume that no transverse waves will be excited in the inclusions. Based on this assumption, we can model the solid inclusions as an equivalent media that supports only longitudinal wave propagation. Overall, the longitudinal wave propagation will be studied which is also known as acoustic wave propagation in phononic structures.
Basic linear acoustic equations, illustrated here, are limited to small oscillations of pressure in a compressible ideal fluid. The energy is propagated via the interactions of these small oscillations inside acoustic media. The equations for wave propagation can be derived by applying fundamental laws of force balance and material constitutive relation on a small slice of fluid under pressure as shown in Fig. 2.1. We start with the derivation of 1D wave propagation and later generalize the equations to higher dimensions.

Let’s consider a slice of fluid under pressure, $P$, as shown in Fig. 2.1; in its equilibrium state, the slice with surface area $\sigma$ is at position $x$. Let’s assume that such a slice under influence of differential pressure across its boundaries, is displaced in space by distance $x + u(t, x)$ and the initial slice thickness $dx$ is also changed to $dx + du$. Where

$$du = \frac{du}{dx} dx = Sdx,$$

$S$ represents the strain.

We note that $du$ is much smaller than $dx$ because of linear acoustics (small strain approximation). This approximation means that the coordinate of the right side of the displaced slice is $x + u(t, x) + dx$, while that of the left side is $x + u(t, x)$. As a result the net pressure force acting on the slice is given by

$$dF = \sigma(P(x + u(t, x)) - P(x + u(t, x) + dx))$$

Upon expanding the pressure to first order, the above equation becomes

$$dF = -\sigma \frac{dP(x + u(t, x))}{dx} dx;$$

where $\sigma dx = dV$ is the volume of the slice.

Now applying Newton’s first law, yields
\[- \frac{dP(x + u(t, x))}{dx} = \rho \frac{d^2u}{dt^2}\]

Because of the small strain approximation, here we neglected any variations in the density ($\rho$) of the fluid, and approximated mass to be $\rho dV$.

It is to be noted that the pressure in above equation, $P = P_o + p(t,x)$, is representative of both static pressure $P_o$ and dynamic pressure $p(t,x)$. Hence, upon assuming the static pressure to be constant with time and space, $P_o$ can be dropped and we are left with only dynamic pressure $p(t,x)$.

Now from the compressible fluid model, we have the condition that the dynamic pressure is proportional to the local volume variation through the proportionality constant bulk modulus ($B$), i.e.

\[p = -B \frac{du}{dx} = -BS\]

Where strain ($S$) is considered a representative of local dilation. The negative sign indicates that the volume is diminished as the pressure is increased.

Finally upon combining the above material constitutive equation with the equation derived via force balance, we have the following 1D wave acoustic equation for homogenous isotropic media.

\[
\left(\frac{1}{\rho} \frac{d^2p(t,x)}{dx^2}\right) = \frac{1}{B} \frac{d^2p(t,x)}{dt^2}
\]

If the material properties viz. $\rho$ and $B$ vary with space, the above wave equation should be modified as below, to accommodate for the inhomogeneous material distribution.

Eq. 2.1

\[
\nabla \left(\frac{1}{\rho(\bar{x})} \nabla (p(t,\bar{x}))\right) = \frac{1}{B(\bar{x})} \frac{\partial^2 p(t,\bar{x})}{\partial t^2}
\]
In the above eq. 2.1, $x$ is replaced by $\bar{x}(=x \hat{i} + y \hat{j})$ to represent 2D wave propagation and $\nabla$ operator represents the derivative with respect to different directions, i.e.

$$\nabla(p(t, \bar{x})) = \frac{\partial p(t, \bar{x})}{\partial \bar{x}} \hat{i} + \frac{\partial p(t, \bar{x})}{\partial \bar{y}} \hat{j}.$$  

This acoustic wave equation will be solved to study the pressure distribution in phononic structures. In this thesis, the host and inclusion are assumed to be made of air and steel/PVC respectively and each of their material properties viz. density ($\rho$) and bulk modulus ($B$) are given below in Table. 2.1.

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Steel</th>
<th>PVC</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, ($\rho$), [Kg/m$^3$]</td>
<td>7.85*10$^3$</td>
<td>1.56*10$^3$</td>
<td>1.3</td>
</tr>
<tr>
<td>Bulk Modulus, (B) [N/m$^2$]</td>
<td>290759.25*10$^6$</td>
<td>8948.2*10$^6$</td>
<td>0.15*10$^6$</td>
</tr>
<tr>
<td>Longitudinal speed of sound ($C_l$) [m/s]</td>
<td>6086</td>
<td>2395</td>
<td>340</td>
</tr>
</tbody>
</table>

### 2.2. Periodic geometry

#### 2.2.1. Direct space

Here we will describe some of the fundamental geometric properties phononic structures. A phononic structure in 2D, given in Fig.1.1b, can be represented by a set of infinite translation operations on a unit-cell in direct space; where the translation vectors are defined as

$$R = n_1 \bar{a}_1 + n_2 \bar{a}_2$$  

The two non-collinear vectors $\bar{a}_1, \bar{a}_2$ are called lattice vectors and $n_1, n_2$ belong to the set of integers. The number of lattice vectors depend on the dimensions of the periodic structure, for example as shown in Fig.2.2, a 2D phononic structure requires two lattice vectors.
A unit-cell of a phononic structure is composed of continuous host and discrete inclusions, as shown by shaded regions in Fig.2.2, where the spatial distribution of inclusions inside the unit-cell is called basis; for example, the shaded unit-cells in Fig.2.2(a), either have one or two total inclusions that form the basis. The unit-cell with basis, along with a set of lattice vectors \(a_1, a_2\), are essential to completely define a phononic structure.

It is to be noted that the choice of unit-cell is not unique; as shown in Fig.2.2 (a), the two shaded unit-cells with basis inclusions marked in either blue or red, can both recreate the same phononic structure. A unique primitive unit-cell is one with lowest number of inclusions in its basis (or the one with smallest area). For example, upon comparing the two unit-cells in Fig.2.2(a), we can clearly say the unit-cell marked in red with only one basis inclusion is a prime candidates for primitive unit-cell.

A primitive unit-cell may be constructed by following the rules laid out by Wigner-Seitz [35]. (1) Draw lines that connect a given inclusion to all of its neighboring inclusions and (2) Draw perpendicular bisectors to these lines. The space enclosed within these constructions yield the primitive unit-cell. Sometimes it is more convenient to deal with unit-cell that is non-primitive, as it exhibits the symmetry of the lattice more clearly. For example, the non-primitive unit-cells marked in blue of Fig.2.2(a), exhibits the rectangular symmetry of the lattice more clearly than the primitive unit-cell with six edges marked in red.
Now based on the translational properties of the phononic structurers there can be two lattice classifications viz. (a) Bravais lattice and (b) non-Bravais lattices. In a Bravais lattice all inclusions are equivalent, meaning, for an observer located on any of the inclusion, the phononic structure appears exactly the same. Or in other words, if the crystal is translated by any vector joining two inclusions, say T in Fig.2.2(a), the structure appears exactly the same as it did before the translation (this property is called translational symmetry). On the other hand, in a non-Bravais lattice, some inclusions are non-equivalent, for example in Fig.2.2(b) after translating the structure over T, the spatial distribution of the translated lattice does not align with that of the original lattice.

In two-dimensions, there are five such Bravais lattices; four of them are marked in Fig.2.3 along with their primitive unit-cell representations. Each of the Bravais lattice have different translational, rotation, inversion and mirror symmetries (all of which are together called as point-group symmetry). Because of the differences in point-group symmetries and the fact that wave propagation is strongly related to the symmetry of the lattice, each of these Bravais lattices exhibit very different wave behavior; plots of dispersions characteristics of phononic structure in each of the different Bravais lattice configuration will be shown in later sections (refer Sec. 2.4).

Another important geometric feature that defines the phononic structures is the filling fraction, which is a ratio of the area occupied by the unit-cell to that of the area of inclusions in the basis. The filling fractions (ff) of four different 2D Bravais lattice configurations, with cylindrical inclusions of radius $R_o$, are given in Eq. 2.3. Dispersion relations are also sensitive to filling fraction and studies in this regard will be also shown in further sections (refer Sec. 2.5).
$$f_{\text{square or rectangle}} = \frac{\pi R_o^2}{|a_1||a_2|};$$

Eq. 2.3

$$f_{\text{hexagon or center-rectangle}} = \frac{\pi R_o^2}{|a_1||a_2|\sin(\varphi)};$$

2.2.2. Reciprocal space

The reciprocal lattice is described as the set of all vectors $G$ that satisfy the following identity

Eq. 2.4

$$e^{-iG\cdot R} = 1$$

Where $G = m_1\vec{b}_1 + m_2\vec{b}_2$, with $\vec{b}_1$, $\vec{b}_2$ being the lattice vectors in reciprocal space and as before $R = n_1\vec{a}_1 + n_2\vec{a}_2$ represents a general lattice vector in direct space.

Simplification of the above relations leads to the following relation between direct and reciprocal lattice vectors. From Eq. 2.5 it can be seen that the reciprocal lattice vectors $(\vec{b}_2,\vec{b}_1)$ are orthogonal to the lattice vectors in real space $(\vec{a}_1,\vec{a}_2)$.

Eq. 2.5

$$\vec{b}_i\cdot\vec{a}_j = 2\pi\delta_{ij}$$

Equivalent to the lattice vectors $(\vec{a}_1,\vec{a}_2)$ in direct space, infinite translation operations of the reciprocal lattice vectors $(\vec{b}_1,\vec{b}_2)$ defines the reciprocal space that which has similar symmetry properties as that of direct space. For example, the reciprocal lattice (formed by $\vec{b}_1,\vec{b}_2$) of Bravais and non-Bravais lattice (given in Fig.2.2) are shown below in Fig.2.4. Since reciprocal lattice vectors $(\vec{b}_1,\vec{b}_2)$ depend on the direct lattice vectors $(\vec{a}_1,\vec{a}_2)$, different unit-cell definitions lead to different reciprocal spaces. Primitive and non-primitive unit-cells in Fig.2.2(a), leads to reciprocal spaces Fig.2.4(a,b), while unit-cell in Fig.2.2(b) yields reciprocal space given in Fig.2.4(c). Similar
to direct lattice, a primitive unit-cell can also be drawn in reciprocal space (shown in Fig. 2.4, as shaded regions) and such unit-cells are called as the 1st Brillouin zone.

The importance of the Brillouin zone stems from the Bloch wave description of wavefield in periodic media (Eq. 2.12); it is found that the solutions can be completely characterized by Bloch wave behavior in 1st Brillouin zone [36]. Or in other words, the knowledge of Bloch waves (used for describing the wavefield) inside the 1st Brillouin zone is sufficient to construct all the Bloch waves of the periodic media.

Now upon considering symmetry properties, the 1st Brillouin zone can be further reduced into the smaller area and it is found that extracting Bloch wave vectors across these high-symmetry boundaries is adequate to characterize the wave field in entire periodic structure. Such reduced 1st Brillouin zones are marked in Fig. 2.4 by a black closed polygon and the high symmetry points are marked as Γ, X, M and N.

It has to be noted that if a non-primitive unit-cell is used in direct lattice (blue unit-cell marked in Fig. 2.2(a)), then the 1st Brillouin zone (required for calculation of Bloch waves) has to be extracted from its corresponding reciprocal space (Fig. 2.4(b)); using Brillouin zone definition of a non-primitive unit-cell to extract Bloch wave vectors, may sometimes be computationally efficient [37].
2.3 Plane wave expansion method

Plane wave expansion method is one of the most commonly used methods to solve wave propagation in phononic structures [15,38]. It is a versatile method that can handle different lattice geometries/shape of inclusions and can be used in any number of dimensions. Our focus will be on 2D phononic structure with periodicity in $xy$ reference plane, while the material properties along $z$-reference axis are invariant. Though the inclusions are made of solid, only longitudinal wave propagation is considered due to the large impedance contrast between inclusions and host (as explained in Sec. 2.1). Hence, here we show the application of PWE method to 2D acoustic wave equation (Eq. 2.1) and extract band structure of phononic structure. The band structures provide abundance of information on the wave propagation characteristics, hence they will be studied in detail using PWE method.

The basic idea of PWE method is to expand the periodically varying material properties ($\rho$ and $B$) and pressure wave field ($p(t,x)$) of the acoustic wave equation (Eq. 2.1) in Fourier series. The $\vec{x}$ in Eq. 2.1, defined as $xt + yf$, is a 2D spatial vector that belongs to the unit-cell with origin at the center.

The basis functions used in the Fourier series expansion, to represent the periodically varying material properties and wavefield, are a set of all plane waves and the wavenumber of these plane waves are considered to be linear combinations of reciprocal lattice vectors ($G = m_1\vec{b}_1 + m_2\vec{b}_2$). For example, the inverse of the density, $\rho^{-1}(\vec{x})$ in Eq. 2.1 can be represented as

Eq. 2.6

$$\rho^{-1}(\vec{x}) = \sum_G \rho_G^{-1} e^{i\vec{G} \cdot \vec{x}}$$

Where the sum is taken over all integers, $m_1$ & $m_2$. To cross-check this expansion, let’s increment the position vector by a period i.e. $\vec{x}' = \vec{x} + \vec{a}_1$. In this case, the density remains unchanged, since

Eq. 2.7

$$\rho^{-1}(\vec{x}+\vec{a}_1) = \sum_G \rho_G^{-1} e^{i\vec{G} \cdot (\vec{x}+\vec{a}_1)} = \sum_G \rho_G^{-1} e^{i\vec{G} \cdot \vec{x}} e^{i\vec{G} \cdot \vec{a}_1} = \sum_G \rho_G^{-1} e^{i\vec{G} \cdot \vec{x}} e^{i2\pi m_1} = \rho^{-1}(\vec{x})$$

Similarly the inverse of bulk modulus in Eq. 2.1 can be represented as

Eq. 2.8

$$B^{-1}(\vec{x}) = \sum_G B_G^{-1} e^{i\vec{G} \cdot \vec{x}}$$
The Fourier amplitude \( \rho_{G}^{-1} \), in above Eq. 2.7 for each \( G \), is given by the integral

\[
\rho_{G}^{-1} = \frac{1}{A_{uc}} \int_{A_{uc}} \rho^{-1}(\mathbf{x}) e^{-iG \cdot \mathbf{x}} d\mathbf{x}
\]

where the integral is evaluated over the unit-cell, which has an area \( A \).

Similarly, for \( B_{G}^{-1} \) (in Eq. 2.8) we have

\[
B_{G}^{-1} = \frac{1}{A_{uc}} \int_{A_{uc}} B^{-1}(\mathbf{x}) e^{-iG \cdot \mathbf{x}} d\mathbf{x}
\]

In order to factor time out of wave equation, Eq.2.1, we impose a time-harmonic solution on the pressure, so that

\[
p(t,\mathbf{x}) = p(\mathbf{x}) e^{-ita}
\]

And now since the problem is periodic, we can also write the displacement \( p(\mathbf{x}) \) as a Fourier series of the form

\[
p(\mathbf{x}) = e^{iK \cdot \mathbf{x}} \sum_{G} p_{G} e^{iG \cdot \mathbf{x}}
\]

where \( K \) is the Bloch wave vector. The reason for introducing \( e^{iK \cdot \mathbf{x}} \) is to satisfy the Bloch condition which states that the solution is entirely periodic, except for the phase shift across the unit-cell. To show that this condition is satisfied by Eq. 2.12, we add a generic lattice vector \( R = n_{1}\mathbf{a}_{1} + n_{2}\mathbf{a}_{2} \) to \( \mathbf{x} \).

\[
p(\mathbf{x}+\mathbf{R}) = e^{iK \cdot (\mathbf{x}+\mathbf{R})} \sum_{G} p_{G} e^{iG \cdot (\mathbf{x}+\mathbf{R})}
\]

Substitution of this new spatial vector, as shown below, yields that the pressure solution represented via Eq. 2.13 is only phase shifted and that it indeed satisfies Bloch condition.

\[
e^{iK \cdot \mathbf{R}} e^{iK \cdot \mathbf{x}} \sum_{G} p_{G} e^{iG \cdot \mathbf{x}} e^{iG \cdot \mathbf{R}}
\]
\[ e^{i\mathbf{K} \cdot \mathbf{x}} = e^{i\mathbf{K} \cdot \mathbf{x}} \sum_{\mathcal{G}} p_{\mathcal{G}} e^{i\mathbf{G} \cdot \mathbf{r}} e^{i(m_{1n} + m_{2n})} \]

\[ = e^{i\mathbf{K} \cdot \mathbf{x}} p(\mathbf{x}); \text{ Since } n_1, n_2, m_1, m_2 \text{ are all integers.} \]

Now, as required in the wave equation Eq. 2.1, we take the spatial derivative of pressure, i.e.

Eq. 2.14

\[ \nabla(p) = \frac{dp(\mathbf{x})}{dx} \mathbf{x} + \frac{dp(\mathbf{x})}{dy} \mathbf{y} \]

\[ \nabla(p) = \sum_{\mathcal{G}} i \left( (K_x + G_x) \mathbf{x} + (K_y + G_y) \mathbf{y} \right) p_{\mathcal{G}} e^{i(\mathbf{G} \cdot \mathbf{x})} \]

And upon following the order of operators in Eq. 2.1, we have

Eq. 2.15

\[ \rho^{-1}(\mathbf{x}) \nabla(p) = \sum_{\mathcal{G}} \sum_{H} i \left( (K_x + G_x) \mathbf{x} + (K_y + G_y) \mathbf{y} \right) p_{\mathcal{G}} \rho^{-1}_{H} e^{i(\mathbf{G} + \mathbf{H}) \cdot \mathbf{x}} \]

where \( H \) is used to highlight the difference between the expansions of material properties and the displacement. Further taking the derivative of the above equation yields,

Eq. 2.16

\[ \nabla \left( \rho^{-1}(\mathbf{x}) \nabla(p) \right) = -\sum_{\mathcal{G}} \sum_{H} \left( (K_x + G_x)(K_x + G_x + H_x) \mathbf{x} + (K_y + G_y)(K_y + G_y + H_y) \mathbf{y} \right) p_{\mathcal{G}} \rho^{-1}_{H} e^{i(\mathbf{G} + \mathbf{H}) \cdot \mathbf{x}} \]

By defining new vector \( \mathcal{G}_d \), such that \( \mathcal{G}_d = \mathcal{G} + \mathbf{H} \), we can shift the order of the vector \( \mathbf{H} \) by \( \mathcal{G} \). Based on the above definition, Eq. 2.16 becomes

\[ = -\sum_{\mathcal{G}_d} \sum_{\mathcal{G}} \left( (K + \mathcal{G})(K + \mathcal{G}_d) \right) p_{\mathcal{G}} \rho^{-1}_{\mathcal{G}_d} e^{i(K + \mathcal{G}_d) \cdot \mathbf{x}} \]

Upon substitution of above Eq. 2.16 into the acoustic wave equation, Eq. 2.1, Eq. 2.1 finally transforms to an eigen value problem (EVP) as shown below
Eq. 2.17 \[ \sum_{G} \sum_{G'} \left( \left( (K + G) \cdot (K + G_d) \right) \rho_G^{-1} - \omega^2 K_{G-G_d}^{-1} \right) p_G e^{i(G'-K) \cdot \tau} = 0 \]

Since the above equation must apply to for all \( \mathcal{G} \), for every \( G \) we require

Eq. 2.18 \[ \sum_{G} \left( \left( (\mathcal{K} + \mathcal{G}) \cdot (\mathcal{K} + \mathcal{G}_d) \right) \rho_{G_d}^{-1} - \omega^2 K_{G-d}^{-1} \right) p_\mathcal{G} = 0 \]

This EVP (in Eq. 2.18) is an infinite set of linear equations which, given a value for Bloch wave vector \( \mathcal{K} \), we can truncate to find approximate values for eigen frequencies. The conventional procedure is to scan the wave vector along the edges of the irreducible 1\(^{st}\) Brillouin zone (as described in Sec. 2.2.2). To construct a band structure, we evaluate eigen frequencies for different wave vectors along high-symmetry directions and plot them on vertical axis along with their corresponding Bloch wave directions on horizontal axis. If there is a Bloch wave frequency that is not allowed in all of the high symmetry wave vector directions, then that frequency is considered a bandgap and waves within this spectral region will not propagate inside the phononic structure.

2.3.1. Example calculation using PWE method

Until now we have reviewed PWE method that can be used for a generic shape inclusion or unit-cell. As an illustrative example, here we will extract the band structure of phononic structure with circular inclusions in a non-Bravais lattice; the primitive unit-cell of this configuration is shown in Fig.2.2b (marked in red) along with its two basis inclusions of radius \( R_o \). For this configuration we pick a filling ratio of 0.64 and the radius of inclusions are assumed to be 0.041 [m]. The magnitude of lattice vectors \( a_1 \), \( a_2 \) (or also known as lattice constants) and the staggered distance (s) in Fig.2.2b are opted to be 0.1772, 0.0929 and 0.0649 [m] respectively. The material properties of host and inclusion are provided in Table 2.1.

With discontinuous material distribution inside unit-cell (Fig.2.2b), we can split the integrals of the Fourier coefficients (Eq. 2.6, 2.8). In this scenario, the density equation (Eq. 2.6) yields the following
For the case when $G = 0$

\[
\rho_G^{-1} = \frac{1}{A_{uc}} \left( \int_{A_{host}} \rho_{host}^{-1} d\vec{x} + \sum_{inc} \int_{A_{inc}} \rho_{inc}^{-1} d\vec{x} \right)
\]

\[
= \frac{1}{A_{uc}} \left( \rho_{host}^{-1} A_{uc} + (\rho_{inc}^{-1} - \rho_{host}^{-1}) \sum_{inc} A_{inc} \right)
\]

since, $A_{uc} = A_{host} + \sum_{inc} A_{inc}$

and for the case when $G \neq 0$, we have

\[
\rho_G^{-1} = \frac{1}{A_{uc}} \left( \int_{A_{host}} \rho_{host}^{-1} e^{-i\vec{G} \cdot \vec{x}} d\vec{x} + \sum_{inc} \int_{A_{inc}} \rho_{inc}^{-1} e^{-i\vec{G} \cdot \vec{x}} d\vec{x} \right)
\]

since, $A_{uc} = A_{host} + \sum_{inc} A_{inc}$

\[
\rho_G^{-1} = \frac{1}{A_{uc}} \left( \rho_{host}^{-1} \int_{A_{host}} e^{-i\vec{G} \cdot \vec{x}} d\vec{x} + (\rho_{inc}^{-1} - \rho_{host}^{-1}) \sum_{inc} \int_{A_{inc}} e^{-i\vec{G} \cdot \vec{x}} d\vec{x} \right)
\]

Since area integral of a plane wave over unit-cell area is zero ($\rho_{host}^{-1} \int_{A_{uc}} e^{-i\vec{G} \cdot \vec{x}} d\vec{x} = 0$), the above equation yields,

\[
\rho_G^{-1} = \frac{1}{A_{uc}} (\rho_{inc}^{-1} - \rho_{host}^{-1}) \sum_{inc} \int_{A_{inc}} e^{-i\vec{G} \cdot \vec{x}} d\vec{x}
\]

Upon translating and modifying the co-ordinate system, we have

\[
\rho_G^{-1} = \frac{1}{A_{inc}} (\rho_{inc}^{-1} - \rho_{host}^{-1}) \sum_{inc} \int_{A_{inc}} e^{-i\vec{G} \cdot \vec{x}} \frac{r}{|G|^2} d\theta dr
\]

and the above integral can be simplified into
\[
\rho_{\Gamma}^{-1} = \frac{1}{A_{\text{inc}}} \left( \rho_{\text{inc}}^{-1} - \rho_{\text{host}}^{-1} \right) \frac{2\pi}{|G|} \sum R_{\text{inc}} e^{-i \bar{G} \cdot \vec{R}_{\text{inc}}} J_1 (|G| R_{\text{inc}})
\]

where \( J_1 \) is the Bessel functions of first kind. In all the above equations, for the unit-cell considered in Fig.2.2(b), \( R_{\text{inc}}, \vec{C}_{\text{inc}} \) represent the radius and the vector describing the center of the two basis inclusions in \( xy \)-plane. Similar expansions can be derived for \( B_{\Gamma}^{-1} \). We now have everything we need to solve the EVP (Eq. 2.18) for the eigen frequencies.

Eq. 2.18 may be re-written as a generalized matrix eigenvalue problem, as

\[
A \rho_{\Gamma} = \omega^2 B \rho_{\Gamma}
\]

Where the components of the matrix \( A_{ij} \) are given as

\[
A_{ij} = \left(K + \bar{G}_j \right) \left(K + \bar{G}_i \right) \rho_{\Gamma_{\alpha-\alpha_i}}^{-1}
\]

and the components of the matrix \( B_{ij} \) are

\[
B_{ij} = B_{\Gamma_{\alpha-\Gamma_j}}^{-1}
\]

As explained before, in order to solve the problem numerically, we truncate the expansion by using finite number of plane waves thus allowing us to construct the matrices \( A \) and \( B \). Based on Eq. 2.5, the reciprocal lattice vectors (in Fig.2.4(c)) for a non-Bravais lattice with rectangular unit-cell (in Fig.2.2(b)) is given by

\[
\bar{b}_1 = \frac{2\pi}{a_1} i; \bar{b}_2 = \frac{2\pi}{a_2} j
\]

We restrict the reciprocal vector \( (\bar{G} = m_1 b_1 + m_2 b_2) \) by allowing \( m_1, m_2 \) to reside in the range \((-p, -p) < (m1, m2) < (p, p)\), where \( p \) belongs to integer set. This gives a total of \((2p + 1)^2\) distinct plane waves, which in turn leads to \( A, B \) being \((2p + 1)^2\) square matrices. The irreducible 1st Brillouin zone for the unit-cell in Fig.2.2(b) is given in Fig.2.4(c); where the high symmetry
24 points are given by \( \Gamma = (0 \ 0), \ X = \begin{pmatrix} \pi \\ a_x \end{pmatrix} \ 0, \ M = \begin{pmatrix} \pi \\ a_x \\ a_y \end{pmatrix}, \ N = \begin{pmatrix} 0 \\ \pi \\ a_y \end{pmatrix} \). The Bloch wave vector is swept along the edges of the polygon (\( \Gamma\)-X-M-N-\( \Gamma \)) and eigen frequencies are extracted.

**Fourier representation of periodic material properties**

One of the major drawbacks of PWE method is that a large number of plane waves are required for convergence of eigen frequencies in a complex structure, but in this thesis (with simple geometries being considered) we obtain good convergence with relatively low number of plane waves. As a sanity check, here we use a total of 361 plane waves and plot the material distribution inside the unit-cell (as given in Fig.2.5) using Fourier expansions.

The \( xy \) axis in the Fig.2.5 represents spatial co-ordinates and along \( z \)-axis we plot the material properties evaluated via Fourier series. The inverse of density is plotted in Fig 2.5(a), while inverse of bulk modulus is given in Fig.2.5(b). Upon comparing the spatial distribution of material in the unit-cell with reference inclusions, marked as black/red ellipses in \( xy \) plane, we can say that Fourier expansion with 361 plane waves provide very good convergence. Also the magnitude of material properties (both inverse of density and Bulk modulus) match quite well with values given in Table 2.1.
**Acoustic band structure and acoustic bandgaps (ABGs)**

Having good convergence of material properties (Fig. 2.5), the EVP in Eq. 2.21 is evaluated for eigen frequencies along each Bloch wave vector using ‘eig’ function in Matlab. From the results generated, the acoustic band structure, (Fig. 2.6) is created. Each branch in dispersion diagram corresponds to different propagating modes of the system; first ten modes are plotted here. For reference, we also plot (in Fig. 2.6) the unit-cell along with the irreducible 1st Brillouin zone used in PWE calculations.

The discontinuities in this band structure are called bandgaps and there can be two kinds of bandgaps in phononic structures; the first is called *partial/directional* ABG, which are gaps in dispersion diagrams that extend only in one direction and the second kind is called *complete* ABG, which extend in all wave directions. In the following plot, the upper and lower bounds of *partial/directional* ABGs along main symmetry directions of the phononic structures, viz. Γ-X and Γ-N, are marked with black arrows (where Γ-X and Γ-N represent x, y directions in direct space) and *complete* ABGs are marked by horizontal red regions. When wave frequencies lie inside *complete* ABG, there are no excited Bloch waves that allow wave propagation through a phononic structure.

![Band structure of Non-Bravais lattice](image)

Figure 2.6: Band structure of Non-Bravais lattice. In the left panel, grey regions represent *partial/directional* bandgaps along x-direction while red bands correspond to *complete* bandgaps. In the right panel, unit-cell definition and high symmetry directions (along which band structure is evaluated) are given.
Bandgap formation mechanism

The general mechanism of bandgaps formation is based on the complex interactions between scattered wave fields from different inclusions inside the phononic structures. If the wave length of the scattered field is of the order of lattice constant, then destructive interference occurs and wave propagation is not possible [39].

To illustrate this theory, we first calculate the frequency of a wave with half wavelength equal to lattice constant. By assuming linear dispersion theory, this frequency can be found out to be

\[ f_{BG} = \frac{C|\vec{K}|}{2\pi}; \]

where \( C \) is the longitudinal speed of sound in host media (air in our case) and \( |\vec{K}| \) is the desired wave number (inverse of the wavelength). Wave number corresponding to lattice constant is extracted from the irreducible 1st Brillouin zone. Moreover, since the distribution of inclusions in \( xy \) plane is anisotropic – the frequency at which destructive inference occurs, will be different in different directions; as a result, lattice constants along different directions are extracted by picking wavenumbers at different high symmetry points of Brillouin zone.

In this example, we pick the wave number \( |\vec{K}| \), at X, N symmetry points (17.72 [1/m] & 33.8 [1/m]) of the Brillouin zone and the frequencies evaluated via assuming linear dispersion theory are, 959[Hz] & 1829 [Hz] respectively. It can be noticed these frequencies lie well inside the partial ABG frequencies obtained via the PWE method along \( \Gamma-X \) and \( \Gamma-N \) directions (1087-2276[Hz] and 1705-1846[Hz], as given in Fig.2.6). This correlation explains that the ABG are formed at wave frequencies with wavelengths comparable to lattice constant and that the basis of ABG formation phenomena is via destructive interference. It should be noted that, the complete ABG is formed in the region 1705-1846[Hz] where there is an overlap of partial ABGs in different wave directions.

2.3.2. Convergence of PWE method

The ABG results generated above are highly dependent on the number of plane waves used for representing the material distribution. Hence in some cases, when the inclusions of phononic structure are either too close or too small, a large number of plane waves are required for satisfactory convergence of eigen frequencies. To illustrate this problem, here in Fig.2.7, we plot the spatial distribution of material properties extracted via Fourier series using different number
of plane waves, for a phononic structure with a high filling fraction, 0.79. The material distribution is given in the Fig.2.7; on the left (Fig.2.7(a,c)) we use 361 plane waves, while on the right (Fig.2.7(b,d)) we use 961 plane waves. Upon comparing the result in Fig.2.7, the difference is clear and that Fourier expansion with higher number of plane waves better represent the actual material properties in terms of both magnitude and spatial distribution. Therefore, one has to carefully select the parameters of the PWE method, to obtain decent results.

2.4. Effect of lattice symmetry on acoustic band structure

As described earlier (Sec. 1.2), the band structure of a phononic structure depends on the geometry of the lattice. To study this effect, we plot band structures of different Bravais lattices, viz. rectangle, square, hexagon and center-rectangle lattices (given in Fig.2.3). The geometric parameters of each lattice are chosen such that the filling fraction and radius of inclusion of all of

Figure 2.7: Effect of the number of plane waves used in Fourier expansions on the approximated spatial distribution and intensity of material properties inside unit-cell.
the configurations are the same with values 0.4725 and 0.041[m] respectively. Under these constraints, the lattice vectors and unit-cells of each configuration are evaluated and provided in Table. 2.2; as shown in Fig.2.3, the reference co-ordinate system is at the center of unit-cell.

Table 2.2: Geometric properties of different lattice symmetries

<table>
<thead>
<tr>
<th>Lattice-symmetry</th>
<th>Lattice Vectors</th>
<th>Ratio of lattice constants and angle between lattice vectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>$\bar{a}_1 = 0.1337\hat{i}$</td>
<td>$\bar{a}_2 = 0.0836\hat{j}$</td>
</tr>
<tr>
<td>Square</td>
<td>$\bar{a}_1 = 0.1057\hat{i}$</td>
<td>$\bar{a}_2 = 0.1057\hat{j}$</td>
</tr>
<tr>
<td>Hexagon</td>
<td>$\bar{a}_1 = 0.0984\hat{i} - 0.0568\hat{j}$</td>
<td>$\bar{a}_2 = 0.1136\hat{j}$</td>
</tr>
<tr>
<td>Center Rectangle</td>
<td>$\bar{a}_1 = 0.0668\hat{i} - 0.0836\hat{j}$</td>
<td>$\bar{a}_2 = 0.1672\hat{j}$</td>
</tr>
</tbody>
</table>

Further, for the cases of rectangular and center-rectangle configurations, the ratio of the magnitudes of lattice vectors are specified (given in last column Table. 2.2) so as to render these configurations unique. Primitive unit-cells of these lattice configurations are given in Fig.2.3 and their corresponding irreducible 1st Brillouin zones given in Fig.2.8; these zones are used for extracting high symmetry wave vector directions for PWE calculations.
2.4.1. Discussions

From the above band structure plots, we can see that the spectral locations of complete ABGs of different lattice configurations are vastly distinct. This is because of the variations in the geometry and symmetry of primitive unit-cells of different lattice configurations. As a result of different primitive unit-cells (in direct lattice), the high symmetry points of the corresponding 1st Brillouin zone (in reciprocal lattice) are also different, yielding bandgaps at different spectral locations; as said earlier (Sec. 2.3.1), the bandgap’s spectral location is directly related to the high symmetry points through the relation, \( f_{BG} = \frac{c |\mathbf{K}|}{2\pi} \), and hence each lattice symmetry exhibits different bandgaps.

As before, the partial bandgaps are represented by arrows and absolute bandgaps are represented by horizontal red shaded regions. It should be noted that the partial bandgaps marked along \( \Gamma-J \) in hexagon (Fig 2.8c) and \( \Gamma-X / \Gamma-N \) in center-rectangle (Fig 2.8d), have one band (2nd band) – that would produce transmission. However, these are deaf bands [40]; because of the symmetry in their mode shapes, they cannot be excited by a plane wave in their corresponding directions. Hence these bands are neglected while evaluating the partial bandgaps.

We now provide a table of partial ABGs of different lattice configurations evaluated via PWE method and compare them against the frequencies obtained via assuming linear dispersion relations. We specifically look at partial bandgaps along \( \Gamma-X \) and \( \Gamma-N \) wave vector directions that
correspond to x, y directions respectively. For some cases, such as square (Fig 2.8b) and hexagon (Fig. 2.8c)) we don’t evaluate ABG along y direction; however, because of the symmetry in 1st Brillouin zone we can obtain ABG along Γ-N from corresponding Γ-M (Fig.2.8b) and Γ-J (Fig.2.8c) directions.

The consolidated results of ABGs generated via PWE calculations and ABGs evaluated via linear dispersion relations, \( f_{BG} = \frac{c|\mathbf{g}|}{2\pi} \), are provided in Table 2.3. These results show that the ABGs calculated via linear dispersion relations lie completely inside the partial ABGs calculated via PWE method for all configurations. The co-relation between these results show that the destructive interference is the primary mechanism behind ABG formation mechanism in phononic structures. Now since the partial ABGs occur at distinct frequencies for different configurations, the complete ABG that form at integer multiples of the partial bandgaps, also occurs at different frequencies (as provided in Table 2.3).

<table>
<thead>
<tr>
<th>Lattice-symmetry</th>
<th>First complete bandgap frequency, (Hz)</th>
<th>partial bandgap along (Ga-X or X direction), (Hz)</th>
<th>Partial bandgap along (Ga-N or Y direction), (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PWE method, midgap frequency and frequency range</td>
<td>Linear Dispersion relation, ( f_{BG} = C_o k_A/2\pi )</td>
<td>PWE method, midgap frequency and frequency range</td>
</tr>
<tr>
<td></td>
<td>Linear Dispersion relation, ( f_{BG} = C_o k_A/2\pi )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangle</td>
<td>1962</td>
<td>1277 (377-2177)</td>
<td>1271 (1934 (1627-2242)</td>
</tr>
<tr>
<td>Square</td>
<td>1852</td>
<td>1518 (993-2043)</td>
<td>1608 (1518 (993-2043)</td>
</tr>
<tr>
<td>Hexagon</td>
<td>2931</td>
<td>1503 (1183-1824)</td>
<td>1727 (2262 (1505-3020)</td>
</tr>
<tr>
<td>Center-Rectangle</td>
<td>4639</td>
<td>2070 (1557-2584)</td>
<td>2084 (2078 (1695-2461)</td>
</tr>
</tbody>
</table>
These results, viz. strong dependence of bandgap’s spectral location on lattice symmetry, suggest that a structure capable of transforming between different Bravais lattice-types may possess strong adaptable wave characteristics. While there are some works done in the past which focus on reducing the symmetry of the lattice (refer Sec. 1.2.1) to achieve bandgap adaptation, there has not been any work dedicated on changing the lattice geometry between different Bravais lattice-types that have distinct dispersion characteristics (Fig.2.8).

2.5. Effect of filling fraction on acoustic band structure

Another important factor that affects bandgap is the filling fraction of the lattice. So, here we study the evolution of complete ABG with increasing filling fraction for each lattice configuration. For this analysis, we pick the radius of the inclusion of all lattice configurations to be fixed at (i.e. 0.041[m]) and for the case of rectangular and center-rectangular lattice configurations we introduce additional conditions (same as before, refer last column of Table. 2.2). With all of these constraints and filling fraction equations given in Eq. 2.3, the magnitude of lattice vectors has to be decreased to increase filling fraction.

Without loss of generality, we study the ABG properties with filling fraction changing from 0.1 to 0.75 and the results are provided in Fig.2.9. Instead of looking at the entire band structure, we only extract the spectral information of complete ABGs from the band structure and represent them as markers in Fig.2.9; where circle and square marker represent lower and upper bandgap edges of complete bandgap.

As give in Fig.2.9, the filling faction is increased up to 0.75 for all lattice configurations except for rectangular configuration; this is because of the constraints imposed on the rectangular configuration and closed packing condition is reached at filling fraction of 0.475.

2.5.1. Discussion

These results show a general trend that, increasing the filling fraction leads to increase in the spectral width of low frequency ABGs and some new ABGs at higher frequencies may sometimes
form or close depending on the filling fraction. Also from Fig.2.9, it can be concluded that the results in Fig.2.8 – i.e. different Bravais lattices produce ABGs at different spectral locations – remain valid at all filling fractions. Even though these results (Fig.2.9) may lead to broadband ABG characteristics, being able to create ABGs at only selective frequencies is not yet feasible.

2.6. Conclusion

In this chapter we have presented basic tools required to study wave propagation in phononic structures and have used these tools to study the effect of important geometric properties on bandgaps. The results generated in this chapter further validate our hypothesis (given in Sec. 1.4) that, a phononic structure with a practical mechanism that is capable of shifting periodic geometries between different lattice symmetries may lead to phenomenal broadband adaptive wave characteristics.
CHAPTER 3.
ORIGAMI-BASED RECONFIGURABLE PHONONIC STRUCTURES

In order to achieve our research goal (Sec.1.4), here we synthesize a new class of reconfigurable phononic structures, viz. origami structure (OS), which can shift between different Bravais lattice-types. Such transformation, which can achieve distinct changes in point-group symmetry of lattice, leads to phenomenal bandgap adaptation.

Origami is an ancient paper folding art, in which a single piece of paper can be folded from a crease pattern to take different shapes be it an animal, or tessellation or an abstract form (Fig.3.1) [41–48]. In addition, origami folding can be a simple one-degree-of-freedom action for rigid-foldable designs; thus minimal local actuation can lead to effective global shape changes. Another key feature of origami is that its scale independent geometric folding allows it to be fabricated at vastly different length scales without losing reconfigurability. Overall the versatile, scalable and practical to implement origami design will be exploited to build our ideal reconfigurable phononic structure for achieving broadband adaptive wave transmissions.

Figure 3.1: Origami applications in (a) art [49] (b) engineering structures [50].
3.1. Design of origami structures

The basic ingredient of origami structures (OS), proposed in this thesis, is an origami sheet composed of inclusions attached either on the facets or vertices of origami sheet. The design considered here is shown in Fig.3.2; cylindrical solid inclusions are attached between vertices of two origami sheets with air as host media and 2D acoustic wave propagation in the plane transverse to the inclusion’s axis is studied.

3.1.1. Miura-origami

Origami sheet can be modelled as a periodic tessellation of identical degree-4 vertices, which consists of four crease lines that meets at one point. Among the vast many possible origami tessellations – attributed to seven geometric design parameters viz. four crease lengths and three independent angles between them – here in this thesis we pick a classic design called Miura-Ori that consists of only three independent parameters to construct OS. An unfolded Miura-ori sheet with crease pattern is given in Fig.3.3a. This sheet is characterized by parallel creases in one direction intersected by zigzag creases with alternating plus and minus angles; crease can be either a mountain (upward, marked in red) or a valley (downward, marked in blue) fold.
3.1.2. Kinematics of miura-ori folding

The Miura-Ori sheet being periodic in nature, one can focus on a unit-vertex shown in Fig.3.3(b) to understand the overall kinematics. The geometric design of the vertex is determined by three parameters that remain unchanged regardless of folding, they are the length of the two crease lines \((a, b)\) and the sector angle between these two creases \((\gamma)\). The dihedral folding angle \((\theta)\), defined between the facet and \(xy\) reference plane can uniquely determine the folding configuration. The Miura-Ori is flat when \(\theta = 0^\circ\) and fully folded when \(\theta = 90^\circ\). Therefore, the outer geometry of the Miura-Ori unit-vertex is related to the folding angle as well as the vertices design parameters. The position of the vertices can be uniquely determined based on Eq. 3.1. Since the origami structure considered in this thesis (Fig.3.2) has inclusions attached at the vertices of the sheet, their topology distribution is directly related to the positions of the vertices projected onto the \(xy\) reference plane (red ellipses in Fig.3.3b).

\[
H = a \sin \theta \sin \gamma
\]

\[
L = \frac{2b \cos \theta \sin \gamma}{\sqrt{1 - \sin^2 \theta \sin^2 \gamma}}
\]

Eq. 3.1

\[
W = 2a \sqrt{1 - \sin^2 \theta \sin^2 \gamma}
\]

\[
s = \frac{b \cos \gamma}{\sqrt{1 - \sin^2 \theta \sin^2 \gamma}}
\]

Figure 3.3:(a) Unfolded miura-Ori sheet with crease pattern; red (blue) creases indicate mountain (valley) fold (b) Miura-origami unit vertex parameters
3.1.3. Parametric analysis

A comprehensive parametric analysis is conducted to survey the achievable lattice topology transformations via Miura-ori folding. We found that it is possible to redistribute the inclusions between four (Fig. 2.3) of the five Bravais lattice types. The only exception is the oblique lattice type that requires a more generic, single collinear degree-4 vertices design, which would not be discussed here.

For the purpose of parametric study, two dimensionless parameters (denoted as $A$ and $B$) are defined that correlate the lattice topology to the folding kinematics of Miura-Ori (Eq. 3.2).

Assume $n$ is any positive integer, the following relations (Eq. 3.3) of $A$ and $B$ should be satisfied to obtain various Bravais lattices. Notice that the hexagonal lattice is a special case of center-rectangular lattice, and the square is a special case of rectangular.

$$A = \frac{S}{L/2} = \frac{1}{\tan \gamma \cos \theta}$$

Eq. 3.2

$$B = \frac{W/2}{L/2} = \frac{a(1 - \sin^2 \theta \sin^2 \gamma)}{b \cos \theta \sin \gamma} = \frac{a}{b} \cos \gamma \left( A + \frac{1}{A} \right)$$

Eq. 3.3

Rectangle: $A/B = n$

Square: $A = n; B = 1$

Hexagon: $A = (2n-1)/\sqrt{3}; B = 2/\sqrt{3}$

Center-Rectangle: $A/B = n-1/2$
The physical significance of \( n \) in the dimensionless parameters (\( A \) and \( B \)) can be explained through Fig.3.4(a-b). As an example we choose hexagonal lattice configuration which requires \( A = (2n - 1)/\sqrt{3} \), \( B = 2/\sqrt{3} \) (where \( n \) is an integer). Even though the hexagonal lattice formed by the inclusions are identical in Fig.3.4(a-b), the parameters of underlying Miura-ori sheets that depend on unit-vertex parameters (Fig.3.3b) are different and that \( n \) is a measure of geometry.

Further, the equations in 3.3 can be simplified into the following Eq. 3.4

\[ n = 1, B = \frac{2}{\sqrt{3}}, A = \frac{1}{\sqrt{3}} \]

\[ n = 2, B = \frac{2}{\sqrt{3}}, A = \frac{3}{\sqrt{3}} \]
Since the folding angle ($\theta$), has a range of $0^\circ - 90^\circ$, $\cos^2 \theta$ in Eq. 3.4 can range from 0 to 1. Based on these bounds, the formulations in Eq. 3.4 provide us with a large range of combinations of sector angle ($\gamma$) and crease length ratio $a/b$ where Miura-Ori folding can enable switching between rectangular and center-rectangular lattices multiple times (shaded region in the parametric plot of Figs.3.5(a-b)). When $\gamma$ and $a/b$ reach specific combinations, one of the center-rectangular (CR) or rectangular (R) configurations becomes a hexagonal (H) or square (S), respectively (dashed curves in Figs.3.5(a-b)). There exists a subset of $\gamma$ and $a/b$ combinations (dash-dotted curve) where it’s possible to switch between hexagonal (H) and square (S) lattice types. It is not possible to switch between two different square lattices; however, it is possible to switch between two hexagonal lattices with different lattice constants (thickened dashed-dotted curve in Fig.3.5a).
Figure 3.5: Achievable lattice configurations via Miura-Ori rigid folding based on different crease designs. (a) Combinations of $\gamma$ and a/b that can achieve center-rectangular (CR) lattice configuration. White regions correspond to Miura-Ori designs that cannot achieve CR; and the shaded regions correspond to crease designs that can achieve CR multiple times as the Miura folds from $\theta = 0^\circ$ to $90^\circ$. For visual clarity, regions with CR$>3$ are not plotted separately. Dashed curves in this plot correspond to the subsets of $\gamma$ and a/b combinations where one of the achievable CR configurations is actually hexagonal (H). Again for clarity, only the curves based on $n = 1$ to 6 are plotted. (b) Similar plot as (a), but for rectangular (R) and square (S) configurations. When considering the lattice transformation of a specific Miura-Ori design, both of the two plots need to be considered. In (c-f), we show how folding shifts the lattice configuration between square (S) and Hexagon (H) in OS with a triangular crease design (marked by triangular marker in a and b).
Folding illustrations of lattice transformation in different miura-ori configurations

As an illustration of topology transformation between different Bravais lattices in OS, here in Figs.3.6(a-e) we show the lattice distribution of OS with five different crease designs. The legends in Figs.3.6(a-e) provide the origami parameters and folding angle used to achieve the corresponding Bravais lattice configurations.

Through the parametric analysis shown in Fig.3.5(a-b) and via the folding illustrations shown here in Fig.3.6(a-e), we can say that the OS with Miura-origami design can be designed to transform between four (Fig.2.3) of the five Bravais lattices.

Figure 3.6:(a-e) Topology transformation in OS with different crease designs. Different markers (square, pentagon, circle, triangle and star) represent different crease designs.
3.2. Study of adaptation in wave behavior caused by origami folding

3.2.1. Design parameters

To demonstrate the effect of lattice transformation on wave propagation, in this proof-of-concept investigation, we pick OS with triangle crease design given in Fig. 3.5a; where the crease lengths \( a (= b) \) and sector angle \( \gamma \) are assigned to be 0.15 \([m]\) and 60° respectively. The circular cylindrical inclusions attached to the vertices are assumed to be made of steel with a radius of 0.041 \([m]\) and the OS is suspended in air (as given in Fig. 3.2). The longitudinal velocity of sound \( (c_l) \) and density \( (\rho) \) of cylindrical inclusions (air host) are provided in Table 2.1. Based on the geometric parameters, a maximum folding angle of 71° is possible before the closed packing condition of inclusions is reached. To illustrate the effect of folding operation on the topology of cylindrical inclusions, we plot the 2D cross section plane of OS transverse to the insertion rod axis as shown in Fig.3.5(c-f). The lattice topology of inclusions changes from a hexagon (at 0°, Fig.3.5c) to square (at 55°, Fig.3.5d) and finally to hexagon (at 70°, Fig.3.5f) while transforming into a reduced-symmetry non-Bravais lattice at intermediate angles (for example, at 65°, Fig.3.5e) during folding. Since the initial configuration is a hexagon we refer to this particular design as hexagonal-OS.

An animation of the hexagonal-OS unit-cell is shown in movie M1 [51]. Cross section views of OS at different instances in the movie M1, where hexagon and square configurations are formed, are shown in Fig.3.7 (a-c).

![Cross section views of OS](image)

Figure 3.7: Instances of the top view of OS in movie M1 at different folding angles (a) Hexagon lattice at 0° (b) Square lattice at 55° (c) Hexagon lattice at 70°. (d-f) fabricated OS at different folding angles corresponding to (a-c)
To realize the folding induced lattice transformation, we also built an OS composed of 3D printed Miura-ori sheet with PVC tubes as inclusions, attached onto its vertices (Fig.3.7(d-f)). To build the origami sheet a print-stack-assemble concept is used; wherein the individual pieces of the assembly are shown here in Fig.3.8(a). There are a total of 4 different pieces (viz., male facet, female facet, stripe A and stripe B) which can be individually printed and assembled to form the origami sheet. The facets of the origami sheet are snap-fitted into each other and are provided with a unique hinge mechanism that allows facets to revolve with respect to each other (Fig.3.8(b)). Moreover, an innovative stripe design is implemented which can hold the PVC pipes up-right and move them as per the trajectory of origami vertices during the folding process (Fig.3.8(b)).

Through the above demonstrations we can say that the example OS design considered in Fig.3.7 is capable of

(a) shifting between different Bravais lattice symmetries (Hexagon and Square)
(b) has a practical and efficient 1DOF topology transformation mechanism

Overall, we have successfully designed an OS as per our new idea (as given in Sec. 1.4) and above features should make this OS design achieve adaptable wave transmission characteristics. Such kind of lattice transformation between different Bravais lattices has never been done before and here we achieve such transformation via an effective origami folding mechanism. Further, since lattice symmetry plays an important role in wave behavior, such reconfigurations can lead to large adaptation in wave propagation features especially in terms of bandgaps, as will be shown in later sections.
3.2.2. Wave propagation analysis

To study the effect of the unique topology transformation induced by origami folding on the wave transmission in hexagonal-OS (Fig.3.2), we develop a numerical tool that combines wave propagation theory with origami folding kinematics. The acoustic wave propagation behavior through the 2D plane, transverse to inclusion axis, is evaluated by solving the governing pressure wave equation (Eq. 2.1) with the assumption that the material properties of OS are invariant along the z-direction.

At a given folding angle ($\theta$), the periodical repetition of material properties can be represented by a rectangular primitive unit-cell with lattice vectors ($a_1, a_2$) and two basis inclusions ($C_1, C_2$), as shown in inset of Fig.3.9a. As mentioned earlier, hexagonal-OS transform into a hexagonal lattice at $0^\circ$ and $70^\circ$ and into a square lattice at $55^\circ$. While these Bravais configurations require only one inclusion in the basis, at intermediate folding angles the lattice topology turn into a non-Bravais lattice and in these folding configurations two basis inclusions are necessary to represent the lattice topology (Sec. 2.2). Hence a general two inclusion basis is selected, for construction of a unit-cell, to be able to represent all folding configurations. Now, the effect of topology transformation induced by folding can be introduced into the unit lattice parameters via the origami kinematic relations (Eq. 3.1) yielding,

\[
a_1 = L\hat{x}
\]

\[
a_2 = \frac{W}{2} \hat{y}
\]

Eq. 3.6

\[
C_1 = \frac{|a_1|}{2} \hat{a}_1 + \frac{|a_2|}{2} \hat{a}_2
\]

\[
C_2 = \left( s - \frac{3|a_2|}{2} \right) \hat{a}_2
\]
For a given folding angle, the PWE method described in Sec 2.3 -- for solving wave equation in a non-Bravais lattice -- is used to formulate EVP and is solved for eigen frequencies along different wave vectors spanning the edges of the irreducible 1st Brillouin zone (provided in inset of Fig.3.9a). The acoustic band structure of hexagonal-OS at folding angle of 65° is shown in Fig.3.9a; three complete bandgaps are found for this reduced-symmetry non-Bravais lattice.

**Validation of PWE method via COMSOL**

To validate the dispersion characteristics obtained via PWE method, here we evaluate the transmission spectra of finite hexagonal-OS at 65° folding along x-direction via commercial software COMSOL and the results are compared against the band structure (along ΓX direction) obtained using PWE method. In this spirit, the cross section of periodically distributed circular steel rods in air at 65° folding angle (Fig.3.5e) are modeled in acoustic-structural interaction.
module of COMSOL as a strip with finite length and periodic boundaries along x and y directions respectively, refer Fig.3.10(c). The following are the geometric parameters of the COMSOL model: unit-cell length along x & y direction are $a_1=0.1772$[m] & $a_2=0.0929$[m] respectively, the radius of steel inclusion is 0.041[m] and the staggered distance between the two adjacent columns of inclusions is $s=0.028$[m].

To evaluate the x-direction wave propagation behavior, plane pressure wave is applied as input on left edge and radiation boundary conditions are imposed on right edge to avoid reflections. Through the frequency domain analysis, the transmission spectrum – defined as the ratio of sound pressure level (dB) at output to input edge – is extracted and shown as a dashed curve in the top
left corner of Fig.3.10. We also provide the band structure (represented by a solid line) along ΓX wave direction, predicted using PWE method, in Fig.3.10. From the top two plots in Fig.3.10, it is clearly evident that the transmission dips (where wave propagation is blocked along x-direction) generated via commercial software correspond very well with the bandgaps predicted via PWE along ΓX wave vector. Also, the steady state acoustic pressures maps at different points along the transmission spectra (for example, A, B & C in Fig.3.10) clearly illustrates the wave blocking and propagation phenomena. Thus this COMSOL simulation serves as a way to validate the PWE method and also provides a tool to extract the magnitude of transmission and sound pressure distribution inside the OS at a chosen frequency.

Upon validating the PWE method, numerical simulations are performed across the range of possible folding angles (0°-71°) of hexagonal-OS and all results are consolidated into Fig.3.9b; where horizontal axis represents the folding angle and on vertical axis we plot normalized bandgap edge frequencies. To generate Fig.3.9b, a folding angle increment of 5° in 0°-45° region is used – where filling fraction is low and bandgap widths are small – and a 1° increment is used in 46°-71° region – where bandgap is highly sensitive to folding angle. Results in Fig.3.9b, at standard Bravais lattice configurations viz. hexagon (0° & 70°) and square (55°) match well with existing literature [18,19].

**Evolution of bandgap via folding: Observations**

In Fig. 3.9b, the ABG originating at hexagonal lattice configuration at 0° (with mid-gap frequency ~1.06, where mid-gap frequency is calculated as the average of the upper and lower edge frequencies of PBG) gradually narrows with increasing folding angle and totally ceases to exist around 25°. Another ABG related to hexagonal lattice appears at a folding angle close to 65°, and starts to grow in width as the folding angle increases to 70° (mid-gap frequency ~1.60). In a completely disjoint folding regime, two separate PBGs related to square lattice appear and then disappear as the folding angle increase from 50° to 66°, and their mid-gap frequencies are ~0.82 and ~2.77, respectively. Such dramatic and discrete variation in ABG spectral locations (especially in 25°-50° and 65°-66° folding regimes), together with the large ABG widths at higher folding angles make the origami structures ideal for adaptable broadband wave applications.
**Evolution of bandgap via folding: Discussions**

To gain a deeper physical insight into such discrete ABG adaptations with respect to folding, it has to be understood that during folding (refer Fig. 3.5(c-f)) (a) the lattice topology of hexagonal-OS changes between hexagon and square and (b) filling fraction increases. Figure 3.11 illustrates the evolution of low frequency bandgap in both hexagonal (blue markers) and square (red markers) lattices due to increase in filling fraction. It can be observed that mid-gap frequency of the first bandgap in both square and hexagon symmetries form at different spectral locations for all filling fractions.

Therefore, the hexagonal OS that can undergo transformation between these two lattice symmetries during folding can observe discontinuous ABGs (black markers in Fig. 3.11); i.e. ABGs jump between different spectral locations as folding angle is increased (represented by horizontal pink lines in Fig. 3.11). Specifically, the discrete shifts in spectral location of first ABG during folding are due to the phase transformation in the hexagonal-OS’s lattice topology between

![Graph](image)

**Figure 3.11:** Comparison of OS-bandgaps with pure square and hexagonal lattices, each with filling fractions corresponding that to OS.
different symmetries. Since different lattice symmetries have different dispersion characteristics, the hexagonal-OS that is capable of shifting between them possess rich adaptable characteristics.

3.2.3. Sensitivity analysis

In order to study the robustness of ABGs, here we perform sensitivity analysis of ABGs to spatial mistuning. For this study, two levels of disorder are added to the spatial co-ordinates of inclusions; one with a standard deviation ($\sigma$) of $a^{\ast}1.7\%$ and other with $\sigma$ of $a^{\ast}3.65\%$. Mistuning is introduced into the lattice distribution via disturbing the spatial position of inclusions from their perfectly periodic distribution by a uniform random variable with desired $\sigma$. The value of $\sigma= a^{\ast}1.7\%$ is the maximum mistuning allowed, for entire folding angle regime ($0^\circ$-$70^\circ$) to be accessed without the inclusions interfering with each other. On the other hand with $\sigma= a^{\ast}3.65\%$, closed packing condition would reach at $65^\circ$ folding angle.

Since it is not possible to model an infinite disordered system, here we approximate the mistuned system to quasi-periodic structure with super-cell. For example, super-cells with 100 inclusions (marked as squares) used for modelling different disordered OSs are shown in Fig.3.12. Desired amount of mistuning is added to these super-cells and then we approximate the infinite disorder system as repetitions of these super-cells. By using super-cell technique (explained in appendix A)

![Figure 3.12](image.png)

Figure 3.12: Definition of super-cell used for modelling mistuned OS with disorder ($\sigma$) of (a) $a^{\ast}1.7\%$ (b) $a^{\ast}3.65\%$. Inclusions marked in green are mistuned, while in red are periodically distributed (plotted for reference).
we generate results of a quasi-periodic system. Though these results are only approximate, a large enough super-cell should lead us to the exact behavior of truly mistuned system [52].

Super-cell PWE method (Appendix A) is used for extracting the band structure of two mistuned OSs, using a super-cell with 100 inclusions (as shown in Fig.3.12) and results are plotted in Fig.3.13(b-c); it should be noted that the band structure calculations for OS with a*3.65% mistuning stops at 65° because closed packing is reached. A total of 1681 PW are used in simulations to achieve desired convergence.

Upon comparing the mistuned OS’s bandgap (Fig.3.13(b-c)) with periodic OS bandgap (Fig.3.15(a)), it can be observed that the width of ABGs reduces with increase in mistuning. Although this effect is more prominent for high frequency ABGs, the width of low frequency ABGs are left almost unaltered. Henceforth, the presence of low frequency ABGs under mistuning states that the adaptability of OS is a robust phenomenon.

3.2.4. Effect of initial lattice configuration

So far we have looked at the results of hexagonal-OS, i.e. the lattice configuration at 0° folding angle is a hexagon. Now to study the effect of initial lattice configuration on wave adaptation behavior, we picked a different initial configuration of honeycomb (which will be referred to as
honeycomb-OS) that which can be easily constructed from hexagonal-OS by simply removing few lattice inclusions as represented by dotted circles in inset of Fig.3.14. The consolidated bandgap results of honeycomb-OS at different folding angles in Fig.3.14 are calculated via PWE method, with unit-cell as shown by shaded region in the inset of Fig.3.14. Further, the bandgap edge frequencies on the vertical axis are reduced via same normalization factors as used in Fig.3.9b.

In contrast to the highly sensitive bandgaps of hexagonal-OS (Figs.3.9b), the low frequency ABGs of honeycomb-OS (Fig.3.14) exists for most of the folding operation and remains almost invariant. Such unique invariable transmission characteristics are never observed in topology transformation studies, and it can be useful if both the structure shape change and acoustic blocking are desired functionalities.

On another note, there are two ABGs within the area highlighted by dotted ellipse in Fig.3.14 and the modes between them have localized modeshapes/wavefunctions inside the waveguide of

Figure 3.14: ABG adaptation in OS with honeycomb as initial lattice configuration. Square (circle) markers represent the lower (upper) edge frequencies of bandgap. Distribution of solid circles in the inset represent the lattice topology and shaded region the unit-cell of honeycomb-OS at 0°, 55° and 70° folding angles (left to right).
honeycomb-OS; where the waveguide is formed via aligned missing (dotted) inclusions during folding process. Based on previous work [2,4,33,53], these observations suggest that the two bandgaps inside the dotted ellipse are created as a result of the hexagonal-OS’s bandgap – in $65^\circ$-$70^\circ$ folding angle region (Fig.3.9b) – being split by the localized modes formed inside the waveguide of honeycomb-OS. This phenomena of localized modes in origami structures with defects will be later used in Ch. 5 to study tunable waveguiding.

3.3. Features of origami structure

The interesting results discussed in the previous sections, both adaptive and invariant wave properties, are all generated via folding OS with a specific Miura-ori design (triangle crease design in Fig.3.5a,b). But because of the versatility in designing origami, as shown in Fig.3.5(a,b), various other lattice transformations between non-Bravais lattice and different 2D Bravais lattice-types (for example, Figs.3.6(a-e)) is also possible. Such transformations via origami folding – coupled with significant topology and symmetry variations – could lead to a large design space for tunable wave characteristics. Another key feature of OS is that its scale independent geometric folding allows it to be fabricated at vastly different length scales without losing reconfigurability, making it a viable design for a range of spectral applications. Apart from the versatility and scalability, the global topology transformation, enabled by a one degree-of-freedom folding mechanism in rigid-foldable origamis, can be controlled with minimal local effort [46,54–58]. Overall the versatile, scalable and practical to implement OS design could bring on-demand wave propagation to an entirely new level.

3.4. Conclusion

As per our research goal (Sec.1.4), an ideal reconfigurable origami structures is developed and wave adaptation behavior caused due to folding is studied for the first time. Unique topology transformation between distinct lattice-symmetries – via simple origami folding – is found to be the principle behind phenomenal bandgap transformation. Through analytical and numerical investigations, such drastic wave adaptability is evaluated via PWE method and the findings are
validated via transmission diagrams obtained using commercial software, COMSOL. Apart from the radical bandgap adaptation, almost invariant transmission properties are discovered in origami structures with different initial lattice configuration. All of the interesting wave behavior coupled with large design space, scalable geometry and controllable transformation, makes origami structure a novel design for wide spectra of acoustic applications bringing on-demand wave propagation tailoring to a new level.
CHAPTER 4.
ORIGAMI SONIC BARRIERS FOR TRAFFIC NOISE MITIGATION

In this chapter we utilize the unique tunable complete bandgaps features of origami phononic structures (detailed in Ch. 3) to explore an innovative design of sonic barrier to block dynamically changing traffic noise spectra. To achieve above task, after the introduction we first describe the design of origami sonic barrier followed by analytical and numerical simulation results that demonstrate the tunable bandgap features. We then explain the details of the design and fabrication of a scaled-down origami sonic barrier and present test results that show adaptability in sound blocking features.

4.1. Introduction

With the increase in urban population, the number of vehicles on the road has increased exponentially and the associated traffic noise pollution is peaking. Noise pollution is defined as harmful level of sound that disturbs the natural rhythm of human body and traffic noise is considered one of the major sources of noise pollution in an urban environment. Several studies have shown that high intensity noise is the cause of many health issues such as sleep apnea, stress, fatigue and hypertension [59,60]. Apart from health issues, traffic noise also interferes with cognitive functions including attention, concentration, memory, reading ability, and sound discrimination – leading to less productive work environment.

The main source of traffic noise, which is the vehicle pass-by noise, comes from sources such as engine, intake / exhaust manifolds, tire-road interaction, road surface quality and other engine accessories [61]. It is further known that the frequencies of the noise sources depend on the following two factors (a) type of vehicle: heavy-duty vehicles such as freight trucks, buses and
lorries produce low frequency noise, while light vehicles such as automobiles, motor cycles create high frequency sound and (b) speed of vehicle: vehicles travelling at low speed (for example on highways during rush hour traffic) contributes to low frequency traffic noise, while on the other hand, vehicles travelling at high speed (for example on highways during off-peak traffic) lead to traffic noise dominated by high frequency content. It has been quantified that these variations in traffic conditions cause the dominant frequency of the noise spectra to shift between 500-1200 [Hz] [62–64]. Hence, an effective sonic barrier needs to be able to adapt and attenuate the dynamically changing dominant traffic noise spectra to reduce the harmful effects of noise pollution.

While the traditional barriers such as vertical walls that act as barriers can attenuate the traffic noise across the entire spectrum, they are heavy and block the flow of wind which leads to high loads and moments on the foundation upon which it is built (Fig.4.1), therefore limiting its applications. Moreover, the incidence of oblique waves onto these barriers leads to diffraction at the top edge that creates an additional path for noise propagation across the barrier. Further, it should be noted that the design of vertical walls to block the sound across a broad range of frequencies is an overkill strategy, as the traffic conditions that create noise across the entire spectrum seldom occurs. On the other hand, previous studies [27,65–78] and some on-road installations (Fig.4.2c) have shown that the periodic structures can be used as sound barriers. Some of the benefits of periodic sonic barriers include (a) being light weight and permeable to wind (b) less amount of load transfer to foundation on which it is built and (c) having aesthetically pleasing view [65,66]. However, one of the major drawbacks of this design is that, with fixed periodicity, the periodic barriers can only block traffic noise spectra corresponding to certain frequency range that is dictated by Bragg’s effect. In order to elevate this problem, recent studies proposed periodic

Figure 4.1: Examples of vertical wall barriers used as sonic barriers to block noise generated (a) on road (b) by trains (c) by machinery
barriers that possess multi-physical phenomena properties that improve the attenuation range [66,69,71,73,77]. However, the attenuation phenomena, such as absorption and resonance, in the modern barriers (Fig.4.2c) require the use of sophisticated barrier material and the construction process is complicated.

Building on the benefits of periodic barriers and to efficiently block the dynamically changing traffic noise spectra, we propose a new kind of origami sonic barrier (OSB) that constitutes of periodically arranged cylindrical inclusions attached on top of origami sheet. In this setting, the origami folding kinematics can induce reconfiguration in the periodicity of inclusions between different geometric patterns (Bravais lattices), as described in Sec.3.1.3. Since different periodic patterns block different frequency wave propagation, the origami folding induced reconfiguration of OSB can be exploited to block the dynamically changing dominant noise spectra that depends on the traffic conditions. Moreover, the origami folding is a simple one-degree of freedom action and thus minimal local actuation can lead to effective global shape changes. Overall, the OSB with the benefits of periodic barriers, optimal design that can adapt to traffic conditions and practical-to-implement foldable origami design is exploited to build ideal reconfigurable sonic barrier for efficiently managing complex traffic noise pattern.

4.2. Background

A typical periodic sonic barrier is illustrated through the cartoon in Fig.4.2a, where a sonic barrier is composed of periodically arranged cylindrical inclusions in air. As illustrated in Fig.4.2b, traffic

![Figure 4.2](image)

Figure 4.2: Periodic arrangement of pipes used as sonic barriers. (a,b) illustrations of traffic noise propagation with and without sonic barrier (c) Periodic pipe noise barrier installed in Eindhoven by Van Campen Industries.
noise (red solid curves in Fig.4.2b) without a barrier would reach the buildings creating noise pollution, while a sonic barrier would reflect the traffic noise (red dotted curves in Fig.4.2a) back into the road creating a much safer environment on the other side of the barrier.

To illustrate the functionality of sonic barriers, we plot acoustic pressure maps of sound wave propagation through air with and without sonic barriers (Fig.4.3). In Fig.2a, we show the 2D wave propagation of a 500[Hz] sound wave through sonic barrier that is composed of inclusions arranged in square lattice pattern (Fig.4.3a). As a reference, we also show sound propagation through air without any sound barrier (Fig.4.3b). In these acoustic pressure maps, different colored regions indicate different pressure intensity; where green, red, blue indicating zero, positive and negative pressure regions respectively. The presences of dark red and blue contour regions (below the sound barrier in Fig.4.3a) demonstrate high intensity sound wave propagation, while almost completely green region (above the sound barrier in Fig.4.3b) imply very little to no sound propagation. These results in Fig.4.3(a,b) illustrate the wave blocking phenomena of sonic barrier.

Upon further study, it can be found that the blocking frequency of sonic barrier is strongly dependent on the lattice pattern of the inclusions. For example in Fig.4.3(c,d) we plot the acoustic

Figure 4.3:Acoustic pressure map of sound through air with sonic barrier, which are composed of scatterers arranged in (a) square lattice pattern (c) hexagonal lattice pattern. In (a,c) 500Hz and 1000Hz sound wave propagation is studied respectively, while in (b,d), corresponding acoustic pressure maps without sonic barriers are provided.
wave propagation through air with and without sonic barrier that is composed of scatterers arranged in hexagonal lattice; upon comparing Fig.4.3(c,d) it can be clearly seen that the 1000[Hz] sound wave is blocked by the sonic barrier. Overall, from the numerical results in Fig.4.3(a,c) it can be said that the sound frequency (500[Hz]) blocked by the scatterers arranged in square lattice is entirely different from the sound frequency (1000[Hz]) blocked by same scatterers arranged in hexagonal lattice – demonstrating that lattice geometry can be exploited to control the sound blocking properties of sonic barrier and here we use these features to design origami sonic barrier (OSB).

4.3. Design of origami sonic barrier

The proposed design of reconfigurable origami sonic barrier (OSB) is given in Fig.4.4 that can achieve desired transformation between a square and hexagon lattice topologies, as given in Fig.4.3(a,c), to block the dynamically changing traffic noise spectra. The cylindrical inclusions are made of rigid PVC material and air is the host media in which sound propagation is evaluated. The geometric parameters of origami sheet (as given in Fig.4.5a) viz. the radius \( R_o \) of circular rods, the crease lengths \( a(=b) \) and sector angle \( \gamma \) are designed to be 0.1477 m, 0.56 m and 60\(^\circ\) respectively, based on the parametric analysis done in Sec. 3.1.3. For the chosen parameter set, the lattice topology of the cylindrical inclusions would shift between hexagon (Fig.4.4a) to square (Fig.4.4b) and finally to hexagon (Fig.4.4c) when the folding angle is shifted from 0\(^\circ\) to 55\(^\circ\) to 70\(^\circ\) respectively.
Figure 4.4:(a-c) are illustrations of different folding configurations of origami sonic barrier (OSB) and their corresponding cross-section views. The pink polygons in cross section views act as a guide to identify different lattice patterns and show that the lattice transforms from (b) hexagon to a (c) square and to a (d) hexagon when the folding angle is shifted from (b) 0° to (c) 55° and to (d) 70°.
4.4. Analytical and numerical investigations

4.4.1. Analytical investigation via plane wave expansion method

The effect of lattice reconfiguration on bandgap features of OSB design in Fig. 4.4 is studied by solving the first principle wave equation (Eq. 2.1) via PWE method (as described in Sec. 2.3). The OSB is assumed to be large in the axis parallel to the inclusions and we only solve for the acoustic wave propagation through the plane perpendicular to the inclusions axis. The OSB being periodic in \(xy\) plane, its wave behavior can be evaluated by solving the wave equations inside a unit-cell with appropriate periodic boundary conditions (Fig. 4.5b). This unit-cell, as represented by dotted rectangle in Fig. 4.5b, consists of basis that is formed by two inclusions marked with centers at \(C_1\), \(C_2\) and is composed of two lattice vectors \(a_1\), \(a_2\); tessellations of this unit-cell combined with basis inclusions along \(a_1\) and \(a_2\) direction will recreate the sonic barrier. The relationship between the unit-cell parameters and origami unit vertex parameters, required for evaluating Fourier coefficients, are given in Eq. 3.1. The material properties of rigid PVC inclusions and host air can be found in Table. 2.1.

Because of the high impedance mismatch between the solid inclusions and fluid host, most of the wave energy incident onto the inclusions will be reflected. With little energy transfer into the inclusions, we can ignore any transverse vibrations and model the solid material as an equivalent

Figure 4.5: (a) is origami unit vertex labelled with important geometric parameters, (b) is description of unit-cell along with reduced 1st Brillouin zone and (c,d) are models developed in COMSOL for evaluating insertion loss (IL) spectra of origami sonic barrier(OSB) at two different folding configuration.
media that can only sustain longitudinal wave propagation. Further, the host is assumed to be ideal and does not dissipate any wave energy passing through it.

Dispersion diagrams, evaluated via PWE method, of OSB in two different configurations (Fig.4.4(b,c)) are plotted in Fig.4.6(b,d) respectively. Discontinuities in dispersion diagrams across all high symmetry edges of reduced 1st Brillouin zone, represent a complete bandgaps (refer Sec. 2.2.2 for more details) and are highlighted as region between horizontal dash-dot (blue) lines in Fig.4.6(b,d); in such spectral regions sound propagation is blocked for all wave incidence directions. From these plots, it can be clearly seen that the bandgap of OSB at 55° and 70° folding configurations occur around 500 and 1000[Hz] respectively. The significant shift in bandgap during folding, especially in 500-1200[Hz] range, could be very useful for attenuating the dynamically changing traffic noise spectra.

4.4.2. Numerical investigation via finite element analysis

The dispersion diagrams extracted via PWE method are for infinitely periodic structure, but in real world, finite sample effects have to be taken into account. For this reason, numerical simulations are performed on a finite sample in commercial FEA software COMSOL.

For this study, the 2D cross section of OSB in two different configurations, as given in Fig.4.5(c,d), are modelled and evaluated using the acoustics-structural interface package of COMSOL. As

Figure 4.6:(a,c) are the insertion loss (IL) spectra evaluated via numerical simulations in COMSOL and (b,d) are the dispersion diagrams generated via PWE method of origami sonic barrier (OSB) at 55° and 70° folding angle respectively. In (a,c) two different IL curves correspond to 0° and 45° wave incidence.
before, the circular solid inclusions and fluid host are provided with material properties corresponding to PVC and air as given in Table 2.1. In this model the location of a point source and receivers are also shown; where the source is placed at a distance of 2[m] from the first row of OSB and the sound pressure level (SPL) is measured at receivers, which are placed at a distance of 4[m] from the source along x-direction. The two receivers are positioned such that, each of the source-receiver line makes an angle of 0° and 45° with respect to x-axis; measurements from these two different receivers are required to calculate the insertion loss (IL) of transmission along different wave directions. In this model, the four outside boundaries of the model are provided with radiation boundary condition to absorb any reflections.

A frequency domain analysis is performed across the frequency range 10-1500[Hz] and the insertion loss (IL), which is extracted as the difference of SPL (in dB) without and with sonic barrier, is plotted in Fig.4.6(a,c). Each figure contain two IL curves, solid (purple) and dash-dotted (green), representing 0° and 45° wave excitation respectively and higher value of IL in each of these curves represent good sound blocking performance; in general a sound barrier with IL of 10[dB] is considered of good quality [72] – a vertical dash-dot (red) line is plotted in Fig.4.6(a,c) that act as a guide indicating the threshold IL value. For qualitative reference, a 10[dB] drop in IL would be equivalent to cutting noise pressure levels by 90% or to put it in other words, the traffic noise that could otherwise be heard as far as a mile away would now only be perceived from a distance of 0.3 miles. The regions where IL is greater than threshold for both wave incidence directions are marked by horizontal dash-dot (black) lines in Fig.4.6(a-c). It can be clearly seen that the sound blocking phenomena of OSB at 55° and 70° configurations (spectral region between horizontal dash-dot (black) lines) occurs in different spectral regions with IL peaking at 500 and 1000[Hz] respectively. These spectral regions with high IL match very well with the complete bandgaps predicted via PWE method and provide further evidence of adaptability in sound blocking performance of OSB.

Overall, the above analytical and numerical investigation results show that for traffic noise dominated with low frequency content (around 500[Hz]) the OSB can be configured to 55° folding angle, while on the other hand, OSB in 70° configuration can be used to block traffic noise dominated by high frequency content (around 1000[Hz]) – making origami sonic barrier effective in blocking the dynamically changing traffic noise spectra. It should be noted that above analytical
and numerical results, in terms of dispersion plots and IL spectra, are independent of the density ratio between solid inclusions and fluid host if the density ratios are higher than 100 [23] (the density ratio of the system used in this paper is around 1,200).

4.5. Experimental investigation

4.5.1. Scaled-down model

To experimentally demonstrate lattice reconfiguration and adaptable sound blocking features, we have fabricated and performed tests on a scaled-down model of OSB in anechoic chamber (Fig. 4.7). More precisely a one-seventh model is constructed with the following origami unit

![Figure 4.7](image)

Figure 4.7: Different folding configurations of scaled-down origami sonic barrier (OSB). (a-c) and (d-f) are isometric and top views of origami sonic barrier (OSB) at $0^\circ$, $55^\circ$ and $70^\circ$ folding angles respectively.
vertex parameters; crease lengths \((a=b)\) of 0.08[m] and sector angle \((\gamma)\) of 60\(^{\circ}\). Being a linear system, the result of this proof-of-concept scaled-down model should be enough to demonstrate the concept and understand the behavior of full-scale sonic barrier.

### 4.5.2. Fabrication details

The following raw materials and process are used during fabrication: facets of origami sheet are made by waterjet cutting the aluminum sheet of thickness 0.05[in] and the individual facets are arranged together to form an origami sheet using ultra-high molecular weight (UHMW) polythene adhesive sheet of thickness 0.005[in]; during the folding process the UHMW polythene fold at the crease lines while the facets remain rigid. 1¼[in] PVC pipes with outer radius of 0.0211[m] are used as cylindrical inclusions and 360\(^{\circ}\) friction hinges are attached to the origami vertices that hold the PVC pipes up-right and move them as per the trajectory of origami vertices during the folding process. Finally, a caster pipe cap assembly is used as connector between PVC pipes and friction hinges.

The OSB that is folded into different configurations is shown in Fig.4.7, where different views of the barrier are given. In the iso-view (Fig.4.7(a-c)), different parts of the construction are labelled and a close up view of origami sheet at 0\(^{\circ}\), 55\(^{\circ}\) and 70\(^{\circ}\) folding configurations is given; it should be noted that the origami sheet is flat on the table at 0\(^{\circ}\) folding angle (Fig.4.7a) and as the folding angle increases the quadrilateral facets gradually raise making an angle with the table (Fig.4b-c). Through the top views (Fig.4.7(d-f)), it can be clearly seen that the periodic spatial distribution of the inclusions in \(xy\)-plane transforms from a hexagon, to a square and finally to a hexagon Bravais lattices when folding angle changes from 0\(^{\circ}\) to 55\(^{\circ}\) to 70\(^{\circ}\), as predicted through kinematic relations given in Eq.3.1; in these figures, green polygons are drawn as a guide to identify the lattice shapes. In this top view, we can also see the walls of the anechoic chamber in which the experiment is conducted.

In order to measure the insertion loss (IL) due to transmission of a 2D propagating wave, sufficient foam is added at top and bottom of the pipes (as shown in the side view, Fig.4.8(a)), so that the oblique waves incident onto the sonic barrier from the source are absorbed. Further, to minimize
the dispersion of incident wave in the z-direction, a horn (Piezo source #KSN1141A) with rectangular aspect ratio is chosen as source that would focus the sound energy predominantly in $xy$-plane. Finally, to record the pressure of wave after passing through the sonic barrier, a microphone (Larson-Davis $\frac{1}{4}$[in] microphone #2520) is placed behind the sonic barrier (shown in Fig.4.8(a)). While the horn and microphone are placed at the same level in $z$-direction equi-distant from top and bottom foams, they are placed at a distance of 18[cm] in front and 2[cm] behind the sonic barrier along the $x$-direction respectively, as shown in schematic Fig.4.8b.

4.5.3. Test method

Random white noise is generated through the horn and pressure is recorded using the microphone, where NI DAQ and LabVIEW are used for both generation and acquisition of analog signals at a sampling rate of 1[MHz]. 0.1[s] time series signal from the microphone is extracted for calculating FFT spectra and the spectra is averaged over 20[s] of data to improve the signal-to-noise ratio. For each OSB configuration, the test is repeated two times with horn-mic configuration (a) parallel to $x$-axis and passing through the center of the sample (viz. $0^\circ$ excitation) and (b) at an angle of $45^\circ$ with respect to $x$-axis (viz. $45^\circ$ excitation), as shown in schematic Fig.4.8b and during the test in Fig.4.8(c,d); these different experiments are required to test the barrier performance to incident waves traveling in different directions.
The experimental results of 0°, 45° wave excitation of OSB at 55° and 70° folding angle configurations, are shown in Fig. 4.9(a,c), where the IL (in dB) curves are calculated as the ratio of pressure spectra without and with the sonic barrier. Since the horn has a smooth response in 2-30[kHz] range, the IL of transmission is shown from 2.5[kHz] and is cut-off at 10.5[kHz]. Moreover, since the frequency (ω) can be normalized as ωx/c (where, ‘c’ is speed of sound in air and ‘x’ representing the lattice constant parameter) while solving the wave equations Eq.2.1, this spectral range of 2.5-10.5[kHz] in these validation tests of 1:7 scaled-down model also corresponds to the frequency range of 350-1500[Hz] in the actual size sonic barrier, which is where the traffic noise spectra is known to vary.

4.5.4. Results and discussion

In Fig.4.9(a,c) the solid (purple) and dash-dotted (green) curves represent the spectra corresponding to 0°, 45° excitation respectively and the vertical dash-dotted (red) line is the threshold limit set for transmission loss (12[dB]); drop in intensity of 6500[Hz] sound wave, recorded during the experiment on OSB at 70°, is posted online in this youtube video [79]. Further, the horizontal dash-dotted (black) lines are provided that act as a guide to identify the spectral regions where the IL of barrier is greater than the threshold for both directions of excitation.
In Fig.4.9(b,d), dispersion diagrams (generated via PWE) of the scaled-down sonic barrier are plotted, where the region between the horizontal dash-dot (blue) lines represents the region of complete bandgap. The material properties used for evaluating band structure via PWE method (as described in Sec.2.3) in the 2D cross-section plane are given in Table 2.1 and the outer radius of PVC inclusions are as assigned to be 0.0211[m]. Because of the large impedance mismatch between PVC inclusions and air host, most of the wave energy incident on the inclusion-host interface is reflected back into the air and hence with little energy transfer into the inclusions, we modelled hollow PVC pipes as solid PVC rod. Further, the air is assumed to be ideal with non-dispersive characteristics while evaluating the band structure. As will be shown later, the PWE results match well with the experimental data, validating the accuracy of the above assumptions.

Upon comparing the experimental IL spectra (Fig.4.9(a,c)) to that the dispersion diagrams (Fig.4.9(b,d)), it can be clearly observed that the complete bandgap predicted via PWE method aligns very well with the peaks of the IL that are inside the spectral region marked by horizontal black dash-dotted lines. Further, upon comparing the IL of OSB at different folding configurations (Fig.4.9(a,c)), it can be observed that the sound blocking performance occurs in different spectral regions – i.e., the scaled-down OSB at 55° folding angle can block low frequency waves (around 3.5[kHz]), while the same barrier at 70° folding angle can block the high frequency wave propagation i.e. around 7[kHz].

![Figure 4.10](image)

Figure 4.10: Average insertion loss (IL) spectra obtained from data recorded by microphone i.e. displaced along positive and negative y-direction from initial position. (a,b) are IL spectra of scaled-down OSB at 55° and 70° folding configurations.
Apart from the experimental results in Fig.4.9(a,c), to test the sensitivity of the complete bandgap on the microphone’s spatial location along $y$-direction, few more experiments are conducted by moving the microphone along positive and negative $y$-directions. The microphone is displaced by a distance equal to half the lattice constant on either sides of its initial position i.e. given in Fig.4.8b. Pressure is recorded at each mic location and the average of the three measurements (left, center and right) are taken to evaluate the IL spectra as shown in Fig.4.10; where the legend of Fig.4.10(a,b) is same as in Fig.4.9(a,c). The average IL spectra and their overlap spectral regions (region between the black dashed-lines), where the IL is high for both angles of wave incidence, in Fig.4.10(a,b) are very similar to the IL spectra evaluated using one mic location given in Fig.4.9(a,c). This co-relation between the overlap regions of average IL spectra and the IL spectra extracted from single microphone provides additional evidence of the existence of complete bandgaps.

Based on the experiments of scaled-down model (Figs.4.9(a,c) and 4.10) and the simulation results (Fig.4.6), it can be concluded with confidence that the sound blocking features of an actual size origami sonic barrier can be tuned in the frequency range 500-1200[Hz]. The above statements imply that the origami sonic barrier configured to $55^\circ$ folding angle with square lattice can be used to block low frequency dominated traffic noise (that occur on highways during rush-hour or on roads with heavy-vehicle traffic), while on the other hand the reconfigured sonic barrier to $70^\circ$ folding angle with hexagonal lattice can block high frequency dominated traffic noise (that occur on highways during off-peak traffic and on roads with automobile traffic).

One of the other important feature of origami sonic barrier is that the reconfiguration mechanism that cause the wave adaptability is a one-degree of freedom action and thus requires low actuation effort to precisely reconfigure the barrier. Further, with inherent rugged top edge profile, the OSB can better-diffuse the diffracted wave (compared to a vertical wall barrier of same height), leading to reduced transmission of oblique incident wave across the barrier. Additional the OSB with its corrugated façade, perpendicular to wave propagation, leads to better diffusivity of wave that is reflected into the road; such phenomena of radiating the sound energy in many directions is an important property that is required for reflective sound barriers. Hence, the origami sonic barrier (OSB) with the advantages of a periodic barrier, coupled with better diffusion properties and
tunable wave blocking behavior at limited actuation, will be an effective innovation for attenuating complex traffic noise.

4.6. Validation of 2D finite element model

To check the validity of 2D COMSOL model that is used to predict IL spectra of an actual size sonic barrier in Fig.4.6(a,c), here we extract IL spectra of scaled-down OSB in Fig.4.7 using 2D COMSOL model and compare the results with test data in Fig.4.9.

Without loss of generality, for this co-relation, we only study the $0^\circ$ wave incidence case of both origami folding configurations. The COMSOL model is shown in Fig.4.11, where we try to emulate the horn via a diverging 2D cross-section profile with a line source at neck. The distances of the input source and output microphone are the same as given in Fig.4.8b and the output boundaries are given radiation boundary conditions to imitate the anechoic chamber. Frequency spectra analysis is conducted with and without the sonic barrier for $0^\circ$ wave incidence and the results (marked by gold dashed-dot line) are given in Fig.4.11(a,b) for both origami configurations. For reference, the experimental results of the same setup (marked by purple solid line) are also given in Fig.4.11(a,b). The trend and magnitude of the IL spectra predicted by COMSOL are in the ballpark of the experimental results indicating that that 2D cross-section COMSOL model is

Figure 4.11: Acoustic pressure map extracted from finite element simulations done in COMSOL. (a,b) show the pressure distribution of wave propagating through scaled-down origami sonic barrier at $55^\circ$ and $70^\circ$ folding configurations. The excitation frequencies are provided in the inset.
Figure 4.12: Comparison of insertion loss (IL) spectra of scaled-down origami sonic barrier extracted from COMSOL simulations against test spectra. (a,b) are IL spectra corresponding to 55° and 70° folding configurations.

an accurate representation of OSB. Further, the acoustic pressure map given in Fig. 4.11(a,b) also demonstrate the sound blocking performance at frequencies inside the complete bandgap.

4.7. Conclusions

A concept of origami sonic barrier is proposed, which exploits origami folding for lattice reconfiguration and achieve adaptable sound blocking characteristics. Analytical and numerical investigation results generated using PWE method and COMSOL software show that the dispersion characteristics and the insertion loss of the origami sonic barriers can be tuned in the desired frequency range to mitigate dominant traffic noise spectra. The experiments performed on a scaled-down model demonstrate that the origami sonic barrier can be re-configured between different Bravais lattices and that a full-scale origami sonic barrier can be tuned to attenuate the complex noise pollution spectra whose dominant frequency shifts in the range of 500 to 1200[Hz]. Further, the experimental results were used to validate the analytical and numerical models. The tunable wave characteristics together with practical actuation mechanism and better diffusion properties make origami sonic barrier a prime candidate to attenuate traffic noise.
CHAPTER 5.
TUNABLE WAVEGUIDING IN ORIGAMI PHONONIC STRUCTURES

In a second wave control mechanism, we apply the unique complete bandgap tuning features, of origami phononic structures to explore tunable waveguiding. To achieve such a task, we first explore the effect of folding on waveguide modes in origami structures using analytical tools such as the super-cell plane wave expansion method. Later, numerical and experimental investigations are performed on finite origami structures and we demonstrate broadband adaption in waveguide frequency.

5.1. Introduction

Waveguiding is a mechanism of restricting wave energy to a desired path with minimum losses. It is found useful in many applications such as, in filters, multiplexers, de-multiplexers, energy harvesting, structural health monitoring, fluid sensors and also in fundamental research to enhance the interaction between sound and light [4,80–82]. Such effective wave control mechanism can be achieved in phononic structures where the bandgap features, resulting due to periodicity, can create an efficient background for steering wave energy.

It is well known that the impurities in a phononic structure can lead to spatially localized modes with frequencies inside the bandgaps. Such defects can be arranged in a desired path to steer the wave energy and eventually creating a waveguide. Because of the wide spread applications of waveguides, studies were performed in the past to tune the waveguide frequency, so as to use the same waveguide for steering different spectral content. Some of the works include, either changing the geometric properties or changing the material properties of the defects [4,33,39,80,83–87]. However, in such works, the adaptation in waveguide frequency is incremental and is strictly restricted within the bandgap defined by the periodicity of phononic structure.
To advance the state of the art and achieve broadband adaptation in waveguide frequency, we propose a novel concept based on the origami architecture. This origami phononic structure is composed of cylindrical inclusions that are attached onto the vertices of origami sheet, where folding the underlying origami sheet will change the spatial distribution of attached inclusions and result in change of lattice periodicity of inclusions. As the spectral content transmitted through the waveguide depends on the periodicity surrounding it, the folding induced lattice transformation in origami phononic structures can lead to adaptation in waveguide frequency. In fact, in origami

![Lattice reconfiguration in origami phononic structure.](image)

Figure 5.1: Lattice reconfiguration in origami phononic structure. (a-c) are illustrations of origami structure at different folding angles i.e. at (a) 0°, (b) 55° and (c) 70° respectively and (g-i) are corresponding views of fabricated test-setup. (d-f) are the cross-section profiles displaying the spatial distribution of cylindrical inclusions attached on top of origami vertices; green polygons are drawn to identify lattice topologies and it can be seen that lattice transforms between square and hexagonal lattices during folding operation. (j) is a close up view of origami unit-vertex showing various geometric parameters.
structures that can transform between different Bravais-lattice types, for example between a square and hexagon lattice, the waveguide frequency can be drastically tuned. Hence, in this chapter, we explore the design of origami phononic structures that can effectively and drastically tune the frequency of waveguide by reconfiguring the lattice topology between different Bravais-lattice types.

5.2. Design of origami phononic structure with waveguides

The proposed origami phononic structures are created by attaching cylindrical inclusions on top of miura-origami sheet, as shown in Fig. 5.1. In Ch. 3, we have shown that, it is possible to design an origami phononic structure whose inclusions can transform between any four Bravais-lattice types (viz. square, rectangle, center-rectangle and hexagon lattices). Here for the purposes of demonstrating waveguide tuning, we select the parameters of origami sheet (viz. crease lengths (a, b) of 0.08[m], sector angle of 60° and inclusions with radius of 0.0211[m]) that can transform the lattice between a square and a hexagon. For these set of parameters the lattice transforms from a hexagon (Fig. 5.1d) to a square (Fig. 5.1e) and then to a hexagon (Fig. 5.1f) as the folding angle is shifted from 0° to 55° to 70°. Such kind of lattice transformation, as will be shown, will lead to phenomenal waveguide tuning.

A waveguide in this study is created by removing inclusions along a desired path; for example in Fig. 5.2(a,b) the top views of a straight-geometry waveguide in origami phononic structure at two different folding configurations are given, where the location of inclusions removed to form waveguide are identified with grey solid circles. It can be clearly seen that by folding the origami structure (from 55° to 70°) the periodicity around the waveguide is changed. Now to study the effect of this change in periodicity on the waveguide frequency, we perform band-structure analysis via super-cell plane wave expansion method and frequency domain analysis using COMSOL software.
5.3. Analytical And numerical investigations

5.3.1. Analytical investigation via super-cell plane wave expansion method

Often times it is desirable to study phononic structures that break periodicity such as structures with defects. One method of extracting dispersion characteristics is to assume such structures to be quasi-periodic and model them as periodic repetitions of super-cell; where the boundary of a super-cell includes large collection of inclusions along with the defects (Appendix A).

In this case, the origami phononic structure with waveguide can be modelled as a periodic repetition of black rectangular super-cells, as shown in Fig.5.2(a,b), which includes several inclusions and defects that form the waveguide. Greater the number of inclusions around the defects in the super-cell, greater is the convergence of the band-structure results; for the simulations in this study, 16 complete inclusions around the waveguide are chosen as the basis of unit-cell to achieve desired convergence at reasonable computation cost.

With the definitions of super-cell and lattice vectors \( (a_1, a_2) \) given in Fig.5.2(a,b) and Eq. (5.1), the acoustic wave propagation behavior through the 2D plane, transverse to inclusion axis, is evaluated by solving the governing pressure wave equation Eq. (2.1) – for band structure via plane wave

![Image](image)

Figure 5.2: Cross-section profiles of origami phononic structure in different folding configurations (a) 55° and (b) 70° with straight-geometry waveguide. (c,d) are corresponding bandgap structures evaluated via plane wave expansion method, assuming that origami phononic structure is made from periodic repetitions of rectangular super-cell and lattice vectors as given in (a,b). The modes represented by red dots in (c,d) have localized modeshape with sound energy confined inside the waveguide; drastic variation in spectral location of localized modes in (c) and (d) indicate that the waveguide can be tuned to different frequency wave propagation via changing the folding angle.
expansion method (PWE) – with the assumption that the material properties of origami structure are invariant along the z-direction.

\[
\mathbf{a}_1 = L \mathbf{\hat{x}} \\
\mathbf{a}_2 = \frac{9W}{2} \mathbf{\hat{y}}
\]

Eq. 5.1

Figures 5.2(c,d) are the dispersion diagrams of origami phononic structure with waveguide at two folding configurations 55° and 70° respectively. In order to find the waveguide modes (modes with localized wave functions inside the waveguide), we need to compare the band structure diagrams in Fig.5.2(c,d) with that of perfectly periodic origami phononic structures without a waveguide (which can be found in Fig.4.9(b,d)). For the purpose of comparison, we extract complete bandgaps and mark them as red dashed-dot rectangular regions in Fig.5.2(c,d). It can be clearly seen that some modes, marked by large red dots, are formed inside the spectral region where a bandgap is predicted for periodic case. Based on [2,4,33,53], these observations suggest that the modes inside the red rectangle have localized wave functions inside the waveguide.

5.3.2. Numerical investigation via finite element analysis

In order to validate the spectral and localization properties of waveguide modes, we perform numerical simulations on a finite origami phononic structure using the acoustics-structural numerical analysis package of COMSOL. 2D cross-section profiles used for simulations are given in Fig.5.3(a,b), where we study waveguide with complex-geometry. As stated before, the periodicity around the waveguide is changed when the folding angle is varied and this effect can be clearly seen in Fig.5.3(a,b), where (a), (b) correspond to 55° and 70° folding configuration respectively.

In the COMSOL model, plane incident wave is excited at the bottom of the simulation domain (in front of the origami structure) while a line receiver is placed at a distance of 1[cm] behind the origami structure. The receiver measures the pressure and evaluates the sound pressure level (SPL,
Figure 5.3: Numerical acoustics simulations are performed on cross-section profile of origami phononic structure with complex-geometry waveguide; where (a),(b) are COMSOL models correspond to folding configurations at 55° and 70° respectively. Thick solid lines and arrow direction at the bottom of (a,b) represent the incident plane wave and outer boundaries are provided with radiation boundary conditions to reduce reflections. (c,d) are corresponding contour maps of sound pressure level (SPL, in dB) along line-receiver that is positioned at the output of waveguides across the width of structure (marked as horizontal dashed line in (a,b)) and (e,f) are zoomed in spectral regions of contour maps in dB) along the y-direction. The outer boundaries of the simulation domain are provided with radiation boundary conditions to absorb any reflections.

Frequency domain analysis is performed across the frequency range of interest and contour maps of SPL are plotted in Fig.5.3(c-f); where (c), (d) are results of SPL across the width of the structure at the output of waveguide at 55° and 70° respectively and (e), (f) are zoomed in spectral regions where waveguiding is predicted to happen via band structure analysis (as given by red rectangular regions in Fig.5.2(c,d)). From the zoomed-in plots, it can be observed that the SPL is localized around the output of the waveguide indicating that the sound energy is guided through the complex-shaped waveguide. It can further be noticed that the spectral region where waveguiding
occurs is different for different configurations, indicating that the waveguide frequency can be tuned via changing the folding angle. Moreover, since the periodicity around the waveguide shifted between different Bravais-lattice types, in this case from a square to a hexagon, we have achieved significant adaptation in waveguide frequency (waveguide frequency doubled for the example case considered here).

5.4. Experimental Investigation

5.4.1. Test setup

An experiment has been set up to validate the waveguide tuning. Top and side views of the origami phononic structure at 55° and 70° folding configurations are shown in Fig.5.4 and the corresponding close-up views of the origami sheet are given in Fig.5.1(h-i). 1 ¼ [in] PVC pipes with outer radius of 0.0211[m] are chosen as inclusion and 0.05[in] thick aluminum sheets are water jet cut to form facets of origami sheet (geometric parameters are provided in Sec.5.2). Different facets are connected together by high-density polythene adhesive tape to form the
origami sheet and inclusions are attached to the vertices of the origami sheet via 360° friction hinges. To hold the origami structure in a given folding configuration, stencils corresponding to the spatial distribution of inclusions are carved out using laser on Masonite boards and are slipped onto the pipes as seen in Fig.5.4.

The experiments are performed on the origami structure at 55° and 70° folding configurations and two sets of tests are performed on each configuration, (a) with waveguide (Fig.5.4(b,d,f,h)) and (b) without waveguide (Fig.5.4(a,c,e,g)) – the results from these sets will be compared to demonstrate waveguiding phenomena. Since this test setup is built with three rows of inclusions along the x-direction, waveguiding will be demonstrated in a simple-geometry waveguide.

As before (Sec. 5.3), waveguide is formed by removing inclusions (at spatial location marked by green circles in Fig.5.4(b,f)) and it can be seen that as the folding angle is changed from 55° to 70°, the periodicity around the waveguide is shifted from a square to hexagon. As a guide to the eye and for better legibility, lattice topology of the two folding configurations are marked by green polygons in top views of the origami structure without waveguide (Fig.5.4(a,e)). The incident wave is produced by a horn, which is placed at an angle such that the incident wave (represented by red arrow in Fig.5.4) makes an angle of 45° with respect to the x-axis. To record the pressure, a microphone is located at the center of the origami structure in the y-direction (which also coincides with the y-coordinate of the output of the waveguide) and behind the origami structure at distance of 2[cm].

5.4.2 Results and discussions

The SPL spectra of transmission is calculated, as the ratio of pressure measured by microphone to the reference pressure $20e^6$[Pa], for the cases with and without waveguide and are given in Fig.5.5(a,b). Firstly, for the case without waveguide (green dash-dot curves in Fig.5.5(a,b)), it should be noticed that there are regions of very low transmission at spectral locations that are coincident with complete bandgaps regions (represented by red rectangle) of origami phononic structure without waveguide. Now for the case with waveguide (purple solid curve in Fig.5.5(a,b)), the transmission measured at the output of the waveguide is substantially increased inside the red rectangle compared to the case without waveguide – as predicted by simulation results in
Fig. 5.2, 5.3 – indicating that the wave energy is confined and propagating through the waveguide; these regions are highlighted as light blue shaded rectangular regions. Finally, it can be clearly seen that the spectral region where wave guiding occurs is drastically shifted by changing the folding configuration. Overall, these experimental results validate the analytical and numerical predictions and corroborate the findings that, folding in origami structures can be used as a means to drastically tune the waveguide frequencies. These results also provide us confidence to use the analytical and numerical 2D models to gain insights into the dynamics of wave propagation in origami phononic structures with waveguides.

5.5. Conclusion

In this research, we proposed a novel phononic structure based on origami architecture that can shift between different Bravais-lattice types for tunable waveguiding. Analytical and numerical simulation are performed on 2D models with simple and complex waveguide to predict the spectral range of waveguiding and also to demonstrate that the waveguide frequency can be drastically tuned via folding. Further, experimental investigation revealed that the spectral range of
waveguiding can indeed be tuned and the results matched very well with theoretical findings. Overall, this practical and scalable origami phononic structure with tunable waveguiding will be useful in various applications such as filters, multiplexers, de-multiplexers, energy harvesting, fluid sensors and structural health monitoring. In addition, this architecture could also be a primary candidate for fabricating phoxonic crystals with tunable waveguides where interactions between photons and phonons can be studied.
CHAPTER 6.
ADAPTIVE UNIDIRECTIONAL TRANSMISSION IN ASYMMETRIC ORIGAMI STRUCTURES

In this chapter we explore the adaptable wave features of origami structures in terms of directional bandgap tuning to explore tunable unidirectional transmission. We first discuss the novel design of asymmetric origami structure and later through numerical simulations, we demonstrate how directional bandgaps of origami structures are exploited towards tunable unidirectional transmission.

6.1. Background

Unlike the design of sonic barrier in Ch.4, where the propagation of wave is blocked on either side of the barrier (front and rear), there are applications that are required to block wave propagation only in one-direction while allowing transmission in other direction. This kind of unconventional wave propagation is called unidirectional transmission [32,88–93]. There are tremendous opportunities [7,32] for unidirectional transmission in acoustic domain, such as in stealth technologies, for enhancing sonar communication, for soundproofing and for building next generation information processing systems. In all of the above applications, having the ability to tune the asymmetric transmission spectra is necessary to adapt/adjust to changing input and environmental conditions.

A typical architecture of achieving unidirectional transmission consists of two components (as given in schematic Fig.6.1). The first component is (a) either nonlinear media (NLM) for changing the frequency of the incident wave (blue region in Fig.6.1 - which converts part of the incident spectral content with frequency \( \omega \) to \( \omega' \)) or a spatial mode-convertor (MC) for changing the
direction of the incident wave (green region in Fig.6.1 - which changes the direction of incident wave from $k$ to $k'$) and the second component is (b) an anisotropic filter (orange region in Fig.6.1).

In this kind of design, the spectral or spatial mode of wave incident normal to the NLM-end or the MC-end (wave travelling from $A$ to $B$) would be converted into either a different frequency ($\omega'$) or different spatial mode ($k'$) that would surpass the barrier created by the filter (yellow region in the dispersion diagram); while on the other hand, the same wave incident on the filter-end would be blocked (wave travelling from $B$ to $A$) as the incident wave’s frequency ($\omega$) or direction ($k$) without any conversion, would fall in the bandgap of filter (grey region in the dispersion diagram) [32,88,94–99]. While these previous approaches could achieve unidirectional transmission, the tunability of the asymmetric transmission spectra is limited since it requires modifying the periodic properties of the structure and there is no robust controllable mechanism to achieve such a task.

To tackle this problem, we exploit the tunable directional bandgaps features of origami structures to design novel asymmetric structure and achieve adaptable unidirectional wave transmission.

**Figure 6.1:** Schematic that explains the principle behind unidirectional transmission. Top left and right panels illustrate the mechanism behind unidirectional transmission exploiting non-linear properties and diffraction phenomena respectively. Bottom panel shows a dispersion diagram that indicate the outcome of the wave propagation with respect to its spectral and spatial information. Waves inside grey region are blocked while waves inside yellow region are passed inside the periodic filter marked by orange color in top panels.
The design of spatially asymmetric origami structures is shown in Fig. 6.2, works on the diffraction principle detailed in Fig. 6.1. In this design, cylindrical inclusions are attached between vertices of two origami sheets with air as the host medium, where the spatial asymmetry along the \( x \)-direction is clearly illustrated in the cross-section plot, Fig. 6.2c,d.

This configuration is composed of two parts: (a) attached onto the left half of the origami sheet (represented by green inclusions), is a distribution of different diameter cylindrical inclusions that which acts as lens and change the spatial mode/direction of the incident wave [100]; and (b) attached onto the right half of the origami sheet (represented by red inclusions), is a periodic distribution of same-diameter cylindrical inclusions, which behaves as a anisotropic filter and blocks waves travelling in \( x \)-direction. The origami sheet parameters are same as given in Sec.3.2, i.e. the crease lengths \( a (=b) \) and sector angle (\( \gamma \)) are assigned to be 0.15 [m] and 60\(^0\) respectively.
The radius of all inclusions on the filter side (red inclusions in Fig. 6.2) are 0.041[m], while the radius of the inclusions in the diffraction media (green inclusions in Fig. 6.2) are linearly reduced along y-direction on either side of the central axis; where the radius of inclusions along the central axis is 0.041[m] and the radius of three inclusion moving away from central axis (in Fig. 6.2c) are 0.0342[m], 0.0273[m] and 0.0205 [m] respectively. It should be noted that the linear gradient is chosen arbitrarily to demonstrate unidirectional transmission in this proof-of-concept model. The diffraction media and the filter of finite asymmetric origami structures are made from ten columns of inclusions each, as represented by green and red inclusions respectively in Fig. 6.2.

With this novel arrangement, the reciprocity of waves with frequency inside the directional bandgap of the periodic media is broken. As illustrated in Fig. 6.2c, the right propagating plane wave front (red solid line) is continually refracted in the gradient media and the outgoing mode-shifted wave (red dashed line) – whose wave front not parallel to the incident plane wave – would cross the barrier imposed by the directional filter properties of the periodic media, whereas (in Fig. 6.2c) the left propagating plane wave (blue solid) would be blocked (blue dash-dot line) by the periodic media because it is in the bandgap region. Based on the principle of linear acoustic reciprocity [101] – which states that, despite interchanging the locations of a source (e.g., A in Fig. 6.1) and a receiver (B in Fig. 6.1), in an unchanging environment, the transmission signal will remain the same – the symmetry in wave propagation is broken and unidirectional transmission is achieved in asymmetric origami structures (Fig. 6.2). Apart from exhibiting such unusual wave
behavior, the innovative asymmetric origami structure will advance the state of the art by achieving tunable unidirectional transmission spectra. The origami folding, which can transform the lattice configurations of the periodic filter (represented by red inclusions in Fig. 6.2) between various Bravais lattice configurations (as shown in inset of Fig. 6.3a,b), will have a strong influence on the \( x \)-directional ABG of filter. Since the unidirectional transmission occurs in the spectral region where directional bandgaps are present, folding can lead to adaptation in asymmetric transmission spectra.

While we studied the shift in spectral location of complete bandgaps due to folding in Sec. 3.2, here we will look at the effect of folding on directional bandgap. The band structure of the periodic filter (red inclusions in Fig. 6.2(c,d)) at 55° and 70° is shown in Fig. 6.3(a,b). The directional bandgaps along \( x \)-direction, which can be identified as discontinuities in dispersion diagrams along

![Figure 6.4: Transmission spectra extracted via COMSOL simulations demonstrating tunable unidirectional transmission in asymmetric origami structures. (a-c) are the transmission spectra of the asymmetric origami structure design at 55° and 70° folding angles. The grey spectral regions, representative of unidirectional transmission, clearly depend on the folding configurations.](image)
Ga-X direction, are represented as grey regions in Fig.6.3(a,b). It can be clearly seen that the grey region (directional bandgaps of the filter) shift between different spectral regions as the folding proceeds. We will exploit this feature to achieve adaptability in asymmetric transmission.

### 6.3. Demonstration of unidirectional transmission via numerical simulation

To show that origami folding offers an effective means to control the unidirectional transmission, the asymmetric origami structure at 55° and 70° folding angles, given in Figs.6.2(c,d), are modelled and evaluated in acoustics-structural interface package of COMSOL. The model has a finite size in x-direction and the input-edge is excited by plane pressure wave while the output-edge is provided radiation boundary condition to absorb any reflections. A frequency domain analysis is performed across a range of frequencies and the transmission spectrum, which is the ratio of sound pressure level at output edge to input edge, for right and left propagating waves are plotted in Figs.6.4(a,b), represented by red and blue curves respectively.

It is can be observed that the transmission spectra of the right propagating wave (red) is much higher than the left propagating wave (blue) in some frequency regions (represented by grey rectangular regions), which is evidence of unidirectional transmission. Moreover, the frequency regions where such unusual behavior occurs coincide well with the x-directional bandgaps illustrated in Figs.6.3(a,b). Further, upon comparing Figs.6.4(a,b), it is clear that the spectra of unidirectional transmission can be adaptive, i.e., the unidirectional wave frequency ranges (gray shaded areas in Fig.6.4(a,b)) can be significantly changed via topology reconfiguration of the underlying architecture induced by on-demand origami folding.

While the combined design of diffraction media and periodic filter attached onto origami sheet (given in Fig.6.2) is shown to achieve adaptable asymmetric transmission during folding, the intensity of the asymmetric transmission (i.e. difference in magnitude of blue and red curves in grey regions in Fig.6.4) can be improved by performing further optimization studies on the diffraction media.
6.4. Conclusions

State of the art in unidirectional transmission devices is advanced by designing an asymmetric origami structures with tunable unidirectional transmission features. Adaptation in directional bandgaps of origami structure coupled with asymmetric design is exploited to achieve unidirectional transmission. COMSOL simulations performed on a finite size model indicate that unidirectional transmission is possible in asymmetric origami structures and that the spectral region of asymmetric transmission can also be tuned via folding.
SUMMARY OF SCHOLARLY CONTRIBUTIONS AND BROADER IMPACT

Scholarly contributions

In this thesis, we aim to advance the state of the art in on-demand wave tailoring. Through analytical, numerical and experimental investigations, we have shown that origami structures that exhibit unique topology transformation can drastically change the wave propagation behavior in terms of bandgap. The massive adaptation in transmission coupled with large design space, scalable geometry and controllable transformation, makes origami structure a novel design for wide spectra of acoustic applications bringing on-demand wave tailoring to a new level.

We have utilized origami design for exploring unique wave control mechanisms such as for traffic noise mitigation, waveguiding and unidirectional transmission. So far the advances in these fields are limited by lack of substantial adaption either in sound blocking frequency or waveguide frequency or unidirectional transmission spectra, respectively. In this thesis, we have significantly advance the state of the art in these unconventional wave control mechanisms via achieving broadband tailoring in their frequency spectra.

Broader impact

The fundamental knowledge gained in this research will advance the field of acoustics and create new knowledge in reconfigurable adaptive structures research. The innovations and tools developed for the modeling, synthesis, analysis, fabrication and experimental investigations of the origami platform will enable unprecedented transformation of geometric topology uncovering new wave phenomena. The outcomes and finding will be useful to engineering applications, such as for sound barriers and signal processing elements like filters and unidirectional waveguides, especially under system variations. Moreover due to the equivalence in wave equation of sound and light, the phononic crystal design proposed in this thesis can also be used to use to build photonic crystal and manipulate light wave propagation.
APPENDIX

Super-cell PWE method

Often times it is desirable to study structures that break periodicity such as structures with disorder or defects. One method of extracting dispersion characteristics is to assume such structures to be quasi-periodic and model them as periodic repetitions of super-cell. A super-cell is a collection of primitive unit-cells and either the defect or disorder is modelled into it.

For example, a phononic structure with single defect can be modelled as a periodic repetition of super-cells (marked in blue), as shown in Fig. A1. A total of 9 primitive unit-cells (marked by dotted black boundaries) are chosen to construct the super-cell around the defect, for both square and hexagon lattice symmetries (Fig. A1(a), (b)).

In this super-cell the definition of unit lattice vector changes to

\[
\hat{a}_1 = N_1 a_1 \text{ and } \hat{a}_2 = N_2 a_2;
\]

where \(N_1, N_2\) are determined by the size of the super-cell (for example in Fig. A1, \(N_1 = N_2 = 3\)).

Figure A1: Super-cell used for modelling defect structures.
As before the reciprocal lattice vectors \((b_1', \ b_2')\) are related to the lattice vectors in direct space \((a_1', \ a_2')\) as follows

\[
a_i' \cdot b_j' = 2\pi \delta_{ij};
\]

Based on above definitions, the plane waves \(G\) that are used to represent the periodic distribution of material properties and wave field are given as

\[
G' = m_1 b_1' + m_2 b_2';
\]

Finally the Fourier coefficients in the material expansion of a super-cell are given as

\[
\tau_{G=0} = \frac{1}{A_{cell}} \left( \sum_{inc=1}^{\text{total inclusions}} \left( A_{inc} (\tau_{inc} - \tau_{BG}) \right) + A_{cell} \tau_{BG} \right)
\]

\[
\tau_{G \neq 0} = \frac{1}{A_{cell}} \left( \sum_{inc=1}^{\text{total inclusions}} \left( J \left| G \right|^2 R_{inc} e^{-iG \cdot \varrho_{inc}} \frac{2\pi}{|G|} (\tau_{inc} - \tau_{BG}) R_{inc} \right) \right)
\]

Where \(J\) is the Bessel function of the 1\textsuperscript{st} kind of order 1, “\(inc\)” represents the inclusions in basis, \(BG\) referring to the host media and \(R_{inc}\) the radius of the circular inclusion. The summation is over the number of inclusions in basis (for example in Fig.A1, there are 9 inclusions in the basis).

The above new definitions of super-cell in conjunction with PWE method can be used to study the sensitivity of bandgaps to spatial disorder or material/geometric defects in phononic structures. In general, if large numbers of primitive cells are considered, then the analysis on the supercell will accurately reflect the behavior of a phononic structure with defect or disorder.
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