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The benefits of expediting government gold sales ☆

Dale W. Henderson [a,*], Stephen W. Salant [b], John S. Irons [c], Sebastian Thomas [d]

[a] *Federal Reserve Board, United States*
[b] *University of Michigan, United States*
[c] *Center for American Progress, United States*
[d] *RCM Global Investors, United States*

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Abstract

Additional gold can be made available either by mining at high cost (approximately $250 per ounce in 1997 dollars) or by mobilizing government stocks at zero cost. Governments own massive above-ground stocks but loan out only a small percentage of these stocks. Making all government gold available for private uses immediately through some combination of sales and loans maximizes total welfare from private uses, a consequence of the first welfare theorem. We simulate a calibrated version of our model to quantify the effects of liquidating government stocks on alternative dates. If governments sell immediately rather than never, total welfare increases by $340 billion; if they make an unanticipated sale in 20 years, $105 billion of that amount is lost. By depressing prices, such sales benefit depletion and service users but injure private owners of stocks above and below-ground. However, the injury to above-ground stock owners is more than offset by the benefits to service users—often the same individuals. Mine owners would be the principal losers; however, they could be compensated (twice over) from government sales revenue without any need for tax increases.

Keywords: Model of gold market; Government gold stocks; Gold mining; Uses of gold

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* Corresponding author. Federal Reserve Board, Washington, DC 20551, United States. Tel.: +1 202 452 2343; fax: +1 202 452 6424.
E-mail address: dale.henderson@frb.gov (D.W. Henderson).

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1. Introduction

Gold has both private uses and government uses. Private uses can be divided into two categories: *depletion uses* which reduce the stock and *service uses* which do not. For example, gold is depleted when it is used in dentistry and some industrial processes. Gold yields services when it is held as jewelry, bars, coins, and medals but remains available for the future. Governments hold gold as a monetary asset, as a "war chest," and as a strategic material.

Additional gold for any private use can be obtained either from mines or from government stocks. The cost of extracting gold from the mines is approximately $250 per ounce in 1997 dollars. However, the resource cost of drawing down government stocks is zero. Governments lend out relatively little of their massive above-ground stocks. Making these stocks available to the private sector sooner increases welfare from private uses (net of extraction costs). In addition, governments themselves maximize their discounted revenues by immediate sales. We assess the size and distribution among market participants of the increase in welfare that would arise if governments sold their gold early in the horizon (or equivalently if governments loaned out their gold early in the horizon and sold later).

For simplicity, we focus on policies under which governments sell all their gold outright in some given year. We show, however, that the same equilibrium and payoffs result from a variety of policies involving different combinations of government gold sales and government gold loans. A gold loan involves lending gold today and receiving back the same amount of gold plus a loan fee next year. Welfare from private uses is maximized when governments make all their gold available immediately through some combination of sales and loans. Although private agents are indifferent to which combination is chosen, governments may not be. Under some policies involving lending, governments continue to *own* stocks of gold for two hundred years or more, even though they do not *possess* those stocks.

Making all government gold available for private uses immediately eliminates two types of inefficiencies. A *production inefficiency* occurs whenever a given intertemporal pattern of private uses can be supplied at lower cost, for example, by beginning to use costless government gold now and postponing any costly extraction from the mines until government stocks are exhausted (Herfindahl, 1967). A *use inefficiency* occurs whenever welfare can be raised by altering the intertemporal pattern of private uses and can arise even if there is no possibility of a production inefficiency as, for example, when all gold can be obtained at the same constant marginal cost. Since gold has service uses as well as depletion uses, use inefficiencies can be avoided only if government gold is made available to service users immediately.[1]

A simple "thought experiment" can be used to isolate the production inefficiency. Suppose governments offered to sell their above-ground stocks to mine owners in exchange for epsilon less than $250 per ounce (our estimate of the marginal cost of extraction) and title to an equal amount of underground gold. Since the cost of government gold is slightly less than the cost of mining, mine owners would fill orders by purchasing above-ground government gold as long as it lasted. Under this policy, government gold reserves would remain unchanged in magnitude but would increasingly reside

[1] If gold had only depletion uses like the resource in the classic Hotelling (1931) analysis, use inefficiencies could be avoided even if governments did not make gold available immediately; it would be sufficient for them to commit now to make their gold available to private users gradually over time.

underground. Moreover, since the marginal cost to mine owners of obtaining gold from the new source is essentially the same as before, they would not alter their pricing. Hence, the welfare of private participants in the gold market would not change. However, government revenues would change, and their increase measures the production inefficiency which is eliminated.

To obtain a "model-free" estimate of the production inefficiency, one needs–in addition to assumptions about the marginal cost of extraction and the interest rate–a forecast of the future mining path. Suppose, as a first approximation, that mine owners steadily bought government gold for their customers at $250 per ounce in the amount of 73 million ounces per year (the mining rate in recent years), the 1107 million ounces of current government stocks would last for 15 years. If governments invested the receipts from their sales at a 2.5% annual interest rate, they would earn $232 billion in present value terms!

In order to obtain additional information that is of importance for policy, we construct a (publicly available) simulation model.[2] We use this model (1) to refine our estimate of the production inefficiency, (2) to calculate the magnitude of the use inefficiency, and (3) to disaggregate by interest group the gains and losses which would result if both inefficiencies were simultaneously eliminated by an immediate sale or a sale at any specified date. Of necessity, our model embodies additional assumptions about demand parameters, growth rates, market structure, and price expectations.[3] Some of the alternative additional assumptions that we consider significantly affect either the estimated size of the use inefficiency or conclusions about the price and distribution effects of government gold sales or both. However, none yield estimates of the production inefficiency that are appreciably different from the model-free estimate because the forecasted paths for mining in the absence of a government sale are not appreciably different from the one assumed in making the model-free estimate.

Using our *reference set* of parameters in our simulation model, we conclude that total welfare, as measured by discounted economic surplus in 1997 dollars, increases by $340 billion if the sale is immediate. The refined estimate of the gain from removing the production inefficiency is $247 billion (72.6% of the surplus gain)—an amount only $15 billion (6%) larger than our model-free estimate.[4] We estimate that removing the use inefficiency raises surplus by another $93 billion (27.4% of the surplus gain). If government sales are delayed 20 years, $105 billion of the $340 billion is lost.

Gold sales benefit depletion users, service users, and the governments themselves while injuring to a lesser extent owners of the mines and above-ground stocks. Delaying sales reduces both the gains of the depletion users and governments and the losses of mine owners.

To the extent that the individuals who enjoy the services of gold jewelry and art also own these items, it is appropriate to aggregate their gains and losses. In aggregate, service users and owners of the above-ground stock benefit by $71 billion if there is an immediate sale. They suffer a net loss of $16 billion if the sale is delayed 20 years.

Mine owners are the only losers if governments sell immediately (assuming that the gains and losses of service users and above-ground stock owners are aggregated). With an immediate sale, government revenue is large enough that mine owners can be compensated twice over without raising taxes.

[2] The web site for the model is at www−personal.umich.edu/~ssalant/gold/.

[3] Readers wishing to explore the consequences of alternative input parameters are invited to go to the model web site.

[4] This 6% discrepancy arises because the annual mining path predicted by the model exhibits some variation while in constructing our model-free estimate we assume that mining remains steady at 73 million ounces per year.

Although government revenue available to pay compensation is smaller if the sale is delayed 20 years, the losses of mine owners fall by virtually the same amount. As a result, government revenue left over after covering both the loss of the mine owners and the net loss of services users and above-ground stock owners falls by only 10%.

Our estimates understate the total welfare gains from gold sales for at least two reasons: they leave out both the gain from delaying the substantial pollution damages caused by gold mining and refining and any reductions in tax distortions which may arise if additional revenues from gold sales induce tax decreases.[5]

As seems reasonable, if one country unilaterally sells its gold *before* other governments can sell or even announce a sale, that country earns a larger discounted revenue than it does if other governments sell at the same time or earlier. According to our simulation model, the U.S. earns 17% more if it makes a unilateral sale.

The remainder of this paper comprises five sections. In Section 2, we describe the model and its competitive equilibrium. In Section 3, we discuss the effects of government gold policies on competitive equilibrium and the payoffs of market participants. In Section 4, we calibrate the simulation model and use it to estimate the effects of alternative government gold policies. Section 5 contains the conclusions.

2. The model

In this section, we discuss the market participants in our model and describe the unique competitive equilibrium that results if their price-taking behaviors are consistent.

2.1. The behavior of private market participants

In our model there are four groups of private market participants: depletion users, service users, above-ground stock owners, and mine owners. Since our objective is to simulate our model, we find a discrete-time formulation[6] convenient. We interpret a period as a year and have extended the finite horizon far enough that lengthening it further would have negligible effects on the results we report.

2.1.1. Depletion users

In each year, depletion users buy q_t units of gold at the price of P_t per unit (measured in background good or constant “dollars”). Their depletion uses reduce the gold stock. The inverse demand function for depletion users is denoted

$$P_t = P(q_t, t), \quad \text{(inverse depletion demand)} \tag{1}$$

where

$$\frac{\partial P_t}{\partial q_t} < 0, \quad \frac{\partial P_t}{\partial t} \geq 0, \quad P(0,t) = \infty, \quad t = 0, \ldots, T. \tag{2}$$

[5] The negative externalities associated with gold mining include mercury, arsenic, and cyanide poisoning and damages caused by the human wastes of the miners. Descriptions of gold mining in the United States and Brazil by Duncan (1997) and the World Bank (1991), respectively, indicate that these externalities can be large. Young (2000) cites the existence of these externalities as one important reason for initiating a national debate about government gold policy in the United States.

[6] It is straightforward to recast our analysis in continuous time or as a planning problem.

The price such users are willing to pay in year t falls as the quantity depleted increases, is non-decreasing with time, and approaches infinity as the quantity depleted approaches zero. We also require that the sequence of inverse demand curves is bounded in the sense that $P(q_t, t) \leq \mathbf{P}(q_t)$ for $q_t > 0$ for some function $\mathbf{P}(q_t)$. The ordinary demand function for depletion users is the inverse of Eq. (1):

$$q_t = q(P_t, t), \quad \text{(depletion demand)}. \tag{3}$$

2.1.2. Service users

In each year, service users borrow A_t units of gold for one year at a loan fee of R_t per unit (measured in background good or constant "dollars").[7] Service uses do not reduce A_t, which is referred to as the service stock.[8] The inverse demand function for service users is denoted

$$R_t = R(A_t, t), \quad \text{(inverse service demand)} \tag{4}$$

where

$$\frac{\partial R_t}{\partial A_t} < 0, \quad \frac{\partial R_t}{\partial t} \geq 0, \quad R(0, t) = \infty, \quad t = 0, \ldots, T. \tag{5}$$

The loan fee such users are willing to pay in a given year falls as the size of the stock borrowed increases, is non-decreasing with time, and approaches infinity as the stock borrowed approaches zero. The ordinary demand function for service users is the inverse of Eq. (4):

$$A_t = A(R_t, t) \quad \text{(service demand)}. \tag{6}$$

2.1.3. Owners of the above-ground stock

The problem faced by a representative owner of the service stock is to find the

$$\max_{A_t \geq 0} \sum_{t=0}^{T} I^t [R_t A_t - P_t (A_{t+1} - A_t)], \quad \text{subject to } A_0 = \bar{A},$$

where A_t and $\bar{A}$ are the stock in year t and the exogenous initial stock, respectively. Owners of the above-ground stock are assumed to hold $\bar{A}$ units initially:

$$A_0 = \bar{A} \quad \text{(initial condition for } A_t). \tag{7}$$

Within any year t, first the inherited above-ground stock is loaned out and then mining, depletion, and sales or purchases by above-ground stock owners occur so that $A_{t+1} - A_t \gtreqless 0$. Since year T is the last year of the finite horizon, above-ground stock owners can neither loan out nor sell gold carried forward into year $T+1$. As a result, in year T they will sell the gold they have carried forward into that year and will not buy any gold, so no above-ground stock is carried forward into year $T+1$:

$$A_{T+1} = 0 \quad \text{(terminal condition for } A_t). \tag{8}$$

Therefore, all gold above ground at the start of year T or newly mined during that year is depleted.

[7] In reality, the loan fee is typically denominated in gold. Our assumption that it is denominated in background good or constant "dollars" simplifies the analysis without affecting the conclusions.

[8] Gold from the service stock is assumed to be a perfect substitute for gold from the mines in any use. In our model, we abstract from the costs of transforming newly mined gold into jewelry, bars, or coins, but in the final section, we discuss extending our model to allow for such 'adjustment' costs.

Let $I = 1\frac{1}{1+i} \in (0,1)$ denote the market discount factor, where i is the exogenous real interest rate. In deciding whether to sell off an additional unit in any year t (rather than in the following year), each owner compares the revenue advantage from selling in the earlier year ($P_t - \mathrm{IP}_{t+1}$) to the disadvantage of selling earlier—the foregone loan fee (or foregone enjoyment) (IR_{t+1}).

Since owners take current and future prices and loan fees as given, optimal behavior insures that the following conditions hold:

$$\begin{aligned}&A_{t+1} = 0, \quad \text{only if } P_t - \mathrm{IP}_{t+1} \geq \mathrm{IR}_{t+1},\\ &0 < A_{t+1} < \infty, \quad \text{only if } P_t - \mathrm{IP}_{t+1} = \mathrm{IR}_{t+1},\\ &A_{t+1} = \infty, \quad \text{only if } P_t - \mathrm{IP}_{t+1} \leq \mathrm{IR}_{t+1}. \quad \text{(carry forward conditions for } A_t\text{)}\end{aligned} \tag{9}$$

2.1.4. Mine owners

The problem faced by the representative mine owner is to

$$\max_{h_t \geq 0} \sum_{t=0}^{T} I^t (P_t - c) h_t \quad \text{subject to } \bar{H} - \sum_{t=0}^{t} h_t \geq 0,$$

where h_t and $\bar{H}$ are respectively the amount extracted in year t and the exogenous initial below-ground reserves. Owners of the below-ground stock are assumed to hold $\bar{H}$ units of reserves initially:

$$H_0 = \bar{H} \quad \text{(initial condition for } H_t\text{)}. \tag{10}$$

In any year t, they enter with stock H_t and can reduce their holdings by extracting and selling $h_t \in [0, H_t]$. We assume that the cost of extraction (c) is independent of the amount extracted and is insensitive to the size of reserves. Since year T is the last year of the finite horizon, mine owners cannot sell what they extract in year $T+1$.

Therefore, it is not optimal to enter year $T+1$ with any gold underground as long as there has been a strictly positive net price ($P_t - c > 0$) in some prior year:

$$H_{T+1} = 0 \quad \text{(exhaustion of mines condition)}. \tag{11}$$

In deciding whether to extract and sell an additional unit in any year t (rather than in the following year), each mine owner compares the revenue advantage from selling the unit in the earlier year ($P_t - \mathrm{IP}_{t+1}$) to the disadvantage of extracting in the earlier year (Iic).

Since owners take current and future prices as given, optimal behavior insures that the following conditions hold:

$$\begin{aligned}&h_t > 0,\ h_{t+1} = 0 \quad \text{only if } P_t - \mathrm{IP}_{t+1} \geq Iic,\\ &h_t > 0,\ h_{t+1} > 0 \quad \text{only if } P_t - \mathrm{IP}_{t+1} = Iic, \quad \text{(adjacent year mining conditions)}\\ &h_t = 0,\ h_{t+1} > 0 \quad \text{only if } P_t - \mathrm{IP}_{t+1} \leq Iic.\end{aligned} \tag{12}$$

Notice that in our formulation the revenue advantage from selling a unit in the earlier year ($P_t - \mathrm{IP}_{t+1}$) is the same whether the unit sold comes from above or below-ground.

2.2. Competitive equilibrium

If all privately owned gold is initially underground and no official stocks are ever released, then the economy must pass through two phases. During each phase, some stocks acquired from the mines are held above ground for their services. Mining occurs only during the first phase. Once the mines are exhausted, the next phase begins and the service stock is drawn down to satisfy depletion demand.

During each phase, the price rises by less than the rate of interest. In the first, or mining, phase, both mine owners and holders of the above-ground stock could make a gross gain of $P_t - \mathrm{IP}_{t+1} > 0$ in discounted terms by selling in year t rather than in year $t+1$. Each type of agent is indifferent between reaping this gross gain or not because selling in period t would also involve an offsetting cost. Mine owners would incur the cost of extraction a year earlier, and above-ground stock holders would forego the rental income or service flow that they would receive if they waited to sell until $t+1$. Since each type of agent has the same gross gain from selling at t instead of $t+1$ and each type has an offsetting cost of selling in the earlier period, the offsetting costs must be equal: $R(A^*_{t+1}, t+1) = ic$. That is, the above-ground stock must adjust so that the rental is constant during the mining phase. As a consequence, if the rental demand curve shifts out over time, the above-ground stock must grow.

In the second phase, there is no mining. The above-ground stock is drawn down, and the rental rate rises above ic.

3. Government policy

In this section we discuss government gold policies which can involve both sales and lending. Sales could in theory occur before, during, or after the mining phase. Because of its relevance to the current situation, we focus attention on the case of sales during the mining phase. Then we discuss equivalent policies involving lending. We end by explaining why selling sooner increases discounted government revenue.

3.1. A sale during the mining phase

Suppose that governments sell their gold during the mining phase and that the magnitude of the sale is sufficient that mines suspend operations, inducing a three-phase equilibrium.[9] Denote the year of the government sale of $\bar{G}$ as $t_S = 0, \ldots, T$.

Fig. 1 shows what happens if an unanticipated government sale in year 20 interrupts the first phase of a two-phase equilibrium. At the time of the sale ($t_S = 20$), the price drops, and the first phase of a three-phase equilibrium begins. The solid line in the five panels of Fig. 1 describes what happens with no government sale. The dotted lines illustrate what occurs following the sale. The first phase, during which there is no mining but there is still gold below-ground, runs from year $t_S = 20$ to year $t_B - 1 = 40$. The second phase during which there is mining runs from year $t_B = 41$ to year $t_E = 56$. The third phase during which there is no more gold below ground and depletion is fed from the service stock runs from year $t_E + 1 = 57$ to year $T = 399$.

[9] If government stocks are sufficiently large, there will be no mining in the year of a sale of these stocks. More formally, if $\bar{G} > A^*_{t_S+1} - A_{t_S} + q(f_{t_S,\, t_S}, t_S)$, $t_S \in 0, \ldots, t_E$, where t_E is the year in which mining ends in the absence of a sale, then there will be no mining in year t_S. By hypothesis, $\bar{G} + A_{t_S} - q(f_{t_S, t_S}, t_S) > A^*_{t_S+1}$, $t_S \in 0, \ldots, t_E$. Hence, $\bar{G} + A_{t_S} + h_{t_S} - q(f_{t_S}, t_S, t_S) > A^*_{t_S+1}$, $t_S \in 0, \ldots, t_E$ for any $h_{t_S} \geq 0$. But the left-hand side is simply $A_{t_S}+1$, so our condition implies that $A_{t_S+1} > A^*_{t_S+1}$. Under this circumstance, $R(A^*_{t_S+1}, t+1) < ic$ so that $h_{t_S} = 0$: mining must stop when there is a government sale that satisfies our condition.

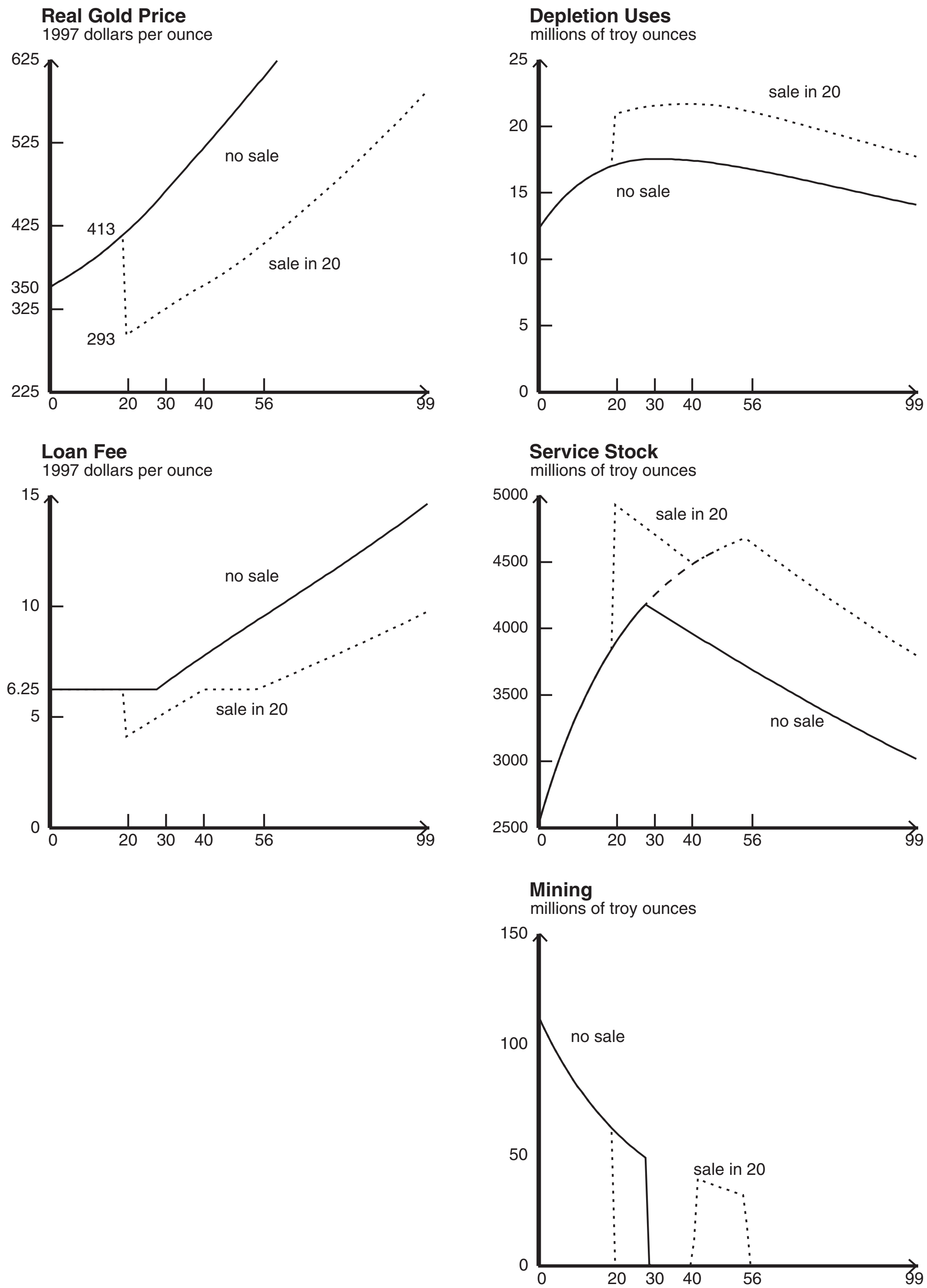

Fig. 1. Time paths: sale in year 20 compared to no sale.

In year $t_S=20$ the price (top left) drops from about \$413 per ounce before the sale to about \$293 and remains below the no sale path thereafter. Depletion (top right) is higher after the sale because the price is lower. The service stock (middle right) increases sharply in year $t_S+1=21$ because it now includes the gold the government sold.

During the first phase after a sale in $t_S=20$, the mines (bottom right) are shut down ($h_t=0$). During this phase the service stock is drawn down to satisfy depletion demand in each year. In years $t_S+1=21$ through $t_B-1=40$, the service stock (middle right) carried into the year is greater than A_t^*, so the loan fee (middle left) is below ic: $A_t>A_t^*$, $t=t_S+1,\ldots,t_B-1$, so the mines (bottom right) are closed ($h_t=0$, $t=t_S,\ldots,t_B-1$). We represent A_t^* in Fig. 1 (middle right) by the rising solid line and the rising dashed and dotted continuations of that line.

3.1.1. Equivalent policies involving loans

Instead of selling all their gold at some date t_S, governments could achieve identical results by committing at t_S to a loan policy with two features: (1) lend all holdings henceforth and (2) own in every subsequent year a specified amount never larger than the above-ground private stock would have been in each year following a government sale at t_S.

To verify that the same path of prices and loan fees would prevail under the loan policy, we show that no excess supply or demand would occur in any year given the price and loan fee paths in the equilibrium with an immediate sale. Given these paths, loan demand and depletion demand are unchanged.

First, we verify that this unchanged loan demand is satisfied in every year with the loan policy. With an immediate sale, loan demand is satisfied in every year because the entire amount (A_t) is loaned out by the private sector. With the loan policy, that same amount is loaned out in every year—part by the private sector and the rest by the governments. Given the price and loan paths, private agents are willing to offer the necessary path of gold loans because they are indifferent as long as no one carries gold into year $T+1$.

Second, we verify that depletion demand is satisfied in every year with the loan policy. With an immediate sale, depletion demand is satisfied by a combination of gold from the mines and gold from the decumulation of the above-ground private stock. With the loan policy, since the price path is unchanged, the optimal path of extraction is the same as with an immediate sale. Moreover, the combined accumulation (or decumulation) of above-ground stock by governments and the private sector is the same in every year as with an immediate sale. Hence, depletion demand is satisfied under the loan policy.

As an illustration, we consider a class of loan policies that yield the dotted path in Fig. 1. Under each policy in this class, the government commits to holding a path of government stocks which lies weakly above the horizontal axis and weakly below the dotted path of the equilibrium service stock in the middle right panel of Fig. 1. The remaining above-ground stock is willingly held by the private sector. We consider three different policies in the class.

If governments wanted to own their existing stocks of gold for as long as possible, they could announce in year 20 that all government holdings would henceforth be loaned out and that these holdings would be constant at $\bar{G}$ until the dotted line in the middle right panel of Fig. 1 dropped to $\bar{G}$ and would then coincide with that dotted line. Under that policy, no government gold would be sold before year 370.

Alternatively, if the governments preferred, they could *purchase* at t_S the *entire* private stock while committing to (1) loaning out their holdings and (2) adjusting their holdings each year to maintain them at the levels indicated by the dotted line in the middle right panel of Fig. 1. Under this scenario, the governments acquire the entire above-ground gold stock and adjust it over time. Until year 370, their stocks remain larger than the massive amounts they already hold; only after that date would their reserves fall below current levels.

Finally, a complete sale at t_S can be regarded as a third member of this class of equivalent policies. This particular policy involves a path of government holdings after t_S coincident in every year with the horizontal axis in the middle right panel of Fig. 1. In this case, the governments have no above-ground stocks to lend out; the private sector makes the stocks available to service users.

The existence of a class of government policies that yield the same outcomes depends on the indifference of holders of above-ground stocks. A private holder of an ounce of gold does not care whether he sells it immediately and lets interest accumulate thereafter or loans it out for a while, selling it in year T or earlier. The latter strategy yields exactly the same compensation, partly from loan fees and partly from the capital gain realized when the gold is eventually sold. Once governments participate in the market, they are likewise indifferent between selling at the outset and lending. It is this *joint* indifference which insures that a class of government policies induces the same equilibrium.

3.2. Selling sooner increases government revenue

In this subsection we use Fig. 2 to provide an intuitively appealing explanation for why selling sooner increases discounted government revenue by comparing the revenue from a sale in year 0

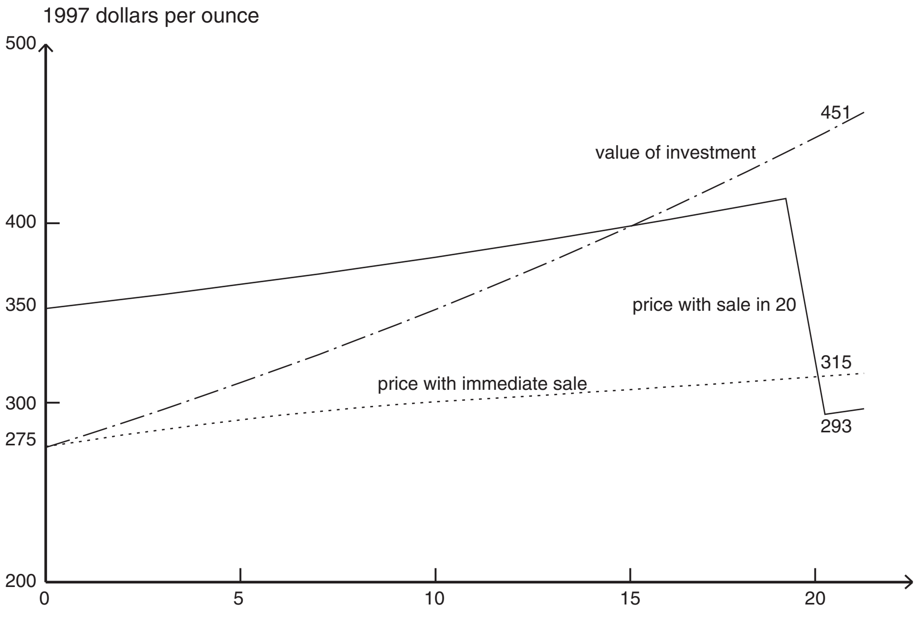

Fig. 2. Government revenue: immediate sale compared to sale in year 20.

with that from a sale in year 20.[10] If governments sell in year 0 (dotted line) the price falls from \$350 to \$275, increases at a rate less than the rate of interest, and reaches \$315 in year 20. The equilibrium price following the sale increases at less than the rate of interest because private owners of the service stock receive another form of compensation which supplements the modest capital gain: loan fees. If governments invest their revenue from the sale in year 0, it grows at the real rate of interest and reaches \$451 in year 20 (dot/dash line), a level \$136 above the \$315 market price then. The \$136 difference reflects the capitalized value of the loan fees that private owners of the above-ground stock would receive if they loaned out their gold and then sold it for \$315 in year 20. Since private owners receive loan fees, they are indifferent between selling initially or in year 20. However, since governments do not receive loan fees because they do not lend out the gold they hold, they are much better off if they sell at the outset and invest the proceeds.

Indeed, \$136 per ounce understates the full gain because the price in year 20 would be *below* \$315 if governments postponed their sale until then. If governments do not sell until year 20 (solid line) the price is higher during the intervening 20 years and, as a consequence, depletion is smaller. Therefore, after a sale in year 20, the total stock, including gold both below and above-ground, is larger than it would be if there were a sale in year 0, and so is the service stock augmented by gold from the government sale. The equilibrium price at any given date can be written as a function of these two stocks and must fall if both increase.[11] Hence, the larger stocks which result if the sale is delayed 20 years cause the price following a sale in year 20 to fall to \$293, a price \$22 lower than \$315. Hence, the full gain (capitalized to year 20) to governments from selling immediately instead of in year 20 is \$158 per ounce (\$451–\$293) as indicated by the gap between the dot–dash line and the solid line.[12]

It is interesting to review the historical record in light of this analysis. During the 1970s, the U.S. Treasury conducted several auctions of relatively small amounts of gold. We compared the average prices received at these auctions by capitalizing them all to September 1997 (using the one-month U.S. Treasury bill rate) and found that the average price at the May 1978 auction was the lowest. The actual average price at the May 1978 auction was \$180 per ounce. Selling at even this price turned out to be more profitable than leaving gold in government inventories until September 1997. By September 1997 the price of gold had risen to about \$310 per ounce. However, this increase did not go even halfway toward keeping up with inflation as measured by the increase in the U.S. CPI. In order for the gold price to have remained constant in real terms, the dollar price in September 1997 would have to have risen to about \$451 per ounce. Furthermore, it was possible to do better than keeping up with inflation by investing in one-month U.S. Treasury bills. Had the \$180 amount received from selling an ounce of gold in May 1978 been continually reinvested in one-month U.S. Treasury bills, by September 1997 it would have grown to about \$709, an amount more than twice the then prevailing price of about \$310 per ounce.

[10] Fig. 2, like Fig. 1, is constructed using the reference set of parameters shown in Table 1.

[11] Both rise, for example, if an increase in the above-ground stock more than offsets any decrease in the below-ground stock. Proof of the claim in the text is available in Henderson, Irons, Salant, and Thomas (1997).

[12] It is interesting to note that in this example most (136/158 = 86%) of the gain from expediting government sales arises because of our assumptions that gold has service uses and that governments do not loan out their holdings prior to selling. This source of gain does not arise in Salant and Henderson (1978) where we abstract from service uses of gold.

4. Quantitative analysis of alternative policies

In this section, we calibrate a simulation model consistent with our theoretical specification and use it to quantify the positive and normative effects of alternative government policies.

4.1. Calibration of the model

We assume that the real interest rate and the constant marginal cost of mining are stationary over time and that the demands of depletion users and service users have the following functional forms:

$$q_t = a\gamma_{q,t}P_t^{-\epsilon}, \quad A_t = b\gamma_{A,t}R_t^{-\rho} \tag{13}$$

where $\gamma_{q,t}$ and $\gamma_{A,t}$ are given by[13]

$$\gamma_{g,t} = 2 - y_q^t, \quad 0<\gamma_q<1, \quad \gamma_{A,t} = 2 - \gamma_A^t, \quad 0<\gamma_A<1. \tag{14}$$

Given these functional forms, our model has eleven parameters: T, i, c, $\bar{A}$, γ_q, γ_A, $\bar{H}$, ϵ, a, ρ, and b. For any admissible values of these eleven parameters, there exist unique solution paths for the four endogenous variables: p_t, q, R_t, and A_t.[14] The parameter values employed in our simulations and some data used in calibration are displayed in Table 1.

The values for 5 parameters (T, i, $\bar{A}$, γ_q, and γ_A) reported in Panel A are used in all simulations. We assume a horizon of 400 years (T=399), a horizon sufficiently long that our predictions for the first one hundred years are unaffected if the horizon is lengthened. Our choice for i (2.5%) is well within the range of estimates reported in the macroeconomics literature. Our choices for c and $\bar{A}$ are based on data from the gold market reported in the literature.[15] We assume that the demand shift parameters $\gamma_{q,t}$ and $\gamma_{A,t}$ increase monotonically over time according to the functional forms in Eq. (14). Under this assumption, model predictions are roughly consistent with two features of the data: constant or rising depletion during years of rising gold prices and increases in the service stock during the mining phase. We interpret $\gamma_{q,t}$ and $\gamma_{A,t}$ as population indexes and assume, for simplicity, that they are always equal. Given the values of γ_q and γ_A in panel A, $\gamma_{q,t}$ and $\gamma_{A,t}$ begin at a value of 1 and reach 93.5% of their asymptotic value of 2 after 50 years.

The values of the remaining 6 parameters ($\bar{H}$, c, ρ, ϵ, b, and a) are changed among simulations. As reported in Panel B, we choose a "reference set" of parameters, indicated by the subscript rs, and make the simplifying assumption that $\epsilon_{rs}=\rho_{rs}$. We check the sensitivity of our results to changes in $\bar{H}$ and c because there is considerable uncertainty about these two key parameters and to the relaxation of the simplifying assumption that $\epsilon_{rs}=\rho$. $\bar{H}_{rs}$ is among the highest estimates of $\bar{H}$ available in the literature. Nonetheless, we consider a value of $\bar{H}$ that is 25% higher than $\bar{H}_{rs}$ because estimates of the below-ground stock have been revised upward more frequently than downward in recent years. Technological change might cause c to fall, but the increased bargaining power of mine workers in South Africa and other gold producing countries might cause it to rise. We consider a value of c that is 20% lower than c_{rs}

[13] The inverse demand curve is, therefore, $P_t = \left(\frac{q_t}{a\gamma_{q,t}}\right)^{-1/\epsilon} = \left(\frac{a\gamma_{q,t}}{q_t}\right)^{1/\epsilon}$ where $\epsilon>0$. It has the properties in Eq. (2); in addition, $P(q_t,t) \le P_{(q_t)} = \left(\frac{2a}{q_t}\right)^{1/\epsilon}$ for $q_t>0$.

[14] Uniqueness of competitive equilibrium follows from two facts: (1) every competitive equilibrium solves the associated planning problem and (2) the associated planning problem for our model has a unique solution since inverse demand functions are downward sloping, and cost functions are assumed to be linear.

[15] See Appendix A for a discussion of this and other data choices.

Table 1
Values of parameters and data

Panel A: Parameters Used in All Simulations						
$T=399^{a}$					$\gamma_q=0.96$	
$i=0.025$					$\gamma_A=0.96$	
$\bar{A}=2468^{b}$						
Panel B: Parameters Changed Among Simulations						
Parameter set	Parameters					
	$\bar{H}$	c	ρ	ϵ	b	a
Reference set	2292^{b}	250^{c}	0.57	0.57	6.50×10^{10}	3.36×10^{8}
$\bar{H}>\bar{H}_{rs}$	2865^{b}	250^{c}	0.39	0.39	5.89×10^{9}	1.20×10^{8}
$c<c_{rs}$	2292^{b}	200^{c}	0.40	0.40	4.38×10^{10}	1.27×10^{8}
$\bar{H}>\bar{H}_{rs}$, $c<c_{rs}$	2865^{b}	200^{c}	0.29	0.29	4.60×10^{10}	6.73×10^{7}
$\rho<\rho_{rs}$	2292^{b}	250^{c}	0.35	1.64	4.35×10^{10}	1.74×10^{11}
Panel C: Data on Price and Depletion						
$\bar{P}=350^{c}$					$\bar{q}=12^{c}$ per year	
Panel D: Data on Government Gold Stocks						
$\bar{G}=1107^{c}$					U.S. Gold $=262^{c}$	

[a] Years.
[b] Millions of troy ounces.
[c] Dollars per troy ounce.

not because we think the $c<c_{rs}$ case is more likely but because it yields a more conservative estimate of the welfare gain from a government sale. For the same reason, we consider a set of parameter values in which $\rho<\rho_{rs}=\epsilon_{rs}<\epsilon$.

For each alternative set of parameters, the values for four demand parameters (the demand elasticities, ϵ and ρ, and the demand constants, a and b) are obtained by calibrating the model. Our method of calibration involves setting values for some of the endogenous variables and solving for the demand parameters that are consistent with those values using an iterative technique. In particular, for every trial pair of the elasticities, we set a so that $\bar{P}=P(\bar{q},0)$ and b so that $ic=R(\bar{A},0)$, where the values of $\bar{P}$, $\bar{q}$, i, c, and $\bar{A}$ are given in Table 1. The final values for the demand parameters are chosen to satisfy the terminal condition $A_{T+1}\approx 0$.[16] Therefore, by design, with each set of parameters our model 'predicts' an initial price of $P_0=\bar{P}$, initial depletion of $q_0=\bar{q}$, and a mining phase in progress with service stock equal to $A_0^{*}=\bar{A}$, that is, $ic=R(\bar{A},0)$.

4.2. Estimates of the effects of alternative policies

In this subsection we describe simulations of the effects of an unanticipated sale of all government gold at different points in time under each of the parameter sets displayed in Table 1 and the effects of an unanticipated immediate sale of only U.S. gold under the reference set.

[16] In each case, we arrived at the final values chosen for the elasticities ϵ and ρ starting from several different widely-spaced initial values.

4.2.1. The effects of selling government gold under the reference set

We begin by considering the effects of selling all government gold, $\bar{G}$ in Panel D of Table 1, in year t_S, $t_S=0, \ldots, T$ under the reference set of parameters. Since the sale of government gold is unanticipated, the paths of all the variables before t_S are the same as they would be if government gold were withheld forever.

The five panels of Fig. 3 show the effects on the gold market of two extreme government gold policies, no sale of any government gold (the solid lines) and an immediate sale of all government gold (the dotted lines). With an immediate sale, the price (top left) drops at once from \$350 to about \$275 per ounce and remains below the no sale path thereafter; as a result, depletion uses (top right) are higher in every year. With an immediate sale, the loan fee (middle left) is lower initially and in most years, and is never higher; the service stock (middle right) is higher initially and in most years, and is never lower. With no sale, mining (bottom right) occurs initially and declines slowly until year 29 when the mines are exhausted. In contrast, with an immediate sale, the mines shut down at once, reopen again in year 11 and are exhausted in year 54. With an immediate sale, cumulative depletion exceeds the sum of the initial service stock and the below-ground stock for the first time in year 368, so under the loan policy that is equivalent to an immediate sale and involves postponing government sales for as long as possible, the first sale of government gold takes place in that year.

4.2.2. Two breakdowns of the welfare gain

We provide two breakdowns of the estimated gain in total welfare from selling government gold earlier. The first breakdown is by type of market participant. The gain in total welfare is obtained by adding up the gains for each type of market participant. For depletion users and service users, the gains in a given year are the increases in the areas under their demand curves (net of any expenses required to purchase or rent the quantity chosen) while the total gains are the discounted sums of those increases in areas. For the government, service stock owners, and mine owners, the gains in a given year are the increases in revenues in that year (net of any cost of generating those revenues), while the total gains are the discounted sums of those increases in net revenues.

The second breakdown is by type of inefficiency reduced. We decompose the total welfare gain from a government sale in year t_S into two components, the gain arising from removing the production inefficiency and the gain arising from removing the use inefficiency. To isolate the former, we compute the largest reduction in discounted costs which a planner could achieve while replicating the depletion and above-ground stock paths of the "no sale" equilibrium if *beginning in year* t_S he could replace gold taken from below ground at high cost with gold taken from government vaults at no cost. The gain from removing the use inefficiency is then calculated as the difference between the total welfare gain from the sale at t_S and the amount attributed to the production inefficiency.

The upper left and right panels of Table 2 show the estimated effects on welfare from private uses for three comparisons of alternative government selling policies under the reference set of parameters. The first columns show how welfare changes with an immediate sale of all government gold versus no sale. Total welfare increases by \$340 billion. The largest share of the increase in welfare (87.5%) takes the form of government revenue in the first instance. Depletion users and service users gain, but private above-ground stock owners and mine owners lose. The breakdown by type of inefficiency reduced shows that 72.6% of the total welfare gain comes from eliminating the production inefficiency.

The second columns show how welfare changes with a sale of all gold in year 20 versus no sale. The gain from reducing the production inefficiency is smaller but represents about the same share (72.1%) of

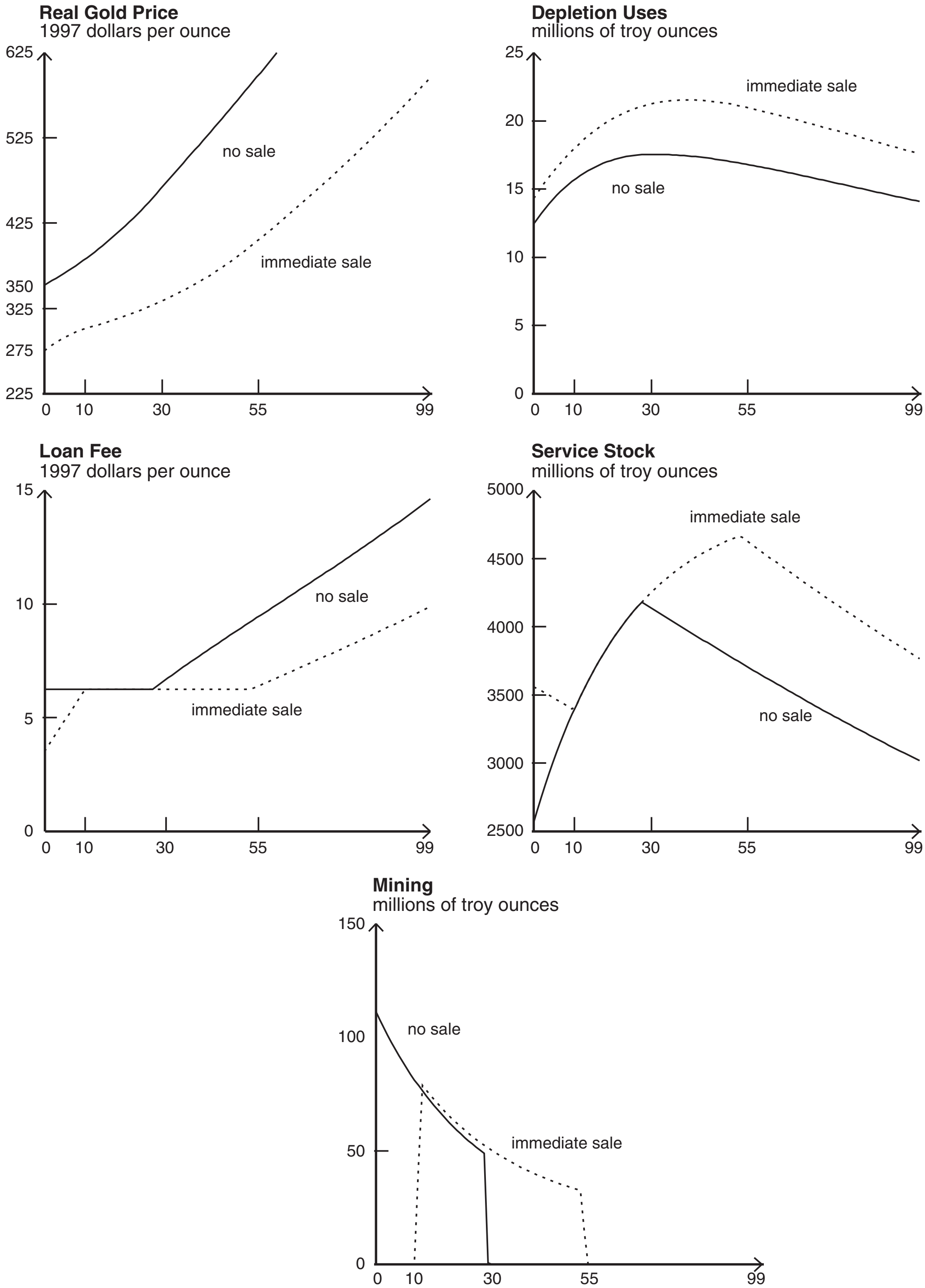

Fig. 3. Time paths: immediate sale compared to no sale.

Table 2
Estimated effects on economic welfare

	Immediate vs. No	20 vs. No	Immediate vs. 20	Immediate vs. No	20 vs. No	Immediate vs. 20
	Reference Case (billions of 1997 dollars)			Reference Case (% of total)		
Total	**340**	**234**	**105**	**100**	**100**	**100**
Government Revenue	297	193	104	87.5	82.5	98.7
Depletion Users	104	88	17	30.7	37.3	15.8
Service Users	260	272	−12	76.6	116.1	−11.3
Stock Owners	−189	−288	99	−55.5	−122.8	94.1
Mine Owners	−133	−30	−102	−39.2	−13.2	−97.3
Production Inefficiency	247	169	78	72.6	72.1	73.8
Use Inefficiencies	93	65	27	27.4	27.9	26.2
	$\bar{H}$ > Reference $\bar{H}$ (% change from reference case)			ρ < Reference ρ (% change from reference case)		
Total	**−1.6**	**−2.1**	**−0.5**	**−0.5**	**−1.6**	**1.7**
Government Revenue	−2.6	−3.3	−1.0	−0.7	−1.7	1.3
Depletion Users	10.8	15.3	0.8	−10.9	−14.6	−2.6
Service Users	9.5	14.6	−1.9	11.3	22.1	−12.7
Stock Owners	−6.3	−12.2	6.8	−2.3	−6.9	8.1
Mine Owners	−13.0	−16.5	−5.1	2.1	−0.5	7.7
Production Inefficiency	0.0	0.0	0.0	0.0	0.0	0.0
Use Inefficiencies	−1.6	−2.1	−0.5	−0.5	−1.6	1.7
	c < Reference c (% change from reference case)			$\bar{H}$ > Ref $\bar{H}$, c < Ref c (% change from reference case)		
Total	**−5.9**	**0.0**	**−19.2**	**−7.7**	**−2.3**	**−19.7**
Government Revenue	−10.3	−6.4	−18.9	−13.0	−9.9	−19.8
Depletion Users	21.5	27.6	7.9	39.9	53.8	9.0
Service Users	27.9	42.3	−4.1	36.1	54.8	−5.3
Stock Owners	−24.5	−56.5	46.6	−30.9	69.1	54.0
Mine Owners	−20.5	−7.0	−50.7	−39.9	31.9	−57.5
Production Inefficiency	−14.5	−14.4	−14.8	−14.5	−14.4	−14.7
Use Inefficiencies	8.6	14.4	−4.4	6.8	12.1	−5

the total. Some may find it implausible that governments would never sell their gold, so in the third columns we present the welfare effects of an immediate sale of all government gold versus a sale in year 20. Total welfare is $105 billion higher with an immediate sale. The gain from reducing the production inefficiency is smaller than in the other two comparisons but represents about the same share (73.8%) of the total.

Fig. 4 shows the estimated effects on welfare from private uses of the sale of all government gold in every year t, $t=0, \ldots, T$ versus no sale under the reference set of parameters. The gains in total welfare, government revenue, and depletion users' welfare and the losses in mine owners' welfare decline monotonically with delay. In contrast, the gains in the welfare of service users and the losses in welfare of service stock owners do not change monotonically with the date of the government sale. Given our

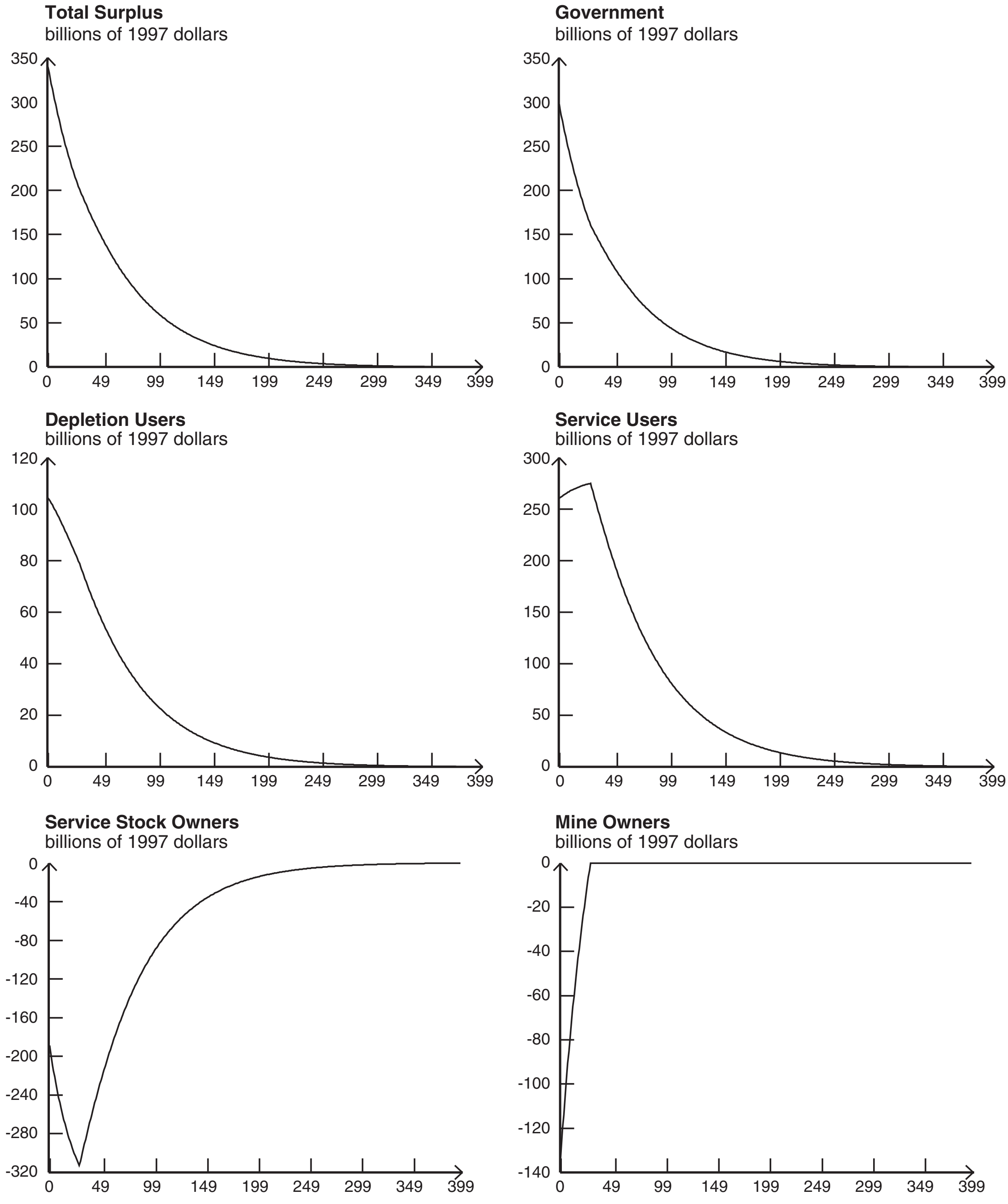

Fig. 4. Differences in economic welfare: welfare with sale of all government gold in a given year minus welfare with no sale.

assumption that the demand schedules shift outward over time because of population growth, postponing a government sale has both positive and negative effects on the welfare of the two types of agents if it occurs during the original mining phase. The postponement of a government sale reduces the present value of the capital loss per unit suffered by owners of the service stock. However, in the presence of population growth, a delay in the sale means that the capital loss *per unit* is imposed on larger holdings since $\{A_t^*\}$ is increasing.

4.2.3. Sensitivity analysis

Now we investigate the sensitivity of the simulated price and welfare effects of different government policies to changes in some key parameters. In particular, we compare these effects under each of four alternative parameter sets to their effects under the reference set. The comparison of welfare results is reported in the four panels at the bottom of Table 2. For each alternative parameter set, differences in welfare changes from those in the reference case for a given policy comparison are reported *as percentages of the total welfare change for that policy comparison under the reference set (top left row)*. To be concrete, consider the entries in the middle column for the $\rho<\rho_{\mathrm{rs}}$ parameter set (middle right). The Table says that, under this alternative parameter set, the change in total welfare resulting from an immediate sale rather than a sale in year 20 is smaller than the \$234 billion increase under the reference set by 1.6% of \$234 billion (\$3.74 billion). The change in government revenue is smaller than the \$193 billion increase under the reference set by 1.7% of \$234 billion (\$3.98 billion). Because we express each difference as a percent of the same number (\$234 billion), the sum of the percentage differences for the various market participants adds to -1.6% as does the sum for the production and the use inefficiencies.

An important part of the explanation of the difference in results between the reference set of parameters and each alternative set is the difference in demand parameters (ϵ, ρ, a, and b) required to match the data ($\bar{P}=P(\bar{q},\,0)$ and $ic=R(\bar{A},\,0)$) in the calibration process. We explain the difference in demand parameters for each alternative set.

Recall that under the policies involving government lending and postponement of government sales for as long as possible, the first sale of government gold takes place in the year in which cumulative depletion exceeds the sum of the initial above-ground and below-ground stocks for the first time under the assumption that there is an outright sale. As stated above, this year is year 368 for an outright immediate sale under the reference set. This year is year 358 or later for outright immediate sales under all the alternative parameter sets considered.

4.2.3.1. $\bar{H}>\bar{H}_{\mathrm{rs}}$. First, we compare the results for the reference set with those for the alternative set with $\bar{H}>\bar{H}_{\mathrm{rs}}$ by 25%. With no sale, for both sets the price starts at $\bar{P}$ and remains above it for most of the horizon. With the $\bar{H}>\bar{H}_{\mathrm{rs}}$ set, more gold must be depleted over the same horizon, so demand at prices above $\bar{P}$ must be higher, and, therefore, ϵ and a must be smaller than in the reference set as shown in Table 1. Since $\rho=\epsilon$ by assumption, ρ and b must also be smaller.

With an immediate sale, the price drop is greater (to \$267 rather than \$275) with the $\bar{H}>\bar{H}_{\mathrm{rs}}$ set as is plausible because depletion demand is more inelastic and the price path must rise more steeply in the early years because the loan fee is smaller given that ρ and b are smaller. The differences for total welfare (Table 2, middle left) and for the production inefficiency versus use inefficiency breakdowns are not very sensitive to the change in $\bar{H}$. The results for the market-participant breakdowns for the immediate sale versus no sale and for a sale in year 20 versus no sale are relatively easy to interpret.

Since the price falls by more with the $\bar{H}>\bar{H}_{\mathrm{rs}}$ set, the increases in government revenue are smaller and service stock owners' and mine owners' losses are larger. Although the inverse depletion demand and service demand schedules are steeper, the price and loan fee fall by enough more that depletion users' and service users' welfare gains are larger.

4.2.3.2. $\rho<\rho_{\mathrm{rs}}$. Next, we compare results for the reference set of parameters with those for the alternative set with $\rho<\rho_{\mathrm{rs}}$. If ρ is smaller, then b must be smaller if the ρ, b pair is to imply $ic=R(\bar{A}, 0)$. The same amount of gold must be depleted over the same horizon with both sets, so it makes sense that there should be less depletion in early years and more in later years with the $\rho<\rho_{\mathrm{rs}}$ set. If the inverse depletion demand function is to be consistent with these changes in the depletion path while continuing to satisfy $\bar{P}=P(\bar{q}, 0)$, ϵ and a must be larger in the $\rho<\rho_{\mathrm{rs}}$ set.

With an immediate sale the price drop is slightly larger (to \$273 rather than \$275) with the $\rho<\rho_{\mathrm{rs}}$ set as is plausible because even though depletion demand is more elastic the price path must rise more steeply in the early years because the loan fee is smaller given that ρ and b are smaller. Total welfare gains and production-inefficiency versus use-inefficiency breakdowns (Table 2, middle right) are not very sensitive to the change in ρ. The results for the market-participant breakdowns for the immediate sale versus no sale and for a sale in year 20 versus no sale comparisons are relatively easy to interpret. The changes in the differences in government revenue and mine owners' welfare are quite small. The changes in the welfare differences for depletions users, service stock owners, and service users are somewhat larger. Depletion users gain less because even though the depletion demand curve is flatter and the price drops in the first few years are greater, the price drops in later years are smaller. Service users gain more, and service stock owners' losses are greater because even though the inverse service demand curve is steeper, the loan fee falls by enough more in enough of the early years that count most.

4.2.3.3. $c<c_{\mathrm{rs}}$. Now, we compare results for the reference set of parameters with those for the alternative set with $c<c_{\mathrm{rs}}$. The change in results is considerably greater for this alternative set than for the alternative sets with $\bar{H}>\bar{H}_{\mathrm{rs}}$ and with $\rho<\rho_{\mathrm{rs}}$ Since $ic<ic_{\mathrm{rs}}$, the price path must rise faster with the $c<c_{\mathrm{rs}}$ set as long as the mines are open and with our parameters continues to rise faster for many years thereafter. The same amount of gold must be depleted with both parameter sets, so demand at prices above $\bar{P}$ must be higher and ϵ and a must both be smaller with the $c<c_{\mathrm{rs}}$ set as shown in Table 1. Since $\rho=\epsilon$ by assumption, ρ and b must also be smaller.

With an immediate sale, the price drop is considerably greater (to \$243 rather than \$275) with the $c<c_{\mathrm{rs}}$ set as is plausible because depletion demand is more inelastic and the price path must rise more steeply in the early years because the loan fee is lower given that ρ and b are smaller. The total welfare gains (Table 2, bottom left) for an immediate sale versus either of the other two alternatives are noticeably reduced with a lower c. As might be expected, the gain from reducing or eliminating the production inefficiency is much less with the $c<c_{\mathrm{rs}}$ set. The gain from reducing or eliminating use inefficiencies is greater for the two comparisons involving no sale. The results for the market-participant breakdowns for the immediate sale versus no sale and for a sale in year 20 versus no sale are relatively easy to interpret. The price falls by more with the $c<c_{\mathrm{rs}}$ set, so the increase in government revenue is smaller, the gains of depletion users are larger, and the losses of stock owners and mine owners are larger. The path of the loan fee is enough lower in the first part of the horizon that counts most that the gains of service users are greater with the $c<c_{\mathrm{rs}}$ set.

4.2.3.4. $\bar{H} > \bar{H}_{rs}$ and $c < c_{rs}$. Finally, we compare results for the reference set of parameters with those for the alternative set with both $\bar{H} > \bar{H}_{rs}$ and $c < c_{rs}$. The total welfare gains (Table 2, bottom right panel) for an immediate sale versus either of the other two alternatives are lower with this alternative set than with any other parameter set considered, as is plausible because both raising $\bar{H}$ and lowering c tend to reduce the total welfare gain for reasons given in the discussion of the $\bar{H} > \bar{H}_{rs}$ and $c < c_{rs}$ sets. The results for the case with both $\bar{H} > \bar{H}_{rs}$ and $c < c_{rs}$ are roughly the same as the sum of the results for the case with only $\bar{H} > \bar{H}_{rs}$ and those for the case with only $c < c_{rs}$. That is, for the range of parameters we consider the nonlinearities in the model are not very important.

4.2.4. The incentive for a government to be the first to sell

If one government unilaterally sells its gold before other governments can sell or even announce a sale of their reserves, that government earns a larger discounted revenue than it would if other governments sold at the same time. For example, Fig. 5 shows what happens if the U.S. unilaterally and immediately sells all its gold. The gold price drops only \$19 per ounce–from \$350 to \$331–instead of dropping \$75 per ounce–from \$350 to \$275–as it does if all governments sell at the same time. Since the U.S. gold stock is 262 million troy ounces (Table 1, Panel D)–24% of all government gold–U.S. revenue is about \$87 billion when the U.S. manages to sell before announced sales of the other countries but only about \$72 billion when all gold is sold, a difference of about \$15 billion or about 17%.

The extra benefit the U.S. enjoys from its preemptive sale comes not at the expense of the other governments but instead at the expense of the private agents who purchase U.S. gold. If all of the other governments hold unanticipated sales on the day following the U.S. sale, the price drops to \$275 then. The other governments are no worse off than if all sales had occurred together on the previous day, but private purchasers of U.S. gold each sustain a capital loss of \$56 per ounce the moment the other governments announce their sales.

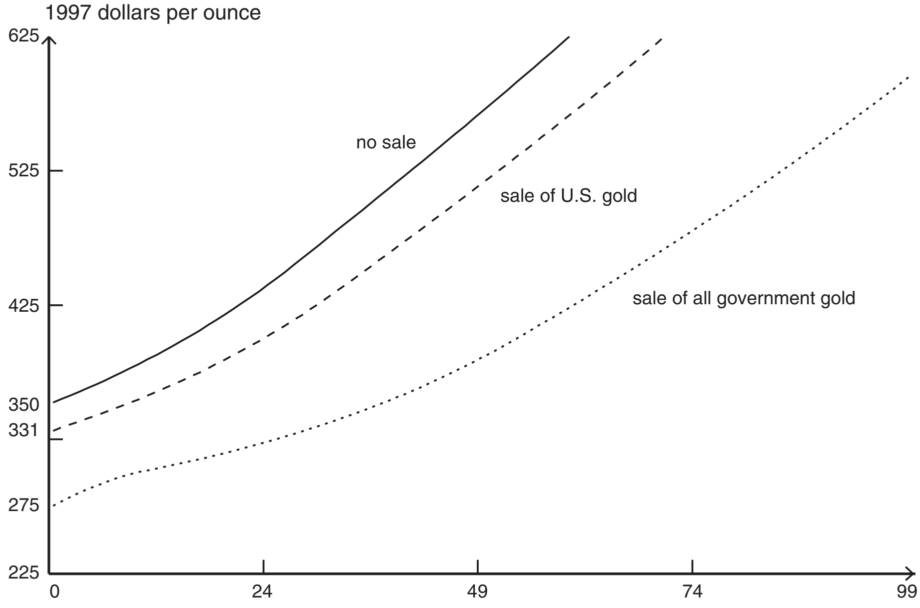

Fig. 5. Predicted paths for real gold price under alternative assumptions about government gold sales.

If the announcement of a U.S. sale *raises* the market's probability assessment that the other governments will sell soon, the U.S. gain from a preemptive sale is reduced, to zero in the limiting case. The increased probability of further sales would cause the price to fall below $331 at the time of the U.S. sale, to $275 in the limiting case in which the market regards sales of all gold by other governments as certain to follow the U.S. sale.

In September 1999, European governments announced an agreement to limit their gold sales and lending for the next five years.[17] Their announcement was followed by a significant rise in the gold price. This sequence of events is consistent with the view that over the several years preceding actual or announced sales by Belgium, the Netherlands, Switzerland, and the United Kingdom and increased gold lending had caused the market to revise upward its probability assessment of further sales and lending and that the agreement caused the market to revise downward its assessment.

5. Conclusions

In our previous paper on government gold sales (Salant & Henderson, 1978), we showed that the risk of sales not only depressed the gold price but led it to rise by more than the rate of interest. At the outset of that earlier work (p. 630), we explicitly abstracted from two prominent characteristics of the gold market: the service flow generated by gold jewelry (and art objects) and the high extraction cost of gold. We did so since neither characteristic was responsible for the phenomenon we sought to illuminate.

In this paper, our goal is different. We quantify the huge welfare losses caused by the continued postponement of government sales and determine the distribution of those losses among groups of market participants. Delayed government sales (1) deprive service users of available gold (one of two use inefficiencies) and (2) force the private sector to rely on a source of supply with higher extraction costs (the production inefficiency). Since service flows and extraction costs are central to the efficiency loss, ignoring them is no longer possible. To make our analysis accessible, we simplify elsewhere. Since the private market had, until the last decade, seemingly been lulled into giving little weight to the possibility of substantial government sales, we ignore the uncertainty central to our previous paper—taking pains to calibrate our model using data from this earlier period when the risk of government sales seemed small.[18]

Welfare from private uses is maximized by making all the gold currently held by governments immediately available to private agents who value its depletion and service uses. As our simulation results show, postponing government gold sales another 20 years fritters away $105 billion or roughly one-third of the potential gain. The losses from delay are not evenly distributed among the various

[17] For the texts of the September 1999 Washington Agreement and the more recent September 2004 agreement, see World Gold Council (2005).

[18] Readers interested in understanding how the two analyses fit together are referred to Appendix B. Other assumptions could also usefully be relaxed if realism was preferred to simplicity. It would probably be relatively easy to relax our assumptions that every deposit of gold has the same marginal cost of extraction and that this common marginal cost of extraction is invariant to the amount extracted in a year. It would also probably be relatively easy to relax our assumption that once a unit of gold has been taken from belowground it can be depleted or added to the service stock without incurring any 'adjustment' costs. A crucial building block for an extension to the case of adjustment costs is provided by Flood and Garber (1984) who posit rising marginal costs of adjustment for transforming units of aboveground gold into units of service stock.

groups involved. Governments lose by delaying sales. The simulations suggest that they lose \$104 billion by delaying their sales another 20 years. The remaining \$1 billion are lost directly by the private market. In the simulations, depletion users always gain and mine owners always lose. However, service users and private above-ground stock owners may gain or lose under our assumption that population is growing.

In our analysis, government gold ownership does not have any direct benefits. There is a view that it does for at least three reasons: (1) gold reserves would be necessary if gold ever again played an important role in international monetary arrangements; (2) gold is an important part of a “war chest” for times of international crisis; and (3) gold is irreplaceable in certain strategic uses. To the extent that there is some truth to this view, it suggests that governments might prefer gold loans to gold sales.

Any benefits of government ownership of gold are lost at once with an immediate sale of all government gold. However, any such benefits are lost much later with the equivalent policy of immediate lending of all remaining government gold and gradual sales of government gold beginning after a delay of some years. It is clear that if governments lent out all their gold but wanted to keep open the possibility of using it in a crisis, they would have to structure loan contracts so that they could get their gold back immediately in a crisis. Difficulties might arise in incorporating the necessary provisions into loan contracts. In addition, costs would be incurred in administering gold loans and gradual gold sales. However, these difficulties and costs might well be small enough that it would still be worthwhile for governments to make most or all of their gold available for private uses immediately through gold loans.

In any event, the importance of gold as a possible future reserve asset, as part of a war chest, and as a strategic material has clearly diminished in recent years and will, in all likelihood, continue to diminish. Our analysis reveals just how large the benefits from government gold ownership would have to be for the policy of massive government stockpiling to be justified.

Appendix A

In this appendix we explain how some of the values in Table 1 are obtained. We represent the name “Gold Field Mineral Services Ltd.” as “GFMS”.

A weighted average of cash costs in the different producing countries for 1992 as reported in, GFMS (1993) is \$300 per troy ounce. However, we chose \$250 per troy ounce as our reference value for c, partly because of the suggestions of several gold market analysts. The value for $\bar{A}$ is constructed using data from GFMS (1993) and GFMS (1996). Fig. 30 on p.32 of GFMS (1993) contains an estimate of total production through the end of 1992 (121,500 metric tons). Following Sehnke (1997), we assume that the total stock still available at the end of 1992 was only 0.85 times this amount. To obtain our estimate of the private stock at the end of 1992, we deduct the estimate of government stocks as of the end of 1991 (35,556 metric tons) in Table 1on p. 11 of O’Callaghan (1993), which was the only estimate we found for a time near the end of 1992 which included the holdings of the government of the former Soviet Union. In the text of p. 32, GFMS (1993) provides an estimate of the amount of gold in jewelry at the end of 1992 (“around” 50,000 metric tons). We adopt this estimate and assume that the gold in medals, coins, bars, and physical investment at the end of 1992 was equal to our estimate of the total private stock minus the amount in jewelry. To obtain our estimate of $\bar{A}$ at the end of 1995, we add estimates of the increases for the years 1993, 1994, and 1995 of gold in jewelry from Table 5 on p. 45, medals from Table 9 on p. 57, coins from Table 10 on p. 58, and bars from Table 11 on p. 63 all in GFMS (1996) and of gold in physical investment from the text on p. 62 in GFMS (1994), on p. 61 in GFMS (1995), and on p. 61 in GFMS (1996) and convert the result to troy ounces using the conversion factor 32,150.7 troy ounces permetric ton.

Estimating the path of world population for many years in the future is a very difficult task. Several methods of estimation are considered in Cohen (1995). Our assumption that $\gamma_q=\gamma_A=0.96$, which implies that world population levels off at twice its current value by about 2050, is consistent with one of these methods.

Our estimate is the sum of the individual items (71,300 metric tons), not the rounded total, for the end of 1996 in the column labeled "Reserve Base" in Sehnke (1996) times the conversion factor of 32,150.7 troy ounces per metric ton. The comparable figure for the end of 1995 (60,800 metric tons) in Sehnke (1996) is 15% smaller primarily because of a change in the estimate for South Africa.

The value of $\bar{P}$ is a round number close to an average of market prices for the first half of 1997. The value of $\bar{q}$ is an average of depletion uses for the years 1993, 1994, and 1995 as reported in GFMS (1996).

The values for $\bar{G}$ and U.S. gold are the holdings of all governments and international institutions and the holdings of the U.S. government as of the end of November 1996 as reported in International Monetary Fund (1996).

Appendix B

In this appendix, we outline a modified version of the model. In this modified version, private agents anticipate each period that a sale of $\bar{G}$ will occur with probability α given that no sale has occurred already as in Salant and Henderson (1978). As a consequence, the two recursions are different, but they reduce to the recursions of the text when $\alpha=0$.

The loan recursion, the counterpart to Eq. (9), becomes

$$P_t - I[(1-\alpha)P_{t+1} + \alpha f(A_{t+1}+\bar{G}, H_{t+1}, t+1)] = IR(A_{t+1}, t+1).$$

This recursion must hold at every year $t=0,\ldots,T-1$. $f(x_t, y_t, t)$ is the price immediately following a government sale at t if the augmented above-ground stock is x_t and the below-ground stock is y_t. It could be calculated and stored numerically by simulating our model from alternative initial conditions, beginning in year $t+1$. For someone to hold gold above ground, he would have to earn as much selling one unit today and banking the proceeds as he would by renting out that unit for the year and then receiving the probability-weighted average of (1) P_{t+1} in the absence of a government sale or (2) $f(A_{t+1}+\bar{G}, H_{t+1}, t+1)$ following a government sale at $t+1$.

The mining recursion, the counterpart to Eq. (12), becomes

$$I[(1-\alpha)(P_{t+1}-c) + \alpha\kappa(A_{t+1}+\bar{G}, H_{t+1}, t+1)] = P_t - c,$$

which, after simplification, can be written in more familiar form as

$$P_t = I[(1-\alpha)P_{t+1} + \alpha\kappa(A_{t+1}+\bar{G}, H_{t+1}, t+1) + c(\alpha+i)].$$

This recursion holds in the interior of the mining phase. Prior to that phase, the left-hand side will be larger; subsequent to it, the left-hand side will be smaller. $\kappa(x_t, y_t, t)$ is the shadow price immediately following the government sale at $t+1$ if the augmented above-ground stock is x_t and the below-ground stock is y_t. This function could be calculated and stored numerically by simulating our model from alternative initial conditions beginning in year $t+1$. For someone to extract at t he would have to earn as much extracting and selling one unit at t as he would expect to earn (1) extracting and selling at $t+1$ in the absence of a government sale or (2) extracting *after the resumption of mining* following a government sale at $t+1$.

All of the other equations are the same in the modified model. In particular, the initial conditions (7) and (10), terminal conditions (8) and (11), and transition equations ($H_{t+1}=H_t-h_t$ and $A_{t+1}=A_t+h_t-q(P_t)$) are the same.

For sufficiently small α, the results would approximate the results of the simulations we have reported. For larger values of α, the *qualitative* conclusions would be the same but the *quantitative* conclusions would be different. The higher α, the smaller the gain in expected total welfare that would result from making government gold available. On the other hand, if a government sale is anticipated to occur with high probability but does not occur, there is a welfare *loss*.

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