The Default Specification Principle

Armin Nikkhah Shirazi
University of Michigan, Ann Arbor
armin@umich.edu

March 8th, 2018

APS March Meeting, Los Angeles

What Does It Say and What Does It Mean?

- *What it says*: **"The absence of an explicit specification entails all possible default specification outputs"**
- *What it means*: Roughly, it means that the failure to explicitly specify something entails that of any of the outputs which could possibly obtain as a consequence of carrying out that specification, all are available as "live possibilities"

The principle can be interpreted either ontically or epistemically

Why is This Useful?

Although in a certain sense tautological, its value lies in that its formal expression requires the formulation of a novel but simple framework, **default specification theory**, which

- when applied against a background of *space*, leads to the the concept of probability
- when applied against a background of *space and time*, leads to a generalization of the Born Rule

I will use the language of *category theory* to express the framework.

Why Use Categorical Language?

The principle

- Treats *the absence of something* as if it were something
- is *modal* (i.e. it distinguishes between explicit and default specifications, each of which is distinct from no [i.e. the absence of) specifications]

First order logic (and therefore, much of contemporary mathematics) is inadequate to represent the principle formally, but category theory can handle it, as long as the definitions for categories are met (Objects, arrows, identities, associativity etc.)

Default Specification Theory: Definitions I

Category of Explicit Sets *ExSet* (easy to define: subcategory of *Set*) has two objects:

- The set of explicit specifications
- The set of explicit specification outputs

Usually, at least one explicit specification output is mapped to each explicit specification so

- arrows from outputs to specifications are *surjections*
- We are, however, more interested in the "inverse":
  Consider inverse surjection in terms of a *fiber* of explicit specification outputs on each element of the set of explicit specifications: ***The Actuality fiber***

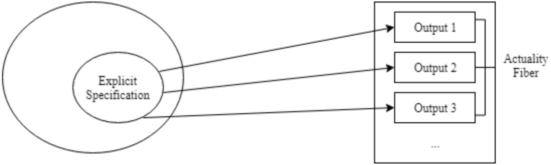


Figure: A fiber of outputs *as actualities* on an Explicit Specification

Default Specification Theory: Definitions II

Category of Default objects *Default* (in simplest case, also subcategory of Sets). Also has two objects:

- The set of *Absences* of explicit specifications
- The set of default specification outputs

Relationship between them parallel to *ExSet* (though default specification output set can have additional structure), so again:

- arrows from outputs to absences of specifications are *surjections*
- Consider the inverse image, that is, the *fiber* of default specification outputs on each absence of an explicit specification: ***The Actualizability fiber*** (Actualizable=Capable of becoming actual)

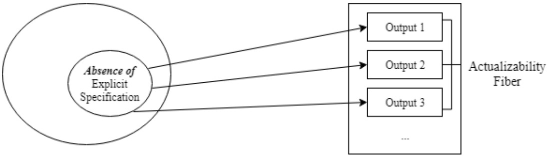


Figure: A fiber of outputs *as actualizabilities* on *the absence* of an Explicit Specification. **This is just the default specification principle.**

Default Specification Theory: The Bottom Line

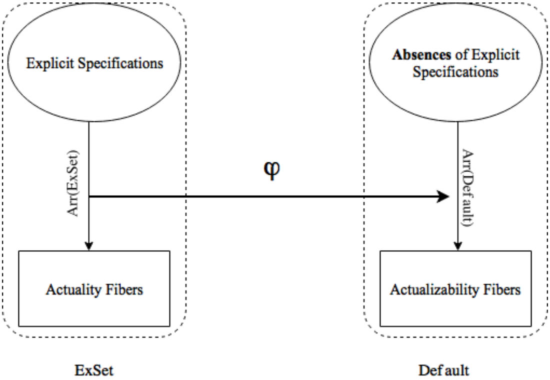
- Definitions give rise to arrow categories: $Arr(ExSet)$ and $Arr(Default)$.
- At bottom, default specification theory is about relating the arrow categories via a functor φ:

$$Arr(ExSet) \xrightarrow{\varphi} Arr(Default)$$

- Framed differently, ***default specification theory is about the relationship between morphisms involving actuality fibers and those involving actualizability fibers.***
- The actuality/actualizability distinction is *independent* of the ontic/epistemic distinction

Default Specification Theory: Structure I

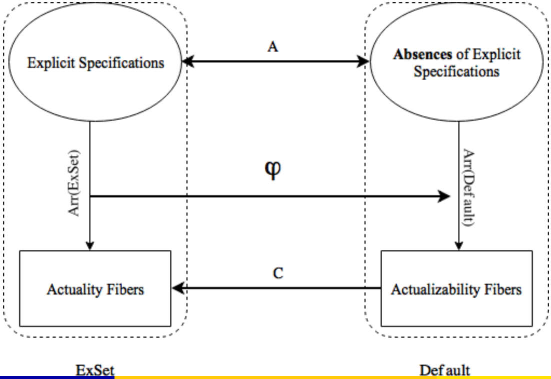
Mixing internal and external diagrams makes this explicit:



Default Specification Theory: Structure II

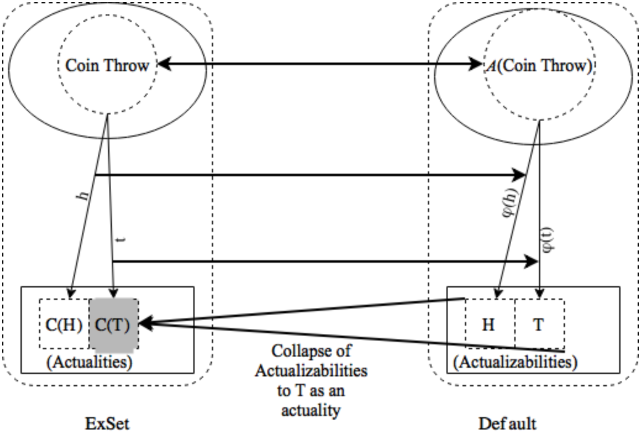
Structure permits definition of two functors between *ExSet* and *Default*:

- The *Absence Functor A*, an isomorphism which maps each explicit specification to its absence
- The *Collapse Functor C*, (usually) an epimorphism which maps a set of default specifications to a single explicit specification. **It models the transformation of a set of non-actual possibilities to a single actuality.**



Example I: Coin Throw

Can be interpreted ontically or epistemically:



- Ontic: Absence represents no throw event yet, collapse represents throw event
- Epistemic: Absence represents ignorance, collapse represents updating of knowledge

Connection to Probability Theory

Actualizability fibers have the right conceptual properties to be sample spaces! It means we can *mathematically* distinguish between the *concept* of probability and non-probabilistic unit measures by adding a *zeroth axiom* to the axioms of Probability.

Let $\Omega = \bigcup_{i=1}^{N} E_i$ be a set where N is either finite or countably infinite, $\mathcal{A} \subseteq \mathcal{P}(\Omega)$ a set of its mutually exclusive subsets E_i, and call the pair $(\Omega, \mathcal{A})$ a measurable space. A real-valued function $P : \mathcal{A} \to \mathbb{R}$ satisfying

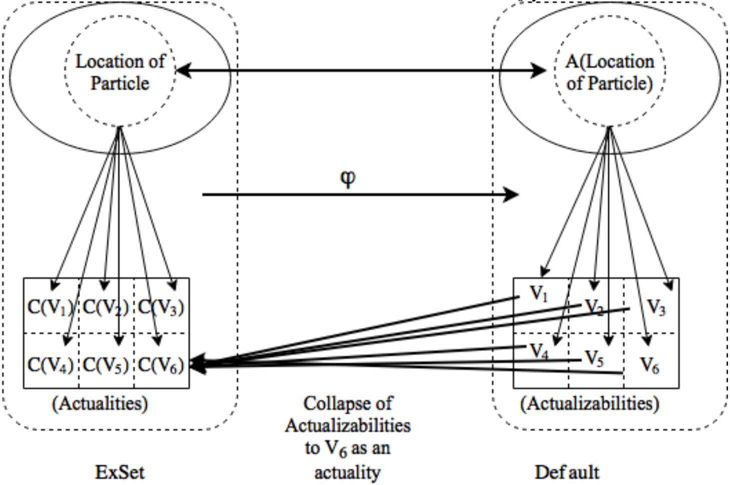
- Axiom 0: Ω is an actualizability fiber
- Axiom 1: $0 \leq P(E_i) \leq 1$
- Axiom 2: $P(\Omega) = 1$
- Axiom 3: $P \bigcup_{i=1}^{N} E_i = \sum_{i=1}^{N} P(E_i)$

is called a *probability*.

Notice: Axiom 0 is false $\Leftrightarrow$ P is a *non-probabilistic* unit measure (e.g. unit length, mass etc.) because then Ω does not represent a set of unactualized potentialities.

Example II: Particle in a Box (Epistemic)

Particle is somewhere in box, but ignorant of location. Partition box volume V into elements V_i to obtain the probability $P(V_i) = \int_{V_i} f dV$ where $f = f(V)$ is the probability density function. In the limit $V_i \rightarrow 0$ this becomes $P(\mathbf{x}) = \int_V f(\mathbf{x}) dx^3$



Example III: Particle in the Box (Ontic)

In example II, particle existed somewhere in box prior to observation, so collapse was purely epistemic. Now assume that particle really does not have a position in space prior to being observed (except that it will be found inside the box if we try to observe it) **and also that it does not have a position in time** prior to being observed.

We must apply the default specification principle to *space and time*.

- *To space*: The absence of a position in space entails all possible positions in space *as actualizabilities*. Can use this to define a "probability density" term over space, just as in non-quantum case. i.e. $P(V_i) = \int_{V_i} f(V)dV$
- *To time:* This involves *two distinct applications*:
  - The absence of a position in time entails all possible positions in time *as actualizabilities*. The object's literal timelessness (in spacetime) entails that its past and future are already *determined* as actualizabilities.
  - The absence of a position in time implies *the absence of an association with a temporal directionality*. By the principle, this entails an association with *both the forward and the backward time directions* as actualizabilities

Example III: Particle in the Box (Ontic) Continued

Applying the principle to both space and to time yields something like a “probability density" *for each time direction*: $f^+(V,+t)$ and $f^-(V,-t)$ (compare to just $f(V)$ for probability density). The probability of finding the particle in some volume element V_i is then given by *combining them*:

$$P(V_i) = \int_{V_i} f^-(V,-t) \circ f^+(V,+t)dV \tag{1}$$

where $\circ$ is the operation that turns the integrand into a probability density (i.e a term independent of time direction). In the limit $V_i \to 0$ this becomes

$$P(\mathbf{x}) = \int_{V} f^-(\mathbf{x},-t) \circ f^+(\mathbf{x},+t)dx^3 \tag{2}$$

This is a generalization of the Born rule!

Connection to Born Rule

To reduce (2) to the Born rule, we need to additionally assume that the functions are *symmetric to each other under time reversal and periodic in time with the same period*:

$$f^{\pm}(V, \pm t) = f^{\pm}(V, \pm t \pm T) \tag{3}$$

Then we can expand in a Fourier series:

$$f^{+}(\mathbf{x}, +t) = \sum^{\infty} c_n^{*}(\mathbf{x}) e^{+i\frac{n}{T}t} \equiv \Psi^{*}(\mathbf{x}, t) \tag{4}$$

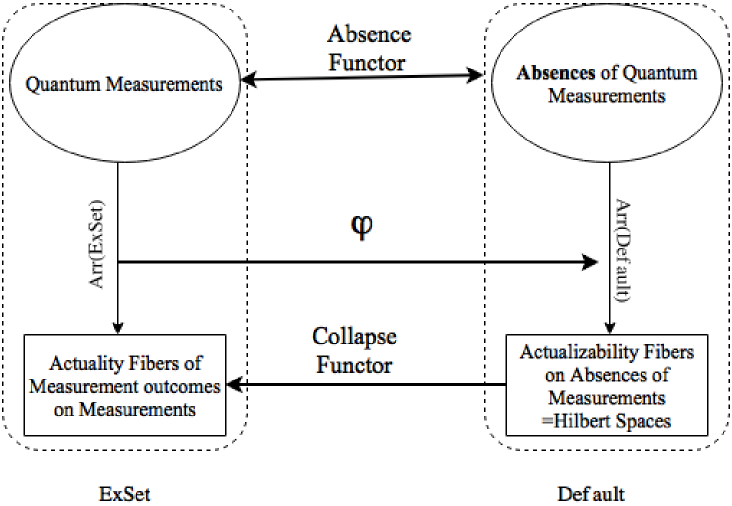
$$f^{-}(\mathbf{x}, -t) = \sum^{\infty} c_n(\mathbf{x}) e^{-i\frac{n}{T}t} \equiv \Psi(\mathbf{x}, t) \tag{5}$$

Because the directionality of time is given by the sign of the time term, and the time term is in an exponent, summing over both directions in time implies that the coefficients associated with each must be multiplied. Hence, $\circ$ *is the multiplication operation* and we have

$$P(\mathbf{x}) = \int_V \Psi^{*}(\mathbf{x}, t)\Psi(\mathbf{x}, t) dx^3 \tag{6}$$

Default Specification and Hilbert Space

This implies that the Hilbert space is a set of actualizability fibers:



Axiomatically Incorporating Default Specification

This is straightforward except for one thing [1]:

- Axiom 0: The states of quantum systems are represented by “actualizability fibers” Ψ which are elements of $\mathcal{H}$, a complex Hilbert space
- Axiom 1: Observables are represented by linear Hermitian operators acting on the elements in $\mathcal{H}$
- Axiom 2: The Measurement of the property of a state is represented by the collapse functor, which maps all *summands* of an “actualizability fiber" into the set of actuality fibers to a state which is isomorphic to an eigenstate in $\mathcal{H}$
- Axiom 3: The Probability of finding a quantum system in a state isomorphic to a given eigenstate upon a measurement is given by the Born Rule
- Axiom 4: The time evolution of a state under the absence of a measurement is unitary and given by the Hamiltonian

[1] To make this work, co-membership in an actualizability fiber must be replaced by *summation*, and this needs to be mathematically justified.

Deeper Challenges

Every discovery is bound to raise new and deeper questions. Most significant discovery here is that the Born Rule can be conceptually understood (if not rigorously derived) by assuming that quantum systems not only lack definite positions in space, but also definite positions *in time* prior to a measurement. This is more obvious under the path integral approach:*The absence of a spacetime history entails all possible spacetime histories as actualizabilities.*

If that is so, then in what sense can quantum systems be said to *exist* prior to a measurement? **We cannot even begin to answer this question until we have a physics-based criterion for existence.** Defining one in a consistent manner is, in my view, one of the most important and urgent theoretical challenges in fundamental physics.

Thank You!