Incumbent Repositioning with Decision Biases

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Research Summary: Incumbent firms often reposition themselves in response to competitive entrants, but when doing so the firm incurs repositioning costs. To model strategic interactions between incumbents and entrants this article proposes an approach that incorporates both repositioning costs and associated decision biases, which have been identified in the economics and strategy literatures as critical aspects of strategic change, but have been largely ignored in game-theoretic treatments at the strategy level. Using formal models, we analytically characterize the impacts of repositioning costs and biases on firms’ equilibrium strategies and profits. Including these costs and biases changes the nature of strategic dynamics as well as introduces new implications for strategic choice.

Managerial Summary: In a baseline setting, where both an entrant and an incumbent are unbiased, the incumbent’s repositioning ability can benefit the entrant, but hurt the incumbent. As a result, the market leader’s (incumbent’s) superior repositioning ability does not necessarily create an advantage for the leader relative to the entrant. This is true even if the entrant is biased in estimating the incumbent’s repositioning ability and the incumbent is sophisticated (namely, aware of the entrant’s bias and having a correct assessment of it). Indeed, when an entrant is biased, this unequivocally reduces its own performance, but this bias can hurt the incumbent more, thus enhancing the entrant’s relative advantage. In a similar vein, when an incumbent is biased in its estimation of the entrant, this can actually help the incumbent’s relative advantage. Perhaps more interestingly, although entrant bias and incumbent bias always harm the entrant and incumbent, respectively, both the entrant and the incumbent can earn more than in our baseline setting where both firms are unbiased. Furthermore, as opposed to the case where the incumbent is unaware of entrant bias, the incumbent is not necessarily better off being less biased, i.e., aware, but with an inaccurate assessment, of entrant bias. Interestingly,
both the incumbent and the entrant can be harmed by the incumbent’s reduced bias.

*Keywords*: repositioning costs, overestimation, underestimation, behavioral strategy, awareness.

**Introduction**

Entrants have repeatedly been identified as one important determinant of the market dynamic, especially the profitability of incumbents (Bain, 1956; Kumar and Sudharshan, 1988; Bresnahan and Reiss, 1991; Simon, 2005; Berry and Reiss, 2007; McCann and Vroom, 2010). Incumbents can be affected, for instance, when entrants attempt to penetrate the market by reducing prices, which in effect intensifies competition among firms (Rumelt and Teece, 1994; Chen, 1996; Simon, 2005; Besanko et al., 2009; McCann and Vroom, 2010). To avoid confrontation that could lead to destructive price wars, incumbent firms often reposition their products or brands, especially in industries with low barriers to entry (Trout and Ries, 1982; Smiley, 1988; Carpenter, 1989; Hauser and Shugan, 2008; Ellickson et al., 2012). One well-known example of repositioning is Johnson & Johnson’s Tylenol brand of analgesics. Tylenol once dominated the over-the-counter market for pain relief by establishing the drug as effective with few side effects. After its competitor Advil entered the market in 1984, Tylenol revised its marketing to emphasize Tylenol’s gentleness (Hauser and Shugan, 2008). Another example is the shift in pricing format of US local retailers in response to the entry of Walmart in the 1990s (Ellickson et al., 2012).

While repositioning can be advantageous, it may come at a cost because past strategic decisions, which often require prior commitments (Ghemawat 1991), may need to be changed. The costs associated with this repositioning (“repositioning costs”) typically include investments to overcome within-firm managerial resistance to change, to rework channel relationships, and to educate (or advertise to) consumers about the new positioning (Menon and Yao, 2017). Repositioning costs have substantial implications for the competitive interplay between firms (Ellickson et al., 2012; Wang and Shaver, 2014, 2016). Consider the case of Mobileye (Yoffie, 2014), a technology leader in vision technologies for advanced driver assistance systems (ADAS), which was highlighted in Menon and Yao (2017, page 1954):

Rather than selling its technologies directly to original equipment manufacturers (OEMs), Mobileye initially partnered with Tier 1 automotive suppliers such as TRW and Autoliv who, in turn, sold the OEMs integrated ADAS using Mobileye and non-Mobileye vision technologies. Mobileye subsequently instituted an exclusivity policy under which it would only work with Tier 1 suppliers who were not developing (or were no longer developing) non-Mobileye
ADAS. While this policy change resulted in the loss of some partners (e.g., Autoliv), it led
the remaining partners (e.g., TRW) to emphasize the development of complementary parts
(i.e., non-camera technology) for a Mobileye-centered ADAS system while deemphasizing de-
velopment of substitutes for Mobileye’s vision technology. The success of automotive vision
technologies depends on the technology’s ability to accurately identify objects under varied
driving conditions and accurate identification requires an object identification database that
improves with extensive in-field use of the technology. Over time, then, the cost to a Mobil-
eye current partner of repositioning itself to be a direct competitor of Mobileye is increasing,
and, hence, Mobileye’s policy change arguably benefits Mobileye by reducing the field of
potential vision-technology competitors despite exploding ADAS demand.

Despite the importance of repositioning costs, the implications of this construct have received limited
attention in studies of the competitive interplay between incumbents and entrants (Menon and Yao,
2017).

In an attempt to address this void, this paper explores how repositioning costs affect firms’ decisions
and the associated profits in a competitive entry environment. To this end, we first develop a base-
line model that builds on the pioneering work by Ghemawat (1991) on firm’s commitment to making
irreversible capacity decisions, but extends it to the positioning context with differentiated firms. The
model consists of one incumbent (she) and one entrant (he), where the incumbent is the market leader
and initially a monopoly. Prior to the entry of the new firm, the incumbent has the best possible market
position; however, after arrival of the entrant, the incumbent can adjust her position in order to differen-
tiate herself from him. The repositioning costs of this adjustment depend on the incumbent’s ability to
change her activities, which we term “repositioning ability.” When a firm has a strong repositioning abil-
ity, repositioning costs will be lower, for a given degree of repositioning. This ability may be related to a
firm’s possession of advanced machinery or technologies that facilitate the reconfiguration of its business
processes (Eisenhardt and Martin, 2000). The repositioning ability of an incumbent may be low for a
variety of reasons such as existing commitments. Examples here include Apple’s iPhone’s commitment
to use its own operating system and distribution channel (Ghemawat, 1991; Hill and Rothaermel, 2003;
Pacheco-De-Almeida et al., 2008), newspaper businesses unable to reposition when Craigslist enters the
local market (Seamans and Zhu, 2017), and Loblaw Companies Limited, a major grocery chain in Cana-
da, that built large superstores before Walmart entered the market and could not efficiently switch to a
small store format (Besanko et al., 2009).

Repositioning costs are an important influence on the competitive interactions between incumbents and entrants. In the case of an exogenous entrant position, the incumbent’s profit always increases when she possess a superior repositioning ability, which allows her to efficiently differentiate herself from the entrant. When the entrant position is endogenous, however, the incumbent’s superior repositioning ability also reduces the need for the entrant to differentiate, thus increasing competitive pressure on the incumbent and potentially reducing incumbent profit. These different outcomes result in two key implications. First, the entrant’s equilibrium profit can increase rather than decrease in relation to the incumbent’s repositioning ability. That is, contrary to the intuition that a firm benefits from competing with a weak rival, as opposed to a strong one, the new entrant’s performance can actually be worse when his rival, the incumbent, becomes weaker in her repositioning ability. Second, the incumbent faces less competition when her repositioning ability is lower. Thus, if the benefit from reduced competition is greater than the loss due to lower repositioning ability, the incumbent can be better off. As a result, despite the extra repositioning costs incurred by the incumbent, the incumbent can still earn more profit than the entrant. Interestingly, the incumbent’s relative advantage, defined as the incumbent’s relative performance over the entrant (Lieberman and Montgomery, 1988, 1998; Lehmann-Grube, 1997), prevails, especially when the incumbent’s repositioning ability is relatively low. In other words, the incumbent’s low repositioning ability can serve as a relative advantage, which is often an important strategic concern for firms (Barney, 1991; Pacheco-de Almeida and Zemsky, 2007; Chatain and Zemsky, 2007; Drnevich and Kriauciu纳斯, 2011; Alcácer et al., 2015).

Entrants are often unable to precisely assess the incumbent’s repositioning ability due to the prevalence of cognitive biases, systematic errors in how executives process information in strategic decision making (Schwenk, 1984; Horn et al., 2005; Menon, 2017; Menon and Yao, 2018). Cognitive bias prevents managers from completely avoiding errors in estimating a competitor’s ability (Prescott and Visscher, 1977; Camerer, 1991; Camerer and Lovallo, 1999; Goldfarb and Yang, 2009). For example, using controlled laboratory studies, Moore and Cain (2007) and Cain et al. (2015) find that entrants to a market tend to systematically overestimate or underestimate the skills of their rivals. In a similar vein, using the data from US local telephone markets shortly after the Telecommunications Act of 1996, Goldfarb and Xiao (2011) demonstrate that a new entrant’s ability to predict incumbent’s behavior varies and depends on the manager experience and education of the new entrant. Similar bias exists for diversifying entrants as well as de novo ones. In the early 1990s, when Anheuser-Busch diversified to enter the
snack food business, the beer giant greatly underestimated the repositioning ability of the incumbent, Frito-Lay, which efficiently repositioned itself (Stalk Jr and Lachenauer, 2004; Horn et al., 2005).

Against this backdrop, we develop our main model characterizing a biased entrant who either underestimates or overestimates the repositioning ability of the incumbent. We use the terms “underestimation bias” ("overestimation bias") when the entrant underestimates (overestimates) the incumbent’s repositioning ability. Consistent with intuition, entrant bias leads to suboptimal positioning decisions, inevitably decreasing the entrant’s performance. We find, however, that entrant bias can either increase or decrease the incumbent’s performance, depending on the type of bias. In particular, the entrant’s underestimation bias helps the incumbent, whereas the overestimation bias hurts her. Interestingly, overestimation bias can harm the incumbent more than it does the entrant. One implication of this result is that the entrant’s overestimation bias can boost his performance relative to that of the incumbent, while underestimation bias does not; overestimation bias can thus confer a relative advantage to the entrant.

In the discussion so far, we have assumed that the incumbent is unbiased and can fully foresee the entrant’s bias (consequently the entrant’s actions). However, incumbents can inaccurately assess the entrant’s decisions. For example, Harley Davidson as an incumbent motorcycle manufacturer inaccurately assessed Honda’s entry into the US lightweight motorcycle segment during the early 1960s (Pascale and Christiansen, 1989; Menon, 2017). In our context, this can mean that the incumbent is either unaware of entrant bias or inaccurately perceives the level of entrant bias, despite having awareness. Although the entrant is biased on some level, the incumbent behaves as though the entrant were unbiased or differently biased.

Given these misperceptions, we first investigate a setting where the incumbent does not accurately assess entrant bias, despite having an awareness of entrant bias, i.e., entrant bias anticipated by the incumbent is different from the actual bias of the entrant. In this setting, we find that, although incumbent bias unanimously results in self harm, ironically it can boost the incumbent’s performance relative to that of the entrant. That is, compared to the case where the incumbent perfectly assesses entrant bias, the biased incumbent can achieve a higher relative performance. To clarify, incumbent bias can be a drag not only for the incumbent, but also the entrant. The drag for the entrant can be greater than the incumbent’s losses due to her decision bias, enhancing the leader advantage. Perhaps more interestingly, in this setting where neither the incumbent nor the entrant has an accurate assessment of the other, both firms can actually earn more than in the baseline model where both firms are accurate.
This means that, although the respective biases of the incumbent and the entrant always result in harm to the individual firm, firms can be better off when they both behave in a biased rather than an unbiased way.

We further consider a setting in which the incumbent is completely unaware of the entrant’s decision bias, rendering her even more biased. Interestingly, this increased bias does not necessarily inflict further self harm on the incumbent. In fact, the incumbent’s complete lack of awareness can simultaneously benefit both the incumbent and the entrant, leading to better performances for both firms compared to the case where the incumbent is aware of entrant bias (but with an inaccurate assessment), depending on the magnitude of entrant’s and incumbent’s respective biases.

**The Literature and Our Contributions**

This paper contributes to the literature on incumbent strategies in response to entrants entering the market. These responses are among a firm’s most important strategic decisions and have long been a central issue in economics, strategy, and marketing (Modigliani, 1958; Dixit, 1979; Milgrom and Roberts, 1982; Economides, 1984; Donnenfeld and Weber, 1995; Hauser and Shugan, 2008; McCann and Vroom, 2010). Recent studies have focused on one post-entry defending strategy of the incumbent, repositioning, and conclude that repositioning costs are fundamental to strategic interactions of firms’ activity systems (Ellickson et al., 2012). To model this, our baseline model follows the tradition of Hotelling (1929) and numerous other works he inspired (e.g., Dixit and Stiglitz 1977, Thomas and Weigelt 2000, and Alcácer et al. 2015). In Hotelling’s model without repositioning costs, each firm’s equilibrium strategy is to locate at the ends of the market, maximally differentiating itself from the competitor (d’Aspremont et al., 1979). Most real-world examples, however, act in a manner more consistent with our theoretical predication that firms are not necessarily located at the two ends of the Hotelling line. This paper accordingly helps reconcile the gap between theory and practice. Our baseline model also shows that a higher repositioning ability can translate to a higher profit for the entrant (competitor), but a lower (absolute and relative) profit for the incumbent.

Although anticipating and studying competitor moves is a key aspect of strategy practice, the formal modeling of repositioning costs in strategy began only recently with Menon and Yao (2017). Their “starting point is Pankaj Ghemawat’s (1991) theory of commitment as the essential element in identifying strategic choices. Ghemawat persuasively argues that a strategic choice is one that involves commitment
and that committed choice creates the persistent pattern of action typically characterized as strategy.”

Following this argument, Menon and Yao (2017) analytically investigate the interaction between an incumbent (innovator) and an entrant (follower). Within their setting, the incumbent develops and introduces a new product generation and then chooses to offer either generous or stingy licensing terms to a follower who can imitate the innovation or become a complementor. The follower’s choices are modeled as involving possible repositioning costs because the activity systems supporting imitation versus complementarity are different. We complement this pioneering research by investigating the post-entry repositioning of the incumbent, one prevalent strategy in practice.

More importantly, we focus on settings where the entrant and the incumbent are not necessarily rational and may have different levels of foresight. This follows the idea that “there has long been a recognition in game theory that an empirically realistic treatment of the problem of strategic interaction, one that can account for real-world outcomes of strategic interactions, will have to take into account the beliefs and cognitions of the actors involved” (Menon, 2017). When firms’ biases are considered, this leads to counter-intuitive results on firms’ absolute performance. First, although entrant bias and incumbent bias (unawareness) always hurt the entrant and incumbent, respectively, both firms can earn more than in our baseline setting where both are fully cognizant of the other. Second, as opposed to the case where the incumbent is fully unaware of entrant bias, the incumbent is not necessarily better off by being aware (but with an inaccurate assessment) of this bias. Indeed, both the incumbent and the entrant can be hurt by the incumbent’s awareness.

Also relevant to our work here is the rich literature on leader or follower relative performance, which forms the basis for the definition of relative advantage, a major area of research in the field of strategic management (Lieberman and Montgomery, 1988; Barney, 1991; Lieberman and Montgomery, 1998; Kerin et al., 1992; Golder and Tellis, 1993; Tyagi, 2000; Hawk et al., 2013). We enrich this literature by incorporating repositioning costs and decision biases into a classical model with differentiation and market entry. By doing so, we make three key contributions. First, we find that, despite intuition to the contrary, the incumbent can gain more profit than the entrant when the incumbent’s repositioning ability is relatively low rather than high. The incumbent’s repositioning costs can serve as a leader advantage. In other words, when the incumbent’s repositioning ability benefits herself, it may also benefit the entrant the same or even more. This result contributes to the literature which often focuses only on the impacts of the dynamic capability (or repositioning ability) on the focal firm’s absolute performance (Eisenhardt and Martin, 2000). Second, although the entrant’s estimation bias always
hurts the entrant, this bias can enhance the advantage of the entrant (as a follower) relative to the incumbent. Third, the relative advantage of the incumbent is not necessarily lower than that of the entrant when she is biased versus unbiased in perceiving the entrant’s action. The incumbent’s lack of awareness as a decision bias can actually help her own relative advantage. In summary, the repositioning ability and awareness of the leader are not necessarily a leader advantage, whereas the estimating bias of the follower can be a follower advantage.

**Baseline Model**

We develop our baseline model with an unbiased entrant and incumbent based on the standard Hotelling’s formulation (Hotelling, 1929; d’Aspremont et al., 1979). In particular, we consider a market where the ideal points of customers are distributed uniformly in $[-0.5, 0.5]$, and consumers with ideal point $t \in [-0.5, 0.5]$ value a firm of position $x$: $R - (x - t)^2$. Here, the firm’s position $x$ can be either geographic, as in a store’s or a restaurant’s physical location or a space of preferences, as in the sweetness of a soft drink. $R$ is the reservation price of customers, which is assumed to be the same for all customers, and high enough so that all customers buy from the firm. $(x - t)^2$ is the disutility incurred by consumers if they are geographically distant from the firm or their ideal product does not match the product offered by a firm. The above framework or similar ones have been widely adopted in the literature of position/price competition (Tyagi, 2000; Sajeesh and Raju, 2010; Liu and Tyagi, 2011).

Before the new entrant enters the market, the incumbent is the monopoly in the market. For a monopoly firm positioning at $x$ and pricing at $p$, the valuation of consumers with ideal point $t$ is $R - (x - t)^2 - p$. Consequently, the total disutility for all consumers $\int_{-0.5}^{0.5} (x - t)^2 dt = \frac{1}{12} + x^2$. The incumbent (monopoly) positions at the best position $x = 0$, where the consumers total disutility is minimized (Makadok and Ross, 2013).

After the entrant enters the market, the incumbent can move its original position to a new one $x_i$ but will incur a cost

$$k|x_i - 0| = k|x_i|,$$

where $k(\geq 0)$ is the repositioning parameter. The repositioning costs described in (1) increases with respect to repositioning distance $|x_i|$. In other words, the repositioning costs are distance-based (Montgomery and Wernerfelt, 1988) and increases with the firm’s distance between the origin (the initial
position from which the incumbent is moving) and destination (the position to which the incumbent is moving). Such costs often reflect changes in the set of resources and capabilities required to execute the origin versus destination activity systems, or account for difficulties in changing from the initial activity system of the firm, for example, difficulties in unwinding and changing current operations and related commitments (Upton, 1995; Menon and Yao, 2017).

The parameter \( k \) in (1) can represent the incumbent’s repositioning ability: the higher is \( k \), the lower is the repositioning ability. For example, \( k = 0 \) represents the full flexibility for repositioning that is costless, whereas \( k = +\infty \) is the extreme where the incumbent incurs an infinite cost for repositioning.

After the incumbent chooses her new position, the incumbent and new entrant decide their prices \( p_i \) and \( p_e \) simultaneously so that the firms maximize their own profits. This follows the long traditional argument in the literature of position competition (Hotelling, 1929; d’Aspremont et al., 1979; Makadok and Ross, 2013) that price decisions are believed to be more flexible than position decisions; price can be easily changed, but it is often difficult to adjust strategy or product positions. We also follow the traditional approach in this literature to solve the problem backward by first evaluating firms’ price decisions and then solving their re/positioning decisions. Given the incumbent position \( x_i \) and price \( p_i \) as well as the entrant position \( x_e \) and price \( p_e \), consumers make their purchasing decisions. For the sake of presentation, we present the case where only the new entrant repositions herself to the right of the incumbent \( (x_i < x_e) \). The case of \( x_i > x_e \) is discussed in Appendix A.

Given \( x_i < x_e \), the customers with ideal point \( t \) such that \( R - (x_i - t)^2 - p_i > R - (x_e - t)^2 - p_e \iff t < \frac{p_e - p_i}{2(x_e - x_i)} + \frac{x_i + x_e}{2} \) purchase from the incumbent. Otherwise, consumer purchase from the entrant. Consequently, the demand of the incumbent firm \( D_i \) and the demand for the new entrant \( D_e \) are

\[
(D_i, D_e) = \left( \frac{1}{2} + \frac{p_e - p_i}{2(x_e - x_i)} + \frac{x_i + x_e}{2} \cdot \frac{1}{2}, \frac{1}{2} - \frac{p_e - p_i}{2(x_e - x_i)} - \frac{x_i + x_e}{2} \right).
\]  

The incumbent and the entrant’s pricing problems are therefore

\[
\max_{p_i}(p_i - c) \left( \frac{1}{2} + \frac{p_e - p_i}{2(x_e - x_i)} + \frac{x_i + x_e}{2} \right) \quad \text{and} \quad \max_{p_e}(p_e - c) \left( \frac{1}{2} - \frac{p_e - p_i}{2(x_e - x_i)} - \frac{x_i + x_e}{2} \right),
\]  

respectively, where \( c(\geq 0) \) is the unit production cost for both firms. Solving (3), we obtain equilibrium
prices and demands for the incumbent and the new entrant:

\[(p_i, p_e) = \left( c + \frac{(x_e - x_i)(3 + x_i + x_e)}{3}, c + \frac{(x_e - x_i)(3 - x_i - x_e)}{3} \right). \tag{4} \]

Note that the profit margins of both firms \(p_i - c\) and \(p_e - c\) increase in their position difference \(x_e - x_i\), which means that differentiation between positions has a potential to boost the margins, and consequently profits for both firms.

Given (3)-(4), we next solve the incumbent repositioning problem:

\[
\pi^k_i(x_e) := \max_{x_i} (p_i - c) \left( \frac{1}{2} + \frac{p_e - p_i}{2(x_e - x_i)} + \frac{x_i + x_e}{2} \right) - k|x_i|
= \max_{x_i} \frac{(x_e - x_i)(3 + x_i + x_e)^2}{18} - k|x_i|. \tag{5}
\]

In solving (5), the incumbent would like to position herself close to position \(x = 0\) to limit her repositioning costs. Moreover, positioning closer to \(x = 0\) is more efficient. In addition, if the incumbent can either observe or foresee the entrant’s position \(x_e\), the incumbent also would likely differentiate herself from the entrant for a higher differentiation \((x_e - x_i)\). Overall, the incumbent needs to balance repositioning costs, position efficiency, and differentiation all together to reach the optimal position \(x^k_i\).

Next, we characterize the incumbent’s optimal position level \(x^k_i\) and the associated incumbent’s profit \(\pi^k_i(x_e)\) in Lemma 1.

**Lemma 1.** Given the entrant’s position \(x_e(>0)\):

a) The incumbent’s optimal position is

\[
x^k_i(x_e) = \begin{cases} 
0 & \text{if } k > \frac{9-x_e^2}{18} \\
-6-x_e+\sqrt{(3+2x_e)^2+54k} & \text{if } k \leq \frac{9-x_e^2}{18}. \tag{6}
\end{cases}
\]

Moreover, \(\frac{\partial x^k_i(x_e)}{\partial k} > 0\) and \(\frac{\partial x^k_i(x_e)}{\partial x_e} > 0\) when \(k \leq \frac{9-x_e^2}{18}\).

b) The incumbent’s optimal profit \(\pi^k_i(x_e)\) satisfies \(\frac{\partial \pi^k_i(x_e)}{\partial k} < 0\) and \(\frac{\partial \pi^k_i(x_e)}{\partial x_e} > 0\).

Lemma 1a shows that, when the entrant enters the market, it is optimal for the incumbent to remain at the original position if the repositioning parameter is relatively high \((k > \frac{9-x_e^2}{18})\). However, when the repositioning parameter is relatively low \((k \leq \frac{9-x_e^2}{18})\), the incumbent repositions herself, and distances herself from the entrant for a higher differentiation in order to lessen price competition. That is, \(x^k_i(x_e) < \)
0. Moreover, the closer the entrant to the incumbent’s original position, the higher the magnitude of the incumbent’s repositioning $|x_i^k(x_e)| = -x_i^k(x_e)$. That is, $\frac{\partial x_i^k(x_e)}{\partial x_e} > 0$. For the entrant, positioning away from the incumbent’s original position $(x = 0)$ results in less competition for the incumbent so that the incumbent’s incentive to reposition is relatively low. Moreover, for a given incumbent position $x_e$, the incumbent’s new position $x_i^k(x_e)$ always increases in her repositioning parameter $k$, i.e., $\frac{\partial x_i^k(x_e)}{\partial k} > 0$: the higher the repositioning parameter, the lower the magnitude of repositioning $|x_i^k(x_e)|$. Lemma 1b shows that the incumbent’s optimal profit increases in the entrant’s position $x_e$ ($\frac{\partial \pi^k(x_e)}{\partial x_e} > 0$). In this context, a higher entrant position means a greater distance from the incumbent’s position, benefiting the incumbent. Moreover, Lemma 1b shows that the incumbent’s profit decreases in her own repositioning parameter ($\frac{\partial \pi^k(x_e)}{\partial k} < 0$), which means the incumbent benefits from her own repositioning ability: the higher her repositioning ability, the higher her profit, given the entrant’s position.

Anticipating the incumbent’s repositioning strategy, the new entrant accordingly decides his position. In particular, if the new entrant perfectly foresees the incumbent’s repositioning parameter $(k)$, then he is able to infer the incumbent’s reposition strategy described in (6). Consequently, the entrant’s profit in (4) becomes

$$
\pi^k_e = \max_{x_e} \left( p_e - c \left( \frac{1}{2} - \frac{p_e - p_i}{2(x_e - x_i)} \right) \right) \left( \frac{1}{2} - \frac{x_i + x_e}{2} \right),
$$

where $x_i^k(x_e)$ is from (6). Like the incumbent, the entrant needs to differentiate himself from his rival (the incumbent) while attempting to position himself close to an efficient position. Moreover, the entrant strategically anticipates that his position decision can affect the incumbent’s repositioning decision (Lemma 1). Thus, although the entrant does not incur repositioning costs directly, his equilibrium position $x_e^k$ and the ensuing profit $\pi^k_e$ are affected by the incumbent’s repositioning parameter $k$. Next, we characterize the entrant’s equilibrium position $x_e^k$ in Lemma 2 below.

**Lemma 2.** Let $x_e^k$ denote the solution of (7) for any given $k$. Then, there exists a repositioning parameter $\hat{k}$ such that $x_e^k = 1$ for $k > \hat{k}$, whereas $x_e^k$ increases in $k$ for $k \leq \hat{k}$.

Lemma 2 characterizes the entrant’s equilibrium position $x_e^k$ and how it changes with the repositioning parameter $k$; see Figure 1 for an illustration. For a relatively large repositioning parameter such that $k > \hat{k}$, repositioning does not happen. Thus, the entrant’s position $x_e^k(= 1)$ is independent of the
repositioning parameter $k$. However, for a relatively small repositioning parameter such that $k < \hat{k}$, the incumbent repositions with a new position increasing in $k$. In other words, as repositioning becomes easier ($k$ is smaller), the entrant’s position $x^k_e$ becomes closer to (or less differentiated from) the incumbent’s original position $x = 0$. In the extreme of costless repositioning ($k = 0$), the entrant positions himself at the incumbent’s original position $x^k_e = 0$ by anticipating the incumbent’s repositioning behavior.

The repositioning parameter $\hat{k}$ is the value where the incumbent is indifferent to both the non-repositioning strategy and the repositioning strategy. However, as shown in Figure 1, the entrant’s position jumps as the incumbent switches from a repositioning strategy to a non-repositioning strategy, i.e., $x^k_{e^-} < x^k_{e^+}$. As the incumbent switches from a repositioning strategy to a non-repositioning strategy around $\hat{k}$, the entrant’s positioning decision is affected significantly. This observation helps explain the forthcoming propositions.

— Place Figure 1 here —

**Proposition 1.** When $k > \hat{k}$, where $\hat{k}$ is defined in Lemma 2, the entrant’s equilibrium profit $\pi^k_e = \frac{2}{9}$. However, when $k < \hat{k}$, $\pi^k_e$ decreases in $k$.

Proposition 1 demonstrates that the entrant’s profit $\pi^k_e = \frac{2}{9}$ is independent of the incumbent’s repositioning costs when the repositioning parameter is high such that $k > \hat{k}$; see Figure 1 for an illustration. When her repositioning parameter is high, the incumbent does not reposition herself and does not incur repositioning costs (Lemma 2). However, when the repositioning parameter is low ($k < \hat{k}$), the incumbent repositions herself and the entrant’s profit decreases in $k$. Stated differently, a higher repositioning ability of the incumbent can translate to a higher profit for the entrant. This is somewhat counter-intuitive in the sense that one may expect that the entrant is better off when his rival, the incumbent, becomes weaker in terms of repositioning ability. When the incumbent’s repositioning ability is relatively low, the incumbent tends to move away from her current (i.e., the most efficient) position, leaving the entrant a relatively efficient position. Indeed, in the extreme of costless repositioning ($k = 0$), the entrant’s profit reaches the maximal value $\pi^k_e = \frac{8}{9}$ as shown in Figure 1.

Given the entrant’s equilibrium position and profit, we now turn our attention to the equilibrium decision of the incumbent. From (6) and Lemma 2, the incumbent’s equilibrium position is

$$x^k_i := \begin{cases} 0 & \text{if } k > \hat{k} \\ \frac{-6 - x^k_{e^-} + \sqrt{(3 + 2x^k_{e^-})^2 + 54k}}{3} & \text{if } k \leq \hat{k}. \end{cases}$$

(8)
That is, when the incumbent chooses to reposition herself \((k \leq \hat{k})\), her equilibrium reposition depends on the entrant’s repositioning \(x_e^k\). This means that the repositioning parameter affects her own positioning decision not only directly as indicated in Lemma 1, but also indirectly via the entrant positioning \(x_e^k\). Lemma 3 below characterizes how \(x_i^k\) is affected by the repositioning parameter \(k\).

**Lemma 3.** The incumbent’s equilibrium position \(x_i^k\) increases in \(k\) when \(k < \hat{k}\).

Lemma 3 shows that when repositioning occurs \((k < \hat{k})\), the larger the repositioning parameter, the smaller the repositioning magnitude \(|x_i^k|\). That is, the incumbent’s equilibrium position \(x_i^k\) increases in \(k\). This result is qualitatively consistent with Lemma 2 where the entrant position is exogenous, albeit for an additional driver. In the case of an exogenous entrant position, the incumbent is reluctant to reposition because of the high costs associated with a high \(k\). But, when the entrant’s position is endogenous, the entrant would position himself away from the incumbent with a higher repositioning parameter, putting less pressure on the incumbent. The low repositioning ability renders a less competitive environment, and consequently less incentive for the incumbent to differentiate.

After characterizing the incumbent’s repositioning strategy in Lemma 3, we next evaluate the incumbent equilibrium profit \(\pi_i^k := \pi_i^k(x_e^k)\), where \(\pi_i^k(\cdot)\) and \(x_e^k\) are defined in (5) and Lemma 2 respectively.

**Proposition 2.** The incumbent’s equilibrium profit \(\pi_i^k = \frac{1}{2}\) when the incumbent’s repositioning parameter is relatively high \((k > \hat{k})\). However, \(\pi_i^k\) can increase in \(k\) when the incumbent’s repositioning parameter is relatively low \((k \leq \hat{k})\).

Proposition 2 states when the incumbent does not reposition herself \((k > \hat{k})\), her profit \(\pi_i^k = \frac{1}{2}\) is independent of the repositioning parameter \(k\). Moreover, in contrast to Lemma 1, Proposition 2 shows that in equilibrium with an endogenous entrant’s position, the incumbent can be better off for a relatively higher \(k\) when \(k \leq \hat{k}\); a lower repositioning ability can imply a higher incumbent profit. Intuitively, recall that a relatively high repositioning parameter \(k\) would render a less competitive environment for the incumbent. The magnitude of this benefit, due to reduced competition, can be higher than the losses due to higher repositioning costs. To illustrate, recall that the incumbent’s switch from a repositioning strategy to a non-repositioning strategy can significantly affect the entrant’s positioning decision for a small increase in \(k\) around \(\hat{k}\). As a result, the incumbent can be better off as \(k\) increases. This insight that an inflexible commitment can create advantages to a firm has had a storied tradition in business strategy (Ghemawat, 1991) along with the application of game theory to social sciences and economics (e.g., Schelling 1980; Sutton 1991; Chatain and Zemsky 2007). This insight is further elaborated in
Menon and Yao (2017), who analyze the interaction between an incumbent (innovator) and an entrant (follower) involving repositioning costs. We extend this insight in a different, yet important setting, where the incumbent incurs repositioning costs after the competitive entry (Wang and Shaver, 2014; Seamans and Zhu, 2017).

After characterizing the entrant’s and incumbent’s profits, we evaluate the incumbent’s relative performance to the new entrant, i.e., \( \Delta_k := \pi^i_k - \pi^e_k \). In our setting, the incumbent firm is the leader of the market, and she has a pioneering advantage in the sense that the incumbent initially positions herself at the best market location. However, the incumbent can have lower performance than the entrant.

**Proposition 3.** When \( k > \hat{k} \), the incumbent earns more profit than the entrant, i.e., \( \Delta_k > 0 \). However, when \( k \leq \hat{k} \), the incumbent can earn less profit than the entrant, i.e., \( \Delta_k < 0 \).

Proposition 3 demonstrates that, when the repositioning parameter is relatively high (\( k > \hat{k} \)), the leader advantage prevails. Because the incumbent initially occupies the best market location, her first-mover advantage remains as the repositioning parameter is high. However, when the repositioning parameter is relatively small (\( k \leq \hat{k} \)), although the incumbent firm is more operationally efficient on repositioning, her leader advantage can disappear because, if repositioning occurs, the incumbent positions at a less efficient location than the entrant’s location. One implication of this result is that \( \Delta_k \) can decrease in \( k \) as \( k \) becomes higher. Counter to intuition, the incumbent’s low repositioning ability can help her gain relative advantage over her rival. Overall, Proposition 3 states that the incumbent’s leader advantage can disappear when the entrant precisely foresees the incumbent’s repositioning decision. This begs the question of how the entrant’s foresight affects this result. To explore it, we dive deeper to study the case of a biased entrant.

**Biased Entrant**

We now examine the case where the incumbent’s costs are not perfectly known by the entrant. In particular, the new entrant is biased and behaves as if the incumbent’s repositioning parameter is \( \theta k \) rather than \( k \) where \( \theta \geq 0 \). That is, when \( \theta < 1 \) (resp. \( \theta > 1 \)), the entrant estimates the incumbent’s repositioning costs as less (more) than they really are, i.e., the entrant overestimates (underestimates) the repositioning ability of the incumbent. The level of entrant’s underestimation bias can be characterized by \( \theta - 1 \), whereas the level of entrant’s overestimation bias can be characterized by \( 1 - \theta \). In this way, \( |\theta - 1| \) indicates the level of estimation bias.
Given the incumbent’s repositioning parameter $k$, the biased entrant characterized by the bias parameter $\theta$ anticipates the incumbent’s repositioning decision as

$$x^\theta_k(x_e) = \begin{cases} 0 & \text{if } k > \frac{9-x_e^2}{18}\theta \\ \frac{-6-x_e + \sqrt{(3+2x_e)^2 + 54k}}{3} & \text{if } k \leq \frac{9-x_e^2}{18}\theta, (\leq 0) \end{cases}$$

although the incumbent’s decision is actually characterized by (6). As a result, the entrant solves

$$\max_{x_e} \frac{[x_e - x^\theta_k(x_e)][3 - x^\theta_k(x_e) - x_e]^2}{18}$$

for his optimal position $\hat{x}_e(\theta)$, i.e., the solution of (9). Given this, from (6), the incumbent’s new position is

$$\hat{x}_i(\theta) := x^k_i(\hat{x}_e(\theta)) = \begin{cases} 0 & \text{if } k > \frac{9-\hat{x}_e(\theta)^2}{18}\theta \\ \frac{-6-\hat{x}_e(\theta) + \sqrt{(3+2\hat{x}_e(\theta))^2 + 54k}}{3} & \text{if } k \leq \frac{9-\hat{x}_e(\theta)^2}{18}\theta, (\leq 0) \end{cases}.$$ 

We start with the case where the incumbent knows the incumbent’s position $x^\theta_k(x_e)$ before repositioning. In practice, such knowledge can be obtained from direct observation of the entrant’s strategic position. When the entrant’s position is unobserved to the incumbent, she may foresee the entrant’s position, which reflects an awareness of the entrant bias and an accurate assessment of the level of entrant bias. We relax our assumption later in the sections “Biased Entrant and Imprecise Incumbent” and “Biased Entrant and Unaware Incumbent.”

Next, we evaluate the equilibrium profits for the incumbent and the entrant. Given the equilibrium positions $\hat{x}_e(\theta)$ and $\hat{x}_i(\theta)$ along with (7) and (5), the entrant’s and the incumbent’s equilibrium profits are

$$\hat{\pi}_e(\theta) := \frac{[\hat{x}_e(\theta) - x^k_i(\hat{x}_e(\theta))][3 - x^k_i(\hat{x}_e(\theta)) - \hat{x}_e(\theta)]^2}{18}$$

and

$$\hat{\pi}_i(\theta) := \pi^k_i(\hat{x}_e(\theta)) = \max_{x_i} \frac{[\hat{x}_e(\theta) - x_i][3 + x_i + \hat{x}_e(\theta)]^2}{18} - k|x_i|,$$

respectively. Although the entrant behaves as if he is solving (9) for his optimal decision, the entrant’s profit is characterized by (11) rather than (9). Moreover, although the incumbent is not biased, her
equilibrium profit can be affected by the bias parameter $\theta$ via the entrant’s biased decision $\hat{x}_e(\theta)$.

**Proposition 4.** Given the entrant bias parameter $\theta$:

a) The entrant’s equilibrium profit $\hat{\pi}_e(\theta)$ decreases in $\theta$ when $\theta > 1$, while $\hat{\pi}_e(\theta)$ increases in $\theta$ when $\theta < 1$.

b) The incumbent’s profit $\hat{\pi}_i(\theta)$ increases in $\theta$.

Proposition 4a shows that the entrant’s profit $\hat{\pi}_e(\theta)$ decreases in the bias parameter $\theta$ when the entrant has an underestimation bias ($\theta > 1$), but increases in the bias parameter $\theta$ when the entrant has an overestimation bias ($\theta < 1$). Consequently, regardless of the type of bias, the entrant’s profit decreases in the level of bias ($|\theta - 1|$). However, Proposition 4b shows that the incumbent’s profit increases in the bias parameter $\theta$. Essentially, the incumbent prefers the entrant to choose a position that is distant from her because her profits increase with differentiation. When the entrant underestimates the incumbent’s reposition ability, he stays away from the incumbent and the incumbent thus benefits from the greater differentiation. Similarly, when the entrant overestimates the ability, he locates himself close to the incumbent hoping the incumbent will reposition herself farther away. Given this, we check whether the entrant’s bias can lead to a higher relative performance for the entrant.

**Proposition 5.** Define $\hat{\Delta}(\theta) := \hat{\pi}_i(\theta) - \hat{\pi}_e(\theta)$. Then, when $\theta > 1$, then $\hat{\Delta}(\theta)$ increases in the level of the entrant’s bias $(\theta - 1)$, i.e., $\theta > 1 \implies \hat{\Delta}'(\theta) > 0$. However, when $\theta < 1$, $\hat{\Delta}(\theta)$ can decrease in the level of the entrant’s bias $(1 - \theta)$.

Proposition 5 demonstrates that the entrant’s underestimation bias ($\theta > 1$) always drags down the entrant’s relative performance, namely, the follower advantage $-\hat{\Delta}(\theta)$. However, the entrant’s overestimation bias ($\theta < 1$) can boost his relative advantage over the incumbent. For insights, recall that when the entrant overestimates the incumbent’s repositioning ability, the biased entrant would like to position himself closer to the incumbent, putting more competition pressure, a drag, on the incumbent. When this drag on the incumbent is significant and outweighs the entrant’s losses due to his decision bias, i.e., when the incumbent’s profit is more sensitive to entrant bias than entrant profit, the entrant’s relative advantage can be enhanced. Thus, overestimation bias can hurt the incumbent even more than it does the entrant, and it is possible that the entrant with the overestimation bias can make more profit than the incumbent even when the unbiased entrant cannot. This is interesting because, given that the unbiased entrant can foresee the incumbent’s decision, at the least, the unbiased entrant can mimic the
biased entrant’s decision and earn the same profit as the biased entrant. However, ironically, Proposition 5 indicates that although the unbiased entrant can increase his own profit by reacting optimally, his optimal reaction nevertheless can increase his competitor’s profit even more, depending on the type of entrant bias.

Proposition 5 also implies that the insight when the entrant is unbiased, namely the low repositioning ability of the incumbent can be a leader advantage (Proposition 3), remains when the entrant is biased in (under)estimating the incumbent’s repositioning ability. This also means that the entrant (follower) advantage can persist when the entrant has an overestimation bias while the incumbent incurs repositioning costs. We can show that, when the incumbent does not incur repositioning costs, the entrant’s follower advantage is sustained as long as his position is closer than the incumbent’s position to the center of the Hotelling line (see Appendix A for details).

Biased Entrant and Imprecise Incumbent

So far we have assumed that the incumbent is not only aware of entrant bias, but also has a precise assessment of that bias. We now extend our analysis to the case where the incumbent’s assessment of the entrant bias is inaccurate. In particular, although entrant bias is captured by the parameter \( \theta \), the incumbent assesses the incumbent’s parameter as \( \hat{\theta} (\neq \theta) \). As a result, the entrant described by the bias parameter \( \theta \) positions himself at \( \hat{x}_e(\theta) \), i.e., the solution of (9). However, the incumbent still positions herself at \( \hat{x}_i(\hat{\theta}) \), where \( \hat{x}_i(\cdot) \) is from (10). The entrant’s and the incumbent’s equilibrium profits are therefore

\[
\hat{\pi}_e(\theta, \hat{\theta}) := \frac{[\hat{x}_e(\theta) - \hat{x}_i(\hat{\theta})][3 - \hat{x}_i(\hat{\theta}) - \hat{x}_e(\theta)]^2}{18},
\]

and

\[
\hat{\pi}_i(\theta, \hat{\theta}) := \frac{[\hat{x}_e(\theta) - \hat{x}_i(\hat{\theta})][3 + \hat{x}_i(\hat{\theta}) + \hat{x}_e(\theta)]^2}{18} - k|\hat{x}_i(\hat{\theta})|,
\]

respectively. The incumbent’s leader advantage is consequently

\[
\hat{\Delta}(\theta, \hat{\theta}) := \hat{\pi}_i(\theta, \hat{\theta}) - \hat{\pi}_e(\theta, \hat{\theta}).
\]
We next study the impact of the incumbent’s imprecise bias (i.e., the inaccurate assessment of the entrant’s behavior) on the absolute and relative performances defined in (13)-(15), as we have done before.

**Proposition 6.** The incumbent’s imprecise bias always leads to a lower profit for the incumbent, i.e., $\hat{\pi}_i(\theta, \hat{\theta}) \geq \hat{\pi}_i(\theta)$ for any $\hat{\theta}$. However, the incumbent’s imprecise bias can lead to a higher relative performance for the incumbent, i.e., $\hat{\Delta}(\theta, \hat{\theta}) > \hat{\Delta}(\theta)$ when $\hat{\theta} > \theta$.

Proposition 6 shows that, although incumbent bias always leads to a lower profit for the incumbent, it can actually boost the incumbent’s leader advantage – the incumbent’s performance relative to the entrant. Although the incumbent’s bias hurts the incumbent, it can hurt the entrant even more than it does the incumbent when $\hat{\theta} > \theta$ (see Proposition 7 below), thus boosting the incumbent’s relative performance.

**Proposition 7.** The incumbent’s imprecise bias hurts the entrant, i.e., $\hat{\pi}_e(\theta, \hat{\theta}) < \hat{\pi}_e(\theta)$ when $\hat{\theta} > \theta$, but it benefits the entrant, i.e., $\hat{\pi}_e(\theta, \hat{\theta}) > \hat{\pi}_e(\theta)$ when $\hat{\theta} < \theta$.

Thus, incumbent bias can either hurt or benefit her rival (entrant) depending on the relative value of $\hat{\theta}$ and $\theta$; entrant bias can also benefit the incumbent (Proposition 4). This raises the question of whether firms can earn more when they are both biased versus both unbiased.

**Proposition 8.** Although entrant bias and incumbent bias always lead to lower performance for the entrant and incumbent respectively, i.e., $(\hat{\pi}_e(\theta), \hat{\pi}_i(\theta, \hat{\theta})) \leq (\pi^k_e, \hat{\pi}_i(\theta))$, biased firms can earn more than sophisticated firms, i.e., $(\hat{\pi}_e(\theta, \hat{\theta}), \hat{\pi}_i(\theta, \hat{\theta})) > (\pi^k_e, \pi^k_i)$.

Proposition 8 indicates that although entrant bias and incumbent bias are harmful for each individual firm, each can be better off when they are both biased versus both unbiased. That is, each firm can be better off by their simultaneous (but not singular) biases. For insights, in our setting, in addition to repositioning costs incurred by the incumbent, firm performance is affected by their individual locations, which determine efficiency, as well as their relative location, which determines differentiation. When the entrant underestimates the incumbent’s repositioning ability, the entrant tends to position himself farther from the incumbent, benefiting the incumbent. At the same time, incumbent bias can lead to a distancing of the entrant positioning from the entrant, thus benefiting the entrant. When these benefits outweigh the losses due to biases, both firms are better off. Consider the case where the repositioning parameter $k = 0$. In our baseline model, the entrant’s and incumbent’ equilibrium profits
are \((\pi^k_e, \pi^k_i) = (\frac{8}{9}, \frac{2}{9})\) with the associated equilibrium positions \((x^k_e, x^k_i) = (0, -1)\). The entrant is hurt by his own bias because \(\hat{\pi}_e(\theta) \leq \hat{\pi}_e(\theta = 1) = \pi^k_e\) (Proposition 4). The incumbent bias also always hurts herself \((\hat{\pi}_i(\theta, \hat{\theta}) \leq \hat{\pi}_i(\theta))\), because the rational incumbent optimally chooses her optimal position in order to obtain her optimal profit in (12), but the biased incumbent does not in (18). However, if the entrant expects that the incumbent’s repositioning parameter is higher than \(\hat{k}\) with \(\theta = \infty\), then the entrant positions at \(x^k_e(\theta) = 1\) (Lemma 2), while the entrant still positions at \(\hat{x}_i(\hat{\theta}) = -1\) when \(\hat{\theta} < \infty\). As a result, from (16)-(17), both biased entrant and biased incumbent earn more than unbiased firms, i.e., \((\hat{\pi}_e(\theta, \hat{\theta}), \hat{\pi}_i(\theta, \hat{\theta})) = (1, 1) > (\frac{8}{9}, \frac{2}{9}) = (\pi^k_e, \pi^k_i)\). In this example, we see that the decision biases of the incumbent and the entrant can lead to a situation where firms are well differentiated. Although such differentiation can entail less efficient positions (far from the center of the Hotelling line), thus hurting firms (especially the incumbent originally located at the most efficient position), this example illustrates that the benefits of increasing differentiation can offset the losses of reduced efficiency for both firms.

### Biased Entrant and Unaware Incumbent

In the last section, we investigate the case where the incumbent is biased in assessing the entrant behavior, i.e., the incumbent is partially cognizant in the sense that the incumbent is still aware of entrant bias, but has an inaccurate assessment. In this section, we extend our scope to the case with a more biased incumbent who is totally unaware of entrant bias. The objective of this section is two-fold. First, we check the robustness of our main insights in section “Biased Entrant and Imprecise Incumbent,” namely incumbent bias can enhance her relative advantage while hurting the entrant, and decision biases can lead to a mutually beneficial situation for both firms (Propositions 6-8). Second, we derive new insights driven by the incremental bias of the unaware (versus aware but imprecise) incumbent.

We now describe the equilibrium outcome when the entrant is biased and the incumbent is unaware of entrant bias. For this case, from (8), the incumbent positions herself at \(x^k_i\) while expecting the entrant’s position to be \(x^k_e\). Consequently, the equilibrium positions for the entrant and the incumbent are \(\hat{x}_e(\theta)\) and \(x^k_i\) respectively. For this setting where the entrant has biased estimates of the incumbent’s repositioning ability \((\theta \neq 1)\) while the incumbent is biased in terms of not foreseeing entrant bias, the
entrant’s and the incumbent’s equilibrium profits are

\[ \tilde{\pi}_e(\theta) := \frac{[\hat{x}_e(\theta) - x^k_i][3 - x^k_i - \hat{x}_e(\theta)]^2}{18} \]

and

\[ \tilde{\pi}_i(\theta) := \frac{[\hat{x}_e(\theta) - x^k_i][3 + x^k_i + \hat{x}_e(\theta)]^2}{18} - k|x^k_i|, \]

respectively. Consequently, the incumbent’s leader advantage is

\[ \tilde{\Delta}(\theta) := \tilde{\pi}_i(\theta) - \tilde{\pi}_e(\theta). \]

We next study the impact of incumbent bias (unawareness) on both her absolute performance and relative performance defined in (16)-(18).

**Proposition 9.**

a) The incumbent’s unawareness always leads to a lower profit for the incumbent, i.e., \( \hat{\pi}_i(\theta) \geq \tilde{\pi}_i(\theta) \) for any \( \theta \). However, the incumbent’s unawareness can lead to a higher relative performance for the incumbent, i.e., \( \tilde{\Delta}(\theta) > \hat{\Delta}(\theta) \).

b) The incumbent’s unawareness hurts the entrant, i.e., \( \tilde{\pi}_e(\theta) < \hat{\pi}_e(\theta) \) when \( \theta < 1 \).

c) Although the entrant’s estimation bias and the incumbent’s unawareness always lead to lower performances for the entrant and incumbent respectively, i.e., \( (\hat{\pi}_e(\theta), \hat{\pi}_i(\theta)) \leq (\tilde{\pi}_e(\theta), \tilde{\pi}_i(\theta)) \) for any \( \theta \), biased firms can earn more than the sophisticated firms, i.e., \( (\tilde{\pi}_e(\theta), \tilde{\pi}_i(\theta)) > (\hat{\pi}_e(\theta), \hat{\pi}_i(\theta)) \).

Propositions 9a-b show that, although a lack of awareness leads to a lower profit for the incumbent, this lack of awareness as a decision bias can actually boost the incumbent’s leader advantage while hurting the entrant. Proposition 9c further shows that, when both firms are biased they can earn more than unbiased firms. Thus, the main insights from the setting where the incumbent has estimation bias (Propositions 6-8) continuously hold for a setting where the incumbent has a bias due to a lack of awareness. Given this, we compare the equilibrium profits for these two settings to evaluate the impact of the incumbent’s lack of awareness versus estimation bias.

**Proposition 10.** The increasing bias of the incumbent can benefit both the entrant and the incumbent, i.e., \( (\tilde{\pi}_e(\theta), \tilde{\pi}_i(\theta)) > (\hat{\pi}_e(\theta, \hat{\theta}), \hat{\pi}_i(\theta, \hat{\theta})) \), when \( \hat{\theta} < 1 < \theta \).
entrant bias than when the incumbent is aware (but with an inaccurate assessment) of entrant bias. When the incumbent foresees an overestimation bias ($\hat{\theta} < 1$), she would expect the entrant to position himself close to her so that the incumbent has more incentive to reposition than when the incumbent is unaware of entrant bias, rendering a less competitive environment and benefiting the entrant. Moreover, when the entrant has an underestimation bias ($\theta > 1$) but the incumbent expects the entrant to have an overestimation bias ($\hat{\theta} < 1$), the incumbent’s estimation of entrant behavior is so biased that the incumbent can be better off by being completely unaware of entrant bias. This means that, interestingly, the incumbent’s (increasing) bias can even benefit the incumbent, per se, which differs from our previous result that a firm’s own bias always hurts the firm (Propositions 4, 6, and 9). Indeed, Proposition 10 shows that with increasing bias the incumbent can benefit both herself and the entrant, depending on incumbent bias and entrant bias.

Concluding Remarks

This paper investigates an incumbent’s repositioning costs and associated decision biases within a market entry setting. To this end, we first explore a baseline setting where both the incumbent and entrant are unbiased; the entrant perfectly assesses the incumbent’s repositioning ability, and the incumbent also knows the entrant’s assessment. We find that the incumbent’s repositioning ability can benefit, rather than harm the entrant, and can harm, rather than benefit, the incumbent. This opens the opportunity for the new entrant to perform better than an incumbent with a relatively high repositioning ability, which implies that the incumbent’s repositioning ability does not necessarily contribute to a leader advantage. In addition to being a clean engine to think about repositioning costs and strategic interactions, this baseline model also has the potential to bring a fresh perspective to other applied phenomena. For example, consider strategic interactions between (multinational) firms competing across geographies, where the transfer of learning plays an important role (Kalnins and Mayer, 2004; Alcácer and Zhao, 2012; Alcácer et al., 2015; Seamans and Zhu, 2017). Suppose the incumbent (entrant) has a sister incumbent (sister entrant) owned by the same parent corporation. If the sister incumbent has experience with the entry of the sister entrant, then the incumbent’s own repositioning costs can be significantly reduced as a result of learning across sister units (Alcácer et al., 2015; Seamans and Zhu, 2017). In such a context, our analysis indicates that reduced repositioning costs can lead to (i) lower performance for the incumbent, and (ii) a greater benefit for the entrant than for the incumbent because the incumbent’s relative
performance is reduced (rather than enhanced). In sight of our results, researchers and practitioners should consider beyond the positive or negative effect of repositioning ability on the focal incumbent firm’s absolute performance, which is the primary focus of the existing literature; they should also consider the possible negative effect on her relative performance vis-à-vis competitive entrants, which is often an important strategic concern for firms (Barney, 1991; Chatain and Zemsky, 2007; Drnevich and Kriauciuunas, 2011; Alcácer et al., 2015).

In order to investigate the impact of decision bias on the dynamics between incumbents and entrants, we consider an entrant with either overestimation or underestimation bias regarding the incumbent’s repositioning ability. In a setting with a rational incumbent, who is not only aware of entrant bias, but also has an accurate assessment of his bias, we find it is possible that the biased entrant can make more profits than the incumbent, while an unbiased entrant cannot. That is, entrant bias can be a relative advantage for the entrant. In particular, we find that, although underestimation bias never helps the entrant gain better relative performance, overestimation bias does. This insight is a contribution to the literature on bias in estimating the skill of others (e.g., see Moore and Cain 2007 Goldfarb and Xiao 2011, and Cain et al. 2015), which is often considered detrimental. For example, Anheuser-Busch underestimated the repositioning ability of the incumbent (Frito-Lay), whose helm was just taken by Roger Enrico (Stalk Jr and Lachenauer, 2004), leading to a significant loss for Anheuser-Busch. If Anheuser-Busch had overestimated (rather than underestimated) the incumbent, then our analysis indicates that, although the overestimation bias would still have been harmful for Anheuser-Busch, it could have actually hurt Frito-Lay more, thus boosting Anheuser-Busch’s relative advantage.

Presumably, an incumbent can also be biased with regard to her expectations of the entrant’s behavior. We find that when the incumbent is biased in her assessment of entrant bias, this can create a relative advantage for her. A more striking finding is that both the entrant and incumbent can earn more than in the baseline model where neither firm is biased. Decision bias is therefore not necessarily detrimental for firms, particularly when both the entrant and the incumbent are biased. We also note that the incumbent is more biased when she is completely unaware of entrant bias. In this setting, the incumbent’s increasing bias can interestingly benefit both herself and the entrant.

In the above analyses, we assume that repositioning costs are linear in the magnitude of repositioning. We introduce one extension along this direction by assuming that repositioning costs are fixed and independent with respect to the magnitude of repositioning (Appendix B). This may happen when repositioning is greatly facilitated by flexible manufacturing systems, which consist of comput-
er numerically-controlled machines and other programmable automation that enable the production of different products on the same system. Exploring this extension, we find that our insights that repositioning costs can be a leader advantage for the incumbent (Proposition 13), whereas estimation bias can be a follower advantage for the entrant (Proposition 15), remain true, although quantitative details differ depending on the modeling parameters.

Our core argument is that repositioning costs and the associated biases should be central to analyses of strategic dynamics in the context of market entry and incumbent repositioning. This paper takes one of the first analytical steps to examine the impact of these factors and generates counter-intuitive results. Our model sheds new light on the implications of repositioning costs and decision biases with respect to competitive advantage. While we believe that our analytical results apply broadly to different types of post-entry repositioning across different contexts, future work could extend our study to empirical contexts. For example, future work can follow approaches in the empirical literature (Galasso and Simcoe, 2011; Wang and Shaver, 2014, 2016) to classify different decision biases and different repositioning costs and then accordingly determine market entry decisions and the associated absolute and relative performances of firms. Such an empirical effort would not only test the predictions of the current model, but also offer guidelines for the design and adoption of strategies aimed at enhancing repositioning ability and curtailing the biases of executives.

Figure 1: The equilibrium position $x_e^k$ and profit $\pi_e^k$ for the entrant.
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References


Appendix A: Proofs and Derivations

Justification of (4) When \( x_i < x_e \), the equilibrium prices of the incumbent and entrant are \((p_i, p_e) = (c + \frac{(x_e - x_i)[3 + (x_i + x_e)]}{3}, c + \frac{(x_e - x_i)[3 - (x_i + x_e)]}{3})\) with associated demands \((D_i, D_e) = (\frac{1}{2} + \frac{x_i + x_e}{6}, \frac{1}{2} - \frac{x_i + x_e}{6})\).

When \( x_i > x_e \), the equilibrium prices are \((p_i, p_e) = (c + \frac{(x_i - x_e)[3 + (x_i + x_e)]}{3}, c + \frac{(x_i - x_e)[3 - (x_i + x_e)]}{3})\) with associated demands \((D_i, D_e) = (\frac{1}{2} - \frac{x_i + x_e}{6}, \frac{1}{2} + \frac{x_i + x_e}{6})\). For this case, the incumbent’s profit is

\[
\frac{(x_i - x_e)(3 - x_i - x_e)^2}{18} - k|x_i|
\]

while the entrant’s profit is

\[
\frac{(x_i - x_e)(3 + x_i + x_e)^2}{18}
\]

For both cases \( x_i + x_e \leq 3 \) holds for non-trivial equilibria with non-negative demands. This is indeed the case and is checked in the proof of Lemma 3.

Proof of Lemma 1. a) In this proof, we present the details only for the case \( x_e > 0 \), and \( x_e < 0 \) follows a similar proof.

We first prove that the incumbent’s optimal position is such that \( x_i \leq 0 \) when \( x_e > 0 \). By contradiction, suppose that the incumbent’s optimal decision \( x_i > 0 \) when \( x_e > 0 \). We then have two possible cases: (i) \( x_i < x_e \) and (ii) \( x_i > x_e \). For case (i), the incumbent’s profit in (5) becomes \( \frac{(x_e - x_i)(3 + x_i + x_e)^2}{18} - k x_i \).

However, given \( x_i > 0 \), the incumbent can be better off by positioning at \(-x_i < x_e\) with the associated profit \( \frac{(x_e + x_i)(3 - x_i + x_e)^2}{18} - k x_i \), because

\[
\frac{(x_e + x_i)(3 - x_i + x_e)^2}{18} - k x_i - \left[\frac{(x_e - x_i)(3 + x_i + x_e)^2}{18} - k x_i\right] = \frac{x_i(9 + x_i^2 - 2)}{9} > 0
\]

where the inequality follows from \( x_i + x_e \leq 3 \). For case (ii), the incumbent’s profit in (19) becomes

\[
\frac{(x_i - x_e)(3 - x_i - x_e)^2}{18} - k x_i
\]

which is less than the incumbent profit \( \frac{(x_e + x_i)(3 - x_i + x_e)^2}{18} - k x_i \) by positioning at \(-x_i\).

We now derive the incumbent’s optimal decision. When \( x_e > 0 (\Rightarrow x_i \leq 0) \) the objective function of (5) becomes \( \frac{(x_e - x_i)(3 + x_i + x_e)^2}{18} + k x_i \), which is concave in \( x_i \) with the first-order-condition \( k = \frac{(3 + x_i + x_e)(3 - x_i - x_e)}{18} \Rightarrow x_i = -\frac{6 - x_e + \sqrt{(3 + 2x_e)^2 + 54k}}{3} \). The solution \( x_i = -\frac{6 - x_e - \sqrt{(3 + 2x_e)^2 + 54k}}{3} \) does not satisfy \( x_i + x_e \leq 3 \), and

\[
-\frac{6 - x_e + \sqrt{(3 + 2x_e)^2 + 54k}}{3} \leq 0 \iff k \leq \frac{9 - x_e^2}{18}
\]

The incumbent’s optimal solution is consequently characterized by (6). Moreover, \( -\frac{6 - x_e + \sqrt{(3 + 2x_e)^2 + 54k}}{3} \) increases in \( k \) and \( x_e \) respectively.

b) For any \( x_i \), the objective function in (5) decreases in \( k \). From the envelope theorem, the incumbent’s optimal profit then also decreases in \( k \), i.e., \( \frac{\partial p^*(x_e)}{\partial k} < 0 \). Moreover, for any \( x_i \), the derivative of the
objective function in (5) with respect to $x_e \frac{(3+x_i+x_e)(3+3x_e-x_i)}{18}$ is positive because $x_i+x_e \leq 3 \implies x_i \leq 3$. Consequently, from the envelope theorem, the incumbent’s optimal profit also increases in $x_e$, i.e.,

$$\frac{\partial \pi_k(x_e)}{\partial x_e} > 0.$$ **Proof of Proposition 2.** If the incumbent does not reposition herself ($k \geq \frac{9-x^2_e}{18}$), then the entrant’s profit in (7): $\pi_e^k = \max_{x_e} \frac{x_e(3-x_e)^2}{18} = \frac{2}{5}$ with the equilibrium entrant position $x_e^k = 1$. However, if the incumbent repositions herself ($k < \frac{9-x^2_e}{18}$), then the entrant’s profit in (7) becomes $\pi_e(k, x_e) = \frac{1}{18}(x_e - \frac{\sqrt{54k+3(2x_e+3)^2}}{3} - x_e - 6)[3 - x_e - \frac{\sqrt{54k+3(2x_e+3)^2}}{3} - x_e - 6]^2$ from (6). Here, $\pi_e(k, x_e)$ is supermodular in $k$ and $x_e$, i.e., $\frac{\partial \pi_e(k, x_e)}{\partial x_e \partial k} = \frac{18k(2x_e-9)+4x_e+6}{[54k+(2x_e+3)^2]^{3/2}} > 0$, because $6 + 4x_e + k(2x_e - 9) > x_e[x_e(9-2x_e)+90] + \frac{3}{2} > 0$ from $x_i + x_e < 3$ and $k < \frac{9-x^2_e}{18}$. The entrant equilibrium position $x_e^k$ therefore increases in $k$ when the incumbent repositions herself. For a special case of $k = 0$, the entrant’s equilibrium profit is $\max_{x_e} \frac{(x_e-\frac{2}{3})(3-x_e)}{18} = \frac{8}{9}$ with the associated entrant position $x_e^0 = 0$ from $x_e^k(x_e) = \frac{x_e-3}{3}$ (Lemma 1a).

We now establish the existence of $\hat{k}$. First, the incumbent repositions (does not reposition) herself when $k = 0 \ (k = \infty)$. Second, suppose there exist repositioning parameters $k_1$ and $k_2$ ($> k_1$) such that the incumbent repositions (does not reposition) herself when $k = k_2 \ (k = k_1)$. Then, $k_1 \geq \frac{9-(k_1^2)^2}{18}$ while $k_2 < \frac{9-(k_2^2)^2}{18}$ from Lemma 1a. Consequently, $\frac{9-(k_1^2)^2}{18} < k_1 < k_2 < \frac{9-(k_2^2)^2}{18} \implies x_e^k > x_e^{k_2}$, which contradicts that $x_e^k$ increases in $k$.

**Proof of Proposition 1.** When the incumbent does not reposition herself ($k > \hat{k}$), the entrant’s equilibrium profit is $\pi_e^k = \frac{2}{5}$ from the proof of Lemma 2. However, when the incumbent repositions herself ($k \leq \hat{k}$), the entrant’s profit is $\pi_e(k, x_e) = \frac{|x_e-x_e^k(x_e)||3-x_e^k(x_e)-x_e|^2}{18}$, where $x_e^k(x_e)$ is shown as in (6). From $\frac{\partial \pi_e(k, x_e)}{\partial x_e^{k}(x_e)} = -\frac{|3+x_e-3x_e^k(x_e)||3-x_e^k(x_e)|}{18} < 0$ and $\frac{\partial x_e^{k}(x_e)}{\partial k} > 0$ (Lemma 1a), $\pi_e(k, x_e)$ decreases in $x_e$ for any $k$, implying the entrant’s equilibrium profit $\pi_e^k$ decreases in $k$.

**Proof of Lemma 3.** From (8), when $k < \hat{k}$, $\frac{dx_k}{dk} = \frac{(3+2x_e^k)}{3\sqrt{(3+2x_e^k)^2}} - \frac{1}{3} > 0$, because $\frac{dx_k}{dk} > 0$ (Lemma 2) and $k < \frac{9-(k^2)^2}{18} < \frac{3+2x_e^k}{18} \implies \frac{3+2x_e^k}{18} > \frac{1}{3}$.

Furthermore, in equilibrium, $x_i^k + x_e^k < 3$ always holds because $x_i^k + x_e^k$ increases in $k$ from $x_e^k$ increases in $k$ (Lemma 2), while $x_i^k + x_e^k = 1$ when $k = \infty$.

**Proof of Proposition 2.** When $k > \hat{k}$, the equilibrium positions are $(x_e^k, x_i^k) = (1, 0)$ from the proof of Lemma 2. As a result, the incumbent’s equilibrium profit $\pi_i^k = \frac{(x_e-x_i)(3+x_i+x_e)^2}{18} - k|x_i| = \frac{1}{2}$. However, the incumbent’s profit $\pi_i^k$ can increase when $k \leq \hat{k}$. It is sufficient to consider the repositioning parameter around $\hat{k}$, where the incumbent does not reposition (repositions) herself when $k$ is relatively low (high), i.e., $k = \hat{k}^- \ (k = \hat{k}^+)$ For an instantaneous increase of the repositioning parameter from $k^-$ to $k^+$, the
entrant’s position significantly increases and \( x_e^k < x_e^{k'} \); see Figure 1. Consequently, as \( k \) increases from \( k^- \) to \( k^+ \), the incumbent’s equilibrium profit increases around \( k \), i.e., \( \pi^k_i (x_e^k) > \pi^k_i (x_e^{k-}) \), because \( \pi^k_i (x_e) \) increases in \( x_e \) (Lemma 1b).

**Proof of Proposition 3.** When \( k > \hat{k} \), from the proofs of Lemma 2 and Proposition 1, \( \pi^k_i = \frac{1}{2} > \pi^k_e = \frac{2}{5} \implies \Delta^k > 0 \). When \( k \leq \hat{k} \), it is sufficient to consider \( k = 0 \), where \( \pi^k_e = \frac{8}{9} > \pi^k_i = \frac{(x_e-x_i)(3+x_e+x_e)^2}{18} - k|x_i| = \frac{2}{5} \implies \Delta^k < 0 \).

**Proof of Proposition 4.** a) We first show that the entrant’s profit \( \pi_e(\theta) \) in (11) is concave in the entrant’s position \( \hat{x}_e(\theta) \). When the repositioning parameter is relatively low \( (k < \frac{9-\hat{x}_e^2}{18}) \), we define \( A = \sqrt{54k + (2\hat{x}_e + 3)^2} \); then \( g(\hat{x}_e, k) := \frac{d^2\hat{x}_e}{dx_e^2} = \frac{8[729k^2+27k(4\hat{x}_e(A+3\hat{x}_e)-81-6A)+(2\hat{x}_e-3)(2\hat{x}_e+3)^2(A+2\hat{x}_e+3)]}{81A^4} \) from (10). Furthermore, \( \frac{\partial g(\hat{x}_e, k)}{\partial \theta} = \frac{16[729k^2(4A+18\hat{x}_e-9)+(2\hat{x}_e-3)A+(2\hat{x}_e+3)(2\hat{x}_e+3)](4A+5\hat{x}_e+15)+3(4A+69))}{81A^4} \) \( > 0 \), and \( g(\hat{x}_e, k)|_{\hat{x}_e=1} = \frac{8[25\sqrt{54k+25}+125+27k(2\sqrt{54k+25}+69-27k)]}{81(54k+25)^{1.4}} > 0 \) when \( k < \frac{9-\hat{x}_e^2}{18} \leq \frac{1}{2} \). That is, \( g(\hat{x}_e, k) < 0 \) always holds when \( k < \frac{9-\hat{x}_e^2}{18} \). When the repositioning parameter is relatively high \( (k \geq \frac{9-\hat{x}_e^2}{18}) \), the entrant’s profit \( \frac{\pi_e(3-x_e)^2}{18} \) is also concave in \( \hat{x}_e \).

We now show that the entrant’s position \( \hat{x}_e(\theta) \) increases in \( \theta \). The entrant’s position \( \hat{x}_e(\theta) \) increases in \( k \) (Lemma 2). The entrant also makes decisions based on \( \theta k \) rather than \( k \); see (9). The entrant’s position \( \hat{x}_e(\theta) \) accordingly increases in \( \theta \).

Consequently, the entrant’s profit \( \pi_e(\theta) \) is concave in \( \theta \). Since the entrant makes a higher profit when he is unbiased \( (\theta = 1) \) versus biased, the entrant’s profit \( \pi_e(\theta) \) decreases (increases) in \( \theta \) when \( \theta > 1 \) \( (\theta < 1) \).

b) From (12), the incumbent’s profit \( \hat{\pi}_i(\theta) \) is affected by \( \theta \) via \( \hat{x}_e(\theta) \), which increases in \( \theta \) from part a). Moreover, the incumbent’s profit increases in the entrant’s position \( \hat{x}_e(\theta) \) (Lemma 6b). As a result, the incumbent’s profit \( \hat{\pi}_i(\theta) \) increases in \( \theta \).

**Proof of Proposition 5.** When \( \theta > 1 \), \( \hat{\pi}_i(\theta) \) increases in \( \theta \) (Proposition 4b) while \( \hat{\pi}_e(\theta) \) decreases in \( \theta \) (Proposition 4a). Thus, \( \hat{\Delta}(\theta) = \hat{\pi}_i(\theta) - \hat{\pi}_e(\theta) \) increases in \( \theta - 1 \) when \( \theta > 1 \). When \( \theta < 1 \),

\[
\hat{\Delta}'(\theta)|_{\theta=1} = \hat{\pi}'_i(\theta)|_{\theta=1} - \hat{\pi}'_e(\theta)|_{\theta=1} = \left( \frac{d\hat{\pi}_i(\theta)}{d\hat{x}_e(\theta)} \right) \left( \frac{d\hat{x}_e(\theta)}{d\theta} \right) \bigg|_{\theta=1} - \left( \frac{d\hat{\pi}_e(\theta)}{d\hat{x}_e(\theta)} \right) \left( \frac{d\hat{x}_e(\theta)}{d\theta} \right) \bigg|_{\theta=1} = \left( \frac{3 + \hat{x}_i(\theta) + \hat{x}_e(\theta))}{18} \right) \left( \frac{3 + 3\hat{x}_e(\theta) - \hat{x}_i(\theta)}{18} \right) \frac{d\hat{x}_e(\theta)}{d\theta} \bigg|_{\theta=1} > 0,
\]

where the second equality follows from \( \left( \frac{d\hat{\pi}_e(\theta)}{d\hat{x}_e(\theta)} \right) \bigg|_{\theta=1} = 0 \) (the envelope theorem). Thus, \( \hat{\Delta}(\theta) \) decreases in
1 − θ around θ = 1.

**Follower Advantage without Repositioning Costs.** Let \( x_i^0 \) represent the incumbent’s position after repositioning. Given \( x_i^0 \) and \( \hat{x}_e(\theta) \), from (11)-(12), the entrant’s profit is \( \pi_e = \frac{[\hat{x}_e(\theta) - x_i^0][3 - x_i^0 - \hat{x}_e(\theta)]^2}{18} \) while the entrant’s profit is \( \pi_i = \frac{[\hat{x}_e(\theta) - x_i^0][3 + x_i^0 + \hat{x}_e(\theta)]^2}{18} \) if firms engage in a price competition again. As a result, \( \pi_e - \pi_i = 2(\frac{\hat{x}_e(\theta) - x_i^0}{3}) \left[ 3 - \frac{\hat{x}_e(\theta)}{2} \right] > 0 \iff |x_i^0| > |\hat{x}_e(\theta)|. \)

**Proof of Proposition 6.** The entrant’s profit defined in (12) solves the optimization problem and consequently is higher than the profit defined in (14). That is, \( \hat{\pi}_i(\theta) \geq \hat{\pi}_i(\theta, \hat{\theta}) \) for any \( \hat{\theta} \). We next compare the relative performances \( \hat{\Delta}(\theta, \hat{\theta}) \) and \( \hat{\Delta}(\theta) \):

\[
\frac{\partial \hat{\Delta}(\theta, \hat{\theta})}{\partial \hat{\theta}} \bigg|_{\hat{\theta} = \theta} - \frac{d \hat{\Delta}(\theta)}{d \theta} \bigg|_{\hat{\theta} = \theta} = \frac{\partial \hat{\pi}_i(\theta, \hat{\theta})}{\partial \hat{\theta}} \bigg|_{\hat{\theta} = \theta} - \frac{\partial \hat{\pi}_e(\theta, \hat{\theta})}{\partial \hat{\theta}} \bigg|_{\hat{\theta} = \theta}
\]

\[
= - \frac{\partial \hat{\pi}_e(\theta, \hat{\theta})}{\partial \hat{\theta}} \bigg|_{\hat{\theta} = \theta} \frac{\partial \hat{\Delta}(\theta, \hat{\theta})}{\partial \hat{\theta}} \bigg|_{\hat{\theta} = \theta} \frac{d \hat{x}_i(\hat{\theta})}{d \theta} \bigg|_{\hat{\theta} = \theta}
\]

\[
= \frac{[3 + \hat{x}_e(\theta) - 3\hat{x}_i(\theta)][3 - \hat{x}_i(\theta) - \hat{x}_e(\theta)]}{18} \frac{d \hat{x}_i(\hat{\theta})}{d \theta} \bigg|_{\hat{\theta} = \theta}
\]

\[
= \frac{[3 + \hat{x}_e(\theta) - 3\hat{x}_i(\theta)][3 - \hat{x}_i(\theta) - \hat{x}_e(\theta)]}{18} \frac{d \hat{x}_i(\hat{\theta})}{d \theta} \bigg|_{\hat{\theta} = \theta}
\]

\[
> 0,
\]

where the second equality is from the envelope theorem, the third and fourth equalities are from (13), the last equality is from (10), and the last inequality is from \( \frac{d \hat{x}_i(\hat{\theta})}{d \hat{x}_e(\theta)} > 0 \) (Lemma 1b) and \( \frac{\hat{x}_e(\hat{\theta})}{\hat{x}_e(\hat{\theta})} > 0 \) (the proof of Proposition 4a). We therefore conclude because \( \hat{\Delta}(\theta, \hat{\theta} = \theta) = \hat{\Delta}(\theta) \).

**Proof of Proposition 7.** From (13),

\[
\frac{\partial \hat{\pi}_e(\theta, \hat{\theta})}{\partial \hat{\theta}} = \frac{\partial \hat{\pi}_e(\theta, \hat{\theta})}{\partial \hat{x}_i(\hat{\theta})} \frac{d \hat{x}_i(\hat{\theta})}{d \theta} \bigg|_{\hat{\theta} = \theta}
\]

\[
= - \frac{[3 + \hat{x}_e(\theta) - 3\hat{x}_i(\theta)][3 - \hat{x}_i(\theta) - \hat{x}_e(\theta)]}{18} \frac{d \hat{x}_i(\hat{\theta})}{d \theta} \bigg|_{\hat{\theta} = \theta}
\]

\[
= - \frac{[3 + \hat{x}_e(\theta) - 3\hat{x}_i(\theta)][3 - \hat{x}_i(\theta) - \hat{x}_e(\theta)]}{18} \frac{d \hat{x}_i(\hat{\theta})}{d \theta} \bigg|_{\hat{\theta} = \theta}
\]

\[
< 0,
\]

where the inequality is from \( \frac{d \hat{x}_i(\hat{\theta})}{d \hat{x}_e(\theta)} > 0 \) (Lemma 1b) and \( \frac{\hat{x}_e(\hat{\theta})}{\hat{x}_e(\hat{\theta})} > 0 \) (the proof of Proposition 4a). We
can therefore conclude because \( \hat{\pi}_e(\theta, \hat{\theta}) = \hat{\pi}_e(\theta) \) when \( \hat{\theta} = \hat{\theta} \).

**Proof of Proposition 8.** First, entrant bias hurts the entrant because \( \hat{\pi}_e(\theta) \leq \pi_e^k \) from (7) and (11). Moreover, incumbent bias hurts the incumbent \( \hat{\pi}_i(\theta, \hat{\theta}) \leq \hat{\pi}_i(\theta) \) from Proposition 6. Second, we show that biased firms can earn more profit than unbiased firms. It is sufficient to consider \( k = 0 \). From (13)-(14), we find that
\[
\hat{\pi}_e(\theta, \hat{\theta}) = \frac{|\hat{\pi}_e(\theta) - \hat{\pi}_e(\hat{\theta})|3 - \hat{\pi}_e(\hat{\theta}) - \hat{\pi}_e(\theta)|}{18} > \pi_e^k = \frac{8}{9} \text{ and } \hat{\pi}_i(\theta, \hat{\theta}) = \frac{|\hat{\pi}_i(\theta) - \hat{\pi}_i(\hat{\theta})|3 + \hat{\pi}_i(\hat{\theta}) + \hat{\pi}_i(\theta)|}{18} > \pi_i^k = \frac{2}{9} \text{ hold when } \hat{\pi}_i(\hat{\theta}) \text{ is relatively low and } \hat{\pi}_e(\theta) \text{ is relatively high.}
\]

**Proof of Proposition 9.** a) The incumbent who is aware of entrant bias has the equilibrium profit in (12) \( \hat{\pi}_i(\theta) = \max_{x_i} \left[ \frac{|\hat{\pi}_e(\theta) - x_i|3 + x_i + \hat{x}_e(\theta)|}{18} - k|x_i| \right] \geq \frac{|\hat{\pi}_e(\theta) - x_i^k|3 + x_i^k + \hat{x}_e(\theta)|}{18} - k|x_i^k| = \hat{\pi}_i(\theta) \). That is, the unawareness of incumbent leads to a lower profit for the incumbent.

For the relative profit, consider \( \theta = 1 - \epsilon \), where \( \epsilon > 0 \) is significantly small. As the incumbent becomes unaware of entrant bias, the incumbent’s position changes (increases). However, the incumbent’s loss from unawareness is significantly small (zero from the envelope theorem) because \( \epsilon \) is significantly small and \( \theta \approx 1 \). However, the entrant’s loss from the incumbent’s position is comparatively large. Moreover, the entrant’s loss dominates the incumbent’s loss and incumbent unawareness helps her relative performance.

b) When \( \theta < 1 \), the incumbent unawareness leads to an increasing incumbent position, which hurts the entrant.

c) First, entrant bias hurts the entrant because \( \hat{\pi}_e(\theta) \leq \pi_e^k \) from (7) and (11). Moreover, incumbent bias hurts the incumbent \( \hat{\pi}_i(\theta) \leq \hat{\pi}_i(\theta) \) from part a) in this proof. Second, we show that biased firm can earn more profit than unbiased firms. It is sufficient to consider \( k = 0 \). From (16)-(17),
\[
\hat{\pi}_e(\theta) = \frac{|\hat{\pi}_e(\theta) - x_i^k|3 + x_i^k + \hat{x}_e(\theta)|}{18} > \pi_e^k = \frac{8}{9} \text{ and } \hat{\pi}_i(\theta) = \frac{|\hat{\pi}_i(\theta) - x_i^k|3 + x_i^k + \hat{x}_e(\theta)|}{18} > \pi_i^k = \frac{2}{9} \text{ hold when } x_i^k \text{ is relatively low and } \hat{x}_e(\theta) \text{ is relatively high.}
\]

**Proof of Proposition 10.** Since \( \frac{\partial \hat{\pi}_e(\theta, \hat{\theta})}{\partial \hat{\theta}} < 0 \) (the proof of Proposition 7), \( \hat{\pi}_e(\theta) = \hat{\pi}_e(\theta, \hat{\theta} = 1) > \hat{\pi}_e(\theta, \hat{\theta}) \) when \( \hat{\theta} < 1 \). The objective function of (14) shows that the incumbent’s profit is concave in \( \hat{x}_i(\hat{\theta}) \) for any \( \hat{x}_e(\theta) \) from the proof of Lemma 1. For any entrant position \( \hat{x}_e(\theta) \), we define \( \hat{x}_i^*(\theta) = \arg \max_{x_i} \left[ \frac{|\hat{\pi}_e(\theta) - x_i|3 + x_i + \hat{x}_e(\theta)|}{18} - k|x_i| \right] \). When \( \theta > 1 \), \( \hat{x}_e(\theta) > \hat{x}_e(1) = x_e^k \) because \( \hat{x}_e(\theta) \) increases in \( \theta \) from the proof of Proposition 4. Thus, \( \hat{x}_i^*(\theta) = x_e^k(\hat{x}_e(\theta)) > x_e^k(x_e^k) = x_i^k \) because \( x_i^k(x_e) \) increases in \( x_e \) (Lemma 1) and \( \hat{x}_e(\theta) > x_e^k \). Furthermore, \( x_i^k = \hat{x}_i(\hat{\theta} = 1) > \hat{x}_i(\hat{\theta}) \) when \( \hat{\theta} < 1 \) because \( \hat{x}_i(\hat{\theta}) \) increases in \( \hat{\theta} \) from the proof of Proposition 7. When \( \theta > 1 > \hat{\theta} \), \( \hat{x}_i^*(\theta) > x_i^k > \hat{x}_i(\hat{\theta}) \), and consequently \( \hat{\pi}_i(\theta) > \hat{\pi}_i(\theta, \hat{\theta}) \) from (14) and (17).
Appendix B: Fixed Repositioning Costs

In this extension, we assume that the incumbent’s repositioning costs are independent with the magnitude of changes. That is, the incumbent repositioning problem is:

$$\pi^k_i(x_e) : = \max_{x_i} \frac{(x_e - x_i)(3 + x_i + x_e)^2}{18} - \mathbb{I}_{x_i \neq 0}k,$$  \hspace{1cm} (21)

where $\mathbb{I}$ is the indicator function. If the incumbent repositions herself, then she will incur fixed repositioning costs $k$; otherwise, the incumbent does not incur repositioning costs. Next, we characterize the incumbent’s optimal position level $x^k_i$ and the associated incumbent’s profit $\pi^k_i(x_e)$.

Lemma 4. Given the entrant’s position $x_e(> 0)$:

a) The incumbent’s optimal position is

$$x^k_i(x_e) = \begin{cases} 
0 & \text{if } k \geq \frac{(3-2x_e)^2(12+5x_e)}{486} \\
\frac{x_e-3}{3} & \text{if } k < \frac{(3-2x_e)^2(12+5x_e)}{486}.
\end{cases}$$  \hspace{1cm} (22)

Moreover, $\frac{\partial x^k_i(x_e)}{\partial x_e} > 0$ when $k < \frac{(3-2x_e)^2(12+5x_e)}{486}$.

b) The incumbent’s optimal profit $\pi^k_i(x_e)$ satisfies $\frac{\partial \pi^k_i(x_e)}{\partial k} \leq 0$ and $\frac{\partial \pi^k_i(x_e)}{\partial x_e} \geq 0$.

Anticipating the incumbent’s repositioning strategy above, the new entrant can accordingly decide his position. The entrant’s profit then becomes

$$\pi^k_e : = \max_{x_e} \frac{[x_e - x^k_i(x_e)][3 - x^k_i(x_e) - x_e]^2}{18},$$  \hspace{1cm} (23)

where $x^k_i(x_e)$ is from (22). We next characterize the entrant’s equilibrium position $x^k_e$ in Lemma 5 below.

Lemma 5. Let $x^k_e$ denote the solution of (28) for any given $k$. Then, $x^k_e = 1$ for $k > \frac{2}{9}$ whereas $x^k_e = 0$ for $k \leq \frac{2}{9}$.

Lemma 5 characterizes the new entrant’s equilibrium position and how it changes with the repositioning parameter $k$. For a relatively large $k$ ($k > \frac{2}{9}$), the incumbent does not reposition herself and $x^k_e = 1$. However, for a relatively small $k$ ($k \leq \frac{2}{9}$), the incumbent repositions himself, and her new position is $x^k_e = 0$. In other words, when the repositioning is getting easier ($k$ is smaller), the entrant position $x^k_e$ is closer to (or less differentiated with) the incumbent’s original position $x = 0$. 

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Proposition 11. When \( k > \frac{2}{5} \), the entrant’s equilibrium profit is \( \pi^k_e = \frac{2}{9} \). However, \( \pi^k_e = \frac{8}{9} \) when \( k \leq \frac{2}{5} \).

Because the incumbent does not reposition herself when the repositioning parameter is high (Lemma 5), the entrant’s profit \( \pi^k_e = \frac{2}{9} \) when \( k > \frac{2}{5} \). However, when the incumbent repositions \( (k \leq \frac{2}{5}) \), the entrant’s profit \( \pi^k_e = \frac{8}{9} \). That is, a higher incumbent’s repositioning ability (a lower \( k \)) can translate to a higher profit for the entrant.

Given the entrant’s equilibrium position and profit, we now turn our attention to the incumbent. From (22) and the entrant equilibrium position described in Lemma 5, the incumbent’s position is

\[
x^k_i := \begin{cases} 0 & \text{if } k > \frac{2}{5} \\ -1 & \text{if } k \leq \frac{2}{5} \end{cases}
\]

in the equilibrium. We next evaluate the incumbent’s equilibrium profit \( \pi^k_i := \pi^k_i(x^k_e) \), where \( \pi^k_i(x^k_e) \) and \( x^k_e \) are from (21) and Lemma 5, respectively.

Proposition 12. The incumbent’s equilibrium profit is \( \pi^k_i = \frac{1}{2} \) when the incumbent’s repositioning parameter is relatively high \( (k > \frac{2}{5}) \). However, \( \pi^k_i = \frac{2}{5} - k \) when \( k \leq \frac{2}{5} \).

Proposition 12 shows the incumbent can benefit for a higher \( k \) (from \( k \leq \frac{2}{5} \) to \( k > \frac{2}{5} \)) in the equilibrium with an endogenous entrant position. This means that higher repositioning costs can imply a higher incumbent profit. After characterizing the entrant’s and the incumbent’s profits, we next evaluate the relative performance.

Proposition 13. The incumbent earns more profit than the entrant when the incumbent’s repositioning parameter is high \( (k > \frac{2}{5}) \). However, the incumbent can earn less profit than the entrant when the incumbent’s repositioning parameter is relatively low \( (k \leq \frac{2}{5}) \).

Proposition 13 shows that the incumbent makes a higher profit versus the new entrant when his repositioning parameter is relatively high \( (k > \frac{2}{5}) \). However, when the repositioning parameter is relatively small \( (k \leq \frac{2}{5}) \), although the incumbent firm is more efficient in repositioning herself, her position advantage over the entrant can disappear so that she makes less profit than the entrant.

We now study the case where the incumbent’s cost is not perfectly known by the entrant. Given the incumbent’s repositioning parameter \( k \), the entrant solves the following problem

\[
\max_{x_e} \frac{[x_e - x^0_i(x_e)][3 - x^0_i(x_e) - x_e]^2}{18},
\]

(25)
where $x_{i}^{\theta k}(x_e)$ is the incumbent’s repositioning decision for a repositioning parameter $\theta k$, i.e.,

$$x_{i}^{\theta k}(x_e) = \begin{cases} 
0 & \text{if } k \geq \frac{(3-2x_e)^2(12+5x_e)}{4860} \\
\frac{x_e-3}{3} & \text{if } k < \frac{(3-2x_e)^2(12+5x_e)}{4860}.
\end{cases} \quad (26)$$

Given (25)-(26), we next characterize the entrant’s equilibrium position and the associated equilibrium profit:

$$\pi_{xe}^{k}(\theta) := \left[ x_{xe}^{k}(\theta) - x_{i}^{\theta k}(x_{xe}(\theta)) \right] \left[ 3 - x_{i}^{k}(x_{xe}(\theta)) - x_{xe}^{k}(\theta) \right] \frac{18}{2}. \quad (27)$$

**Lemma 6.** Given the repositioning parameter $k$:

a) Let $x_{xe}^{k}(\theta)$ denote the solution of (25). Then, $x_{xe}^{k}(\theta)$ increases in $\theta$.

b) The entrant’s equilibrium profit $\pi_{xe}^{k}(\theta)$ decreases in $\theta$ when he has the underestimation bias ($\theta > 1$), while $\pi_{xe}^{k}(\theta)$ increases in $\theta$ when he has the overestimation bias ($\theta < 1$).

Given Lemma 6, we next evaluate the incumbent’s equilibrium profit:

$$\pi_{i}^{k}(\theta) := \pi_{i}^{k}(x_{xe}^{k}(\theta)) = \max_{x_i} \frac{[x_{xe}^{k}(\theta) - x_{i}][3 + x_{i} + x_{xe}^{k}(\theta)]^2}{18} - 1_{x_i \neq k}. \quad (28)$$

We next evaluate the impacts of the entrant’s bias on the incumbent’s bias parameter $\theta$.

**Proposition 14.** The incumbent’s profit $\pi_{i}^{k}(\theta)$ increases in $\theta$.

Proposition 14 shows that the incumbent’s profit increases in the bias parameter $\theta$. This implies that, when the entrant underestimates the incumbent’s repositioning ability ($\theta > 1$), the incumbent’s profit $\pi_{i}^{k}(\theta)$ increases in the level of entrant’s bias $\theta - 1$.

**Proposition 15.** Define $\Delta_{k}^{k}(\theta) := \pi_{i}^{k}(\theta) - \pi_{i}^{k}(\theta)$. Then, when $\theta > 1$, then $\Delta_{k}^{k}(\theta)$ decreases in the level of the entrant’s bias $(\theta - 1)$, i.e., $\theta > 1 \implies \frac{d\Delta_{k}^{k}(\theta)}{d\theta} > 0$. However, when $\theta < 1$, $\Delta_{k}^{k}(\theta)$ can increase in the level of the entrant’s bias $(1 - \theta)$.

Proposition 15 shows that the entrant’s underestimation bias ($\theta > 1$) always drags down the entrant’s relative performance, namely, the follower advantage $\Delta_{k}^{k}(\theta)$. However, the entrant’s overestimation bias ($\theta < 1$) can boost the entrant’s follower advantage.
Proofs of Appendix B

Proof of Lemma 4. a) If the incumbent does not reposition, then her profit is \( \frac{x_e(3+x_e)^2}{18} \). However, if the incumbent repositions herself, then her profit is \( \max_{x_i} \frac{(x_i-x_e)(3+x_i+x_e)^2}{18} - k \). For the second case, the first-order-condition yields \( x_i = -3 - x_e \) or \( x_i = \frac{2x_e-3}{3} \). Note that \( x_i = -3 - x_e \) leads to \( \pi_i^e(x_e) = -k \leq 0 \), whereas \( \pi_i^e(x_e) = \frac{2(3+2x_e)^3}{243} - k \) when \( x_i = \frac{2x_e-3}{3} \). Consequently, \( x_i = \frac{2x_e-3}{3} \) is the unique solution if \( \frac{2(3+2x_e)^3}{243} - k > \frac{x_i(3+x_e)^2}{18} \iff k < \frac{(3-x_e)^2(12+5x_e)}{486} \). Otherwise, the incumbent does not reposition and \( x_i(k, x_e) = 0 \). Last, \( x_i = \frac{2x_e-3}{3} \) increases in \( x_e \).

b) When \( k \) is relatively low (\( k \leq \frac{(3-2x_e)^2(12+5x_e)}{486} \)), the incumbent’s profit \( \frac{x_e(3+x_e)^2}{18} \) increases in \( x_e \). However, when \( k \) is relatively high (\( k > \frac{(3-2x_e)^2(12+5x_e)}{486} \)), the incumbent’s profit \( \frac{2(3+2x_e)^3}{243} - k \) decreases in \( k \) and increases in \( x_e \).

Proof of Lemma 5. From Lemma 4, we know that when \( k \leq \frac{(3-2x_e)^2(12+5x_e)}{486} \), the incumbent’s position is \( x_i = \frac{2x_e-3}{3} \). As a result, the entrant’s profit \( \frac{8(3-x_e)^2(3+2x_e)}{243} \) has the first-order-solution \( x_e = 0 \) and the associated profit \( \frac{8}{9} \). However, as \( k > \frac{(3-2x_e)^2(12+5x_e)}{486} \), the incumbent’s position is \( x_i = 0 \). As a result, the entrant’s position \( \frac{(3-x_e)^2}{18} \) has the first-order-solution \( x_e = 1 \) and the associated profit \( \frac{2}{9} \). Since \( \frac{8}{9} > \frac{2}{9} \), we can see that if \( k < \frac{(3-2x_e)^2(12+5x_e)}{486} \), then the optimal entrant’s decision is \( x_e^k = 0 \). However, \( \frac{(3-2x_e)^2(12+5x_e)}{486} \)

Proof of Proposition 11. When \( k > \frac{2}{9} \), \( (x_e, x_i) = (1, 0) \implies \pi_i^k = \frac{2}{9} \). However, when \( k \leq \frac{2}{9} \), \( (x_e, x_i) = (0, -1) \implies \pi_i^k = \frac{8}{9} \).

Proof of Proposition 12. When \( k > \frac{2}{9} \), \( (x_e, x_i) = (1, 0) \implies \pi_i^k = \frac{8}{9} \). However, when \( k \leq \frac{2}{9} \), \( (x_e, x_i) = (0, -1) \implies \pi_i^k = \frac{2}{9} - k \).

Proof of Proposition 13. From Propositions 11-12, \( \pi_i^k = \frac{8}{9} > \pi_i^k = \frac{2}{9} \) when \( k > \frac{2}{9} \). However, when \( k \leq \frac{2}{9} \), \( \pi_i^k = \frac{2}{9} - k < \pi_i^k = \frac{8}{9} \).

Proof of Lemma 6. a) When \( k < \frac{(3-2x_e)^2(12+5x_e)}{486} \), the incumbent’s position is \( x_i = \frac{2x_e-3}{3} \). The entrant therefore has the first-order-solution \( x_e = 0 \) with the associated profit \( \frac{8}{9} \). As \( k \geq \frac{(3-2x_e)^2(12+5x_e)}{486} \), the incumbent’s position is \( x_i = 0 \). As a result, the entrant has the first-order-solution \( x_e = 1 \) and the associated profit \( \frac{2}{9} \). Since \( \frac{8}{9} > \frac{2}{9} \), we can see that if \( k < \frac{(3-2x_e)^2(12+5x_e)}{486} \), then the optimal entrant’s decision is \( x_e^k = 0 \). However, \( \frac{(3-2x_e)^2(12+5x_e)}{486} \)

b) When \( k > \frac{2}{9} \), \( x_e^k(\theta) = 1 \) and \( \left( \frac{(3-2x_e)^2(12+5x_e)}{486} \right)_{x_e=1} = \frac{17}{486} \). Thus, when \( k > \frac{2}{9} \), the optimal decision is \( x_e = 1 \). Consequently, \( x_e^k(\theta) \) increases in \( \theta \).

b) When \( k > \frac{2}{9} \), \( x_e^k(\theta) = 1 \) and \( \left( \frac{(3-2x_e)^2(12+5x_e)}{486} \right)_{x_e=1} = \frac{17}{486} \). However, when \( k < \frac{2}{9} \), \( x_e^k(\theta) = 0 \).
Consequently, \( \frac{(3-2x_e)^2(12+5x_e)}{486} \bigg|_{x_e=0} = \frac{2}{9} (\frac{17}{486}). \)

We first analyze the case of \( \theta > 1 \) so that \( \frac{2}{9} > \frac{2}{49}. \) We have two subcases: (i) \( \frac{17}{486} < \frac{2}{99}. \) For this subcase, \((x_e, x_i) = (0, -1)\) and \((\pi_e, \pi_i) = (\frac{8}{9}, \frac{2}{9} - k)\) if \( k < \frac{2}{99}, \) then \((x_i, x_e) = (1, 0)\) and \((\pi_e, \pi_i) = (\frac{2}{9}, \frac{8}{9} - k).\) (ii) \( \frac{17}{486} > \frac{2}{99}. \) For this subcase if \( k < \frac{2}{99} \) then \((x_e, x_i) = (0, -1)\) and \((\pi_e, \pi_i) = (\frac{8}{9}, \frac{2}{9} - k)\); if \( \frac{2}{99} < k < \frac{17}{486} \) then \((x_e, x_i) = (0, -\frac{2}{3})\) and \((\pi_e, \pi_i) = (\frac{160}{243}, \frac{49}{243} - k);\) if \( k > \frac{17}{486} \) then \((x_e, x_i) = (1, 0)\) and \((\pi_e, \pi_i) = (\frac{2}{9}, \frac{8}{9} - k).\) We now analyze the case of \( \theta < 1 \) so that \( \frac{2}{9} < \frac{2}{99}. \) If \( k < \frac{2}{9} \) then \((x_e, x_i) = (0, -1)\) and \((\pi_e, \pi_i) = (\frac{8}{9}, \frac{2}{9} - k)\); if \( \frac{2}{9} < k < \frac{2}{99} \) then \((x_e, x_i) = (0, 0)\) and \((\pi_e, \pi_i) = (0, 0);\) if \( k > \frac{2}{99} \) then \((x_e, x_i) = (1, 0)\) and \((\pi_e, \pi_i) = (\frac{2}{9}, \frac{8}{9}).\) As a result, when \( \theta > 1,\)

\[
(\pi_e^k(\theta), \pi_i^k(\theta)) = \begin{cases} 
\left( \frac{8}{9}, \frac{2}{9} - k \right) & \text{if } k \leq \frac{2}{99} \\
\left( \frac{160}{243}, \frac{49}{243} \right) & \text{if } \frac{2}{99} < k \leq \frac{17}{486} \\
\left( \frac{2}{9}, \frac{8}{9} \right) & \text{if } k \geq \max\left\{ \frac{2}{99}, \frac{17}{486} \right\}.
\end{cases}
\tag{29}
\]

However, when \( \theta < 1,\)

\[
(\pi_e^k(\theta), \pi_i^k(\theta)) = \begin{cases} 
\left( \frac{8}{9}, \frac{2}{9} - k \right) & \text{if } k \leq \frac{2}{9} \\
(0, 0) & \text{if } \frac{2}{9} < k \leq \frac{2}{99} \\
\left( \frac{2}{9}, \frac{8}{9} \right) & \text{if } k > \frac{2}{99}.
\end{cases}
\tag{30}
\]

We can see that the entrant’s equilibrium profit \( \pi_e^k(\theta) \) decreases in \( \theta \) when \( \theta > 1, \) whereas \( \pi_e^k(\theta) \) increases in \( \theta \) when \( \theta < 1. \)

\[\square\]

**Proof of Proposition 14.** From the proof of Lemma 6, we can conclude.

\[\square\]

**Proof of Proposition 15.** When \( \theta > 1, \) then \( \Delta^k(\theta) \) increases in \( \theta \) (the proof of Lemma 6). However, when \( \theta < 1, \) then \( \Delta^k(\theta) = 0 \) if \( \frac{2}{9} < k < \frac{2}{99}, \) while \( \Delta^k(\theta) = -\frac{6}{9} \) if \( k > \frac{2}{99}. \) That is, \( \Delta^k(\theta) \) decreases as \( \theta \) increases. Accordingly, \( \Delta^k(\theta) \) can increase in \( 1 - \theta. \)

\[\square\]