

Giant Thermal Expansion in 2D and 3D Cellular Materials

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When temperature increases, the volume of an object changes. This property was quantified as the coefficient of thermal expansion only a few hundred years ago. Part of the reason is that the change of volume due to the variation of temperature is in general extremely small and imperceptible. Here, abnormal giant linear thermal expansions in different types of two-ingredient microstructured hierarchical and self-similar cellular materials are reported. The cellular materials can be 2D or 3D, and isotropic or anisotropic, with a positive or negative thermal expansion due to the convex or/and concave shape in their representative volume elements respectively. The magnitude of the thermal expansion coefficient can be several times larger than the highest value reported in the literature. This study suggests an innovative approach to develop temperature-sensitive functional materials and devices.

Most materials have a positive thermal expansion coefficient (PTEC) and they expand isotropically when heated. The thermal expansion coefficient (TEC) of solid materials is usually in the order of $3 \times 10^{-6} \text{ K}^{-1}$ for ceramics, 10^{-5} K^{-1} for metals, and 10^{-4} K^{-1} for polymers.^[1,2] Very few unusual materials^[3–7] have a negative thermal expansion coefficient (NTEC) and their lattice dimensions shrink with heating. Large negative thermal expansion is usually anisotropic,^[4–6] or even shrinking in one direction and expanding in another direction. Although quite a large isotropic NTEC $\alpha = -1.2 \times 10^{-3} \text{ K}^{-1}$ has been found for a solid polyacrylamide film,^[7] the magnitude of isotropic thermal expansion coefficient of solid materials without pores is usually very limited.^[3,8]

Many researchers^[9–14] aim to find materials with a negative thermal expansion coefficient because such materials are of great research interest and have important applications, e.g., activators or sensors, due to the coupled thermal-mechanical behaviour.^[15] It has been recognized that the thermal expansion coefficients of one-phase or two-phase solid materials that do not contain a pore phase are always very limited in magnitude, and that three-phase materials^[2,16–18] containing a pore phase could have a much larger thermal expansion coefficient than the one-phase or two-phase solid materials. Thus, people have designed some cellular

materials with an improved magnitude of NTEC.^[19–22] Here we study different new types of microstructured two-ingredient hierarchical and self-similar 2D and 3D cellular materials that can be not only isotropic (note that “isotropic” means $\alpha_x = \alpha_y$ for 2D cellular materials and $\alpha_x = \alpha_y = \alpha_z$ for 3D cellular materials) or anisotropic, but also have either a negative or a positive linear thermal expansion coefficient with a magnitude significantly larger than any reported value in the literature.

To enhance the magnitudes of thermal expansion coefficients, the 2D and 3D cellular materials in this paper are made of two different solid ingredients A and B (Figures 1 and 2). It is assumed

that ingredient A is a ceramic with a Young's modulus of $E_A = 2 \times 10^{11} \text{ N m}^{-2}$ and a thermal expansion coefficient of $\alpha_A = 3 \times 10^{-6} \text{ K}^{-1}$, and ingredient B is a polymer with a Young's modulus of $E_B = 3 \times 10^9 \text{ N m}^{-2}$ and a thermal expansion coefficient of $\alpha_B = 200 \times 10^{-6} \text{ K}^{-1}$. Figure 1c–e shows the geometrical structures of the periodic representative volume elements (RVEs) of the first-level two-ingredient 2D cellular materials with isotropic NTEC, isotropic PTEC and anisotropic TEC, respectively. The two straight and inclined struts are made of ingredient A, perpendicular to each other, and have rigid connection in the middle. All the other struts in the RVEs of the first-level (i.e., level-1) 2D cellular materials are made of ingredient B. Figure 2 shows the geometrical structures of the periodic RVEs of the first-level two-ingredient 3D cellular materials with isotropic NTEC, isotropic PTEC, and anisotropic TEC, respectively. The four straight and inclined struts are made of ingredient A and have rigid connection in the middle. All the other struts in the RVEs of the first-level 3D cellular materials are made of ingredient B. When the effect of thermal expansion is absent, the shape of all the nonstraight struts in the RVEs of both the first-level 2D and 3D materials is assumed to be a chevron with a span of $L = 1.0$ and an amplitude of

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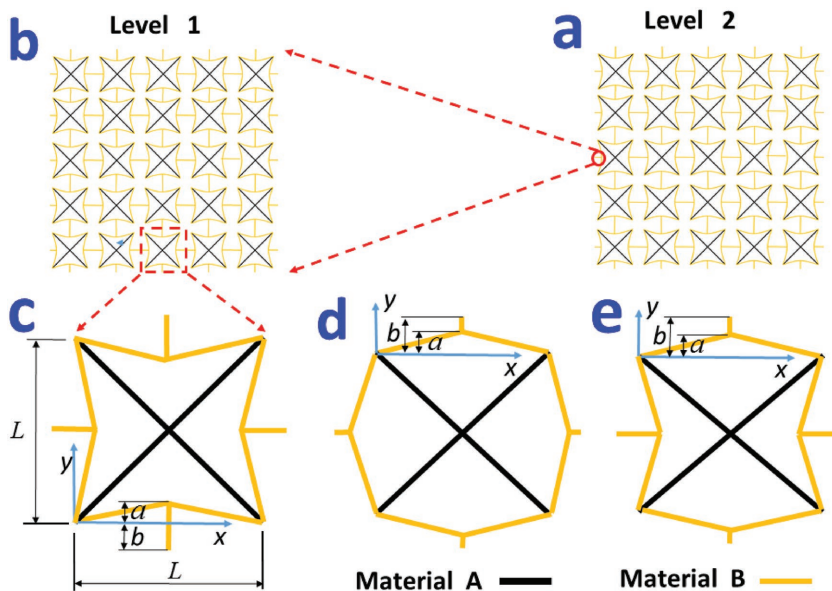


Figure 1. Geometrical structures of two-ingredient hierarchical 2D cellular materials. a) Overview of the second-level 2D cellular material consisting of 5×5 RVEs with nonstraight struts made of the level-1 material shown in (b). b) Overview of the first-level 2D cellular material consisting of 5×5 identical RVEs shown in (c) (or d or e). c) RVE of isotropic negative thermal expansion coefficient. d) RVE of isotropic positive thermal expansion coefficient. e) RVE of anisotropic thermal expansion coefficients.

a, as shown in Figure 3a. Moreover, all the dimensions in Figures 1–3, including the x , y , and z axes, are normalized by L . In addition, all the chevron struts are assumed to have a uniform thickness t for 2D cellular materials or a uniform square

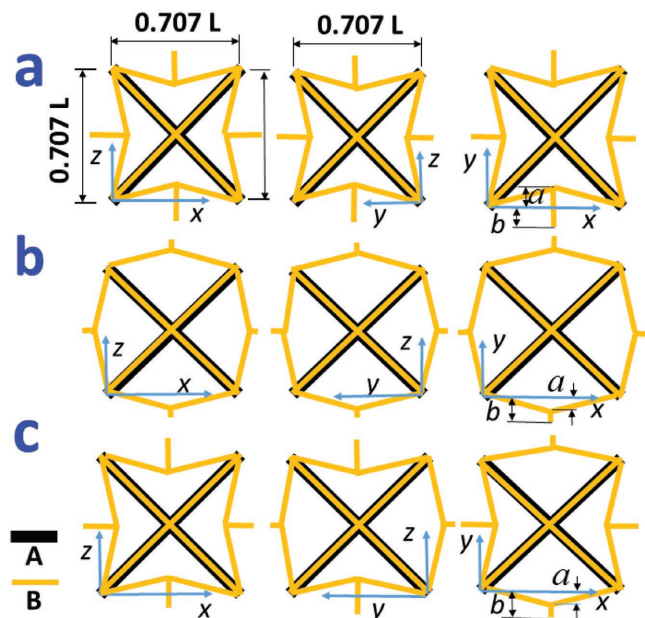


Figure 2. Geometrical structures of two-ingredient 3D cellular materials. a) RVE of isotropic negative thermal expansion coefficient. b) RVE of isotropic positive thermal expansion coefficient. c) RVE of anisotropic thermal expansion coefficients.

cross-section of side t for 3D cellular materials. The chevron struts (made of ingredient B) are assumed to be pin-connected with the straight struts of the cross (made of ingredient A) in the middle.

When there is a temperature change ΔT , the change of the amplitude Δa of the chevron struts can be obtained from Equations (S1–3) and (S1–4) (see the Supporting Information), and the magnitude of the isotropic negative (Figures 1c and 2a), or isotropic positive (Figures 1d and 2b), or anisotropic (Figures 1e and 2c) linear thermal expansion coefficients of both the first-level 2D and 3D cellular materials can be obtained as $\alpha_1 \approx k_1 \Delta \alpha$, where $\Delta \alpha = \alpha_B - \alpha_A \approx \alpha_B = 2 \times 10^{-4} \text{ K}^{-1}$, and the linear thermal expansion magnification factor k_1 is defined as $k_1 = \alpha_1 / \Delta \alpha$ and given as

$$k_1 = \frac{1.667 \Delta a}{\Delta \alpha \Delta T} \quad (1)$$

for the first-level 2D cellular materials, and

$$k_1 = \frac{2.205 \Delta a}{\Delta \alpha \Delta T} \quad (2)$$

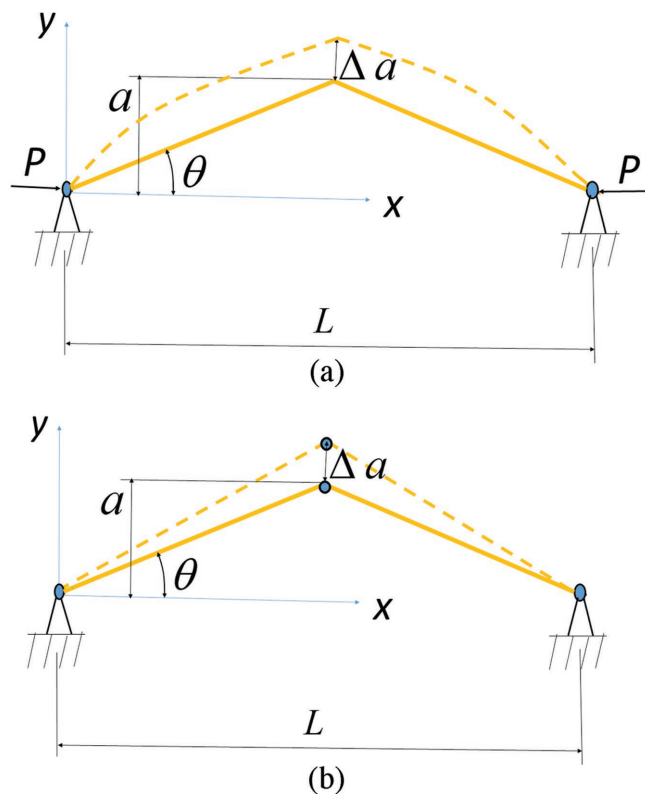


Figure 3. a) The configuration of the chevron struts made of ingredient B before and after thermal deformation. b) The configuration of the pin-jointed struts made of ingredient B before and after thermal deformation.

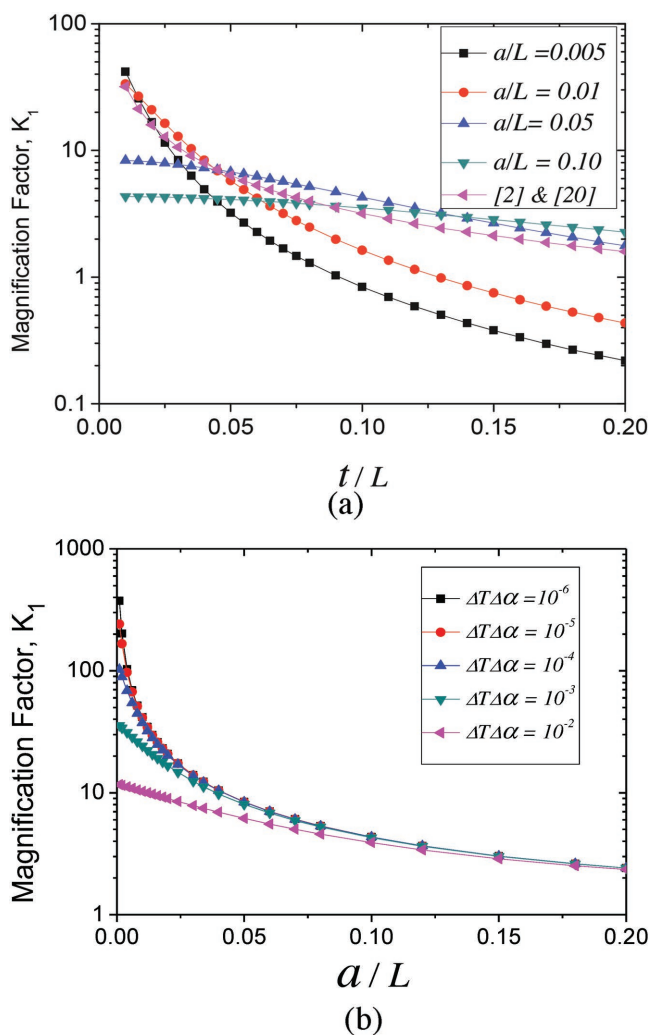


Figure 4. Magnification factor k_1 of the positive or negative, isotropic or anisotropic linear thermal expansion coefficients of the first-level 2D cellular materials: a) with chevron struts which are pin-connected to the cross; b) with pin-jointed struts.

for the first-level 3D cellular materials. In Equations (1) and (2), $\Delta\alpha\Delta T$ is the thermal strain of the chevron struts relative to the cross in the middle. The detailed derivation of $\Delta\alpha$, α_1 , and k_1 is given in Section S1.1 (Supporting Information).

Figure 4a shows the effects of the dimensionless amplitude a/L and the aspect ratio t/L on the magnification factor k_1 of the positive or negative, isotropic or anisotropic linear thermal expansion coefficients of the first-level 2D cellular materials with chevron struts which are pin-connected to the cross in the middle. It is noted that the values of k_1 in Figure 4a obtained from Equations (1) and (2), and Equations (S1–3) and (S1–4) (Supporting Information) remain almost the same when $0.005 \text{ K} \leq \Delta T \leq 100 \text{ K}$. The results presented in Figure 4a are obtained from the small deformation theory and a single Timoshenko beam element, and the combined effects of thermal expansion, strut bending, transverse shearing and axial compression on the deformation of the chevron struts have all been considered. We also found that the transverse

shear deformation of the chevron struts has negligible effect on the values of k_1 .

When a/L is 0.005 and $t/L = 0.01$, the magnification factor k_1 of the first-level 2D cellular materials can have a value of 41.75. Thus, the magnitude of the linear isotropic or anisotropic NTEC or PTEC of the first-level 2D cellular materials can reach $\alpha_1 = k_1\Delta\alpha = 41.75 \times 2 \times 10^{-4} = 8.35 \times 10^{-3} \text{ K}^{-1}$. This magnitude is much larger than the values of thermal expansion coefficient reported in refs. [3–8]. Moreover, if ingredient B is chosen as a polyacrylamide whose thermal expansion coefficient is $\alpha_B = -1.2 \times 10^{-3} \text{ K}^{-1}$ (see ref. [7]), the magnitude of the linear isotropic or anisotropic NTEC or PTEC of the first-level 2D cellular materials could reach $|\alpha_1| = |k_1\Delta\alpha| = 41.75 \times 1.2 \times 10^{-3} = 5.01 \times 10^{-2} \text{ K}^{-1}$, suggesting that the giant thermal expansion could be further enhanced by a larger thermal expansion coefficient of ingredient B. With the increase of t/L , the value of k_1 reduces very quickly because the ratio of the axial compressive strain ε to the relative thermal strain $\Delta\alpha\Delta T$ increases rapidly with t/L . When a/L is 0.05 and $t/L \leq 0.025$, the thermal expansion magnification factor k_1 of the first-level 2D cellular materials can achieve a value of 7.943 and thus the effective linear thermal expansion coefficient can be $\alpha_1 = k_1\Delta\alpha \geq 7.943 \times 2 \times 10^{-4} = 1.5886 \times 10^{-3} \text{ K}^{-1}$ over the range of $0 \leq \Delta T \leq 5 \text{ K}$.

It is worth mentioning that the derivation of the results in Figure 4a is independent of the Young's modulus of ingredient B if the deformation of the chevron struts is linear elastic, and that the axial compression of the chevron struts can significantly reduce the value of k_1 . For different types of the first-level 3D cellular materials shown in Figure 2, the linear thermal expansion magnification factors of the isotropic or anisotropic NTEC or PTEC are 1.322 times those of their first-level 2D cellular counterparts. It is also noted that the thermal deformation of the chevron struts is in general a geometrical nonlinearity problem. To validate the accuracy and applicability of the results in Figure 4a, we have performed finite deformation geometrical nonlinearity analysis (see Section S1.2, Supporting Information). The values of k_1 obtained from the geometrical nonlinearity analysis are almost the same as those presented in Figure 4a. This is partly because even when $\Delta\alpha\Delta T = 2 \times 10^{-3}$ (i.e., $\Delta\alpha = 2 \times 10^{-4} \text{ K}^{-1}$ and $\Delta T = 10 \text{ K}$), the strain in the solid chevron struts is still smaller than 0.2%. Moreover, we have also performed finite element simulation to validate the theoretical results using the commercial finite element software ABAQUS. Half a chevron strut is partitioned into 800 plane stress CPS4T elements and the obtained results are shown in Tables S1 and S2 (Supporting Information) to validate the theoretical results presented in Figure 4a,b. For the cases when the chevron struts are pin-jointed with the cross, the FEM results in Table S1 (Supporting Information) show quite good agreement with the theoretical results presented in Figure 4a although the FEM results indicate that the larger the value of $\Delta\alpha\Delta T$, the smaller the magnification factor k_1 . For the cases when the chevron struts are replaced by two pin-jointed struts (as shown in Figure 3b), the FEM results in Table S2 (Supporting Information) are identical to the theoretical results presented in Figure 4b. When the chevron struts are rigidly connected instead of pin-jointed to the cross in the middle

of the 2D or 3D cellular materials, the values of a/L remain unchanged, the thermal expansion magnification factors presented in Figure 4a are still valid when the value of $2t/L$ (instead of t/L) are used to find the corresponding k_1 .

Now, if the chevron strut in Figure 3a is replaced by two pin-jointed struts as shown in Figure 3b, the values of the thermal expansion magnification factor k_1 of the first-level 2D cellular materials are obtained (see Section S1.3, Supporting Information) and presented in Figure 4b. For the first-level 2D cellular materials when a/L is 0.001, the thermal expansion magnification factor k_1 is 375 and the linear TEC is $\alpha_1 = k_1\Delta\alpha = 375 \times 2 \times 10^{-4} = 0.075 \text{ K}^{-1}$ if $\Delta T \leq 0.005 \text{ K}$; and k_1 becomes 102.3 and $\alpha_1 = k_1\Delta\alpha = 102.3 \times 2 \times 10^{-4} = 0.0204 \text{ K}^{-1}$ if $\Delta T \leq 0.5 \text{ K}$. These magnitudes are much larger than any reported thermal expansion coefficients in literature, including the results in refs. [2,20], which is the highest value reported to the best of our knowledge. It is worth pointing out that the magnification factors of pin-jointed struts (Figure 4b) are sensitive to the dimensionless amplitude a/L and the temperature change ΔT . On the other side, they are entirely independent of the aspect ratio t/L . When a/L is 0.001 and $t/L \geq 0.05$, the single-level 2D cellular materials still have a reasonable stiffness $E_1 \approx 1.2 \times 10^3 \text{ N m}^{-2}$ from Equation (S2-5) (Supporting Information). However, if a/L is too small, the 2D cellular materials with pin-jointed struts may not have a sufficient stiffness to support their self-weight and to enable the expected thermal expansion function. In general, the larger the range of the relative thermal strain $\Delta\alpha\Delta T$ (or temperature change ΔT), the smaller is the thermal expansion magnification factor k_1 . The values of k_1 of the first-level 3D cellular materials shown in Figure 2 are 1.322 times those of their first-level 2D counterparts. Figure 4b shows that when $a/L = 0.02$ and $\Delta T \leq 0.5 \text{ K}$, k_1 is always larger than 20 for the first-level 2D cellular materials and larger than 26.44 for first-level 3D cellular materials. The upbound value of k_1 depends on the specific stiffness, namely stiffness-to-weight ratio, of the cellular structures. In other words, the giant thermal expansion of our proposed cellular material could be further improved by using materials with higher specific stiffness and TEC as component B.

Structural hierarchy can not only enhance the mechanical properties of materials,^[23-27] but may also enhance the magnitude of the linear thermal expansion coefficient of 2D and 3D cellular materials. Both the two-ingredient 2D and 3D hierarchical materials are thus assumed to be self-similar, as demonstrated in Figure 1. To enable the expected thermal expansion function, the minimum mechanical stiffness of the cellular materials is assumed to be about $E_n \approx 1.0 \times 10^3 \text{ N m}^{-2}$ (this is because the relative density of a hierarchical cellular material is much lower than a normal solid material). For a two-level hierarchical 2D cellular material with pin-jointed chevron struts (as shown in Figure 3a), if $t/L = 0.045$ and $a/L = 0.01$ (or 0.05), magnification factor of the linear thermal expansion coefficient can be obtained as $k_2 = (k_1)^2 = 6.91^2 = 47.7$ (or $7.03^2 = 49.4$) when $\Delta T \leq 5 \text{ K}$. In this case, the stiffness of the two-level hierarchical 2D cellular materials is about $E_n \approx 1.58 \times 10^3 \text{ N m}^{-2}$. If the dimensionless amplitude $0.05 \leq a/L \leq 0.1$ and $0.045 \leq t/L \leq 0.1$, the magnification factor of the linear thermal expansion

coefficient of a two-level hierarchical and self-similar 2D cellular material shown in Figure 1 can easily achieve a value of $k_2 = (k_1)^2 > 3.5^2 = 12.25$ when $\Delta T \leq 5 \text{ K}$, which is significantly greater than that of its single-level counterpart with the same a/L and t/L . It is noted that for 2D or 3D cellular materials with chevron struts, the magnification factor of the linear thermal expansion coefficient strongly depends on the values of both a/L and t/L ; in contrast, their mechanical stiffness is mainly dependent on the aspect ratio t/L and entirely independent of the amplitude a/L .

We have also studied the case when the chevron struts in the hierarchical and self-similar 2D and 3D cellular materials are replaced by pin-jointed struts. Figure 4b and Table S2 (Supporting Information) show that if $t = 0.04L = 0.04$ and $a = 0.05L = 0.05$ for a two-level hierarchical and self-similar 2D cellular material, the magnification factor of the linear thermal expansion coefficient can be obtained as $k_2 = (k_1)^2 \geq 8.375^2 = 70.14$ when $\Delta T \leq 0.5 \text{ K}$, or $k_2 = (k_1)^2 \geq 8.034^2 = 64.55$ when $\Delta T \leq 5 \text{ K}$. In this case, the mechanical stiffness of the two-level hierarchical 2D cellular materials can be obtained from Equation (S2-6) (Supporting Information) and given as

$$E_n = \frac{(8ta^2)^n}{(L^2 + 4a^2)^{3n/2}} E_B = \frac{(8 \times 0.04 \times 0.05^2)^2}{(1 + 4 \times 0.05^2)^3} \times 3 \times 10^9 = 1.864 \times 10^3 \text{ N m}^{-2}.$$

If $t = 0.1L = 0.1$ and $a = 0.06L = 0.06$, for a two-level hierarchical and self-similar 3D cellular material, the magnification factor of the linear thermal expansion coefficient can be obtained as $k_2 = (1.322k_1)^2 \geq 1.322^2 \times 7.02^2 = 86.127$, thus $\alpha_2 = k_2\Delta\alpha \geq 86.127 \times 2 \times 10^{-4} = 0.0172 \text{ K}^{-1}$ when $\Delta T \leq 0.5 \text{ K}$, or $k_2 = (k_1)^2 \geq 1.322^2 \times 6.815^2 = 81.17$ and $\alpha_2 = k_2\Delta\alpha \geq 81.17 \times 2 \times 10^{-4} = 0.0162$ when $\Delta T \leq 5 \text{ K}$. In this case, the strain or deformation of the two-level hierarchical and self-similar 3D cellular materials will be $\alpha_2\Delta T = 0.0172 \times 0.5 = 0.86\%$ (or $\alpha_2\Delta T = 0.0162 \times 5 = 8.1\%$) although the thermal strain in the struts is just 0.02% (or 0.2%). The mechanical stiffness of the two-level hierarchical and self-similar 3D cellular materials can be obtained from Equation (S2-8) (Supporting Information) and given as

$$E_n = \frac{(17.64t^2a^2)^n}{(L^2 + 4a^2)^{3n/2}} E_B = \frac{(17.64 \times 0.1^2 \times 0.06^2)^2}{(1 + 4 \times 0.06^2)^3} \times 3 \times 10^9 = 1.16 \times$$

10^3 N m^{-2} . If $t = 0.15L = 0.15$ and $a = 0.08L = 0.08$, for a three-level hierarchical and self-similar 2D cellular materials, the magnification factor of the linear thermal expansion coefficient can be obtained as $k_3 = (k_1)^3 \geq 5.241^3 = 143.96$ (i.e., $\alpha_3 = k_3\Delta\alpha \geq 143.96 \times 2 \times 10^{-4} = 0.0288 \text{ K}^{-1}$) when $\Delta T \leq 5 \text{ K}$. In this case, the stiffness of the three-level hierarchical 2D cellular materials can be obtained from Equation (S2-6) (Supporting Information) and given as

$$E_n = \frac{(8ta^2)^n}{(L^2 + 4a^2)^{3n/2}} E_B = \frac{(8 \times 0.15 \times 0.08^2)^3}{(1 + 4 \times 0.08^2)^{4.5}} \times 3 \times 10^9 = 1.21 \times 10^3 \text{ N m}^{-2}.$$

For three-level hierarchical and self-similar 3D cellular materials, if $t = 0.2L = 0.2$ and $a = 0.1L = 0.1$, the magnification factor of the linear thermal expansion coefficients is obtained as $k_3 = (1.322k_1)^3 \geq 1.322^3 \times 4.28^3 = 181.1$ (i.e., $\alpha_3 = k_3\Delta\alpha \geq 181.1 \times 2 \times 10^{-4} = 0.0362 \text{ K}^{-1}$) when $\Delta T \leq 5 \text{ K}$, and the stiffness of the three-level hierarchical and self-similar 3D cellular materials is given as

$$E_n = \frac{(17.64t^2a^2)^n}{(L^2 + 4a^2)^{3n/2}} E_B = \frac{(17.64 \times 0.2^2 \times 0.1^2)^3}{(1 + 4 \times 0.1^2)^{4.5}} \times 3 \times 10^9 = 0.883 \times$$

10^3 N m^{-2} .

Lakes^[2,20] has designed isotropic single-level 2D hexagonal honeycomb and 3D tetrakaidecahedral open cell foam with curved struts made of two different ingredients. They both could have a very large isotropic positive or negative thermal expansion coefficient given by

$$\alpha_{\text{RVE}} = \frac{l}{h_1 + h_2} \left[\frac{1}{2 \tan(\theta/2)} - \frac{1}{\theta} \right] (\alpha_2 - \alpha_1) = \frac{l}{t} \left[\frac{1}{2 \tan(\theta/2)} - \frac{1}{\theta} \right] \Delta\alpha = \frac{l}{t} f(\theta) \Delta\alpha,$$

where l and t are the length and thickness of the curved struts, θ is the angle of the curved struts and $\Delta\alpha$ is the difference of the thermal expansion coefficients of the two ingredients.^[20] When $\theta = \pi$, the magnification factors of the cellular materials designed by Lakes^[2,20] are included in Figure 4a for comparison. It is noted that θ should be smaller than 229.18° (i.e., 4 radians), otherwise the curved struts will overlap, and thus the maximum possible value of $f(\theta)$ is smaller than 0.48. If the aspect ratio $t/l = 0.01$, the maximum possible magnification factor of the NTEC or PNTec of both the 2D and 3D cellular materials designed by Lakes^[2,20] can be obtained as $k_1 = \frac{\alpha_{\text{RVE}}}{\Delta\alpha} = \frac{l}{t} f(\theta) < 48$, thus $\alpha_1 = \alpha_{\text{RVE}} = k_1 \Delta\alpha < 48 \times 2 \times 10^{-4} = 0.0096 \text{ K}^{-1}$, which is significantly smaller than some of our above reported results for the single-level 2D or hierarchical self-similar 2D and 3D cellular materials in this paper (as can be seen, k can be easily much larger than 50 in the materials designed in this paper). It is also noted that the aspect ratio t/l should be in general larger than 0.01 in the 2D and 3D cellular materials designed by Lakes.^[2,20] This is because the Young's modulus is

$$E_1 = 2.3 \left(\frac{t}{L} \right)^3 E_s \quad (3)$$

for hexagonal honeycombs,^[1,20] and

$$E_1 = \frac{0.76 \rho^2}{1 + 1.09 \rho} E_s = \frac{0.1608 (t/L)^4}{1 + 0.5014 (t/L)^2} E_s \quad (4)$$

for tetrakaidecahedral open cell foams.^[20,28] In the cellular materials designed by Lakes,^[2,20] if the two ingredients are chosen as a metal and a ceramic (with a Young's modulus $E_s = 2 \times 10^{11} \text{ N m}^{-2}$), the cellular materials may be sufficiently stiff, but $\Delta\alpha$ would be in the order of 10^{-5} K^{-1} and the resultant thermal expansion coefficient would still be very small compared to our results of the 2D or 3D cellular materials in this paper. If their two-ingredients are a ceramic and a polymer and $t/l = 0.01$, the Young's modulus of the cellular materials will be $6.9 \times 10^3 \text{ N m}^{-2}$ for honeycombs and 4.82 N m^{-2} for open-celled foams. Thus, the aspect ratio t/l cannot be smaller than 0.01, otherwise the single-level cellular materials designed by Lakes^[2,20] do not have a sufficient stiffness to support the self-weight and to enable the expected function of thermal expansion.

Here, it has been demonstrated that although structural hierarchy can enhance the magnitude of the linear thermal expansion coefficient for cellular materials, it is impossible to achieve an "unbounded" value due to the limit of a required minimum mechanical stiffness. The magnitude of the linear thermal

expansion coefficient of an n th-level hierarchical and self-similar cellular material is obtained as $\alpha_n = \Delta\alpha \cdot (k_1)^n = (\alpha_B - \alpha_A)(k_1)^n$ for 2D and $\alpha_n = \Delta\alpha \cdot (1.322k_1)^2 = (\alpha_B - \alpha_A) \cdot (1.322k_1)^n$ for 3D if the shape of the RVE is convex, and $\alpha_n = \Delta\alpha \cdot (-k_1)^n = (\alpha_B - \alpha_A)(-k_1)^n$ for 2D and $\alpha_n = \Delta\alpha \cdot (-1.322k_1)^2 = (\alpha_B - \alpha_A) \cdot (-1.322k_1)^n$ for 3D if the shape of the RVE is concave. The magnitude of the isotropic NTEC, isotropic PTEC, and anisotropic TEC of the cellular materials in this paper could achieve a value nearly 0.1 K^{-1} and significantly larger than the maximum possible value of TEC reported in literature, e.g., the maximum possible result of the 2D and 3D cellular materials designed by Lakes.^[2,20] The Young's modulus of ingredient B has no effect on the results of k_n and α_n , but strongly affects the stiffness of the cellular materials. The normal-auxeticity mechanical phase transition has recently been found in graphene, an atomic-thick 2D hexagonal carbon.^[29] The results in this paper could apply to multiscale metamaterials design^[30] spanning from macro- down to micro- and nanoscales and our study opens a new avenue to developing more sensitive functional materials or devices. Although there might be some technical challenges to manufacture the designed pin-jointed structures at the micro-scale, their broad applications could be foreseen.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

Conflict of Interest

The authors declare no conflict of interest.

Keywords

2D cellular materials, 3D cellular materials, structural hierarchy, thermal expansion

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