One-Dimensional Fourier Imaging and *k*-Space

OVERVIEW

We have thus far marched through the entire mechanism of how to generate an MR signal emanating from an object, or from a human body, for that matter. The signal discussed up to this point comes from the entire object and has no spatial discrimination. Nevertheless, magnetic resonance imaging offers the possibility to obtain spatially resolved anatomical information. This is accomplished by taking advantage of the Larmor relationship, which dictates that the frequency of the spins depends on the local magnetic field. Variations in the local field were found in the previous unit (*UNIT B4.1*) to cause a loss of signal; however, with the judicious application of a varying local magnetic field (referred to as a magnetic field gradient), it becomes possible to spatially separate the different signal components by ensuring that each spatial location has a unique frequency assigned to it. Under this circumstance, it is possible to obtain a one-to-one mapping of the signal from a given frequency to a given spatial location. The image is then created using the well-known Fourier transform reconstruction method. The Fourier transform takes the MR signal as acquired in the time domain and converts it to the frequency domain where 1-D spatially resolved information can be obtained.

Position Encoding

If a (spatially) linearly varying field is added to the static field, then the z component of the field becomes:

$$B_z(z,t) = B_0 + zG(t) \tag{B4.2.1}$$

The quantity G is a constant gradient in the z direction:

$$G \equiv \partial B_z / \partial z \tag{B4.2.2}$$

and for now G is set equal to G_z. The time dependence of $G(t)$ reminds us that the gradient may undergo quite general modifications during the course of an experiment. In the rotating frame, the frequency depends on G via:

$$\omega_G(z,t) = \gamma z G(t) \tag{B4.2.3}$$

where $\gamma \equiv 2\pi\bar{\gamma}$. The use of a gradient to establish a relation, such as Equation B4.2.3, between the position of spins along some direction and their precessional rates is referred to as frequency encoding along that direction. The object of the standard MR imaging of humans is to extract information about the hydrogen distribution of the sample, in this case by looking at the resulting signal due to frequency encoded proton spins.

The 1-D Imaging Equation and the Fourier Transform

The phase plays the key role in providing the spatial information. The phase for a constant frequency is in the simplest case given by:

Contributed by Weili Lin
Current Protocols in Magnetic Resonance Imaging (2001) B4.2.1-B4.2.12

$$\phi(z,t) = \omega_0 t \tag{B4.2.4}$$

In the rotating reference frame there is no phase change relative to the frequency ω_0 but only with respect to the changes introduced by the gradient field. The accumulated phase up to time t due to the applied gradient is given by

$$\begin{aligned} \phi_G(z,t) &= -\int_0^t dt' \omega_G(z,t') \\ &= -\gamma z \int_0^t dt' G(t') \\ &= -\gamma G z t \end{aligned} \tag{B4.2.5}$$

where G is a constant gradient applied instantaneously after the applied RF pulse at $t = 0$.

The signal acquired as a function of time is the sum of all signal contributions from each isochromat of spins with spin density $\rho(z)$:

$$\begin{aligned} s(t) &= \int dz\, \rho(z) e^{i\phi_G(z,t)} \\ &= \int dz\, \rho(z) e^{-i\gamma G z t} \end{aligned} \tag{B4.2.6}$$

Equation B4.2.6 is often referred to as the *1-D imaging equation*. It shows that the signal is related to the Fourier transform of the sought after spin density.

k-Space and the Fourier Transform

The time domain is usually referred to as the *k*-space domain simply because it correlates more easily with the usual definitions of the Fourier transform. By setting:

$$k = \not{\gamma} G t \tag{B4.2.7}$$

the expression for the signal becomes:

$$s(k) = \int dz \rho(z) e^{-i2\pi k z} \tag{B4.2.8}$$

where the time dependence resides implicitly in the spatial frequency k. This expression shows that, when linear gradients are implemented, the signal $s(k)$ is the Fourier transform of the spin density of the sample. The spin density is said to be Fourier encoded along z by the linear gradient.

Obtaining the Image

The fact that the signal and the density are related by such a well studied linear transform is a boon to MRI. The foremost property of the Fourier transform is its well-defined inverse. Given $s(k)$ for all k, the spin density of the sample can be found by taking the inverse Fourier transform of the signal:

$$\rho(z) = \int dk s(k) e^{+i2\pi k z} \tag{B4.2.9}$$

The signal $s(k)$ and the image $\rho(z)$ are a "Fourier transform pair."

A Simple Example of Position Encoding

Imagine that there are two objects to be imaged, well separated in space by a distance *d*. In the presence of a spatially uniform external static magnetic field, both objects will precess at the same frequency, the Larmor frequency. After the application of an RF pulse, all spins will precess at exactly the same frequency in the transverse plane. Up to this point, there is no spatial discrimination of the two objects. By adding a spatially varying magnetic field in addition to the presence of the static magnetic field, we have purposely introduced a slight variation in the main magnetic field so that objects at different spatial locations will have a slightly different resonance frequency (see Fig. B4.2.1).

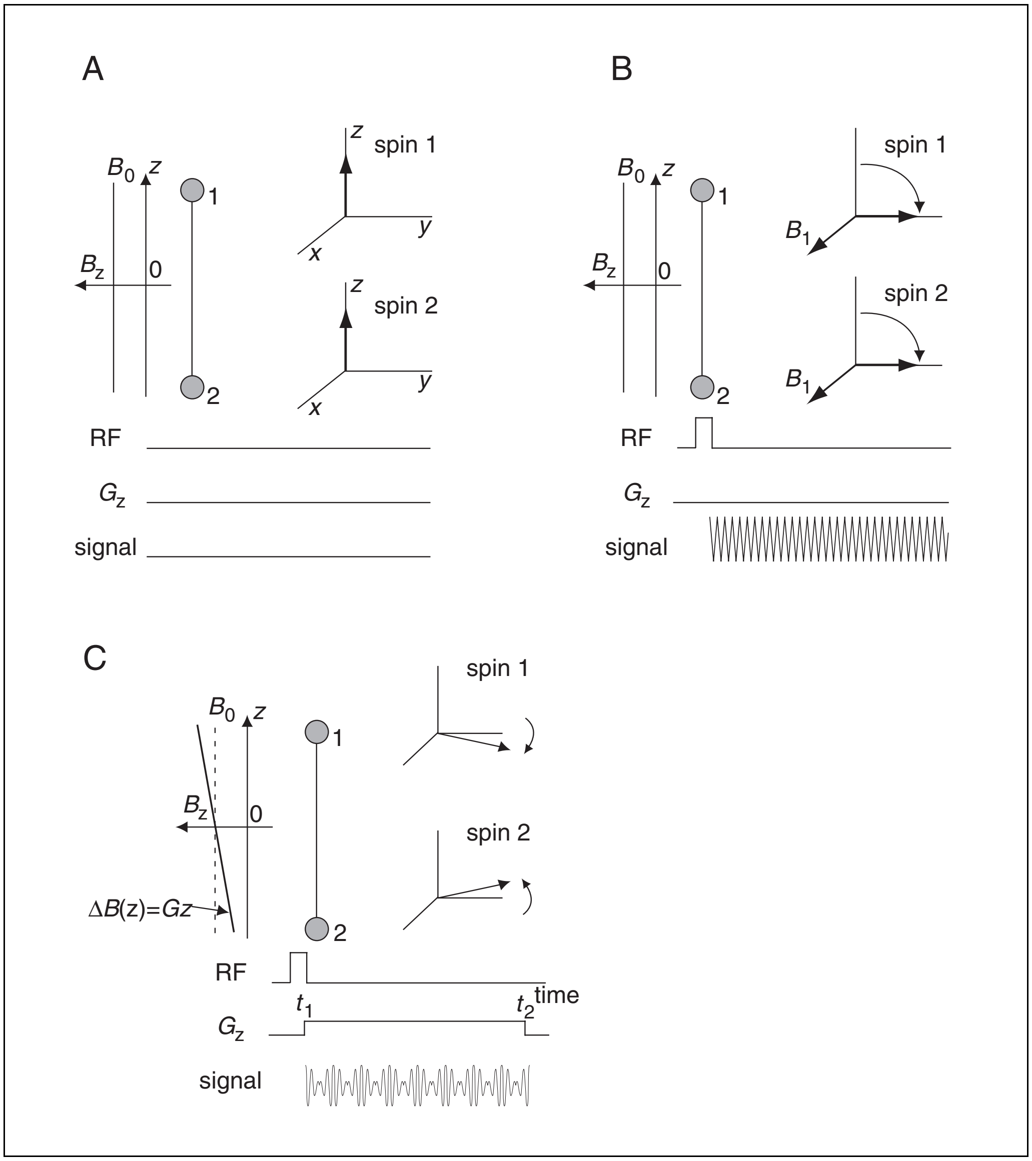

Figure B4.2.1 Example of a 1-D MRI experiment. The positions of two-point spins (depicted as circles on a “spin dumbbell” with no MRI signal from the connecting rod) are to be determined by imaging. The *z*-component of the laboratory magnetic field is plotted to the left of the dumbbells. The rotating frame orientation of the spin in each dumbbell is displayed at the right of the diagram. The sequence diagrams underneath represent the rotating frame input RF (assumed to be along x' and producing a $\pi/2$ rotation), the applied gradient, and the laboratory received signal (with T_2 decay neglected), before demodulation, as a function of time. Steps (**A**), (**B**), and (**C**) are described in the text. Note that G_z has the constant value G in part (**C**) between the times t_1 and t_2.

Acquiring the data using a gradient echo design as in *UNIT B4.1*, and applying the Fourier transform converts the k-space (or time) data to the frequency domain. The difference in frequency between the two points will be γGd (where d is the distance between two spins) and we will see two spikes at two frequencies separated by this amount. Each spike will represent the signal from each point. The amount of signal in the two frequency locations will be directly proportional to the spin density at those locations.

k-Space Coverage

In the FID (free induction decay) acquisition method, data are collected only for positive times. In the gradient echo acquisition method, both negative and positive times around the echo time are collected. The concept of k-space or spatial frequency represents the type of information content in the signal. The lower the value of $|k|$, the more information is obtained about broad structures, while the higher $|k|$, the more information is obtained about the edges of structures. It is useful to show how much of data are covered during the experiment. This we do by showing what values of k are being covered during different parts of the sequence, and we refer to this as the k-space coverage and illustrate it in a k-space diagram.

Of course, in reality, the human body consists of contiguous tissues rather than distinct objects. As one might guess when trying to image many structures, the requirements to obtain spatially resolved information for such an object are more complex. Suffice it to say here that the sampling must be long enough to regionalize information, and it must be dense enough to gain a sufficient coverage of interest or field-of-view. Bearing these two caveats in mind, several examples of data acquisition or k-space coverage are shown in Figures B4.2.3 and B4.2.4. The data must be sampled, and it can be sampled for only a finite length of time (a continuous recording of the data cannot be saved). These two conditions put restrictions on the quality of the output.

TECHNICAL DISCUSSION

In this section, detailed derivations of the above concepts will be given relating the application of gradients to encode position and how this generates a signal which can be considered as a Fourier transform of the original object signal distribution. Finally, the concepts of k-space will be introduced and an example of k-space coverage will be given for both gradient echo and spin echo experiments.

Frequency Encoding and the Fourier Transform

The goal of imaging is to determine the available MR signal, which in turn is related to the spin density $\rho(z)$ of a sample as well as its relaxation times. For the discussion below, the effects of relaxation times are ignored for now. Since we are considering a one-dimensional example with an object lying along the z-direction, $\rho(z)$ is a function of position. Therefore, it is possible that a means can be employed to provide the spatially resolved information based on $\rho(z)$. In order to achieve this goal, the first step is to connect the spin precession to its position, and the second is to recognize that this connection implies that the signal is a well-known linear Fourier transform of the spin density.

Frequency Encoding of the Spin Position

The Larmor frequency of a spin will be linearly proportional to its position z if the static field is augmented with a linearly varying field. The maximum of the linearly changing field at any point in the system is usually considerably smaller in magnitude than the static field. The magnetization is considered to have been tipped, initially, into the transverse plane, so that it is already precessing freely with angular frequency ω_0 before the gradient field is brought into play. (Throughout the discussion of this unit, it is assumed that the RF pulses and the gradient fields are not applied at the same time.)

If a (spatially) linearly varying field is added to the static field, then the z component of the field is:

$$B_z(z,t) = B_0 + zG(t) \tag{B4.2.10}$$

The quantity G is a constant gradient in the z direction:

$$G_z \equiv \partial B_z / \partial z \tag{B4.2.11}$$

and for now G is set equal to G_z. The time dependence of $G(t)$ reminds us that the gradient may undergo quite general modifications during the course of an experiment. Define the variation in the angular frequency of the spins by:

$$\omega(z,t) = \omega_0 + \omega_G(z,t) \tag{B4.2.12}$$

From Equation B4.2.10, the deviation from the Larmor frequency is linear in both z and G:

$$\omega_G(z,t) = \gamma z G(t) \tag{B4.2.13}$$

The use of a gradient to establish a relation, such as Equation B4.2.13, between the position of spins along some direction and their precessional rates is referred to as frequency encoding along that direction. The goal of the standard MR imaging of humans is to extract information about the hydrogen distribution of the sample, in this case by looking at the signal due to frequency encoded proton spins.

The accumulated phase, up to time t, due to the applied gradient is:

$$\begin{aligned} \phi_G(z,t) &= -\int_0^t dt' \omega_G(z,t') \\ &= -\gamma z \int_0^t dt' G(t') \end{aligned} \tag{B4.2.14}$$

where it is recalled that the gradient is assumed to be applied only after the initial RF excitation at $t = 0$.

The 1-D Imaging Equation and the Fourier Transform

The signal acquired as a function of time with demodulating frequency $\Omega = \omega_0$ is given by:

$$s(t) = \int dz \rho(z) e^{i\phi_G(z,t)} \tag{B4.2.15}$$

where the phase, after demodulation, is determined by the gradient field. Equation B4.2.15 is applicable to measurements with gradient fields that have arbitrary z dependence; it is often referred to as the *1-D imaging equation*. It can be generalized by replacing ϕ_G by the angle ϕ due to all gradient and RF field variations. Remember that any initial (spatially constant) phase is ignored.

The explicit z dependence in the phase, Equation B4.2.14, for the linear field leads to:

$$s(k) = \int dz\, \rho(z) e^{-i2\pi kz} \qquad \textbf{(B4.2.16)}$$

where the time dependence resides implicitly in the spatial frequency $k = k(t)$ with:

$$k(t) = \bar{\gamma} \int_0^t dt' G(t') \qquad \textbf{(B4.2.17)}$$

This expression shows that, when linear gradients are implemented, the signal $s(k)$ is the Fourier transform of the spin density of the sample. The spin density is said to be Fourier encoded along z by the linear gradient.

The fact that the signal and the density are related by such a well studied linear transform is a boon to MRI. The foremost property of the Fourier transform is its well-defined inverse. Given $s(k)$ for all k, the spin density of the sample can be found by taking the inverse Fourier transform of the signal:

$$\rho(z) = \int dk\, s(k) e^{+i2\pi kz} \qquad \textbf{(B4.2.18)}$$

The signal $s(k)$ and the image $\rho(z)$ are a "Fourier transform pair." It is important to understand that any direction could have been chosen for this one-dimensional example. In other units, the "frequency encoding direction" or "read direction" is chosen as the x direction, rather than the z direction.

The Coverage of *k*-Space

It is most useful to formulate imaging arguments in terms of image (z) space and data (k) space, in view of the Fourier transform connection. The inverse transform implies that the spin density can be reconstructed from the signal, if the latter is collected over a sufficiently large set of k values. The integration in Equation B4.2.18 requires "good coverage" of k-space.

The dependence on time and the applied gradient amplitude is the key to covering a large enough range in k. In the case where the applied gradient is constant over the whole time interval $(0, t)$, Equation B4.2.17 reduces to:

$$k = \bar{\gamma} Gt \qquad \textbf{(B4.2.19)}$$

Therefore, to collect a uniform distribution of points in k-space, it is only necessary to sample the signal at a constant rate in the presence of a constant gradient. Sampling both negative and positive values for k can be achieved by changing the sign of the gradient (see the section on gradient echo later in this unit).

If the signal could be measured continuously over a very long time and an accurate integration carried out in Equation B4.2.18, then a faithful picture of the spin density would be found; however, several factors prevent collection of continuous data over all

k-space. First, there is the fact that the signal must be sampled in a finite amount of time, and second, there is the fact that relaxation effects wipe out the signal within a finite period. In actuality, the data collected will be a truncated and discretized version of $s(k)$, and it is necessary to carefully study the discrete Fourier transform to understand the impact these two modifications have upon the resulting image relative to the continuous analytic form given in Equation B4.2.18.

Simple Two-Spin Example

Consider a pair of classical spins (a "dumbbell" of two-point spins) lying along the z axis, at $z = \pm z_0$. Although individual classical spins are not experimentally relevant, the simplicity of the model is useful in demonstrating the 1-D imaging method. Suppose the spins have reached equilibrium alignment along the static field (Fig. B4.2.1, panel A). If an RF pulse is applied to tip the spins into the transverse plane, a single-frequency signal results in the absence of a gradient. If the signal shown in Figure B4.2.1, panel B were demodulated at the Larmor frequency, it would be constant.

If the experiment is run again in the presence of the field gradient (Fig. B4.2.1, panel C), during the time interval (t_1, t_2), the precessional rates of the two spins will differ slightly from the Larmor frequency. For $G > 0$ and gyromagnetic ratio γ the spin at z_0 rotates clockwise and the spin at $-z_0$ rotates counterclockwise at the same rate, in the rotating reference frame; therefore, while the constant gradient G is applied, with $t_1 < t < t_2$, the spin at z_0 has rotated through an angle, $\phi(z_0, t) = -\gamma G z_0(t - t_1)$, and the spin at $-z_0$ through an angle, $\phi(-z_0, t) = \gamma G z_0(t - t_1)$.

The signal in this case can be directly calculated since the integrals over the spin density reduce to the sum of the contributions from each spin. Let $t_1 = 0$ for convenience. The net signal is:

$$
\begin{aligned}
s(t) &= s_0 e^{-i\gamma G z_0 t} + s_0 e^{i\gamma G z_0 t} \\
&= 2s_0 \cos(\gamma G t z_0) \qquad 0 < t < t_2
\end{aligned}
\qquad \textbf{(B4.2.20)}
$$

or:

$$
s(k) = 2s_0 \cos(2\pi k z_0) \qquad 0 < k < k_2 \equiv \gamma\!\!\!-\, G t_2 \qquad \textbf{(B4.2.21)}
$$

with k from Equation B4.2.19 and s_0 the (common) magnitude of the signal associated with each spin. Equation B4.2.20 exhibits the expected beat envelope from two identical sources at different frequencies. (The superposition of the envelope on the fast carrier frequency corresponds to the laboratory signal shown in Fig. B4.2.1, panel C.) Given this signal and knowledge of the applied gradient, the distribution of the spins follows. In this case, the beat frequency $\gamma\!\!\!-\, G z_0$ implies the distance between the spins is $2z_0$. Albeit a very simple example, this demonstrates how the frequencies imposed by gradients yield information about the spatial distribution of the spins in a sample.

It is useful to verify that the signal leads directly to $\rho(z)$ through the inverse Fourier transform. By changing the sign on G, Equation B4.2.21 may be considered to be valid for all k; a later discussion is directed to techniques for acquiring data at both positive and negative k values. From Equation B4.2.18:

$$\begin{aligned}\rho(z) &= \int_{-\infty}^{\infty} dk\, 2s_0 \cos(2\pi k z_0) e^{i2\pi kz} \\ &= s_0 \int_{-\infty}^{\infty} dk \left(e^{i2\pi k(z+z_0)} + e^{i2\pi k(z-z_0)}\right) \\ &= s_0 \left[\delta(z+z_0) + \delta(z-z_0)\right]\end{aligned} \tag{B4.2.22}$$

with:

$$\delta(z) \equiv \int_{-\infty}^{\infty} dk e^{i2\pi kz} \tag{B4.2.23}$$

In the last step, it is recognized that the integrals over the exponentials are Dirac delta functions. For now, it only needs to be noted that the delta function $\delta(z - z_0)$ is, qualitatively, a spike at $z = z_0$ with, in the limit, an infinite height, zero width, and unit area. Thus, the spin density, Equation B4.2.23, is exactly what is expected, for two point spins at $\pm z_0$.

Gradient Echo and *k*-Space Diagrams

Suppose the two spins in the dumbbell example are replaced by a "cylinder" of an arbitrary *z*-distribution of spins (Fig. B4.2.2). The positions of the spins are frequency encoded by an applied gradient, and Fourier techniques are available, as discussed, to image the spin distribution. Figure B4.2.2 carries us through an MR experiment where the equilibrium magnetization M_0 is attained (see Fig. B4.2.2. panel A) and acted upon by a 90° RF pulse along x' to create a transverse magnetization also of amplitude M_0 over the whole sample. The decaying signal in Figure B4.2.2, panel B, represents the FID in the laboratory frame. Applying a gradient along $\hat{z}$ causes a more rapid decay exhibiting beat frequencies (Fig. B4.2.2, panel C). The transverse components of spins at different *z*-locations are shown projected onto the *x*-*y* plane to illustrate the dephasing of the signal. By reversing the gradient polarity, an echo can be formed at $t' = 0$ (Fig. B4.2.2, panel D), as explained below.

Signal measurements over a sufficient range of *k* are needed to reasonably reconstruct the spin density. The principal technique for obtaining negative, as well as positive, *k* values is the "gradient echo" method. A single, constant gradient (such as in Fig. B4.2.2, panel C) is restricted in the *k* interval it generates, while a combination of gradients (such as in Fig. B4.2.2, panel D) can be used to expand the *k* range from negative to positive *k* values. The time at which the data corresponding to the $k = 0$ point (the echo) are measured can be moved, for example, to the center of a particular gradient "lobe."

In particular, combinations of gradient lobes permit the recovery of a signal loss due to the presence of the gradients themselves. The net signal from a set of spins will begin to disappear, after a gradient field is applied, for the same reason as that underlying T'_2 decay. Since the applied gradient is a field inhomogeneity, the spins dephase, as illustrated by the "fan-out" around $\phi = 0$ in Figure B4.2.2, panel C, for constant gradients. (For a given sign on the gradient, both positive and negative phases are found for any distribution of spins straddling the origin, $z = 0$.) A series of gradient pulses can be used to form an echo similar to the spin, or RF, echo described in *UNIT B4.1*. A simple example of a gradient echo is developed below.

It should be noted that the loss of signal due to the presence of the gradient is expected from the property of the Fourier transform. Roughly speaking, the Fourier transform is a (continuous) sum of signal contributions with different phases that interfere with each other. The longer the cylinder of spins, the faster the Fourier transform drops off with *k*

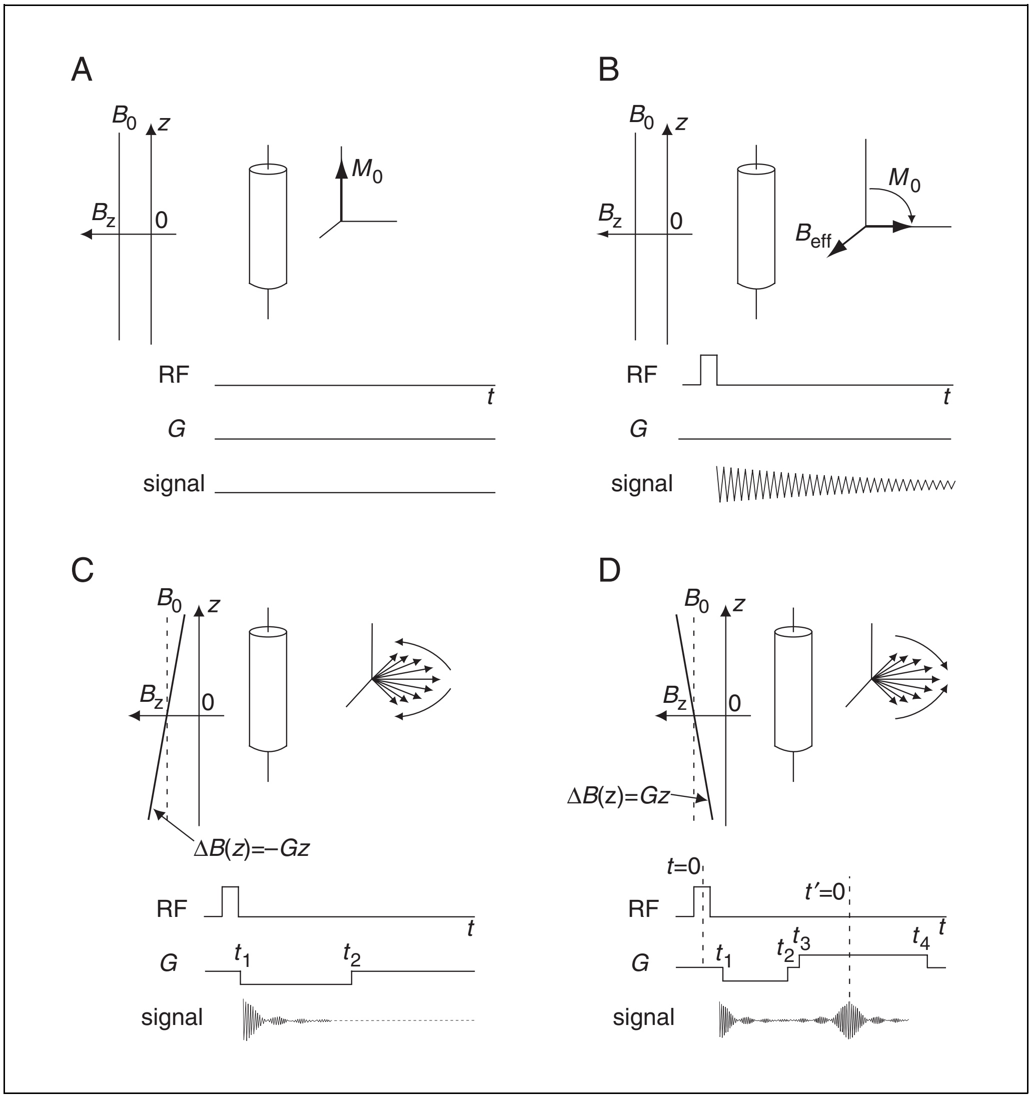

Figure B4.2.2 A consecutive set of time events in a 1-D MRI experiment. This illustration follows the description of Figure B4.2.1 except the two-point spins are replaced by a "cylinder" containing an arbitrary distribution of spins. Parts (**A**) and (**B**) are the same as in the previous figure except that in (B) the signal includes T_2 decay, and (**C**) and (**D**) are described in the text. The magnetic field plotted at the left of the cylinder in (C) refers to the field in the time interval (t_1, t_2), and, in (D), to the interval (t_3, t_4). For comparison, the spin isochromats are pictured lying in a single plane, despite their different z coordinates. Note that the amplitude of the gradient is $-G$ in part (C) and has both a negative and a positive lobe of strength G in part (D).

(i.e., with time in the imaging experiment). It is observed in Figure B4.2.2, panel C that the signal is (half of) a sinc function with a falloff in signal as expected from the dephasing arguments. Rephasing can be achieved, as discussed below, such that the full sinc function signal is produced (Fig. B4.2.2, panel D). In essence, the details of the dephasing determine the Fourier transform, and this dephasing represents the encoded information content.

The associated coverage of k values can be described by a k-space diagram. The k-space diagram for the simple boxcar gradient in Figure B4.2.1, panel C is shown in Figure B4.2.3. For a constant gradient, the k value is proportional to the imaging time according to Equation B4.2.19. In this case, only half of k-space is covered for a gradient with a given sign. Other examples are exhibited below for gradient echo experiments. The effect on the signal due to the gradient manipulation is studied through the expression for the

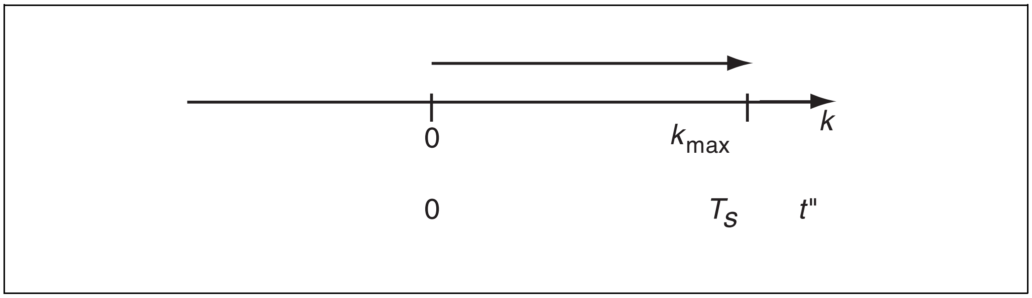

Figure B4.2.3 The k-space coverage for an FID imaging experiment. The time $t'' = (t - t_1)$ is defined from the beginning of the gradient. The total sampling time is T_S. Notice that only half of k-space is covered by this experiment. There is no progression through k-space until time t_1 in Figure B4.2.1, panel C which is equivalent to $t'' = 0$.

signal strength (Equation B4.2.15). Recall that the relation between the phase and an arbitrary linear gradient in the z-direction is given in Equation B4.2.14.

The Gradient Echo

The gradient dephasing of spins in an object is already discussed in *UNIT B4.1*. We repeat this discussion and carry it further to show how the data are sampled in the time domain and equivalently in k-space. Let us analyze the specific gradient sequence included in Figure B4.2.2, panel D.

A constant negative gradient ($G_z = -G$ for $G > 0$) is present in the time interval (t_1, t_2). From Equation B4.2.14 the phase accumulation due to the gradients for a spin at z, and at a time t, during the application of the first gradient lobe is:

$$\phi_G(z,t) = +\gamma G z(t - t_1) \qquad t_1 < t < t_2 \tag{B4.2.24}$$

The sign is consistent with the counterclockwise precession expected for a negative gradient and positive z-coordinate.

The second gradient lobe is positive $G_z = G$ in terms of the same parameter G during the time interval (t_3, t_4). The phase behavior relative to the y'-axis at any time in this interval is:

$$\phi_G(z,t) = +\gamma G_z(t_2 - t_1) - \gamma G z(t - t_3) \qquad t_3 < t < t_4 \tag{B4.2.25}$$

Notice that there is no phase change during the period when the applied gradient is zero.

The phase in Equation B4.2.25 returns to zero independent of z (Fig. B4.2.2, panel D) when:

$$t = t_3 + t_2 - t_1 \equiv T_E \tag{B4.2.26}$$

This time T_E is the time of the gradient echo. The echo corresponds to that time during the second gradient lobe where the evolved area under the second lobe just cancels the area of the first lobe—i.e., when the zeroth moment of $G(t)$ vanishes:

$$\int G(t)\,dt = 0 \tag{B4.2.27}$$

The gradient echo analysis in imaging revolves around this criterion.

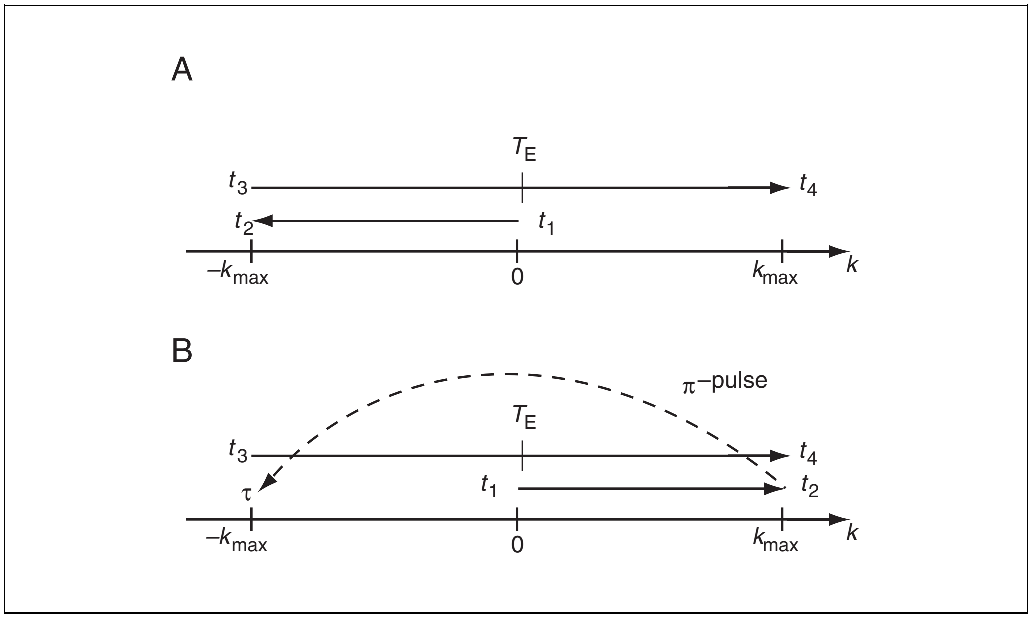

Figure B4.2.4 The k-space coverage for the basic gradient echo experiment and the two variants of the spin echo experiment presented in the text. Diagram (**A**) applies to the basic gradient echo experiment and to the spin echo variant where all frequency encoding occurs after the π-pulse. Diagram (**B**) shows the spin echo variant where both gradient lobes are positive. In all cases, $k_{max} = \gamma G(t_4 - t_3)/2$. The dashed line represents the action of the π-pulse which changes k_{max} to $-k_{max}$.

Returning to the example, the second gradient can be applied so that the echo occurs at its center as in Figure B4.2.2, panel D; that is, when $(t_4 - t_3)/2 = t_2 - t_1$. It is useful to define a new time coordinate with its origin at the echo:

$$t' \equiv t - t_3 - (t_2 - t_1) = t - T_E \qquad \textbf{(B4.2.28)}$$

Thus, Equation B4.2.25 can be rewritten:

$$\phi_G(z,t) = -\gamma G z t' \qquad -(t_4 - t_3)/2 < t' < (t_4 - t_3)/2 \qquad \textbf{(B4.2.29)}$$

with an obvious zero at $t' = 0$. It is during the time interval indicated in Equation B4.2.29 that the data are usually sampled.

The signal, Equation B4.2.15, can be written as a function of t' over the region in which $G_z = G$. During the time the second gradient is turned on, the signal is:

$$\begin{aligned} s(t') &= \int dz \rho(z) e^{-i\gamma G z t'} \qquad -(t_4 - t_3)/2 < t' < (t_4 - t_3)/2 \\ s(k) &= \int dz \rho(z) e^{-i2\pi k z} \end{aligned} \qquad \textbf{(B4.2.30)}$$

where $k = \gamma G t'$ from Equation B4.2.17. With the time-shift to t', the signal Equation B4.2.30 is expressed in a form convenient to analyze the data as referenced from the center of the second lobe. (The second lobe is called the "rephasing gradient lobe" of the read gradient, while the first lobe is referred to as the "dephasing lobe" for that gradient.) Such time-shifts are regularly used in MR analysis.

The above experiment thus leads to a range of negative and positive k-space points for measurements made symmetrically about the echo. The negative gradient lobe has been utilized to create an echo in the middle of the positive rephasing gradient where $k = 0$. A k-space diagram for the gradient echo experiment is shown in Figure B4.2.4, panel A. The signal in k-space is a simple transcription of Equation B4.2.30:

$$s(k) = \int dz \rho(z) e^{-i2\pi kz} \qquad -k_{\max} < k < k_{\max} \tag{B4.2.31}$$

where $k_{\max} = \gamma G(t_4 - t_3)/2$.

In contrast to the k-space coverage for a gradient echo experiment which refocuses the phase through the application of a bipolar gradient structure, a π-pulse is employed for a spin echo experiment to refocus the phase so that the gradient induced phase dispersion can be recovered at the spin echo (Fig. B4.2.4, panel B).

KEY REFERENCES

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