Managing Epistemic Uncertainty in Design Models through Type-2 Fuzzy Logic Multidisciplinary Optimization

by

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ABSTRACT

Humans have a natural ability to operate in dynamic environments and perform complex tasks with little perceived effort. An experienced ship designer can intuitively understand the general consequences of design choices and the general attributes of a good vessel. A person's knowledge is often ill-structured, subjective, and imprecise, but still incredibly effective at capturing general patterns of the real-world or of a design space. Computers on the other hand, can rapidly perform a large number of precise computations using well-structured, objective mathematical models, providing detailed analyses and formal evaluations of a specific set of design candidates. In ship design, which involves generating knowledge for decision-making through time, engineers interactively use their own mental models and information gathered from computer-based optimization tools to make decisions which steer a vessel's design.

In recent decades, the belief that large synthesis codes can help achieve cutting-edge ship performance has led to an increased popularity of optimization methods, potentially leading to rewarding results. And while optimization has proven fruitful to structural engineering and the aerospace industry, its applicability to early-stage design is more limited for three main reasons. First, mathematical models are by definition a reduction which cannot properly describe all aspects of the ship design problem. Second, in multidisciplinary optimization, a low-fidelity model may incorrectly drive a design, biasing the system level solution. Finally, early-stage design is plagued with limited information, limiting the designer's ability to develop models to inform decisions.

This research extends previously done work by incorporating type-2 fuzzy logic into a human-centric multidisciplinary optimization framework. The original framework used type-1 fuzzy logic to incorporate human expertise into optimization models through linguistic variables. However, a type-1 system does not properly account for the uncertainty associated with linguistic terms, and thus does not properly represent the uncertainty associated with a human mental model. This limitation is corrected with the type-2 fuzzy logic multidisciplinary optimization presented in this work, which more accurately models a designer's ability to "communicate, reason and make rational decisions in an environment of imprecision, uncertainty, incompleteness of information and partiality of truth" (Mendel et al., 2010). It uses fuzzy definitions of linguistic variables and rule banks to incorporate "human intelligence" into design models, and better handles the linguistic uncertainty inherent to human knowledge and communication. A general mathematical optimization proof of concept and a planing craft case study are presented in this dissertation to show how mathematical models can be enhanced by incorporating expert opinion into them. Additionally, the planing craft case study shows how human mental models can be leveraged to quickly estimate plausible values of ship parameters when no model exists, increasing the designer's ability to run optimization methods when information is limited.

CHAPTER I

Introduction

1.1 Background and Motivation

The design of ships and other complex engineered systems involves generating knowledge for decision-making through time. In the initial stages of the design process, when decisions must be made with relatively little information, designers leverage their professional knowledge and simplified mathematical models (or synthesis tools) to gather information and help them make choices. Additionally, increases in computational capability over the past several decades have pushed many organizations to adopt mathematical modeling and optimization in their design process.

While optimization has had many successes in the analysis of simpler, well defined products, its success in ship design has been more mitigated. Ships are unique among engineered products in their complexity and architectural characteristics, rivaled only by large infrastructure projects and spacecraft. While the largest aircrafts carry a few hundred passengers for no more than 20 hours, the largest military ships must sustain several thousand passengers for months at a time, adding tremendous technical and physical complexity for which traditional mathematical models are ill-suited. Mathematical models are well-suited for problems with precise, well-defined solutions, but suffer in ship design which is just as much an art as it is a science in that often no clear, objective, solution exists.

Mathematical models suffer for three main reasons. First, they are reductions, which by definition, don not fully describe all aspects of the real-world problem they are trying to represent. Second, in multidisciplinary optimization in which multiple models with varying levels of fidelity are used together, the global solution may be falsely driven by the low fidelity model. Third, realistic mathematical models are difficult to develop in environments with limited information, which are inherent to the early stages of ship design. Before expanding on these research problems in Section 1.2, a very brief review of human knowledge and mathematical modeling will be given.

1.1.1 Human Knowledge

The first and most important tool a designer can rely on is his or her set of mental models and subjective knowledge, developed over years of professional experience. These tools provide designers with intuition about the typical attributes of a good ship, helping them quickly and automatically steer a design process for a given set of stakeholder requirements. For example, a slender vessel is known to have smaller resistance but worse seakeeping than one with a larger beam. Hence, when designing for speed, larger length-to-beam ratios are intuitively favored, automatically ruling out a large number of designs which do not exhibit these attributes. While this is a simplified example, it shows that experienced human designers are very capable of making quick, effortless decisions without any explicit computation. A Michigan professor in the Naval Architecture and Marine Engineering department remembers how his first boss could produce a very accurate first sketch of a new vessel design in a matter of hours, almost automatically.

1.1.2 Mathematical Models

The second tool is a set of (often computerized) mathematical models which are an "abstract description of the real world giving an approximate representation of more complex functions of physical systems" (Papalambros and Wilde, 2017). These are more precise than mental models, and they attempt to capture and structure knowledge about a system into a form which is usable to make a decision. In general, they describe the relationship between design variables (e.g. a vessel's principle characteristics) and some performance measure or objective (e.g. a vessel's resistance). For example, the resistance of a ship is a complicated phenomenon influenced by many factors. Measuring it in full-scale experiments would be cost prohibitive, so naval architects use their physics knowledge and reduced-scale experiments to develop mathematical models which approximate the resistance characteristics of a vessel (Zubaly, 2009). A resistance model breaks drag down into frictional resistance, wave-making resistance, form-drag, and air resistance, structuring knowledge such that the vessel's resistance can be easily estimated using design variables, scaling parameters like the Reynolds and Froude numbers, and coefficients determined through the model test. These mathematical models, which range from low fidelity regressions of historical data to high fidelity simulations (Jouhaud et al., 2007; Yang and Sen, 1996), are used to extract objective knowledge of a design space and understand its trade-offs. Designers run these models through design software and optimization routines, evaluating different test cases to gather information and better understand the design space.

While human mental models and subjective preferences are nearly impossible to describe using mathematical models, they are often used to evaluate design solutions. The actual design decisions result from the analysis and interpretation of the mathematical model's results by the human designer—what is commonly known as the "engineer's judgment." It is the iterative interplay between decision-making by the human designer and findings from design tools which determines a design's success. After all, decision-making is a cognitive process resulting in the choice of a course of action among several alternative possibilities.

However, the trend in naval design is to remove the cognitive-process and replace it with large synthesis codes and optimization. Recent advances in computational capability and market competitiveness have pushed organizations to automate the design process using complex computer codes capable of rapidly producing and evaluating a large number of solutions (Gray et al., 2013), with the goal of finding the most promising one. This has distanced designers from decision making, both because they have less control over decisions now made by computers, but also because their removal to a position of oversight limits their understanding of the design details. Kassel et al. (2010) discuss the need for "an automated tool [... which] could rapidly produce a full range of feasible ship arrangements from a basic shell of a ship, and then a vulnerability assessment could be performed on each of these many design variations. Thus, the designer is instantly aware of the vulnerability implications of the sizing and arrangement of the ship." The mathematical structure of optimization allows a quick and effortless analysis of the product, helping the designer find variable values that maximize a product's performance. Although optimization has been particularly useful to the design of engineered systems, it is ill-adapted to ship design, in which the mathematical representation has difficulty capturing all aspects of the real-world problem.

1.2 Research Problems

This dissertation was motivated by three main research problems which limit the applicability of optimization to ship design. They are presented here while Section 1.3 will present several answers to these problems.

1. The first research problem solved in this thesis stems from a model's inherent property of being a reduction. It is often said that all models are wrong, but many are useful. A model cannot capture every detail of a real world problem, so simplifications and abstract representations are required. Properly modeling every aspect of a problem is impossible due to time and budget constraints, because some of the data are unavailable for legal or privacy reasons, or simply because the designer is unaware of them. A typical solution to this problem has been to develop and link codes of ever increasing fidelity. However, this method is constrained by time and budget limits, and simply by the mathematical formulation of the model which is not always adequate at describing architectural problems and subjective goals inherent to ship design. Instead, this thesis proposes that engineers' expertise be incorporated directly into engineering models to increase their information content and ability to guide designs.

2. The second major problem addressed in this thesis is particular to multidisciplinary optimization (MDO), in which a top level optimizer coordinates the objectives of multiple, interdependent, lower level disciplines, in order to identify the best overall solution. The models of the lower level disciplines could be a combination of high fidelity and low fidelity models. For example, an MDO might use a semi-analytical model, an extensive computer model like CFD, and a regression model (Besnard et al., 2007). In this case, the low fidelity disciplines might incorrectly drive a solution because their models cannot capture the design environment adequately, thus providing an inaccurate solution. Any time and money spent developing high fidelity tools could therefore be lost when they are integrated with low fidelity models. As an answer, this thesis proposes a method of adaptively weighing the influence of discipline optimizers based on the expected validity of their results. The fusion logic used in this thesis provides an increased ability to coordinate the multiple lower level disciplines

compared to traditional MDO methods.

3. The last research problem addressed in this thesis deals with the limited amount of information which is available in early-stage design before a ship concept is chosen for further investigation. This notion has been formalized by Andrews (2012) as the "wicked problem," referring to the difficulty elucidating a vessel's requirements without first selecting solution concepts for the vessel. With traditional optimization, these concepts require a significant jump in model fidelity, which are time-consuming and expensive to develop, and predicate the solution based on the model's assumptions and a sunk-cost bias. Instead, a high-level, easily-implemented model, even if imprecise, would help the designer quickly understand the general trends of the solution concepts before implementing a more detailed analysis. A vessel's arrangements, for example, are affected by the shape of the hull form, the choice of the powering and propulsion system, and many other decisions which may evolve over the course of the design process. Thus, it is impossible to determine the exact model of arrangements and resulting trim-related performance until later in the design process when many design decision have been made. By allowing designers to build models earlier, even if they are imprecise, design decisions may be made with more confidence.

1.3 Dissertation Contributions

This thesis expands the scope of optimization models to make them more applicable to ship design, particularly in the early-stages of the process. As discussed in this chapter, current methods are ill-suited for the design of complex engineered systems. The typical response has been to develop optimization models with ever-increasing fidelity, complexity, and cost. While these methods are useful to analyze a single system in late-stage design, the model precision they require makes them unable to explore the design space as a whole. This dissertation proposes a new method which incorporates human expertise directly into the optimization model through type-2 fuzzy logic to leverage the wealth of knowledge people gain throughout their professional careers. Type-2 fuzzy logic was inspired by a human's ability to operate in an "environment of imprecision, uncertainty, incompleteness of information" (Mendel et al., 2010) and thus is an appropriate tool to model human reasoning and communication abilities through linguistic variables. Ship design is as much of an art as it is a science, so an optimization method which operates on vague linguistic variables and incorporates mental models has the potential to better handle open-ended problem which do not have a single optimal solution.

Several specific contributions coming out of this dissertation have been identified which lead to enhanced modeling and optimization capabilities for early-stage design. They are presented here and will be expanded upon throughout this dissertation.

1. Formulated a method of increasing a model's information content by incorporating ill-structured human knowledge into optimization models. The method properly accounts for linguistic variable vagueness through type-2 fuzzy logic.

2. Developed a multidisciplinary optimization method inspired from hierarchical type-2 fuzzy logic controllers which allows contribution 1 to be applied to multidisciplinary systems.

3. Developed fusion logic methods which provide flexible aggregation of discipline-level preferences into a system-level preference, and can account for subjective linguistic requirements.

4. Developed a method of leveraging human mental models to estimate plausible values of ship parameters when only limited models or information exists.

5. Developed a method of differentiating Pareto optimal points by incorporating subjective preferences to objective mathematical models.

1.4 Dissertation Structure

This dissertation is divided into seven chapters. The first provides an introduction to the problems selected for research and the contributions towards solving these problems.

Chapter 2 offers a background on modeling and optimization, both in the general sense and from an engineering point of view. Multidisciplinary optimization theory is discussed along with its applications to engineering. Finally, the need to go beyond mathematical modeling and optimization to better describe the attributes of a real-world problem is discussed.

Chapter 3 discusses uncertainties associated with decision-making, engineering, and design. Both aleatory and epistemic uncertainty are introduced. Methods of modeling these uncertainties are presented, along with several engineering applications.

Chapter 4 provides a thorough background on fuzzy sets, fuzzy logic systems, and fuzzy logic controllers. Type-2 fuzzy logic was chosen as the theoretical background to introduce linguistic variables into models and optimization, and this chapter provides the technical background to understand the methods presented in this thesis.

Chapter 5 introduces the type-2 fuzzy logic multidisciplinary optimization and illustrates it on the optimization of a generic set of mathematical objective functions. The structure of the method is first given. Then the translation between a numerical and linguistic variable is covered. The membership functions and rule banks, which encode human knowledge, are presented. Several methods of fusion logic, which aggregate the preferences of each discipline, are developed.

Chapter 6 illustrates the type-2 fuzzy logic multidisciplinary optimization on a practical application which involves optimizing a planing craft with respect to sea-keeping and resistance. The method of estimating ship parameters from human mental models when no engineering model exists is first presented. Then, the resistance and seakeeping disciplines are presented along with the linguistic performance parameters the use to evaluate a design.

Chapter 7 closes the dissertation with a conclusion, briefly summarizing the main findings and contributions of this work, and provides opportunities for future work.

CHAPTER II

Modeling and Optimization: Steering a Design

2.1 Introduction

This thesis presents a method of incorporating human expertise into engineering models which use linguistic variables to increase the flexibility of design optimization routines. While optimization research is abundant, its use in industry is still limited. Martins (2012) attributes this to engineers having little practical knowledge of optimization, and to the "backlash due to 'overselling' of numerical optimization." This backlash may stem from a misunderstanding of optimization's intended purpose. With increasing optimization code complexity and capability, the trend in naval design has been to remove the decision-maker's cognitive process and replace it with large optimization tools, distancing designers from the decision-making process and reducing the control they can have over the design. However, the main goal of optimization should be to gather information, and engineers should not have a blind faith in the tool's ability to accurately represent a design space.

Other limitations of optimization exist which have motivated this thesis. The first is the difficulty creating engineering models to use in optimization, and the second is the difficulty of communication which exists between optimization routines and human designers. The interaction between the designer and the design process plays an important role in the understanding of a product and one's ability to control it.

Although successfully applied across many industries, optimization would be much more widely-used if better ways can be found to keep designers in the loop. This chapter reviews modeling and optimization, both in the broadest sense and mathematical definition, and reviews several applications to engineering design. Then, several existing methods of keeping the designer in the loop are discussed.

2.2 Modeling and Optimization: a Broad Definition

When thinking of modeling and optimization, most engineers think of a formal mathematical process which seeks the best solution to a problem. However, optimization is a much broader concept defined by the Oxford Dictionary as the action of making the best or most effective use of a situation or resource, and has been practiced by human beings long before computers existed. A model is just an abstract representation which captures elements of knowledge about a system or process. It is a lens through which we see the world, and thus is the basis for all decision making. Many different models exist, with varying levels of precision. They can range from vague mental models capturing our general understanding of the world around us, to extremely precise mathematical models describing all attributes of a system. The wheel was invented to optimize the transportation of goods and people, and was likely inspired by observing that a round object is much easier to move than a square one. This observation, or mental model, was sufficient to its invention. In the 17th century, through observation, theorization, but not without standing on the shoulders of giants, Sir Isaac Newton developed his laws of motion, which provided a mathematical explanation (or model) of the ease with which a wheel can be moved. In the late 19th century, Frederick Taylor analyzed work flows and conceptualized his theory of scientific management (Taylor, 1911) to optimize worker productivity. In the early 20th century, Henry Ford combined the principles of scientific management with the moving assembly line used by meat-packers in Chicago and Cincinnati, to reduce the production time of the Model T Ford from 12 hours to about 90 minutes. Before implementing his new method to the entire assembly plant, Mr. Ford modeled it, tested it, and tweaked it on a mock-up (or model) where employees pulled the car in assembly through a series of work stations which mimicked what would become the modern assembly plant.

Several observations can be made from these four examples. From the wheel example, we learn that models can be derived from observation, and can lead to useful inventions without a complete understanding of the theory which supports them. Wheels were successfully used for 4000 years before Newton's laws of motion explained why they work. In ancient Mesopotamia, a simple but sufficient model of a wheel was simply an intuitive linguistic rule stating that a round object is easier to move than a square one. Newton's second law of motion for rotations, however, gave a precise mathematical model of the wheel's dynamics. From Taylor and Ford's work, we learn that modeling and optimization can be applied to soft sciences (i.e. labor productivity), and that models and experiments are vital to better understand a process without incurring large risks in case of failure. The experience gained from learning previously developed techniques, from trial-and-error, and from analysis can lead to many gains.

2.3 Modeling and Optimization: a Mathematical Definition

While the broad definition of modeling and optimization is somewhat fuzzy, its mathematical counterpart is well known and precisely formulated. Model-based design optimization iterates between a mathematical model and an optimizer to seek the design variable values that give the best design performance, or objective, as is shown in Figure 2.1.

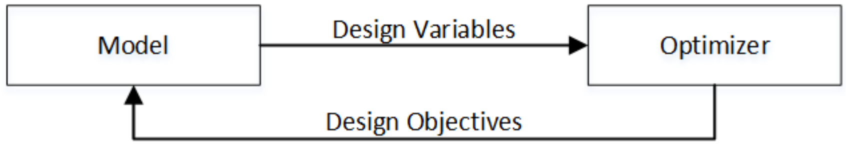


Figure 2.1: Model-based design optimization process.

At the most basic level, the optimizer seeks the design variable values $\mathbf{x}$ which minimize a single objective function $f(\mathbf{x})$ subject to constraints $\mathbf{g}(\mathbf{x})$ and $\mathbf{h}(\mathbf{x})$ (Equation 2.1).

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f(\mathbf{x}) \\ \text{subject to:} \quad & \mathbf{g}(\mathbf{x}) \leq \mathbf{0} \\ & \mathbf{h}(\mathbf{x}) = \mathbf{0} \end{aligned} \tag{2.1}$$

Many optimization methods have been developed to find the highest performing variable value on the design space, and extensive literature exists on the topic, so we will not dwell on it long. Several categorizations exist, with a common one grouping them into gradient and non-gradient-based optimizers. A gradient-based optimizer, such as the steepest decent method, uses the principles of calculus to find the optimum point. They are extremely quick and their convergence can be proven, but are limited to continuous, differentiable problems, and will get stuck in local minima. Non-gradient based optimizers do not require derivatives, thus can be used on non-continuous, non-differentiable problems. Evolutionary algorithms are the most famous optimization methods in this category. Genetic algorithms (GA) and particle swarm optimizers (PSO) are two popular evolutionary optimization methods extensively studied in the literature. They are more robust and capable of finding global optima, but are much slower than gradient-based methods.

The model abstracts a problem through series of mathematical relations leading to the objective function, which represents a relationship between design variables and performance parameters the designer seeks, and the constraints which represent requirements to be met. For example, the designer could seek to design a wheel which is most easily spun per unit torque, but still big enough to carry a cart. The designer would translate this requirement to designing the wheel with the smallest moment of inertia, subject to a minimum radius.

Mathematical models are all around us. They can be derived from first principles, from physics-based simulations (e.g. finite element analysis or computational fluid dynamics), or from regressions if experimental data is available. For the many problems that have a known mathematical relationship between design variables and objective function, mathematical optimization is an excellent tool. However, its results should still be validated as they are only as good as the model and the optimizer itself. Recall that models are reductions that usually only account for a subset of the features that belong to the real world problem they are trying to represent. The features they do capture are also just approximated, and will never capture them completely.

2.4 Multidisciplinary Design Optimization

Following the recent improvements of computational capabilities available to the private and public sector, there has been a push in many engineering fields to develop tools capable of thoroughly exploring system design spaces to increase their performance and reduce cost. The U.S. Navy uses the Advanced Surface Ship Evaluation Tool (ASSET) which can already rapidly produce multiple ship designs parametrically from historical data, but the Navy has "expressed a desire for even more sophisticated design and analysis tools capable of facilitating advanced design methods" (Gray et al., 2013).

One class of engineering design tools that is particularly useful in complex engineering problems like aerospace and naval architecture is multidisciplinary design optimization (MDO). In such problems, the management of interdependencies between diverse disciplines or teams with conflicting and changing requirements becomes an extremely important consideration because "the performance of a multidisciplinary system is not only driven by the performance of the individual disciplines but also by their interactions" (Martins and Lambe, 2012). The Rockwell International (now Boeing) B-1 Lancer is a bomber which exemplifies this concept. The B-1A was the first prototype developed and could reach speeds of Mach 2.2 at high altitude. The program was canceled but later resurrected with the B1-B, which had a top speed of Mach 1.25 but a significantly increased payload which allowed it to carry more fuel and weapons (Pace, 1998). The interaction between the payload and the aerodynamics discipline forced the Rockwell's engineers to make design choices by prioritizing one of the disciplines at the other's detriment, with large consequences on the product's commercial viability. MDOs provide a structured method of dealing with these interactions by coupling multiple disciplines through numerical optimization, which coordinates the inter-discipline influence and trade-offs (Hannapel and Vlahopoulos, 2010).

Multidisciplinary optimizers are composed of two components: the analysis blocks (or models) and the optimizer. Analysis blocks take design variable values as inputs and return an objective function and constraint values. They are usually interdependent, meaning that their output is dependent on the output of the other discipline blocks, and can range from low fidelity regressions of historical data to high fidelity physics-based simulations like computational fluid dynamics (Jouhaud et al., 2007; Yang and Sen, 1996). The optimizer block uses the objective function and constraint values to select the design variable values to evaluate at the next iteration of the process, guiding the design search. MDOs are separated into two kinds of architecture. Monolithic architectures treat all subsystems as a whole by unifying them into a single optimization problem. Distributed architectures separate each subsystem and solve them individually while coordinating the search of each discipline to determine a consistent solution. While these methods have few direct parallels to the optimization developed in this thesis, they are presented below for completeness and to show the relations between the components of a multidisciplinary system.

2.4.1 Monolithic Multidisciplinary Design Optimization Formulation

Single-level, or monolithic architectures are the simplest and most common multidisciplinary optimization formulations. While the analysis blocks of all disciplines are separate, only one optimizer solves the design variable search. (Cramer et al., 1994) identify two monolithic MDO formulations, multidisciplinary feasible (MDF) and individual discipline feasible (IDF), but several more exist. MDF algorithms (Equation 2.2) perform a full analysis of the design variable vector for every discipline at each iteration of the optimization, which can be computationally expensive but guarantees the consistency of the solution at each step of the algorithm. Thus if the optimization is stopped early, the design variable evaluated by each discipline will be identical and the solution will be consistent. If the discipline analysis blocks are computationally expensive, the IDF formulation is more advantageous. It includes coupling variables and consistency constraints which ensure a consistent solution once the algorithm has converged. This eliminates the need to solve each discipline analysis block at each iteration of the algorithm, which can lead to significant computational savings, especially if the analysis blocks use high fidelity physics based simulations.

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f(\mathbf{x}, \mathbf{y}(\mathbf{x})) \\ \text{subject to:} \quad & \mathbf{g}(\mathbf{x}, \mathbf{y}(\mathbf{x})) \leq \mathbf{0} \\ & \mathbf{h}(\mathbf{x}, \mathbf{y}(\mathbf{x})) = \mathbf{0} \end{aligned} \tag{2.2}$$

2.4.2 Distributed Multidisciplinay Design Optimization Formulation

Distributed, or multi-level, multidisciplinary optimization formulations decompose the problem into multiple subproblems which can represent components, engineering disciplines, or even different companies (Rabeau et al., 2007). Modern engineering practices like concurrent engineering decompose a design problem, where many groups solve sub-problems in parallel, only communicating infrequently. This is motivated by geographical barriers (a contractor with rare expertise might be located far from the company), by political pressures to distribute job creation throughout a region (as is the case for U.S. military procurement), but also to reduce time to market.

One distributed MDO method commonly used in product development is analytical target cascading (ATC), also known as requirements flow-down (Allison et al., 2005). It breaks down a problem into a hierarchy of optimizers. The bottom level optimizers solve sub-system or discipline level problems according to local targets set by the top level optimizer (i.e. the ship design manager). The top level optimizer manages the multiple, interdependent, lower level disciplines, in order to identify the local targets that provide the best integrated solution. Additionally, the top level seeks system level performance targets analogous to stakeholder requirements, like cost, range, or top speed. The ATC formulation, provided by Kim et al. (2003), is given in Equations 2.3-2.5. Local and linking variables are denoted by $\mathbf{x}$ and $\mathbf{y}$ respectively. Design consistency is achieved by passing targets from a parent optimizer to its children, who return responses back. Tolerances on consistency constraints are given by ϵ. Subscripts sup, s, ss denote the super-system, system and sub-system respectively. Superscripts U denote targets passed down from an optimizers one level up, whereas subscripts L denote responses returned up from an optimizer one level down. r_i represents an analysis tool, such as a regression or simulation, used to evaluate a design value.

Equation 2.3 gives the problem formulation of the top-level, or super-system, which coordinates the system (or disciplines) level searches by providing targets to the system level optimizers to achieve a coherent solution.

$$
\begin{aligned}
&\text{minimize} && ||\mathbf{R}_{sup} - \mathbf{T}_{sup}|| + \boldsymbol{\epsilon_R} + \boldsymbol{\epsilon_y} \\
&\text{with respect to} && \mathbf{x}_{sup},\ \mathbf{y}_s,\ \mathbf{R}_s,\ \boldsymbol{\epsilon_R},\ \boldsymbol{\epsilon_y} \\
&\text{where} && \mathbf{R}_{sup} = r_{sup}(\mathbf{R}_s, \mathbf{x}_{sup}) \\
&\text{subject to} && \sum_{k \in C_{sup}} ||\mathbf{R}_{s,k} - \mathbf{R}^L_{s,k}|| \leq \boldsymbol{\epsilon_R} \\
& && \sum_{k \in C_{sup}} ||\mathbf{y}_{s,k} - \mathbf{y}^L_{s,k}|| \leq \boldsymbol{\epsilon_y} \\
& && \mathbf{g}_{sup}(\mathbf{R}_{sup}, \mathbf{x}_{sup}) \leq \mathbf{0}
\end{aligned}
\tag{2.3}
$$

The system level optimization problem is given in Equation 2.4. It minimizes the deviation between its target provided by the super-system $R^U_{s,j}$ and its response $R_{s,j}$ subject to problem and consistency constraints. The consistency constraints depend on values returned by the children of the optimizer

$$
\begin{aligned}
&\text{minimize} && ||\mathbf{R}_{s,j} - \mathbf{R}^U_{s,j}|| + ||\mathbf{y}_{s,j} - \mathbf{y}^U_{s,j}|| + \boldsymbol{\epsilon_R} + \boldsymbol{\epsilon_y} \\
&\text{with respect to} && \mathbf{x}_{s,j},\ \mathbf{y}_{s,j},\ \mathbf{y}_{ss},\ \mathbf{R}_{ss},\ \boldsymbol{\epsilon_R},\ \boldsymbol{\epsilon_y} \\
&\text{where} && \mathbf{R}_{s,j} = r_{s,j}(\mathbf{R}_{ss}, \mathbf{x}_{s,j}, \mathbf{y}_{s,j}) \\
&\text{subject to} && \sum_{k \in C_{s,j}} ||\mathbf{R}_{ss,k} - \mathbf{R}^L_{ss,k}|| \leq \boldsymbol{\epsilon_R} \\
& && \sum_{k \in C_{s,j}} ||\mathbf{y}_{ss,k} - \mathbf{y}^L_{ss,k}|| \leq \boldsymbol{\epsilon_y} \\
& && \mathbf{g}_{s,j}(\mathbf{R}_{s,j}, \mathbf{x}_{s,j}, \mathbf{y}_{s,j}) \leq \mathbf{0}
\end{aligned}
\tag{2.4}
$$

The subsystem level optimization problem is given in Equation 2.5. It seeks to minimize the deviation between its targets received by the system level, and its responses calculated by analysis functions $r_{ss,j}$. It does not have any consistency constraints since it has no children, but can have a problem constraint.

$$
\begin{aligned}
&\text{minimize} && ||\mathbf{R}_{ss,j} - \mathbf{R}^U_{ss,j}|| + ||\mathbf{y}_{ss,j} - \mathbf{y}^U_{ss,j}|| \\
&\text{with respect to} && \mathbf{x}_{ss,j},\, \mathbf{y}_{ss,j} \\
&\text{where} && \mathbf{R}_{ss,j} = r_{ss,j}(\mathbf{x}_{ss,j}, \mathbf{y}_{ss,j}) \\
&\text{subject to} && \mathbf{g}_{ss,j}(\mathbf{R}_{ss,j}, \mathbf{x}_{ss,j}, \mathbf{y}_{ss,j}) \leq \mathbf{0}
\end{aligned}
\tag{2.5}
$$

2.4.3 Multidisciplinary Optimization in Engineering

Multidisciplinary optimization has been successfully applied to the design of many complex systems. In automotive engineering, ATC was used to design a vehicle chassis according to ride quality and handling (Kim et al., 2003). A hierarchical MDO for automotive design problems is commonly decomposed into engine, drive-train, and vehicle dynamics disciplines (Kim et al., 2002; Kokkolaras et al., 2004), representing the different teams working on the product development. MDO's are also common in aircraft design (Henderson et al., 2012; Vlahopoulos et al., 2011), which was one of the first industries to adopt them. ATC has been used for wing and propulsion design over multiple flight regimes (Allison et al., 2006). MDO has also found many uses in ship design. It has been used in propeller design, where hydrodynamics and structural disciplines interact (Takekoshi et al., 2005; Young et al., 2010), and in propeller-hull systems optimization (Nelson et al., 2013). Hannapel (2012) developed the MDO with Target Values and applied it to reliable and robust optimization of a bulk carrier where transportation cost, annual cargo, and lightship were optimized.

2.5 The Art and Science of Ship Design: the Need to Go Beyond Mathematical Models

As mentioned in the introduction, it is really the iterative interplay between the human designer and the design tools that make a design successful. Experts carry with them extensive domain knowledge about their area of expertise. They use it to analyze results and make design decisions, they refine it with every vessel they work on, and they transfer it with every person they work with. This domain knowledge is subjective, ill-defined, and implicit. Computers, on the other hand, require structured and explicit knowledge (or mathematical models). They can perform computations at a speed much greater than humans, but their ability to convey knowledge is limited by the inflexibility of the models they use, as many processes cannot be described mathematically. A design process is really a collaboration between these two entities. An optimization run without the input and oversight of an experienced user could give untrustworthy and dangerous results, but a human designer working without computer optimization will likely only achieve a converged design rather than a great one. The human-in-the-loop optimization process is illustrated in Figure 2.2. The user runs a model through an optimization to gather information and gain a deeper understanding of the design space. Preferences are extracted, referring to the user refining the next model to run. Major decisions could be made here, such as choosing the propulsion type, the desired payload, or any other decision which will have a major impact on future design. Then the process continues until a satisfactory design has been found.

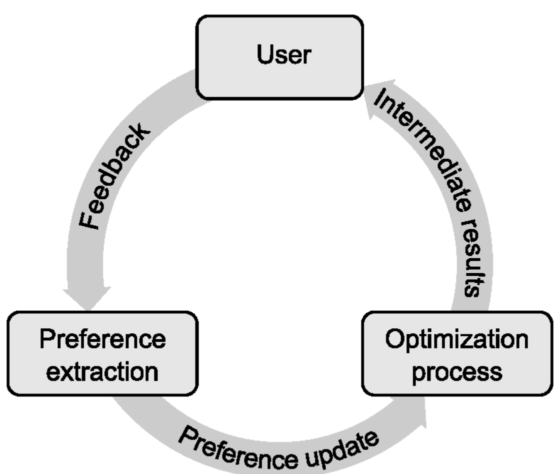


Figure 2.2: Human-in-the-loop optimization. (Meignan et al., 2015)

Still, there are many limitations with mathematical modeling and optimization. They are discussed throughout this thesis, and include (1) the approximate representation of the real world models are trying to represent, (2) the difficulty of interacting with and controlling optimization routines, (3) the mistrust towards many optimization methods, which is justified given points one and two, (4) the inability to use optimization before precise models of a vessel have been developed (implying that design decisions have already been made).

The constant increase in available computing power has pushed back the limits on model detail and complexity. For naval design, Kassel et al. (2010) discuss the need for a tool to "rapidly produce a full range of feasible ship arrangements from a basic shell of a ship" and analyze the "vulnerability implications of the sizing and arrangement of the ship." However, even the best mathematical models cannot accurately capture all aspects of a design problem given that mathematics are best adapted at describing only the most precise and specific of statements, and modeling a whole ship with such precision is impractical. Mathematical models also only have a limited ability to model imprecise concepts and subjective preferences, which are inherent in ship design. Interactive optimization recognizes this limitation and calls on the user to fill this modeling gap by providing feedback about the optimization results during the run, enriching the model and guiding the search process (Meignan et al., 2015). For example, Kim and Cho (2000) developed an interactive genetic algorithm to determine the attributes that make a piece of clothing fashionable. Aesthetics cannot be modeled mathematically, so at each iteration of the algorithm, a user ranking of several clothing designs replaced the traditional objective function used by optimization algorithms. However, this method is prone to tedium and error, and the population size of the algorithm is limited. Interactive optimization improves the interaction between the user and the model, but the mismatch between precise mathematical models and the imprecise human natural language still remains. Fuzzy logic operates on linguistic variables instead of numeric ones, which minimizes the need to translate a human mental model to a mathematical one. Fuzzy logic was originally developed as a method of programming computers using natural language (Zadeh, 1965), making it an adequate tool to formalize personal preferences and experiences into a computer model. A detailed review of its applicability to engineering problems will be given in Chapter III, Section 3.4.2.2 after introducing engineering uncertainty, and fuzzy logic's theoretical background is given in Chapter IV.

2.6 Closing Remarks

This chapter provided a background on modeling, which is a translation of human knowledge into abstract representations of a system of interest. Engineering modeling is typically done through mathematics, and has had significant success in many industries. However, in its most basic form, a model is just a description. It does not need to be mathematical, and if optimization can be performed using the natural language of a human being, many more models could be used to formalize knowledge and make engineering decisions. Interactive optimization and fuzzy logic prove promising in this aspect.

CHAPTER III

Uncertainty in Engineering Design

3.1 Introduction

There are two major factors that influence decisions and actions. The first is choice, or the presence of alternatives. Without choice, a person's path is laid out for them, leaving no other option than to blindly follow it, and removing all control over one's own life choices. The second factor is the presence of uncertainty. Here, uncertainty is defined as the lack of perfect information regarding alternatives and decisions. It is also a prerequisite for risk, since without uncertainty there is no room for failure. While not studied in introductory engineering courses, uncertainty is inherent to any real world problem and forms the basis of all decision making. It is an inherent aspect of choice, and grants humans control over their lives, present and future, since "in a predestinate world, decision would be illusory; in a world of perfect knowledge, empty; in a world without natural order, powerless" (Shackle, 1969). Uncertainty provides the grounds for making decisions which will have a strong impact on a person's future opportunities. Many dimensions of it exist, as described by Klir and Wierman (2006):

> When dealing with real world problems, we can rarely avoid uncertainty. At the empirical level, uncertainty is an inseparable companion of almost

any measurement, resulting from a combination of inevitable measurement errors and resolution limits of measuring instruments. At the cognitive level, it emerges from the vagueness and ambiguity inherent in natural language. At the social level, uncertainty has even strategic uses and it is often created and maintained by people for different purposes (privacy, secrecy, propriety).

Uncertainty is typically classified into two broad categories—aleatory and epistemic. In general, the classification is based on whether the uncertainty is a function of the decision maker's knowledge of the system, and can be reduced by gathering more information, or whether the uncertainty is inherent to the system.

3.2 Types of Uncertainty

3.2.1 Aleatory Uncertainty

Aleatory uncertainty is used to describe the inherent variability of a process. Here, variability is defined as the non-uniformity of outcomes for a sample of events. If a random process is repeated several times, the outcomes will differ, but over many trials will be found to follow a natural distribution representative of the system's dynamics. A typical example is a game of chance, which is designed to use uncertainty. In fact, aleatory is derived from the Latin *alea*, which signifies a gamble. A person betting that a fair die lands a six does not know the outcome of his or her bet, but knows that the long run probability of winning is one sixth.

Aleatory uncertainty is often said to be irreducible because its variations are a function of variables outside of the designer's control, and are sometimes a function of variables acting on the system that are unknown to the designer. Aleatory uncertainty is used to capture the general trends of a system's variability when it is not possible to individually capture every cause of the system's behavior. In a world of perfect information, the outcome of a die could be determined as a function of the die's weight, inertia, initial trajectory, and forces acting on it as a function of time; however, we humans lack the ability to measure and control these quantities, even though we understand them, so we lump them into random variations and let chance decide.

3.2.2 Epistemic Uncertainty

Epistemic uncertainty is uncertainty which stems from a lack of knowledge or understanding about a system, and which could be reduced by gathering additional information. It is closely related to Klir's cognitive and social levels of uncertainty.

From a cognitive perspective, epistemic uncertainty about a system is deeply personal since it depends on the knowledge, beliefs, and perspectives which shape one's mental model (O'Hagan, 2004). A measurement may exhibit epistemic uncertainty. I am certain about my age. My doctoral advisor, who knows how many years ago I started my PhD, right after finishing my Bachelor's, has a reasonably good idea of how old I am. The recruiters I speak to on the phone will have a harder time knowing how old I am. A model may also exhibit epistemic uncertainty. A slow and leisurely bicyclist may not understand the drag reduction benefits of drafting, but to a group of competitive cyclists going over 20 mph on average, the benefits of drafting are much more noticeable.

From a social perspective, epistemic uncertainty can arise in complex system design, where the interdependence of discipline design targets evolve with the design process, and thus can be hard to determine before several analyses have been done to better understand the discipline trade-offs. It can also arise when information is deliberately hidden and in teams, because all communications are uncertain since linguistic variable representations are vague to some degree in the minds of the originators and in the minds of the receivers (Wallsten and Budescu, 1995).

3.3 Modeling Uncertainty

Both aleatory and epistemic uncertainty are a source of risk in design decision making. Many engineering models have sought to capture a system's randomness or a designer's lack of knowledge to understand and mitigate their associated risks.

3.3.1 Modeling Aleatory Uncertainty

Probability theory was formalized by Gerolamo Cardano, who wrote the first book on the topic in the sixteenth century (Hacking, 2006). The individual instances of a random process often do not provide enough information about the process, and it is often useful to summarize its general properties through a probabilistic model, which is a model that captures the inherent variations of a random process and the global trends it would exhibit if repeated many times. Thus, probability models are natural descriptions of aleatory uncertainty. These trends are described by probability distribution functions whose shape is governed by parameters such as the mean, standard deviation, skew, and others, and which capture the relative frequency of events they model.

For example, the weight or size of a manufactured product composed of the sum of many independent components usually follows a normal distribution centered around the mean and having a spread given by its standard deviation. Assuming a normal distribution, 68% of the products are within one standard deviation of the mean, 95% of them are within 2 standard deviations, and 99.7% of them are within three standard deviations of the mean. In naval architecture, wave heights are often described using Weibull or Rayleigh distributions (Dean and Darlymple, 2011). The surface of the ocean, which is often rough and irregular, is easiest described in the frequency domain using probability distributions to capture the likelihood of encountering given wave heights.

Aleatory uncertainty requires that precise probability models be known. For well understood engineering problems, where large amounts of data are available, probability theory and aleatory uncertainty provide a very good approximation of of the natural randomness of the world. They have been successfully applied to many real world problems like structural analysis, weather forecasting, ship seakeeping, financial analysis, and others. For problems that are not well understood, and for which the probability distribution functions cannot be determined, other methods are required. The American scientist Terrence Fine (1973) once said: “I think it is wiser to avoid the use of a probability model when we de not have the necessary data than to fill the gaps arbitrarily; arbitrary assumptions yield to arbitrary conclusions.” In such case, epistemic uncertainty modeling can help capture just how much is known about the problem, to mitigate the effects of uncertainty without assuming arbitrary knowledge in the form of probability distributions.

3.3.2 Modeling Epistemic Uncertainty

Before the 1960s, uncertainty was solely modeled using probability theory and statistics, and thus the only known dimension of uncertainty was aleatory. Starting in the sixties, the development of alternative theories like fuzzy set theory (Zadeh, 1965), evidence theory (Dempster, 1967; Shafer, 1976) and possibility theory (Zadeh, 1978) started providing decision makers with methods to deal with epistemic uncertainty (Klir and Wierman, 2006). These methods do not assume known probability distributions. Instead, they rely on subjective models which mimic the limited information available to a designer. Fuzzy set theory and evidence theory have gained traction in engineering research, and are discussed in the following sections.

3.3.2.1 Fuzzy Set Theory

Fuzzy set theory is a method of representing data with imprecise boundaries, and fuzzy logic is a method of operating on these sets. They were originally developed as a method of programming computers using natural language (Zadeh, 1965), and were inspired by a human's "capability to converse, communicate, reason and make rational decisions in an environment of imprecision, uncertainty, incompleteness of information and partiality of truth" (Mendel et al., 2010). Fuzzy set theory and fuzzy logic formalizes a human's ability to compute with words, called linguistic variables in fuzzy logic (Zadeh, 1996), to improve a computer's ability to model environments with high epistemic uncertainty.

Most human knowledge is described and communicated through words. Words are an abstraction of the more precise quantitative variables in a human's sensory input, like the outside temperature, and which simplify a person's thought process. Humans do not think in terms of probability density functions and precise measurement, and yet perform complex tasks very well. If an engineering problem has tolerance for such imprecision, fuzzy logic can be used to reduce the computational burden of numerical calculations. An experienced driver's ability to operate a vehicle almost subconsciously, without knowing the exact friction coefficient between the road and tires in wet and dry conditions, illustrates the small cognitive burden that computing with words imposes on its user. A detailed background fuzzy logic theory is given in Chapter IV, and is covered in many resources (Zadeh, 1965, 1975; Dubois and Prade, 1980; Mendel et al., 2014).

3.3.2.2 Evidence Theory

Evidence theory, also known as Dempster-Shafer theory, is a mathematical framework which uses probability intervals and degrees of belief to represent epistemic uncertainty. It combines multiple pieces of limited and sometimes conflicting information (i.e. evidence) into an uncertainty interval based on two measures, belief and plausibility, which act as the lower and upper bound probability distribution functions of the possible variable values. The likelihood of an event is assigned a set, whereas probability theory assigns a single probability of an event happening. However, if the belief and plausibility measures of a variable value are identical, they reduce to the classical probability measure, and evidence theory reduces to probability theory.

Both aleatory and epistemic uncertainty model different types of phenomena. They are complementary and together, provide a better understanding of a human's relationship to knowledge. Both these theories have been applied to real-world design optimization problems, which are covered in the rest of this Chapter.

3.4 Uncertainty in Design Optimization

Uncertainty in design can never be completely avoided, whether it is a function of the natural randomness associated with the system itself, or whether it comes from the designer's lack of knowledge and understanding of the system. Perfect knowledge does not exist, but engineers have developed several methods of mitigating the risks associated with the limited information they have about the world. Given thorough developments of probability theory since the 16^{th} century, it should not come as a surprise that methods of handling aleatory uncertainty in design are far more mature than their epistemic counterpart.

3.4.1 Aleatory Uncertainty in Design Optimization

Aleatory uncertainty is widely modeled in optimization applications for which the goal is to improve a product while ensuring its reliability in a random environment. Here, reliability is defined as the likelihood of performing a given goal satisfactorily. Unlike for a deterministic optimization problem, the design variables and parameters are random variables characterized by probability density functions and their moments. At the most basic level, probabilistic optimization problems can be formulated two ways. The first seeks the minimization of an objective function subject to a probabilistic constraint on the performance, and the second maximizes the reliability the product subject to a constraint on an objective (usually cost). These two formulations are given in Equation 3.1 (Elishakoff and Ohsaki, 2010), where $\mathbf{x}$ is a vector of random variables, $F(\mathbf{x})$ is the objective function, $P_f(\mathbf{x})$ is the probability of failure, with a given acceptable value $P_f^a(\mathbf{x})$. $R(\mathbf{x})$ is the reliability, defined as the complement of the probability of failure, and F_a is the constraint on the objective value.

$$\begin{aligned}
&\underset{\mathbf{x}}{\text{minimize}} && F(\mathbf{x}) \\
&\text{subject to} && P_f(\mathbf{x}) \leq P_f^a \\
& && \\
&\underset{\mathbf{x}}{\text{maximize}} && R(\mathbf{x}) = 1 - P_f(\mathbf{x}) \\
&\text{subject to} && F(\mathbf{x}) \leq F_a
\end{aligned} \tag{3.1}$$

Many probabilistic optimization methods have grown out of this general formulation, the two most common being reliability based design optimization and robust design optimization.

3.4.1.1 Reliability-Based Design Optimization

In reliability-based design optimization, the goal is to find the optimal expected design value that satisfies the constraints with a given probability, also called probabilistic constraints, which provide a reliability index of the product's performance. It has its roots in structural design, where it has widely been applied since the random variations and probabilistic models of such problems are well understood and can be defined accurately. In structural optimization, the aleatory uncertainty originates from random variations in material properties which depend on the raw materials and manufacturing process, or from uncertainty in the loads the structure will experience over time. Several formulations of the optimization problem are presented in Enevoldsen and Srensen (1994). Acar and Solanki (2009) perform structural optimization on an aircraft wing, subject to a stress constraint which takes into account the probabilistic aero-elastic loading of the wing.

Reliability-based optimization is also common in multidisciplinary design (MDO). Yu and Du (2006) perform reliable optimization of an aircraft wing with respect to aerodynamics and structural disciplines, where flight parameters (altitude, speed,...) and physical properties (weight, wing area, yield modulus,...) of the aircraft are assumed to be normally distributed. Other aerospace applications are surveyed by Yao et al. (2011). In naval architecture, Hannapel and Vlahopoulos (2010) performed reliable optimization of a bulk carrier which took into account uncertainty in the operating speed and the regression model for vessel deadweight.

Recall from Chapter II that in analytical target cascading MDO, the goal is to find designs that minimize the difference between the system level's design targets and the values returned by the lower-level disciplines, subject to constraints. This problem is adapted to reliability-based design optimization (Equation 3.2) by assuming random design variables modeled by PDFs and finding the optimal design such that the probability of satisfying the M constraints is greater than a chosen value α_m, which denotes the chosen reliability of the system (Liu et al., 2006).

$$\begin{aligned} &\underset{\mathbf{x}}{\text{minimize}} && ||\mathbf{T} - \mathbf{R}|| \\ &\text{subject to} && \Pr[g_m(\mathbf{x}) \leq 0] \geq \alpha_m, \quad m = 1, ..., M \\ &\text{where} && \mathbf{R} = \mathbf{f}(\mathbf{x}) \end{aligned} \tag{3.2}$$

For a two level ATC composed of the top level system optimizer and several discipline-level optimizers, the probabilistic formulation is given below. Here, the probabilistic constraint provides a measure of reliability of the system. The top level optimizer sets targets $\mathbf{T}_{sys}$ for the lower level disciplines, and seeks to minimize the deviation between these targets and the responses $\mathbf{R}_{sys}$ subject to tolerance constraints for the responses ϵ_R and the linking variables ϵ_y and subject to design constraints $\mathbf{g}$.

$\mathbf{x}$ denotes local variables and $\mathbf{y}$ denotes linking variables. The response of the system $\mathbf{R_{sys}}$ is dependent on an analysis funcion r_{sys}, the discipline level variables $\mathbf{x_d}$, and response $\mathbf{R_d}$ provided from the system to discipline k. $\mathbf{R_d^L}$ indicates the response returned to the system from that discipline. C_{sys} indicates the set of all children of the top level optimizer, which is the number of discipline in this two-level formulation. Equation 3.3 states the system-level formulation.

$$\begin{aligned} &\text{minimize} && ||\mathbf{R}_{sys} - \mathbf{T}_{sys}|| + \epsilon_R + \epsilon_y \\ &\text{with respect to} && \mathbf{x}_{sys},\ \mathbf{y}_d,\ \mathbf{R}_d,\ \epsilon_R,\ \epsilon_y \\ &\text{where} && \mathbf{R}_{sys} = r_{sys}(\mathbf{R}_d, \mathbf{x}_d) \\ &\text{subject to} && \sum_{k \in C_{sys}} ||\mathbf{R}_{d,k} - \mathbf{R}^L_{d,k}|| \leq \boldsymbol{\epsilon_R} \\ &&& \sum_{k \in C_{sys}} ||\mathbf{y}_{d,k} - \mathbf{y}^L_{d,k}|| \leq \boldsymbol{\epsilon_y} \\ &&& \Pr[\mathbf{g}_{sys}(\mathbf{R}_{sys}, \mathbf{x}_{sys}) \leq \mathbf{0}] \geq \boldsymbol{\alpha}_{sys} \end{aligned} \tag{3.3}$$

The discipline level formulation is given in Equation 3.4. At this bottom-most level, the optimizer seeks to minimize the deviation between the output, $R_{d,j}$, of their analysis function r_d, j, and the targets set by the upper level system $R^U_{d,j}$, subject to local, probabilistic constraints $\mathbf{g_{d,j}}$.

$$\begin{aligned} &\text{minimize} && ||\mathbf{R}_{d,j} - \mathbf{R}^U_{d,j}|| + ||\mathbf{y}_{d,j} - \mathbf{y}^U_{d,j}|| \\ &\text{with respect to} && \mathbf{x}_{d,j},\ \mathbf{y}_{d,j} \\ &\text{where} && \mathbf{R}_{d,j} = r_{d,j}(\mathbf{x}_{d,j}, \mathbf{y}_{d,j}) \\ &\text{subject to} && \Pr[\mathbf{g}_{d,j}(\mathbf{R}_{d,j}, \mathbf{x}_{d,j}, \mathbf{y}_{d,j}) \leq \mathbf{0}] \geq \boldsymbol{\alpha}_{d,j} \end{aligned} \tag{3.4}$$

3.4.1.2 Robust Design Optimization

Another class of probabilistic optimization methods is robust design, which seeks to ensure that the performance of the objective functions at the optimal solution will not severely deteriorate in the presence of uncertainty (Hannapel, 2012). Uncertainty in the design variables or parameters can take the form of probability distribution functions, or can be handled using interval analysis if not enough information is known to estimate such functions. In such cases, the variables and parameters can take any value within these intervals, and the problem must be feasible for all such values (i.e. all possible cases), thus provides a worst case scenario for the problem. To mitigate the risk associated with the interval uncertainty in the design variables and parameters, designers seek to optimize the worst case scenario, and the problem becomes a nested optimization routine known as an anti-optimization problem (Elishakoff and Ohsaki, 2010). The inner loop seeks the maximization of the objective function within the domain given by the uncertainty interval, and the outer loop finds the independent variables which minimize this worst case scenario.

Robust optimization is commonly applied to multidisciplinary optimization problems. Wang et al. (2009) apply it to the design of the crankshaft and connecting rod of a combustion engine. Hannapel (2012) developed a robust multi-objective optimization of a bulk carrier with three objectives: annual cargo carried, lightship weight, and transport cost. An interval analysis robust analytical target cascading method was developed by Kokkolaras and Papalambros (2008) following the anti-optimization procedure given by Elishakoff et al. (1994). It is given in Equations 3.5-3.7. For a given nominal value of the design variable value X_N, the interval uncertainty on that variable is given by $[(1-\delta_x)x_N \leq x \leq (1+\delta_x)x_N]$. The worst-case scenario, or maximum value, of each constraint on this interval is solved through the anti-optimization problem given in Equation 3.5 for each constraint $g(x)$.

$$g_{max} = \max_{x} \quad g(x) \quad \text{subject to } (1-\delta_x)x_N \leq x \leq (1+\delta_x)x_N \tag{3.5}$$

These constraint values, along with the worst objective value on the uncertainty interval, are used in the outer loop of the robust ATC to find the optimal worst case scenario (i.e. the anti-optimization problem) for a given nominal design variable value X_N. The inner loop then finds the optimal value of X_N^* which minimizes the worst case scenario, denoted by the subscript w. The best case is found through a minimization problem and is denoted by the subscript b. For the two-level ATC, the top (or system) level formulation is given in Equation 3.6, and the bottom level formulation is given in Equation 3.7, where the notation is identical to the reliability based ATC.

$$\begin{aligned}
&\text{minimize} && ||\mathbf{R}_{sys_w} - \mathbf{T}_{sys_w}|| + ||\mathbf{R}_{sys_b} - \mathbf{T}_{sys_b}|| \\
& && + \epsilon_{R_w} + \epsilon_{R_b} + \epsilon_{y_w} + \epsilon_{y_b} \\
&\text{with respect to} && \mathbf{x}_{sys_N}, \mathbf{y}_{d_N}, \mathbf{R}_{d_N}, \epsilon_{R_N}, \epsilon_{y_N} \\
&\text{where} && \{\mathbf{R}_{sys_w}, \mathbf{R}_{sys_b}\} = r_{sys}(\mathbf{R}_{d_N}, \mathbf{x}_{d_N}) \\
&\text{subject to} && \sum_{k \in C_{sys}} ||\mathbf{R}_{d,k_w} - \mathbf{R}^L_{d,k_w}|| \leq \epsilon_{R_w} \\
& && \sum_{k \in C_{sys}} ||\mathbf{R}_{d,k_b} - \mathbf{R}^L_{d,k_b}|| \leq \epsilon_{R_b} \\
& && \sum_{k \in C_{sys}} ||\mathbf{y}_{d,k_w} - \mathbf{y}^L_{d,k_w}|| \leq \epsilon_{y_w} \\
& && \sum_{k \in C_{sys}} ||\mathbf{y}_{d,k_b} - \mathbf{y}^L_{d,k_b}|| \leq \epsilon_{y_b} \\
& && \mathbf{g}_{sys_{max}}(\mathbf{R}_{sys_N}, \mathbf{x}_{sys_N}) \leq \mathbf{0}
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
&\text{minimize} && ||\mathbf{R}_{d,j_w} - \mathbf{R}^U_{d,j_w}|| + ||\mathbf{R}_{d,j_b} - \mathbf{R}^U_{d,j_b}|| + \\
& && + ||\mathbf{y}_{d,j_w} - \mathbf{y}^U_{d,j_w}|| + ||\mathbf{y}_{d,j_w} - \mathbf{y}^U_{d,j_w}|| \\
&\text{with respect to} && \mathbf{x}_{d,j_N}, \mathbf{y}_{d,j_N} \\
&\text{where} && \{R_{d,j_w}, R_{d,j_b}\} = r_{d,j}(\mathbf{x}_{d,j_N}, \mathbf{y}_{d,j_N}) \\
&\text{subject to} && \mathbf{g}_{d,j_{max}}(\mathbf{R}_{d,j_N}, \mathbf{x}_{d,j_N}, \mathbf{y}_{d,j_N}) \leq 0
\end{aligned} \tag{3.7}$$

When a system is well understood, and its inherent variations are well known, reliable and robust optimization provide a good methods for analyzing the design with respect to the aleatory uncertainty surrounding it. If the inherent variations of the system can be modeled by a probability distribution function, designers can use reliability-based optimization which seeks the system optimum that will perform adequately with a selected probability, or confidence. Here, performing adequately, is defined as satisfying all design constraints. If the inherent variations of the system cannot be modeled by probabilistic functions, robust optimization allows designers to find the solution which will have the best, worst case scenario in the presence of interval uncertainty on design variables or parameters, and this problem becomes a minimax optimization problem.

When the system is not well understood, because of a lack of knowledge about it, then the uncertainty is epistemic and other design optimization methods are required.

3.4.2 Epistemic Uncertainty in Design Optimization

As discussed in Section 3.3.2, the modeling of epistemic uncertainty has gained ground since the 1960s, with the development of fuzzy logic, evidence theory, and possibility theory. Of these three theories, fuzzy logic has seen the most applications to engineering, particularly in control systems, and more recently in design optimization. Evidence theory has seen moderate development in optimization, but possibility theory is still lagging behind the other two methods.

3.4.2.1 Evidence Theory in Design Optimization

Recall that evidence theory is a method of combining multiple pieces of limited and often conflicting information (i.e. evidence) provided by experts. It provides lower and upper bounds on the true probability of an event being true. These bounds are known as the belief and plausibility. When enough information is gathered, these bounds become equal, and evidence theory reduces to probability theory. Mourelatos and Zhou (2006) developed an evidence-based design optimization which uses expert opinion to define the most likely distribution of problem parameters and estimate the uncertainty bounds on the probability distribution of the constraints. In their design optimization method, constraint satisfaction plausibility is used instead of constraint probability satisfaction. Their method is demonstrated on the design of a cantilever beam and that of a pressure vessel, and has been shown to provide a more conservative solution compared to a reliability-based design optimization.

Instead of using plausibility as a constraint, Srivastava and Deb (2011) use it as an optimization objective. They developed a bi-objective evolutionary algorithm which minimizes the problem's original objective function, as well as the plausibility of constraint satisfaction failure. Thus, a trade-off between objective function value and plausibility of failure can be obtained to help the designer gain insight about the problem's risk and rewards. Agarwal et al. (2004) developed a multidisciplinary optimization method which uses evidence theory to incorporate expert opinion to estimate the uncertainty bounds on the output of discipline level analysis tools. Surrogate models often replace high-fidelity codes in early-stage design to increase computational speed and reduce cost. An experienced human designer often has intuitive knowledge about the accuracy of an analysis tool over a range of design variables and parameters, which should be leveraged to increase the reliability of an optimization method. Their method is demonstrated on the early-stage sizing of an aircraft.

3.4.2.2 Fuzzy Logic in Design Optimization

Fuzzy logic is unique compared to other optimization methods in that it operates on linguistic variables rather than numerical variables. This makes it particularly well adapted to solving real world problems because the models it uses are no longer constrained to mathematical models. Design optimization is made up of two steps—a model of the product is first created, and then an optimization algorithm seeks the set of design variables which maximizes the design's performance. The models are created by translating designer knowledge learned from years of professional experience, gathered from data, and created through intuition, into a synthesis of the problem being investigated.

Traditional optimization algorithms operate on numerical variables and mathematical equations. Thus, human knowledge has to be translated to mathematical models before leveraging the benefits of optimization, which can cause a loss of information. Mathematics "is unique among languages in its ability to provide precise expression for every thought or concept that can be formulated in its terms" (Alder, 1991). This precision makes it particularly well suited to describe idealized physical phenomena like the orbit of the earth around the sun, or the drag on a flat plate. Design optimization models, however, are usually not as precise. The development of products for real world applications limits the number of simplifying assumptions that can be used, making the design and the analysis tools more uncertain. Additionally, in complex system design, many requirements will not be precisely known until later in the design process when a material solution will have been decided on. This interrelation between product requirements and material solution was coined the "wicked problem" by Andrews (2012) and is typical of military ships and large architectural projects like airports. Finally, the objectives of the optimization (i.e. stakeholder requirements) are often defined in linguistic terms, which are more intuitive to human designers and customers than mathematical variables are.

For these reasons, fuzzy logic has a lot of potential in design optimization. Using fuzzy logic, designers can develop models using a language that is more familiar and intuitive to them, allowing them to better incorporate their personal experiences and preferences in models. Subjective client requirements which are naturally described linguistically, like aesthetics, can be directly handled by the optimization. Overall, the communication between the optimization routine and the human user is simplified.

Type-1 Fuzzy Logic in Design Optimization

Fuzzy logic's ability to cope with imprecise and incomplete information, much like humans can, has helped it find many successful applications to design and optimization, including some real world problems. Two major classes of fuzzy logic exist: type-1 and type-2. Their theory is covered in detail in Chapter IV, but for now the reader can get by by knowing that type-1 logic uses a single, precise definition of a linguistic variable and that type-2 logic can support many simultaneous definitions for its linguistic variables, but is computationally more demanding.

Since its development in 1965, type-1 fuzzy logic has found wide applications in consumer and industrial product control systems, decision support systems (Naim and Hagras, 2014), market forecasting (Li and Parsons, 1998), and automatic train control systems (Mendel et al., 2014). Its ease of implementation and ability to "think like a human" are two of its best advantages. Steinberg developed an automatic aircraft carrier landing system for an F/A-18 using fuzzy logic to incorporate "elements of human pilot 'intelligence' with more conventional automatic control laws" (Steinberg, 1993). In design optimization, fuzzy logic has been used to rank preferences between performance criteria of a multi-objective optimization using linguistic statements, making it easier for the user to communicate the relative importance of problem objectives to the optimization (Yazdi, 2016).

In ship design, type-1 fuzzy logic has been used for set-based design negotiation where it helped assign linguistic preferences to ranges of design variable values with respect to multiple engineering disciplines (resistance, stability, arrangements,...), allowing users to formalize their preferences for the product and negotiate these preference to perform initial sizing of the vessel (Singer, 2003). Fuzzy logic has been used for general arrangements optimization, which are often "characterized by a large number of vaguely defined, conflicting and subjective considerations, opinions, and preferences" (Kirtley, 2009). Cuneo (2013) uses type-1 fuzzy logic to incorporate human intent into MDO algorithms with the goal of increasing their fidelity. Much of the success of an MDO comes from the designer's interpretation and analysis of its results, and it is this interpretation ability that Cuneo wanted to incorporate in MDOs. When discussing the advantages of his method, he states:

> While there are many examples of MDOs being successfully used in naval architecture design, these methods do not address the issues presented in this [method]. Using high fidelity models in applications led to promising results, but suffered from modeling limitations. Methods of introducing uncertainty led to a more robust final result in early stages of design, but MDO results are still dependent on the models used without the added value of a human designer.

The modeling flexibility afforded by fuzzy logic and its linguistic variables is one of its major strengths. The value of human knowledge developed over a lifetime of designing ships cannot be underestimated, and it is this knowledge that fuzzy logic optimization seeks to encode.

Type-2 Fuzzy Logic in Design Optimization

Despite its considerable success, type-1 fuzzy logic still suffers from a major modeling shortcoming. The type-1 membership functions used to describe a fuzzy linguistic variable are not fuzzy themselves. This means that the membership function is a single, precise function with no uncertainty in its definition. It seems paradoxical to describe an imprecise linguistic variable with a precise membership function that implies a perfectly known definition, but this is what type-1 sets do. To resolve these inadequacies, Zadeh introduced interval type-2 and general type-2 fuzzy sets (Zadeh, 1975) in which membership functions are themselves fuzzy, and can lie anywhere between a given uncertainty bound.

There are (at least) four sources of uncertainties which cannot be modeled in type-1 fuzzy logic systems (FLS) due to their crisp membership functions (Mendel and John, 2002): (1) The meanings of the words that are used in the antecedents and consequents of rules can be uncertain (words mean different things to different people). (2) Consequents may have a histogram of values associated with them, especially when knowledge is extracted from a group of experts who do not all agree. (3) Measurements that activate a type-1 FLS may be noisy and therefore uncertain. (4) The data that are used to tune the parameters of a type-1 FLS may also be noisy. Type-2 systems are able to handle the uncertainties listed above and generally perform well on models with vague mathematical descriptions, in pattern recognition problems when linguistic variable definitions are context dependent, or when expert opinion from group members differ (Mendel et al., 2010).

While type-2 fuzzy logic research is still not at full maturity, the modeling benefits resulting from its ability to better capture uncertainty have been demonstrated in several applications. The majority of the research stems from controls engineering, where fuzzy logic controllers (FLC) are used to leverage human operator experience through linguistic control rules, and have successfully operated systems in unstructured and changing environments (Hagras, 2007). Many current applications deal with autonomous robot navigation (Hagras, 2004; Juang and Hsu, 2009), and a survey of FLC design methods is given by Castillo and Melin (2012).

Only one application of type-2 fuzzy logic to ship design is known to the author. Gray (2012) demonstrated that type-2 fuzzy systems enhanced the Set-Based Design (SBD) reduction process through their ability to handle uncertainty associated with engineering discipline linguistic preferences on ranges of design variables. In highly constrained ships, the type 1 SBD system failed to produce a reduced set of possible designs while the extended type-2 SBD system succeeded. Similar benefits were observed in medical expert systems, where type-2 fuzzy sets were able to capture linguistic uncertainties, which stemmed from the group of experts' diverging opinion, to provide diagnosticians with better decision making capabilities (Ozen and Garibaldi, 2003).

3.5 Closing Remarks

This chapter covered several theories of uncertainty and its implications for design optimization. Uncertainty is inherent and sometimes feared in real-world decision making, but is often a great source of excitement to human beings. It is generally classified into aleatory uncertainty, which characterizes the inherent random variations of a well-understood process, and into epistemic uncertainty, which characterizes the lack of knowledge and understanding about a system.

Several theories have been developed to deal with these uncertainties. The known random variations of a system can be modeled by probability theory, and many reliable and robust optimization methods have been developed to design products subject to these variations. The formal study of epistemic uncertainty started much later, in the 1960's, but several theories have already been validated. Fuzzy logic is the most widely applied one. It operates on linguistic variables instead of numerical values and can handle imprecise, partially true statements. It is also a good tool to incorporate human mental models directly into optimization, increasing the modeling capability of such methods compared to purely mathematical models.

This thesis expands on type-2 fuzzy logic for modeling and design optimization. A background of the theory is presented in Chapter IV. A novel type-2 optimization method is presented in Chapter V before being applied to the early-stage design of a planing craft in Chapter VI.

CHAPTER IV

Fuzzy Logic Background

4.1 Introduction

This thesis relies heavily on fuzzy logic to model uncertainty associated with early-stage ship design and incorporate human expertise in engineering models. Developed in 1965 by Lofti Zadeh as a method of programming computers using natural language, fuzzy logic was inspired by a human's ability to "converse, reason and make rational decisions in an environment of imprecision, uncertainty, incompleteness of information, conflicting information, partiality of truth and partiality of possibility — in short, in an environment of imperfect information" (Zadeh, 2008). It's ease of implementation and ability to "think like a human" have helped it find wide applications in commercial and industrial control systems (Singh and Mishra, 2015), decision support systems (Naim and Hagras, 2014), and market forecasting (Li and Parsons, 1998).

Human's often reason using vague linguistic variables and a set of rules, learned from past experiences, which map environmental cues to a decision outcome through pattern recognition. For example, an experienced driver can accurately estimate the stopping distance a car without having an exact model of the vehicle dynamics in the current environmental conditions. They do not need to know the friction coefficient between the tires and the road to know how hard to press the brake in dry, raining, or snowing conditions. Instead, they tend to think of the conditions as not slippery, slippery, or very slippery, and use learned rules to estimate how the car will behave in the current conditions. If the road is wet and the car is heavily loaded, the car will take longer to brake. They'll know not to press the brakes too hard to avoid making the tires slip, and will keep a bigger distance between themselves and the car in front of them.

Fuzzy logic systems mimic this type of heuristic reasoning using linguistic variables, membership functions, a fuzzifier, a rule bank, fuzzy inference and a defuzzifier. A short description of these terms will be given below before being elaborated on in the following sections.

Definition IV.1. A linguistic variable is a variable who's values are words instead of numbers. A linguistic variable for $friction$ could take the values *not slippery*, *slippery*, and *very slippery* for example.

Definition IV.2. A membership function is a mapping from a numerical variable to a linguistic variable value which characterizes the degree of similarity between the numerical variable and the linguistic variable value. For example, the static friction coefficient of a tire on ice, which equals approximately 0.15, could be characterized as *very slippery* with degree one. Rubber on wet snow, with a static friction coefficient around 0.45, could be characterized as *very slippery* with degree 0.75 and *slippery* with degree 0.25.

Definition IV.3. A fuzzifier operates on the membership function to map a numerical variable value to a linguistic variable value. It is the process of using a membership function to calculate the similarity between the numerical variable value and the linguistic variable value.

Definition IV.4. A rule bank is a set of rules that maps one or more input linguistic variables, also called antecedents, to an output linguistic variable, called a consequent.

These are analogous to patterns that humans learn through experience.

Definition IV.5. Fuzzy inference is the process of matching input linguistic variables to rules contained in the rule bank. It determines which rules are activated and their activation level. This is analogous to a human matching new scenarios to similar past experiences or learned mental models.

Definition IV.6. A defuzzifier combines multiple rules selected by the fuzzy inference process into a single decision or course of action. It is analogous to combining all the relevant experiences to figure out how to act in an unknown situation which shares similarities with past experiences.

4.2 Linguistic Variables: from Crisp Sets to Fuzzy Sets

Fuzzy logic is an extension of classical logic where truth propositions can take any number between 0 and 1 to model imprecise and partially true statements. Classical logic describes a binary truth proposition, where a proposition is true if an element e is a member of the set A of interest (i.e. $e \in A = 1$), and a proposition is false if an element e is not part of set A (i.e. $e \in A = 0$). These binary sets are said to be crisp, and they are represented by a step membership function μ_A which maps the element e to the set A if A is completely representative of e. In Figure 4.1, the membership function assigns heights below six feet to the set *not tall* and assigns heights above six feet to the set *tall*. A five feet ten inches person is classified as not tall and a six feet two inches person is classified as tall. Formally, a crisp set, or classical set A is defined as follows.

Definition IV.7. A crisp set is comprised of a domain X of the real numbers (also called the universe of discourse of A) together with a membership function $\mu_A(x)$, where $\mu_A(x)$ is given by the indicator function in Equation 4.1

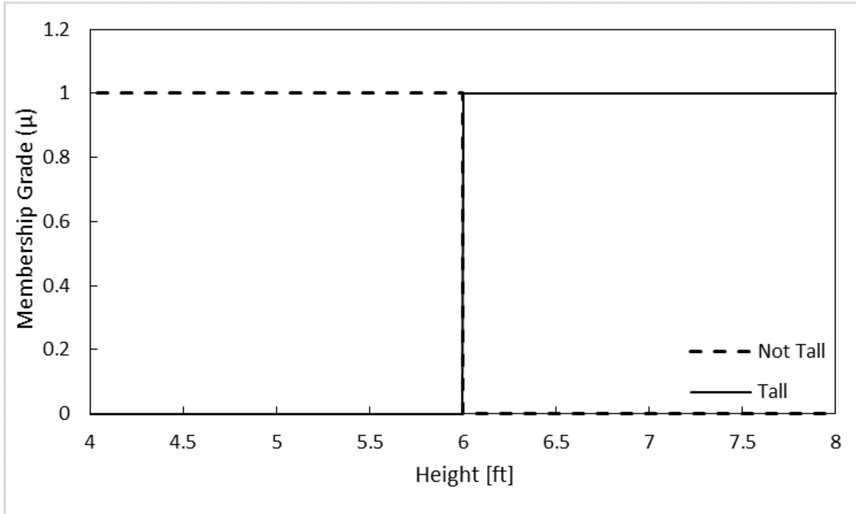


Figure 4.1: Crisp membership function describing human height.

$$\mu_A(x) = \begin{cases} 1 & \text{if} \quad x \in A \\ 0 & \text{if} \quad x \notin A \end{cases} \tag{4.1}$$

Fuzzy set theory expands this concept to allow the degree of membership of an element to a set to lie between 0 and 1 (i.e the membership function is not restricted to an indicator function). Thus, a classical set is a special case of a fuzzy set. Two classes of fuzzy sets exist—type-1 fuzzy sets and type-2 fuzzy sets—which differ by their ability to handle linguistic uncertainties. A type-1 fuzzy set is defined by a single deterministic, or crisp, membership function. Once it is defined, all linguistic uncertainty about the membership function disappears. In type-2 fuzzy sets membership functions are themselves uncertain, or fuzzy, and can take any value within their defined bounds.

4.2.1 Type-1 Fuzzy Sets

Type-1 fuzzy sets are the most widely used, and have been successfully applied to many real world problems. They model linguistic uncertainty through the use of a crisp membership function, which determines the similarity between a numerical variable (i.e. element e) and a linguistic variable. Formally, a fuzzy set is defined by Mendel et al. (2014) as follows.

Definition IV.8. A fuzzy set A is comprised of a domain X of the real numbers (also called the universe of discourse of A) together with a membership function $\mu_A\colon X \to [0,1]$. For each $x \in X$, the value of $\mu_A(x)$ is the degree of membership, or membership grade, of x in A. If $\mu_A(x) = 1$ or $\mu_A(x) = 0 \quad \forall \quad x \in X$, then the fuzzy set is said to be a crisp set.

A type-1 fuzzy set A is described as in Equation 4.2, where the integral operator denotes the collection of all values x in the universe of discourse X, with degree of membership $\mu_A(x)$. Additionally, the slash does not denote division. It associates the value x to a degree of membership in set A.

$$A = \int_{x \in X} \mu_A(x)/x \tag{4.2}$$

The non-binary degree of membership of a numerical variable x to the set A better captures the nuances of real life and the imprecisions associated with human reasoning. The sharp boundary used by the crisp membership function to categorize human height given in Figure 4.1 is unnatural, as there is no sharp boundary describing a change from tall to not tall. Instead, humans tends to think of a gradual change from the height of a tall person to the height of a person who isn't. The fuzzy membership function shown in Figure 4.2 captures this imprecise boundary and gradual change. The five feet ten inches person and the six feet two inches person are both about half tall and half not tall, with the first being slightly more not tall and the second being slightly more tall.

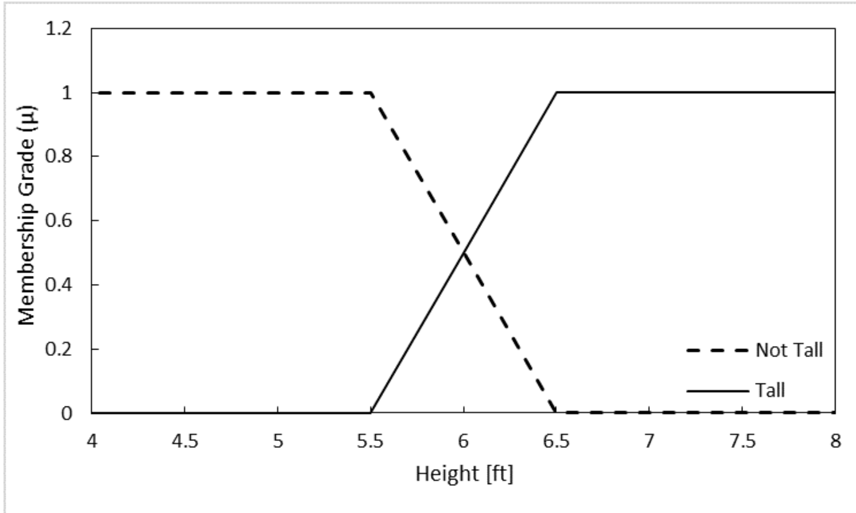


Figure 4.2: Type-1 fuzzy membership function describing human height.

4.2.2 Type-2 Fuzzy Sets

Type-2 fuzzy sets expand their type-1 counterpart to incorporate uncertainty in their membership function definition. A major criticism of type-1 sets is that they are themselves non-fuzzy, which limits their ability to handle uncertainty in their membership function definition. Words mean different things to different people, and are also context dependent. This concept cannot be adequately captured by type-1 sets. Figure 4.2 shows a precise definition of the membership functions for tall (i.e. there is no uncertainty associated with its end points); however, a person's definition of the concept tall tends to be vague and to depend on several factors, like nationality and age. Using a type-1 set with an exact membership function seems counter-intuitive when describing a fuzzy quantity which is by definition imprecise. Type-2 sets, or fuzzy-fuzzy sets, provide a more accurate model of linguistic uncertainty by using a fuzzy definition of the primary membership function (Figure 4.3). They are defined by their primary membership μ_A, which is plotted into the plane of the paper, and their secondary membership u, which is plotted on the vertical axis. The shaded area is referred to as the footprint of uncertainty (FOU) of the primary membership function, and is bounded below by the lower membership function (LMF) and

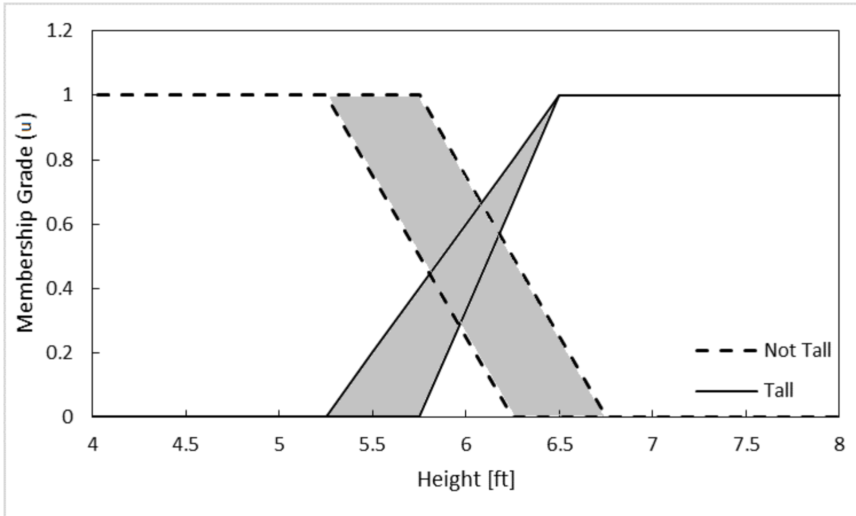


Figure 4.3: Type-2 fuzzy membership function describing human height.

bounded above by the upper membership function (UMF). The even shading implies that all primary memberships between the LMF and UMF are equally likely and thus uniformly weighted. Such sets are denoted interval type-2 fuzzy sets, and make up virtually all applications of type-2 fuzzy sets. Fuzzy sets with non-uniform secondary membership functions are called general type-2 fuzzy sets, but are still in infancy. If all uncertainty about membership function definition disappears, the secondary membership u reduces to μ_A and a type-2 set reduces to a type-1 set, much like a random variable reduces to the mean when the variance goes to zero.

One can also think of a type-2 fuzzy set as a collection of an infinite number of embedded type-1 fuzzy sets contained within the shaded area and bounded between the LMF and UMF. For interval type-2 sets, each of these embedded functions is equally weighted. This concept will be used for fuzzy logic systems type reduction in Section 4.3.

A type-2 fuzzy set A can be described as in Equation 4.3, where the first integral operator denotes the collection of all values x in the universe of discourse X, and the second integral denotes the collection of all secondary membership $u \in U \subseteq [0, 1]$ of its embedded type-1 sets.

$$A = \int_{x \in X} \int_{u \in U_x} \mu_A(x,u)/(x,u) \tag{4.3}$$

For interval type-2 sets, where each secondary, or embedded, membership function is equally likely, $\mu_A(x,u) = 1$ and Equation 4.3 reduces to Equation 4.4 (Mendel et al., 2014).

$$A = \int_{x \in X} \int_{u \in [0,1]} 1/(x,u) = 1/FOU(A) \tag{4.4}$$

4.2.3 Centroid of a Fuzzy Set

The centroid of a fuzzy set is an important calculation in the output processing step of a fuzzy logic system, so this section briefly reviews this concept. A more detailed discussion of centroids can be found in Karnik and Mendel (1998).

4.2.3.1 Centroid of a Type-1 Fuzzy Set

For a type-1 set A, the centroid is simply the center of area of the set (Equation 4.5) where the integral operator takes its definition from calculus as opposed to its meaning in fuzzy set theory.

$$c = \frac{\int x\mu_A(x)\ dx}{\int \mu_A(x)\ dx} \tag{4.5}$$

4.2.3.2 Centroid of a Type-2 Fuzzy Set

The centroid of an interval type-2 fuzzy set (IT2FS) is used for the type reduction step of a fuzzy logic system's output processing. Recall that an IT2FS is composed of an infinite number of equally weighted embedded type-1 fuzzy sets bounded below by the LMF and bounded above by the UMF. The centroid C_A of this superset A is given the union of the centroids c_A of all its embedded sets, and the centroids of the leftmost and rightmost embedded sets provide the bounds on the centroid of the superset. This concept is formally defined in Mendel et al. (2014) as follows.

$$C_A = 1/\bigcup_{\forall A_e} c_A(A_e) = 1/[c_l(A), ..., c_r(A)] = 1/[c_l(A), c_r(A)] \tag{4.6}$$

where A_e are the embedded type-1 sets, and $c_l(A) = min\ c_A(A_e)$ and $c_r(A) = max\ c_A(A_e)$ are the centroids of the leftmost and rightmost embedded sets.

Several algorithms have been developed to calculate the centroid bounds of a type-2 fuzzy sets. The Karnik-Mendel (KM) Algorithm (Karnik and M., 2001), and Enhanced Karnik-Mendel (EKM) Algorithm (Wu and Mendel, 2009) are two common ones.

4.3 Type-2 Fuzzy Logic Systems

Fuzzy sets are the basis of fuzzy logic and its ability to reason with imprecise quantities like a human. This is done by operating on linguistic variables through a fuzzy logic system (FLS), which maps numerical input variables to numerical output variables through a set of linguistic rule. Both type-1 and type-2 FLS follow the same structure. A type-2 FLS is illustrated in Figure 4.4 and is composed of the following four components: the fuzzifier, the rule bank, the inference engine, and the output processor. The output processor of a type-1 FLS only consists of the defuzzifier. For interval type-2 FLS, the output processor first reduces the set to two lower and upper bound type-1 sets (this is the type reducer). The defuzzifier then combines these two type-1 sets into a single numerical value. Interval type-2 FLS are described in more detail below and are illustrated with a simple example which calculates a person's risk of heart disease based on their height and weight. Singer (2003) developed this example for a type-1 FLS, and it is expanded to a type-2 FLS here.

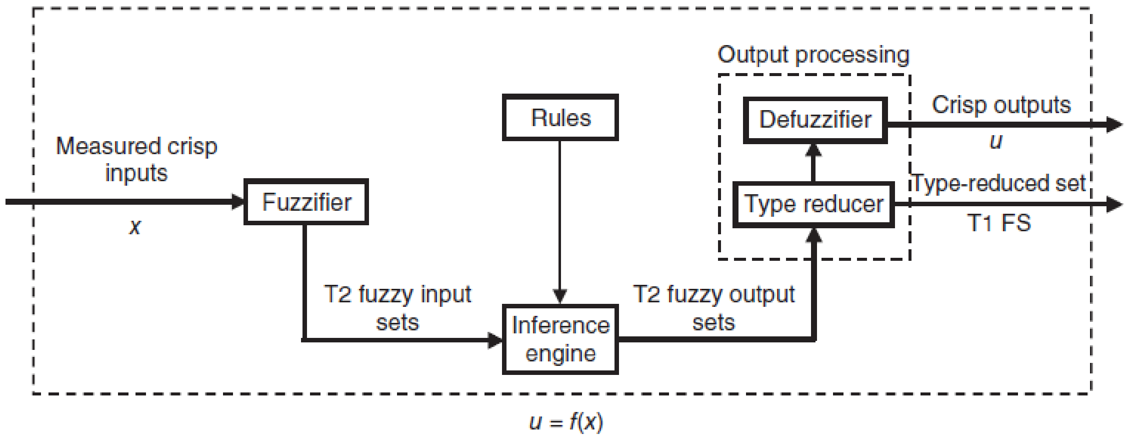


Figure 4.4: Type-2 fuzzy logic system. (Mendel et al., 2006)

4.3.1 Fuzzifier

The first step of the FLS is fuzzification, which converts a crisp numerical input variable to one or more linguistic variables that are most similar to them. This degree of similarity, or membership grade, is assigned by membership functions discussed in Section 4.2. For type-1 fuzzy sets, a crisp variable is mapped to a single membership grade for each fuzzy set. For interval type-2 fuzzy sets, each membership function assigns an upper and lower bound on degrees of membership according to the UMF and LMF of set A (Equation 4.7).

$$FOU_A(x_i) = \left[\underline{\mu}_A(x_i), \bar{\mu}_A(x_i)\right] = \left[LMF_A(x_i), UMF_A(x_i)\right] \tag{4.7}$$

The heart disease example used to illustrate FLS has two inputs: height, measured in feet, and weight, measured in pounds. Height is broken down into three input fuzzy sets identified by the linguistic variables: *short height*, *medium height*, and *tall height*. Weight is also broken down into three input fuzzy sets, which are identified by the *light weight*, *medium weight*, and *heavy weight* linguistic variables. These fuzzy sets are shown in Figures 4.5 and 4.6. Fuzzification performs two mappings, one for height, and one for weight.

Let a person's height be five feet six inches (i.e. 5.5 feet). The type-1 fuzzy set for *short height* maps this crisp variable to a single membership grade (Equation 4.8a) and the type-2 fuzzy set for *medium height* maps it to a range of membership grades, or footprint of uncertainty (Equation 4.8b), as shown in Figure 4.5.

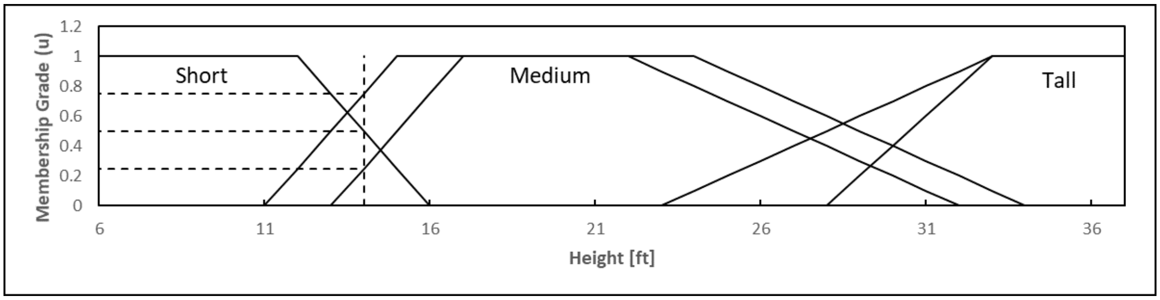


Figure 4.5: Type-2 input fuzzy membership function describing human height.

$$\mu_{short}(5.5) = 0.5 \tag{4.8a}$$

$$\begin{aligned} FOU_{medium}(5.5) &= \Big[\underline{\mu}_{medium}(5.5), \bar{\mu}_{medium}(5.5)\Big] \\ &= \Big[LMF_{medium}(5.5), UMF_{medium}(5.5)\Big] \\ &= \Big[0.25, 0.75\Big] \end{aligned} \tag{4.8b}$$

Let a person's weight be 130 pounds. The type-2 fuzzy set for *light weight* maps this crisp variable to the footprint of uncertainty given in Equation 4.9a and the type-2 fuzzy set for *Medium weight* maps it to the footprint of uncertainty given in Equation 4.9b, as shown in Figure 4.6.

$$\begin{aligned} FOU_{light}(130) &= \Big[\underline{\mu}_{light}(130), \bar{\mu}_{light}(130)\Big] \\ &= \Big[LMF_{light}(130), UMF_{light}(130)\Big] \\ &= \Big[0.50, 0.67\Big] \end{aligned} \tag{4.9a}$$

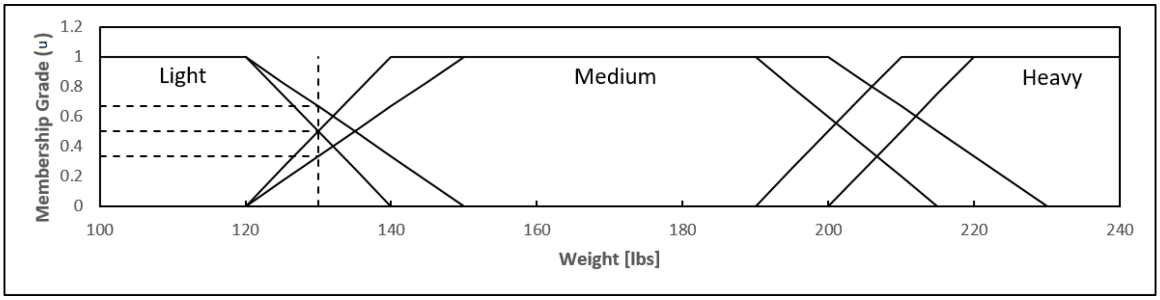


Figure 4.6: Type-2 input fuzzy membership function describing human weight.

$$
\begin{aligned}
FOU_{medium}(130) &= \Big[\underline{\mu}_{medium}(130), \bar{\mu}_{medium}(130)\Big] \\
&= \Big[LMF_{medium}(130), UMF_{medium}(130)\Big] \qquad (4.9b) \\
&= \Big[0.33, 0.50\Big]
\end{aligned}
$$

The heart disease example used to illustrate FLS has one output: heart disease risk factor. This output is broken down into four output fuzzy sets: *low risk*, *average risk*, *moderate risk*, *high risk*. Each set maps to a numerical value for heart disease risk, given by the centroid of each type-2 set, as shown in Figure 4.7. The centroid of a type-2 is the union of the centroids of all its embedded type-1 membership functions, and will be discussed further in Section 4.3.4.

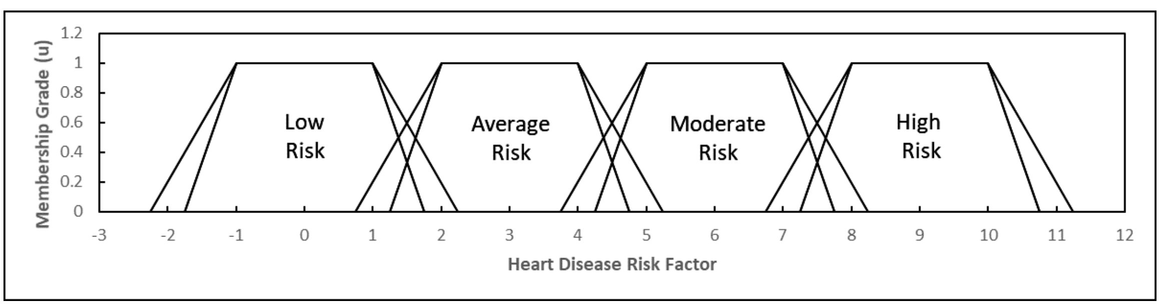


Figure 4.7: Type-2 output fuzzy membership describing human heart disease risk factor.

4.3.2 Rule Bank

The fuzzy rule bank consists of a set of IF-THEN statements that map linguistic inputs (called antecedents) to linguistic outputs (called consequents). These rules can be populated using expert opinion or extracted from data. Two common types of rules are Mamdami type rules, which map linguistic inputs to linguistic outputs (Mamdani and Assilian, 1975), and Sugeno type rules, which map linguistic inputs to a function $f(x)$ of the rules inputs (Sugeno, 1985). This work uses Mamdami type rules because they are more intuitive and suitable to human input. They take the form given in Equation 4.10, where x is the crisp numerical input variable, F is an input linguistic variable (antecedent), and G is the output linguistic variable (consequent).

$$\text{IF } x_i \text{ is } F_j \text{ AND } x_i \text{ is } F_k \text{ THEN } y \text{ is } G \tag{4.10}$$

The rule bank associated with the heart disease example is displayed in Table 4.1. It maps the input height and weight fuzzy sets to the output disease risk factor fuzzy set. For example, a heavy and short person has a high risk of heart disease, but a heavy and tall person has a moderate risk of heart disease since they tend to be leaner.

Table 4.1: Rule bank describing human heart disease risk factor.

		WEIGHT		
		Light	**Medium**	**Heavy**
HEIGHT	**Short**	Low Risk	Moderate Risk	High Risk
	Medium	Low Risk	Average Risk	High Risk
	Tall	Low Risk	Low Risk	Moderate Risk

4.3.3 Fuzzy Inference

The third step of the fuzzy logic system is fuzzy inference, which determines which rules are activated as well as their activation strength. This work uses minimum inference, in which the membership function of the consequent is clipped at the value of the smallest membership value μ of the rules antecedents. Type-2 input membership functions are defined by their footprint of uncertainty at each crisp input x, and the inference process maps it to a footprint of uncertainty on the output fuzzy set. For minimum inference, the lower and upper bounds on firing strength are given in Equations 4.11a and 4.11b for an activated rule S with i antecedents, or inputs.

$$\underline{f}^S = min\Big[\underline{\mu}_{F_1^S}(x_1), \underline{\mu}_{F_2^S}(x_2), ..., \underline{\mu}_{F_i^S}(x_i)\Big] \tag{4.11a}$$

$$\bar{f}^S = min\Big[\bar{\mu}_{F_1^S}(x_1), \bar{\mu}_{F_2^S}(x_2), ..., \bar{\mu}_{F_i^S}(x_i)\Big] \tag{4.11b}$$

In the heart disease example, four rules are activated based on the person's crisp height and weight. These are shaded in Table 4.1 and given below.

- Rule 1: IF Weight is *light* AND Height is *short*, THEN Risk is *low*
- Rule 2: IF Weight is *light* AND Height is *medium*, THEN Risk is *low*
- Rule 3: IF Weight is *medium* AND Height is *short*, THEN Risk is *moderate*
- Rule 4: IF Weight is *medium* AND Height is *medium*, THEN Risk is *average*

The inference process and firing bounds for these four rules is shown in Figure 4.8. Rule 1 has firing bounds $\underline{f}^1 = min[0.5, 0.5] = 0.5$ and $\bar{f}^1 = min[0.5, 0.67] = 0.5$. Rule 2 has firing bounds $\underline{f}^2 = min[0.25, 0.5] = 0.25$ and $\bar{f}^1 = min[0.75, 0.67] = 0.67$. Rule 3 has firing bounds $\underline{f}^3 = min[0.5, 0.33] = 0.33$ and $\bar{f}^3 = min[0.5, 0.5] = 0.5$. Rule 4 has firing bounds $\underline{f}^4 = min[0.25, 0.33] = 0.25$ and $\bar{f}^4 = min[0.75, 0.5] = 0.5$.

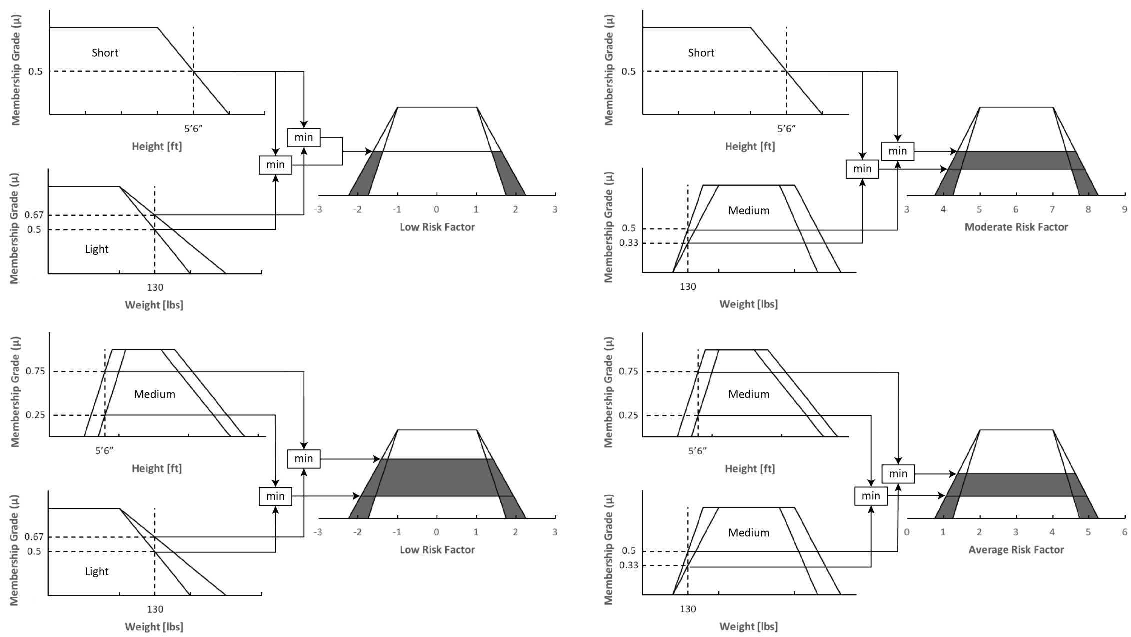


Figure 4.8: Type-2 fuzzy inference process for human heart disease risk factor.

4.3.4 Output Processing

The output processing of an interval type-2 fuzzy logic system consists of type reduction, which reduces a type-2 set to a two type-1 sets, and the defuzzifier which combines these two sets into a single crisp output.

Recall that an interval type-2 fuzzy set is composed of an infinite number of embedded type-1 fuzzy sets, each having secondary membership grade 1. The role of type reduction is to pick out the embedded type-1 sets that bound the uncertainty of the problem. This work uses the center-of sets type reduction method, which is a two step process. The first step consists of finding the centroids of each output fuzzy set $c^S = [c_l^S, c_r^S]$ activated by rule S. In the heart disease example, rules one and two activate the *low risk* output fuzzy set, rule three activates the *moderate risk* output fuzzy set, and rule four activates the *average risk* output fuzzy set. Their centroids are $c^1 = c^2 = [-0.33, 0.33]$, $c^3 = [2.67, 3.33]$, and $c^4 = [5.67, 6.33]$ respectively. In the next step, the rules are combined into an overall left and right centroid. The left centroid is the calculated by finding the $c_l(A_e)$ of the type-2 fuzzy sets with c_l^i on the x-axis and the footprint of uncertainty of the firing levels $[\underline{f}^i, \bar{f}^i]$ on the y-axis, for all activated rules i in S. The right centroid is the calculated by finding the $c_r(A_e)$ of the type-2 fuzzy sets with c_r^i on the x-axis and the footprint of uncertainty of the firing levels $[\underline{f}^i, \bar{f}^i]$ on the y-axis, for all activated rules. For interval type-2 fuzzy sets, all centroids between the $c_l(A_e)$ and $c_r(A_e)$ bounds are equally likely and uniformly weighted, so the defuzzifer combines them by computing the midpoint of its bounds (Equation 4.12).

$$C_A = \frac{c_l(A_e) + c_r(A_e)}{2} \tag{4.12}$$

For the heart disease example, the left and right centroid of the overall output set are given by $[c_l(A_e, c_r(A_e)] = [1.23, 2.90]$ and its midpoint equals 2.07. This is the crisp value returned by the interval type-2 fuzzy logic system.

4.4 Fuzzy Logic Controllers

The type-2 fuzzy logic multidisciplinary optimization presented in this thesis makes use hierarchical fuzzy logic controllers to break down the optimization problem into multiple subproblems each associated with a single discipline. Fuzzy logic control (FLC) theory is a branch of control theory which uses natural language and human inspired IF-THEN rules to regulate the behavior of a system. It maps crisp inputs from one or more sensors to a crisp output, such as engine throttle, by using fuzzy logic systems introduced in Section 4.3. This method has several advantages. First, it can deal with situations that cannot be expressed as completely true or false, but range from truth values between zero and one. Second, its use of natural language to encode IF-THEN control rules allows human experience to be leveraged directly into the control laws and automate tasks in such a way that they are easily understood by the operator (Pedrycz, 1989), increasing the operator's ability to monitor the automated system.

One of the first successful applications of fuzzy logic controllers was developed by Hitachi in 1983 and involved the automation of a train for the Japanese Sendai railway. In this application, the train's location, velocity, and target location were used as crisp inputs to control the powering and breaking of the train based on a set of linguistic rules developed from a skilled human operator's experience (Yasunobu et al., 1983). Since then, fuzzy logic controllers have been applied to many consumer and industrial products, including household appliances, heating systems, camera autofocus mechanisms, automobile cruise control, and robot navigation.

4.4.1 Hierarchical Fuzzy Logic Controllers

On issue with fuzzy logic systems and their use in controls is that the number of IF-THEN rules increase exponentially with the number of crisp variables. Thus, for systems with a large number of variables, computational time explodes. Hierarchical fuzzy logic controllers break down the problem into multiple lower-level independent controllers that each evaluate a smaller set of variables. The behavior of each lower level controller is then integrated by a high-level controller which coordinates the overall behavior of the system. For hierarchical FLC, the number of rules grows linearly instead of exponentially with the number of problem variables, significantly reducing computational time (Mohammadian and Kingham, 2004).

The typical architecture of a hierarchical type-2 fuzzy logic controller is given in Figure 4.9. Each lower level fuzzy logic controller takes a subset of the problem variables x and provides an output behavior preference to the top level controller based on its area of expertise. For a robot navigation problem for example, these areas of expertise could include obstacle avoidance which locates obstacles and informs the top controller to avoid them, and goal seeking which searches for the target location and instructs the top controller to direct the robot towards it (Hagras, 2004). The top controller aggregates and coordinates these preferences into a single plan of action as a function of time. The goal seeking behavior would be triggered the majority of the time, but if an obstacle is located, behavior preferences from the obstacle avoidance controller would send instructions to move out of the way of the obstacle.

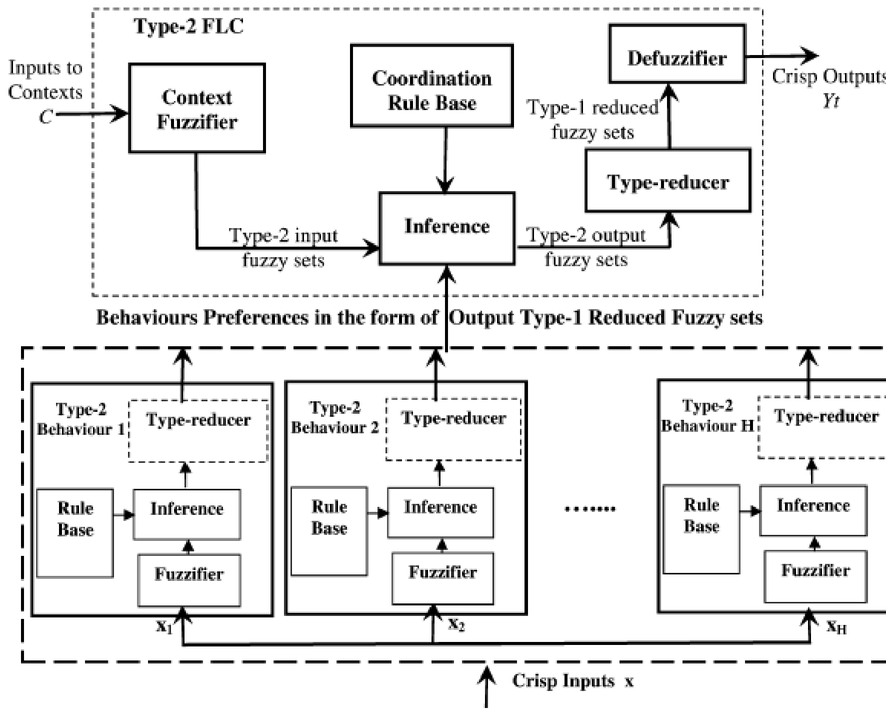


Figure 4.9: Type-2 hierarchical fuzzy logic controller (Hagras, 2004).

Multiple methods of aggregating, or fusing, the preferences of independent FLCs exist. Figure 4.9 uses a fuzzy logic system. Several other aggregation methods, ranging from full compensatory to fully non-compensatory, are discussed in Chen and Hwang (1992). Minimum fusion, or fuzzy intersection (Dubois and Prade, 1980) calculates the minimum of all lower level controller outputs, and is used by Cuneo (2013). It is given in Equation 4.13, where $F_S(\mathbf{x})$ is the system level output and $F_j(\mathbf{x})$ is the output of each lower level controllers.

$$F_S(\mathbf{x}) = \wedge_{j=1}^{N} F_j(\mathbf{x}) \tag{4.13}$$

Other fusion methods include the maximum operator, which calculates the maximum, or fuzzy union, of all lower level controllers, or the average fusion (Equation 4.14). Chen and Hwang (1992) give thirteen other methods of aggregating fuzzy sets.

$$F_S(\mathbf{x}) = \frac{\sum_{j=1}^{N} F_j(\mathbf{x})}{N} \tag{4.14}$$

Hierarchical fuzzy logic controllers were introduced to multidisciplinary optimization by Cuneo (2013). In his method, each discipline uses a separate type-1 fuzzy logic system to assign a preference to each design variable value of the design space with respect to the performance indicators of the discipline. The upper-level controller then aggregates the preferences from each discipline into a system-level preference for each variable instance. This concept was expanded in this thesis to incorporate uncertainty in the multidisciplinary optimization through the use of type-2 fuzzy logic systems and controllers. The method will be explained in detail in Chapter V, but an example flowchart of it is given in Figure 4.10.

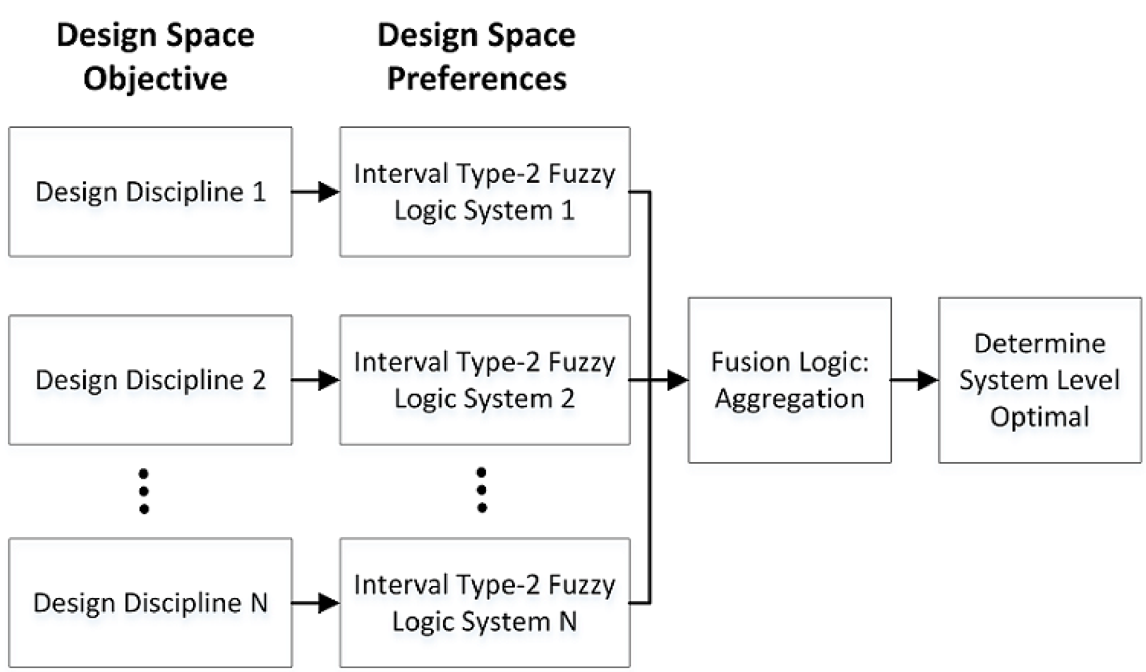


Figure 4.10: Type-2 hierarchical fuzzy logic controller inspired multidisciplinary optimization structure. Adapted from Cuneo (2013) to incorporate uncertainty in the linguistic terms used to define the optimization model.

4.5 Closing Remarks

This chapter provided the theoretical background of this thesis by presenting a method of modeling environments with limited information. By operating on vague linguistic variables instead of numerical variable, fuzzy logic can formalize elements of human intelligence and experience directly into an optimization models and method to enrich their information content. This process is done through type-2 fuzzy sets, fuzzy logic systems, and hierarchical fuzzy logic controllers introduced in the previous pages. Compared to type-1 fuzzy sets which use crisp membership functions, type-2 sets, or fuzzy-fuzzy sets, can better handle limited information associated with early-stage design models and with linguistic reasoning used by human designers, thus is better able to incorporate human experience into engineering models. The next chapters will demonstrate how this is done in practice, first on a generic mathematical problem, and then on the design of a planing craft.

CHAPTER V

Interval Type-2 Fuzzy Logic Multidisciplinary Optimization Under Uncertainty

5.1 Introduction

Chapter II presented a survey of optimization methods developed to model multidisciplinary systems, understand their performance characteristics, and guide design decisions. These methods have been successfully applied to many problems that have well defined models, like the structural optimization of airplane wings; however, their applicability to early-stage ship design is more limited. Detailed mathematical models of a system often do not exist in the early-stages of design, and when they do, their use can predicate solutions since many models are only based on past data. In addition, Cuneo (2013) mentions that traditional optimization methods "depend on the fidelity of the models used to represent the system" instead of relying on the human designer to evaluate solutions, and that a design's success is largely dependent on the user's ability to interpret model results with respect to the design's requirements. In response, he developed a type-1 fuzzy logic multidisciplinary optimization method which incorporate's the designer's knowledge back into the optimization to provide the MDO with a human-like judgment capability. Chapter III discussed several other limitations of traditional MDO methods and of Cuneo's original work which motivated the development of the optimization method presented in this chapter. The main drawback of these methods is their limited ability to represent uncertainties inherent to a real world problem. MDO under uncertainty methods have been developed to capture the aleatory variations of the real world, but are only viable if a statistical model these variations is available. Cuneo's work provided a first attempt at optimization with limited models, but his use of type-1 fuzzy logic to mimic human interpretation does not capture the uncertainties associated with linguistic reasoning and communication used by people.

The type-2 fuzzy logic multidisciplinary optimization method outlined in this chapter extends Cuneo's work to better handle the epistemic uncertainties associated with the limited information available in early-stage design models and the uncertainties associated with linguistic reasoning. It will be explained in detail and illustrated on a generic mathematical optimization proof-of-concept in the following sections, before being illustrated on a practical application which involves the design of a planing craft in Chapter VI. Section 5.2 gives a brief overview of the optimization's structure and components. Section 5.3 shows how human expertise is mapped into an optimization model to reduce the impact of limited information. Section 5.4 and 5.6 demonstrate this mapping on a generic example seeking the simultaneous minimization of three functions, and compares results with several traditional optimization methods.

5.2 Type-2 Fuzzy Logic Multidisciplinary Optimization Structure

By formalizing human expertise directly into an optimization framework, the fuzzy logic multidisciplinary optimization (FLMDO) can fill knowledge gaps associated with limited models. The overall structure of the process (Figure 5.1) is similar to a standard hierarchical MDO with the exception of the human expertise mapping performed in step two of the process. The first two steps of the process correspond to the lower, or discipline, level of the MDO, and the last two steps correspond to the system level.

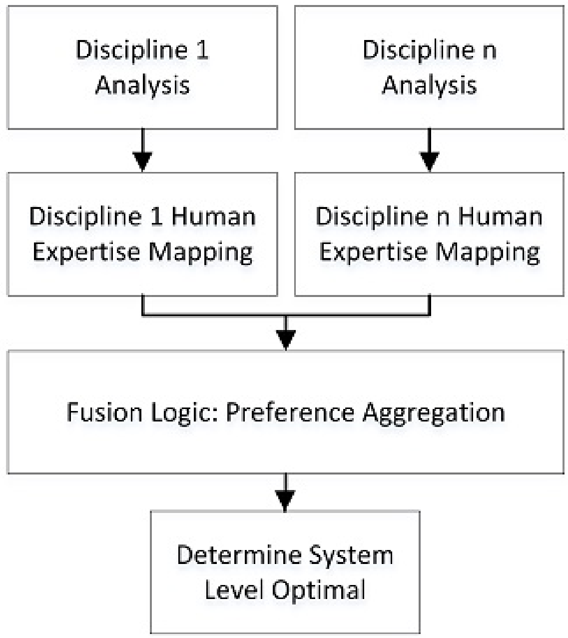


Figure 5.1: Fuzzy logic multidisciplinary optimization structure.

The first step of the process is the discipline analysis, where each engineering discipline evaluates design variable values $\mathbf{x}$, with respect to several performance indicators of interest to them. The outputs of the discipline analysis can be the objective functions of mathematical models, can be the output of high-fidelity physics-based simulation tools, or can be determined less precise mental models like rules of thumb. The slenderness ratio of a vessel is a good approximation for its resistance. Hastie and Dawes (2010) have shown that experts often use rules of thumb developed from experience to make quick decisions, and show high accuracy in their judgement.

The second step of the optimization is human expertise mapping, which consists of assigning preferences to design variable values based on their performance values for each discipline (i.e. the outputs of the discipline analysis). This is done through a fuzzy logic system, and will be explained further in Section 5.3.

The third step aggregates the output preferences of each discipline into an overall system level preference. Several aggregation methods are discussed in Section 5.6.6.

In the final step, the design variable instance with the highest system level performance is found.

5.3 Human Expertise Mapping

In traditional optimization, each design variable value is mapped to one or more performance metrics (i.e. objective function values) capturing the fitness of that design. In type-1 fuzzy logic multidisciplinary optimization (T1-FLMDO), a fuzzy logic system maps a design's performance metrics to a preference value which captures human expertise about the system and enriches the model, but assumes no uncertainty in the human expertise mental model. This assumption is relaxed in type-2 fuzzy logic multidisciplinary optimization (T2-FLMDO) where the performance metrics are mapped to a range of preference values which capture the uncertainty associated with the human expert's opinion about the system. This mapping is illustrated in Figure 5.2 for a single discipline. Performance metric values from the discipline analysis step (top of figure) are used as inputs to the human expertise mapping fuzzy logic system. Two metrics are used as inputs here, but any number of them could be used by the discipline to evaluate solutions. The FLS defines high, medium, and low linguistic preferences for ranges of performance metric values using type-2 fuzzy sets. The exact range of values associated with these linguistic preferences is not exactly however, which capture the uncertainty on the human expert's mental model (middle of Figure 5.2). The shaded transitions between preference regions denote this difficulty of assigning an exact transition point between preference rankings, and are modeled in the FLS by the footprint of uncertainty of the input fuzzy sets. Fuzzification then uses type-2 membership functions to assign each design variable value a set of linguistic preferences with respect to each performance metric used by the discipline analysis.

The linguistic preferences of a design variable value are then combined into an overall discipline level preference using a predefined rule bank, a fuzzy inference and output processing. This assigns a lower and upper bound discipline-level preference to each design variable value as seen in the bottom of Figure 5.2.

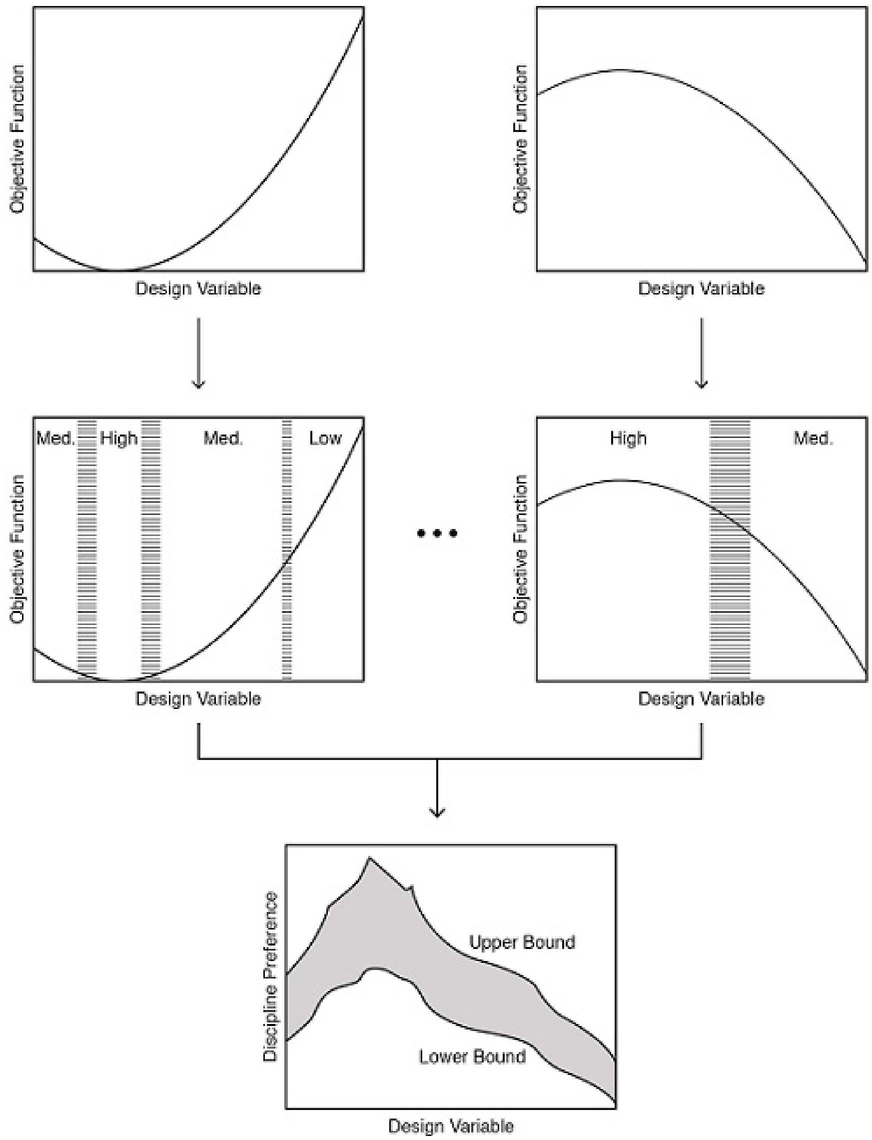


Figure 5.2: Discipline preference mapping capturing human expertise about a system. Adapted from Cuneo (2013) by using type-2 fuzzy logic to handle the uncertain mapping between objective function values and linguistic preferences, as shown by the hatched lines between the low, medium, and high preference. This uncertainty leads to a range of discipline preferences bounded by the lower and upper bound.

This process is performed independently for each discipline, thus for each design variable value a separate fuzzy logic system returns a range of discipline-centric preference values bounded between a lower and upper bound. These bounds capture the uncertainty in the designer's preference mental model. A fusion logic system then aggregates discipline level preferences into a system level preference range, similarly bounded above and below, from which the system level optimal design can be selected. The T2-FLMDO is illustrated in detail for a mathematical optimization problem in following sections.

5.4 Type-2 Fuzzy Logic Multidisciplinary Optimization for Mathematical Problems

This following few sections will illustrate the T2-FLMDO method on a generic mathematical minimization problem with three objective functions each acting as a single discipline. It is important to realize that the method is best suited for real world problems which are hard to define mathematically, or for which designers can use their experience to assign preferences to areas of the design space based on realistic measures learned throughout their professional life. The true potential of the method will be better realized on the planing craft case study presented in Chapter VI, but is initially illustrated on a generic mathematical minimization problem for completeness and to help the reader become more familiar with the method. The performance metrics used by the human expertise mapping step will first be discussed in Section 5.5 for a mathematical problem. Section 5.6 will then demonstrate the complete T2-FLMDO for the problem.

5.5 Human Expertise Mapping for Mathematical Problems

Recall that the goal of the human expertise mapping stage is to incorporate a designer's knowledge directly into a model to enrich it. First, the inputs to the mapping must be selected based on performance measures the designer uses to evaluate a product. In this simple example, the product being evaluated is a mathematical function $f(\mathbf{x})$. Three performance metrics are used to evaluate each independent variable value, which means that the discipline analysis must provide three inputs to the FLS mapping. The first is the function value itself. The second is the l^2-norm of the function gradient. The third metric is the distance between the independent variable value and the variable value with the optimal function value. Design variable values are first assigned a preference (i.e. a linguistic variable) based on their performance with respect to each of these metrics through interval type-2 fuzzy sets. These fuzzy sets are used to assign a high, medium, or low preference for specific ranges of metric values based on some expert knowledge. By using of type-2 fuzzy sets instead of type-1 sets, the method presented in this thesis better captures the uncertainty and imprecision associated with linguistic variables. Type-1 sets assume a precise membership function when mapping numerical variables to linguistic variables, which limits their ability to handle uncertainty, imprecision, and properly model human reasoning. Type-2 sets do not have this limitation since they use fuzzy-fuzzy sets, and thus they are better able to model slight variations of linguistic definitions used by different people, or even by the same person when a definition is affected by context. With type-2 sets, two design with a slight difference in objective function value can be assigned the same membership grade, which is analogous to a humans lumping designs with similar performance into one. After the fuzzification step, a fuzzy logic system (FLS) is used to combine each metrics into an overall preference discipline level preference for each design variable value. The three performance metrics and their associated fuzzy sets discussed next, followed by the FLS.

5.5.1 Objective Function Value Preference

The objective function value preference evaluates each point in the design space based on its objective value relative to the best, mid, and worst discipline objective value on the design space. For this case study, all disciplines seek the minimization of their function, so the best objective value of discipline i refers to the global minimum of its objective function, $f_i(\mathbf{x}^*)$. The worst refers to the maximum of the discipline's objective function value evaluated at all other disciplines' optimal, and is denoted f_i^{max} (Cuneo, 2013). This metric was inspired by the plausible reduction range (PRR) developed by Kim and Vlahopoulos (2012) and has been shown to find points located on the Pareto front. Any design with an objective function value greater than it is given a low preference by the fuzzy logic system. The mid objective value is simply the midpoint between the best and the worst. The best, worst and mid discipline objective values are expressed by equations 5.1a, 5.1b, and 5.1c respectively, where i refers to the discipline being currently evaluated and j refers to the other disciplines, and there are a total of D disciplines.

$$f_i^* = \min \quad f_i(\mathbf{x}) \tag{5.1a}$$

$$f_i^{max} = \max \quad f_i(\mathbf{x}_j^*) \quad \forall \quad j \in D \setminus i \tag{5.1b}$$

$$f_i^{mid} = f_i^* + 0.5(f_i^{max} - f_i^*) \tag{5.1c}$$

These values are used to define type-2 fuzzy sets that capture the designer's preference with respect to the objective function value. Figure 5.3 shows the three fuzzy sets that are used to capture linguistic preferences: high preference, medium preference, low preference.

Objective function values between f^* and f^{mid} are assigned a partially high and partially medium preference, with the high membership decreasing linearly between one around f^* and zero around f^{mid}, and the medium preference increasing linearly

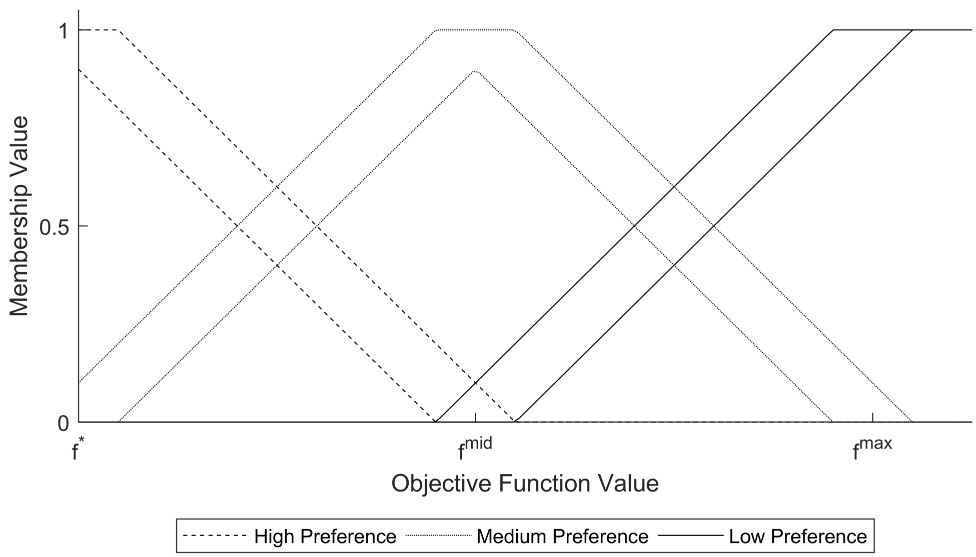


Figure 5.3: Type-2 fuzzy logic input membership function for objective function preference mapping.

between these two points. Objective function values between f^{mid} and f^{max} have partial membership in the medium and low preference fuzzy sets. Values greater than f^{max} are assigned a low preference with membership one, which keeps dominated points from being considered by the algorithm.

The membership functions illustrated in Figure 5.3 are themselves fuzzy. That is, their exact function definition is not known and a given objective function value is assigned a range of membership values bounded between the lower and upper membership function of the type-2 fuzzy sets. These bounds capture several uncertainties which cannot be handled by type-1 fuzzy sets. The first is the uncertainty associated with the use of imprecise linguistic variables which cannot be mapped to a precise number. For example, the uncertainty in the end-points of the membership functions show that any number in the vicinity of f^{max} can have the same membership grade to the medium fuzzy set as f^{max}. The same word can describe a range of similar elements, thus the relation between objective function value and membership value must be fuzzy itself. The second type of uncertainty relates to uncertainty in the input data. Averaging errors for example, which cause imprecisions in the objective function value, are handled by the fuzzy set's footprint of uncertainty, since small shifts in objective value will have equal membership grades.

The membership functions for high, medium, and low objective function preference are given in equations 5.2, 5.3, and 5.4 respectively. The size of the FOU is controlled by the δ_{lb} and δ_{ub} parameters, which shift the LMF and UMF horizontally an amount δ compared to their equivalent type-1 set.

$$lmf_{obj}^{high}\Big(f_i\Big) = \begin{cases} \frac{f_i^* - f_i - \delta_{lb}}{0.5(f_i^{max} - f_i^*)} + 1 & \text{if} \quad f_i \leq f_i^{mid} - \delta_{lb} \\ 0 & \text{if} \quad f_i > f_i^{mid} - \delta_{lb} \end{cases} \tag{5.2a}$$

$$umf_{obj}^{high}\Big(f_i\Big) = \begin{cases} 1 & \text{if} \quad f_i \leq f_i^* + \delta_{ub} \\ \frac{f_i^* - f_i - \delta_{ub}}{0.5(f_i^{max} - f_i^*)} + 1 & \text{if} \quad f_i^* + \delta_{ub} < f_i \leq f_i^{mid} + \delta_{ub} \\ 0 & \text{if} \quad f_i > f_i^{mid} + \delta_{ub} \end{cases} \tag{5.2b}$$

$$lmf_{obj}^{medium}\Big(f_i\Big) = \begin{cases} 0 & \text{if} \quad f_i^* \leq f_i \leq f_i^* + \delta_{lb} \\ \frac{f_i - f_i^* - \delta_{lb}}{0.5(f_i^{max} - f_i^*)} & \text{if} \quad f_i^* + \delta_{lb} < f_i \leq f_i^{mid} \\ \frac{f_i^{max} - f_i - \delta_{lb}}{0.5(f_i^{max} - f_i^*)} & \text{if} \quad f_i^{mid} < f_i \leq f_i^{max} - \delta_{lb} \\ 0 & \text{if} \quad f_i > f_i^{max} - \delta_{lb} \end{cases} \tag{5.3a}$$

$$umf_{obj}^{medium}\Big(f_i\Big) = \begin{cases} \frac{f_i - f_i^* + \delta_{ub}}{0.5(f_i^{max} - f_i^*)} & \text{if} \quad f_i \leq f_i^{mid} - \delta_{ub} \\ 1 & \text{if} \quad f_i^{mid} - \delta_{ub} < f_i \leq f_i^{mid} + \delta_{ub} \\ \frac{f_i^{max} + f_i + \delta_{ub}}{0.5(f_i^{max} - f_i^*)} & \text{if} \quad f_i^{mid} + \delta_{ub} < f_i \leq f_i^{max} + \delta_{ub} \end{cases} \tag{5.3b}$$

$$lmf_{obj}^{low}\left(f_i\right) = \begin{cases} 0 & \text{if} \quad f_i \leq f_i^{mid} + \delta_{lb} \\ \frac{f_i - f_i^{max} - \delta_{lb}}{0.5(f_i^{max} - f_i^*)} & \text{if} \quad f_i^{mid} + \delta_{lb} < f_i \leq f_i^{max} + \delta_{lb} \\ 1 & \text{if} \quad f_i > f_i^{max} + \delta_{lb} \end{cases} \tag{5.4a}$$

$$umf_{obj}^{low}\left(f_i\right) = \begin{cases} 0 & \text{if} \quad f_i \leq f_i^{mid} - \delta_{ub} \\ \frac{f_i - f_i^{max} + \delta_{ub}}{0.5(f_i^{max} - f_i^*)} & \text{if} \quad f_i^{mid} - \delta_{ub} < f_i \leq f_i^{max} - \delta_{ub} \\ 1 & \text{if} \quad f_i > f_i^{max} - \delta_{ub} \end{cases} \tag{5.4b}$$

5.5.2 Gradient Function Value Preference

The gradient function value preference evaluates each point in the design space based on the l^2-norm of its gradient value relative to a value of zero and the maximum l^2-norm of its gradient function. This performance measure is used to give preference to designs with low gradient norm. The intent behind this measure is that designs with high objective functions gradients are at higher risk of performance degradation if small changes in independent variables are made. In areas of the design space where the objective function has a low gradient, a variable change will have a smaller effect on the objective function value, or performance. The l^2-norm of discipline i, evaluated at design value $\mathbf{x}$, is given in Equation 5.5, where $\frac{\partial f}{\partial x_k}\Big|_{\mathbf{x}}$ is the partial derivative of the discipline's objective function with respect to design variable k and there are a total of K design variables.

$$\left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2 = \sqrt{\sum_{k=1}^{K} \left(\frac{\partial f}{\partial x_k}\Big|_{\mathbf{x}}\right)^2} \tag{5.5}$$

Three preference type-2 fuzzy sets are used to evaluate the design variable value with respect to its l^2-norm, and they are denoted using the following linguistic variables: high preference, medium preference, and low preference (Figure 5.4). These sets are defined by the points $\left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2 = 0$, $\left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2 = 1$, and $\left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2 = \left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2^{max}$. Design points with an l^2-norm of zero exhibit no performance degradation for small perturbations in **x** and designs with the maximum l^2-norm exhibit the highest performance degradation for small perturbations in **x**.

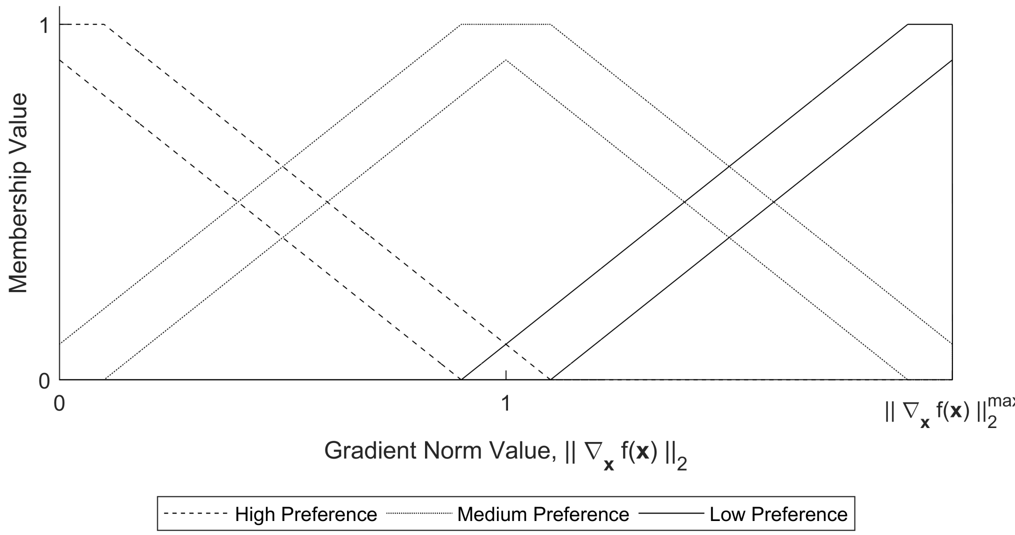


Figure 5.4: Type-2 fuzzy logic input membership function for objective function gradient preference mapping.

The lower and upper membership function for each preference fuzzy set are given in Equations 5.6, 5.7, and 5.8. Gradient norm values between zero and one have partial membership in the high and medium preference sets, and gradient norm values between one and $||\nabla_{\mathbf{x}} f_i||_2 = ||\nabla_{\mathbf{x}} f_i||_2^{max}$ have partial membership in the medium and low preference set. As discussed for the objective function value preference in Section 5.5.1, the use of interval type-2 preference fuzzy sets allows the FLMDO to handle imprecisions in linguistic variables used by the human user, and imprecisions in the input data from the discipline analyzer. The size of the footprint of uncertainty is controlled by the parameters δ_{lb} and δ_{ub} in the membership functions, which shift the LMF and UMF relative to the type-1 membership function. As linguistic or numerical uncertainty increases, the size of the FOU also increases, just like the variance of a data set increases as its becomes more imprecise.

$$lmf_{grad}^{high}\Big(\big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2\Big) = \begin{cases} -\big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 + 1 - \delta_{lb} & \text{if} \quad \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 \leq 1 - \delta_{lb} \\ 0 & \text{if} \quad \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 > 1 - \delta_{lb} \end{cases} \tag{5.6a}$$

$$umf_{grad}^{high}\Big(\big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2\Big) = \begin{cases} 1 & \text{if} \quad \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 \leq \delta_{ub} \\ -\big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 + 1 + \delta_{ub} & \text{if} \quad \delta_{ub} < \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 \leq 1 - \delta_{ub} \\ 0 & \text{if} \quad \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 > 1 - \delta_{ub} \end{cases} \tag{5.6b}$$

$$lmf_{grad}^{medium}\Big(\big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2\Big) = \begin{cases} 0 & \text{if} \quad \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 \leq \delta_{lb} \\ \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 - \delta_{lb} & \text{if} \quad \delta_{lb} < \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 \leq 1 \\ \frac{\big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2^{max} - \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 - \delta_{lb}}{\big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2^{max} - 1} & \text{if} \quad 1 < \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 \leq \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2^{max} - \delta_{lb} \\ 0 & \text{if} \quad \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 > \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2^{max} - \delta_{lb} \end{cases} \tag{5.7a}$$

$$umf_{grad}^{medium}\Big(\big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2\Big) = \begin{cases} \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 + \delta_{ub} & \text{if} \quad \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 \leq 1 - \delta_{ub} \\ 1 & \text{if} \quad 1 - \delta_{ub} < \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 \leq 1 + \delta_{ub} \\ \frac{-\big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2^{max} - \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 + \delta_{ub}}{\big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2^{max} - 1} & \text{if} \quad \big|\big|\nabla_{\mathbf{x}} f_i\big|\big|_2 > 1 + \delta_{ub} \end{cases} \tag{5.7b}$$

$$lmf_{grad}^{low}\left(\left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2\right) = \begin{cases} 0 & \text{if} \quad \left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2 \leq 1 + \delta_{lb} \\ \frac{\left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2 - 1 - \delta_{lb}}{\left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2^{max} - 1} & \text{if} \quad \left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2 > 1 + \delta_{lb} \end{cases} \tag{5.8a}$$

$$umf_{grad}^{low}\left(\left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2\right) = \begin{cases} 0 & \text{if} \quad \left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2 \leq 1 - \delta_{ub} \\ \frac{\left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2 - 1 + \delta_{ub}}{\left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2^{max} - 1} & \text{if} \quad 1 - \delta_{ub} < \left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2 \leq \left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2^{max} - \delta_{ub} \\ 1 & \text{if} \quad \left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2 > \left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2^{max} - \delta_{ub} \end{cases} \tag{5.8b}$$

5.5.3 Distance Function Value Preference

The last performance metric used to evaluate points of a mathematical optimization problem is the distance from the optimum value preference. For this metric, the euclidean distance $d_i(\mathbf{x})$ between each design point $\mathbf{x}$ and the optimal point $\mathbf{x}^*$ is calculated (Equation 5.9), and compared to the smallest distance (i.e. $d_i(\mathbf{x}) = 0$) and the biggest distance $d_i^{max}(\mathbf{x})$ (Equation 5.10).

$$d_i(\mathbf{x}) = \sqrt{\sum_{k=1}^{K}(x_k - x_k^*)^2} \tag{5.9}$$

$$d_i^{max} = \max_{\mathbf{x}} \quad \sqrt{\sum_{k=1}^{K}(x_k - x_k^*)^2} \tag{5.10}$$

Points that are closer to the optimal are preferred, indicating the designer's belief that they will share more characteristics with the optimal point than design values that are farther away. In evolutionary design, for example, engineers tend to use example designs that have worked well in the past as a starting point for subsequent design, and these subsequent design tend to have principle characteristics similar to the example designs.

Two interval type-2 fuzzy sets are used assign preference to design points with respect to their distance from the optimum—high and medium preference—as shown in Figure 5.5.

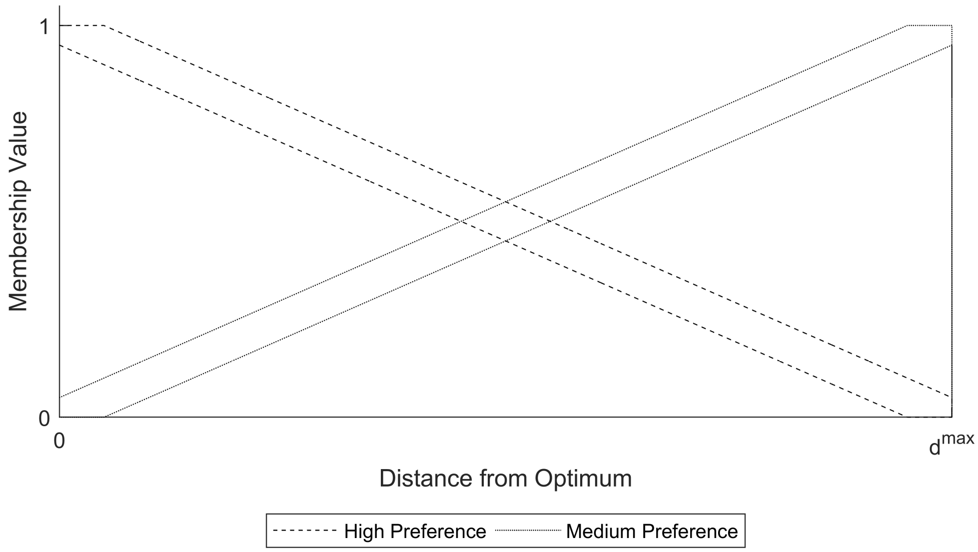


Figure 5.5: Type-2 fuzzy logic input membership function for distance from optimum preference mapping.

The lower and upper membership functions of the high and medium fuzzy sets are given in Equations 5.11 and 5.12 respectively. The optimum design has a distance of zero and a high distance preference near one. As design points move farther from this optimum, their high preference membership value decreases linearly to zero, and their medium preference membership grade increases to one. The exact membership grade is bounded by the fuzzy sets' footprint of uncertainty whose size is controlled by δ_{lb} and δ_{ub}, allowing them to capture uncertainty in linguistic variables and input data.

$$lmf_{dist}^{high}(d_i) = \begin{cases} \frac{d_i^{max} - d_i - \delta_{lb}}{d_i^{max}} & \text{if} \quad d_i \leq d_i^{max} - \delta_{lb} \\ 0 & \text{if} \quad d_i > d_i^{max} - \delta_{lb} \end{cases} \tag{5.11a}$$

$$umf_{dist}^{high}(d_i) = \begin{cases} 1 & \text{if} \quad d_i \leq \delta_{ub} \\ \frac{d_i^{max} - d_i + \delta_{ub}}{d_i^{max}} & \text{if} \quad d_i > \delta_{ub} \end{cases} \tag{5.11b}$$

$$lmf_{dist}^{medium}(d_i) = \begin{cases} 0 & \text{if} \quad d_i \leq \delta_{lb} \\ \frac{d_i - \delta_{lb}}{d_i^{max}} & \text{if} \quad d_i > \delta_{lb} \end{cases} \tag{5.12a}$$

$$umf_{dist}^{medium}(d_i) = \begin{cases} \frac{d_i + \delta_{ub}}{d_i^{max}} & \text{if} \quad d_i \leq d_i^{max} - \delta_{ub} \\ 1 & \text{if} \quad d_i > d_i^{max} - \delta_{ub} \end{cases} \tag{5.12b}$$

5.5.4 Combining Preferences into a Discipline Level Grade

The membership functions presented above are used to capture a discipline's preferences about a system of interest with respect to multiple performance metrics. For a generic mathematical optimization problem, these metrics were the objective function value $f_i^*(\mathbf{x})$, the objective function's gradient value $||\nabla_{\mathbf{x}} f_i(\mathbf{x})||_2$, and the distance to the optimal value $d_i(\mathbf{x})$. The discipline analyzer evaluates these values for each point in the discipline's domain, and passes them to the human expertise mapping step where they are converted into a linguistic variable value using the lower and upper membership functions presented above. Thus, for each point on the domain, and for each metric, two membership grades are calculated which provide the bounds on the footprint of uncertainty. After fuzzification, the input linguisitic preferences of each design point for all three metrics are combined to produce an overall discipline-level preference for each design point. This is done using a rule bank and fuzzy inference, which maps the input linguistic preferences to output linguistic preferences using the minimum inference method presented in Chapter IV. The FLS output processing then uses the center-of-sets type reduction (Karnik and Mendel, 1998) to provide an upper and lower bound on the discipline-level preference for each point in the design space, which is the last step of Figure 5.2.

5.6 Interval Type-2 Fuzzy Logic Multidisciplinary Optimization: a Generic Mathematical Example

This section presents a detailed example of the interval type-2 fuzzy logic multidisciplinary optimization (T2-FLMDO) on a mathematical problem which deals with the minimization of three objective functions simultaneously. First, the problem formulation is given. For each discipline, the discipline analysis step is presented in Section 5.6.2, followed by the human expert mapping step in Section 5.6.3. The fusion logic step, which aggregates the preferences of all disciplines is illustrated in Section 5.6.6.

5.6.1 Problem Formulation

The (T2-FLMDO) is used in this chapter to minimize three functions simultaneously: Equations 5.13, 5.14, 5.15 .

$$\underset{\mathbf{x}}{\text{minimize}} \quad f_1(\mathbf{x}) = x_1^2 + x_2^2 \tag{5.13}$$

$$\underset{\mathbf{x}}{\text{minimize}} \quad f_2(\mathbf{x}) = x_1^4 - 3x_1^2 + x_1 + x_2^4 - 3x_2^2 + x_2 \tag{5.14}$$

$$\underset{\mathbf{x}}{\text{minimize}} \quad f_3(\mathbf{x}) = -\cos(2x_1) + \sin(2x_2) \tag{5.15}$$

5.6.2 Discipline Analysis

The first step of the T2-FLMDO is the discipline analysis. For each design variable value $\mathbf{x}$, it calculates the performance metrics of interest to each discipline, and passes them to the human expertise mapping step of the T2-FLMDO. In this example, each discipline uses the same three performance metrics. The first is the objective function preference, which calculates $f_i(\mathbf{x})$, f_i^*, and f_i^{max}. The second is objective function gradient preference, which calculates $\left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2$ and $\left|\left|\nabla_{\mathbf{x}} f_i\right|\right|_2^{max}$. The third group is the distance from optimum preference, which calculates d_i and d_i^{max}. These values are shown in Table 5.1 and Figures 5.6–5.14.

Table 5.1: Disipline analyzer output.

	Discipline 1	Discipline 2	Discipline 3
x_1^*	0.00	-1.30	0.00
x_2^*	0.00	-1.30	-0.80
f_i^*	0.00	-7.03	-2.00
f_i^{max}	3.38	0.00	0.34
$\|\|\nabla_{\mathbf{x}} f_i\|\|_2^{max}$	5.66	29.70	2.83
d_i^{max}	2.83	4.67	3.44

Discipline 1 Analysis Plots

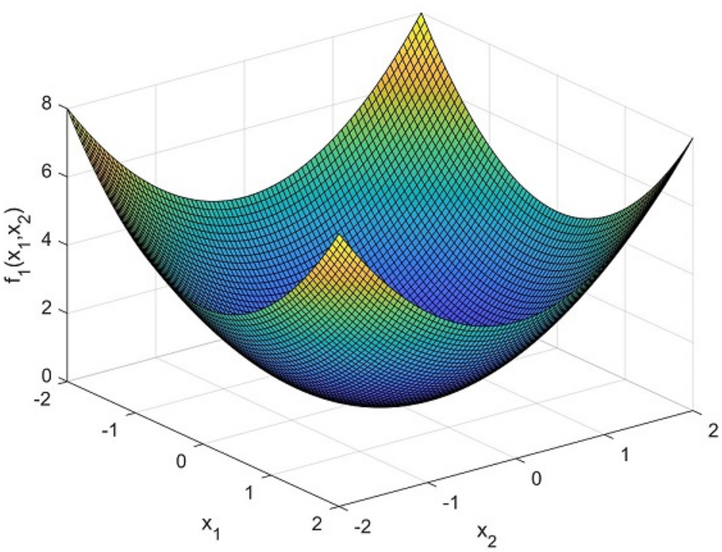


Figure 5.6: Disipline 1 objective function plot.

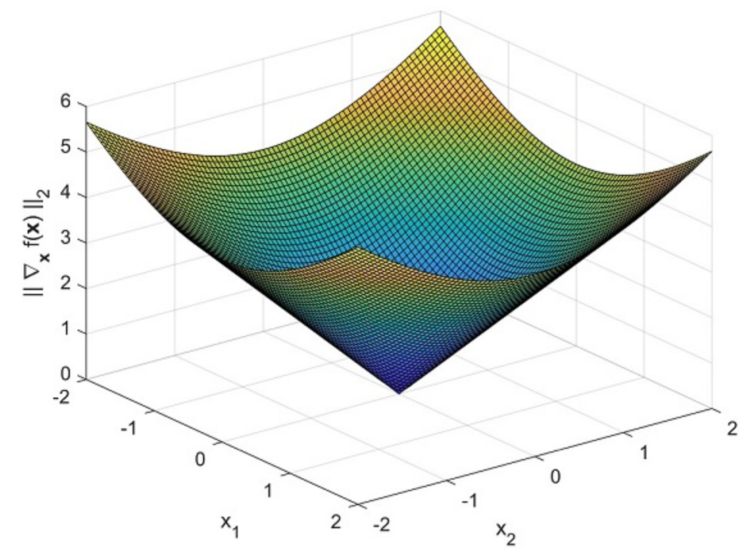


Figure 5.7: Disipline 1 objective function gradient plot.

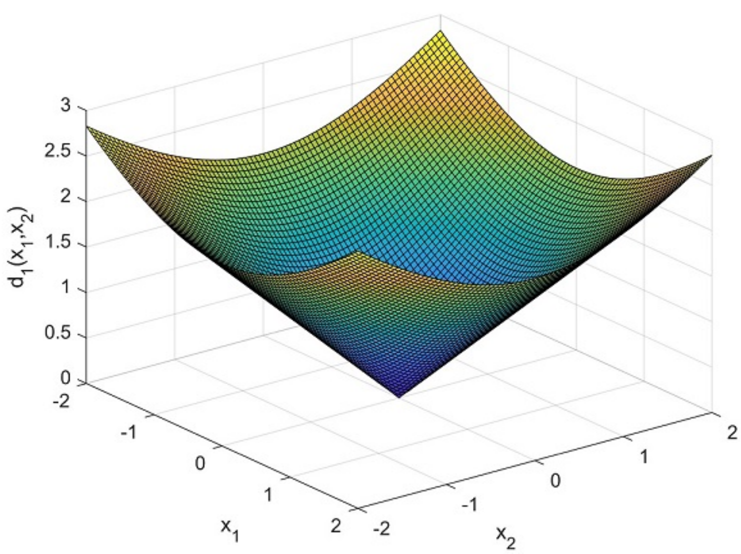


Figure 5.8: Disipline 1 distance from optimum plot.

Discipline 2 Analysis Plots

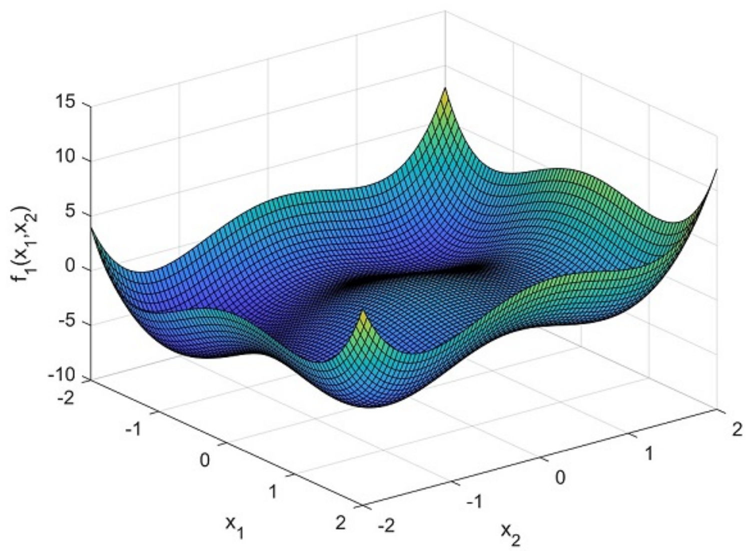


Figure 5.9: Disipline 2 objective function plot.

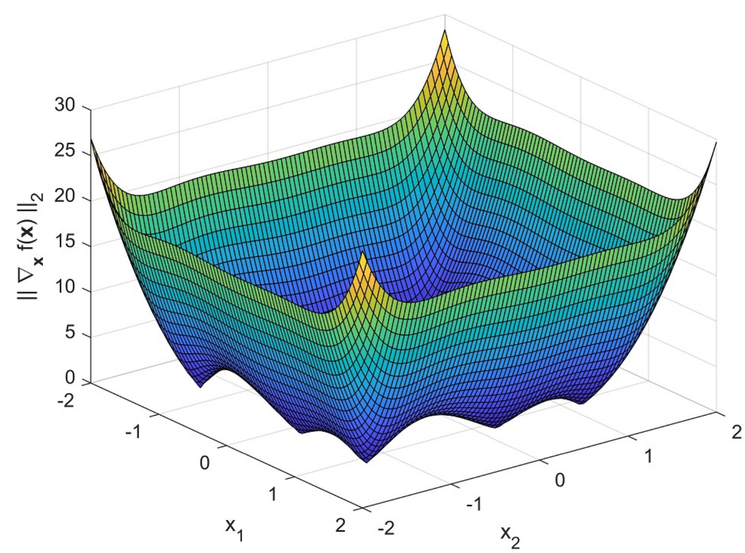


Figure 5.10: Disipline 2 objective function gradient plot.

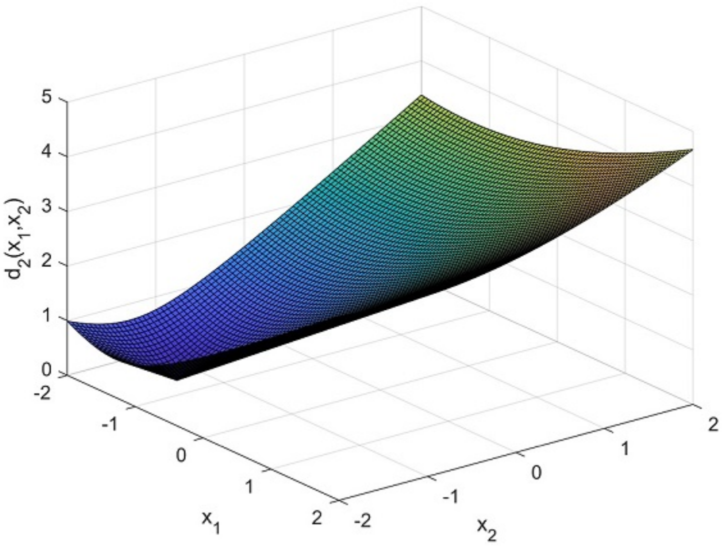


Figure 5.11: Disipline 2 distance from optimum plot.

Discipline 3 Analysis Plots

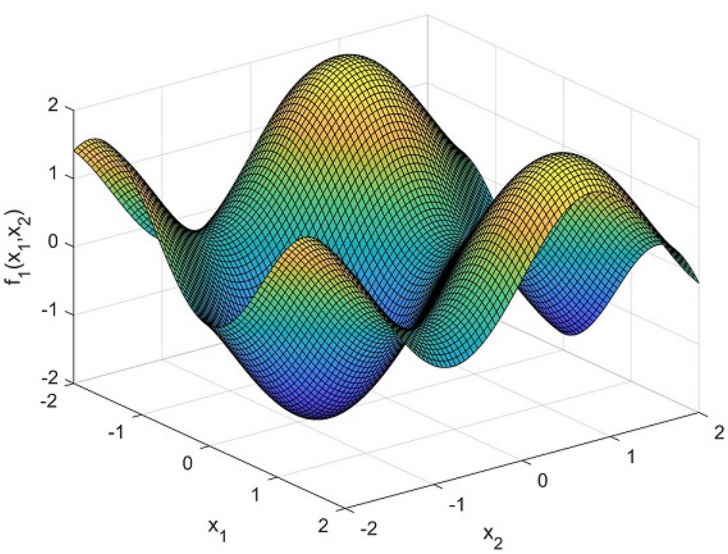


Figure 5.12: Disipline 3 objective function plot.

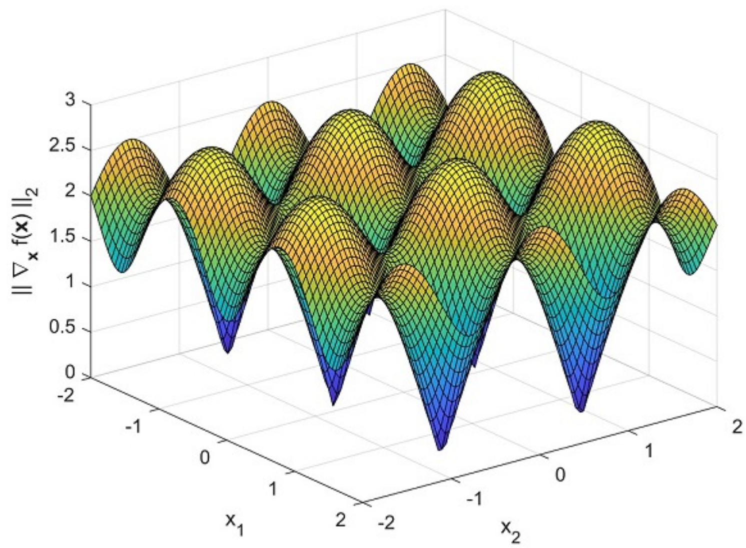


Figure 5.13: Disipline 3 objective function gradient plot.

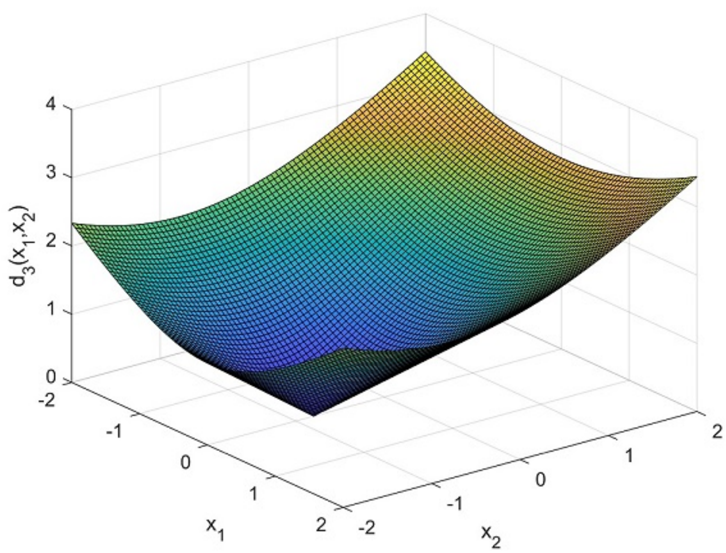
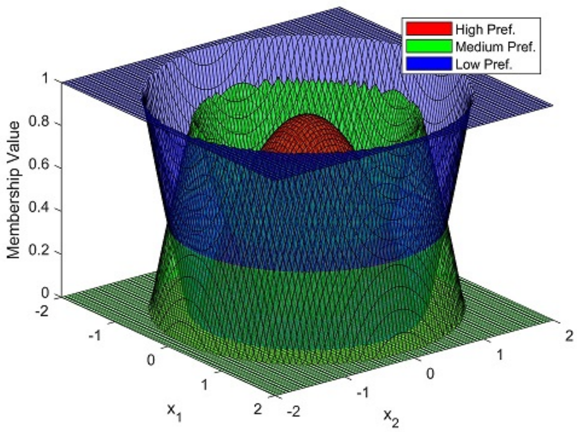


Figure 5.14: Disipline 3 distance from optimum.

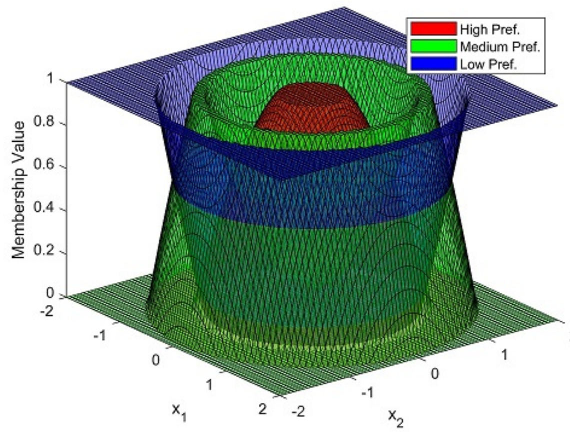
5.6.3 Human Expertise Mapping: Preference Membership Fuzzification

The first step of the human expertise mapping is the preference membership function creation. The FLS fuzzifier takes each design variable value $\mathbf{x}$ and maps it to the interval type-2 preference fuzzy sets described in Section 5.5 based on the values returned by the discipline analyzer. Each design value is assigned to these preference sets with various levels of membership bounded between a lower and upper bound. The fuzzification results for each discipline and perfomance metric are shown in Figures 5.15–5.23. Subfigures (a) are the LMFs and subfigures (b) are the UMFs.

Discipline 1 Linguistic Preference Mapping of Discipline Analysis Results

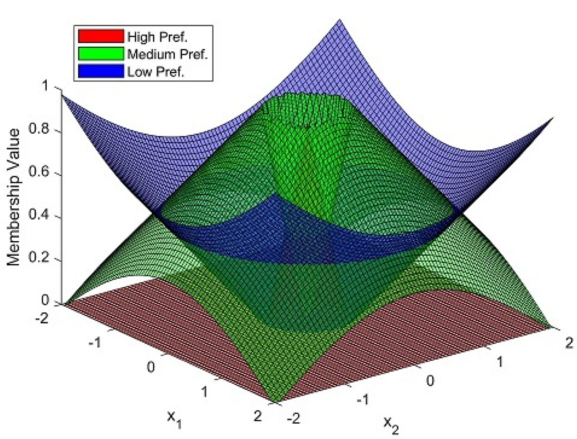


(a) Lower bound

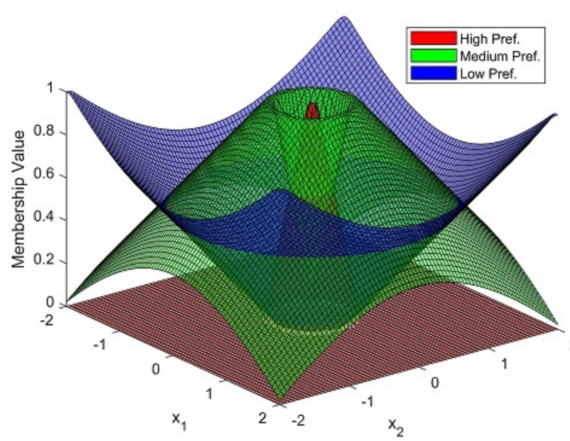


(b) Upper bound

Figure 5.15: Type-2 fuzzy logic linguistic preferences for discipline 1 objective function values.

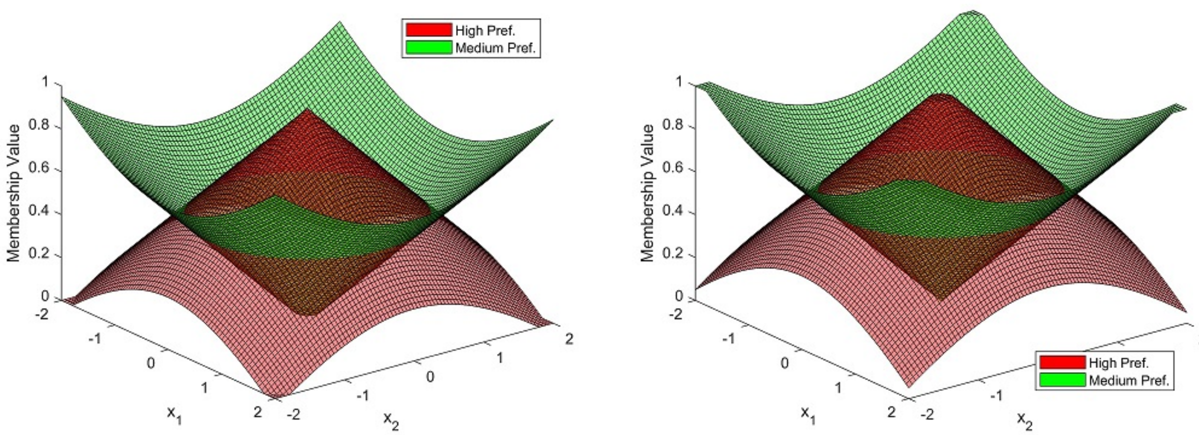


(a) Lower bound



(b) Upper bound

Figure 5.16: Type-2 fuzzy logic linguistic preference mapping for discipline 1 objective function gradient values.



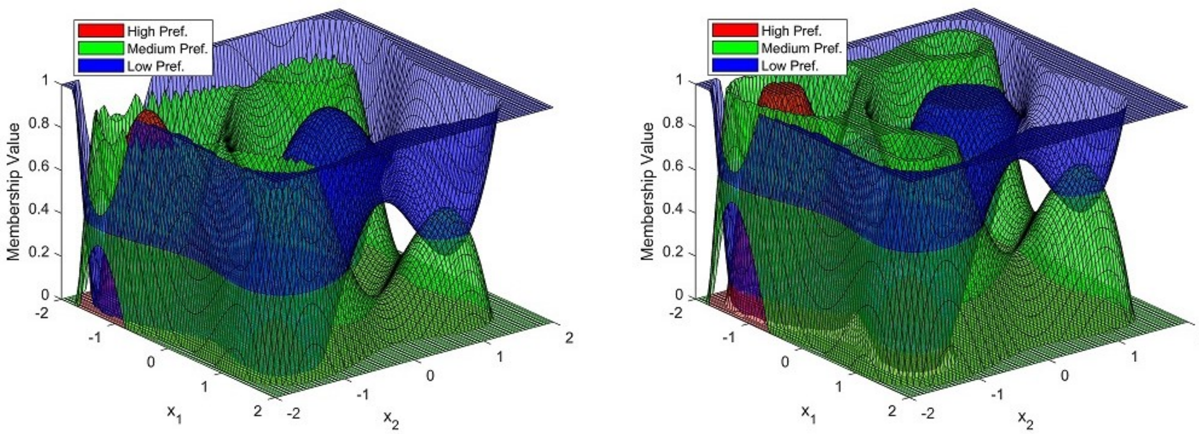
(a) Lower bound (b) Upper bound

Figure 5.17: Type-2 fuzzy logic linguistic preference mapping for discipline 1 distance from optimum values.

Although the primary purpose of these preference plots is to be used as input to the FLS rule bank, they also provide a quick visual representation of the design space with respect to the designer's performance objectives. Designs with a high membership value in the high preference sets should be sought. These are designs at or near the minimum objective function value, designs with a low objective function gradients that are robust with respect to small variations in design variable values, and designs that are close to the optimal.

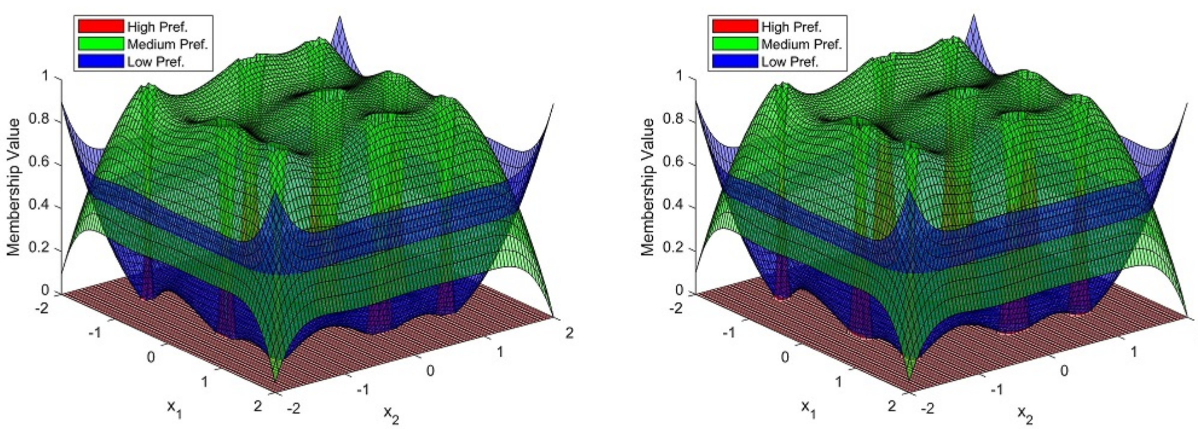
Since a precise definition of designer preferences cannot usually be obtained, the membership functions defined in this work are fuzzy and designer preferences can lie anywhere between the lower and upper bound with equal probability. In the first discipline's objective function preference (Figure 5.15b), the upper bound shows that designs in the vicinity of the true objective function optimal $\mathbf{x} = (0, 0)$ are equally highly preferred with membership value one. The designer sees no difference in objective function preference between these almost identical design points, which models people's tendency to group similar objects assumed to have negligible performance differences into one category. A T1-FLMDO does not allow preferences to be fuzzy, limiting its ability to model human reasoning adequately. This is a limitation of previous work and is corrected with the new T2-FLMDO developed in this thesis. It allows designer preferences to have a fuzzy definition by mapping numerical variables to a range of linguistic membership grades bounded by the footprint of uncertainty.

Discipline 2 Linguistic Preference Mapping of Discipline Analysis Results



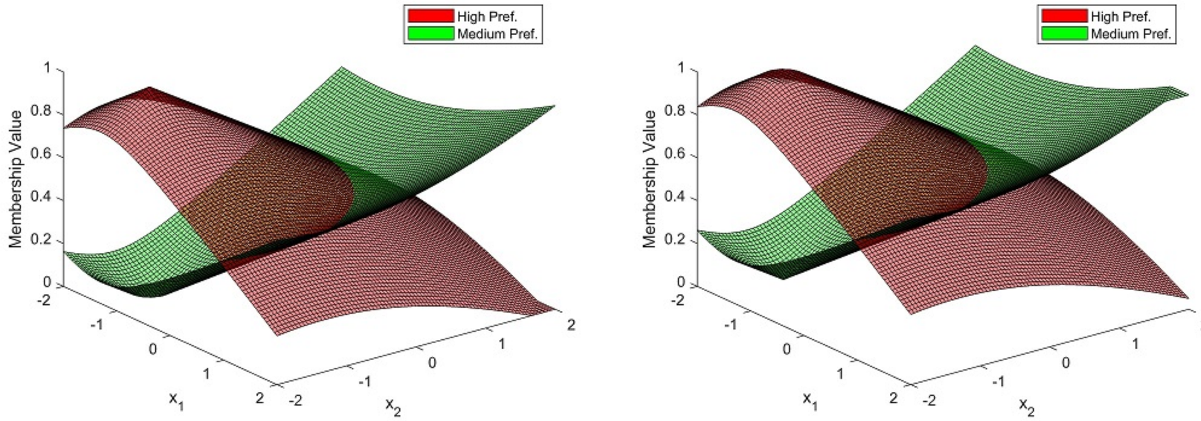
(a) Lower bound (b) Upper bound

Figure 5.18: Type-2 fuzzy logic linguistic preference mapping for discipline 2 objective function values.



(a) Lower bound (b) Upper bound

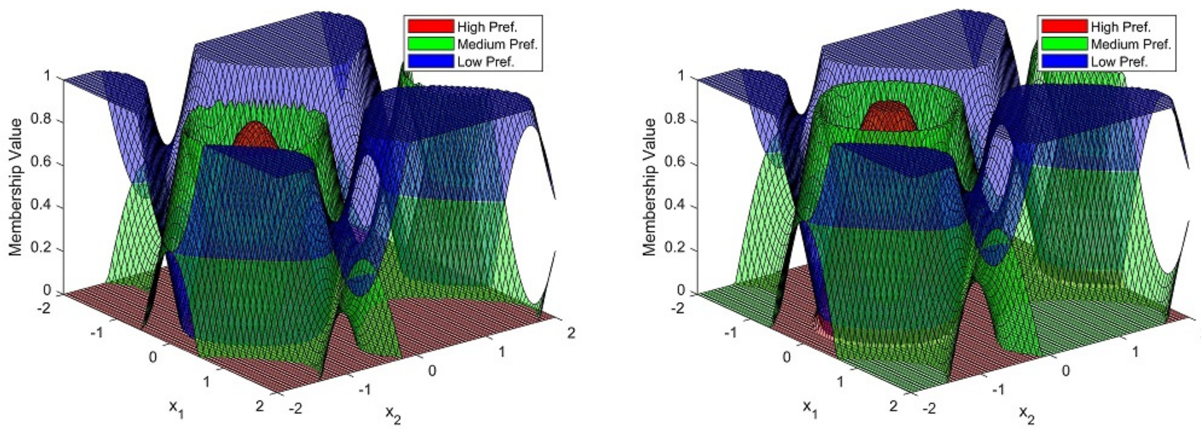
Figure 5.19: Type-2 fuzzy logic linguistic preference mapping for discipline 2 objective function gradient values.



(a) Lower bound (b) Upper bound

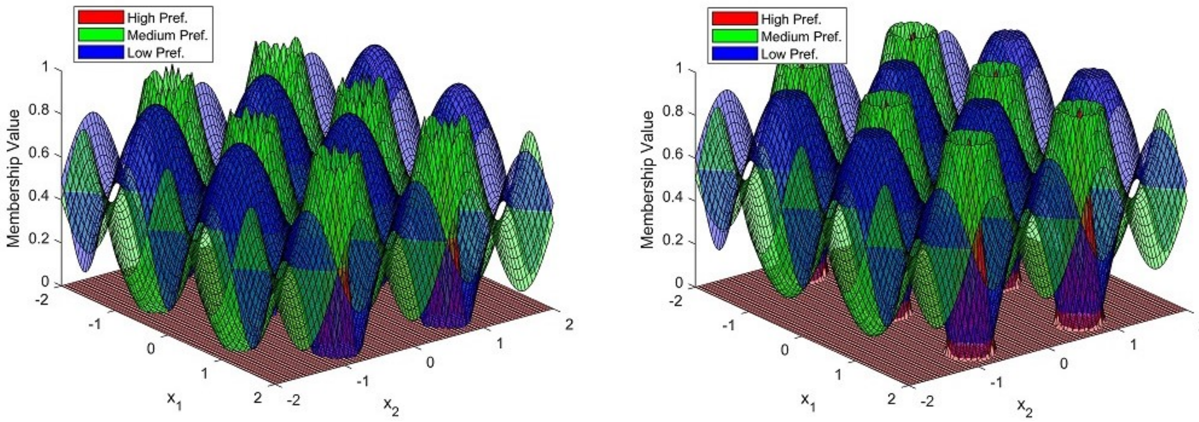
Figure 5.20: Type-2 fuzzy logic linguistic preference mapping for discipline 2 distance from optimum values.

Discipline 3 Linguistic Preference Mapping of Discipline Analysis Results



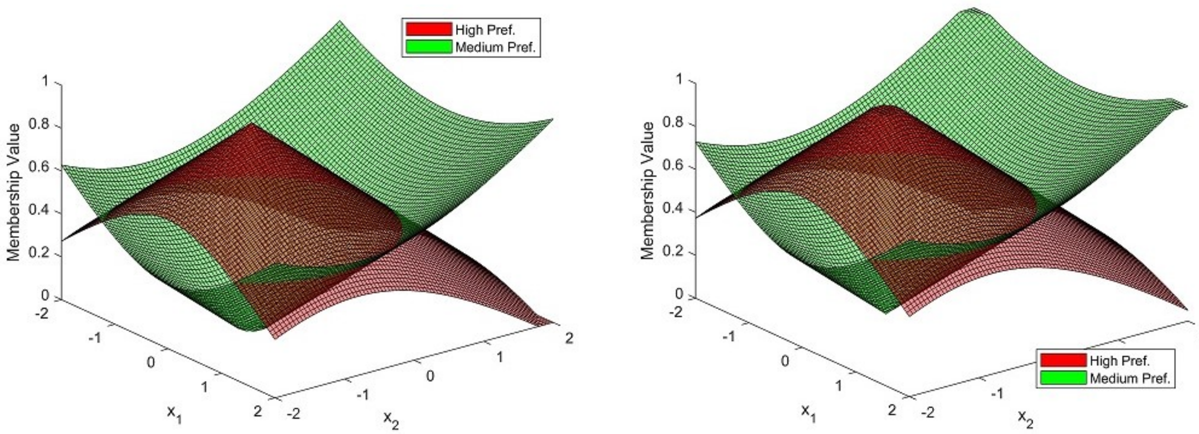
(a) Lower bound (b) Upper bound

Figure 5.21: Type-2 fuzzy logic linguistic preference mapping for discipline 3 objective function values.



(a) Lower bound (b) Upper bound

Figure 5.22: Type-2 fuzzy logic linguistic preference mapping for discipline 3 objective function gradient values.



(a) Lower bound (b) Upper bound

Figure 5.23: Type-2 fuzzy logic linguistic preference mapping for discipline 3 distance from optimum values.

5.6.4 Human Expertise Mapping: Fuzzy Inference

The fuzzification process presented in the previous section maps crisp values from the discipline analysis to three fuzzy preference linguistic variables. These three linguistic variables—objective function preference, gradient function preference, and distance function preference —are then combined into an overall discipline-level preference through a rule bank and fuzzy minimum inference. Recall that a rule bank maps input linguistic variables (i.e. antecedents) to output linguistic variables (i.e. consequents). For the mathematical example, it is given in Tables 5.2 and 5.3. There are 18 possible combinations of input linguistic variable values which map to six possible consequents. The columns represent the three possible objective function preference values, and the rows represent the three possible gradient function preference values. The two tables each represent one of the two distance function variable values.

Table 5.2: Rule bank for high distance function preference.

		Objective Function Preference		
		Low	Medium	High
Gradient Preference	Low	Unpreferred	Medium-Low Preference	Medium-High Preference
	Medium	Low Preference	Medium Preference	High Preference
	High	Low Preference	Medium Preference	High Preference

Table 5.3: Rule bank for medium distance function preference.

		Objective Function Preference		
		Low	Medium	High
Gradient Preference	Low	Unpreferred	Medium-Low Preference	Medium-High Preference
	Medium	Unpreferred	Medium-Low Preference	Medium-High Preference
	High	Low Preference	Medium Preference	High Preference

In the rule bank, the output preferences are more closely correlated to objective function preference than gradient preference function, showing a clear designer bias for the objective function. This bias is consistent with the importance of objective function value in a mathematical optimization problem, which should be emphasized over the other performance metrics.

The output linguistic variables, or consequents of the rule bank, are modeled by six interval type-2 fuzzy sets, each representing the designer's preference level for that design—unpreferred, low preference, medium-low preference, medium preference, medium-high preference, and high preference. This method of assigning preferences to design variables was used by Singer (2003), Gray (2012), and Cuneo (2013) and another variation of it is used in this work (Figure 5.24). Here type-2 fuzzy sets are used instead ot type-1 sets to capture the uncertainty associated with the linguistic description of the output preference. In type-1 sets output membership functions map to a single numerical value through their centroid, but type-2 sets map to a range of centroids.

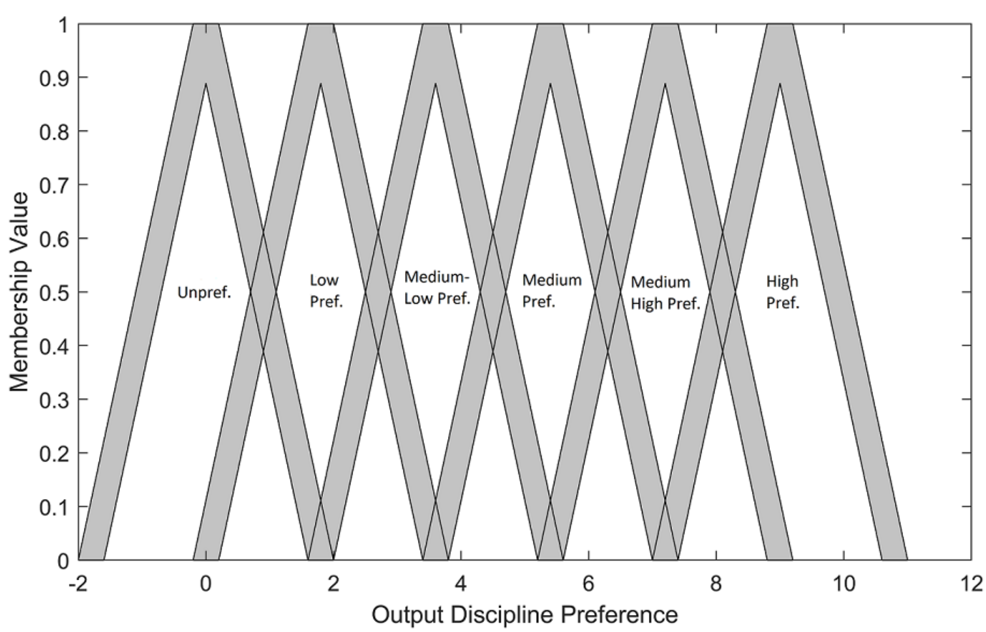


Figure 5.24: Type-2 fuzzy logic output preference membership functions.

Recall that interval type-2 fuzzy sets can be characterized by their left and right centroids which let designers assume a fuzzy mapping between a linguistic variable (output linguistic preference) and a crisp number (output crisp preference). A linguistic variable can map to any crisp value between the left and right centroid with equal probability. The bounds of this mapping can be seen in Table 5.4.

Table 5.4: Type-2 fuzzy logic output preference membership function centroids.

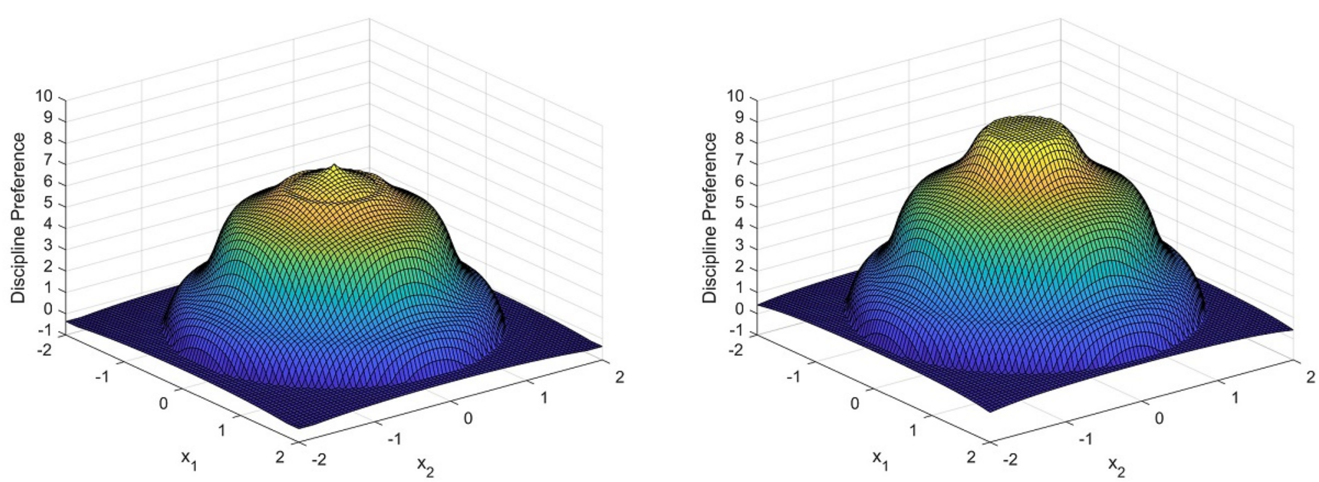
	c_l	c_r
Unpreferred	-0.1	0.1
Low Preference	1.7	1.9
Medium-Low Preference	3.5	3.7
Medium Preference	5.3	5.5
Medium-High Preference	7.1	7.3
High Preference	8.9	9.1

For each design variable value, the rules are activated according to the minimum inference method which calculates two activation levels for each rule: the lower and upper activation levels. The lower activation level is equal to the minimum of the input variables LMFs, and the upper activation level is equal to the minimum of the input variables UMFs. A more detailed description of the fuzzy inference process can be found in Chapter IV Section 4.3.3 of this thesis or in Mendel's primer on type-2 fuzzy sets and systems (Mendel, 2007).

5.6.5 Human Expertise Mapping: Discipline Preferences

The output processing step of the human expertise mapping fuzzy logic system combines the activated rules into two discipline level preference for each design variable value $\mathbf{x}$: a lower bound on discipline preferences and an upper bound on discipline preference. Here, the center-of-sets type reduction is used to calculate these values. After calculating them for each point in the design space, the discipline preference surfaces are plotted (Figures 5.25-5.27). Recall that for interval type-2 fuzzy systems, an infinite number of equally weighted preferences exist between the lower and upper bound at each point in the domain, and the average preference is just the midpoint between the lower and upper bound.

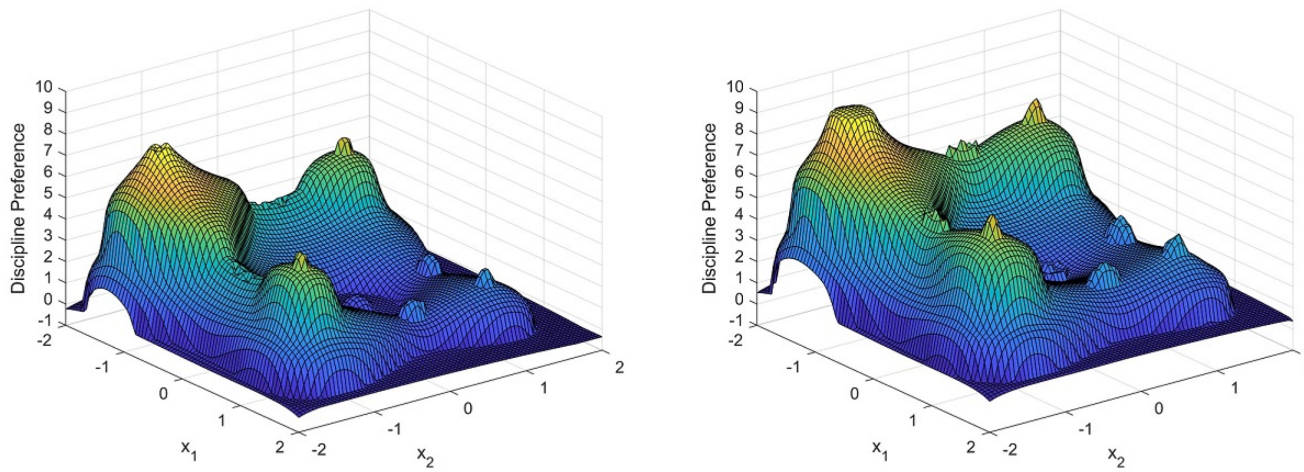
The preference plots show how all three performance metrics—objective function, gradient function, and distance function—are aggregated into an overall preference metric for each discipline that. All six plots show a highest preference for the design with the smallest objective function value. For discipline 1, $\mathbf{x}^* = (0, 0)$; for discipline 2, $\mathbf{x}^* = (-1.3, -1.3)$; and for discipline 3, $\mathbf{x}^* = (0, -0.8)$. Additionally, these points have a zero gradient function value and a zero distance function value, thus are optimal along all three metrics. The plots also show multiple areas of moderate preference that are acceptable to the designer. For discipline 2, $\mathbf{x}^* = (1.15, -1.3)$ or $\mathbf{x}^* = (-1.3, 1.15)$ show relatively high preference in Figure 5.26. These are two local optima with zero gradient and good objective value, thus could provide a good substitute design if the discipline 2 global optimal at $\mathbf{x}^* = (-1.3, -1.3)$ becomes infeasible, or if this value has poor performance with respect to the other disciplines.



(a) Lower bound

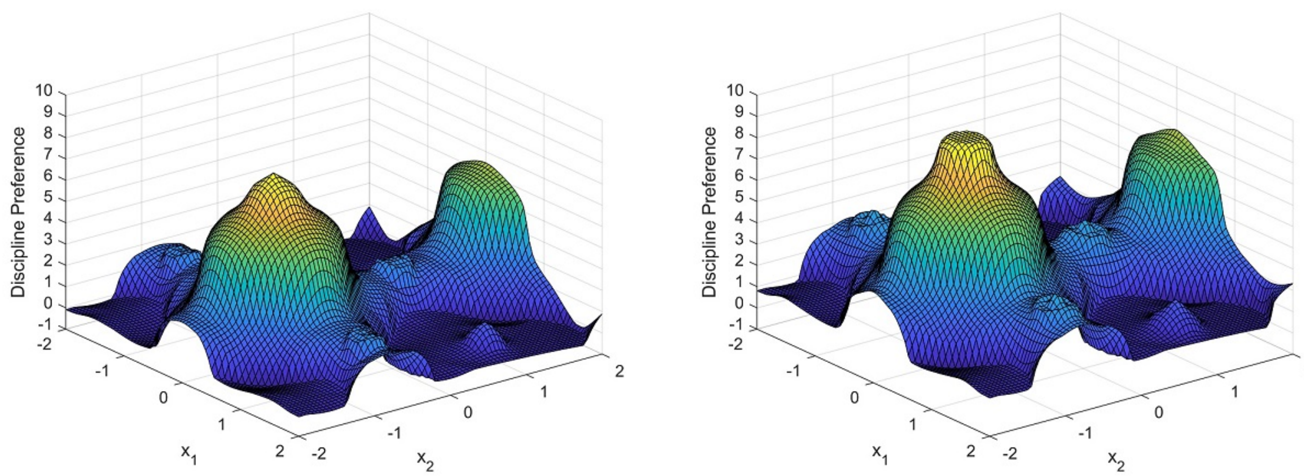
(b) Upper bound

Figure 5.25: Discipline 1 type-2 output preference surface lower and upper bounds.



(a) Lower bound (b) Upper bound

Figure 5.26: Discipline 2 type-2 output preference surface lower and upper bounds.



(a) Lower bound (b) Upper bound

Figure 5.27: Discipline 3 type-2 output preference surface lower and upper bounds.

5.6.6 Fusion Logic

After going through the discipline analysis step and the human expertise mapping step, the three discipline level preference surfaces (Figures 5.25 - 5.27) are aggregated into a system level preference surface. Several aggregation, or fusion logic methods exist, ranging from fully compensatory to fully non-compensatory (Chen and Hwang, 1992). In fuzzy logic, compensatory aggregation allows a low preference from one discipline to be compensated if another discipline has a high preference for that design, and can be used to negotiate design preferences. In non-compensatory aggregation a good performance of one discipline does not compensate for a poor performance of another discipline. Compensatory and non-compensatory aggregation are demonstrated in this section, in addition to one dynamic aggregation which adjusts the fusion method based on the preference levels of each discipline.

5.6.6.1 Compensatory Fusion Logic

Three different compensatory fusion logic methods are discussed here. The first one is the fuzzy intersection, which is a commonly applied aggregation method in the fuzzy logic literature. The intersection of two or more fuzzy sets is calculated by taking the minimum of the sets' membership grades' for each independent variable value (Dubois and Prade, 1980) as shown in equation 5.16, where $F_s(\mathbf{x})$ is the system level preference at design variable $\mathbf{x}$, and $F_i^{mid}(\mathbf{x})$ is the mean preference of discipline i, where there are D disciplines. Recall from Chapter IV that the centroid of a type-2 output fuzzy preference set is given by the union of all its embedded type-1 preference fuzzy sets, and it is this centroid that the mean preference captures for interval type-2 fuzzy logic systems. The embedded type-1 fuzzy sets are bounded below and above, as discussed in Figure 5.27.

$$F_s(\mathbf{x}) = \bigwedge_{i=1}^{D} F_i^{mid}(\mathbf{x}) = \min F_i^{mid}(\mathbf{x}) \quad \forall\, i \in D \tag{5.16}$$

In minimum fusion, the discipline with the smallest preference at each design point gives the cutoff for the system preference, which provides an egalitarian method of aggregating preferences. System level preferences are shown in Figure 5.28. The optimal point is at $\mathbf{x} = (-0.65, -0.86)$ where all three disciplines have moderately high preference, but preferences drop rapidly for decreasing and increasing values of x and y where discipline 1 and 2 have low preference respectively. At the system-level optimal, discipline 1 has an objective value $F_1^{mid} = 1.16$, discipline 2 has an objective value $F_2^{mid} = -4.27$, and discipline 3 has an objective value $F_3^{mid} = -1.26$, which equalizes the outputs of each discipline and provides a compromise of the discipline-level preferences given in Table 5.1.

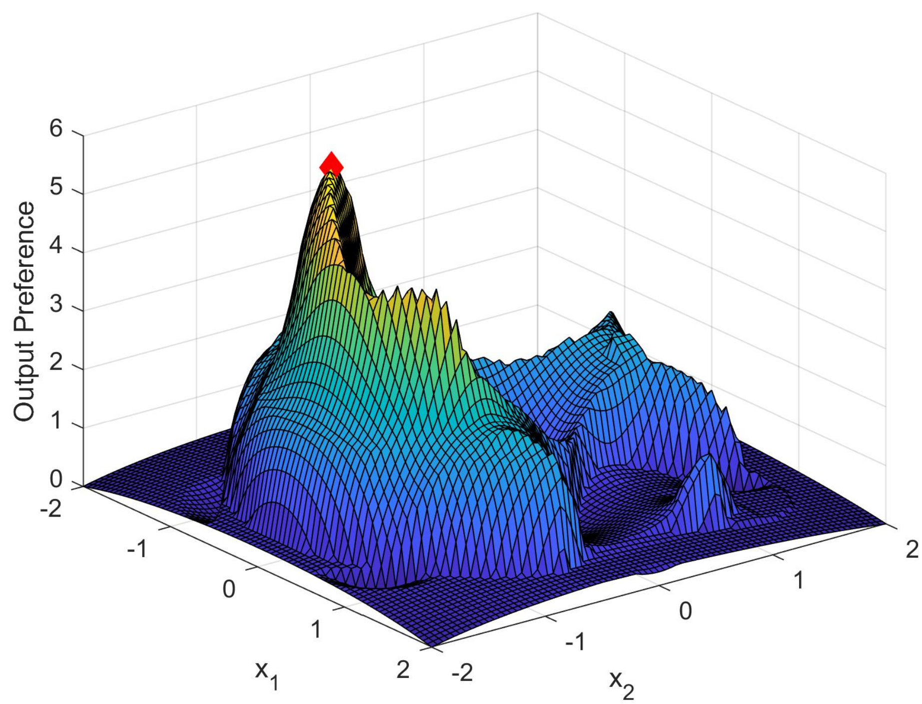


Figure 5.28: Minimum fusion system preference surface.

The two other compensatory fusion logic methods used are the geometric and arithmetic mean, given in Equations 5.17 and 5.18 respectively. The geometric mean scales preferences so that a large or small value does not dominate the system level solution as much as for the arithmetic mean; thus the geometric mean equalizes preferences compared to its arithmetic counterpart, to limit the domination of one discipline's preferences on the overall solution. The system level preference surfaces for the geometric and arithmetic mean fusion logic methods are shown in Figures 5.29 and 5.30 respectively.

$$F_s(\mathbf{x}) = (\prod_{i=1}^{D} F_i^{mid}(\mathbf{x}))^{1/D} \tag{5.17}$$

$$F_s(\mathbf{x}) = \frac{1}{D}\sum_{i=1}^{D} F_i^{mid}(\mathbf{x}) \tag{5.18}$$

Whereas the minimum fusion showed a steep decline in preferences around the optimal point, both mean fusion results show a more gradual decline in preferences around the optimal. These points are not nearly as affected by the decrease in preference of some of the lower performing disciplines. The mean fusions are the least risk-averse of the three compensatory fusion logic methods in that they balance a low preference of one discipline with a high preference of another discipline. The three fusion logic methods give slightly different results, which showcases the added flexibility of the FLMDO in aggregating preferences from each discipline.

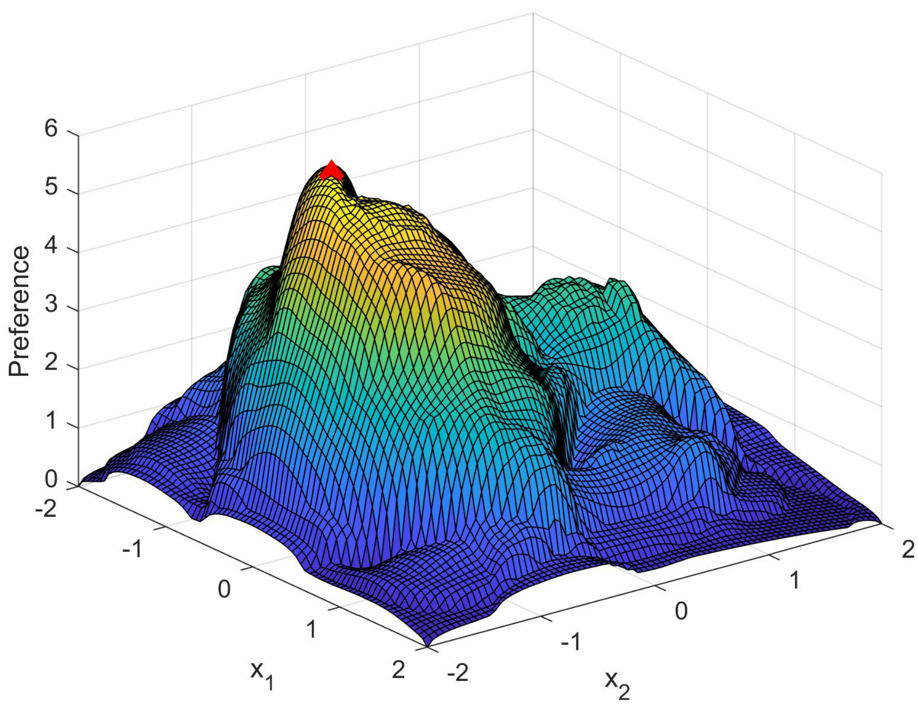


Figure 5.29: Geometric mean fusion system preference surface.

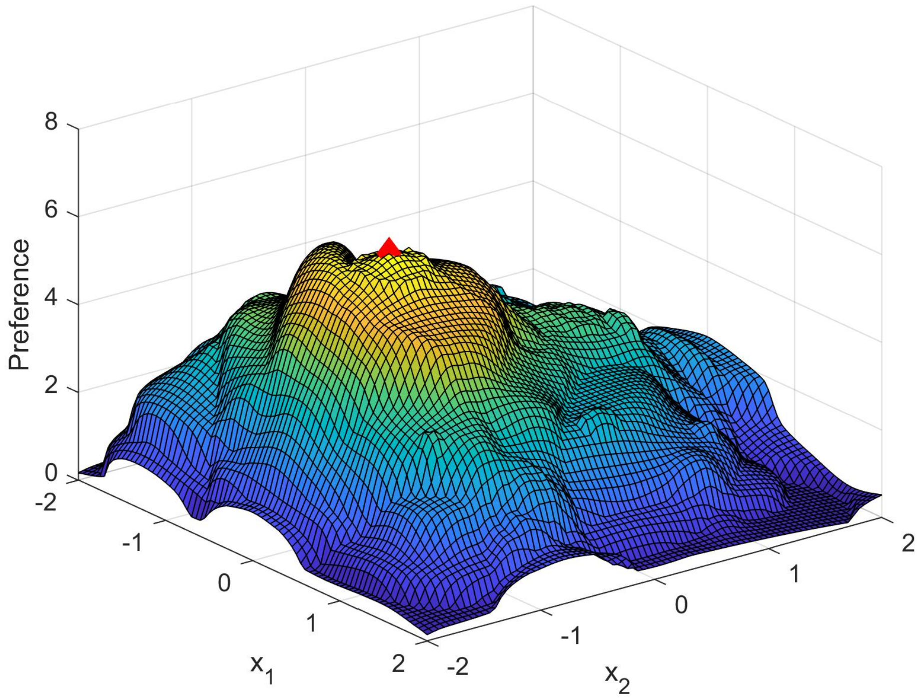


Figure 5.30: Arithmetic mean fusion system preference surface.

5.6.6.2 Non-compensatory Fusion Logic

This section briefly discusses non-compensatory fusion logic, which only considers the preferences of a single discipline irregardless of the other ones when evaluating the overall performance of the system. A pure application of non-compensatory operators do not make sense in the context of multidisciplinary optimization, since they do not account for the preferences of all disciplines. However, they can be useful when applied to the system level fusion in a limited and controlled way, as will be demonstrated in Section 5.6.6.3. While compensatory operators average out performance, such that a design with exceptional performance in discipline 1 will be considered less favorable if it has average performance in discipline 2, a non-compensatory operator will be better able to find designs with exceptional performance in a chosen discipline. If the performance of other disciplines is above a minimum threshold, then a non-compensatory operator will be better able to locate designs that perform very well in the discipline that are most critical to the success of the system. One such fusion logic is given by the union of two or more fuzzy sets (i.e. the OR operator on the sets), which is the maximum of the sets' membership grade at each independent variable (Dubois and Prade, 1980) and is given in Equation 5.19.

$$F_s(\mathbf{x}) = \bigvee_{i=1}^{D} F_i^{mid}(\mathbf{x}) = \max F_i^{mid}(\mathbf{x}) \quad \forall\, i \in D \tag{5.19}$$

The system level preference surface for the fuzzy union fusion logic is given in Figure 5.31. The surface shows three optima, equivalent to the optima of each discipline; however, these optima are meaningless in the context of a multidisciplinary optimization since they do not account for the preferences of all disciplines. Maximum fusion; however, can be useful when applied to the system level fusion in a limited and controlled way, as will be demonstrated in Section 5.6.6.3.

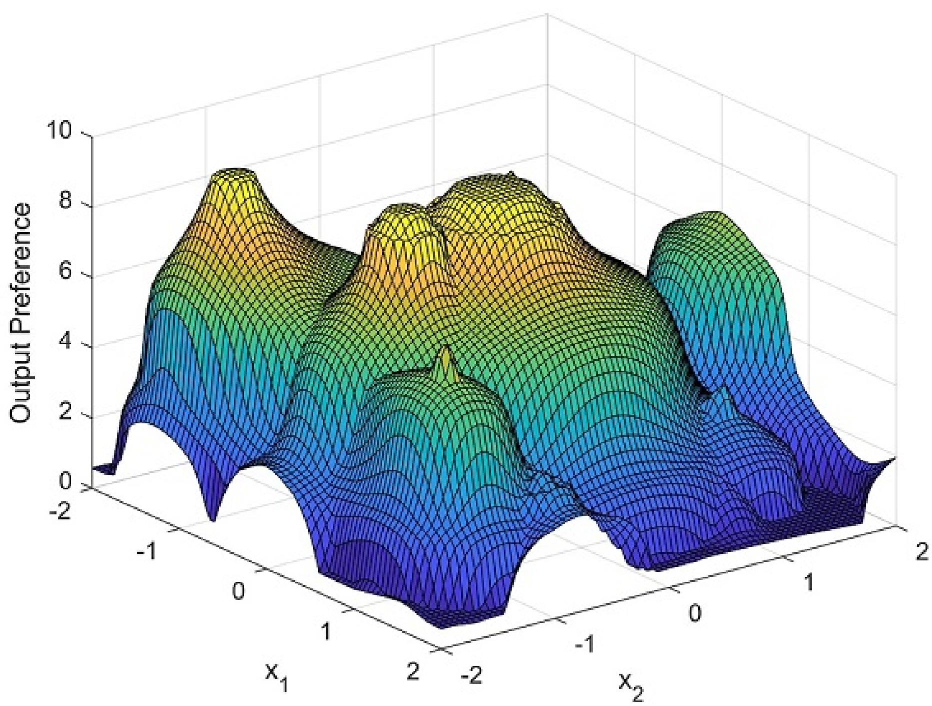


Figure 5.31: Maximum fusion system preference surface.

5.6.6.3 Dynamic Fusion

The compensatory and non-compensatory fusion logic methods presented in Sections 5.6.6.1 and 5.6.6.2 provide a static aggregation of preferences over the entire design space which does not change depending on context. The fusion logic method presented in this section uses a dynamic mapping from situation to preference which can dictate the influence of disciplines based on context, and is more analogous to the hierarchical fuzzy logic controllers used in autonomous robots. Dynamic fusion is often used in autonomous navigation since the priorities of a robot must change regularly based on the current situation or goal it faces (Tunstel et al., 1997). A collision avoidance behavior, for example, is only activated for short periods of time when the vehicle is nearing an obstacle. In this section, this idea is adapted to multidisciplinary optimization to dynamically change the influence of each discipline based on context, which refers here to the preference levels returned by the discipline-level optimization modules. For the design of a ship with respect to seakeeping and resistance, seakeeping might be half as important to the owner than resistance to minimize fuel cost. However, for designs whose seakeeping performance becomes so bad that half of the crew is seasick, seakeeping would become the more important design criteria of the two.

In this proof of concept, three aggregation rules are used depending on the optimization context. The rules range from highest priority (or most specialized) to lowest priority (or most general). The highest priority rule is activated if the preference levels returned by discipline one or three are above a given threshold, and returns the arithmetic mean of discipline one and three. The mid-priority rule is activated when the difference between preference levels of all three disciplines is smaller than a given value, and returns the maximum of all three preferences at that design point. The most general, or lowest priority rule is activated in all other cases, and returns the minimum of all three discipline preferences. This aggregation method is shown in the logic statement of Equation 5.20 where A_1 and A_3 are the discipline one and three preference thresholds for the highest priority rule and t is the difference threshold for the mid-priority rule.

The system level preference surface for the dynamic fusion method is given in Figure 5.32 for $A_1 = A_3 = 7$, which represents 75% of the maximum possible preference, and for $t = 0.5$, which represents 5% of the maximum possible preference. In words, the fusion method states that discipline 2 becomes irrelevant if disciplines one and three have high preference at $\mathbf{x}$ and that if all three disciplines have a vaguely similar opinion about a design point, a there is little risk using a non-compensatory aggregation rule. The optimum of the system level preference is located at $\mathbf{x} = (-0.05, -0.55)$ where discipline one and three have relatively high preference and discipline two has very low preference.

$$
\begin{aligned}
&\textbf{IF} && F_1^{mid}(\mathbf{x}) \geq A_1 \quad \text{AND} \quad F_3^{mid}(\mathbf{x}) \geq A_3 \\
& && \textbf{THEN} \quad F_s(\mathbf{x}) = \frac{F_1^{mid}(\mathbf{x}) + F_3^{mid}(\mathbf{x})}{2} \\
&\textbf{ELSEIF} && |F_1^{mid}(\mathbf{x}) - F_2^{mid}(\mathbf{x})| < t \\
& && \text{AND} \quad |F_1^{mid}(\mathbf{x}) - F_3^{mid}(\mathbf{x})| < t \\
& && \text{AND} \quad |F_1^{mid}(\mathbf{x}) - F_3^{mid}(\mathbf{x})| < t \\
& && \textbf{THEN} \quad F_s(\mathbf{x}) = \bigvee_{i=1}^{D} F_i^{mid}(\mathbf{x}) \\
&\textbf{ELSE} && \\
& && \bigwedge_{i=1}^{D} F_i^{mid}(\mathbf{x})
\end{aligned}
\tag{5.20}
$$

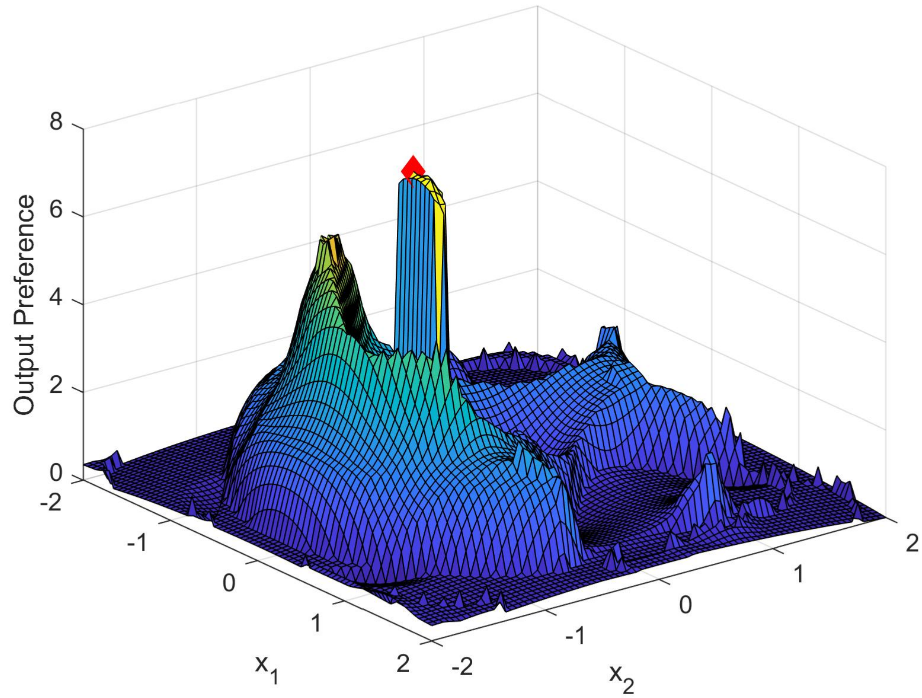


Figure 5.32: Dynamic fusion system preference surface.

5.6.6.4 Fusion Logic with Discipline Preference Uncertainty

The fusion logic methods presented above only consider the mean preference surface of each discipline, given by the union of each discipline's embedded type-1 preference surfaces that are bounded between the lower and upper discipline preferences bounds of Figures 5.25-5.27. To account for the discipline preference uncertainty, the fusion logic must consider the lower and upper preference surfaces of each discipline. In such cases, the system-level optimal is given by uncertainty bounds as shown in Figures 5.33-5.34. The system-level preference upper bound, in red, is calculated by minimum fusion of all the discipline-level preference upper bounds. The system-level preference lower bound, in blue, uses minimum fusion on the discipline-level preference lower bounds. This allows the input membership footprint of uncertainty to be propagated through the fuzzy logic system and the fusion logic to provide bounds on system-level preferences which account for the designer's mental model uncertainty.

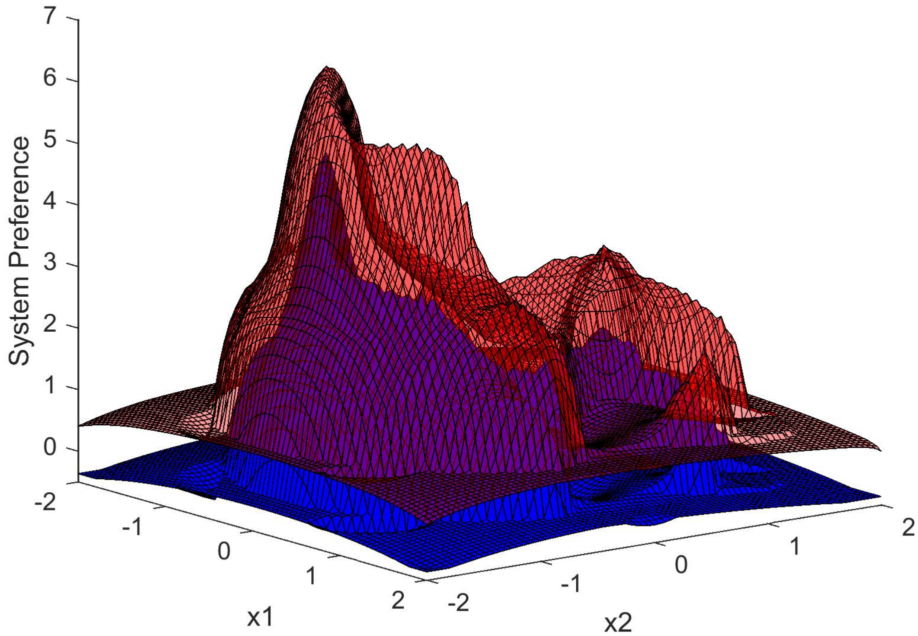


Figure 5.33: Minimum fusion with system preference uncertainty.

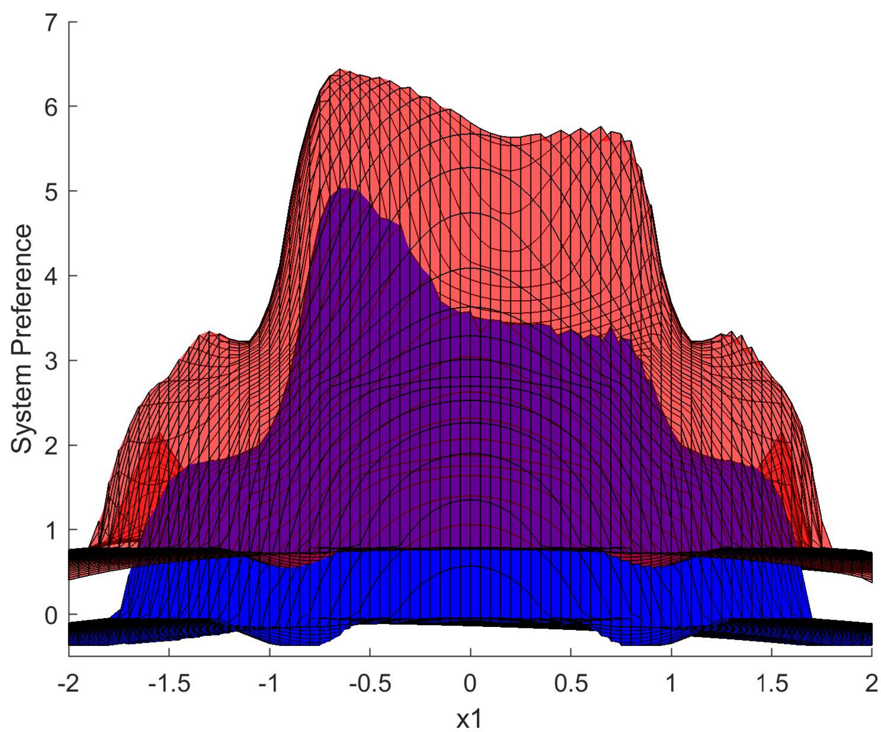


Figure 5.34: Minimum fusion with system preference uncertainty (face view).

5.6.7 System Level Optimals

The last step of the interval type-2 fuzzy logic multidisciplinary optimization is finding the system level optimal, or the design variable value with the highest system level preference $F_s(\mathbf{x}^*)$. In this chapter, the preference surface can be visualized and is entirely known to the decision maker, thus the optimal point can be found easily. In general, a non-gradient based optimization method can be used to find the optimal design value, since fuzzy preference surfaces are not described through mathematical functions and can contain local optima. Genetic algorithms (GA) and particle swarm optimization (PSO) have been tested by the author and have shown positive results. The system level optima for each fusion method were plotted on the preference surface in Figures 5.28-5.30 and 5.32. They are summarized again in the following two subsections, where two sets of FLMDO results are presented. The first is for a type-1 FLMDO (i.e. when no linguistic uncertainty exists) and the second is considers a type-2 FLMDO (i.e. in the presence of linguistic uncertainty).

5.6.7.1 FLMDO Results with no Discipline Preference Uncertainty

The first set of system level optima presented assumes no uncertainty in the fuzzy membership function definition. When this uncertainty disappears, the membership functions' footprints of uncertainty disappear and the fuzzy logic system reduces to being type-1. The optimal points of the T1-FLMDO are shown for all the compensatory fusion logic methods in Table 5.5. These results are compared to two traditional multidisciplinary optimization methods—analytical target cascading (ATC) and all-in-one (AIO). The optimal points are given by (x_1, x_2). The objective value for each of the three disciplines at these points is given by F_1, F_2, F_3, and the system level optimal is given by F_s, which is the sum of all three discipline functions.

Table 5.5: T1-FLMDO results, which assume no linguistic uncertainty.

	x_1	x_2	F_1	F_2	F_3	F_s
Min Fusion	-0.65	-0.86	1.16	-4.27	-1.27	-4.37
Mean$_{\text{geom}}$ Fusion	-0.46	-0.97	1.15	-3.96	-1.54	-4.34
Mean$_{\text{arith}}$ Fusion	-0.01	-0.78	0.61	-2.25	-2.00	-3.64
ATC	-0.80	-1.00	1.64	-5.31	-0.88	-4.55
AIO	-0.84	-1.01	1.73	-5.49	-0.79	-4.55

5.6.7.2 FLMDO Results with Discipline Preference Uncertainty

This set of system level optima presented here assume that the membership functions used in the T2-FLMDO are uncertain, or fuzzy. In such cases, the membership grade of a crisp variable to a fuzzy set can lie anywhere between the lower and upper membership function (i.e. its footprint of uncertainty (FOU)) with equal likelihood, as shown in Figures 5.33, 5.34. Recall from Section 5.5 that the size of the FOU is controlled by the δ_{lb} and δ_{ub} parameters which shift the lower and upper membership functions of the input fuzzy sets left and right relative to the type-1 fuzzy membership function. A 2.5 % FOU refers to a δ_{lb} and δ_{ub} value equal to 2.5% of the midpoint of the input membership functions. For the objective function preference mapping in Figure 5.3, the midpoint of the input membership function is f^{mid}, and for the gradient function preference mapping this value is 1.

The interval T2-FLMDO results are given for the minimum fusion logic method and FOU equal to 2.5%, 5%, and 10% in Tables 5.6 through 5.8. The LMF and UMF give the uncertainty bounds on the optimal. For a 10% FOU on linguistic uncertainty, the best case scenario is given by the point $\mathbf{x} = (-0.65, -0.86)$ with system level objective function value $F_s = -4.37$, and the worst case scenario is given by the point $\mathbf{x} = (-0.67, -0.82)$ with system level objective function value $F_s = -4.31$.

Table 5.6: T2-FLMDO results with 2.5 % FOU linguistic uncertainty.

	x_1	x_2	F_1	F_2	F_3	F_s
Min Fusion T1	-0.65	-0.86	1.16	-4.27	-1.27	-4.37
Min Fusion LMF	-0.65	-0.86	1.16	-4.27	-1.26	-4.37
Min Fusion Mid	-0.66	-0.85	1.16	-4.27	-1.24	-4.35
Min Fusion UMF	-0.66	-0.85	1.16	-4.27	-1.24	-4.35

Table 5.7: T2-FLMDO results with 5 % FOU linguistic uncertainty.

	x_1	x_2	F_1	F_2	F_3	F_s
Min Fusion T1	-0.65	-0.86	1.16	-4.27	-1.27	-4.37
Min Fusion LMF	-0.65	-0.86	1.16	-4.27	-1.26	-4.37
Min Fusion Mid	-0.66	-0.85	1.16	-4.27	-1.24	-4.35
Min Fusion UMF	-0.66	-0.84	1.14	-4.24	-1.24	-4.34

Table 5.8: T2-FLMDO results with 10 % FOU linguistic uncertainty.

	x_1	x_2	F_1	F_2	F_3	F_s
Min Fusion T1	-0.65	-0.86	1.16	-4.27	-1.27	-4.37
Min Fusion LMF	-0.65	-0.86	1.16	-4.27	-1.26	-4.37
Min Fusion Mid	-0.66	-0.85	1.16	-4.27	-1.24	-4.35
Min Fusion UMF	-0.67	-0.82	1.12	-4.20	-1.23	-4.31

Table 5.9 compares the worst case scenario solution of the FLMDO for increasing footprints of uncertainty. Increasing linguistic uncertainty reduces the performance of the optimization, with the objective function value becoming worse. These results compare to the interval uncertainty analytical target cascading method shown in Table 5.10. In the ATC, the uncertainty is on the design variable themselves. For example, a 5% uncertainty means that the independent variables can be shifted up to 5%. Although, type-2 fuzzy logic systems were developed to model epistemic uncertainty and ATC MDOs model aleatory uncertainty, the two sets of results provide a first level of the FLMDO's validation until other MDO methods are developed to handle non-probabilistic uncertainty.

Table 5.9: T2-FLMDO results worst case scenario results.

	x_1	x_2	F_1	F_2	F_3	F_s
FOU=0 (Type-1)	-0.65	-0.86	1.16	-4.27	-1.27	-4.37
FOU=2.5%	-0.66	-0.85	1.16	-4.27	-1.24	-4.35
FOU=5%	-0.66	-0.84	1.14	-4.24	-1.24	-4.34
FOU=10%	-0.67	-0.82	1.12	-4.20	-1.23	-4.31

Table 5.10: ATC results worst case scenario results.

	x_1	x_2	F_1	F_2	F_3	F_s
ATC 0%	-0.80	-1.00	1.64	-5.31	-0.88	-4.55
ATC 2.5%	-0.83	-1.03	1.75	-5.51	-0.79	-4.55
ATC 5%	-0.87	-1.06	1.88	-5.74	-0.68	-4.54

5.7 Closing Remarks

This chapter presented a main contribution of this thesis: an interval type-2 fuzzy logic system which provides a first step to handle epistemic uncertainty inherent to early-stage design. This method uses type-2 fuzzy logic systems to formalize vague linguistic preferences and human expertise into an optimization model. Thus human knowledge can be used formally in early-stage design before detailed models of a vessel are developed. The use of type-2 sets does not assume a precise fuzzy membership function definition, thus models the uncertainty associated with linguistic reasoning and communication used by humans, providing a more realistic model on human expertise compared to type-1 fuzzy systems. As a proof of concept, this chapter illustrated the method on a generic mathematical problem which sought the minimization of three functions. The method showed results comparable to the well-established optimization methods. However, it's potential will be better realized in the following chapter, which illustrates the method on the early-stage design of a planing craft. Detailed ship models often do not exist in early-stage design, and when they do, their mathematical structure limits their ability to adequately represent the system.

The following chapter will show the modeling capabilities of fuzzy logic in early-stage design.

CHAPTER VI

Interval Type-2 Fuzzy Logic Multidisciplinary Optimization for the Early-Stage Design of Planing Craft

6.1 Introduction

This chapter demonstrates the novel interval type-2 fuzzy logic multidisciplinary optimization on a naval architecture application which involves the design of a planing craft. It will show how three sources of uncertainty inherent to the early-stage ship designs can be handled by the method developed in this thesis and presented in Chapter V. The first is the uncertainty associated with limited knowledge about a product when only preliminary models of a vessel's general arrangements exist. A method is demonstrated which leverages human mental models of typical arrangements layout to estimate ranges of ship parameters when only limited information exists. The second is the uncertainty associated with the validity bounds of analysis tools. Regression models are typically valid for given ranges of variable values given by crisp numbers, but it is more realistic to set bounds with fuzzy numbers. The third type of uncertainty originates from the FLMDO's use of linguistic variables to characterize the designer's mental model and preferences. This type of uncertainty was also discussed in the generic mathematical example of Chapter V. Since linguistic variables are vague (i.e. can have a range of meanings depending on the person and the context) it is crucial to consider this last source of uncertainty through type-2 fuzzy sets. These three uncertainties are first discussed in the following subsections, the planing craft problem formulation is given in Section 6.2, which is solved using the T2-FLMDO in Section 6.3. The optimization results are given in Section 6.8. Additionally, Section 6.7 demonstrates how rules of thumb can be formally incorporated into optimization models, and demonstrates a method of aggregating planing accelerations, non-planing accelerations, and slamming into a single seakeeping performance metric

6.1.1 Limited Engineering Models

In early-stage ship design, an inverse relationship exists between design knowledge and design freedom. Initially, knowledge is low and freedom is high. Knowledge increases as design decisions are made, but this reduces freedom on future decisions. The vessel's arrangements, for example, are affected by the shape of the hull form, the choice of the powering and propulsion system, and many other decisions which may evolve over the course of the design process. As a result, it is impossible to determine the exact location of the ship's longitudinal center of gravity until the later stages of the design process. In Section 6.4, this thesis will demonstrate a novel method which leverage mental models of typical arrangement layouts, developed from professional experience to estimate the most likely location of the vessel's LCG.

Estimating the LCG of a planing craft and its subsequent trim, is a critical component of design as it severely affect the craft's drag and seakeeping performance, and it is also the source of several dynamic instabilities which make open water operations less safe. Some risks associated with a poorly placed LCG include the heave and pitch oscillatory instability known as porpoising, which occurs when operating above a critical trim angle and is triggered earlier for vessels with low deadrise, according to empirical studies conducted by Day and Haag (1952). Another instability known as the bow drop occurs when the trim is too low and the forward section of the craft becomes immersed, resulting in localized low pressure which may pull the bow down (Lewandowski, 2004). Other instabilities, such as chine tripping, have been shown to be affected by LCG location and hull form as well (Blount and Codega, 1992).

6.1.2 Model Validity Bounds

Many ship design models are built from empirical data sets, making them only valid for a range of variable values representative of that set. The Savitsky method, for example, is valid for trim values between two and fifteen degrees. However, it is unreasonable to believe that these bounds are sharp, and that the method is valid for trim values just below fifteen degrees but invalid for a trim just above fifteen. Instead, it is likely that the method's validity gradually reduces as trim values increase above the fifteen degree constraint. Given this assumption, the constraint on the model's validity can be said to be vague, or approximate. Fuzzy sets are meant to handle approximate numbers through membership functions which have grade one near the number of interest, and decreasing grade for values farther away from it. A fuzzy set representing the Savitsky method's validity with respect to trim could be modeled by a parallelogram with validity grade one between three and fourteen, and decreasing validity grade outside of these values.

6.1.3 Linguistic Variable Uncertainty

As described throughout this thesis, the fuzzy logic multidisciplinary optimization uses linguistic preference variables to incorporate designer experience into models and optimization processes. Although this allows greater modeling flexibility compared to methods which use only numerical variables, by design it does render the optimization inherently vague. The same word can be used to describe a range of numerical values, and this range changes from person to person and based on context. The sensation of cold is not only dependent on temperature; it is also dependent on a person's energy level, and whether they are moving or still for example. Cold tolerances also vary from person to person. A novel contribution of this thesis is the development of an interval type-2 fuzzy logic multidisciplinary optimization, which better captures linguistic uncertainty than its type-1 fuzzy logic counterpart does. This uncertainty is handled by a fuzzy definition of membership functions, or fuzzy-fuzzy set. Interval type-2 sets use the union of multiple, equally valid membership function definitions to better capture the vague meaning of words used to describe a system.

6.2 Planing Craft Problem Formulation

This section presents the planing craft problem used to illustrate the interval type-2 fuzzy logic multidisciplinary optimization developed in this thesis. In early-stage ship design before the lines of a hull form have been developed, the hydrodynamic properties of a planing craft can be simply described by the vessel's length, beam, deadrise, longitudinal center of gravity, displacement, speed, and significant wave height. These are the quantities considered for this problem, with length, beam, and deadrise used as independent variables, and the others defined as parameters. The optimization goal is to maximize the craft's seakeeping and resistance performance which are defined using fuzzy logic systems that capture the designer's expertise about what constitutes good performance for these two disciplines. The problem formulation is given in Equation 6.1. Note that the tildes in constraint g_3 represent fuzzy numbers.

$$
\begin{aligned}
\underset{L,B,\beta}{\text{maximize:}} \quad & Seakeeping\ Performance \\
& Resistance\ Performance \\
\text{subject to:} \quad & g_1 : 2.5 \leq \frac{L}{B} \leq 7.0 \\
& g_2 : 0.6 \leq C_v \leq 13.0 \\
& g_3 : \tilde{2} \leq \tau \leq \tilde{15} \\
& g_4 : 5 \leq L \leq 30 \\
& g_5 : 2 \leq B \leq 6 \\
& g_6 : 5 \leq \beta \leq 30 \\
\text{given:} \quad & LCG,\ \Delta,\ V_k,\ h_{1/3} \\
\text{where:} \quad & L = \text{length [m]} \\
& B = \text{beam [m]} \\
& \beta = \text{deadrise angle } [^\circ] \\
& \tau = \text{trim } [^\circ] \\
& LCG = \text{longitudinal center of gravity [\% of L from aft]} \\
& \Delta = \text{displacement [kg]} \\
& V_k = \text{speed [kn]} \\
& h_{1/3} = \text{significant wave height [m]} \\
& C_v = \text{speed coefficient } [\tilde{\ }]
\end{aligned}
\tag{6.1}
$$

6.3 Planing Craft Interval Type-2 Fuzzy Logic Multidisciplinary Optimization Method

The interval type-2 fuzzy logic multidisciplinary optimization used to solve the planing craft problem is composed of two parts: the arrangements generator, and the planing craft FLMDO. Both of these use a fuzzy logic system to leverage human knowledge and build engineering models. The arrangements generator calculates an estimate of the vessel's lcg parameter based on the designer's mental model of a typical arrangement layout, which is then passed to the FLMDO to optimize the vessel with respect to seakeeping and resistance.

6.4 Planing Craft Arrangements Generator

By estimating the LCG early, designers have greater insight on the vessel's expected performance, giving them additional foresight and design flexibility to mitigate the risk of poor operational performance. However, few methods exist to estimate it before a detailed general arrangements layout is produced. Regression equations exist which calculate LCG as a function of principal dimensions and ship type (e.g. pleasure craft, work boat, military vessel) but even within a ship type, a large variety of layouts can exist, giving rise to large uncertainties in the weight distribution of a vessel. Within the pleasure craft category, a fishing boat usually has an outboard motor and has heavy fish tanks located in the stern or bow. However, an open-bow day cruiser tends to have an inboard motor, open space for seating, and sometimes a small kitchenette. In this thesis, the arrangements generator uses the human designer's knowledge to estimate possible weight distributions based on his or her knowledge of the most likely arrangement layout for the vessel to be designed.

The arrangements generator uses a fuzzy logic system to determine the most likely location of the longitudinal center of gravity based on the designer's vague mental model of the vessel's general arrangements. This process is illustrated in this section for a two compartment vessel, divided as the machinery and the passenger compartment. If the compartment is assumed to be a standard rectangle, and no additional information about it is available, then the location of its LCG can only be assumed to be in the middle of the box (i.e. 50% of the compartment length). A human designer with a mental model of the compartment's arrangements; however, can easily provide a rough estimate of the LCG location. In this work, these beliefs are captured by type-2 fuzzy input membership functions. The membership function assigns an LCG value a likelihood of being correct based on the designer's mental model of the vessel's typical arrangement layout. In Figure 6.1, representing the engine room compartment model, the LCG is likely to be at midpoint of the compartment (i.e. the center of the box). However, a human designer knows that the engine is typically located near the rear of the compartment, so assigns the LCG a higher likelihood of being near the rear of the compartment.

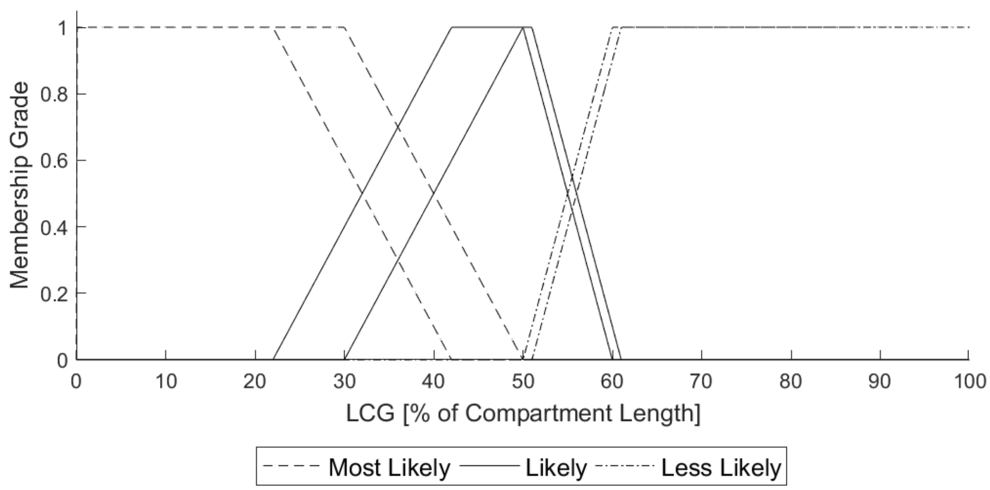


Figure 6.1: Designer belief membership function for machienery arrangements.

Uncertainties associated with the selected propulsion system limit his or her ability to know the exact definition of the "likely" and "most likely" membership functions, and where the transition between the two occurs though. This uncertainty is captured by the parallel dashed and solid lines to the left of Figure 6.1, which provide a measure of LCG dispersion associated with the designer's model and the linguistic terms he or she uses. The bottom lines represent the lower membership functions, or lower bound on the uncertainty, and the top lines represent the upper membership functions, or upper bound on the uncertainty. The large size of a water-jet propulsion system pushes the LCG forward compared to a propeller, for example, but the propulsion system might not be known until later in the design process, hence the large uncertainty on the engine room LCG model around of 40%.

The passenger compartment weight distribution input fuzzy membership functions are built the same way. Here, the weight is believed to be concentrated towards the rear of the space, as shown in Figure 6.2. The narrow beam at the bow limits the placement of passenger spaces at the front of the space, and large vertical accelerations at the bow are a source of passenger discomfort, pushing weight aft. The footprint of uncertainty is large at the rear of the compartment (left of the figure) and small at the front, since the bow is more space constrained than the vessel's midships. This constraint limits uncertainty on space arrangements at the bow.

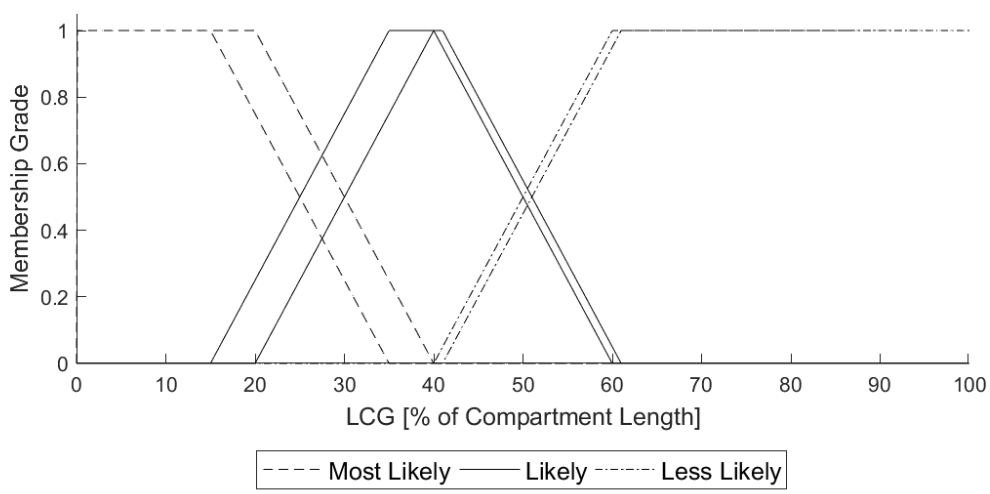


Figure 6.2: Designer belief membership function for passenger arrangements.

Beliefs about each compartment's LCG are aggregated into an overall estimate of the planing craft's LCG through a fuzzy logic rule bank (Table 6.1). In the two compartment vessel, the overall LCG is just the average of the two compartments' LCGs, and the likelihood that this overall LCG is correct is a function of the two

Table 6.1: Arrangements generator rule bank.

		LCG 1 Belief		
		Least Likely	**Likely**	**Most Likely**
LCG 2 Belief	**Least Likely**	Low Likelihood	Medium-Low Likelihood	Medium Likelihood
	Likely	Medium-Low Likelihood	Medium Likelihood	Medium-High Likelihood
	Most Likely	Medium Likelihood	Medium-High Likelihood	High Likelihood

compartments' LCG location likelihood. If compartment 1 is most likely to have its LCG at 30% (most likely membership grade of one) and compartment 2 is likely to have its LCG at 50% (likely membership grade of one), the vessel's overall LCG at 40% of the vessel length has a medium-high likelihood of being the true vessel LCG. If both compartments are most likely to have their LCG at a given value, the average of these two values has a high likelihood of being the true vessel LCG.

The type-2 fuzzy membership sets corresponding to the consequents of the rule bank are given in Figure 6.3. They map the linguistic rule consequents to crisp value scores used by the type-reduction and defuzzification. Higher likelihoods have higher scores. A high likelihood is mapped to a numeric value between 8.9 and 9.1, and a medium likelihood maps to a numeric value between 5.3 and 5.5 for example.

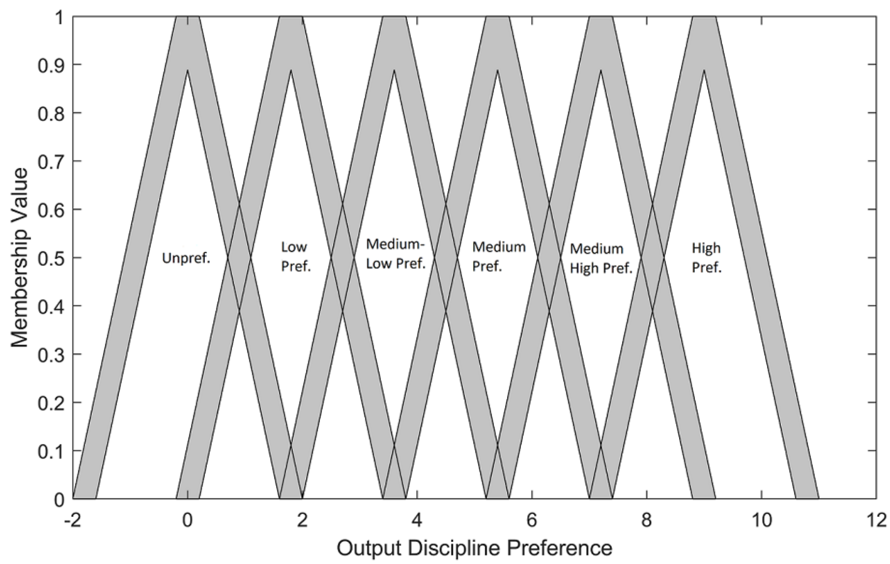


Figure 6.3: Type-2 fuzzy logic output membership fuzzy sets.

The fuzzy logic system used by the arrangements generator produces a model of the vessel's LCG location by aggregating the designer's beliefs about several compartments' arrangements layout. The output of this model is shown in Figure 6.4.

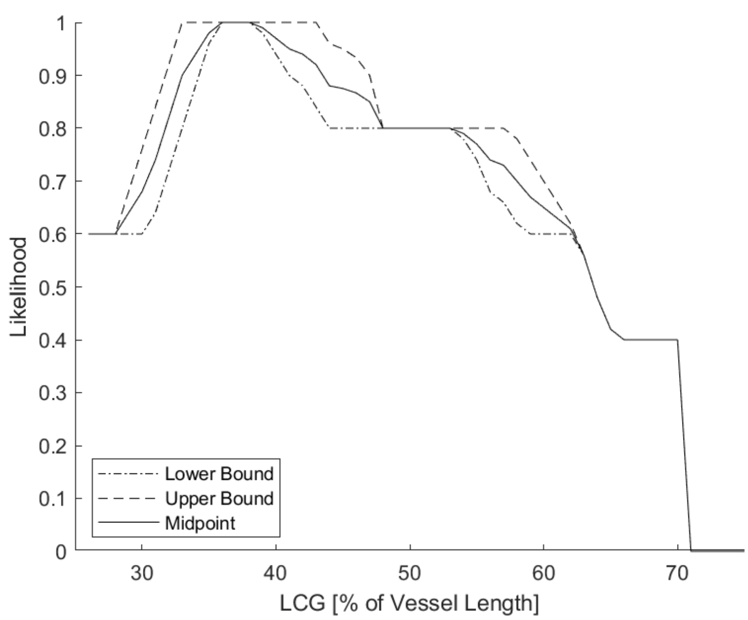


Figure 6.4: Planing craft lcg location fuzzy likelihood curve.

The vessel LCG parameter selected for seakeeping and resistance analysis can be chosen through multiple ways. If only a single lcg value is needed, the average of the LCGs with the highest likelihood can be chosen. In Figure 6.4, the mean of the highest likelihood LCG values is 0.37 for the lower bound and 0.38 for the upper bound. A range of likely LCG values can also be selected by taking the alpha-cut of the set, which is the collection of all values in the universe of discourse of the fuzzy set whose membership degree is greater than or equal to a chosen number alpha (Equation 6.2). A 0.8 alpha-cut on the upper bound of the fuzzy set represented in Figure 6.4 gives $[\mu]_{0.8} = [0.31, 0.47]$.

$$[\mu]_\alpha = \{x \in X \mid \mu(x) \geq \alpha\} \tag{6.2}$$

6.5 Planing Craft Optimizer

The most likely LCG calculated by the arrangements generator is passed as a parameter to the planing craft T2-FLMDO, which optimizes the craft for resistance and seakeeping performance with respect to length, beam, and deadrise. The structure of the FLMDO is laid out in Figure 6.5. Problem inputs and parameters are passed to the resistance and seakeeping discipline analyzers, which calculate performance indicators for their respective discipline. Three metrics are used for resistance: drag, trim, and speed coefficient. Three metrics are used for seakeeping: planing vertical accelerations, non-planing seakeeping performance, and slamming. A design's values with respect to these metrics are then mapped to fuzzy design space preferences through the human expertise mapping step. This mapping is done through two interval type-2 fuzzy logic systems: one for resistance and detailed in Section 6.6, and one for seakeeping which is detailed in Section 6.7. The preferences of both disciplines are aggregated into a system level preference using the fusion logic, from which the optimal design is found.

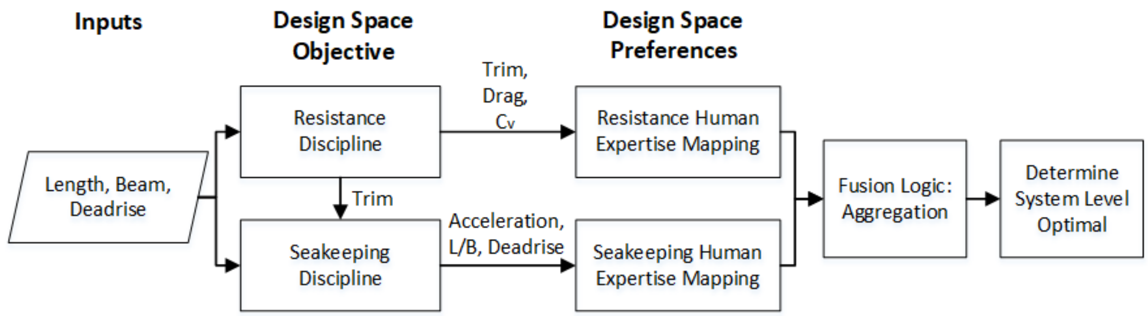


Figure 6.5: Type-2 fuzzy logic MDO structure for the planing craft problem.

6.6 Planing Craft Resistance Discipline

6.6.1 Resistance Discipline Analysis

The purpose of the resistance discipline is to minimize vessel drag subject to trim and speed coefficient constraints. The discipline analysis step of the FLMDO begins by calculating the equilibrium trim, speed coefficient, and drag of the vessel by summing the pressure drag, frictional drag, and dynamic lift acting on the vessel, according to the empirical formulae provided by the Savitsky method. This method assumes a prismatic hullform; which is a hull whose deadrise angle is constant along the vessel length. This assumed hull form and the principle characteristics terminology is given in Figure 6.6.

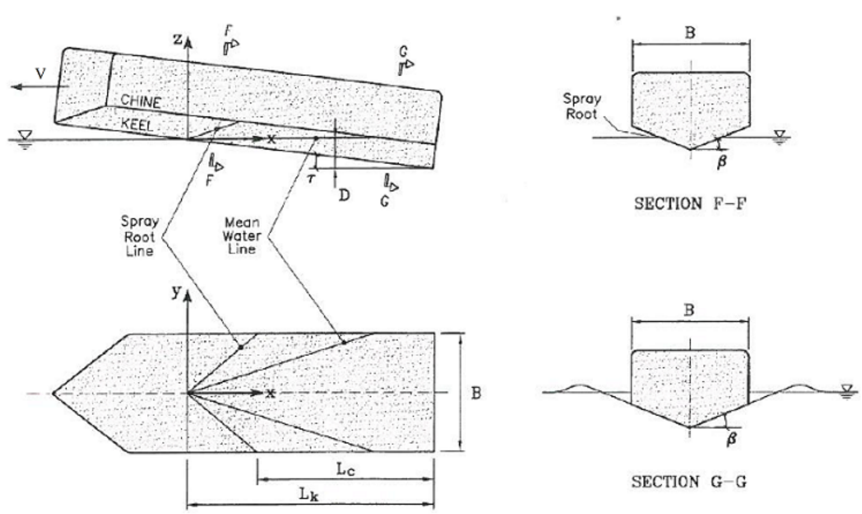


Figure 6.6: Prismatic planing hullform (Savitsky, 1964).

The trim of the vessel in degrees is given by Savitsky (1964) in Equation 6.3 and determined empirically according to Korvin-Kroukovsky et al. (1948). It is valid for $0.6 \leq C_v \leq 13.0$, $2^\circ \leq \tau \leq 15^\circ$, and $\lambda \leq 4$.

$$\tau = \left(\frac{C_{L_o}}{0.012\lambda^{0.5} + \frac{0.0055\lambda^{2.5}}{C_v^2}} \right)^{1/1.1} \tag{6.3}$$

Where the mean wetted length to beam ratio λ is the average of the chine and keel wetted length to beam ratio

$$\lambda = \frac{L_k + L_c}{2B} \tag{6.4}$$

Where the coefficient of speed C_v, or beam Froude number, of the vessel is

$$C_v = \frac{V}{\sqrt{(gB)}} \tag{6.5}$$

And the lift coefficient is given in Equations 6.6 and 6.7 where C_{L_o} is the lift coefficient of a flat plate with equal trim, speed coefficient, and mean wetted length as a surface with deadrise β and lift coefficient C_{L_β}

$$C_{L_o} = C_{L_\beta} + 0.0065\,\beta\, {C_{L_o}}^{0.6} \tag{6.6}$$

$$C_{L_\beta} = \frac{\Delta\, g}{0.5\rho V^2 B^2} \tag{6.7}$$

The total resistance of the planing craft, in Newtons, is given in Equation 6.8 according to Savitsky (1964), where C_f is the Schoenherr turbulent friction coefficient.

$$R = \Delta\, g \tan(\tau) + \frac{\rho V^2 C_f \lambda B^2}{2\cos(\beta)\cos(\tau)} \tag{6.8}$$

6.6.2 Resistance Discipline Human Expertise Mapping

The resistance, trim, and speed coefficient values of a design are then input into the human expertise mapping FLS step. They are first converted to linguistic variables through fuzzy membership functions which convey the designer's preference for specific ranges of these values (Figures 6.7-6.9). The resistance membership function is broken down into five linguistic variables, or fuzzy sets, between R^{min} which is the smallest drag of the population of evaluated designs, and R^{max} which is the drag value of the design which is optimal with respect to vertical accelerations. Designs with smaller resistance are given higher preference. The high resistance preference function has a membership value of one at R^{min}, and linearly goes to zero as drag increases to $R^{1/4}$, which is the quarter point between R^* and R^{max}. The medium preference membership grade increase from zero to one at the same rate between R^* and $R^{1/4}$. Designs with resistance values between the $R^{1/4}$ and R^{mid} are classified as partially good and partially medium. When the resistance value is between the midpoint and the three quarter point, the design is somewhere between the medium and acceptable preference. Any design with a resistance value greater than R^{max} is given a low preference with membership one.

However, the exact transition between the high, good, medium, acceptable, and low resistance preference is not known exactly, which models uncertainty in linguistic terms used to describe designs and the difficulty comparing two designs with similar performance, which can have equal membership grade in a type-2 fuzzy set, unlike for a type-1 set. This uncertainty is captured by the parallel lines in the membership function, which provide the bounds on possible membership grades of a numeric value into a set.

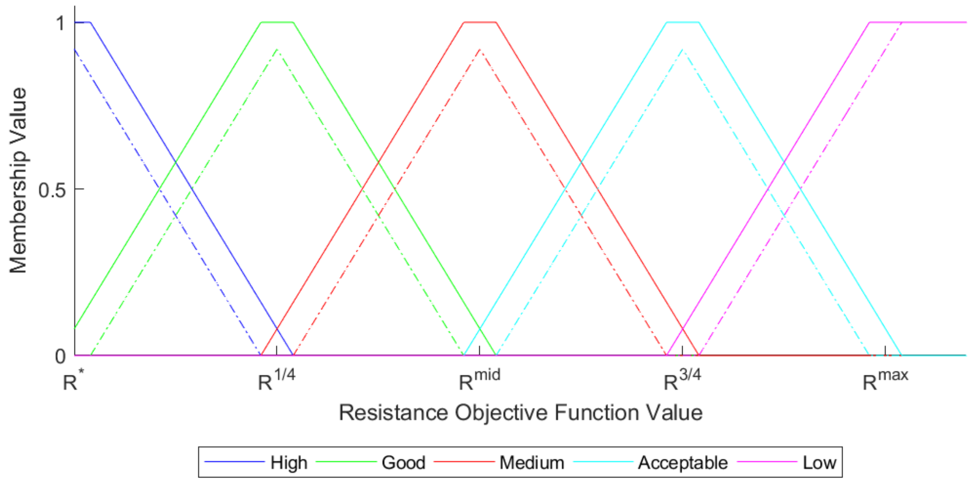


Figure 6.7: Planing craft resistance preference membership function.

A design's trim and speed coefficient are used to penalize designs that are outside of the Savitsky method bounds, which is only valid for trim values between 2° and 15° and for speed coefficients between 0.6 and 13. However, these bounds likely are not crisp. It is hard to believe that the Savitsky method is valid for designs with trim of 2.01° and invalid for designs with trim of 1.99°. The uncertainty associated with the fuzziness of model bounds is modeled by the membership function for trim shown in Figure 6.8. The transition regions between 1.8° and 3°, and between 14° and 15.2° have partial membership in the preferred and unpreferred set. Designs with trim below 1.8° and above 15.2° are unpreferred with membership grade one. Between 3° and 14°, where the Savitsky method is believed to be most accurate, the preferred membership grade equals one.

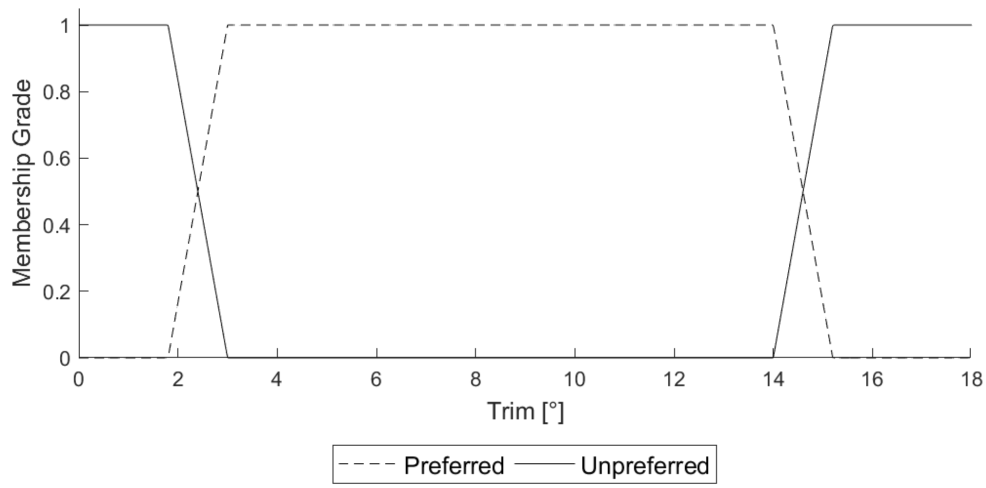


Figure 6.8: Planing craft trim preference membership function.

The speed coefficient membership function in Figure 6.9 follows the same methodology, but assumes that the bounds on the constraint can be described by precise numbers, as constraints are normally described. Thus the membership functions are crisp. Within model bounds the preferred speed coefficient membership grade is equal to one, and it equals to zero anywhere else.

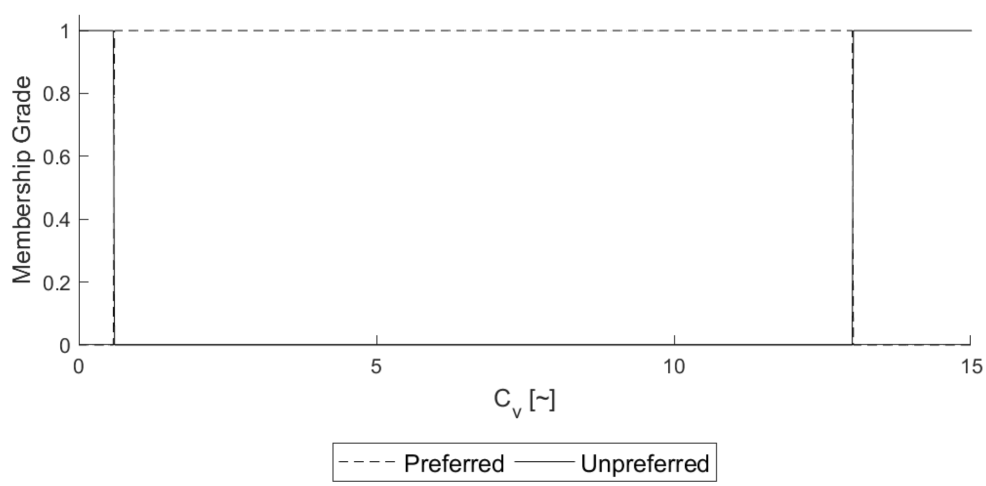


Figure 6.9: Planing craft speed coefficient preference membership function.

The three membership functions, or designer preferences about drag, trim, and speed coefficient values are aggregated into an overall resistance discipline preference using an interval type-2 fuzzy logic system. The rule bank shown in Table 6.2 assigns the output preference according to the drag preference subject to the trim and C_v constraints. If both constraints are unviolated (i.e preferred) with membership degree one, the output linguistic preference equals the drag input linguistic preference. If the constraints are violated, the rule bank consequent is unpreferred. The lower and upper firing level of each rule consequent is calculated through product inference given in equation 6.9, where $\underline{\mu}_{resistance}$ and $\bar{\mu}_{resistance}$ denote the lower and upper membership function of the resistance fuzzy set. Trim and speed coefficient preferences use type-1 fuzzy sets, so have a single membership function definition given by μ_{trim} and μ_{c_v}.

$$\underline{f}^S = \underline{\mu}_{resistance} \cdot \mu_{trim} \cdot \mu_{c_v} \tag{6.9a}$$

$$\bar{f}^S = \bar{\mu}_{resistance} \cdot \mu_{trim} \cdot \mu_{c_v} \tag{6.9b}$$

Table 6.2: Resistance discipline rule bank.

		Drag Preference				
		Low	**Acceptable**	**Medium**	**Good**	**High**
Trim or C_v Preference	**Preferred**	Low Preference	Medium-Low Preference	Medium Preference	Medium-High Preference	High Preference
	Unpreferred	Unpreferred	Unpreferred	Unpreferred	Unpreferred	Unpreferred

The output membership sets, or rule consequents, are identical to the ones used for the arrangements generator in Figure 6.3. All activated rules are combined using the center-of-sets type reduction method, which provides two type-1 fuzzy sets representative of the lower and upper resistance discipline preference bounds for that design variable value.

6.7 Planing Craft Seakeeping Discipline

Planing craft were originally designed to operate in calm-water conditions were seakeeping performance was not a consideration. However, the high speed of such craft have made them attractive to military operations, and subsequent research sought to improve their performance in moderate waves to allow them to operate in open ocean. Their seakeeping performance in high speed planing conditions has been extensively studied both empirically and numerically (Razola et al., 2016; Kim et al., 2008; Akers et al., 1999). Although they are intended for these operational profiles, the performance of planing craft in semi-planing and displacement conditions must also be considered, especially in rough seas where high speed operations might not be achievable (Savitsky, 1985). The seakeeping discipline level FLMDO presented in the next few sections aims to optimize the planing craft for these mixed operational conditions.

6.7.1 Seakeeping Discipline Analysis

In this case study, the seakeeping discipline aims to minimize the craft's vertical accelerations in planing conditions, improve non-planing seakeeping performance, and limit slamming. Each of these performance characteristics are estimated in the seakeeping discipline analysis step. Vertical accelerations in planing conditions are given by Savitsky (1985) in Equation 6.10.

$$a = \frac{1}{c_\Delta}\left[\frac{0.0104\tau}{4}\left(\frac{h_{1/3}}{B} + 0.084\right)\left(\frac{5}{3} - \frac{\beta}{20}\right)\left(\frac{V_k}{\sqrt{L}}\right)^2\left(\frac{L}{B}\right)\right] \tag{6.10}$$

where c_Δ is the beam loading coefficient given by Equation 6.11

$$c_\Delta = \frac{\Delta}{\rho\, g\, B^3} \tag{6.11}$$

Non-planing seakeeping performance is estimated by the length to beam ratio L/B

of the craft (Savitsky, 1985), and slamming performance is estimated by its deadrise angle. For each evaluated design, the discipline analyzer calculates these values and passes them to the human expertise mapping step.

6.7.2 Seakeeping Discipline Human Expertise Mapping

Here, numerical values for planing vertical acceleration, length to beam ratio, and deadrise, are mapped to linguistic variables which capture the designer's preference for these values. The mapping for planing vertical accelerations is given in Figure 6.10. The membership function is broken down into five linguistic preference variables, or fuzzy sets, between a^{min} which is the smallest acceleration of the evaluated designs, and a^{max} which is the acceleration value of the design which is optimal with respect to vertical resistance. Designs with smaller accelerations are given higher preference. The high preference function has a membership value of one at a^{min}, and linearly goes to zero as acceleration increases to $a^{1/4}$. The medium preference membership grade increase from zero to one at the same rate between a^* and $a^{1/4}$. Designs with acceleration values between the one quarter point and the midpoint are classified as partially good and partially medium. When the acceleration value is between the midpoint and the three quarter point, the design is somewhere between a medium and acceptable preference. Any design with an acceleration value greater than a^{max} is given a low preference with membership one.

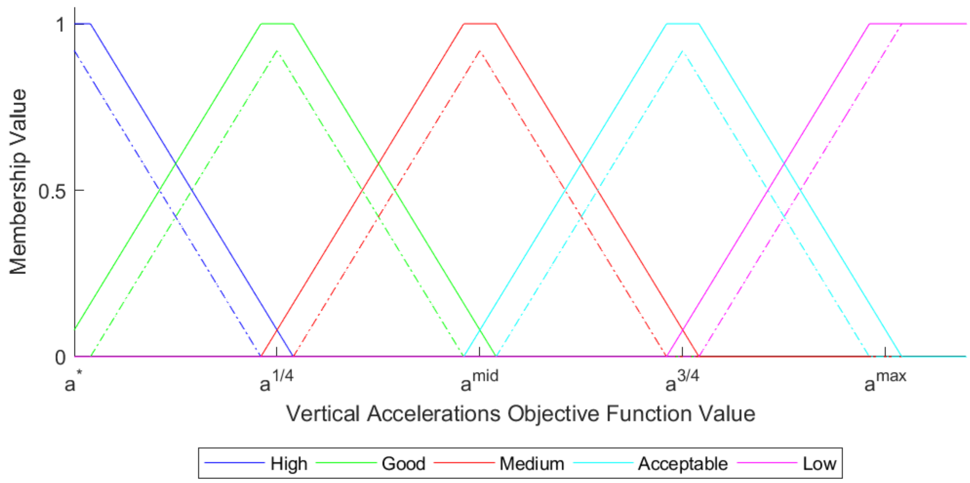


Figure 6.10: Planing craft vertical acceleration preference membership function.

The second performance metric used to evaluate a design's seakeeping is the length to beam ratio, which is used as a surrogate for non-planing seakeeping performance. Vessels with length to beam ratio greater than five reduce the impact of acceleration in semi-planing conditions (Savitsky, 1985) so vessels with larger length to beam ratios are given higher preference with respect to non-planing seakeeping performance. The membership functions encoding these preferences are illustrated in Figure 6.11. Length to beam ratios between 2.5 and 3.5 have low preference. Between 3 and 5, the vessels have medium preference, and above 4.5 the vessels have high L/B preference. The exact transition point between low and medium preference, and between medium and high preference is not known exactly and can occur anywhere within the footprint of uncertainty of the type-2 fuzzy sets.

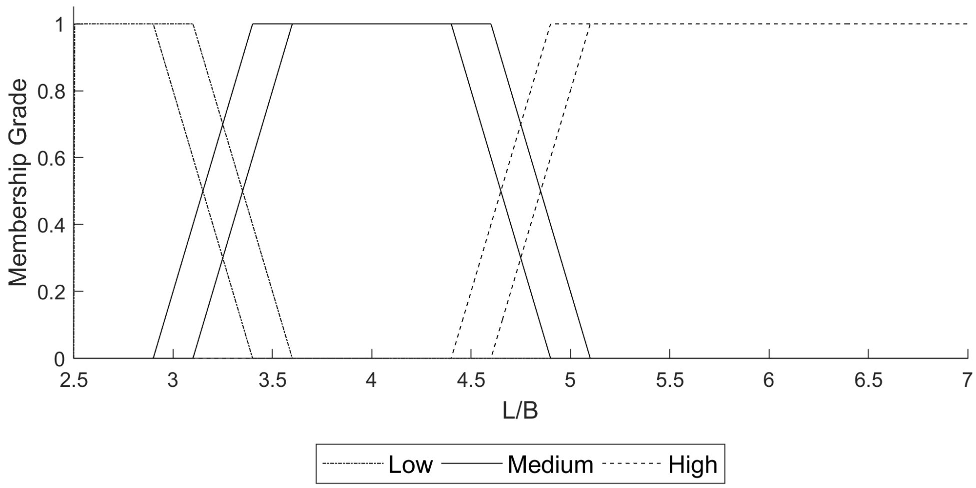


Figure 6.11: Planing craft length to beam ratio preference membership function.

The final set of membership functions used to evaluate a design is the deadrise preference membership function. Deadrise is used as a surrogate for slamming performance. A moderate aft deadrise around 15° is assumed to provide the best compromise between minimizing slamming and minimizing resistance. These preferences are shown in Figure 6.12. Only two linguistic variables are used: high preference for deadrise angles around 15°, and medium preference for values farther away from this value. This choice was made to limit the influence of deadrise on the seakeeping discipline optimizer since it is assumed to be less important than planing and non-planing accelerations.

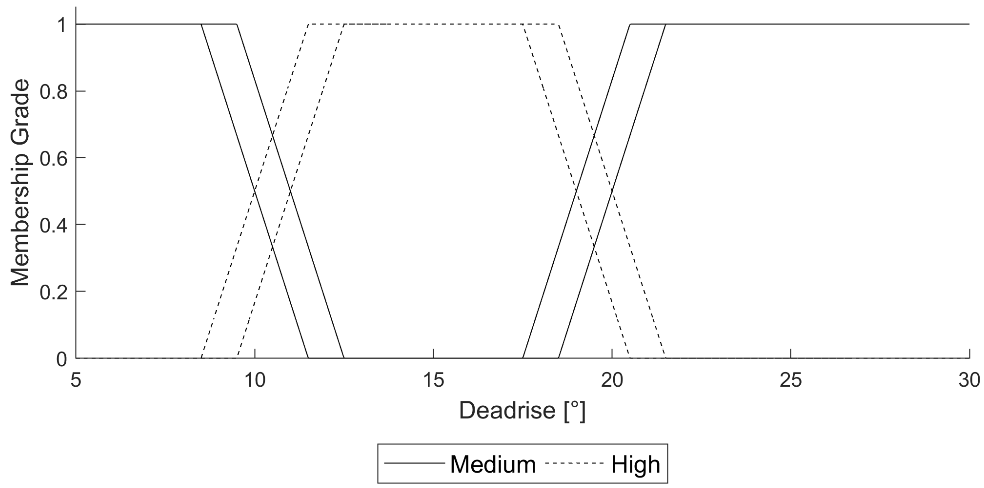


Figure 6.12: Planing craft deadrise preference membership function.

All three seakeeping performance preferences, encoded by their membership functions, are aggregated using the rule bank shown in Tables 6.3 and 6.4. Both the rule banks and membership functions allow designers to easily incorporate their expertise into the optimization model. Table 6.3 shows that the designer places more importance on vertical accelerations than length to beam ratio, since a high acceleration preference gives a high output preference for high and medium length to beam ratio preference. Another way of saying this is that the designer's output preference is more highly correlated with vertical accelerations than with L/B. This rationale can be explained by the primary mission of planing craft to operate at high speed, and because planing accelerations are much more violent on the crew than non-planing accelerations.

The seakeeping fuzzy logic system uses minimum inference and the center-of-sets type reduction to aggregate all three performance metrics into an overall seakeeping score for each design. This score is then fused with the resistance score through minimum or mean fusion logic to establish the system level performance of each design.

Table 6.3: Seakeeping discipline rule bank for high deadrise preference.

L/B Preference		Acceleration Preference: Low	Acceptable	Medium	Good	High
	Low	Low Preference	Medium-Low Preference	Medium-Low Preference	Medium Preference	Medium-High Preference
	Medium	Low Preference	Medium-Low Preference	Medium Preference	Medium-High Preference	High Preference
	High	Low Preference	Medium-Low Preference	Medium Preference	Medium-High Preference	High Preference

Table 6.4: Seakeeping discipline rule bank for medium deadrise preference.

L/B Preference		Acceleration Preference: Low	Acceptable	Medium	Good	High
	Low	Unpreferred	Unpreferred	Medium-Low Preference	Medium Preference	Medium-High Preference
	Medium	Unpreferred	Low Preference	Medium-Low Preference	Medium Preference	Medium-High Preference
	High	Low Preference	Medium-Low Preference	Medium Preference	Medium-High Preference	High Preference

6.8 Planing Craft Optimization Results

Sections 6.4 through 6.7 showed how type-2 fuzzy logic could encode designer expertise to enhance engineering models. Designer mental models are translated into membership functions and rule banks which map numeric design variable values to design preferences with respect to multiple performance criteria. These enhanced models are then used as part of an optimization process to find variable values which best match the designer's preferences.

Section 6.4 showed how a designer's mental model of the planing craft's arrangements could be used to estimate its longitudinal center of gravity. This estimate took the form of the LCG likelihood curve of Figure 6.4. From this curve, either a single LCG can be selected, or a range of likely LCG values can be determined by taking the alpha-cut of the curve. The fuzzy logic multidisciplinary optimization is demonstrated for both these cases in Sections 6.8.1 and 6.8.2.

6.8.1 Unique Longitudinal Center of Gravity

This section assumes that a unique LCG value can be determined from the LCG likelihood curve of Figure 6.4. This value is defined as the mean of the maximum likelihood LCGs. This section examines two sets of results for the planing craft optimization problem given in Equation 6.1. First, a Pareto analysis of the seakeeping and resistance discipline is examined. Then, a single design is selected which provides a compromise between seakeeping and resistance performance.

6.8.1.1 Pareto Analysis

In multidisciplinary optimization, the objectives of different disciplines are often conflicting. In such cases, the performance increase of one discipline comes at a performance cost to at least one of the other disciplines, and the optimization results form a Pareto front. A design variable value $\mathbf{x}^* \in \Omega$ is said to be Pareto optimal (Coello et al., 2004) if for every $\mathbf{x} \in \Omega$ and $i = 1, 2, ..., k$ disciplines

$$f_i(\mathbf{x}) = f_i(\mathbf{x}^*) \quad \forall\, i \in I \tag{6.12}$$

Or there is at least one $i \in I$ such that

$$f_i(\mathbf{x}) > f_i(\mathbf{x}^*) \tag{6.13}$$

The two disciplines used in this study have conflicting objectives. Equation 6.8 for planing craft resistance shows an increase in drag for increasing beam and deadrise. Equation 6.10 for planing craft vertical accelerations shows a decrease in accelerations for increasing beam and deadrise. Figure 6.13 shows the Pareto front of the planing craft seakeeping and resistance performance built with the fuzzy logic multidisciplinary optimizer. It clearly shows the conflicting objectives of the two disciplines, with resistance preference increasing as seakeeping preference decreases, and vice-versa. The uncertainty associated with interval type-2 membership functions is propagated to the Pareto front and visible through the spread on preferences between the lower and upper bound. The min of mean best and mean of mean best system preferences are two system level preferences calculated using the T2-FLMDO fusion logic step and show a solution which balances both disciplines.

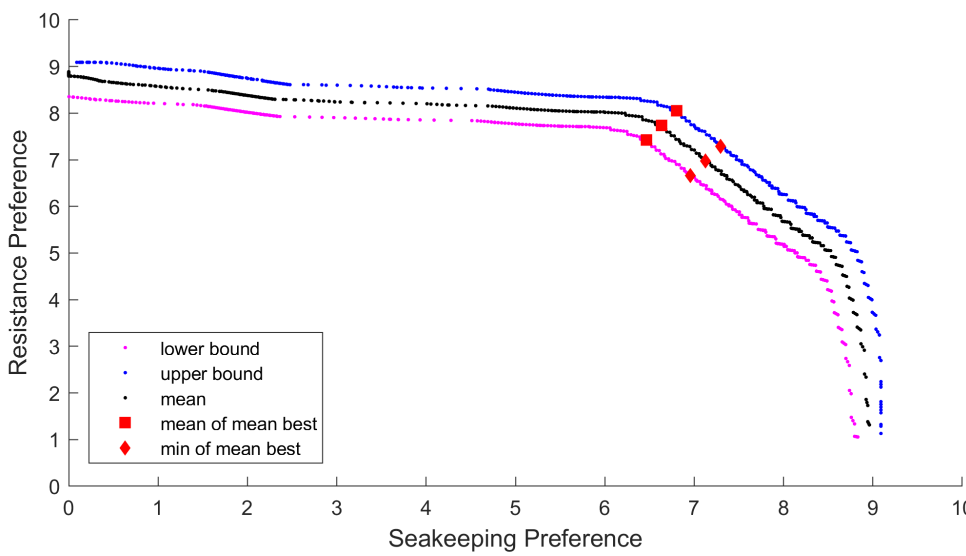


Figure 6.13: Planing craft T2-FLMDO Pareto front.

Several points along the mean Pareto front where sampled and are listed in Table 6.5 (design one exhibits the highest resistance preference and lowest seakeeping preference, and design seven exhibits the lowest resistance preference and highest seakeeping preference). These results show the tradeoff between seakeeping and resistance performance. The vessels with better resistance characteristics tend to be shorter with smaller deadrise angles. At high speed, friction drag becomes a large component of drag so vessels with large wetted lengths exhibit high resistance. Additionally, although deep V-hulls exhibit better seakeeping characteristics, they require more power to plane, and thus exhibit worst resistance characteristics.

Table 6.5: Pareto optimal design points for V = 45 kn, Δ = 30 t and $h_{1/3}$ = 1 m.

	1	**2**	**3**	**4**	**5**	**6**	**7**
Length[m]	10.5	16.0	18.4	22.4	23.0	27.6	28.9
Beam[m]	4.20	3.60	3.40	3.20	3.30	4.10	4.30
Deadrise [°]	5.0	5.0	5.0	14.0	29.0	30.0	30.0
LCG [L from aft]	0.36	0.36	0.36	0.36	0.36	0.36	0.36
Resistance Pref.	8.83	8.29	8.10	7.08	5.07	2.41	1.32
Seakeeping Pref.	0.00	2.54	5.05	7.06	8.46	8.91	8.96
Drag [kN]	36.2	38.6	39.3	43.4	51.4	61.0	64.5
Planing Acc. [g]	1.95	1.45	1.27	0.91	0.66	0.58	0.56

6.8.1.2 Fusion Logic

The Pareto analysis shows the tradeoff between the disciplines' performance. However, the designer must select a single design which provides a compromise between the performances of both disciplines. This is the role of the fusion logic step of the T2-FLMDO. Two fusion logic methods are used and compared in this case study. The first is the fuzzy intersection, or min fusion logic. The second is the mean fusion logic. Table 6.6 shows the optimal design found with each of these method. Both fusion methods, plotted in red in Figure 6.13, provide comparable results which have a good compromise between resistance and seakeeping, as shown by their approximately equal resistance and seakeeping fuzzy preferences. The min fusion gives a slight advantage to seakeeping, while the mean fusion returns a design with a slight advantage on resistance.

Table 6.6: System level optimal design for V = 45 kn, Δ = 30 t and $h_{1/3}$ = 1 m.

	Min Fusion	**Mean Fusion**
Length[m]	22.4	22.4
Beam[m]	3.20	3.20
Deadrise [°]	14.0	6.0
LCG [**L from aft**]	0.36	0.36
Resistance Pref.	7.08	7.73
Seakeeping Pref.	7.06	6.64
System Pref.	7.06	7.19
Drag [**kN**]	43.4	40.8
Planing Acc. [**g**]	0.91	0.99

To better understand the influence of length and beam on the design space, the system level preferences found using min fusion and mean fusion are plotted in Figures 6.14 and 6.15. These show a high preference for vessel lengths between 15 and 22 meters. These lengths provide the best compromise between resistance and seakeeping performance. Vessels shorter than 15 meters tend to favor the resistance at the expense of seakeeping, and vessels longer than 22 meters tend to favor the seakeeping at the expense of resistance, as was shown in Table 6.5. Thus, they have a lower system level preference.

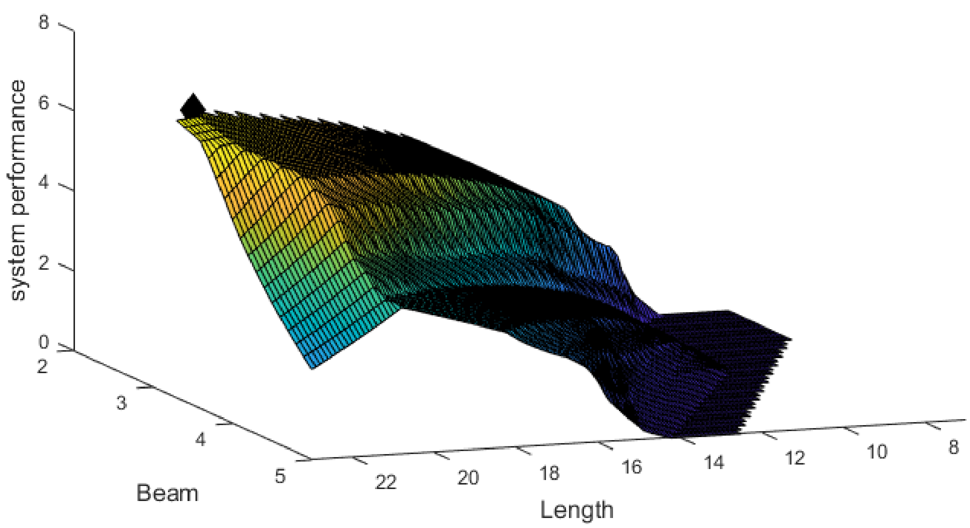


Figure 6.14: Minimum fusion fuzzy logic system preference surface for a 14° deadrise.

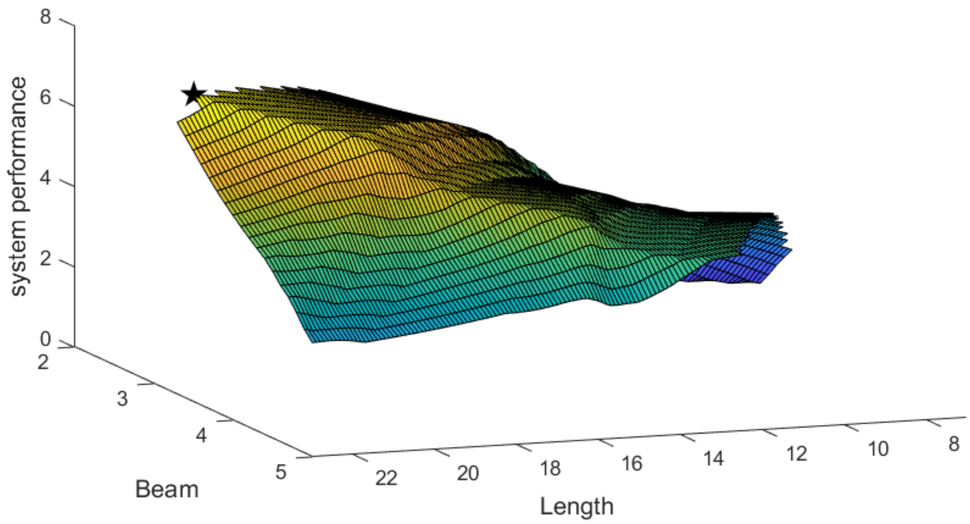


Figure 6.15: Mean fusion fuzzy logic system preference surface for a 6° deadrise.

6.8.2 Longitudinal Center of Gravity Uncertainty

When the longitudinal center of gravity of the planing craft can only be estimated within a set range, a robust design optimization problem is used. It's goal is to find the design with the best possible worst case performance subject to LCG uncertainty. This goal is stated mathematically in Equation 6.14 where F^s denotes the system level preference returned by the fuzzy logic minimum fusion method. Constraint g_1 is handled by a penalty function, and constraints g_2 and g_3 are handled by the fuzzy logic membership functions for trim and speed coefficient given in Section 6.6.

$$
\begin{aligned}
&\underset{L,B,\beta}{\text{maximize}} \quad \underset{lcg \in [lcg_1, lcg_2]}{\text{minimize}} && F^s \\
&\text{subject to:} && g_1 : 2.5 \leq \frac{L}{B} \leq 7.0 \\
& && g_2 : 0.6 \leq C_v \leq 13.0 \\
& && g_3 : \tilde{2} \leq \tau \leq \tilde{15} \\
&\text{given:} && \Delta,\ V_k,\ h_{1/3}
\end{aligned}
\tag{6.14}
$$

The robust planing craft problem is solved with a nested particle swarm optimization algorithm where the inner loop seeks to minimize the system level preference F^s with respect to lcg and the outer loop maximizes the system level preference with respect to length, beam and deadrise. A particle swarm optimization (Kennedy and Eberhart, 1995) is a heuristic algorithm which leverages the interactions between many simple individuals to find the solution to a difficult problem. It was chosen for its ability to operate on non-linear, non-differentiable, and non-convex problems. The use of a fuzzy logic system to formulate the planing craft model rules out any classical optimization method since FLS use linguistic variables rather than mathematical functions to model a system.

The optimization results for the best and worst case LCG scenario are given in Table 6.7 for an LCG range betwen 31% and 47% of the vessel length from the transom, a displacement of 30 tons, a significant wave height of 1 meter, and speeds between 35 and 50 knots.

Table 6.7: Planing craft robust optimization results with $\Delta = 30$ t and $h_{1/3} = 1$ m.

	V = 35 kn		**V = 40 kn**		**V = 45 kn**		**V = 50 kn**	
	Worst	**Best**	**Worst**	**Best**	**Worst**	**Best**	**Worst**	**Best**
Length[m]	27.0	20.2	25.1	22.0	23.5	21.0	19.8	18.3
Beam[m]	3.86	3.77	3.58	3.53	3.36	3.09	2.84	2.99
Deadrise [°]	11.6	11.3	14.5	10.5	17.2	14.7	22.7	14.6
LCG [**L from aft**]	0.32	0.42	0.34	0.38	0.35	0.37	0.37	0.42
Resistance Pref.	6.46	7.61	6.82	7.30	6.57	7.13	6.61	7.14
Seakeeping Pref.	7.65	7.61	7.50	7.30	7.55	7.13	7.61	7.14
System Pref.	6.46	7.61	6.82	7.30	6.57	7.13	6.61	7.14
Drag [**kN**]	37.6	37.2	41.7	40.1	45.7	43.3	48.7	45.8
Planing Acc. [**g**]	0.62	0.64	0.74	0.78	0.84	0.92	0.93	1.03

For every speed case, the uncertainty on the longitudinal center of gravity negatively affected the resistance discipline by increasing the vessel's drag, and resulted in a small increase in the seakeeping discipline preference. The worst case scenario optimization pushed the LCG aft and resulted in longer boats with deeper vee hulls, which are characteristic of vessels with better ride quality but which also have higher drag.

6.9 Closing Remarks

This chapter presented several major contributions of this thesis. A new MDO method was presented which incorporates human knowledge about a system directly into a multidisciplinary optimization model through an interval type-2 fuzzy logic system. This method remedies several limitations associated with traditional mathematical multidisciplinary optimization methods. The first is the limited knowledge about a vessel which is common in early-stage design before detailed models of a craft have been developed, and makes it hard to define a precise model to use within the optimization. The second is the reliance of traditional optimization on mathematical models which make it difficult to incorporate linguistic knowledge into the model. The T2-FLMDO used in this chapter operates on linguistic variables, making it easier to incorporate human knowledge directly into models and enriching them considerably. This allows the designer's prior experience to be better leveraged to fill the gap left by limited models, and allows the designer to have more flexibility when defining the optimization model. However, human expertise models are uncertain themselves, due to the vagueness of linguistic terms humans use to reason and communicate. This uncertainty is handled in this thesis by interval type-2 fuzzy logic which does not assume a precise definition of its membership function.

The interval type-2 FLMDO was demonstrated on the early-stage design of a planing craft with respect to seakeeping and resistance. The vessel's LCG is first estimated using the arrangements generator, a novel contribution of this work which leverages the designer's mental models of the vessel's typical arrangement layout to calculate the most likely LCG. This value is then passed to the resistance and seakeeping discipline which optimize the design by varying length, beam, and deadrise. The seakeeping discipline evaluates a design based on planing accelerations, non-planing seakeeping, and slamming to provide an aggregate seakeeping performance metric of the vessel over all operating regimes. Planing accelerations are evaluated using the Savitsky method whereas non-planing seakeeping and slamming are evaluated using surrogate rules of thumb which model the designer's experience. The resistance discipline evaluates designs based on drag subject to trim and speed coefficient constraints. The system level optimal is calculated by the fusion logic step of the T2-FLMDO, and provides a design which compromises seakeeping and resistance performance.

CHAPTER VII

Conclusion

7.1 Research Problem Review

This dissertation presents a new method which expands the scope of optimization models to make them more applicable to ship design, particularly in the early stages of the process. While successfully applied to many industries with well-defined problems, like structural design, optimization has been less successful in ship design. Ship design is often said to be more of an art than a science. It is plagued by subjective requirements and rising complexity, requiring a lot of input from experienced human designers to find an adequate solution.

However, the trend in naval design has been to gradually remove the designer, replacing them with complex, high-fidelity, linked optimization codes. While these codes are extremely useful in later stages of design to refine a small set of alternatives, their suitability to early-stage design is limited. Three reasons why have motivated the research problems this dissertation resolves:

1. By definition, a mathematical model is a reduction which only covers some of the aspects of the real-world problem it is trying to solve. This is exacerbated as problems become more complex.

2. Varying levels of fidelity in the sub-analyses of a multidisciplinary optimization

may lead the overall solution to be falsely driven by the optimizer with the lowest fidelity.

3. In early-stage design, before choosing a ship concept for further investigation, limited amounts of information are available since no model of the vessel exists. Thus mathematical optimization cannot progress.

7.2 Dissertation Contributions

This dissertation provided a solution to these problems by developing a method of incorporating designer expertise directly into the optimization model through a type-2 fuzzy logic multidisciplinary optimization, which operates on linguistic variables in addition to numerical ones. While mathematics are very good at describing precise concepts and have undeniably been very beneficial to engineering, a new modeling framework is required to incorporate human expertise into models through linguistic variables. Often times in early-stage design, it is difficult to develop precise models of a vessel, and more qualitative descriptions must be used. Type-2 fuzzy logic appropriately handles the imprecision and range of meanings associated with linguistic variables used for these descriptions. Additionally, performance requirements are often given in the form of linguistic specification, requiring them to be translated before being used in optimization. Ship design is as much of an art as it is a science, so the author believes that an optimization method which operates on vague linguistic variable and can incorporate vague mental models will better handle the uncertainty inherent to early-stage design. Mental models can be encoded using words, pictures, or even basic math. This makes them very flexible and adaptable, allowing them to describe many concepts with no clear answer and for which complete information is not available.

Several specific contributions which solve the identified research problem have been laid out throughout this dissertation and are summarized below.

1. Formulated a method of incorporating ill-structured human knowledge into optimization models which properly accounts for linguistic variable vagueness through type-2 fuzzy logic.

2. Developed a multidisciplinary optimization method inspired from hierarchical type-2 fuzzy logic controllers which allows contribution 1 to be applied to multidisciplinary systems.

3. Developed fusion logic methods which provide flexible aggregation of discipline-level preferences into a system-level preference, and can account for subjective requirements.

4. Developed a method of leveraging human experience to estimate plausible values of ship parameters when only limited models or information exists.

5. Developed a method of differentiating Pareto optimal points by incorporating subjective preferences to objective mathematical models.

7.3 Future Work

Several opportunities for future work have been identified to expand upon the contributions of this dissertation.

1. The first and most direct extension of this work is to apply general type-2 fuzzy logic to the current interval type-2 fuzzy logic (IT2FL) multidisciplinary optimization. IT2FL assumes that all membership functions in the footprint of uncertainty are equally weighted; however, if different distribution of weights can be justified by the designer's mental models, then a GT2FL MDO may provide a more accurate model of the linguistic uncertainty surrounding the designer's mental model.

2. The second area of future work suggest developing a type-2 fuzzy logic multi-disciplinary optimizer which does not rely on human users to develop the fuzzy membership functions. These functions could be learned automatically through analysis of significant ship data to suggest design preferences to the user based on objective data. While this method removes the designer from the model creation stage, its use of linguistic variables allows a better user-tool interaction than traditional mathematical tools.

3. The last suggested area of future work is to expand the current fuzzy logic method to the problem of multi-criteria decision making to help with the evaluation and selection of discreet design alternatives. These methods currently use weights and ranks to classify alternatives with respect to performance criteria, and the fuzzy logic method presented in this thesis could aid decision-makers with evaluating alternatives in a more natural way. Additionally, a lot more information is captured in a fuzzy logic system than a single weight value, making the preference model richer.

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