Water Resources Research

Supporting Information for

Application of the Beer-Lambert model to attenuation of photosynthetically-active radiation in a shallow, eutrophic lake

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Contents of this file

Tables S1 and S2

Additional Supporting Information (Files uploaded separately)

Dataset S1 (filename: SI1_Master_Dataset.xlsx)

Introduction

Supporting information includes one dataset (uploaded separately) and two tables. Dataset S1 includes observations and modeled data used for analyses. Tables S1 and S2 contain model fitted parameters and summary statistics. Each data file is a .xlsx-formatted spreadsheet with variable names as headers.

Most data in Dataset S1 were collected between 2012 and 2016, during sampling trips to monitoring stations in western Lake Erie between April and November of each year. Page 1 of Dataset S1 includes data associated with each field monitoring station in western Lake Erie. These data include latitude, longitude, and water depth for each station. Page 2 of Dataset 1 includes field observations of Total Suspended Solids (TSS), Volatile Suspended Solids (VSS), Dissolved Organic Carbon (DOC), Chromophoric Dissolved Organic Matter (CDOM), and Chlorophyll-a (CHL) concentration from each sample used

in multiple regression model development. Additionally, page 2 of Dataset 1 includes calculations for Nonvolatile Suspended Solids (NVSS) and $k_{d(PAR)}$ for all samples. Each NVSS value was calculated as TSS-VSS, while $k_{d(PAR)}$ data for each sample in the table was calculated using the Beer-Lambert model for exponential decay of light with depth.

Data in Tables S1 and S2 were calculated via the stepwise regression development and 10-fold cross-validation of multiple regression models using water quality parameters to predict $k_{d(PAR)}$. These tables include fitted multiple regression model parameters in addition to R^2, adjusted R^2, and RMSE from the 10-fold cross-validation procedure. Table S1 also contains AIC and RSS statistics from the associated stepwise regression analysis.

Table S1: Model fitted parameters and summary statistics for multiple regression models predicting $k_{d(PAR)}$ from water quality variables, showing relative predictive ability of the models, as depicted in Figure 7

		10-Fold Cross-Validation Statistics			Stepwise Regression Statistics	
Parameters	Fitted Model	R^2	Adjusted R^2	RMSE	AIC	RSS
CHL	$k_{d(PAR)}=0.005(CHL)+1.468+\varepsilon$	0.036	0.030	1.193	100.67	321.79
VSS	$k_{d(PAR)}=0.086(VSS)+1.258+\varepsilon$	0.095	0.090	1.150	88.34	302.27
DOC	$k_{d(PAR)}=0.333(DOC)+0.353+\varepsilon$	0.133	0.128	1.140	79.57	289.12
DOC + CHL*	$k_{d(PAR)}=0.311(DOC)+0.002(CHL)+0.353+\varepsilon$	0.141	0.131	1.135	80.16	289.12
CDOM	$k_{d(PAR)}=0.262(CDOM)+1.253+\varepsilon$	0.137	0.133	1.172	78.55	287.62
CDOM + CHL	$k_{d(PAR)}=0.256(CDOM)+0.004(CHL)+1.103+\varepsilon$	0.162	0.152	1.151	75.33	280.09
VSS + DOC	$k_{d(PAR)}=0.067(VSS)+0.287(DOC)+0.216+\varepsilon$	0.188	0.179	1.086	68.99	271.23
VSS + CDOM	$k_{d(PAR)}=0.086(VSS)+0.267(CDOM)+0.838+\varepsilon$	0.232	0.224	1.086	58.08	256.61
NVSS	$k_{d(PAR)}=0.085(NVSS)+0.794+\varepsilon$	0.811	0.810	0.573	-223.24	62.16
TSS + CHL*	$k_{d(PAR)}=0.079(TSS)-0.0002(CHL)+0.473+\varepsilon$	0.844	0.843	0.527	-258.29	51.51
TSS	$k_{d(PAR)}=0.079(TSS)+0.468+\varepsilon$	0.844	0.843	0.522	-260.26	51.51
NVSS + CDOM	$k_{d(PAR)}=0.081(NVSS)+0.137(CDOM)+0.617+\varepsilon$	0.846	0.845	0.506	-261.75	50.607
VSS + NVSS	$k_{d(PAR)}=0.056(VSS)+0.083(NVSS)+0.545+\varepsilon$	0.852	0.850	0.511	-267.93	49.042
NVSS + CHL	$k_{d(PAR)}=0.086(NVSS)+0.007(CHL)+0.537+\varepsilon$	0.863	0.861	0.493	-283.55	45.31
NVSS + DOC	$k_{d(PAR)}=0.082(NVSS)+0.231(DOC)-0.0885+\varepsilon$	0.875	0.873	0.454	-301.65	41.329
TSS + CDOM + CHL*	$k_{d(PAR)}=0.076(TSS)+0.146(CDOM)-0.0005(CHL)+0.303+\varepsilon$	0.884	0.882	0.447	-314.74	38.28
TSS + CDOM	$k_{d(PAR)}=0.076(TSS)+0.145(CDOM)+0.290+\varepsilon$	0.884	0.883	0.440	-316.25	38.376
TSS + DOC	$k_{d(PAR)}=0.076(TSS)+0.185(DOC)-0.214+\varepsilon$	0.884	0.883	0.446	-316.01	38.422
TSS + DOC + CHL	$k_{d(PAR)}=0.077(TSS)+0.201(DOC)-0.002(CHL)-0.220+\varepsilon$	0.888	0.886	0.439	-320.71	37.137
VSS + NVSS + CDOM	$k_{d(PAR)}=0.058(VSS)+0.079(NVSS)+0.141(CDOM)+0.357+\varepsilon$	0.889	0.887	0.432	-322.44	36.813
NVSS + CDOM + CHL	$k_{d(PAR)}=0.083(NVSS)+0.121(CDOM)+0.006(CHL)+0.401+\varepsilon$	0.890	0.888	0.432	-324.4	36.448
VSS + NVSS + DOC	$k_{d(PAR)}=0.044(VSS)+0.081(NVSS)+0.202(DOC)-0.171+\varepsilon$	0.898	0.896	0.414	-339.51	33.757
NVSS + DOC + CHL	$k_{d(PAR)}=0.083(NVSS)+0.184(DOC)+0.005(CHL)-0.096+\varepsilon$	0.899	0.897	0.415	-342.02	33.331

Note: * indicates models for which the fitted value for CHL is not significantly different from zero

Table S2: Model fitted parameters and summary statistics for multiple regression models predicting $k_{d(PAR)}$ from water depth and antecedent averaged wind speed variables, showing relative predictive ability of the models corresponding to the average wind speed correlations depicted in Figure 9

		10-Fold Cross-Validation Statistics		
Parameters	Fitted Model	R^2	Adjusted R^2	RMSE
Water Depth + 2-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.226(Depth)-0.007(WS2)+2.599+ε	0.334	0.322	0.761
Water Depth + 4-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.226(Depth)-0.008(WS4)+2.599+ε	0.334	0.323	0.758
Water Depth + 6-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.226(Depth)-0.001(WS6)+2.566+ε	0.334	0.323	0.747
Water Depth + 8-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.226(Depth)+0.017(WS8)+2.479+ε	0.337	0.325	0.754
Water Depth + 12-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.227(Depth)+0.038(WS12)+2.374+ε	0.340	0.329	0.754
Water Depth + 18-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.226(Depth)+0.035(WS18)+2.387+ε	0.339	0.328	0.755
Water Depth + 24-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.227(Depth)+0.035(WS24)+2.384+ε	0.340	0.328	0.758
Water Depth + 48-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.231(Depth)+0.087(WS48)+2.107+ε	0.355	0.344	0.744
Water Depth + 72-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.232(Depth)+0.109(WS72)+1.992+ε	0.363	0.352	0.739
Water Depth + 96-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.235(Depth)+0.168(WS96)+1.695+ε	0.380	0.369	0.734
Water Depth + 120-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.237(Depth)+0.210(WS120)+1.487+ε	0.388	0.377	0.735
Water Depth + 144-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.237(Depth)+0.220(WS144)+1.434+ε	0.381	0.370	0.730
Water Depth + 168-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.238(Depth)+0.235(WS168)+1.361+ε	0.374	0.363	0.737
Water Depth + 192-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.237(Depth)+0.238(WS192)+1.348+ε	0.370	0.359	0.738
Water Depth + 216-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.237(Depth)+0.249(WS216)+1.294+ε	0.373	0.362	0.737
Water Depth + 240-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.237(Depth)+0.235(WS240)+1.363+ε	0.368	0.357	0.745
Water Depth + 264-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.237(Depth)+0.230(WS264)+1.396+ε	0.364	0.353	0.747
Water Depth + 288-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.236(Depth)+0.201(WS288)+1.547+ε	0.357	0.346	0.743
Water Depth + 312-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.234(Depth)+0.172(WS312)+1.693+ε	0.350	0.339	0.755
Water Depth + 336-hour Avg. Wind Speed	$k_{d(PAR)}$=-0.235(Depth)+0.173(WS336)+1.688+ε	0.349	0.338	0.746