Supplemental figures \& tables

## Supplemental Figures



Figure S1: (a): Correlation matrix of the 9 amino acid phenotypes from METSIM study (b): Co-heritability matrix of the same phenotypes as estimated from PHENIX.
(c): Scaled co-heritability matrix: The elements in the matrix as shown in (b) were divided by the maximum diagonal element.
Ala: Alanine, Gln: Glutamine, Gly: Glycine, His: Histine, Ile: Isoleucine, Leu: Leucine, Phe: Phenylalanine, Tyr: Tyrosine, Val: Valine.


Figure S2: Power for Multi-SKAT tests when phenotypes have compound symmetric correlation structures. Empirical power for minPhen, Hom, Het, PhC, PC-Sel, minP plotted against the number of phenotypes associated with the gene of interest with a total of 5 phenotypes under consideration. Upper row shows the results for $\rho=0.3$ and lower row for $\rho=0.7$. Left column shows results with SKAT kernel $\Sigma_{G}$, and right columns shows results with Burden kernel. $80 \% / 20 \%$ of the causal variants were trait-increasing/trait-decreasing variants.


Figure S3: Power for Multi-SKAT tests when phenotypes have clustered correlation structures. Empirical powers for minPhen, Hom, Het, PhC, PC-Sel, minP are plotted under different levels of association with a total of 6 phenotypes and with clustered correlation structures. Middle column shows the empirical powers for different combinations of phenotypes associated with SKAT kernel $\Sigma_{G}$; the rightmost column shows the corresponding results with Burden kernel; left column shows the corresponding correlation matrices for the phenotypes. The associated phenotypes are indicated in red cross marks across the correlation matrices. $80 \% / 20 \%$ of the causal variants were trait-increasing/trait-decreasing variants.


Figure S4: Power for Multi-SKAT by combining tests with $\Sigma_{P}$ as Hom, Het, PhC, PC-Sel and $\Sigma_{G}$ as SKAT and Burden when phenotypes have compound symmetric correlation structures. Empirical powers for minP-Burden, minP-SKAT and $\operatorname{minP}_{\text {com }}$ are plotted against the number of phenotypes associated with the gene of interest with a total of 5 phenotypes under consideration and $50 \%$ of the variants in the region are causal. Upper row shows the results for $\rho=0.3$ and lower row for $\rho=0.7$. Left column shows results when all the causal variants were traitincreasing variants, and right column shows results when $80 \% / 20 \%$ of the causal variants were trait-increasing/traitdecreasing variants.


Figure S5: QQplot of the p-values of Multi-SKAT omnibus tests without kinship adjustment for the METSIM data $(\mathrm{n}=8545)$. For the ease of viewing, any associations with p-values $<10^{-12}$ have been collapsed to $10^{-12}$


Figure S6: Minor allele frequency (MAF) spectrum for the variants simulated (left panel) and genotyped in METSIM exome array data (right panel). Upper panel shows the MAFs for variants having MAF $<5 \%$. Lower panel zooms in into a region with variants having MAF $<1 \%$


Figure S7: Computation time of Multi-SKAT and existing methods with unrelated individuals and 10 phenotypes. (a) Estimated computation time for different sample sizes when the number of variant was 20. (b) Estimated computation time for different number of variants when the sample size was 5000 . Each dot represents the average from 100 datasets.

## Supplemental Tables

Table S1: Computation time of MultiSKAT tests to analyze a dataset with 5000 individuals, 20 variants and 10 phenotypes. Analysis was done on a 2.80 GHz Intel Xeon CPU.

|  | Method | CPU sec |
| :---: | :---: | :---: |
| Independent samples (Without kinship adjustment) | ```Multi-SKAT (given }\mp@subsup{\Sigma}{P}{}\mathrm{ and }\mp@subsup{\Sigma}{G}{}\mathrm{ ) minP minP``` | 0.014 secs <br> 2.133 secs <br> 3.971 secs |
| Related samples (With kinship adjustment) | $\begin{aligned} & \text { Multi-SKAT (given } \Sigma_{P} \text { and } \Sigma_{G} \text { ) } \\ & \text { minP } \\ & \operatorname{minP}_{\text {com }} \end{aligned}$ | $\begin{gathered} 2.845 \text { secs } \\ 6.961 \mathrm{secs} \\ 10.349 \mathrm{secs} \end{gathered}$ |

Table S2: Backward elimination results for the top 5 genes in Table 2. For a particular gene, each row indicates the phenotype eliminated and the p-value produced correspondingly. The last row indicates the remaining phenotype after backward elimination has been performed. This is the phenotype that drives the signal of association for the particular gene.

| Phenotype deleted | p-values | GLDC <br> Phenotype deleted | p-values | HAL <br> Phenotype deleted | p-values | DHODH <br> Phenotype deleted | p-values | PAH <br> Phenotype deleted | p-values | MED1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Ile | $2.8 \times 10^{-73}$ | Phe | $9.0 \times 10^{-12}$ | Gln | $3.5 \times 10^{-07}$ | Leu | $6.3 \times 10^{-09}$ | Gly | $9.3 \times 10^{-07}$ |
|  | Val | $8.2 \times 10^{-74}$ | Ile | $3.2 \times 10^{-12}$ | His | $2.4 \times 10^{-08}$ | Ile | $3.2 \times 10^{-10}$ | Ala | $3.3 \times 10^{-06}$ |
|  | Leu | $1.3 \times 10^{-74}$ | Leu | $8.6 \times 10^{-14}$ | Ile | $5.7 \times 10^{-09}$ | Ala | $2.5 \times 10^{-10}$ | His | $2.5 \times 10^{-06}$ |
|  | Tyr | $3.3 \times 10^{-74}$ | Ala | $1.0 \times 10^{-14}$ | Phe | $2.3 \times 10^{-09}$ | His | $7.6 \times 10^{-11}$ | Gln | $1.1 \times 10^{-05}$ |
|  | His | $4.8 \times 10^{-74}$ | Val | $2.6 \times 10^{-14}$ | Val | $2.9 \times 10^{-10}$ | Val | $2.6 \times 10^{-09}$ | Ile | $2.3 \times 10^{-05}$ |
|  | Phe | $2.4 \times 10^{-76}$ | Gly | $2.7 \times 10^{-13}$ | Tyr | $1.2 \times 10^{-10}$ | Gln | $6.3 \times 10^{-07}$ | Phe | $1.5 \times 10^{-05}$ |
|  | Ala | $3.2 \times 10^{-71}$ | Gln | $1.2 \times 10^{-11}$ | Gly | $6.7 \times 10^{-11}$ | Gly | $1.4 \times 10^{-06}$ | Val | $1.0 \times 10^{-03}$ |
|  | Gln | $7.4 \times 10^{-64}$ | Tyr | $3.3 \times 10^{-09}$ | Leu | $1.5 \times 10^{-07}$ | Tyr | $6.8 \times 10^{-05}$ | Leu | $4.9 \times 10^{-02}$ |
| Remaining phenotype | Gly |  | His |  | Ala |  | Phe |  | Tyr |  |

Table S3: Smallest 10 p -values and corresponding genes obtained by $\mathrm{PhC}\left(\Sigma_{G}=S K A T\right)$, GAMuT (Projection and Linear kernel) and MSKAT (Q and Q' statistic). Each method produces the same set of top 10 genes, differing slightly by p-values. The tests were performed on unrelated individuals only $(\mathrm{n}=7213)$.

| $\begin{gathered} \mathrm{PhC} \\ \left(\Sigma_{G}=S K A T\right) \end{gathered}$ |  | GAMuT <br> (Projection) |  | $\begin{aligned} & \text { MSKAT } \\ & (\mathrm{Q}) \end{aligned}$ |  | GAMuT <br> (Linear) |  | $\begin{gathered} \text { MSKAT } \\ \left(Q^{\prime}\right) \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gene | p-value | Gene | p-value | Gene | p-value | Gene | p-value | Gene | p-value |
| $G L D C$ | $8.1 \times 10^{-54}$ | $G L D C$ | 0 | $G L D C$ | $8.9 \times 10^{-54}$ | $G L D C$ | $6.2 \times 10^{-15}$ | $G L D C$ | $6.1 \times 10^{-15}$ |
| DHODH | $1.9 \times 10^{-06}$ | DHODH | $2.4 \times 10^{-06}$ | DHODH | $2.1 \times 10^{-06}$ | METTL4 | $3.0 \times 10^{-05}$ | METTL4 | $3.0 \times 10^{-05}$ |
| PAH | $9.9 \times 10^{-06}$ | PAH | $1.0 \times 10^{-05}$ | PAH | $9.9 \times 10^{-06}$ | ASB10 | $4.9 \times 10^{-05}$ | ASB10 | $4.8 \times 10^{-05}$ |
| ALDH1L1 | $6.0 \times 10^{-05}$ | DHODH | $6.1 \times 10^{-05}$ | ALDH1L1 | $5.9 \times 10^{-05}$ | MEOX1 | $6.4 \times 10^{-05}$ | MEOX1 | $6.4 \times 10^{-05}$ |
| HAL | $9.5 \times 10^{-05}$ | HAL | $9.6 \times 10^{-05}$ | HAL | $9.5 \times 10^{-05}$ | PAH | $1.1 \times 10^{-04}$ | PAH | $1.3 \times 10^{-04}$ |
| BCAT2 | $6.2 \times 10^{-04}$ | BCAT2 | $6.3 \times 10^{-04}$ | BCAT2 | $6.1 \times 10^{-04}$ | ABCC8 | $2.5 \times 10^{-04}$ | ABCC8 | $2.5 \times 10^{-04}$ |
| STK33 | $6.7 \times 10^{-04}$ | STK33 | $6.7 \times 10^{-04}$ | STK33 | $6.8 \times 10^{-04}$ | OLFML2A | $2.8 \times 10^{-04}$ | OLFML2A | $2.8 \times 10^{-04}$ |
| TBC1D4 | $1.7 \times 10^{-04}$ | TBC1D4 | $1.6 \times 10^{-04}$ | TBC1D4 | $1.6 \times 10^{-04}$ | OGG1 | $3.9 \times 10^{-04}$ | OGG1 | $4.0 \times 10^{-04}$ |
| ABCC8 | $2.1 \times 10^{-04}$ | ABCC8 | $2.3 \times 10^{-04}$ | ABCC8 | $2.3 \times 10^{-04}$ | CPT1C | $4.7 \times 10^{-04}$ | CPT1C | $4.5 \times 10^{-04}$ |
| MED1 | $1.7 \times 10^{-03}$ | MED1 | $1.7 \times 10^{-03}$ | MED1 | $1.7 \times 10^{-03}$ | DHODH | $2.2 \times 10^{-03}$ | DHODH | $2.2 \times 10^{-03}$ |

Table S4: Functions and clinical implications for the significant and suggestive genes

| Gene | Function | Clinical Implication / Associations |
| :---: | :---: | :---: |
| $G L D C$ | catalyst in glycine cleavage system | glycine encephanlopathy, Autosomal recessive inheritance |
| HAL | catabolism of Histidine | Histidinemia, vitamin D measurement |
| DHODH | catalyzing pyrimidine de novo biosynthesis | postaxial acrofacial dysostosis, total cholesterol |
| PAH | iron containing enzyme | blood metabolite measurements |
| MED1 | coactivator in the transcription of RNA polymerase II-dependent genes | asthma, inflammatory bowel disease |
| STK33 | Serine/threonine protein kinase which phosphorylates VIME | BMI, small cell lung carcinoma |
| ALDH1L1 | catalyzes the conversion of 10formyltetrahydrofolate, NADP, and water to tetrahydrofolate, NADPH, and carbon dioxide | homocytosine, insulin sensitivity |
| BCAT2 | catabolism of the branched chain amino acids leucine, isoleucine and valine | urinary metabolite, eye measurement, reticulocyte |

