Lorentz Transformations and Existence in Minkowski Space

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1 Introduction

The concept of existence is formally disconnected from physics at this time. This paper presents an extraordinarily simple yet significant result which establishes such a connection. The result follows two recent papers [1] [2] which explored implications of the Lorentz transformations in terms of concepts to be introduced momentarily. The result is that existence in Minkowski space, as defined below, is an equivalence relation by absolute dimensionality. Three of its implications are then briefly discussed.

2 Lorentz Contraction implies Dimensional Abatement

Definition 1. *Absolute Dimensionality:* The absolute dimensionality of an object is a dimensionless natural number that refers to the independent *length* dimensions which characterize it.

Definition 2. *Volume-Boundary ratio:* The Volume-Boundary ratio of a compact object with absolute dimensionality $n>1$ is the ratio of its n-dimensional volume to its $n-1$-dimensional boundary.

Definition 3. *Relative Dimensionality:* Relative Dimensionality is the dimensionless ratio of the Volume-Boundary ratio of a compact object with absolute dimensionality $n>1$ to that of a compact reference object, also with absolute dimensionality n.

Definition 4. *Dimensional Diminution:* For an $n-$dimensional compact object, dimensional diminution is the decrease of its *relative* dimensionality compared to its uncontracted state by a dimensionless real factor in the open interval (0,1).

Definition 5. *Dimensional Reduction:* For an $n-$dimensional compact object (n>1), dimensional reduction is the decrease of its absolute dimensionality to $n-1$.

Definition 6. *Dimensional Abatement:* Dimensional Abatement is a less specific umbrella term which can either refer to Dimensional Diminution or to Dimensional Reduction.

Proposition 1. Lorentz contraction implies *dimensional abatement.* More specifically, it implies *dimensional diminution for* $0<v<c$ and *dimensional reduction for* $v=c$.

Proof: Consider a compact body B moving in a frame S and a moving frame S' in which B is at rest. We imagine B in S' as being made out of infinitesimal cubical volume elements oriented, without loss of generality, such that the direction of contraction in S will be normal to one of the sides. It is trivial to show that the Lorentz contraction of each cubical element in S causes it to be dimensionally abated. Since this is true of every infinitesimal volume element of B, it is true of B. ■

3 Time Dilation implies Ontochronic Abatement

Arguably, our understanding of nature has become so deep that in order to make further progress, we need to incorporate the concept of existence into physics. The following existence criterion, taken as an axiom, is an attempt to do so:

Criterion. *A physical object exists in Minkowski space if and only if it is characterized by a timelike spacetime interval.*

Definition 7. *Spacetime Ontic Function:* The spacetime ontic function is a map $\exists_S:\mathfrak{O}\to\{0,1\}$ where $\mathfrak{O}$ is the set of all physical objects taken to be within the domain of physics and $S\subset\mathfrak{O}$ is the subset of $\mathfrak{O}$ of all objects that exist in spacetime. The *spacetime ontic value* of an object is determined by whether it satisfies the existence criterion ($\exists_S(x)=1$) or not ($\exists_S(x)=0$).

Definition 8. *Ontochronicity:* Ontochronicity is the quality of having a duration of physical existence.

Definition 9. *Relative Ontochronicity:* Relative ontochronicity is the dimensionless ratio of the the observed duration of existence of an object compared to that of a reference object, usually the observer.

Definition 10. *Ontochronic Diminution:* Ontochronic diminution is the the decrease of the observed duration of existence of an object in a given time interval by a dimensionless real factor in the open interval (0,1).

Definition 11. *Ontic Reduction:* Ontic reduction is the reduction of the ontic value of an object to 0.

Definition 12. *Ontochronic Abatement:* Ontochronic abatement is a less specific umbrella term which can either refer to ontochronic diminution or to ontic reduction.

Proposition 2. Relativistic time dilation implies *ontochronic abatement.* More specifically, it implies *ontochronic diminution for* $0<v<c$ and *ontic reduction for* $v=c$.

Proof: Follows trivially from re-interpreting the proper time of an object as its observed duration of existence in spacetime, and coordinate time as the duration of existence in spacetime of the observer, between two given spacetime events.■

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4 Four Spacetime Principles

The Reconceptualization of the Lorentz Transformations focuses attention on four spacetime principles for which this author was unable to find prior discussion in the literature, likely due to obviousness. These are two invariance and two symmetry principles:

Principle 1. *Invariance of Absolute Dimensionality:* The absolute dimensionality of any compact body is invariant under spacetime coordinate transformations.

Principle 2. *Homodimensionality of Space:* The dimensionality of every maximally-dimensional spacelike hypersurface of Minkowski space is everywhere the same.

Principle 3. *Invariance of Spacetime Ontic Value:* The spacetime ontic value of any compact body is invariant under spacetime coordinate transformations.

Principle 4. *Homodimensionality of Time:* The dimensionality of every timelike hypersurface of Minkowski space is everywhere the same.

5 Existence in Spacetime as an Equivalence Relation

(i) **Principles 1 and 3 together couple absolute dimensionality to spacetime ontic value.** Propositions 1 and 2 together already show that dimensional and ontochronic diminution couple to each other exactly as Lorentz contraction and time dilation couple to each other, but the two invariance principles together extend this to dimensional and ontic reduction.

(ii) **Principles 2 and 4 together ensure that the coupling of absolute dimensionality to spacetime ontic value holds globally.** In a spacetime in which the homodimensionality of space or of time fails to hold, there could conceivably be regions in which spacelike or timelike hypersurfaces have a different dimensionality inside the region than outside, and in such regions absolute dimensionality and ontic value could decouple. The two homodimensionality principles together ensure that this does not happen. I will call a spacetime in which both homodimensionality principles hold *isodimensional*.

Proposition 3. *Physical existence in Minkowski space is an equivalence relation by absolute dimensionality.*

Proof: An equivalence relation is determined by the properties of reflexivity, symmetry and transitivity. Consider an n-dimensional compact object A subject to the above principles. By the the coupling of spacetime ontic value to absolute dimensionality [5(i)], it must exist in an $n+1$ dimensional Minkowski space region. By the isodimensionality of Minkowski space [5(ii)], this region is, in fact, all of $n+1$ dimensional Minkowski space. In particular, A exists in the $n+1$-dimensional Minkowski space in which its proper time is timelike, so in which it exists. This proves reflexivity. Now consider an m-dimensional compact object B. By the same argument as given for reflexivity, it must exist in an $m+1$ dimensional Minkowski space. Suppose A exists in the same spacetime as B. This implies the proper time of A is timelike in the spacetime in which B exists, or in which the proper time of B is timelike. But that implies the proper time of B is timelike in the spacetime in which the proper time of A is timelike or where A exists, hence B exists in the spacetime of A. This proves symmetry. Finally, consider an l-dimensional compact object C. By the same argument as given for reflexivity, it must exist in an $l+1$-dimensional Minkowski space. Now suppose that B exists in the same spacetime as C, and that A exists in the same spacetime as B. This implies the proper timne of B is timelike in the spacetime in which the proper time of C is timelike, and that the proper time of A is timelike in the spacetime in which the proper time of B is timelike. It follows that the proper time of A is timelike in the spacetime in which the proper time of C is timelike, and hence A exists in the spacetime in which C exists. This proves transitivity.■

6 Discussion

The ontic equivalence relation partitions the set of all objects that physically exist *per se* into *ontic equivalence classes* such that for each $n+1$ dimensional Minkowski space, there is a corresponding equivalence class of n-dimensional objects that exist in it. Three immediate implications of this are:

- **Speed-of-light objects belong to the ontic equivalence class of objects that exist in $2+1$ Minkowski space.** This can now be given as an *explanation* for the impossibility of transforming to the rest frame of a speed-of-light object: if a spacetime observer could transform to a speed-of-light rest frame, he or she would no longer be a *spacetime* observer.
- **The Ontic Equivalence Relation holds more generally.** When defining a non-singular manifold with metric $(\mathcal{M},g)$ we tacitly assume isodimensionality as well as the invariance principles. Point-like singularities can be accommodated by relaxing isodimensionality to *isodimensionality almost everywhere* (i.e. up to a measure zero set of points).
- **Any theory which supposes *both* that the Lorentz Transformations hold *and* that n-dimensional objects exist in an m+1-dimensional region of spacetime, such that n≠m, may be inconsistent.** This may be useful as a guide for theory selection and should be carefully checked in each theory which is a candidate for supposing both.

7 Conclusion

A fuller discussion of the underlying ideas can be found in [1] and [2]. Any set of ideas which reinterprets a familiar framework in a profoundly novel way is bound to raise many new questions and also prompt a re-examination of many other familiar areas not yet re-considered. For this reason, the ideas discussed here may be the beginning of a broader project of re-examining familiar areas of physics in light of the dimensionality and existence concepts introduced here.

References

[1] A. Nikkhah Shirazi *Dimensionality in Physics* Available at Deep Blue, the University of Michigan's repository for scholarly and artistic work: http://hdl.handle.net/2027.42/147435

[2] A. Nikkhah Shirazi *Existence in Physics* Available at Deep Blue, the University of Michigan's repository for scholarly and artistic work: http://hdl.handle.net/2027.42/147436