Online Appendix: Proofs

"Strategic Analysis of Dual Sourcing and Dual Channel with an Unreliable Alternative Supplier "

Proof of Lemma 1

Given the wholesale price W_{cs}, the competitive supplier solves the following problem to maximize its profit:

$$\max_{q_b} \ \Pi_{cs} = (a - bq_b - bq_{cs})q_b + W_{cs}q_{cs},$$

which yields $q_b = \frac{a-bq_{cs}}{2b}$. The OEM's optimal production quantity is given by solving

$$\max_{q_{cs}} \ \Pi_o = (a + m - bq_b - bq_{cs})q_{cs} - W_{cs}q_{cs},$$

which yields $q_{cs} = \frac{a+m-W_{cs}-bq_b}{2b}$. Solving q_b and q_{cs} simultaneously yields:

$$q_b = \frac{a - m + W_{cs}}{3b}; \quad q_{cs} = \frac{a + 2m - 2W_{cs}}{3b}.$$

Substituting them into the supplier's profit function and then solving for optimality with respect to W_{cs}, we have the optimal wholesale price:

$$W_{cs}^B = \frac{5a + 4m}{10}.$$

Bring this back to the equations of quantities and prices, we can derive the optimums.

Proof of Lemma 2

The expected profit functions of the competitive supplier, the OEM and the non-competitive supplier are, respectively

$$\begin{aligned} E\Pi_{cs} &= [a - bq_b - b(q_{cs} + \mu q_{ns})]q_b + W_{cs}q_{cs}, \\ E\Pi_o &= [a + m - bq_b - b(q_{cs} + \mu q_{ns})](q_{cs} + \mu q_{ns}) - b\sigma^2 q_{ns}^2 - W_{cs}q_{cs} - \mu W_{ns}q_{ns}, \\ E\Pi_{ns} &= \mu W_{ns}q_{ns}. \end{aligned}$$

Taking the first order conditions of the profit functions with respect to q_b for the competitive supplier and q_{cs}, q_{ns} for the OEM, and solving them simultaneously, we have:

$$q_b = \frac{a - m + W_{cs}}{3b}; \quad q_{cs} = \frac{2a\sigma^2 + 4m\sigma^2 - (3\mu^2 + 4\sigma^2)W_{cs} + 3\mu W_{ns}}{6b\sigma^2}; \quad q_{ns} = \frac{\mu W_{cs} - W_{ns}}{2b\sigma^2}.$$

Substituting them into the profit functions of the competitive supplier and the non-competitive supplier and then taking the first order conditions, we have the wholesale prices as follows

$$W_{cs} = \frac{10a\sigma^2 + 8m\sigma^2 + 9\mu W_{ns}}{2(10\sigma^2 + 9\mu^2)}; \quad W_{ns} = \frac{\mu W_{cs}}{2}.$$

Solving them two simultaneously, we have

$$W_{cs}^D = \frac{4(5a+4m)\sigma^2}{40\sigma^2 + 27\mu^2}; \quad W_{ns}^D = \frac{2(5a+4m)\sigma^2}{40\sigma^2 + 27\mu^2}.$$

Bring these two optimums back to the functions of quantities and profits, we can derive the optimums.

Proof of Proposition 1 and 2:

Rearranging $E\Pi_{cs}^D$, $E\Pi_o^D$ and Π_{ns}^D with the notation $x = \left(\frac{\mu}{\sigma}\right)^2$, and then taking the derivatives with respect to x, we have

$$\frac{\partial E\Pi_{cs}^D}{\partial x} = -\frac{8(5a+4m)^2(20+27x)}{b(40+27x)^3} < 0;$$
$$\frac{\partial E\Pi_o^D}{\partial x} = \frac{(5a+4m)[a(1161x+200)+4m(621x+616)]}{b(27x+40)^3} > 0.$$

Hence $E\Pi_{cs}^D$ decreases in x while $E\Pi_o^D$ increases in x.

Rearranging Π_{ns}^D with the notation $x = (\frac{\mu}{\sigma})^2$, and then taking the derivatives with respect to x, we have

$$\frac{\partial \Pi_{ns}^D}{\partial x} = \frac{2(5a+4m)^2(40-27x)}{b(40+27x)^3}.$$

$\frac{\partial \Pi_{ns}^D}{\partial x}$ is positive for all $x \in [0, \frac{40}{27})$, and it remains negative for all $x \in (\frac{40}{27}, +\infty)$. Thus Π_{ns}^D is unimodal in x with its unique optimal solution at $x = 40/27$.

Furthermore, $W_{ns}^D = \frac{2(5a+4m)\sigma^2}{40\sigma^2+27\mu^2} = \frac{2(5a+4m)}{40+27x}$ is obviously decreasing in x. Note that

$$q_{ns}^D = \frac{\mu(5a+4m)}{b(40\sigma^2+27\mu^2)} > 0$$

and

$$\left(q_{ns}^D\right)^2 = \frac{\mu^2(5a+4m)}{b(40\sigma^2+27\mu^2)^2} = \frac{x(5a+4m)}{b\sigma^2(40+27x)^2}.$$

Taking the derivative of $\left(q_{ns}^D\right)^2$ w.r.t. x, we have

$$\frac{\partial \left(q_{ns}^D\right)^2}{\partial x} = \frac{(5a+4m)(40-27x)}{b\sigma^2(40+27x)^3}.$$

Hence, $\left(q_{ns}^D\right)^2$ as well as q_{ns}^D is unimodal in x for any given σ, and the maximum is $x = \frac{40}{27}$.

Proof of Lemma 3

In the end market, the expected profit functions of the competitive supplier, the OEM and the non-competitive supplier are, respectively

$$\begin{aligned} E\Pi_{cs} &= (a - bq_b - b\mu q_{ns})q_b, \\ E\Pi_o &= [(a + m - bq_b)\mu - b(\mu^2 + \sigma^2)q_{ns} - \mu W_{ns}]q_{ns}, \\ E\Pi_{ns} &= \mu W_{ns} q_{ns}. \end{aligned}$$

It is easy to show that $E\Pi_{cs}$ and $E\Pi_o$ are concave with respect to q_b and q_{ns}, respectively. This yields the following production quantities of the competitive supplier and the OEM

$$\begin{aligned} q_b &= \frac{2a\sigma^2 + a\mu^2 - m\mu^2 + \mu^2 W_{ns}}{b\,(4\sigma^2 + 3\mu^2)}, \\ q_{ns} &= \frac{\mu(a + 2m - 2W_{ns})}{b\,(4\sigma^2 + 3\mu^2)}. \end{aligned}$$

Substituting them into the non-competitive supplier's profit function, we have the optimal wholesale price $W_{ns}^T = \frac{a+2m}{4}$ and then the optimal quantities and prices by bring this back to the equations.

Proof of Proposition 3

Comparing the competitive supplier's expected profit in the termination scenario with that in the base scenario, we have

$$E\Pi_{cs}^T - \Pi_{cs}^B = -\left[\frac{a^2(55x^2 + 80x)}{80b(3x + 4)^2} + \frac{20amx(5x + 8)}{80b(3x + 4)^2} + \frac{m^2(31x^2 + 96x + 64)}{20b(3x + 4)^2}\right],$$

which is negative.

Comparing the OEM's expected profit in the termination scenario with that in the base scenario, we have

$$E\Pi_o^T - \Pi_o^B = \frac{25a^2x(x + 1) + 100amx(x + 1) - 4m^2(11x^2 + 71x + 64)}{100b(3x + 4)^2},$$

which is a convex quadratic function of a with negative asymmetry axis and negative constant term. Hence, it has one and only one positive root

$$a' = \frac{2m\left(-5x^2 - 5x + 2\sqrt{9x^4 + 33x^3 + 40x^2 + 16x}\right)}{5(x^2 + x)}.$$

It can be shown that this root is decreasing in x. It reaches $\frac{2}{5}m$ when x approaches infinity, and it reaches infinity when x approaches 0. Hence, for a given value of a larger than m: When x is close to 0, a is on the left hand side of the larger root a', hence $E\Pi_o^T - \Pi_o^B < 0$. As x increases, the larger root becomes smaller and smaller, and eventually there exists a unique threshold value x_O such that when $x > x_O$, a begins to be bigger than the larger root a', then we have $E\Pi_o^T - \Pi_o^B > 0$.

Proof of Proposition 4

Comparing the competitive supplier's expected profit in the dual sourcing scenario with that in the termination scenario, we have

$$E\Pi_{cs}^{D} - E\Pi_{cs}^{T} = \frac{a^2\left[-x^2\left(6561x^2+17856x+12160\right)\right]}{16b(3x+4)^2(27x+40)^2} + \frac{a\left[-4mx\left(2187x^3+4680x^2+448x-2560\right)\right]+4m^2\left(2187x^4+18864x^3+50432x^2+54272x+20480\right)}{16b(3x+4)^2(27x+40)^2}.$$

The sign of this function is determined by its numerator, which is a concave quadratic function of a. Notice that it takes a strict positive value at $a=0$, hence it have one and only one root that is larger than 0.

The larger root is given as:

$$a'' = \frac{2m(2560-2187x^3-4680x^2-448x+2\sqrt{3}(81x^2+228x+160)\sqrt{(3x+8)(81x+104)}}{6561x^3+17856x^2+12160x}$$

It can be shown that the larger root is decreasing in x. It reaches $\frac{2}{3}m$ when x approaches infinity and it reaches infinity when x approaches 0.

For a given value of $a>m$, we find that: When $x=0$, a is on the left side of the larger root a'', and hence $E\Pi_{cs}^{D} - E\Pi_{cs}^{T} > 0$. As x increases, the root moves leftward, and finally there exists a unique threshold value of x larger than which a falls on the right side of the larger root, resulting in $E\Pi_{cs}^{D} - E\Pi_{cs}^{T} < 0$. We name this threshold value of x as x_C. This indicates that when x exceeds x_C, the termination scenario is more profitable for the competitive supplier. When x is smaller than x_C, accepting the OEM's dual sourcing strategy is more profitable for the competitive supplier. Lastly, because the root is decreasing in x, we find that when a increases, the threshold value x_C becomes smaller, and vice versa.

Comparing the OEM's expected profit in the dual sourcing scenario with that in the termination scenario, we have

$$E\Pi_{o}^{D} - E\Pi_{o}^{T} = \frac{a^2x^2(2187x^2+5787x+3824)}{4b(4+3x)^2(40+27x)^2} + \frac{4amx(3648+9296x+7839x^2+2187x^3)}{4b(4+3x)^2(40+27x)^2} + \frac{4m^2(2187x^4+10215x^3+17936x^2+14016x+4096)}{4b(4+3x)^2(40+27x)^2},$$

which is apparently larger than 0.

Proof of Proposition 5

We first compare the two roots (a'' and a') in Proposition 3 and 4.

$$a''-a' = \frac{2}{5}m\left[\frac{10\left(2187x^3+6588x^2+5856x+1280\right)}{x\left(6561x^2+17856x+12160\right)} + \frac{10\sqrt{3}\left(81x^2+228x+160\right)\sqrt{(3x+8)(81x+104)}}{x\left(6561x^2+17856x+12160\right)} - \frac{2(3x+4)\sqrt{x(x+1)}}{x(x+1)}\right]$$

It can be shown that the derivatives of both $\frac{10\left(2187x^3+6588x^2+5856x+1280\right)}{x(6561x^2+17856x+12160)}$ (the first term) and

$\left[\frac{10\sqrt{3}\left(81x^2+228x+160\right)\sqrt{(3x+8)(81x+104)}}{x(6561x^2+17856x+12160)} - \frac{2(3x+4)\sqrt{x(x+1)}}{x(x+1)}\right]$ (the second term) are strictly negative. When x goes to infinity, the first term reaches $\frac{10}{3}$ while the second term reaches $-\frac{8}{3}$. This indicates their sum is always larger than $\frac{10}{3} - \frac{8}{3} > 0$. Hence we have $a'' > a'$.

Recall our proof in Proposition 3 and Proposition 4: When x increases, both a'' and a' decrease. For a given value of $a > m$, when x increases from 0 to infinity, a' reaches a first and a'' reaches a later. Thus, x_O is always smaller than x_C for any given a.

Proof of Proposition 6

When the competitive supplier is capacity-constrained, in the dual sourcing scenario, the equilibrium wholesale prices are:

$$\begin{aligned}
W_{cs}^{DK} &= \frac{4(2a\sigma^2 - 3b\sigma^2 K + \sigma^2 m)}{4\sigma^2 + 3\mu^2}, \\
W_{ns}^{DK} &= \frac{2\mu(2a\sigma^2 - 3b\sigma^2 K + \sigma^2 m)}{4\sigma^2 + 3\mu^2};
\end{aligned}$$

the order quantities are

$$\begin{aligned}
q_b^{DK} &= \frac{4a\sigma^2 + a\mu^2 - 4b\sigma^2 K - m\mu^2}{4b\sigma^2 + 3b\mu^2}, \\
q_{cs}^{DK} &= \frac{-a(4\sigma^2 + \mu^2) + bK(8\sigma^2 + 3\mu^2) + m\mu^2}{b(4\sigma^2 + 3\mu^2)}, \\
q_{ns}^{DK} &= \frac{\mu(2a - 3bK + m)}{b(4\sigma^2 + 3\mu^2)};
\end{aligned}$$

and the supply chain parties' profits are

$$\begin{aligned}
E\Pi_o^{DK} &= \frac{(9x+64)b^2K^2 + 2b(2ax - 32a + 13mx)K + (a+2m)^2x^2 + (-4a^2 - 12am + m^2)x + 16a^2}{b(4+3x)^2}, \\
E\Pi_{cs}^{DK} &= \frac{-4(9x+20)b^2K^2 + 4b(20a + 8m + 7ax + 2mx)K + (4a + ax - mx)(ax - 4m - 4a - mx)}{b(4+3x)^2},
\end{aligned}$$

and $E\Pi_{ns}^{DK} = \frac{2x(2a-3bK+m)^2}{b(4+3x)^2}$. Note that $E\Pi_{cs}^{DK}$ is a concave quadratic function of K with a positive symmetry axis. We take the first order condition of $E\Pi_{cs}^{DK}$ with respect to K. This gives $K_{cs}^D = \frac{20a+8m+7ax+2mx}{2b(9x+20)}$, which is smaller than $q_b^D + q_{cs}^D = \frac{20a+8m+13ax+5mx}{b(27x+40)}$.

Proof of Proposition 7

Comparing the profits of the OEM and the competitive supplier between the two scenarios, we have

$$\begin{aligned}
E\Pi_o^{DK} - E\Pi_o^{BK} &= \frac{\mu^2(2a + m - 3bK)\left[K(29b\sigma^2 + 12b\mu^2) - (14a\sigma^2 + 4a\mu^2 - m\sigma^2 - 4m\mu^2)\right]}{b(4\sigma^2 + 3\mu^2)^2}, \\
E\Pi_{cs}^{DK} - E\Pi_{cs}^{BK} &= \frac{\mu^2(2a + m - 3bK)\left[-K(28b\sigma^2 + 15b\mu^2) + (5a\mu^2 + 4m\sigma^2 + m\mu^2 + 12a\sigma^2)\right]}{b(4\sigma^2 + 3\mu^2)^2}.
\end{aligned}$$

Therefore we have the two thresholds:

$$
\begin{aligned}
K_o &= \frac{14a\sigma^2 + 4a\mu^2 - m\sigma^2 - 4m\mu^2}{29b\sigma^2 + 12b\mu^2} = \frac{14a - m + 4(a-m)x}{29b + 12bx}, \\
K_c &= \frac{5a\mu^2 + 4m\sigma^2 + m\mu^2 + 12a\sigma^2}{28b\sigma^2 + 15b\mu^2} = \frac{12a + 4m + (5a+m)x}{28b + 15bx}.
\end{aligned}
$$

It is not difficult to show that $K_o < q_b^D + q_{cs}^D$ and $K_c < q_b^D + q_{cs}^D$, that is, these two thresholds cannot satisfy all the orders and is actually binding. Furthermore,

$$
K_c - K_o = -\frac{(4\sigma^2 + 3\mu^2)(11a\sigma^2 - 36m\sigma^2 - 24m\mu^2)}{b(28\sigma^2 + 15\mu^2)(29\sigma^2 + 12\mu^2)},
$$

so its sign depends on whether $11a\sigma^2 - 36m\sigma^2 - 24m\mu^2 < 0$. i.e., $x > \frac{11a-36m}{24m}$.

Proof of Proposition 8

When the non-competitive supplier has a tight capacity τ, the equilibrium wholesale prices are

$$
\begin{aligned}
W_{cs}^{Dt} &= \frac{2\sigma^2(5a + 4m - 9b\mu\tau)}{20\sigma^2 + 9\mu^2}, \\
W_{ns}^{Dt} &= \frac{2\sigma^2(5a\mu + 4m\mu - 18b\tau\mu^2 - 20b\sigma^2\tau)}{\mu(20\sigma^2 + 9\mu^2)},
\end{aligned}
$$

the production quantities are

$$
\begin{aligned}
q_b^{Dt} &= \frac{(10a - 4m - 6b\tau\mu)\sigma^2 + 3(a-m)\mu^2}{b(20\sigma^2 + 9\mu^2)}, \\
q_{cs}^{Dt} &= \frac{3a\mu^2 + 8m\sigma^2 + 6m\mu^2 - 9b\tau\mu^3 - 8b\sigma^2\tau\mu}{b(20\sigma^2 + 9\mu^2)},
\end{aligned}
$$

and $q_{ns}^{Dt} = \tau$. Note that the binding constraint requires $\tau \le q_{ns}^D = \frac{(5a+4m)\mu}{b(40\sigma^2 + 27\mu^2)}$. Then, taking the first-order derivatives w.r.t. μ and σ respectively, we have

$$
\begin{aligned}
\frac{\partial W_{cs}^{Dt}}{\partial \mu} &= -\frac{18\sigma^2(10a\mu + 8m\mu - 9b\tau\mu^2 + 20b\sigma^2\tau)}{(20\sigma^2 + 9\mu^2)^2} < 0, \\
\frac{\partial W_{cs}^{Dt}}{\partial \sigma} &= \frac{36\sigma\mu^2(5a + 4m - 9b\tau\mu)}{(20\sigma^2 + 9\mu^2)^2} > 0, \\
\frac{\partial q_b^{Dt}}{\partial \mu} &= -\frac{6\sigma^2(10a\mu + 8m\mu - 9b\tau\mu^2 + 20b\sigma^2\tau)}{b(20\sigma^2 + 9\mu^2)^2} < 0, \\
\frac{\partial q_b^{Dt}}{\partial \sigma} &= \frac{12\sigma\mu^2(5a + 4m - 9b\tau\mu)}{b(20\sigma^2 + 9\mu^2)^2} > 0,
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial q_{cs}^{Dt}}{\partial \sigma} &= -\frac{24\sigma\mu^2(5a + 4m - 9b\tau\mu)}{b(20\sigma^2 + 9\mu^2)^2} < 0, \\
\frac{\partial q_{cs}^{Dt}}{\partial \mu} &= \frac{(-160b\sigma^4 - 468b\sigma^2\mu^2 - 81b\mu^4)\tau + 120a\sigma^2\mu + 96m\sigma^2\mu}{b(20\sigma^2 + 9\mu^2)^2} \\
&\ge \frac{\mu(40\sigma^2 - 9\mu^2)(5a + 4m)}{b(20\sigma^2 + 9\mu^2)(40\sigma^2 + 27\mu^2)},
\end{aligned}
$$

the latter of which is positive if $40\sigma^2 - 9\mu^2 > 0$, i.e., $x < 40/9$. Furthermore,

$$\frac{\partial E\Pi_{cs}^{Dt}}{\partial \mu} = -\frac{4\sigma^2(5a+4m-9b\tau\mu)(45a\mu^3+36m\mu^3+200b\sigma^4\tau+270b\sigma^2\tau\mu^2)}{b(20\sigma^2+9\mu^2)^3} < 0,$$

$$\frac{\partial E\Pi_{cs}^{Dt}}{\partial \sigma} = \frac{36\sigma\mu^4(5a+4m-9b\tau\mu)^2}{b(20\sigma^2+9\mu^2)^3} > 0,$$

$$\frac{\partial E\Pi_{o}^{Dt}}{\partial \mu} = \frac{24\sigma^2(3a\mu^2+8m\sigma^2+6m\mu^2+12b\sigma^2\tau\mu)(10a\mu+8m\mu-9b\tau\mu^2+20b\sigma^2\tau)}{b(20\sigma^2+9\mu^2)^3} > 0,$$

$$\frac{\partial E\Pi_{o}^{Dt}}{\partial \sigma} = \frac{2b^2\sigma(8000\sigma^6+10800\sigma^4\mu^2+7452\sigma^2\mu^4+729\mu^6)\tau^2-144b\mu^3\sigma(20a\sigma^2-9a\mu^2-8m\sigma^2-18m\mu^2)\tau}{b(20\sigma^2+9\mu^2)^3} - \frac{48\mu^2\sigma(5a+4m)(3a\mu^2+8m\sigma^2+6m\mu^2)}{b(20\sigma^2+9\mu^2)^3} < 0,$$

and

$$\frac{\partial E\Pi_{ns}^{Dt}}{\partial \mu} = \frac{2\sigma^2\tau[-9(5a+4m)\mu^2-360b\sigma^2\tau\mu+20\sigma^2(5a+4m)]}{(20\sigma^2+9\mu^2)^2}$$

$$\frac{\partial E\Pi_{ns}^{Dt}}{\partial \sigma} = \frac{4\sigma\tau[-400b\tau\sigma^4-360b\tau\mu^2\sigma^2+9\mu^3(5a+4m-18b\tau\mu)]}{(20\sigma^2+9\mu^2)^2}.$$

The numerators of the last two derivatives both include a quadratic function with negative quadratic coefficient and positive constant term, and therefore $E\Pi_{ns}^{Dt}$ is unimodal in both μ and σ.

Proof of Proposition 9

When the non-competitive supplier produces at its full capacity and is able to sell the excess components to a spot market, the equilibrium wholesale prices and quantities are

$$W_{cs}^{Do} = \frac{20a\sigma^2+16m\sigma^2+9\mu^2W}{40\sigma^2+27\mu^2} = \frac{20a+16m+9xW}{40+27x},$$

$$W_{ns}^{Do} = \frac{2(5a\sigma^2+4m\sigma^2+10\sigma^2W+9\mu^2W)}{40\sigma^2+27\mu^2} = \frac{2(5a+4m+10W+9xW)}{40+27x},$$

$$q_b^{Do} = \frac{20a-8m+9ax-9mx+3xW}{b(40+27x)},$$

$$q_{cs}^{Do} = \frac{32m+8ax+28mx+8xW+9x^2W}{2b(40+27x)},$$

$$q_{ns}^{Do} = \frac{x(10a+8m-20W-9xW)}{2b\mu(40+27x)}.$$

Therefore,

$$W_{cs}^{Do} - W = \frac{2(10a+8m-20W-9xW)}{40+27x} > 0$$

if and only if $W < \frac{10a+8m}{20+9x}$. Furthermore, if this condition holds, then $W_{ns}^{Do} = (W_{cs}^{Do}+W)/2 > W$ and $q_{ns}^{Do} > 0$.