

# Jensen-Shannon Complexity and Permutation Entropy Analysis of Geomagnetic Auroral Currents

Adnane Osmane<sup>1</sup>, Andrew P. Dimmock<sup>2</sup>, Tuija I. Pulkkinen<sup>3,4</sup>

<sup>1</sup>Department of Physics, University of Helsinki, Helsinki, Finland

<sup>2</sup>Swedish Institute of Space Physics, Uppsala University, Uppsala, Sweden

<sup>3</sup>Department of Climate and Space Sciences, University of Michigan, Ann Arbor, MI, USA

<sup>4</sup>Aalto University, School of Electrical Engineering, Espoo, Finland

## Key Points:

- Jensen-Shannon complexity plane used to analyse auroral geomagnetic indices.
- Auroral indices are shown to be inconsistent with low-dimensional chaotic processes.
- Maximum in complexity occurs on timescales ranging between 10 and 40 minutes.

This is the author manuscript accepted for publication and has undergone full peer review but has not been through the copyediting, typesetting, pagination and proofreading process, which may lead to differences between this version and the [Version of Record](#). Please cite this article as doi: [10.1029/2018JA026248](https://doi.org/10.1029/2018JA026248)

Corresponding author: Adnane Osmane, [adnane.osmane@helsinki.fi](mailto:adnane.osmane@helsinki.fi)

## Abstract

In this study we determine whether auroral westward currents can be characterised by low dimensional chaotic attractors through the use of the complexity-entropy methodology developed by Rosso, Larrondo, Martin, Plastino, and Fuentes (2007) and based on the permutation entropy developed by Bandt and Pompe (2002). Our results indicate that geomagnetic auroral indices are indistinguishable from stochastic processes from timescales ranging from a few minutes to 10 hours and for embedded dimensions  $d < 8$ . Our results are *inconsistent* with earlier studies of (Baker, Klimas, McPherron, & Büchner, 1990; Pavlos et al., 1992; D. Roberts, Baker, Klimas, & Bargatze, 1991; D. A. Roberts, 1991; Vassiliadis, Sharma, & Papadopoulos, 1991; Vassiliadis, Sharma, Eastman, & Papadopoulos, 1990) indicating that auroral geomagnetic indices could be reduced to low-dimensional systems with chaotic dynamics.

## 1 Introduction

The discovery fifty years ago that fully developed turbulence could in principle be the result of only three instabilities (Ruelle & Takens, 1971), rather than an infinite number Landau (1944), together with the experimental confirmation by Gollub and Swinney (1975) that universal behavior described by a few parameters could be observed in a fluid system, has led to what some authors described as a "chaos revolution" (Lovejoy & Schertzer, 1998). The realisation that nonlinear systems with a very large number of degree of freedoms could be described by low-dimensional dynamical systems naturally found a promising niche in a wide range of space plasma research, and especially in space weather studies, in order to alleviate the computational cost of modelling the Earth's magnetosphere. Following the development of empirical techniques for detecting deterministic chaos by Grassberger and Procaccia (1983), a plethora of studies (Baker et al., 1990; Pavlos et al., 1992; D. Roberts et al., 1991; D. A. Roberts, 1991; Vassiliadis et al., 1991; Vassiliadis et al., 1990) using geomagnetic indices argued that the Earth's magnetospheric dynamics could be reduced to a low-dimensional dynamical systems. However, it was first shown by Osborne and Provenzale (1989) for a general case, and later by Prichard and Price (1992) and Shan, Hansen, Goertz, and Smith (1991) for space weather studies, that the empirical technique could not differentiate between coloured noise and deterministic chaos in geomagnetic time series, due to long autocorrelation times inherent to the former.

44 Nonetheless, the ideas provided by deterministic chaos were extended to nonlinear stochastic  
45 systems by the use of self-organized critical (SOC) models, that is cellular automata  
46 defined by a certain class of discontinuous rules and appropriate boundary conditions  
47 (Lovejoy & Schertzer, 1998). SOC systems were shown to evolve spontaneously to critical  
48 states describable by a low-dimensional dynamical systems (Chang, 1992). Consequently,  
49 Balasis et al. (2006); Consolini and Marcucci (1997); Dobias and Wanliss (2009);  
50 Klimas et al. (2000); Klimas, Vassiliadis, Baker, and Roberts (1996); A. Pulkkinen, Kli-  
51 mas, Vassiliadis, and Uritsky (2006); Uritsky, Pudovkin, and Steen (2001); Uritsky, Kli-  
52 mas, and Vassiliadis (2006); Uritsky and Pudovkin (1998); Valdivia, Klimas, Vassiliadis,  
53 Uritsky, and Takalo (2003); Wanliss, Anh, Yu, and Watson (2005); Wanliss and Dobias  
54 (2007) extended these ideas to magnetospheric systems, demonstrating that nonlinear  
55 stochastic models were a better representation than low-dimensional chaotic attractors.  
56 The emergence of dynamical correlations and non-Markovian features during intense ge-  
57 omagnetic storms, analogous to the emergence of long-range coherence in out-of-equilibrium  
58 systems, implied a reduction in the number of degrees of freedom of the system and in-  
59 herent nonlinearities (Consolini & De Michelis, 2014).

60 In this study, we contribute to the decades old discussions on the properties of geomag-  
61 netic processes by using permutation entropy, a measure developed by Bandt and Pompe  
62 (2002), to quantify complexity in measured time series. Whereas common measures of  
63 complexity, such as the Kolmogorov-Sinai entropy, or the Shannon entropy, ignore tem-  
64 poral order of the values in the time series, entropy measures of ordinal patterns preserve  
65 information of temporal order and provides for an alternative measure of complexity (Riedl,  
66 Muller, & Wessel, 2013). Permutation entropy has now been tested across several sci-  
67 entific disciplines, and is now being used to characterise processes in laboratory and geo-  
68 physical plasma experiments (Consolini & De Michelis, 2014; Maggs & Morales, 2013;  
69 Weck, Schaffner, Brown, & Wicks, 2015).

70 For example, using the complexity-entropy measure developed by Rosso et al. (2007),  
71 Weck et al. (2015) demonstrate that solar wind turbulent fluctuations are stochastic, rather  
72 than chaotic. In the context of geomagnetic activity, it is of primary interest to deter-  
73 mine whether, and under which time scales, geomagnetic currents demonstrate signa-  
74 tures of low-dimensional dynamical systems, if any. Using the methodology developed  
75 by Rosso et al. (2007), we hereafter revisit the question as to whether auroral geomag-  
76 netic indices can be characterised as a low-dimensional chaotic attractor, and use for the

77 first time the Jensen-Shannon complexity on auroral geomagnetic indices. In section 2  
78 we describe the dataset and the complexity-entropy plane used for distinguishing between  
79 stochastic and chaotic time series. In section 3 we present the results. In section 4 and  
80 5 we discuss our findings, their relation to previous studies, and trace out a plan for fu-  
81 ture studies.

## 82 2 Methodology

### 83 2.1 Datasets

84 The data are obtained from the OMNI database (<http://omniweb.gsfc.nasa.gov>), which  
85 provides estimates of solar wind parameters at the bow shock nose (Farris & Russell, 1994)  
86 by propagating observations performed by several spacecraft further upstream (King &  
87 Papitashvili, 2005) as well as measures of geomagnetic activity. We focus primarily on  
88 the *AL* index which is notoriously difficult to predict Newell, Sotirelis, Liou, Meng, and  
89 Rich (2007), presumably because of inherent nonlinearities in its dynamics.

90 *AL* provides an estimate of the maximum westward electrojet intensity using 12 mag-  
91 netometer stations around the northern auroral region (Berthelier & Menvielle, 1993).  
92 Outside of substorm intervals, *AL* can be thought of as a measure of convection, while  
93 during substorms the largest deviations in the horizontal component typically originate  
94 from the substorm current wedge. By nature, *AL* is therefore highly asymmetric and peaks  
95 at low values, reflecting quiet time convection effects and heavy tails associated with sub-  
96 storms occurrences (Newell et al., 2007; Tanskanen, Pulkkinen, Koskinen, & Slavin, 2002).

### 97 2.2 Permutation Entropy

98 Permutation entropy was proposed by Bandt and Pompe (2002) as a complexity  
99 measure for arbitrary timeseries, that is, stationary or non-stationary, deterministic or  
100 stochastic, periodical or noisy. However, it should be pointed that a weak form of sta-  
101 tionary assumption is required, i.e., for  $s < d$ , the probability for  $x_t < x_{t+s}$  should  
102 not depend on  $t$  (Rosso et al., 2013). The Bandt-Pompe permutation entropy is com-  
103 puted on the basis of a probability distribution quantifying the rate of occurrence of *am-*  
104 *plitude orderings* in a time signal  $\mathcal{T}(t) \equiv \{x_t; t = 1, \dots, N\}$  measured at  $N$  evenly spaced  
105 discrete points. Computation of the probability is done for an embedding space of di-  
106 mension  $d$ , which translates in determining patterns of length  $d$  in the order in which

they appear in the timeseries (For readers not familiar with the idea of an embedding dimension and delay we recommend the book by (Ott, 2002, Section 3.8)). The  $d$ -values are called  $d$ -tuples. For instance, for  $d = 3$ , a number of  $d! = 6$  possible sequences are possible, i.e. (1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2) and (3,2,1). For a signal with  $N$  elements, the relative frequency of each of the possible sequences are computed for three successive values of the time series. The number of successive values for embedding dimension  $d$  and signal of  $N$  elements consists in  $1 \leq n \leq N - d + 1$  distinct  $d$ -tuples ordered as  $t_j, t_{j+1} \dots t_{j+d-1}$ . Similarly, for the embedding dimension chosen in our analysis, i.e.  $d = 6$ , a signal with  $N = 10000$  elements has  $1 \leq n \leq N - d + 1 = 9995$  distinct  $d$ -tuples  $t_j, t_{j+1} \dots t_{j+5}$ , or ordinal patterns. Within each  $d$ -tuple, an ordering of the amplitude is obtained as a function of the  $d!$  possible permutations, e.g. 720 permutations for  $d = 6$ . The permutation entropy is then computed for a particular signal by computing the frequency of occurrence of each possible permutations of the amplitude ordering. For a set of probabilities  $P$ , of dimension  $d!$  and probability of occurrence  $p_j \geq 0; j = 1, 2 \dots d!$ , the permutation entropy is defined in terms of the Shannon entropy,  $S$ , as

$$S(P) = - \sum_{j=1}^{d!} p_j \log(p_j). \quad (1)$$

In the following we use the normalized Shannon entropy,  $H$ , defined as

$$H(P) = S(P) / \log(d!) = S(P) / P_e, \quad (2)$$

and the Shannon entropy per symbol,  $h_n$ , defined as

$$h_n(P) = \frac{S(P)}{d - 1}. \quad (3)$$

The denominator in the equation of  $H(P)$  is the maximum Shannon entropy obtained when all states have equal probabilities, i.e.  $p_j = 1/d!; \forall j$  and this maximum probability is here denoted as  $P_e$ . The fundamental underlying idea behind the Bandt-Pompe permutation entropy is that some ordinal patterns may be forbidden, whereas others may be favoured, making the information content less random than in stochastic systems. In theory, one might therefore be able to differentiate stochastic and deterministic fluctuations through the use of the permutation entropy.

However, as with all mathematical tools, one has to be aware of the advantages and limitations. In terms of advantages, the permutation entropy incorporates temporal order and is computationally very fast (Riedl et al., 2013). Additionally it is invariant under

135 any monotonic transformation of the timeseries, e.g. scaling the data has no effect on  
 136 the resulting distribution of permutation patterns. A consequence of the latter property  
 137 is that the permutation entropy does not preserve information of the amplitude in the  
 138 ordinal patterns. But more importantly, finite timeseries constrain the choice of the em-  
 139 bedding dimension  $d$ . Since the number of possible amplitude permutations increases rapidly  
 140 as  $d!$ , the value of  $d$  must be chosen such that  $N \gg d!$ , i.e. the number of points in our  
 141 time series must be sufficiently large for us to sample the relative distribution of each  
 142  $d!$  permutations. In our case, we use  $d = 6$  for timeseries with  $N = 43200$  points, cor-  
 143 responding to 30 days of 1 minute sampling. The maximum embedded delay corresponds  
 144 to  $\tau = 600$  minutes, and the minimum number of segments corresponds to  $N - (d -$   
 145  $1)\tau > 40000$  (Applying our analysis to delays ranging from days to months none of the  
 146 conclusions presented hereafter were modified. However it was not possible to test the  
 147 methodology for timescales ranging from years to solar cycle periods due to data gaps).  
 148 Hence, we use more than 40000 segments to distinguish the frequency of  $d! = 720$  pat-  
 149 terns. Consequently, for a given collection of  $d$ -tuplets, the size in time of the structures,  
 150 or patterns, investigated is  $d\Delta t$ , where  $\Delta t$  is the sampling time. In order to study struc-  
 151 tures with size  $d\Delta t \geq 10$  it is often not practically possible to increase the embedding  
 152 dimension beyond  $d = 7$  since  $8! = 40320$  and one must keep in mind the requirement  
 153 that  $N \gg d!$ . In case of auroral indices, setting an embedding dimension of 8 would  
 154 require a timeseries of length  $N > 4 \cdot 10^6$ , and corresponding to 280 days of 1 minute  
 155 sampled data. Large structures can nonetheless still be investigated by adding an ad-  
 156 ditional parameter  $\tau$  to sub-sample the timeseries. In the sub-sampled signal the inter-  
 157 val between successive data point is  $d\tau$  rather  $\Delta t$ . This technique naturally reduces the  
 158 Nyquist frequency and the number of points to  $N/\tau$ , but preserves the total time of the  
 159 signal (Maggs & Morales, 2013; Weck et al., 2015). For an embedding dimension  $d >$   
 160 2 and embedded delay  $\tau$ , a timeseries with  $N$  points contains  $N - (d - 1)\tau$  segments  
 161 upon which the  $d!$  permutations are computed. Once again, one needs to be very care-  
 162 ful in selecting a sufficiently large number of points to make sure that all possible per-  
 163 mutations can be accounted for. If one only has 1000 segments available to sample the  
 164 relative occurrence of 720 permutations, it is highly unlikely that even if the timeseries  
 165 is stochastic, that all possible permutations would be appropriately sampled. One might  
 166 therefore conclude, incorrectly, that some patterns are forbidden, and that the result-  
 167 ing entropy might be indicative of a deterministic timeseries. The timeseries length must

168 be sufficiently large in order for all permutation patterns to be measured and thereby  
 169 confirm the deterministic properties of the timeseries. This last constraint is particularly  
 170 important when studying coloured noise with very long autocorrelation times. The per-  
 171 mutation entropy analysis can be rendered useless if  $N$  is not sufficiently large and one  
 172 is therefore forced to seek alternative approaches to differentiate stochastic from deter-  
 173 ministic fluctuations.

### 174 2.3 Jensen-Shannon Complexity

175 A solution to supersede this last limitation of the permutation entropy and a means to  
 176 distinguish between long correlated noise and deterministic timeseries, was outlined by  
 177 Rosso et al. (2007). Using Shannon's formulation of entropy for ordinal patterns and the  
 178 Jensen-Shannon complexity as a measure of statistical complexity, Rosso et al. (2007)  
 179 have shown that despite common properties (wide-band power spectrum, irregular be-  
 180 havior of measured signals) it is after all possible to distinguish between stochastic and  
 181 chaotic signals from their location in terms of an entropy-complexity plane. Hence, Rosso  
 182 et al. (2007) combine the complexity measure of Bandt and Pompe and the Jensen-Shannon  
 183 complexity, here defined as

$$C_J^S = D(P) \times H(P) = -2 \frac{S\left(\frac{P+P_e}{2}\right) - \frac{1}{2}S(P) - \frac{1}{2}S(P_e)}{\frac{d!+1}{d!} \log(d!+1) - 2 \log(2d!) + \log(d!)} H(P). \quad (4)$$

184 This complexity measure is the product of the normalised Shannon entropy,  $H(P)$ , and  
 185 the Jensen divergence,

$$D(P) = S\left(\frac{P+P_e}{2}\right) - \frac{1}{2}S(P) - \frac{1}{2}S(P_e), \quad (5)$$

186 hence the Jensen-Shannon denomination. The argument in the denominator serves as  
 187 normalisation constant for the Jensen divergence. The divergence can be interpreted as  
 188 the distance between our distribution of ordinal patterns and the distribution that max-  
 189 imises the Shannon distribution, i.e.  $P_e$  defined above. It is easy to see that it takes the  
 190 value of zero when  $P = P_e$ , that is, when all ordinal patterns are equally likely the Jensen-  
 191 Shannon complexity is zero. Built from the square of the Shannon entropy, it has a parabolic  
 192 shape when plotted against the permutation entropy, but more crucially, the Jensen-Shannon  
 193 complexity can hold multiple values for a fixed Shannon entropy. It is this particular prop-  
 194 erty that allows one to distinguish stochastic noise with long auto-correlation times to  
 195 deterministic and chaotic fluctuations. Thus, a fixed entropy value maps into a range  
 196 of Jensen-Shannon complexity values and one can differentiate between regimes that are

highly deterministic or highly stochastic and everything in-between. For more details on the permutation entropy and Jensen-Shannon complexity we refer to the reviews of Riedl et al. (2013) and Zanin, Zunino, Rosso, and Papo (2012).

### 3 Results

#### 3.1 Choice of chaotic and stochastic timeseries

In this study we benchmark our results for geomagnetic indices with the Lorenz chaotic attractor (Ott, 2002) :

$$\dot{X} = a(Y - X); \quad \dot{Y} = X(c - Z) - Y; \quad \dot{Z} = XY - bZ, \quad (6)$$

and fractional Brown motion (fBm) with Hurst exponent  $h_u \in [0.01, 1]$  (Mandelbrot & Van Ness, 1968). We note that the choice of the Lorenz attractor rather than other well-known chaotic dynamical systems has no effect on our results. The reader can find a longer list of chaotic maps plotted in the complexity-entropy plane in reports by Rosso et al. (2007) and Maggs and Morales (2013) showing a clear demarcation between stochastic and chaotic timeseries. The parameters for the Lorenz attractor are  $a = 10, b = 8/3, c = 28$ . Figure 1 top two panels show the timeseries for  $AL$  in black, and  $\delta AL = AL_{j+1} - AL_j$  in magenta for four months time interval. Time series for the Lorenz strange attractor used for this study are shown as blue traces in Figure 1 and an example for the fBm with Hurst exponent  $h_u = 0.8$  is shown on the bottom right panel of Figure 1 in red. The fBm is stochastic but can nonetheless be structured and contains trends (either persistent or anti-persistent), and is used as the boundary delimiting stochastic and chaotic timeseries.

#### 3.2 Permutation entropy analysis

In Figure 2, we plot the permutation entropy per symbol,  $h_n$ , for a stochastic, a chaotic and the AL time series as a function of embedded delay  $\tau$  and embedded dimension  $3 \leq d \leq 7$ . Fractional Brownian motion with Hurst exponent of 0.75 is on the top panel. As with very other stochastic timeseries the permutation entropy is approximately constant across various  $\tau$  and  $d$ . The small increase in the entropy is due to the fact that our choice of stochastic process has long auto-correlation times. Thus, for very long time delays, patterns become marginally more decorrelated.



225 In contrast, the centre panel for the chaotic Lorenz attractor shows a minimum permu-  
 226 tation entropy for  $\tau = 1$ . The permutation entropy then increases linearly with  $\tau$ , un-  
 227 til for sufficiently large delay of  $\tau > 15$  the patterns become decorrelated. We also no-  
 228 tice from the Lorenz attractor panel that for  $\tau < 10$  the curves for  $d = 7$  and  $d = 6$   
 229 overlap. Hence, embedded dimension  $d = 6$  is sufficient to track all the possible per-  
 230 mutations for the Lorenz attractor. Or, put differently, increasing the embedded dimen-  
 231 sion to  $d > 6$  does not provide more information about the complexity of the timeseries.  
 232 Increasing the embedding dimension increases the range of patterns that are sampled  
 233 in the timeseries. If one is analyzing a dynamical system with embedding dimension of  
 234 6 or 7 with a parameter  $d = 3 - 4$ , one will miss some of the possible permutations.  
 235 Thus, the result plotted for the Lorenz attractor is not surprising, since according to the  
 236 well-known Takens' theorem (Ott, 2002), the embedded dimension must scale as  $d =$   
 237  $2d_S + 1$  where  $d_S$  is the dimension of the strange attractor, which is well-known to be  
 238 between 2.03 and 2.06.

239 It is clear that the profile for  $AL$ , in the bottom panel, resembles the stochastic fractional  
 240 Brownian motion with persistent increments. However, the absence of overlap for the  
 241 various curves, as seen for the Lorenz attractor, does not necessarily imply that  $AL$  is  
 242 stochastic. Instead, it could indicate that a higher embedded dimension, i.e.  $d > 7$ , is  
 243 needed to samples all the patterns (Rosso et al., 2013). However, it is not always pos-  
 244 sible to pick  $d > 7$ , and we resort to the methodology of Rosso et al. (2007) to deter-  
 245 mine possible differences between  $AL$  and stochastic fluctuations.

### 246 3.3 Jensen-Shannon complexity plane

247 Before making use of the complexity-entropy plane, we first present the Jensen-Shannon  
 248 complexity measure for  $AL$  as a function of embedded dimension and delay. The results  
 249 are shown in Figure 3. The Jensen-Shannon complexity is computed against the embed-  
 250 ded delay  $\tau$  on the abscissa for all twelve months of the year 2010. The colour represents  
 251 the embedding dimension ranging between 3 and 6 with the same legend as in Figure  
 252 2. Note that the complexity decreases for growing embedded delay. Hence, ordinal pat-  
 253 terns are more correlated on small timescales ( $\tau < 1$  hour), and become decorrelated  
 254 for  $\tau > 200 - 240$  minutes. Since the embedding dimension of  $AL$  is high, the com-  
 255 plexity curves for  $d = 6$  and  $d = 5$  highlights local maximum and minimum that are  
 256 missed by the  $d = 4$  and  $d = 3$  curve. For all months except May, June, July and Novem-

ber, we notice an enhancement in the complexity for  $14 < \tau < 40$  and  $d = 6$  after the initial monotonic decrease in complexity. A local maximum in complexity is particularly pronounced for the months of August and September. This indication of the presence of correlational structures with timescales ranging between 10 and 40 minutes is not a new result (Osmane, Dimmock, Naderpour, Pulkkinen, & Nykyri, 2015) and will be discussed in the next section.

In Figure 4, we plot the complexity-entropy plane for  $d = 6$ , with minimum and maximum complexity-entropy curves in blue. The complexity-entropy points for fBm are plotted in red circles for  $\tau = 1$  and Hurst exponents ranging between 0.01 and 1 by steps of 0.01. The points for fBm indicate a limit between stochastic and structured timeseries. The complexity-entropy values for the chaotic attractor are computed and plotted in blue triangles for the variable  $X$ , for  $d = 6$  and  $\tau = [1-10]$ . The complexity-values for the Lorentz attractor (cyan squares) have the smallest entropy for  $\tau = 1$  and the largest for  $\tau = 60$ . We clearly see from Figure 4 that the complexity-entropy values for the Lorenz chaotic orbits skim the maximum curve, that is for large entropies, the orbits of a chaotic attractor have large complexity values.

Similarly, we plot on the same figure the complexity-entropy curve for both  $AL$  (black stars) and  $\delta AL = \text{diff}(AL)$  (magenta dots) for  $d = 6$  and time delay values ranging between  $\tau = 4$  and  $\tau = 600$  minutes by increments of 2 minutes. The lowest entropy values for  $AL$  and  $\delta AL$  are computed for  $\tau = 4$ , while the larger entropy values are for large  $\tau > 500$ . Figure 4 indicates that complexity-entropy values of  $AL$  overlap the fBm values for all sub-sampling parameters  $\tau$ . As we increase  $\tau$ , the entropy for  $AL$  increases, and the time-series becomes indistinguishable from fractional Brownian motion with anti-persistent increments, i.e., with Hurst exponents less than 0.5.

It is natural to ask if the observed characterisation of  $AL$  structures ranging between a few minutes to several hours is shared by other auroral current indices. In Figures 5 and 6, we show the dependence of the Jensen-Shannon complexity for all 12 months of 2010 for  $AE$  and  $AU$  respectively. The range of parameters  $d, \tau$  and the legend are the same as in Figure 3. Once again the complexity value is relatively small for both indices. It peaks at low  $\tau$  and decays for large  $\tau$  values. Similarly to  $AL$ , we note that  $AE$  also experiences a local maximum in complexity values for embedded delay ranging between  $\tau \simeq 10$  and  $\tau \simeq 50$ , albeit more pronounced and for different months (with the most

obvious month being April). The complexity for  $AU$  on the other hand experiences local maximum for embedded delay ranging between  $\tau \simeq 30$  and  $\tau \simeq 200$  minutes, with the notable exception of June, July and November where the complexity monotonically decreases until  $\tau \simeq 200$  and plateau at very low values thereafter. In Figure 7 we have reproduced the complexity-entropy plane for  $AL$  (blue stars) and fBm (red dots) and complemented it with values for  $AE$  (black circles) and  $AU$  (magenta lozenges) across the month of August 2010. Similarly as for  $AL$ ,  $AU$  and  $AE$  are highly stochastic across timescales ranging between a few minutes to 10 hours. We note that the same conclusion are equally valid for any other set of months (not shown).

#### 4 Discussion and Conclusion

Using the permutation entropy developed by Bandt and Pompe (2002) and the complexity-entropy plane methodology developed by Rosso et al. (2007) we have demonstrated that geomagnetic indices have larger complexity (structures) and lower entropy (uncertainty) on small timescales of  $\tau < 10$  than on timescales of  $\tau > 10$  minutes. Nonetheless, auroral geomagnetic indices are indistinguishable from stochastic processes, overlapping with fractional Brownian processes on timescales ranging between a few minutes to 10 hours. Our results are therefore *inconsistent* with earlier studies of (Baker et al., 1990; Pavlos et al., 1992; D. Roberts et al., 1991; D. A. Roberts, 1991; Vassiliadis et al., 1991; Vassiliadis et al., 1990) indicating that low-dimensional dynamical systems with chaotic properties might arise in geomagnetic current patterns.

In a similar study, Consolini and De Michelis (2014) also use permutation entropy as a measure of complexity to study the statistical properties of  $SYM-H$  timeseries spanning the period of January 2000 to December 2004. In their study, Consolini and De Michelis (2014) showed that permutation entropy computed on moving time windows was capable of capturing the rapid and local dynamical changes of  $SYM-H$ . During storms, intermittency and the non-stationary nature of the fluctuations of  $SYM-H$  was shown to correlate with lower permutation entropy and higher complexity during quiet times. This result is consistent with Figures 4 and 7 showing that for lower  $\tau$  values, structures in auroral currents have higher complexity and lower entropy.

Following the work of Balasis et al. (2006); Consolini, De Marco, and De Michelis (2013) and Osmane et al. (2015), our study also provides additional means to characterise large-

320 scale and small-scale fluctuations originating in different physical processes. In a statis-  
321 tical study covering 17 years of OMNI data, (Osmane et al., 2015) showed that prob-  
322 ability distribution functions of AL responded in a nontrivial yet coherent fashion to var-  
323 ious solar wind properties and ULF fluctuation amplitudes. For strongly southward IMF,  
324 the AL distribution was characterised by a decrease of the skewness, a shift of the peak  
325 from  $-30$  nT to  $-200$  nT and a broadening of the distribution core. During northward  
326 IMF, the distribution in AL was instead characterised by a large reduction in the stan-  
327 dard deviation and weight in the tail. Despite the different responses of the distribution  
328 function of AL for northward and southward IMF, the non-Gaussian changes were all  
329 occurring on timescales ranging between 10 and 40 minutes, similarly to the larger com-  
330 plexity structures observed in *AL* and *AE* on comparable timescales (i.e. comparable  $\tau$   
331 values), and associated with intermittent fluctuations. In Osmane et al. (2015), the au-  
332 thors argued that the non-Gaussian properties in the PDF of *AL* occurring on timescales  
333 of the order of  $\tau \sim 10 - 20$  minutes could be driven in part by viscous processes (Ax-  
334 ford & Hines, 1961), such as Kelvin-Helmholtz instability (Nykyri & Otto, 2001) and ki-  
335 netic Alfvén waves (Johnson & Cheng, 1997, 2001).

336 Whereas the mapping of auroral indices into the complexity-entropy plane was done in-  
337 dependently of solar wind properties, follow-up studies could combine the methodology  
338 described in (Osmane et al., 2015), and the one presented here to distinguish coherent  
339 geomagnetic responses to upstream solar wind conditions from internal magnetospheric  
340 dynamic processes. Future studies will be extended to other geomagnetic indices and fo-  
341 cus particularly on different solar wind driving conditions that might explain the enhance-  
342 ment in complexity on small timescales of  $\tau < 40$  minutes. Of particular interest, de-  
343 lineating storms in terms of solar wind conditions and statistics might indicate the con-  
344 tribution of magnetospheric dynamics in the triggering of storm activity and the nature  
345 of the nonlinear driving on timescales of minutes where viscous processes take place (Ax-  
346 ford & Hines, 1961; Chaston et al., 2007; Freeman, Warren, & Maguire, 1968; Hasegawa  
347 et al., 2004, 2006; Johnson & Cheng, 1997, 2001; Lee, Johnson, & Ma, 1994; Nykyri &  
348 Otto, 2001; Nykyri et al., 2006), to hours where geomagnetic storms unfold (T. Pulkki-  
349 nen, 2007).

350 Finally, it should be kept in mind that auroral geomagnetic indices do not necessarily  
351 account for the detailed spatial variations of the magnetic field. For instance, as the au-  
352 roral electrojet expands in large storms, high latitude observatories at which *AE* is de-

353 rived experience lower magnetic field variations. Additionally, auroral geomagnetic in-  
 354 dices cannot capture the complexity of the wider magnetospheric system since they are  
 355 constructed as a multi-dimensional mapping of several observatories and reduced to a  
 356 single proxy parameter. In the case of AL, it serves as a proxy for the energy transmit-  
 357 ted into the ionosphere. We can therefore not exclude the possibility that spatial and/or  
 358 temporal variations associated with various magnetospheric processes could be modelled  
 359 in terms of a deterministic set of equations, albeit one that is not as low-dimensional as  
 360 previous authors suggested. Rather, our analysis provides an answer to a much narrower  
 361 question: Can we model fluctuations in auroral geomagnetic indices as low-dimensional  
 362 chaotic attractors, and consequently reduce a system *a priori* composed of a very large  
 363 number degrees of freedom to one with a few degrees of freedom? Though our answer  
 364 is undoubtedly in the negative, our analysis does not preclude the existence of a high-  
 365 dimensional chaotic systems or one based on self-organised critical models (Sharma et  
 366 al, 2016).

### 367 Acknowledgments

368 This work was supported by Academy of Finland grants #267073/2013 and #297688/2015.  
 369 AO would also like to thank James Weygand and Joe Borovsky for useful discussions on  
 370 the use of the complexity-entropy approach to space data. The OMNI data used in this  
 371 paper is available at <http://omniweb.gsfc.nasa.gov> free of charge

### 372 References

- 373 Axford, W. I., & Hines, C. O. (1961). A unifying theory of high-latitude geophysical  
 374 phenomena and geomagnetic storms. *Can. J. Phys.*, *39*, 1433. doi: 10.1139/  
 375 p61-172
- 376 Baker, D., Klimas, A., McPherron, R., & Büchner, J. (1990). The evolution from  
 377 weak to strong geomagnetic activity: An interpretation in terms of determinis-  
 378 tic chaos. *Geophysical research letters*, *17*(1), 41–44.
- 379 Balasis, G., Daglis, I. A., Kapiris, P., Manda, M., Vassiliadis, D., & Eftaxias, K.  
 380 (2006, December). From pre-storm activity to magnetic storms: a transition  
 381 described in terms of fractal dynamics. *Annales Geophysicae*, *24*, 3557-3567.  
 382 doi: 10.5194/angeo-24-3557-2006
- 383 Bandt, C., & Pompe, B. (2002, April). Permutation Entropy: A Natural Complex-

- 384           ity Measure for Time Series. *Physical Review Letters*, 88(17), 174102. doi: 10  
385           .1103/PhysRevLett.88.174102
- 386   Berthelier, A., & Menvielle, M. (1993). Geomagnetic data 1987, iaga indices: aa, am,  
387           kp, dst, ae, rapid variations. *IAGA Bulletin*, 32.
- 388   Chang, T. (1992). Low-dimensional behavior and symmetry breaking of stochas-  
389           tic systems near criticality-can these effects be observed in space and in the  
390           laboratory? *Plasma Science, IEEE Transactions on*, 20(6), 691–694.
- 391   Chaston, C., Wilber, M., Mozer, F., Fujimoto, M., Goldstein, M., Acuna, M., . . .  
392           Fazakerley, A. (2007). Mode conversion and anomalous transport in kelvin-  
393           helmholtz vortices and kinetic alfvén waves at the earth's magnetopause.  
394           *Physical review letters*, 99(17), 175004.
- 395   Consolini, G., De Marco, R., & De Michelis, P. (2013). Intermittency and multi-  
396           fractional brownian character of geomagnetic time series. *Nonlinear Processes*  
397           *in Geophysics*, 20(4), 455–466.
- 398   Consolini, G., & De Michelis, P. (2014). Permutation entropy analysis of com-  
399           plex magnetospheric dynamics. *Journal of Atmospheric and Solar-Terrestrial*  
400           *Physics*, 115, 25–31.
- 401   Consolini, G., & Marcucci, M. (1997). Multifractal structure and intermittence  
402           in the ae index time series. *Nuovo cimento della Società italiana di fisica. C*,  
403           20(6), 939–949.
- 404   Dobias, P., & Wanliss, J. A. (2009, May). Intermittency of storms and substorms: is  
405           it related to the critical behaviour? *Annales Geophysicae*, 27, 2011-2018. doi:  
406           10.5194/angeo-27-2011-2009
- 407   Farris, M. H., & Russell, C. T. (1994, September). Determining the standoff dis-  
408           tance of the bow shock: Mach number dependence and use of models. *J. Geo-*  
409           *phys. Res. Space Phys.*, 99, 17681. doi: 10.1029/94JA01020
- 410   Freeman, J., Warren, C., & Maguire, J. (1968). Plasma flow directions at the  
411           magnetopause on january 13 and 14, 1967. *Journal of Geophysical Research*,  
412           73(17), 5719–5731.
- 413   Gollub, J. P., & Swinney, H. L. (1975). Onset of turbulence in a rotating fluid.  
414           *Physical Review Letters*, 35(14), 927.
- 415   Grassberger, P., & Procaccia, I. (1983). Characterization of strange attractors. *Phys-*  
416           *ical review letters*, 50(5), 346.

- 417 Hasegawa, H., Fujimoto, M., Phan, T.-D., Reme, H., Balogh, A., Dunlop, M., . . .  
 418 TanDokoro, R. (2004). Transport of solar wind into earth's magnetosphere  
 419 through rolled-up kelvin-helmholtz vortices. *Nature*, *430*(7001), 755–758.
- 420 Hasegawa, H., Fujimoto, M., Takagi, K., Saito, Y., Mukai, T., & Rème, H. (2006).  
 421 Single-spacecraft detection of rolled-up kelvin-helmholtz vortices at the flank  
 422 magnetopause. *Journal of Geophysical Research: Space Physics (1978–2012)*,  
 423 *111*(A9).
- 424 Johnson, J. R., & Cheng, C. Z. (1997). Kinetic Alfvén waves and plasma trans-  
 425 port at the magnetopause. *Geophys. Res. Lett.*, *24*, 1423-1426. doi: 10.1029/  
 426 97GL01333
- 427 Johnson, J. R., & Cheng, C. Z. (2001). Stochastic ion heating at the magnetopause  
 428 due to kinetic Alfvén waves. *Geophys. Res. Lett.*, *28*, 4421-4424. doi: 10.1029/  
 429 2001GL013509
- 430 King, J. H., & Papitashvili, N. E. (2005, February). Solar wind spatial scales in and  
 431 comparisons of hourly Wind and ACE plasma and magnetic field data. *J. Geo-*  
 432 *phys. Res. Space Phys.*, *110*, 2104. doi: 10.1029/2004JA010649
- 433 Klimas, A. J., Valdivia, J., Vassiliadis, D., Baker, D. N., Hesse, M., & Takalo, J.  
 434 (2000, August). Self-organised criticality in the substorm phenomenon and  
 435 its relation to localised reconnection in the magnetospheric plasma sheet. *J.*  
 436 *Geophys. Res. Space Phys.*, *105*, 18765-18780. doi: 10.1029/1999JA000319
- 437 Klimas, A. J., Vassiliadis, D., Baker, D. N., & Roberts, D. A. (1996, June). The  
 438 organized nonlinear dynamics of the magnetosphere. *J. Geophys. Res. Space*  
 439 *Phys.*, *101*, 13089-13114. doi: 10.1029/96JA00563
- 440 Landau, L. D. (1944). On the problem of turbulence. In *Dokl. akad. nauk sssr*  
 441 (Vol. 44, pp. 339–349).
- 442 Lee, L., Johnson, J. R., & Ma, Z. (1994). Kinetic alfvén waves as a source of plasma  
 443 transport at the dayside magnetopause. *Journal of Geophysical Research:*  
 444 *Space Physics (1978–2012)*, *99*(A9), 17405–17411.
- 445 Lovejoy, S., & Schertzer, D. (1998). Stochastic chaos and multifractal geophysics.  
 446 *Chaos, Fractals and models*, *96*, 38–52.
- 447 Maggs, J., & Morales, G. (2013). Permutation entropy analysis of temperature fluc-  
 448 tuations from a basic electron heat transport experiment. *Plasma Physics and*  
 449 *Controlled Fusion*, *55*(8), 085015.

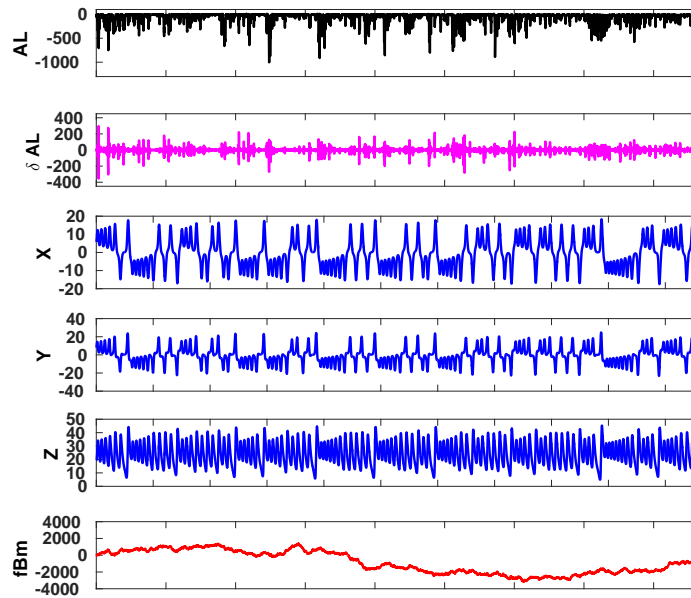


- 450 Mandelbrot, B. B., & Van Ness, J. W. (1968). Fractional brownian motions, frac-  
451 tional noises and applications. *SIAM review*, *10*(4), 422–437.
- 452 Newell, P., Sotirelis, T., Liou, K., Meng, C.-I., & Rich, F. (2007). A nearly universal  
453 solar wind-magnetosphere coupling function inferred from 10 magnetospheric  
454 state variables. *Journal of Geophysical Research: Space Physics (1978–2012)*,  
455 *112*(A1).
- 456 Nykyri, K., & Otto, A. (2001). Plasma transport at the magnetospheric boundary  
457 due to reconnection in kelvin-helmholtz vortices. *Geophys. Res. Lett.*, *28*(18),  
458 3565–3568.
- 459 Nykyri, K., Otto, A., Lavraud, B., Mouikis, C., Kistler, L. M., Balogh, A., & Rème,  
460 H. (2006, October). Cluster observations of reconnection due to the Kelvin-  
461 Helmholtz instability at the dawnside magnetospheric flank. *Ann. Geophys.*,  
462 *24*, 2619-2643. doi: 10.5194/angeo-24-2619-2006
- 463 Osborne, A. R., & Provenzale, A. (1989). Finite correlation dimension for stochas-  
464 tic systems with power-law spectra. *Physica D: Nonlinear Phenomena*, *35*(3),  
465 357–381.
- 466 Osmane, A., Dimmock, A., Naderpour, R., Pulkkinen, T., & Nykyri, N. (2015). The  
467 impact of solar wind ulf  $b_z$  fluctuations on geomagnetic activity for viscous  
468 timescales during strongly northward and southward imf. *J. Geophys. Res.*,  
469 2169-9402. doi: 10.1002/2015JA021505
- 470 Ott, E. (2002). *Chaos in dynamical systems*. Cambridge university press.
- 471 Pavlos, G., Kyriakou, G., Rigas, A., Liatsis, P., Trochoutsos, P., & Tsonis, A.  
472 (1992). Evidence for strange attractor structures in space plasmas. In *An-*  
473 *nales geophysicae* (Vol. 10, pp. 309–322).
- 474 Prichard, D., & Price, C. (1992). Spurious dimension estimates from time series of  
475 geomagnetic indices. *Geophysical research letters*, *19*(15), 1623–1626.
- 476 Pulkkinen, A., Klimas, A., Vassiliadis, D., & Uritsky, V. (2006). Role of stochas-  
477 tic fluctuations in the magnetosphere-ionosphere system: A stochastic model  
478 for the ae index variations. *Journal of Geophysical Research: Space Physics*  
479 *(1978–2012)*, *111*(A10).
- 480 Pulkkinen, T. (2007). Space weather: terrestrial perspective. *Living Rev. Solar*  
481 *Phys*, *4*(1).
- 482 Riedl, M., Muller, A., & Wessel, N. (2013). Practical considerations of permuta-

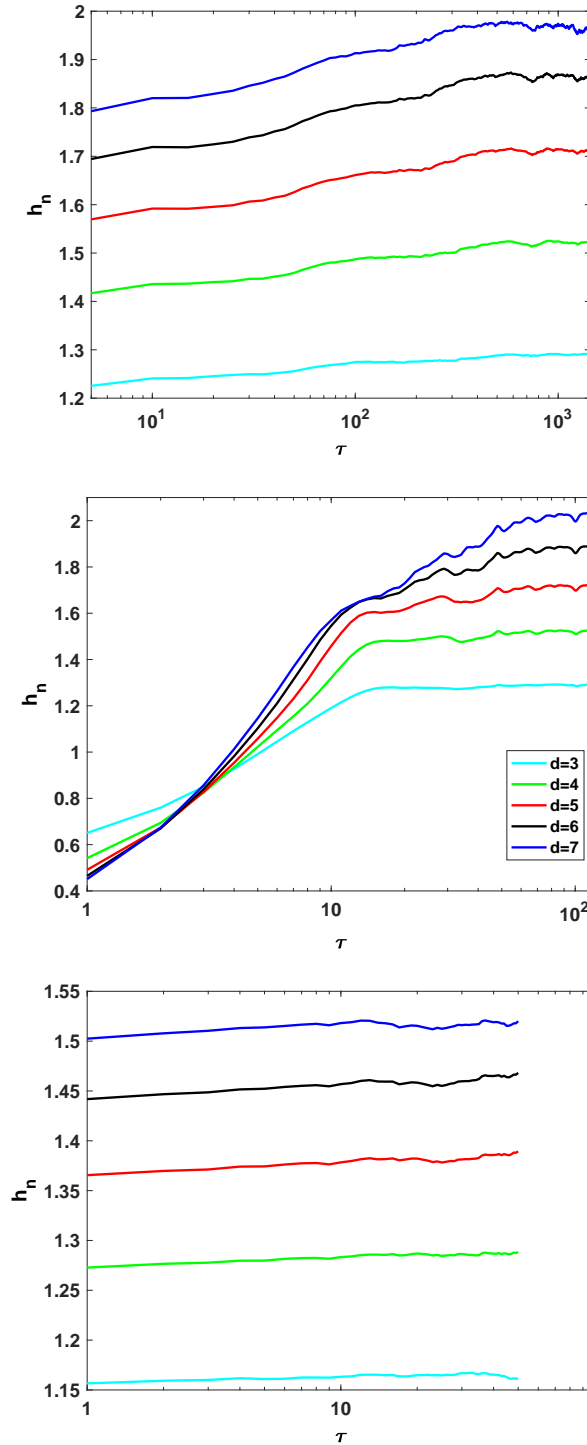


- 483 tion entropy. *Eur. Phys. J. Special Topics*, *222*, 249. doi: 10.1140/epjst/e2013  
484 -01862
- 485 Roberts, D., Baker, D., Klimas, A., & Bargatze, L. (1991). Indications of low dimen-  
486 sionality in magnetospheric dynamics. *Geophysical Research Letters*, *18*(2),  
487 151–154.
- 488 Roberts, D. A. (1991). Is there a strange attractor in the magnetosphere? *Journal of*  
489 *Geophysical Research: Space Physics (1978–2012)*, *96*(A9), 16031–16046.
- 490 Rosso, O. A., Larrondo, H. A., Martin, M. T., Plastino, A., & Fuentes, M. A. (2007,  
491 October). Distinguishing Noise from Chaos. *Physical Review Letters*, *99*(15),  
492 154102. doi: 10.1103/PhysRevLett.99.154102
- 493 Rosso, O. A., Olivares, F., Zunino, L., De Micco, L., Aquino, A. L. L., Plastino, A.,  
494 & Larrondo, H. A. (2013, April). Characterization of chaotic maps using the  
495 permutation Bandt-Pompe probability distribution. *European Physical Journal*  
496 *B*, *86*, 116. doi: 10.1140/epjb/e2013-30764-5
- 497 Ruelle, D., & Takens, F. (1971). On the nature of turbulence. *Commun. math. phys.*,  
498 *20*(3), 167–192.
- 499 Shan, L.-H., Hansen, P., Goertz, C., & Smith, R. (1991). Chaotic appearance of the  
500 ae index. *Geophysical Research Letters*, *18*(2), 147–150.
- 501 Sharma et al, A. S. (2016). 25 years of self-organized criticality: space and labora-  
502 tory plasmas. *Space Sci. Rev.*, *198*, 167-216. doi: 10.1007/s11214-015-0225-0
- 503 Tanskanen, E., Pulkkinen, T., Koskinen, H., & Slavin, J. (2002). Substorm energy  
504 budget during low and high solar activity: 1997 and 1999 compared. *Journal*  
505 *of Geophysical Research: Space Physics (1978–2012)*, *107*(A6), SMP–15.
- 506 Uritsky, V., Pudovkin, M., & Steen, A. (2001). Geomagnetic substorms as perturbed  
507 self-organized critical dynamics of the magnetosphere. *Journal of Atmospheric*  
508 *and Solar-Terrestrial Physics*, *63*(13), 1415–1424.
- 509 Uritsky, V. M., Klimas, A. J., & Vassiliadis, D. (2006). Analysis and prediction  
510 of high-latitude geomagnetic disturbances based on a self-organized criticality  
511 framework. *Advances in Space Research*, *37*(3), 539–546.
- 512 Uritsky, V. M., & Pudovkin, M. I. (1998, December). Low frequency 1/f-like  
513 fluctuations of the AE-index as a possible manifestation of self-organized  
514 criticality in the magnetosphere. *Annales Geophysicae*, *16*, 1580-1588. doi:  
515 10.1007/s00585-998-1580-x

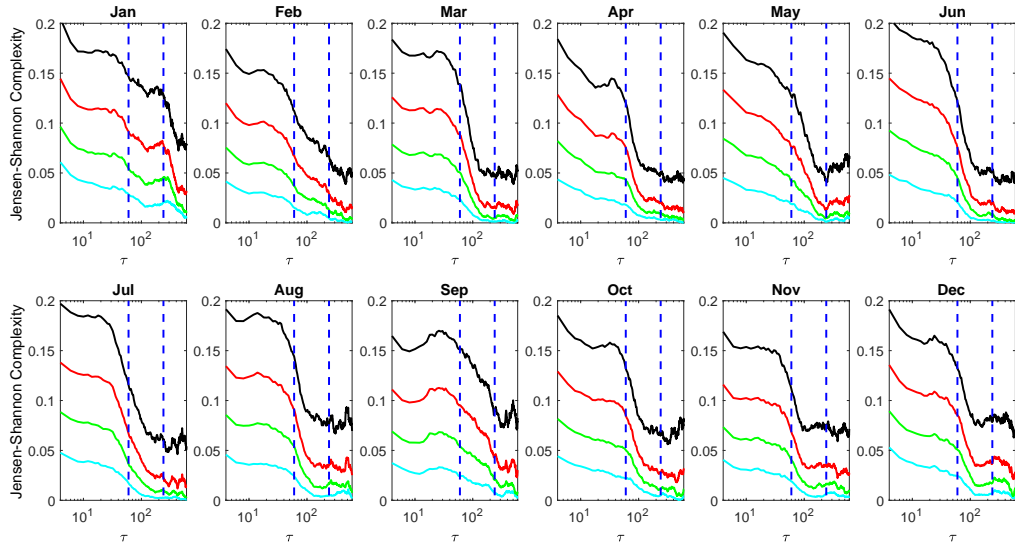
- 516 Valdivia, J., Klimas, A., Vassiliadis, D., Uritsky, V., & Takalo, J. (2003). Self-  
517 organization in a current sheet model. In *Advances in space environment*  
518 *research* (pp. 515–522). Springer.
- 519 Vassiliadis, D., Sharma, A., & Papadopoulos, K. (1991). Lyapunov exponent of mag-  
520 netospheric activity from al time series. *Geophysical research letters*, *18*(8),  
521 1643–1646.
- 522 Vassiliadis, D. V., Sharma, A. S., Eastman, T. E., & Papadopoulos, K. (1990, Octo-  
523 ber). Low-dimensional chaos in magnetospheric activity from AE time series.  
524 *Geophysical research letters*, *17*, 1841-1844. doi: 10.1029/GL017i011p01841
- 525 Wanliss, J. A., Anh, V. V., Yu, Z.-G., & Watson, S. (2005, August). Multifractal  
526 modeling of magnetic storms via symbolic dynamics analysis. *Journal of Geo-*  
527 *physical Research (Space Physics)*, *110*, 8214. doi: 10.1029/2004JA010996
- 528 Wanliss, J. A., & Dobias, P. (2007, April). Space storm as a phase transition. *Jour-*  
529 *nal of Atmospheric and Solar-Terrestrial Physics*, *69*, 675-684. doi: 10.1016/j  
530 .jastp.2007.01.001
- 531 Weck, P. J., Schaffner, D. A., Brown, M. R., & Wicks, R. T. (2015, February).  
532 Permutation entropy and statistical complexity analysis of turbulence in lab-  
533 oratory plasmas and the solar wind. *Physical Review E*, *91*(2), 023101. doi:  
534 10.1103/PhysRevE.91.023101
- 535 Zanin, M., Zunino, L., Rosso, O. A., & Papo, D. (2012, August). Permutation En-  
536 tropy and Its Main Biomedical and Econophysics Applications: A Review. *En-*  
537 *tropy*, *14*, 1553-1577. doi: 10.3390/e14081553



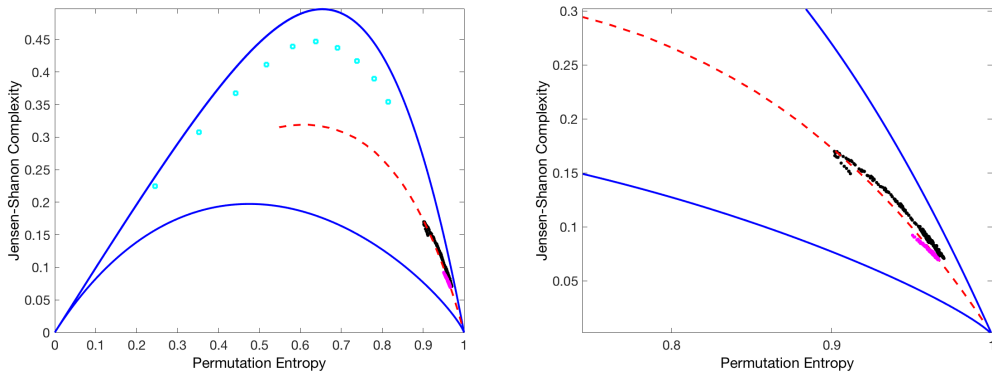
**Figure 1.** Top two panels show AL and  $\delta AL = AL_{i+1} - AL_i$  time series for an interval of four months spanning 01-01-2009 to 30-04-2009. The panels with blue traces show time series for the Lorenz attractor with parameters  $a = 10, b = 8/3, c = 28$ . The bottom panel shows an example of fractional Brownian motion with Hurst exponent  $h_u = 0.8$ .



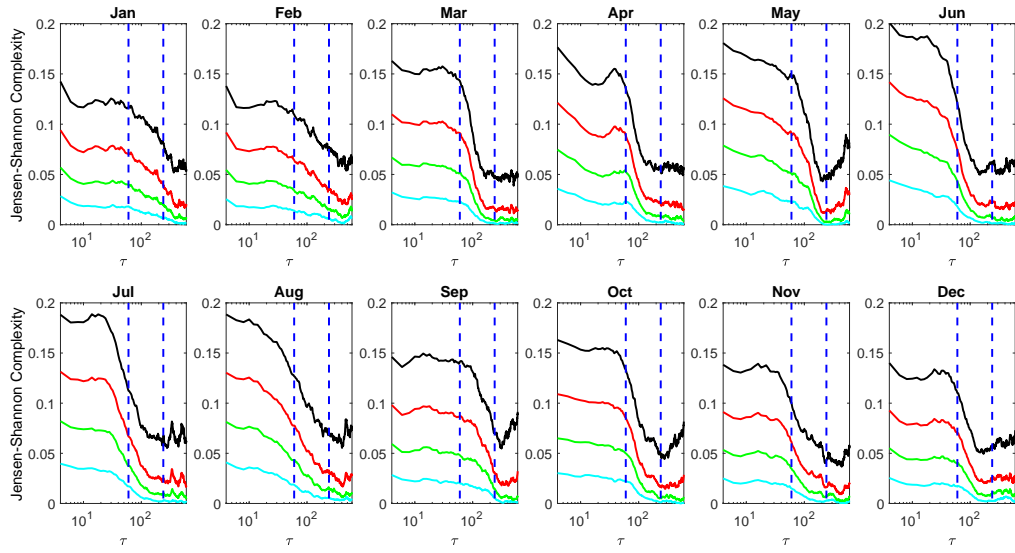
**Figure 2.** Permutation entropy as a function embedded delay  $\tau$  and parametrised for embedding dimension  $3 \leq d \leq 7$  for fractional Brownian motion with Hurst exponent  $H = 0.75$  (top panel), the Lorenz attractor (center panel) and AL (bottom panel).



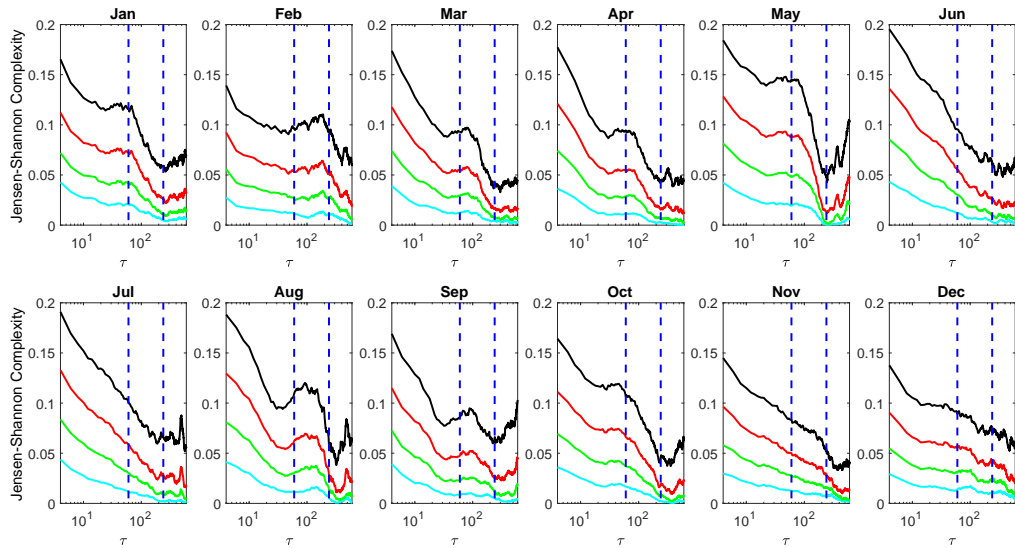
**Figure 3.** Jensen-Shannon Complexity versus embedded delay  $\tau$  for all 12 months of 2010 of  $AL$  values. The legend for the colour is the same as in Figure 2. The embedded delay  $\tau$  has units of minutes and the dashed vertical line indicate the 60 minute and 240 minute marks.



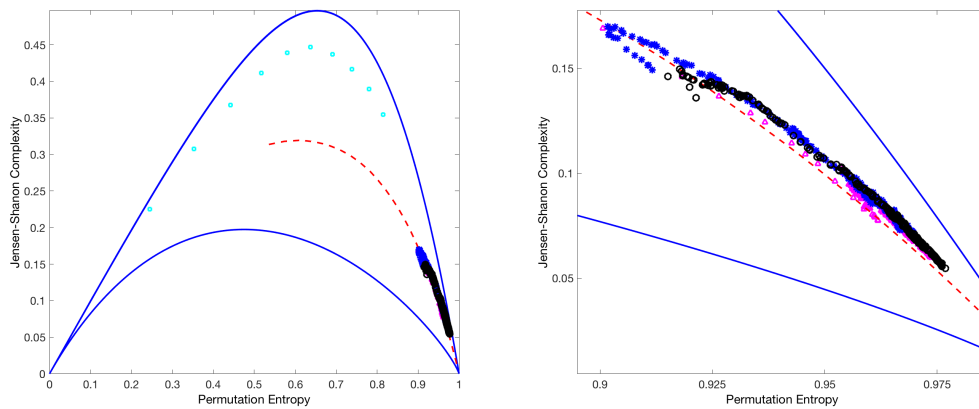
**Figure 4.** Complexity-entropy plane for  $AL$  (black dots),  $\delta AL$  (magenta dots), the Lorentz strange attractor (cyan squares), and fractional Brownian motion (red dash) on the left panel with a zoom in the low complexity high entropy part on the right panel. The blue curves represent the minimum and maximum entropy-complexity curve for an embedded parameter  $d = 6$ . The permutation entropy and Jensen-Shannon complexity values for  $AL$  and  $\delta AL$  are computed for  $4 \leq \tau \leq 600$  minutes. The complexity-values for  $AL$  reside along those for fractional-Brownian motion of Hurst exponents less than 0.5.



**Figure 5.** Jensen-Shannon Complexity versus embedded delay  $\tau$  for all 12 months of 2010 of *AE* values. The legend for the colour is the same as in Figure 2.



**Figure 6.** Jensen-Shannon Complexity versus embedded delay  $\tau$  for all 12 months of 2010 of *AU* values. The legend for the colour is the same as in Figure 2.



**Figure 7.** Complexity-Entropy plane for *AL* (blue stars), *AU* (magenta lozenge), *AE* (black circles) and fractional Brownian motion (red dots) and  $d = 6$ . The lowest entropy values are computed for  $\tau = 4$  minute, while the larger entropy values are for  $\tau = 600$  min. The complexity-values for all three auroral geomagnetic indices reside along those for fractional-Brownian motion with Hurst exponents less than one half.

Figure 1.

Author Manuscript



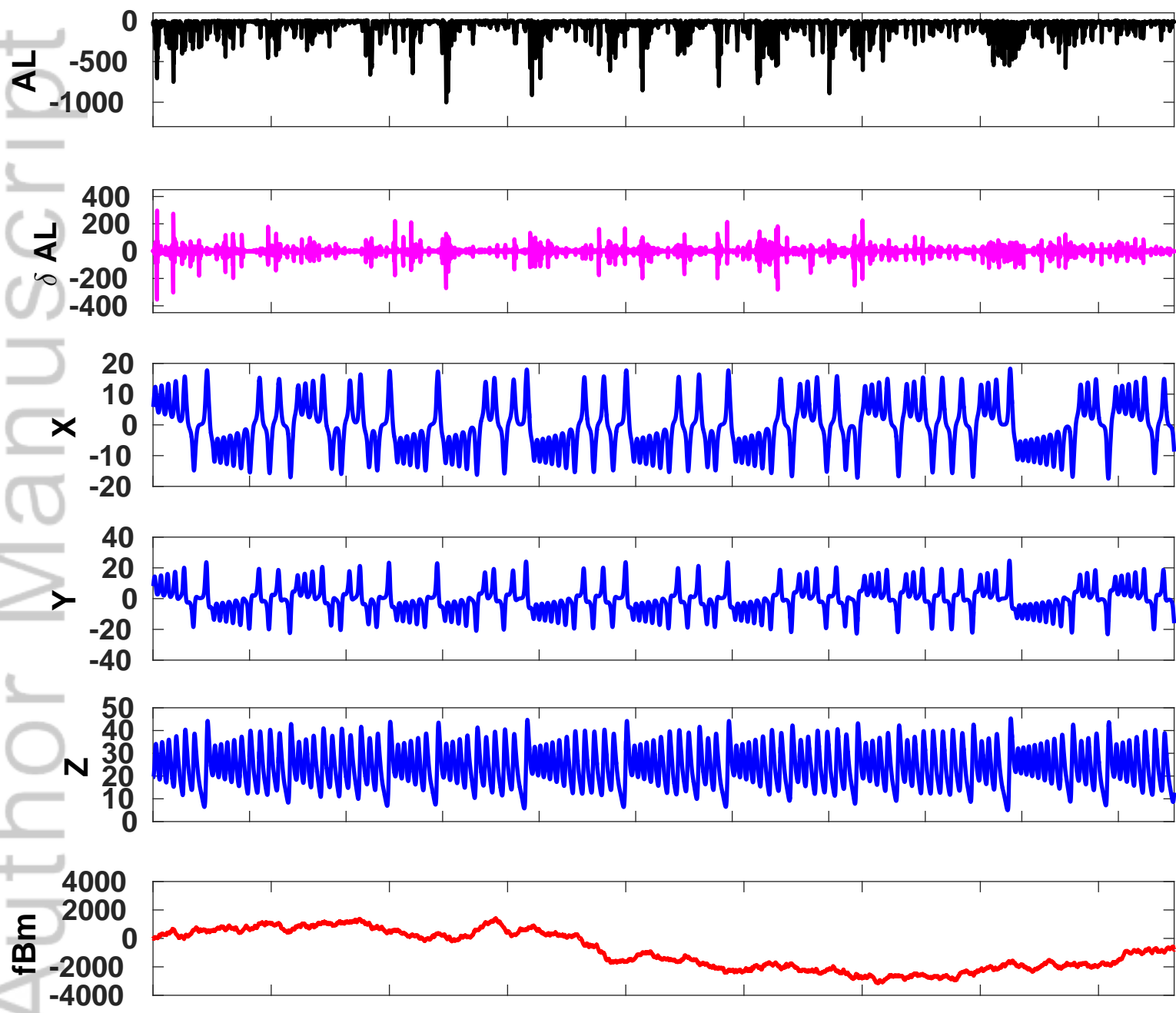


Figure 2a.

Author Manuscript

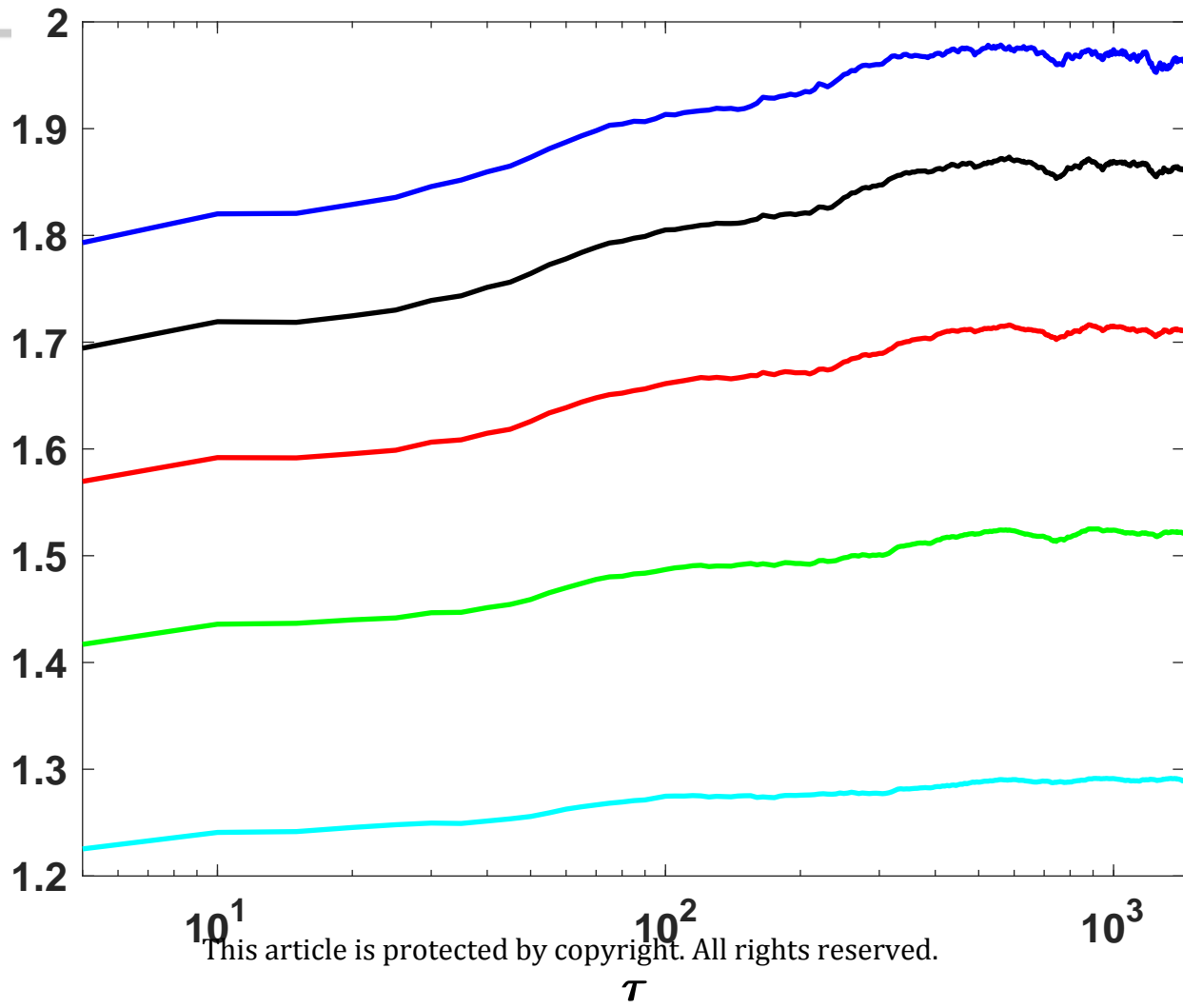


Figure 2b.

Author Manuscript

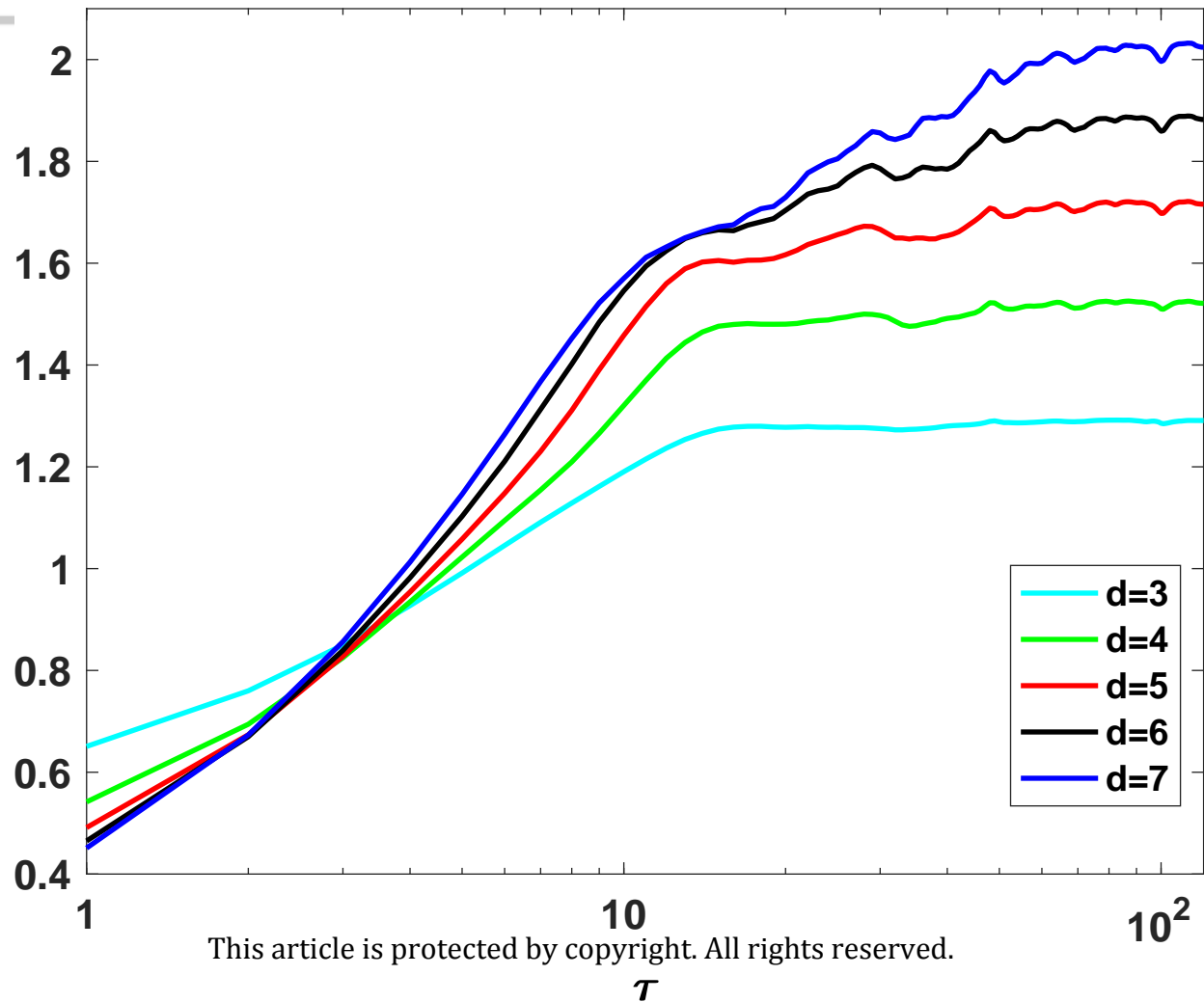


Figure 2c.

Author Manuscript

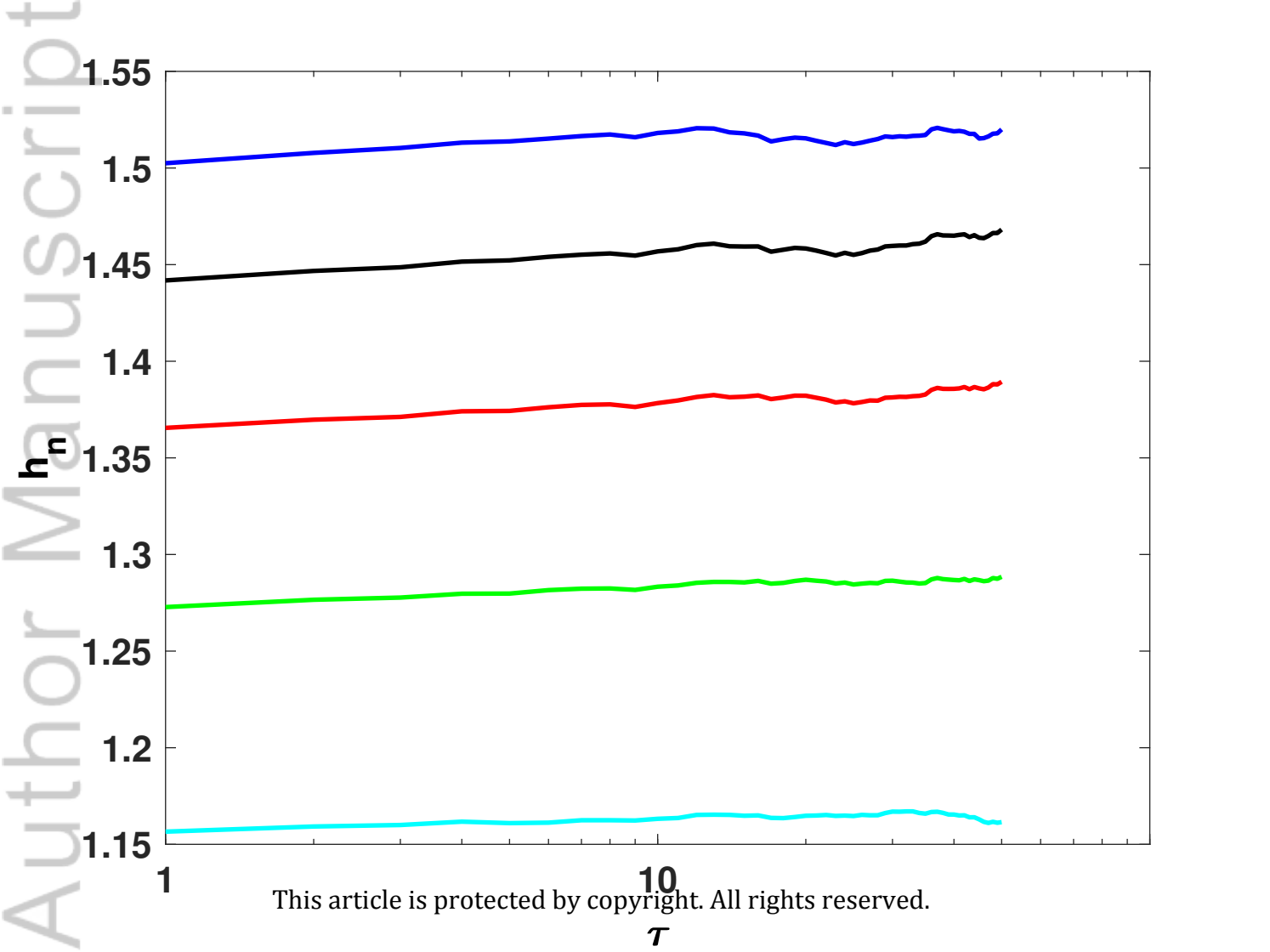


Figure 3.

Author Manuscript



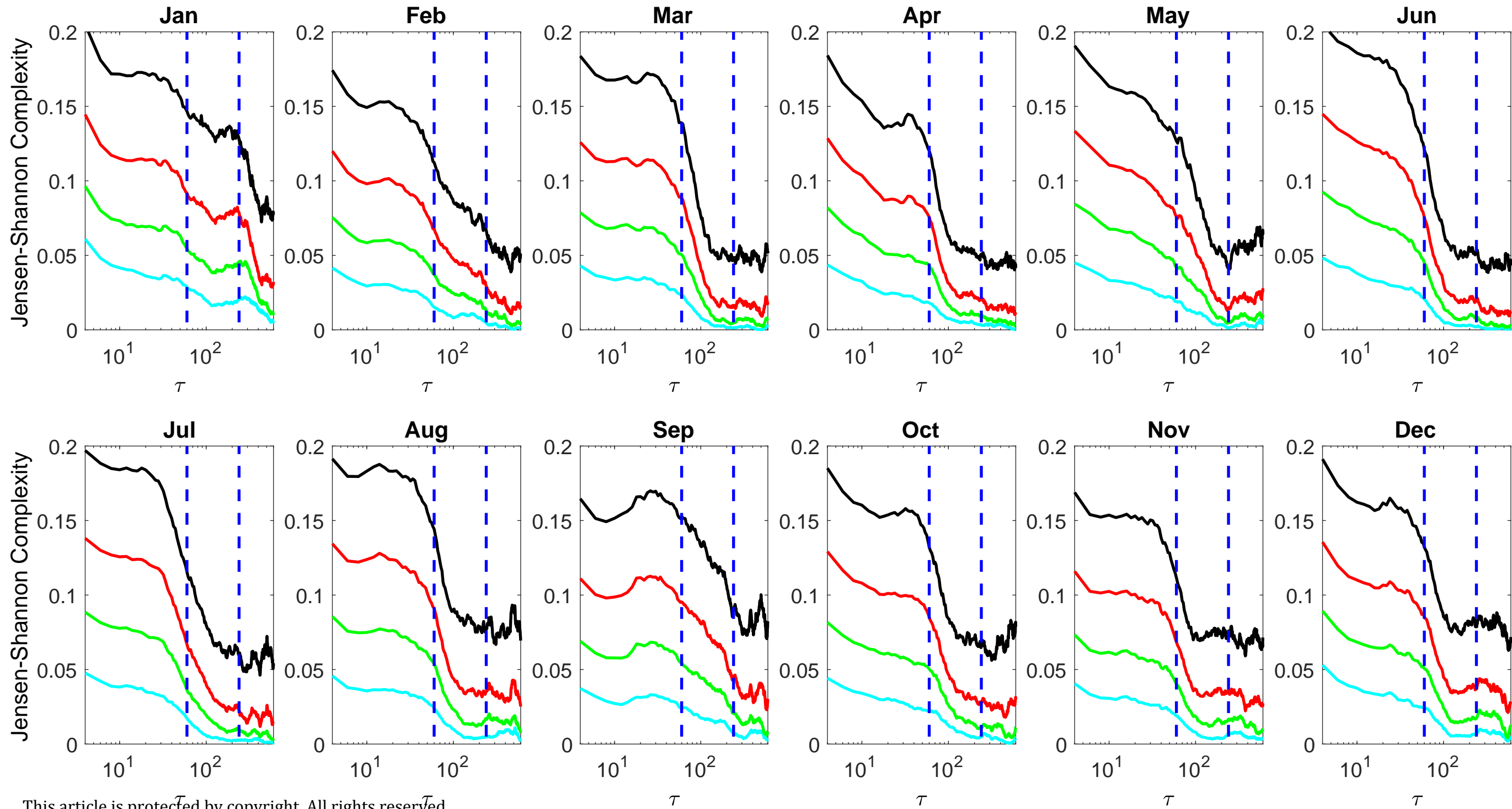


Figure 4a.

Author Manuscript

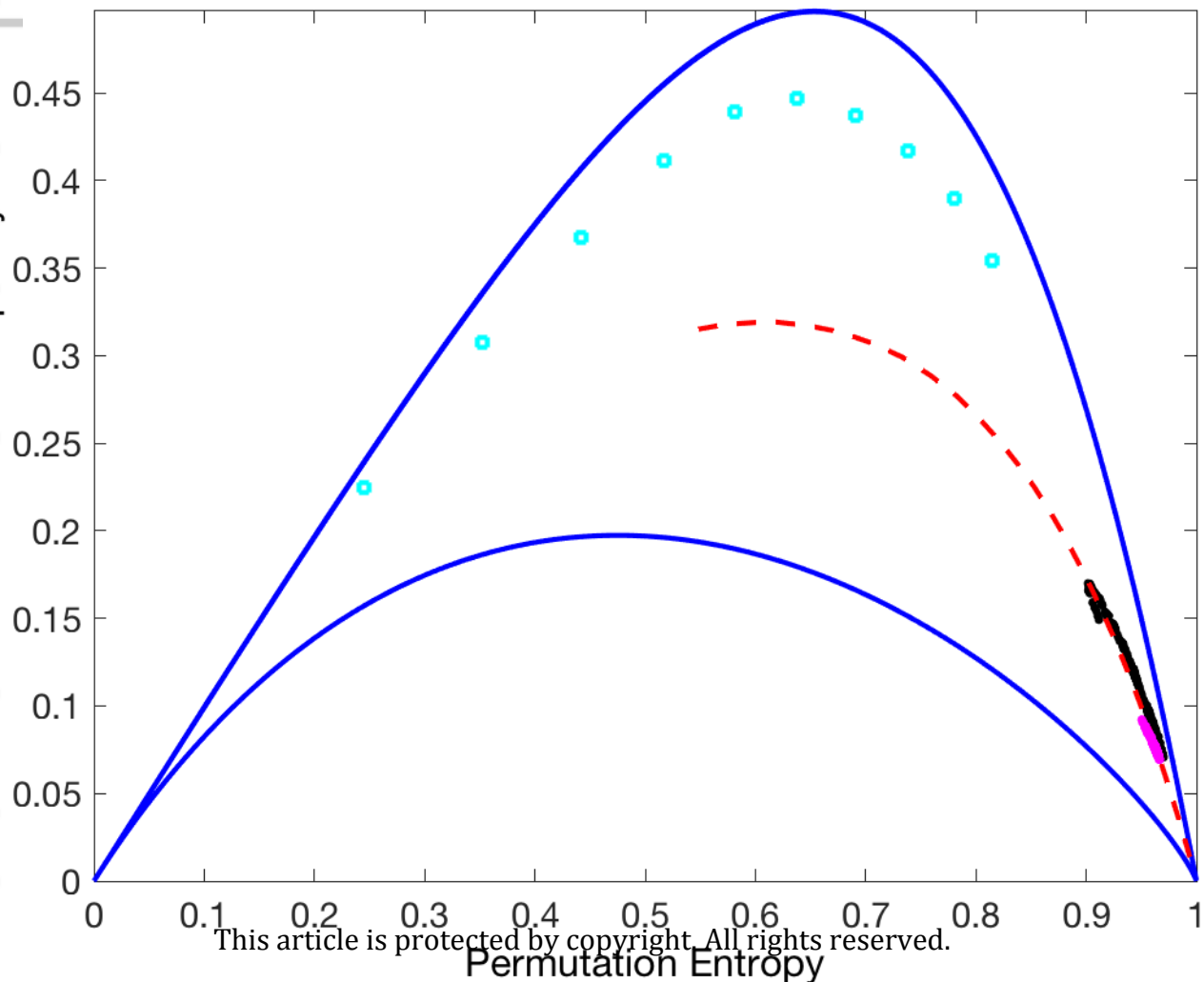


Figure 4b.

Author Manuscript

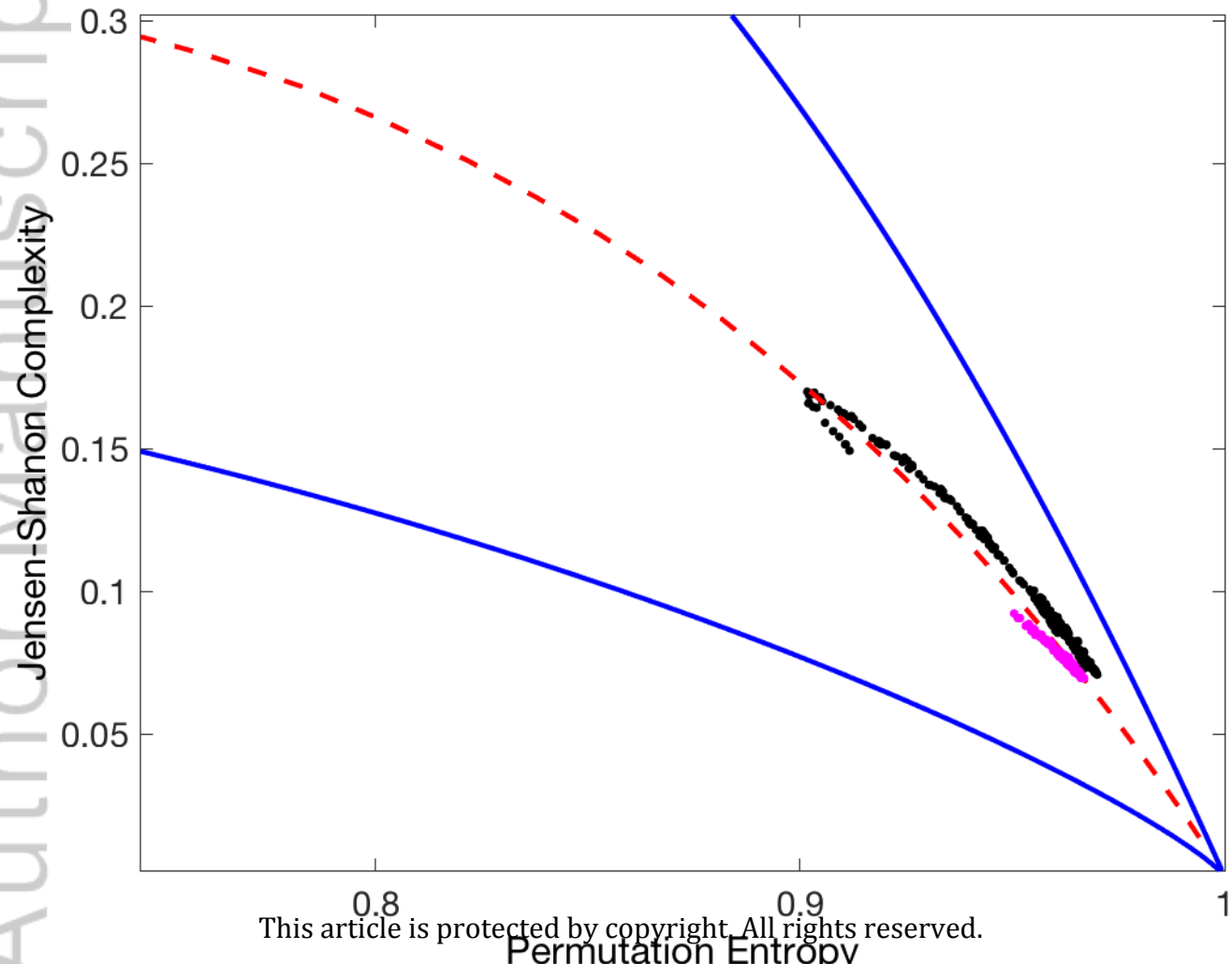


Figure 5.

Author Manuscript

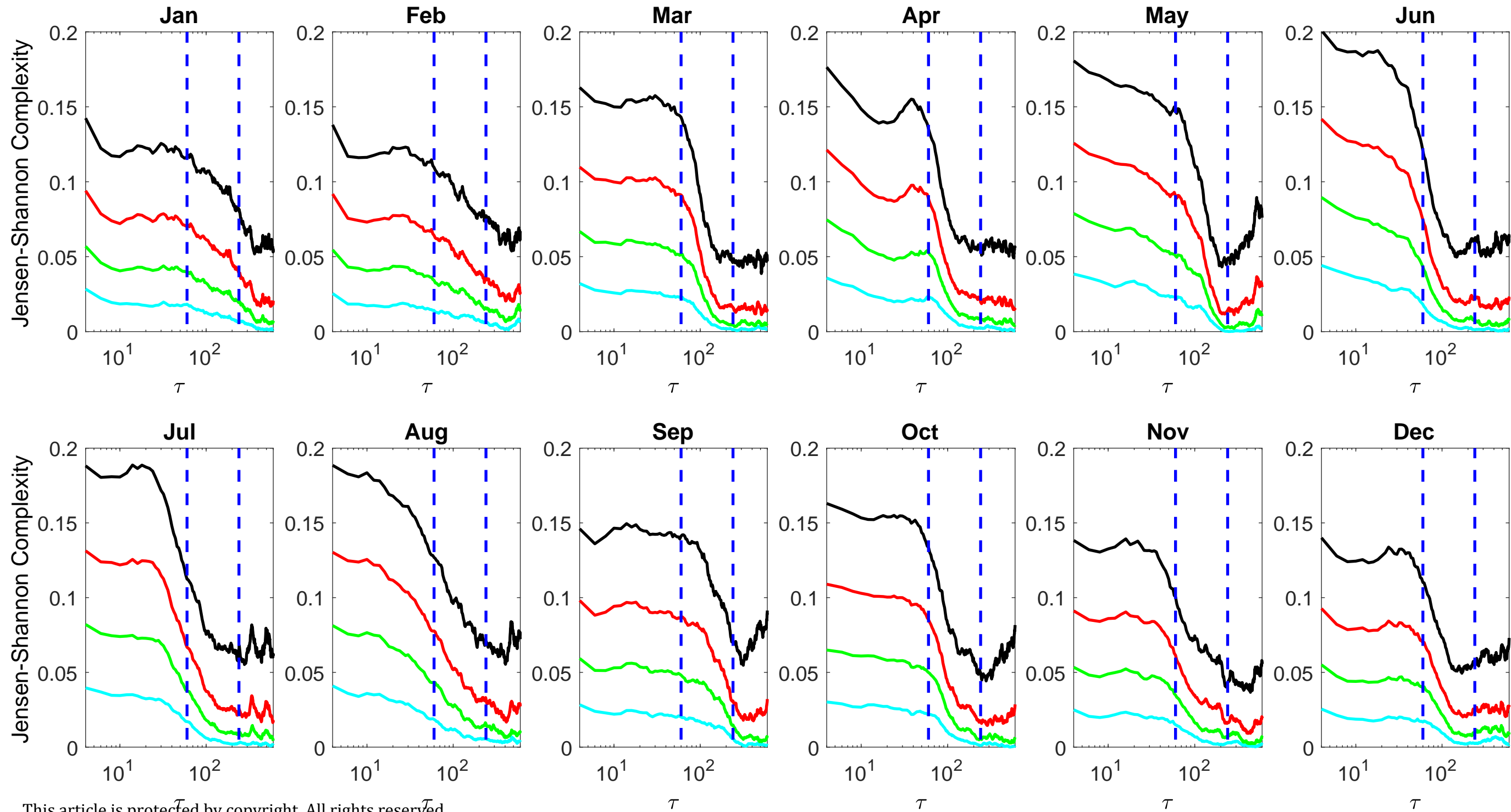


Figure 6.

Author Manuscript



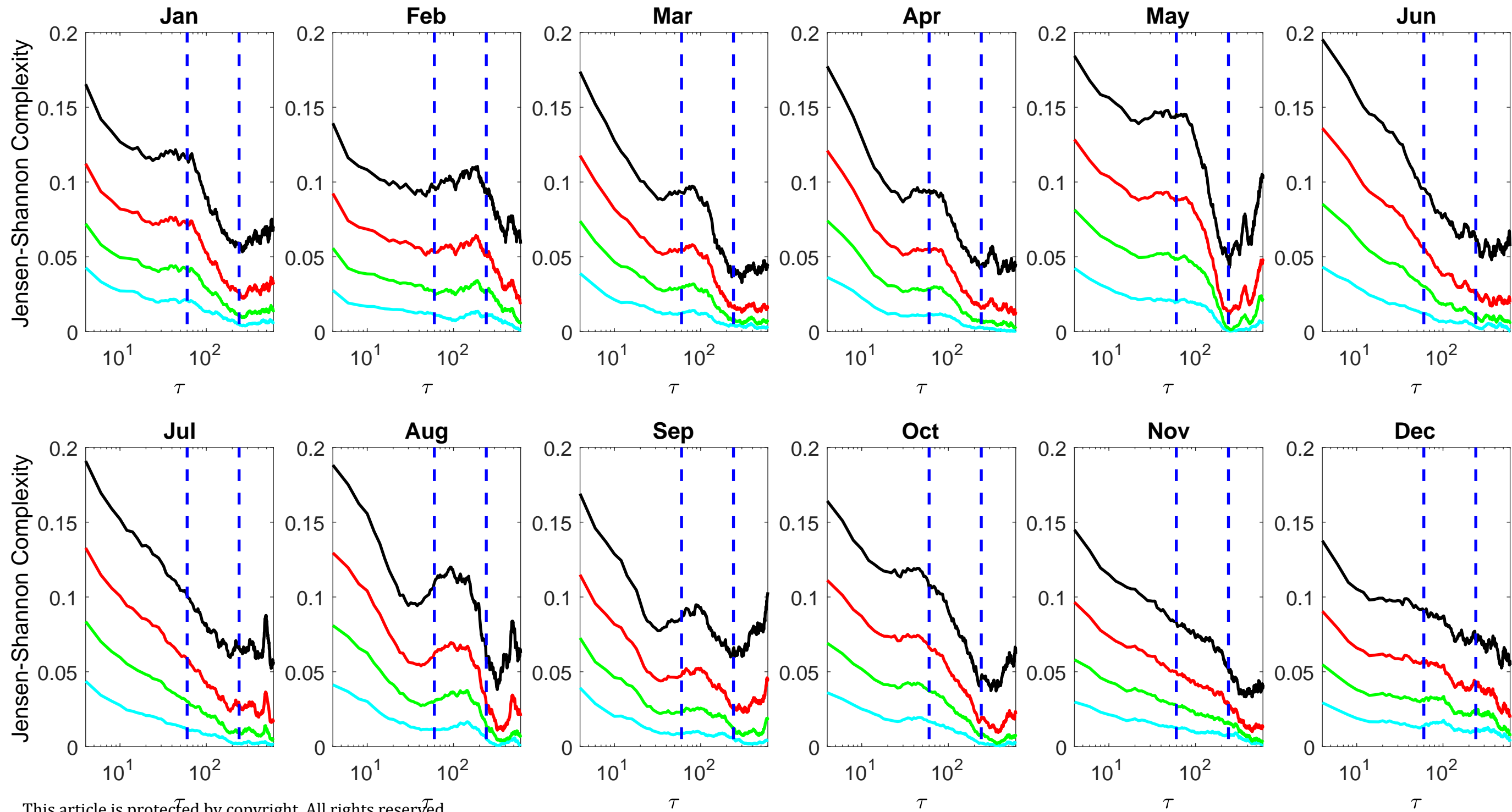


Figure 7a.

Author Manuscript

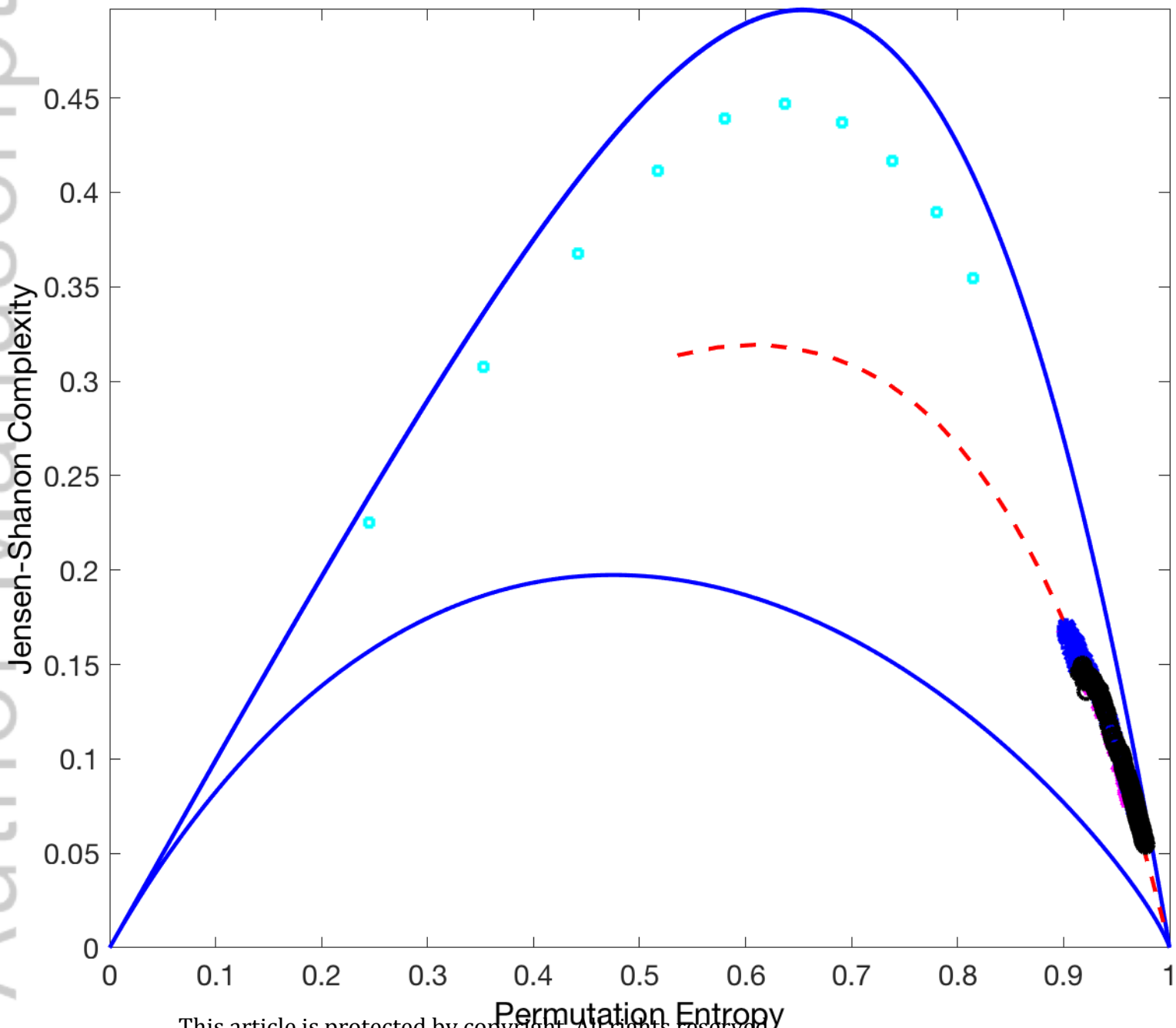
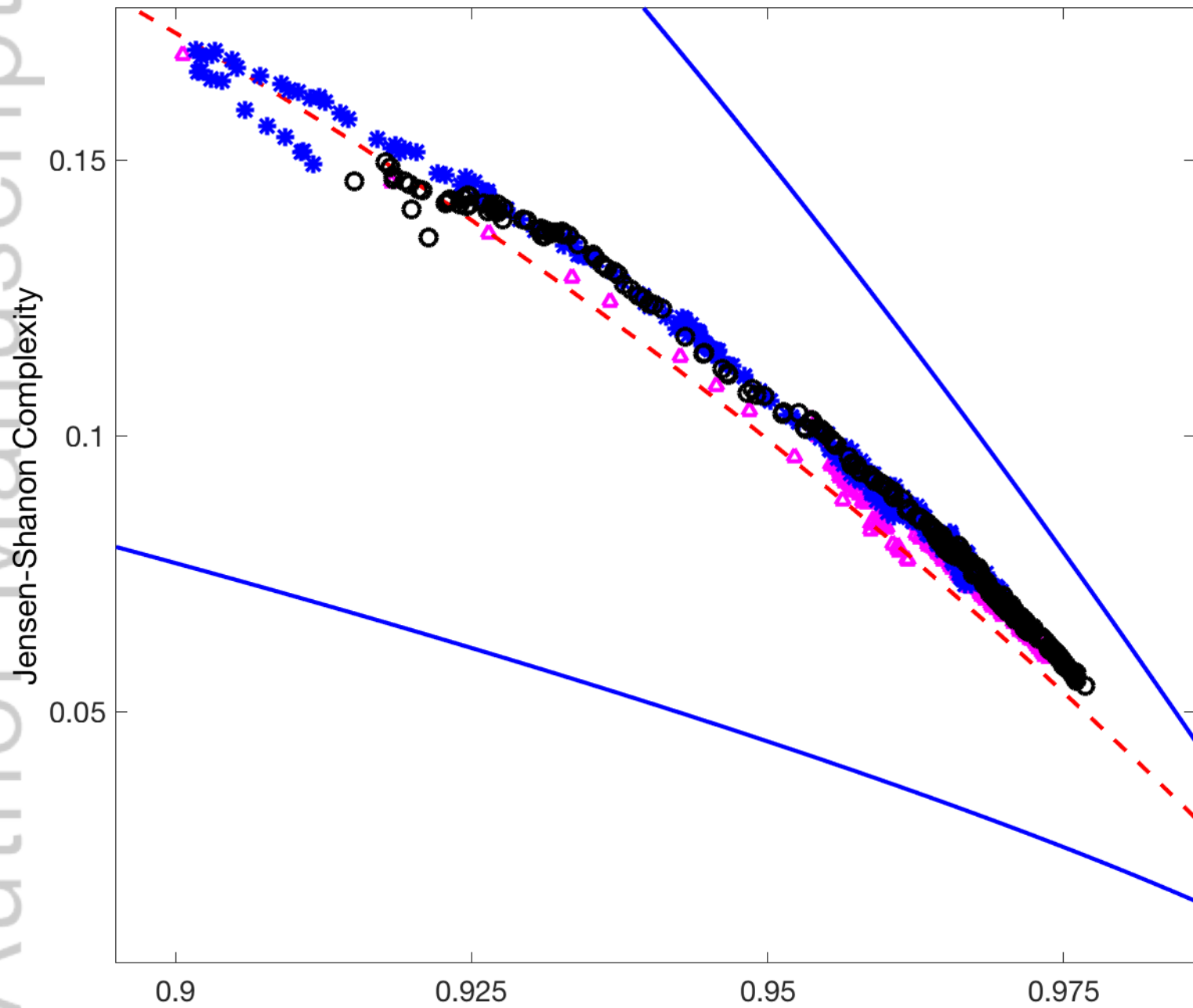
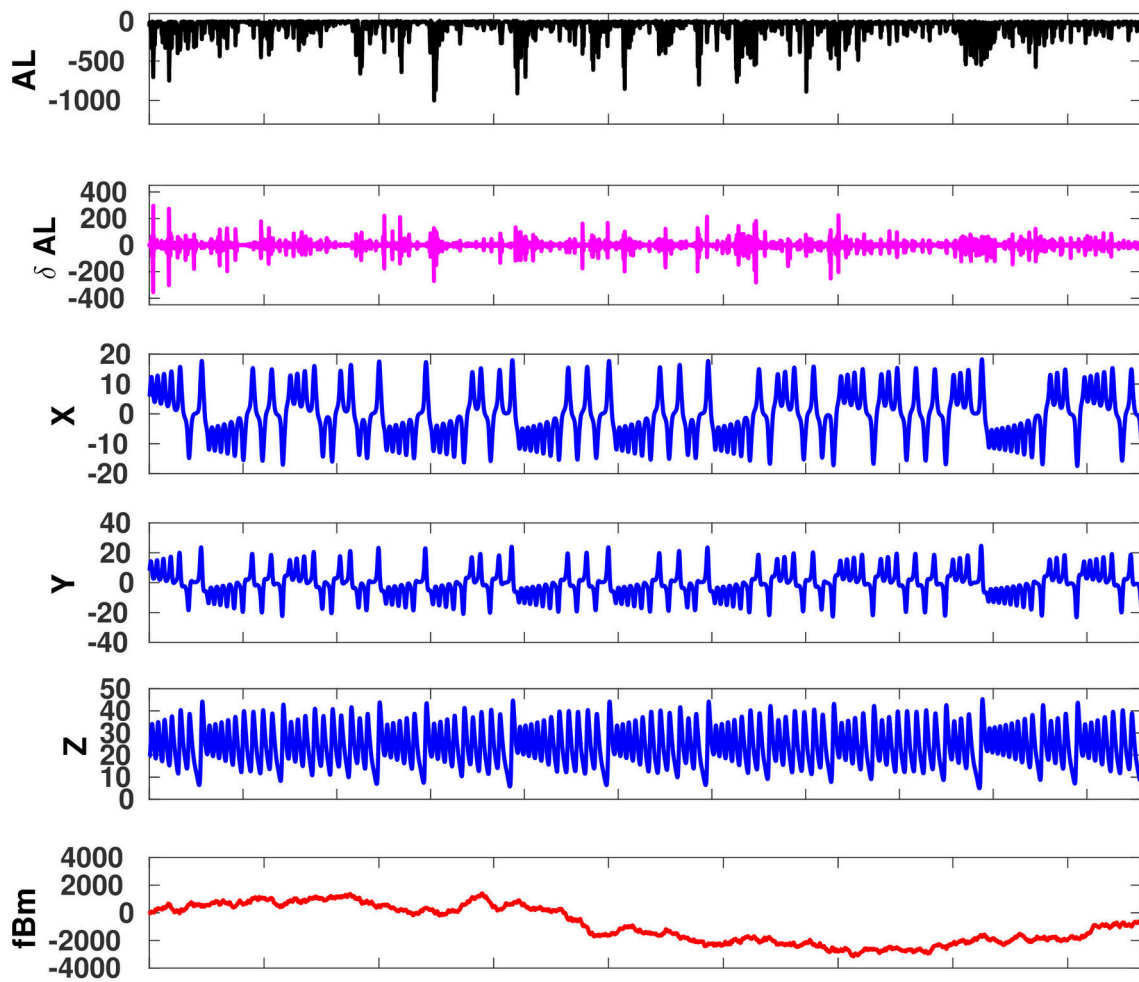


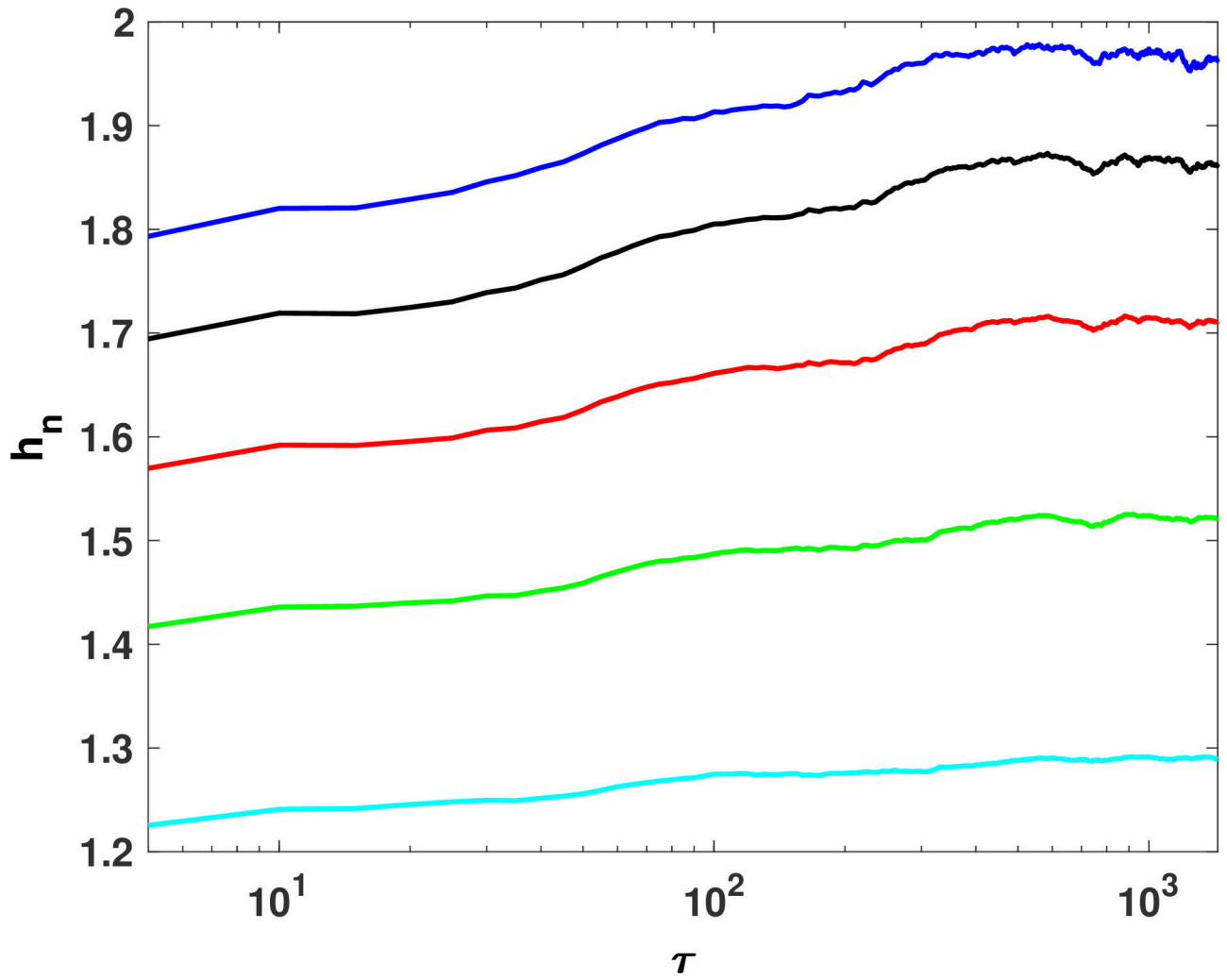
Figure 7b.

Author Manuscript

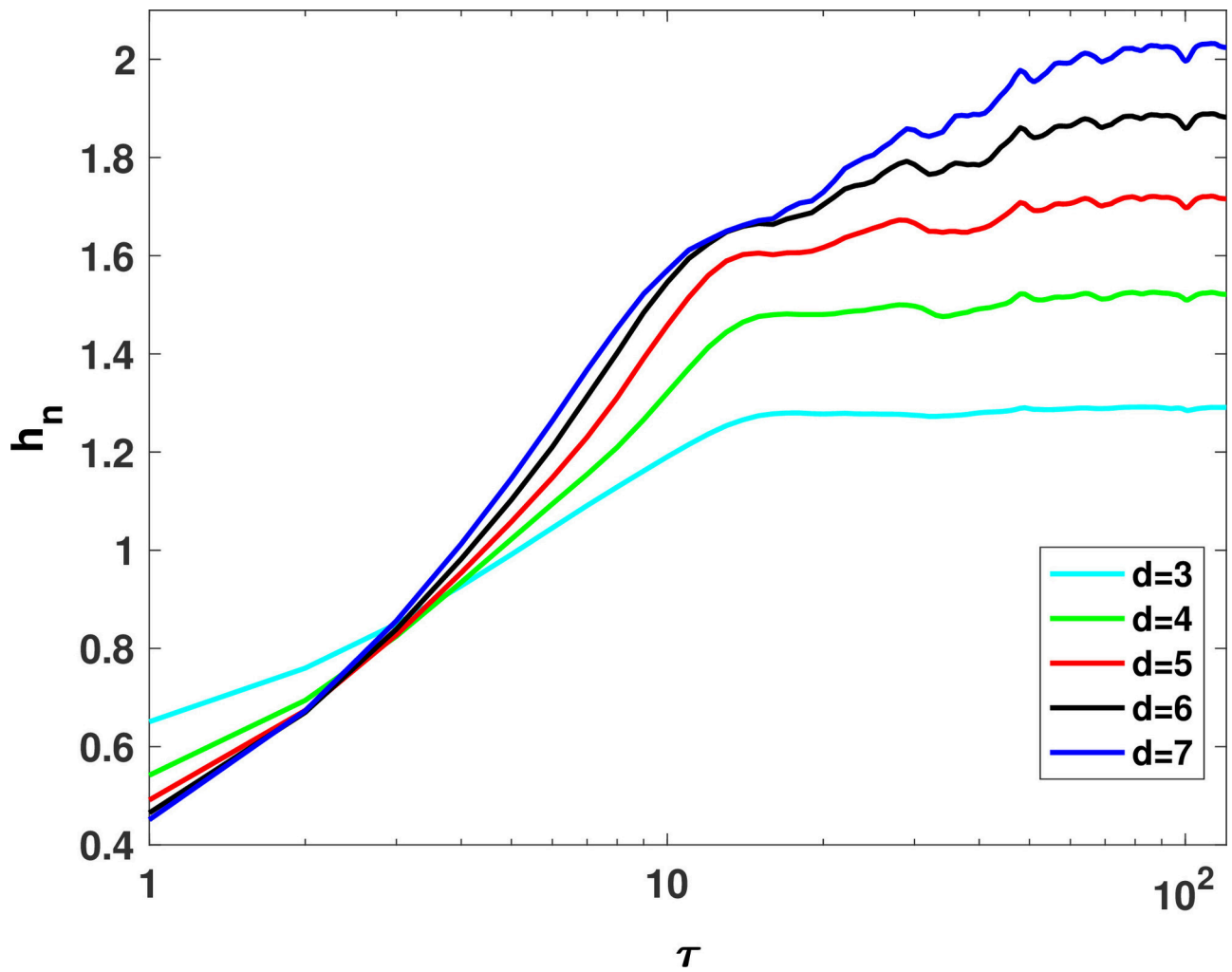




2018ja026248-t-f01-z-.eps

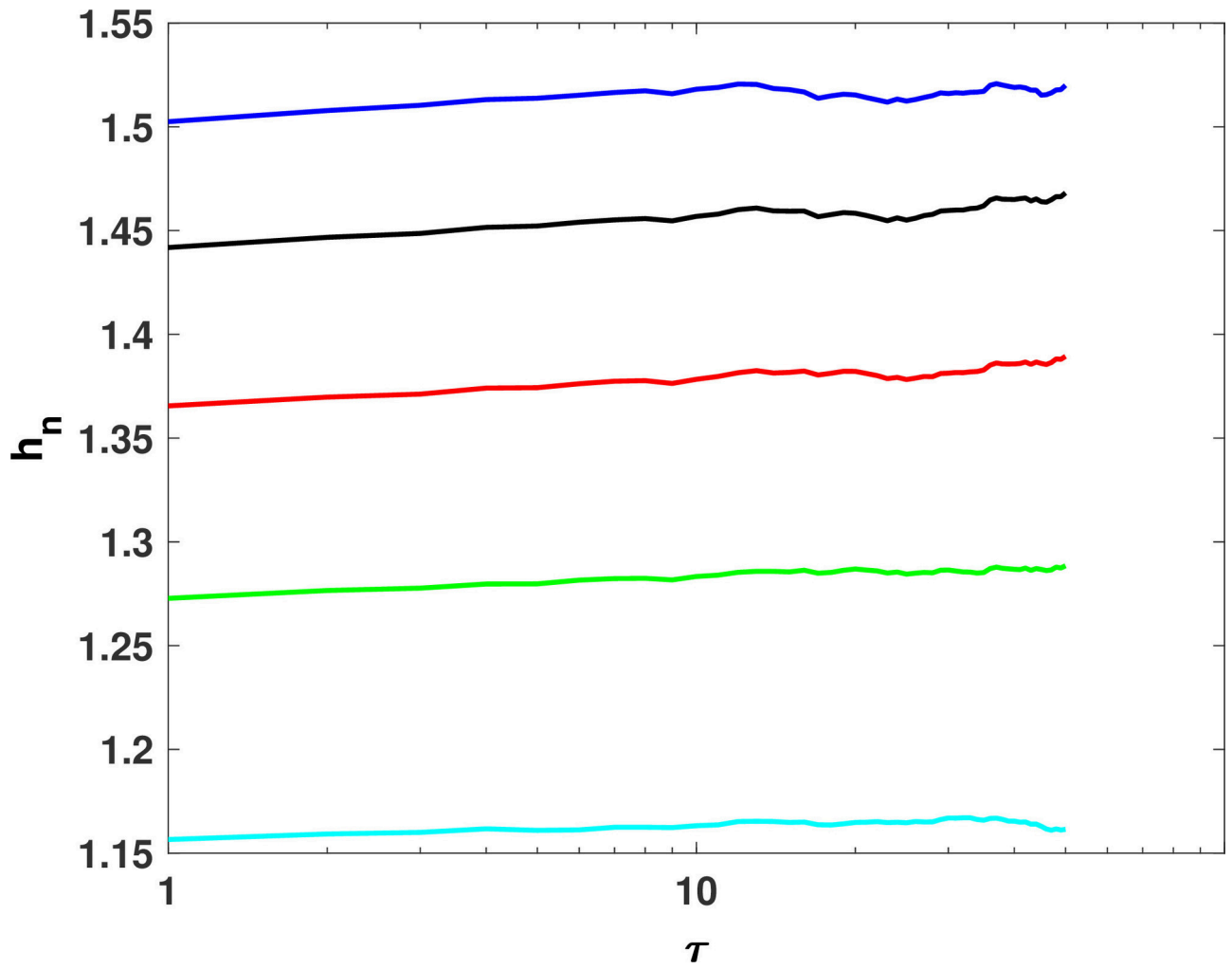


2018ja026248-t-f02-z-eps

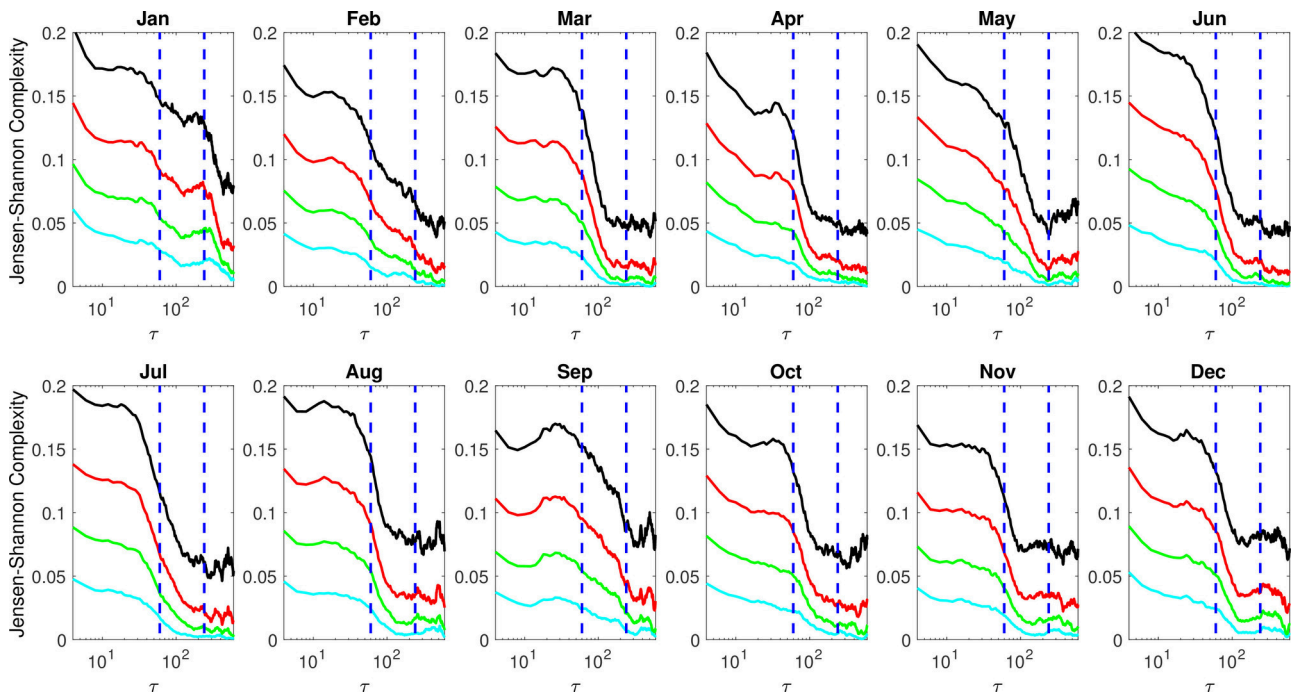


2018ja026248-t-f03-z-.eps

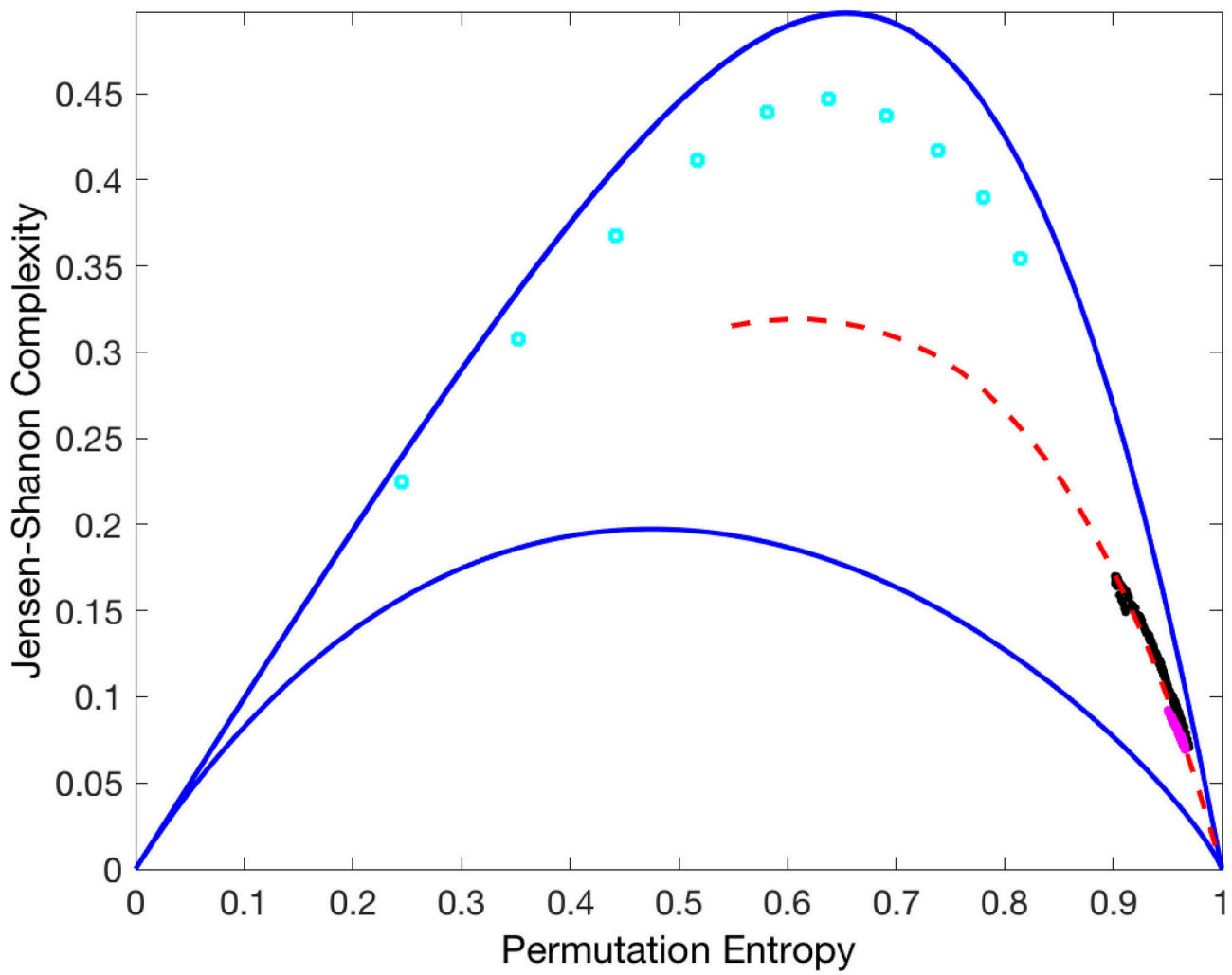




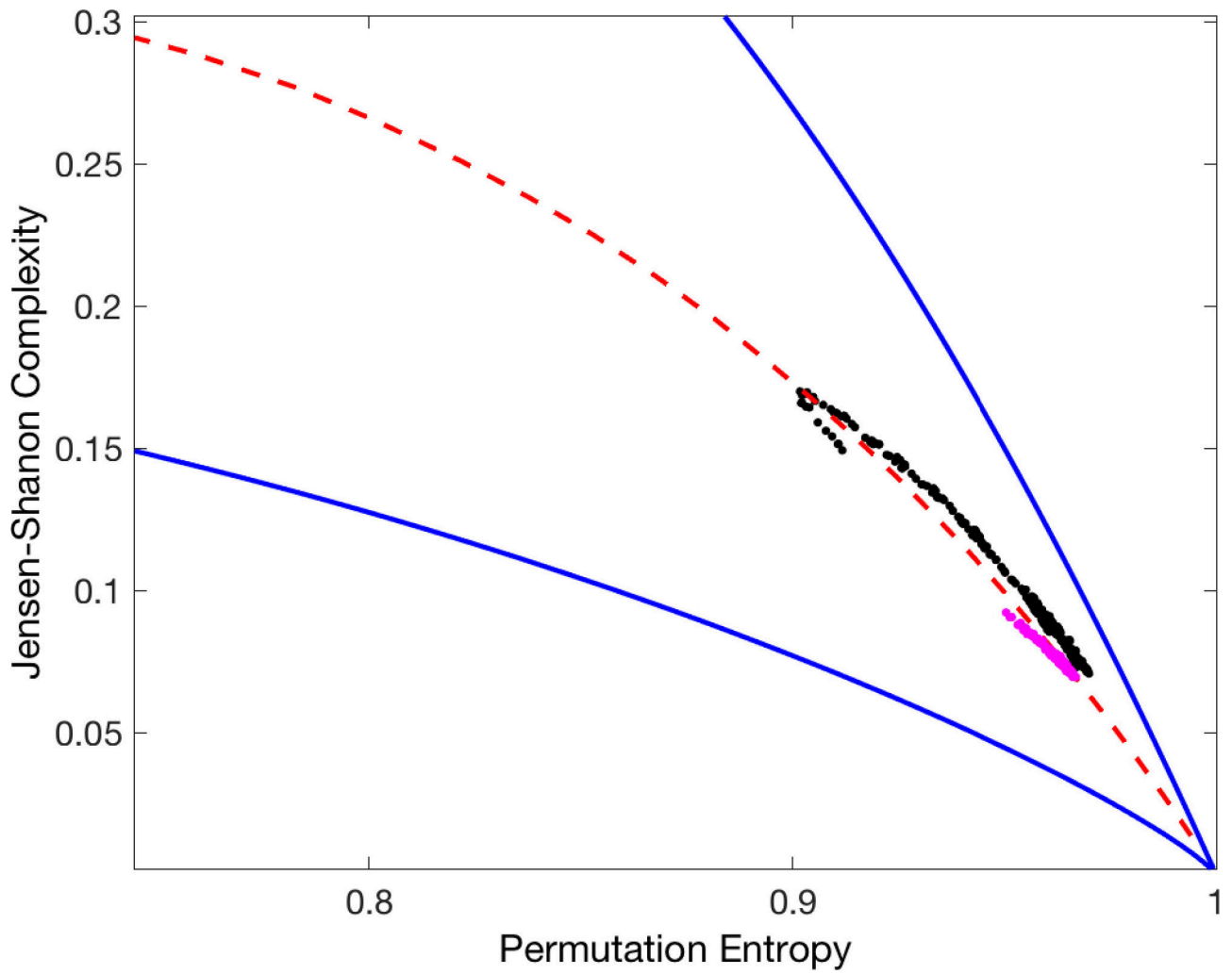
2018ja026248-t-f04-z-.eps



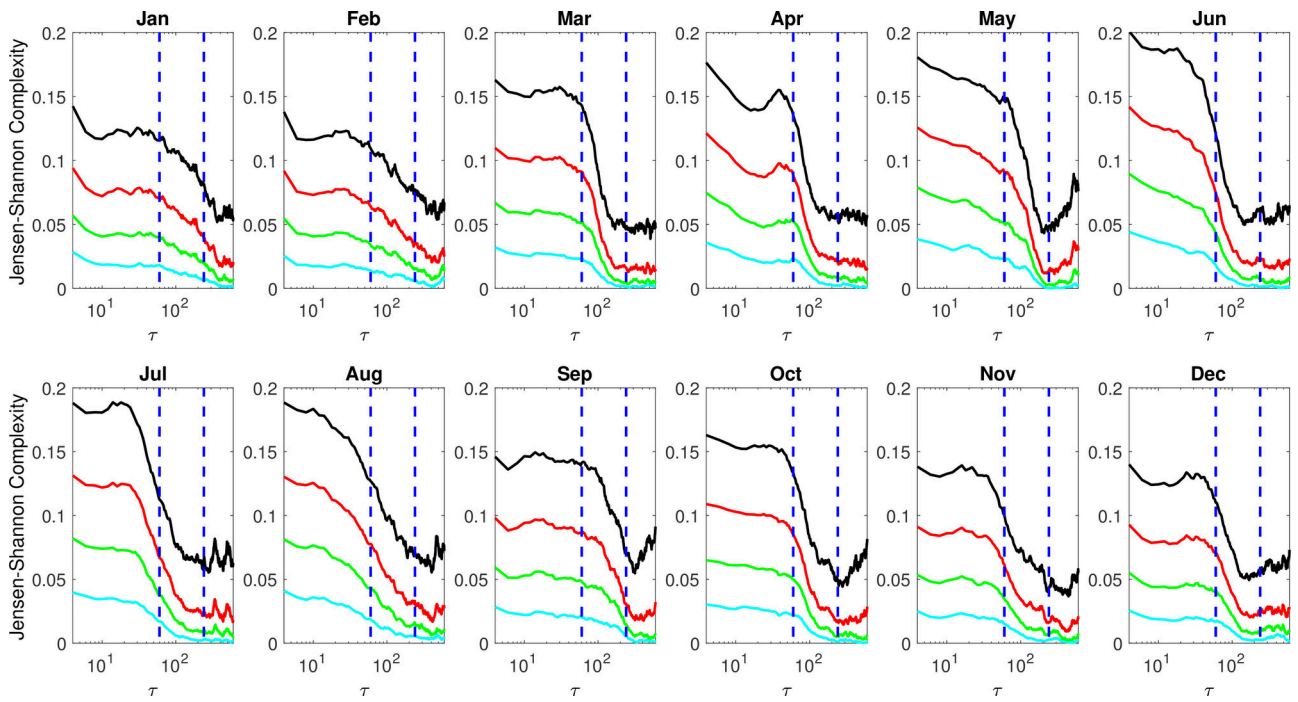
2018ja026248-t-f05-z-eps



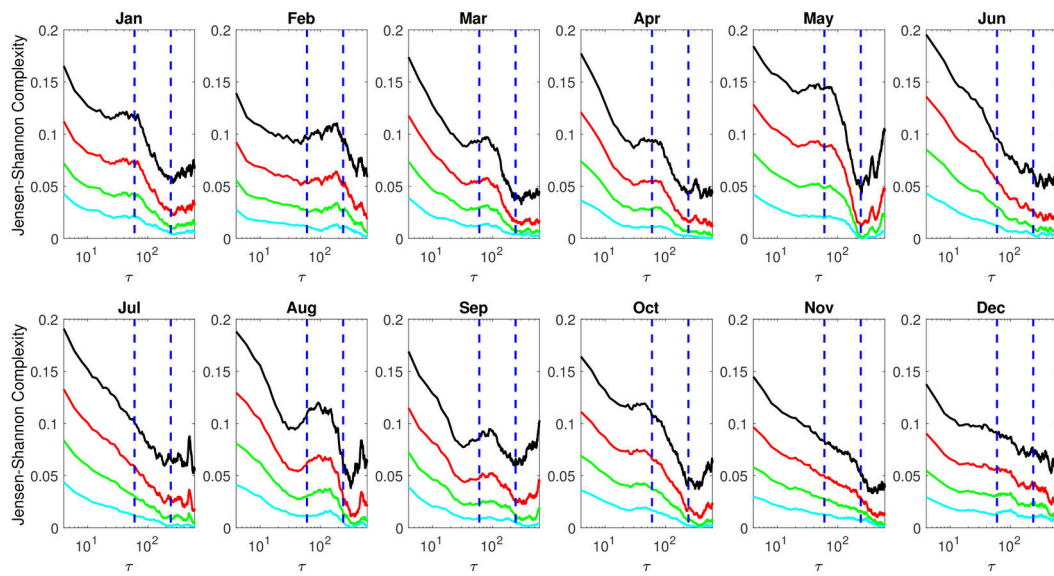
2018ja026248-t-f06-z-eps



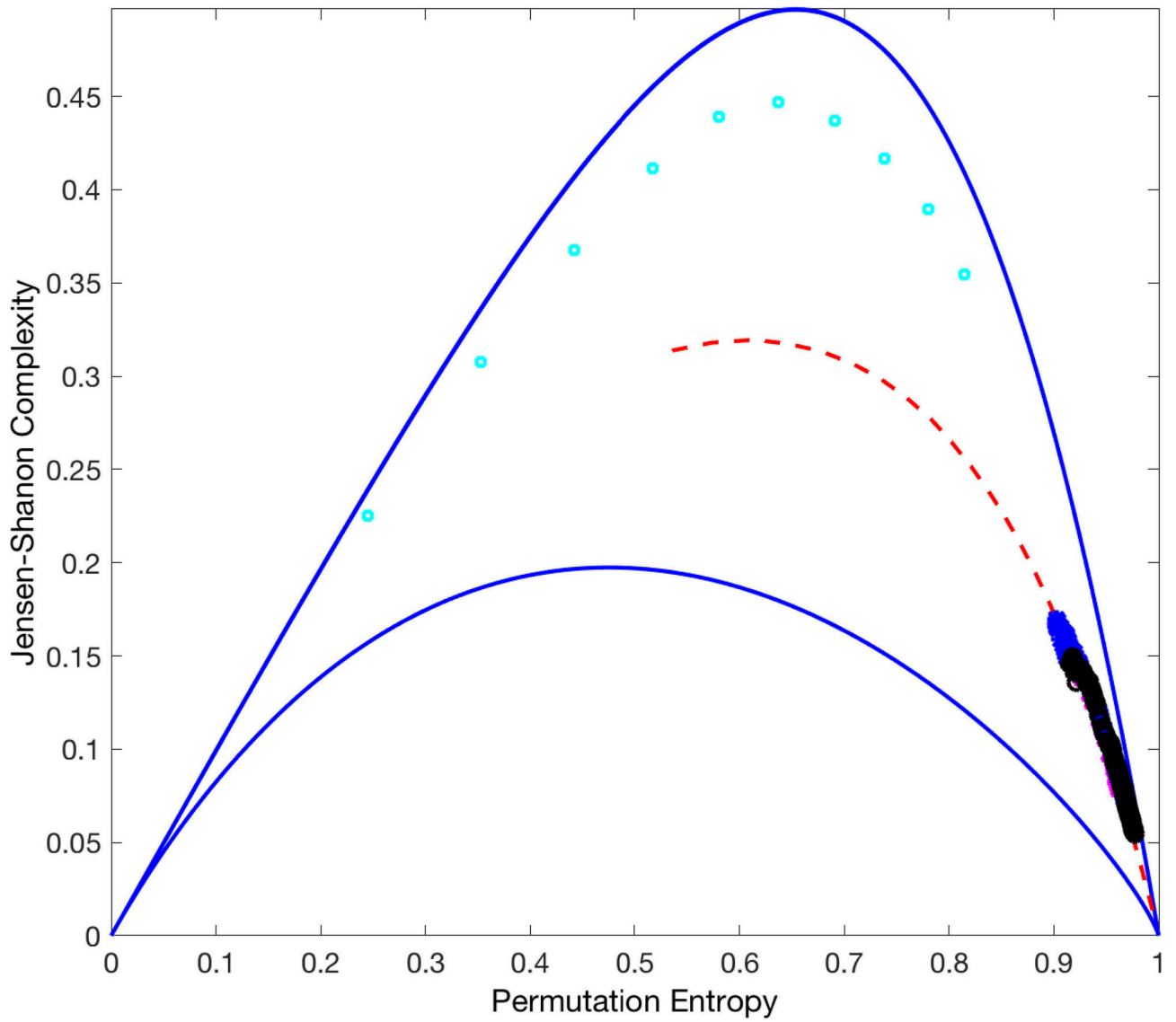
2018ja026248-t-f07-z-.eps



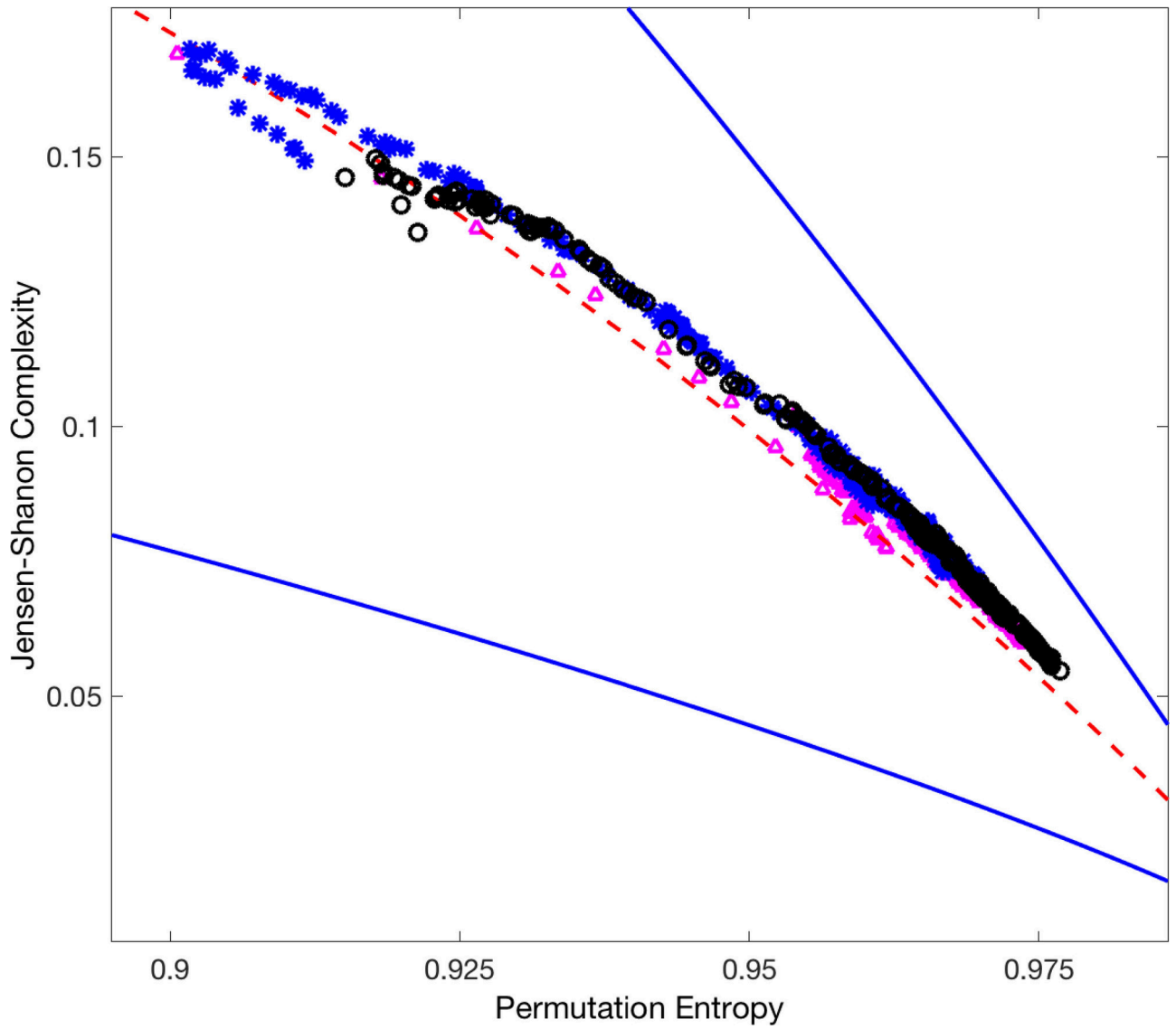
2018ja026248-t-f08-z-eps



2018ja026248-t-f09-z-eps



2018ja026248-t-f10-z-.eps



2018ja026248-t-f11-z-.eps