



Managing Communication Delays and Model Error in Demand Response

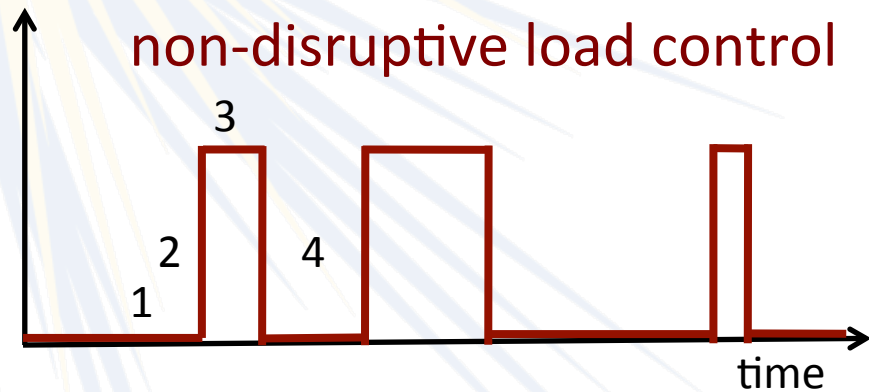
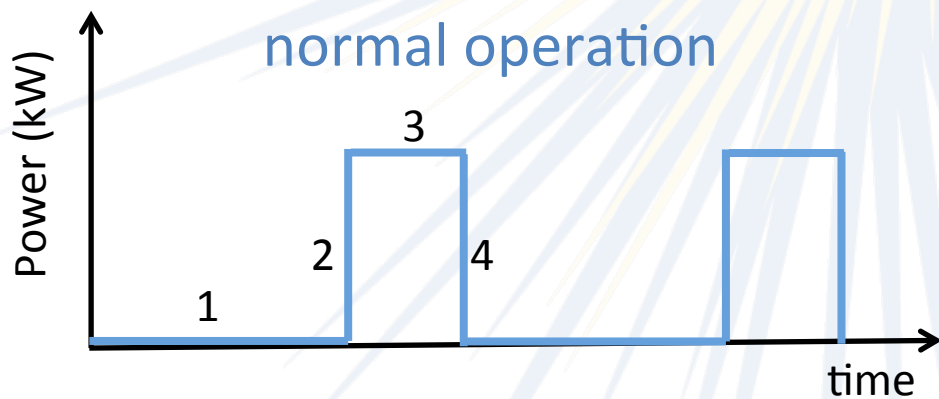
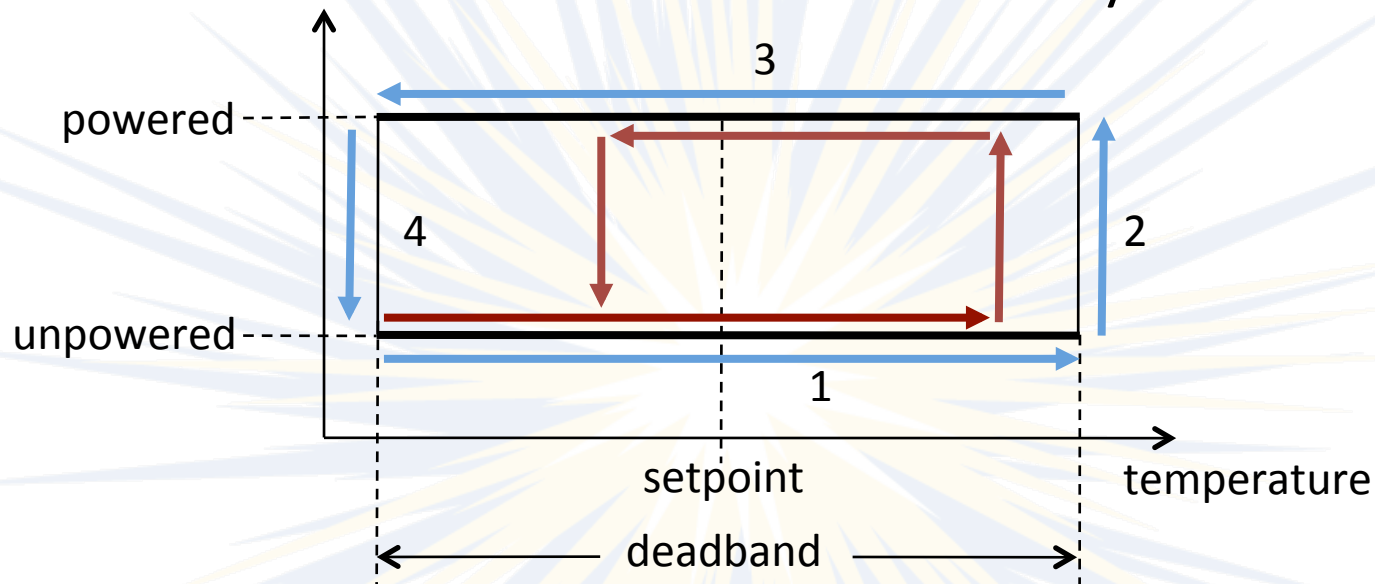
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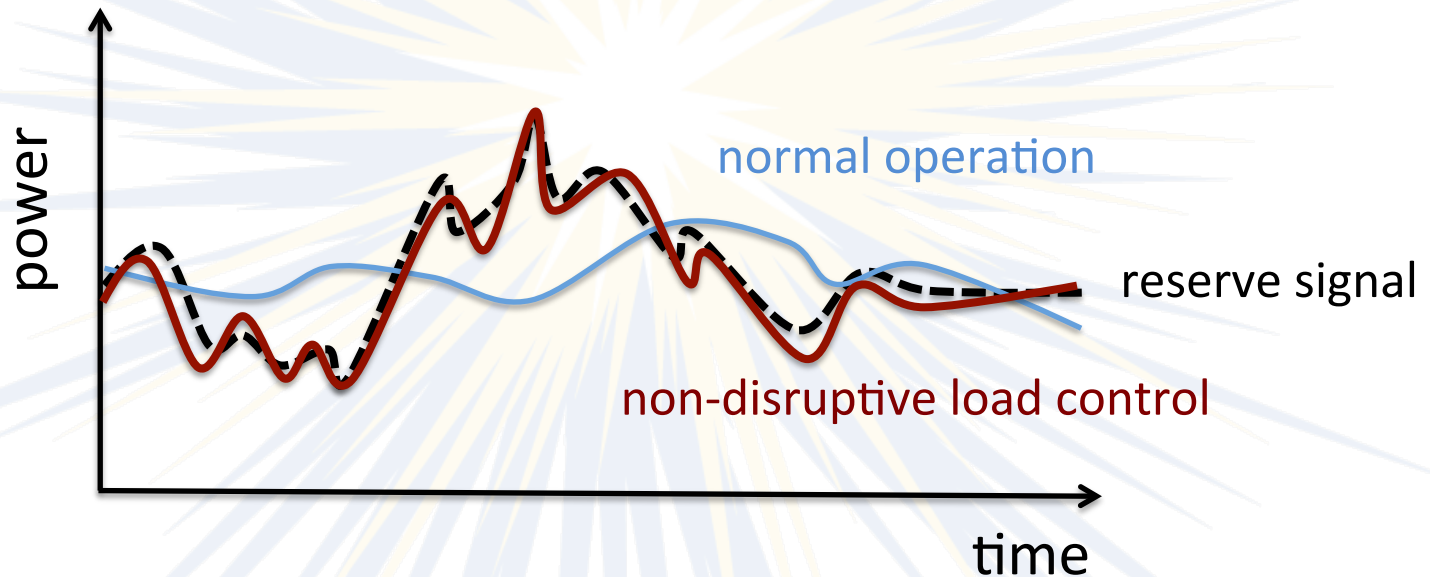
Greg Ledva, Evangelos Vrettos, Silvia Mastellone, Göran Andersson

How can loads provide reserves?

→ your refrigerator, air conditioner, and water heater are already flexible



Thousands of thermostatically controlled loads (TCLs) can track signals and provide reserves

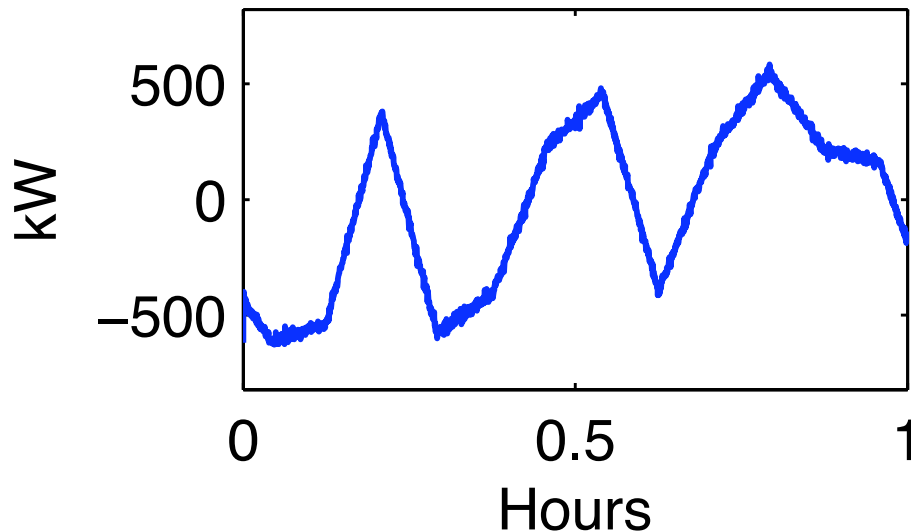


[Mathieu, Koch, and Callaway *IEEE Transactions on Power Systems* 2013]

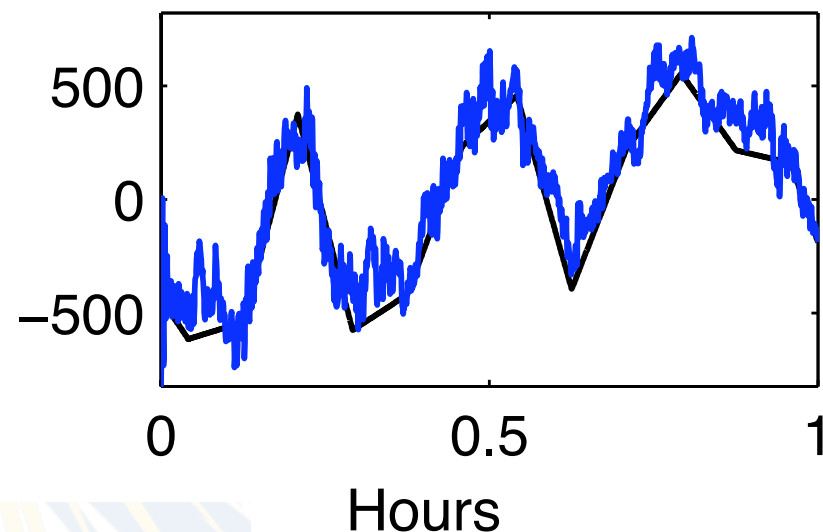
Simulation results:

1000 ACs tracking 5-minute market signal

Controller gets temperature/state of each load every 2 seconds



Controller infers TCL behavior from power measurements at the substation



→ The more the controller knows about the loads, the better it can track a signal

[Mathieu, Koch, and Callaway *IEEE Transactions on Power Systems* 2013]

Data from loads

- Parameters

- the make/model of the load?
- its temperature setpoint/dead-band width?
- some information about the household?

→ Modeling

- Real-time data

- Measurements of the on/off state and/or internal temperature?
- Household smart meter data?
- Power measurements from the distribution network?

→ Feedback control

→ High quality, infrequent

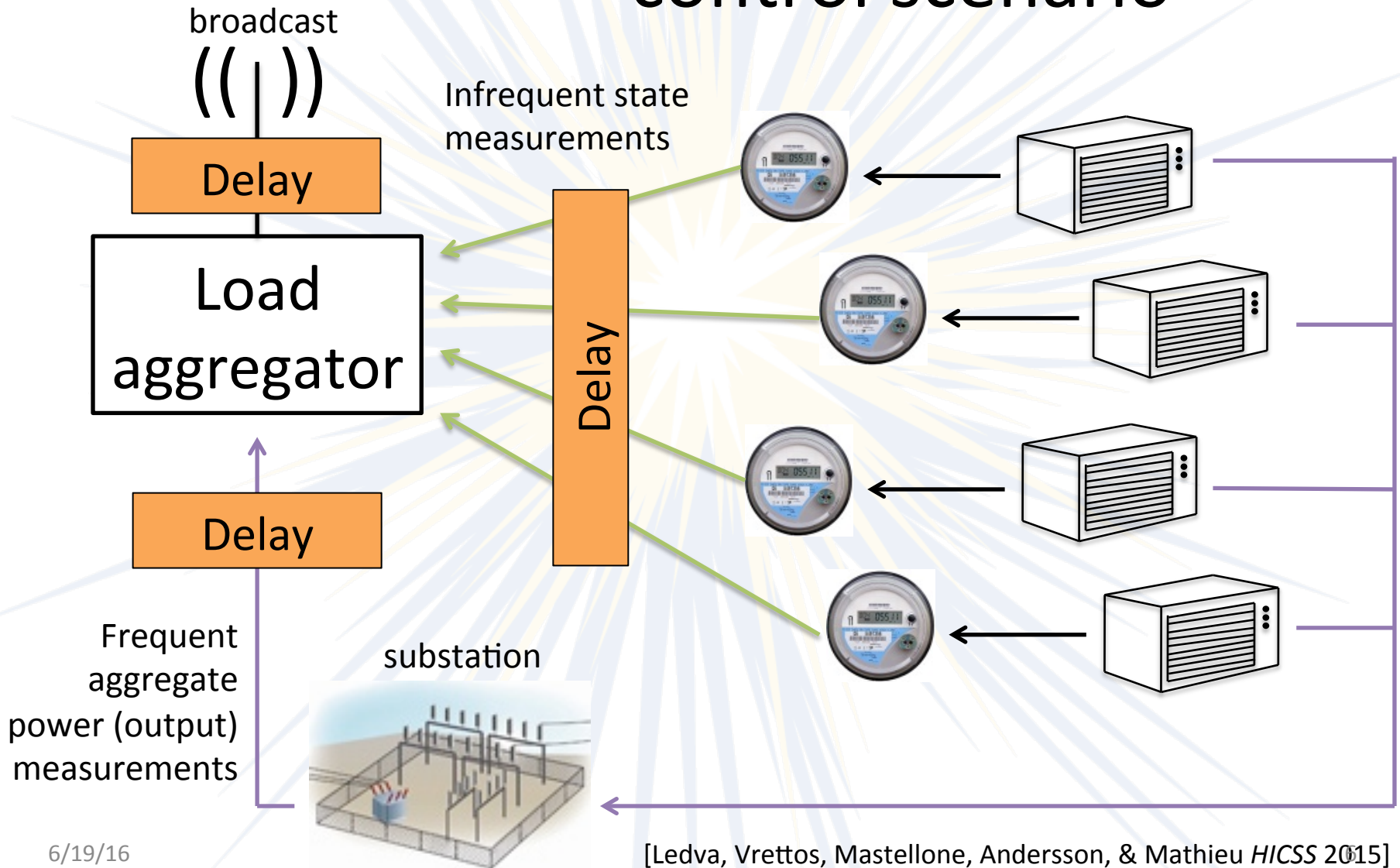
→ Low quality, frequent

- Recorded data

- high resolution power measurements of each load?

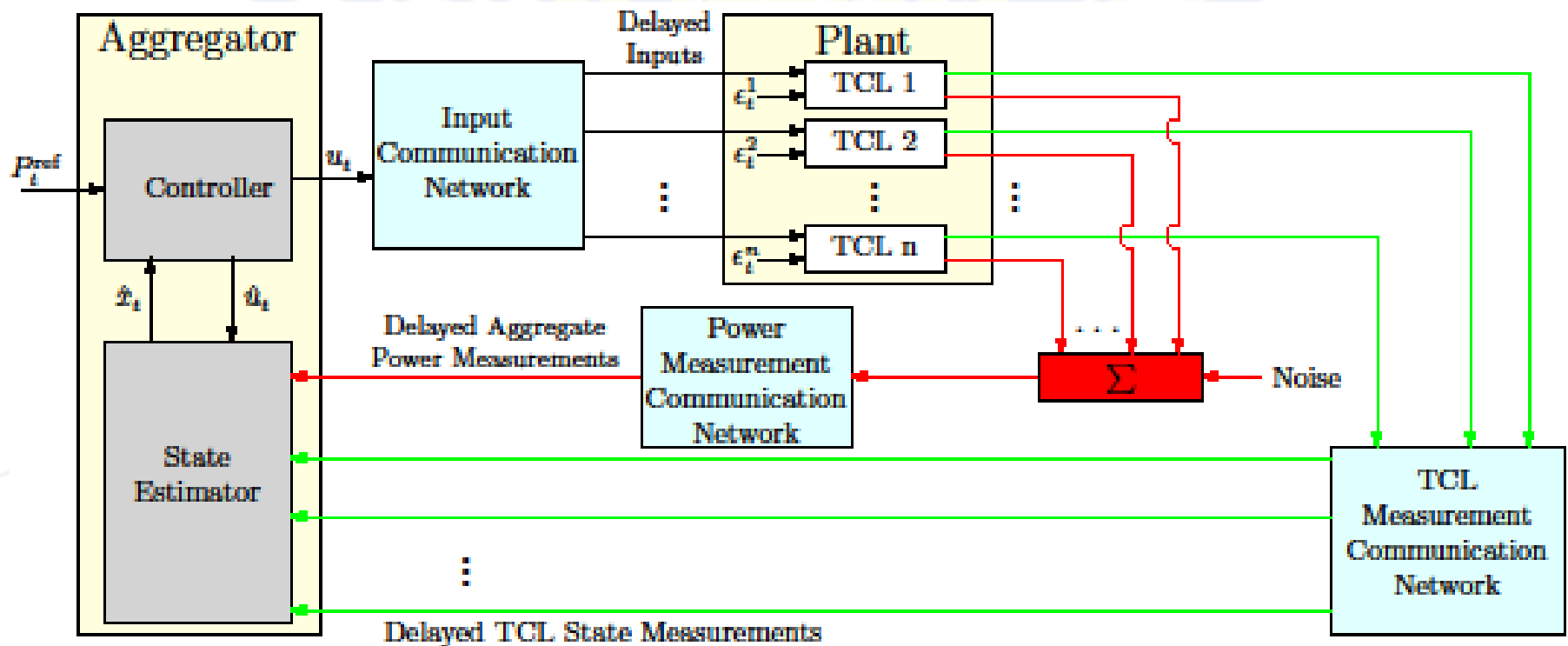
→ Auditing

Communication and control scenario



System block diagram

Delays cause unsynchronized arrivals of inputs at the loads and measurements at the estimator



The challenge

- Design an **estimator** and **controller** to enable loads to track a signal *despite delays*
- Assuming...
 - Control inputs & measurements are time-stamped
 - Delay statistics are known
 - State measurements are taken frequently; measurement *histories* are transmitted infrequently
 - Aggregate power measurements are *very* noisy (though the noise is normally distributed, zero-mean, and the standard deviation is known)

Two-state TCL model

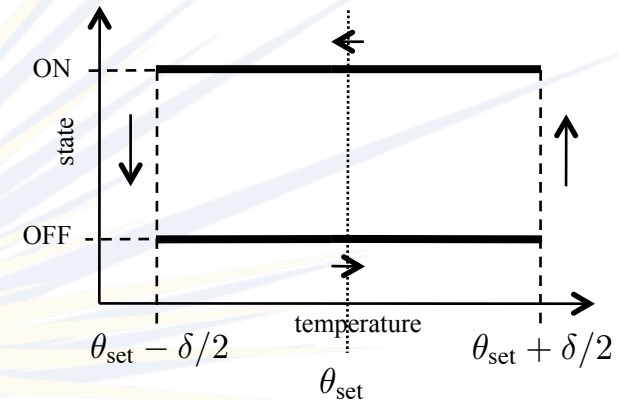
Each TCL i is modeled with a stochastic hybrid difference equation:

Temperature of the space

$$\theta_i(k+1) = a_i \theta_i(k) + (1 - a_i)(\theta_{a,i} - m_i(k)\theta_{g,i}) + \epsilon_i(k)$$

On/off state

$$m_i(k+1) = \begin{cases} 0, & \theta_i(k+1) < \theta_{\text{set},i} - \delta_i/2 \\ 1, & \theta_i(k+1) > \theta_{\text{set},i} + \delta_i/2 \\ m_i(k), & \text{otherwise} \end{cases}$$

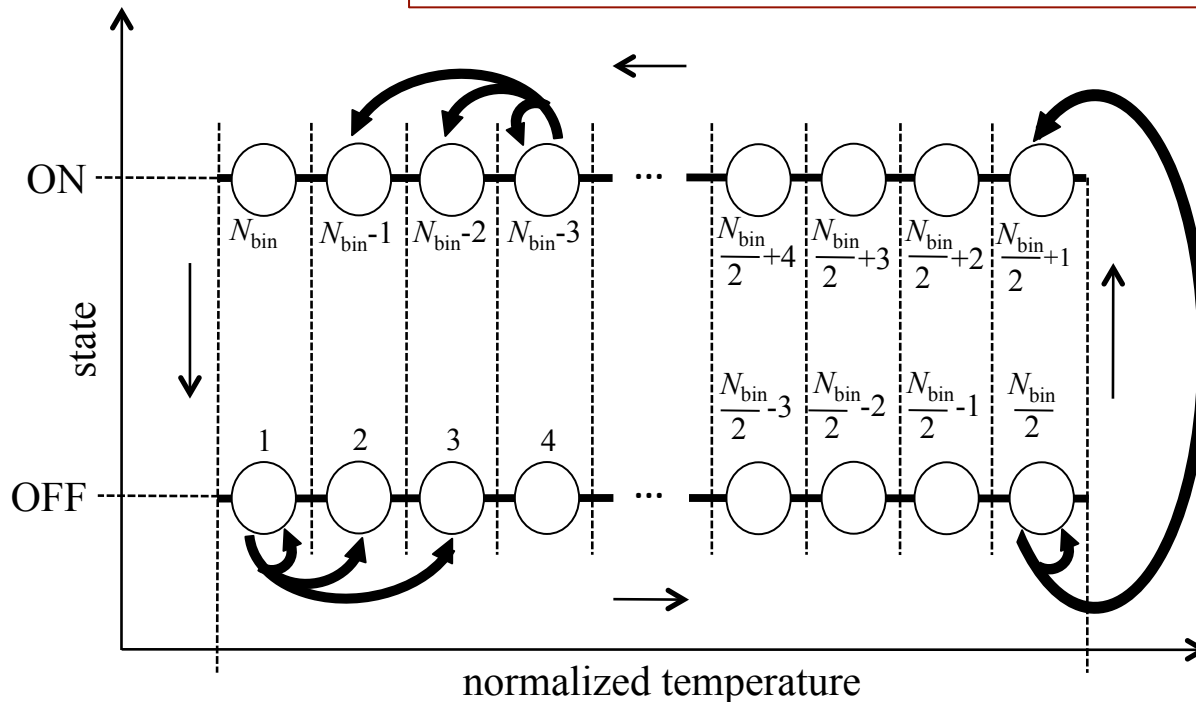


a , thermal parameter
 θ_g , temperature gain
 θ_a , ambient temperature
 ϵ , noise
 θ_{set} , set point
 δ , dead-band width

[Ihara & Schweppe 1981, Mortensen & Haggerty 1990, Uçak & Çağlar 1998]

Aggregate System Model

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{B}_\omega\boldsymbol{\omega}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \boldsymbol{\nu}(k) \end{aligned}$$



Similar models in the literature:

- Lu & Chassin 2004/2005
- Bashash & Fathy 2011/2013
- Kundu & Hiskens 2011
- Zhang et al. 2013

[Mathieu, Koch, and Callaway *IEEE Transactions on Power Systems* 2013]

Estimator Designs

- Based on Kalman Filtering
 - Estimator 1: Parallel filter estimator
 - One Kalman Filter per load
 - Each time a measurement arrives, filter it
 - Synthesize aggregate estimate from individual estimates
 - Estimator 2: Single Kalman Filter Using Aggregate State Predictions
 - Use state measurement histories to estimate *individual* load parameters (two-state model)
 - Use individual load models to predict current state
 - Use predictions as “measurements” in Kalman Filter

Controller Design

- Based on Model Predictive Control
 - Use knowledge of delay distributions and past control inputs

First control sequence:

$$u_1, u_2, u_3, \dots, u_n$$

Second control sequence:

$$u_2, u_3, \dots, u_{n+1}$$

Third control sequence:

$$u_3, u_4, \dots, u_{n+1}$$

$$u_k$$

Input estimate: $\hat{u}_k = u_k \mathcal{P}$

Control Formulation

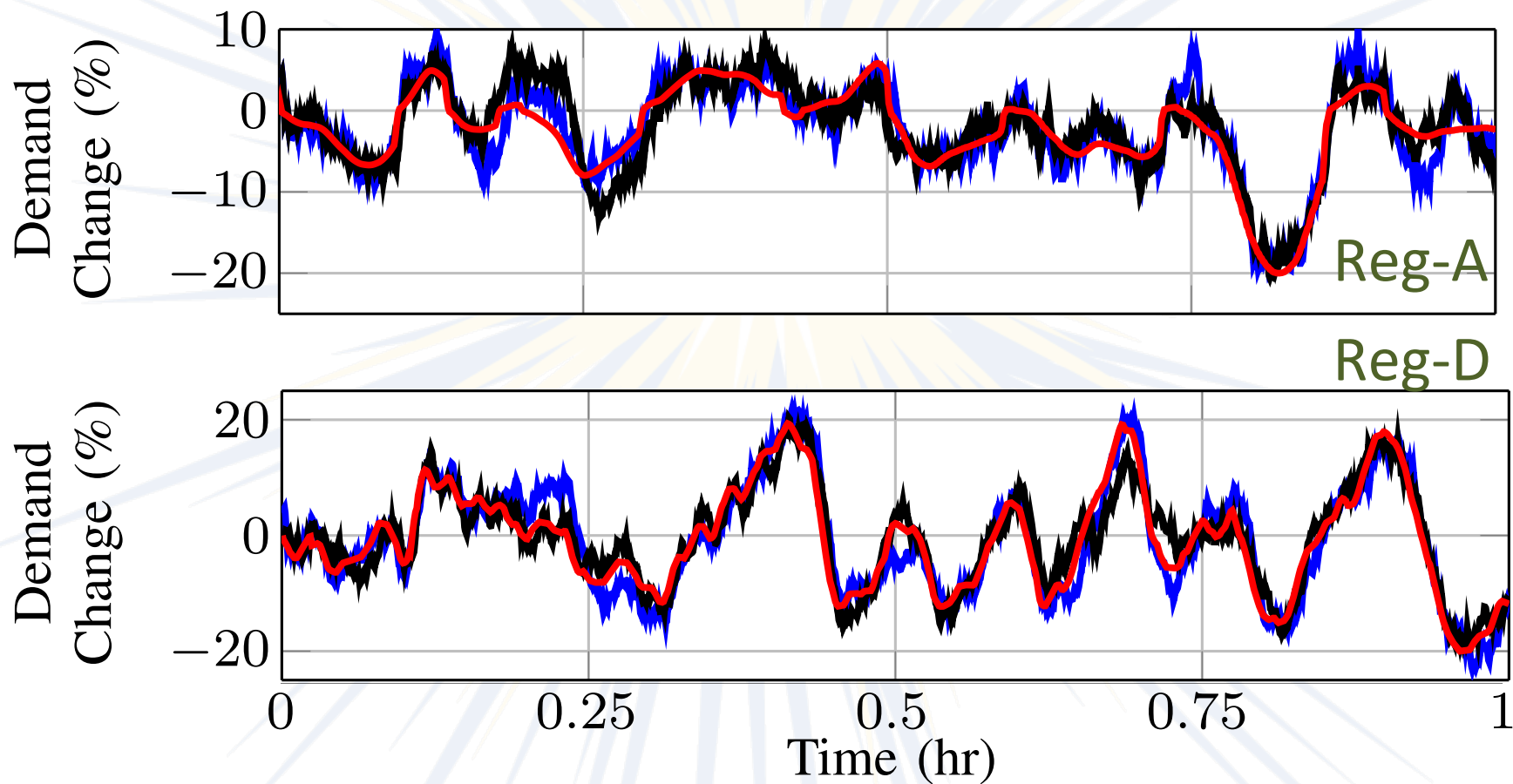
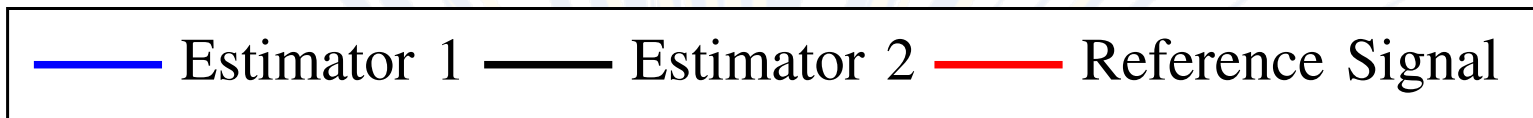
$$\text{minimize}_{u, \delta} \sum_{k=t}^{t+N^{\text{mpc}}-1} \left[\underbrace{c^y (y_k^{\text{err}})^2}_{\text{Tracking error}} + \underbrace{c^\delta (\delta_k^- + \delta_k^+)}_{\text{State constraint deviations}} + \underbrace{\sum_{m=k-N^{\text{mpc}}+1}^k c^u (u_{k|m}^\top u_{k|m})}_{\text{Control effort}} \right]$$

$$\text{s.t.} \quad \begin{aligned} x_{k+1} &= A x_k + B \hat{u}_k \\ \hat{u}_k &= U_k \mathcal{P} \\ y_k^{\text{err}} &= y_k^{P, \text{ref}} - C^P x_k \\ u_{k|m}^i &\leq x_k^i & i \in \{1, \dots, N^x/2\} \\ -u_{k|m}^i &\leq x_k^{N^x+1-i} & i \in \{1, \dots, N^x/2\} \\ 0 - \delta_k^- &\leq x_k \leq 1 + \delta_k^+ \\ 0 &\leq \delta_k^-, \delta_k^+. \end{aligned}$$

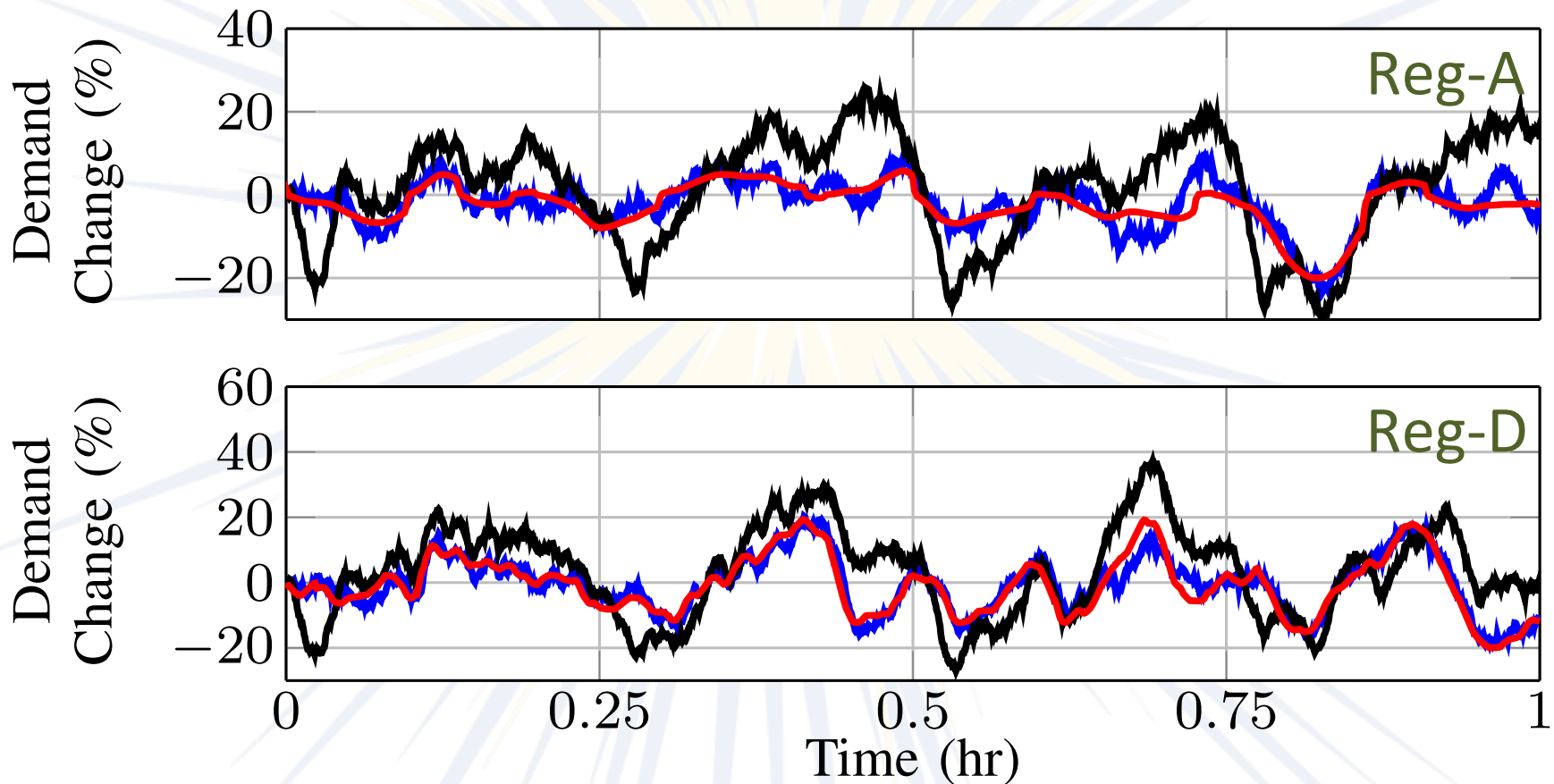
Case Studies

- PJM Regulation Signals, Reg-A & Reg-D
- Average input delay of 20 seconds
- (Delayed) state histories arrive every 15 minutes

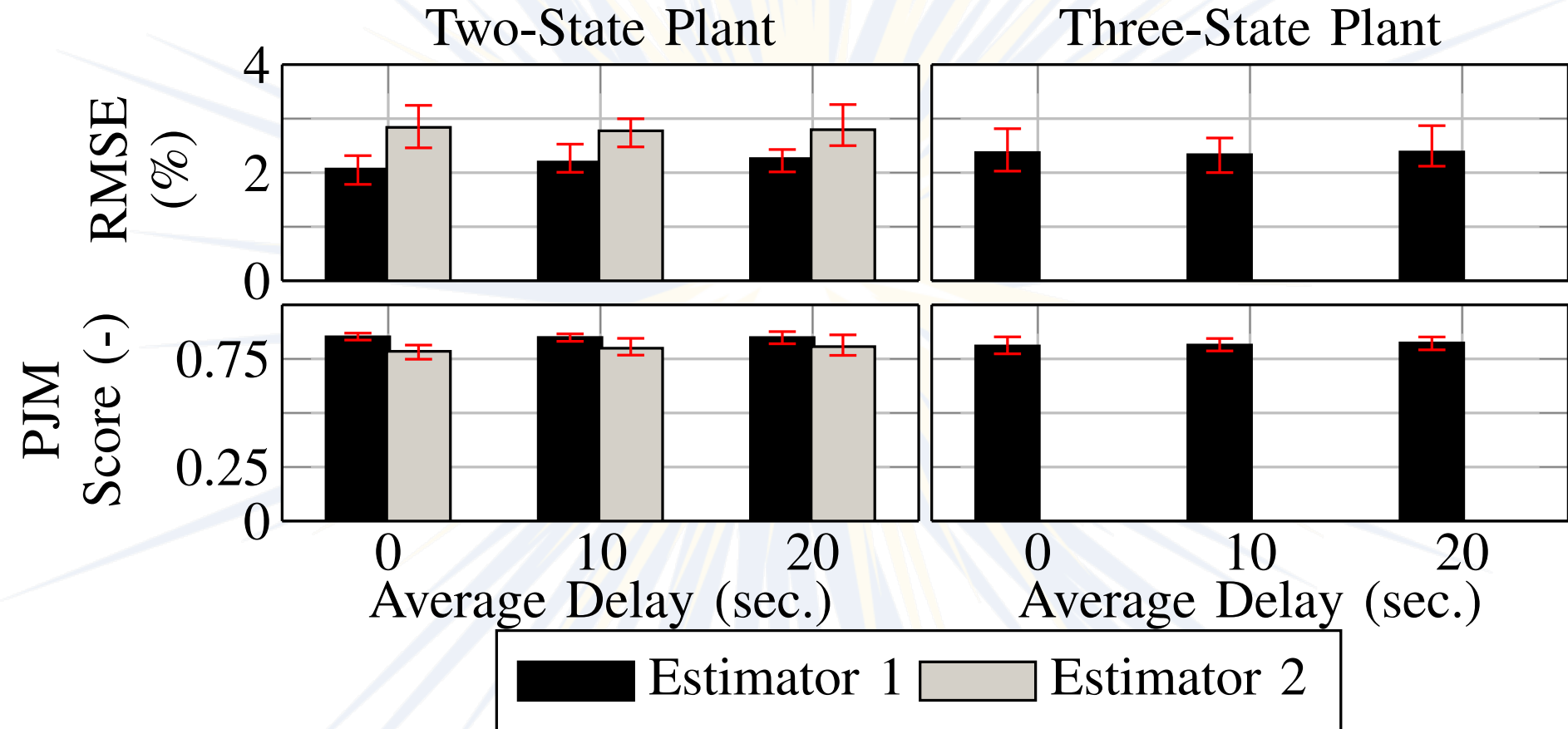
Results



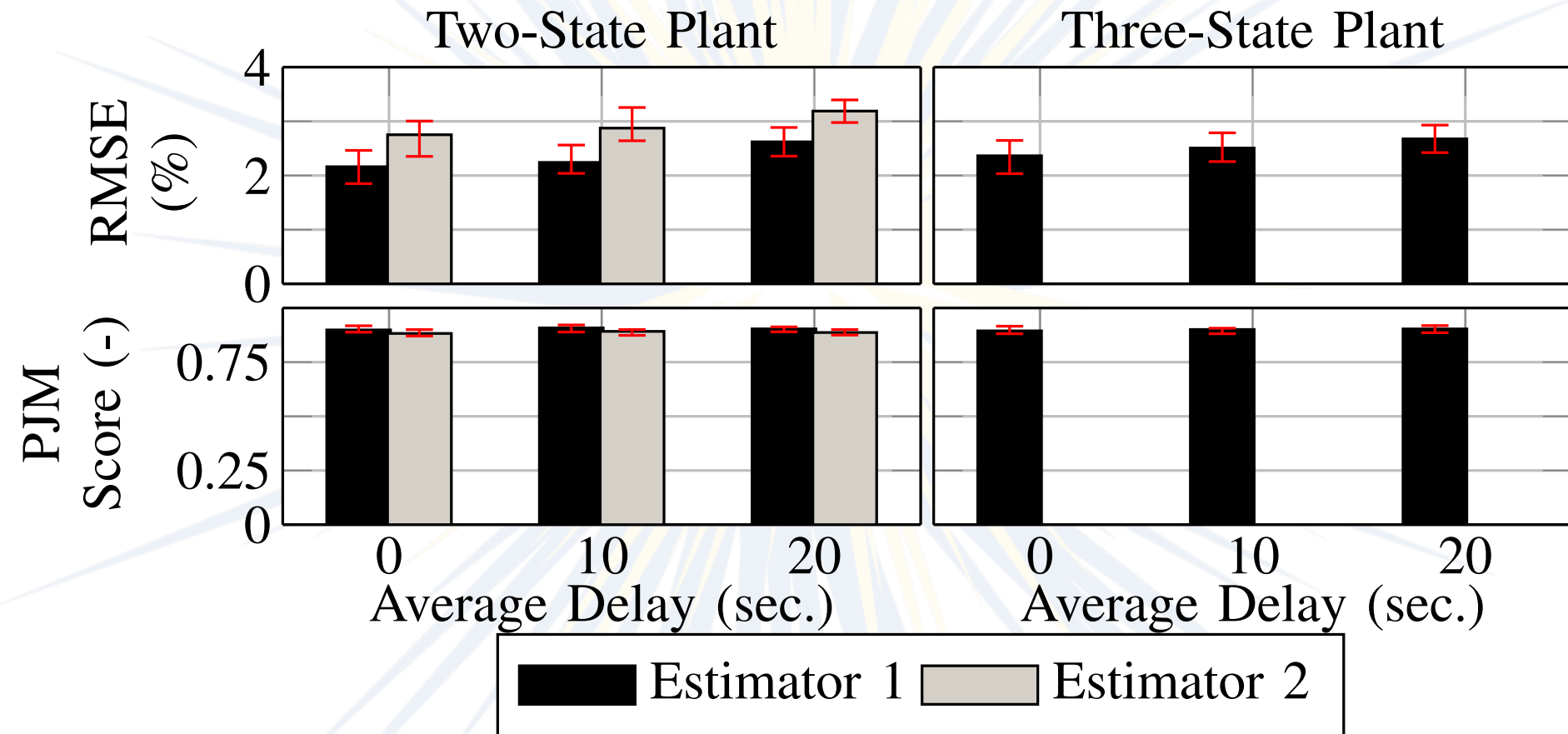
Results: Model Mismatch



Control Results: Reg-A



Control Results: Reg-D



Controller Reformulation

Original Model

$$x_{k+1} = A x_k + B u_k$$

$$y_k = C x_k.$$

Modal Model

$$\begin{bmatrix} 1 \\ \tilde{x}_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \tilde{A} \end{bmatrix}}_{A^*} \underbrace{\begin{bmatrix} 1 \\ \tilde{x}_k \end{bmatrix}}_{x_k^*} + \underbrace{\begin{bmatrix} 0 \\ \tilde{B} \end{bmatrix}}_{B^*} u_k$$

$$y_k = \underbrace{\begin{bmatrix} y_{ss} & \tilde{C} \end{bmatrix}}_{C^*} \begin{bmatrix} 1 \\ \tilde{x}_k \end{bmatrix}$$

Reduced-Order Model

$$\tilde{x}_{k+1} = \tilde{A} \tilde{x}_k + \tilde{B} u_k$$

$$\tilde{y}_k = \tilde{C} \tilde{x}_k.$$

Controller Reformulation

The linear controller uses constant gains generated from an output-regulating Linear Quadratic Regulator (LQR) with reference feedforward.

Linear Controller
$$u_t^{\text{seq}} = -K_{\infty}^x \bar{x}_t - K_{\infty}^w w_t + K_{\infty}^y y_t^{\text{des}}$$

LQR Formulation
$$\min_u \sum_{k=t}^{\infty} \begin{bmatrix} \tilde{x}_k \\ w_k \end{bmatrix}^T \begin{bmatrix} \tilde{C}^T & q^y \tilde{C} & 0 \\ 0 & 0 & q^w \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ w_k \end{bmatrix} + (u_k^{\text{seq}})^T R u_k^{\text{seq}}$$

s.t.
$$\begin{bmatrix} \tilde{x}_{k+1} \\ w_{k+1} \end{bmatrix} = \begin{bmatrix} \tilde{A} & 0 \\ \tilde{C} & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ w_k \end{bmatrix} + \begin{bmatrix} \tilde{B} \\ 0 \end{bmatrix} u_k^{\text{seq}}$$

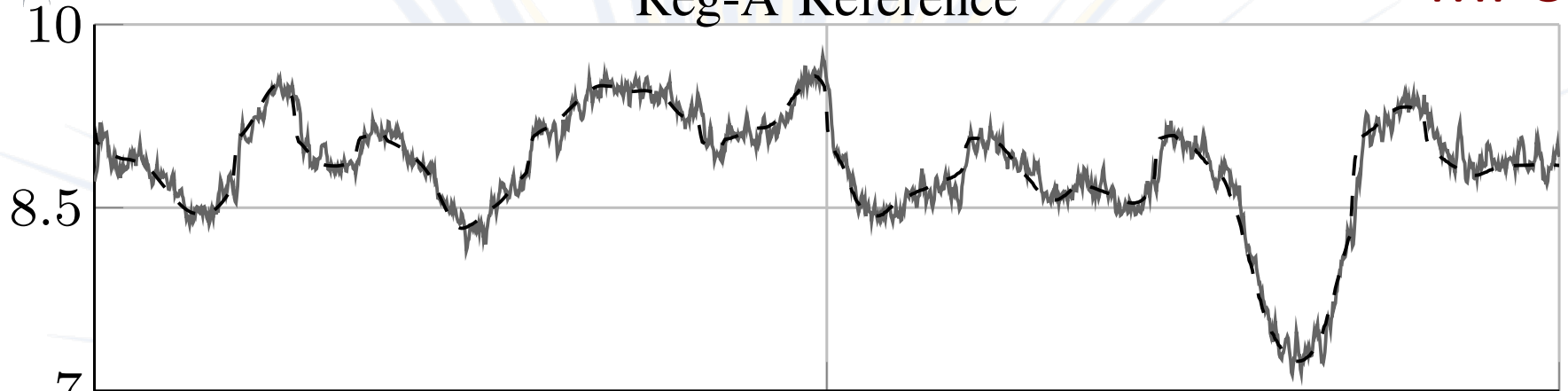
Feedforward Gain
$$K_{\infty}^y = \left(\tilde{C} \{ zI - \tilde{A} + \tilde{B} \tilde{K}_{\infty}^x \}^{-1} \tilde{B} \right)^{-\dagger}$$

Case Studies

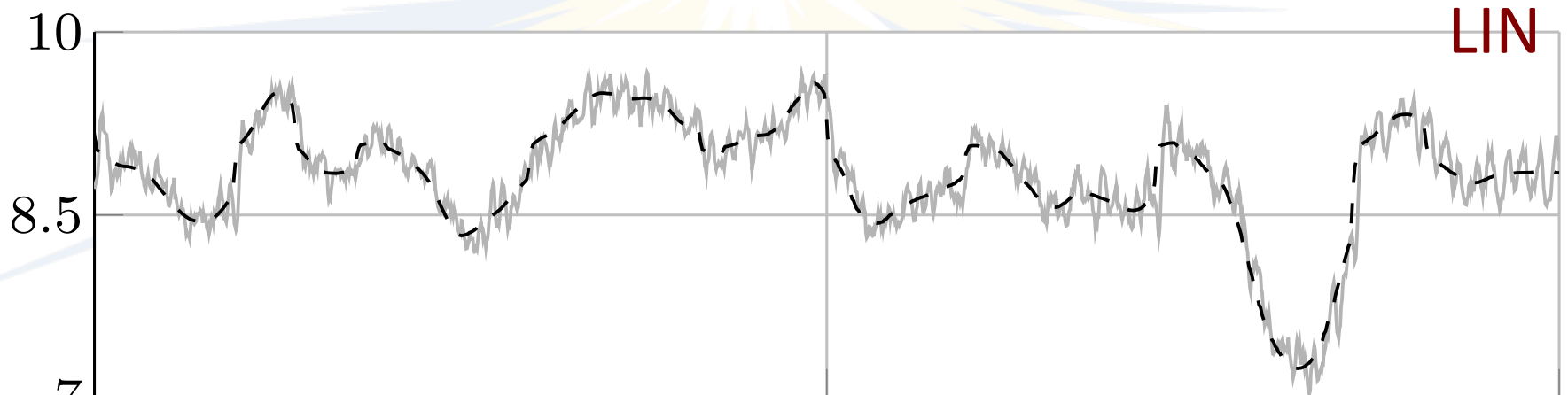
- PJM Regulation Signals, Reg-A & Reg-D
- Average input delay of 20 seconds
- Full state feedback, no measurement delay
- Three-state models used for the plant

Reg-A Reference

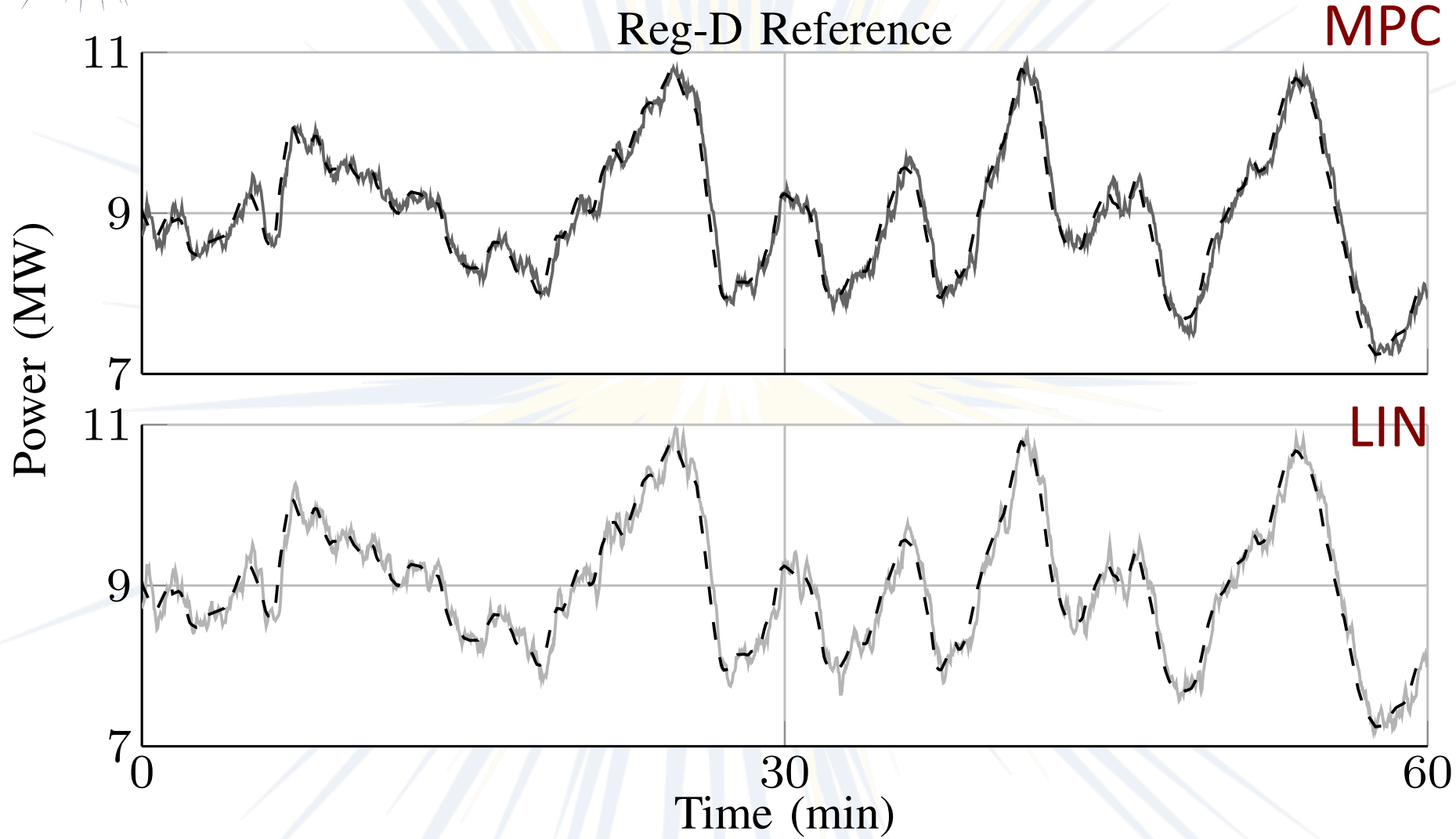
MPC



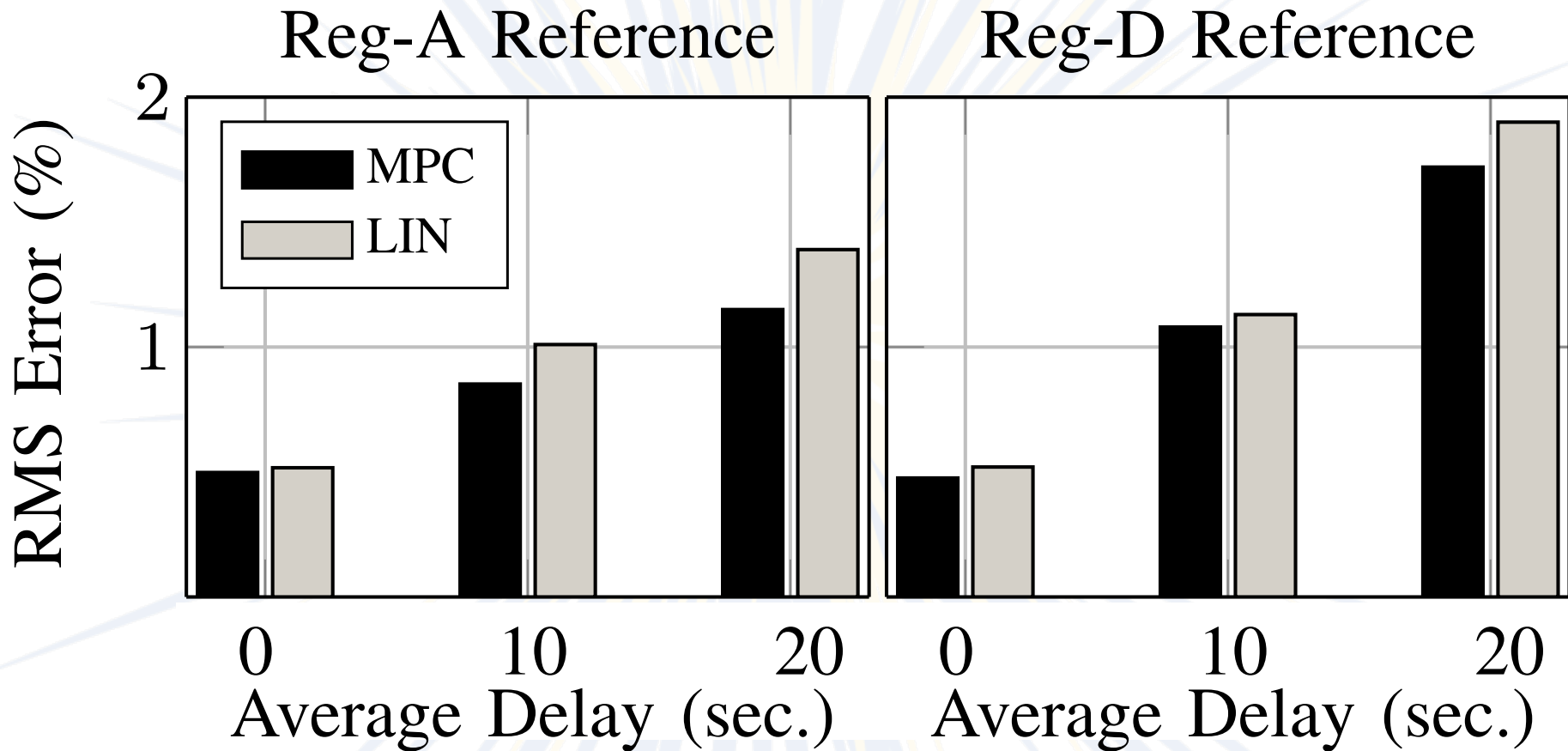
LIN



Time (min)



Results Summary



→ LIN is 100 times faster than MPC

Key takeaways

- Communication network limitations necessitate controller/estimator designs that cope with delays, bandwidth limitations, etc.
- Delays make loads less capable of providing fast services, but we can mitigate these impacts through delay-aware control and estimation.