



# Inference and control of electric loads given sparse measurements and communications delays

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# Benefits and Challenges of the Modern Electric Grid

- Grid sensing and communication systems are becoming more prevalent
  - Cost & privacy concerns
  - Need methods to infer grid/load information from existing measurements
- Renewable energy resources are also becoming more prevalent
  - Most (e.g., wind and solar) are intermittent and uncertain
  - Need new sources of power system reserves

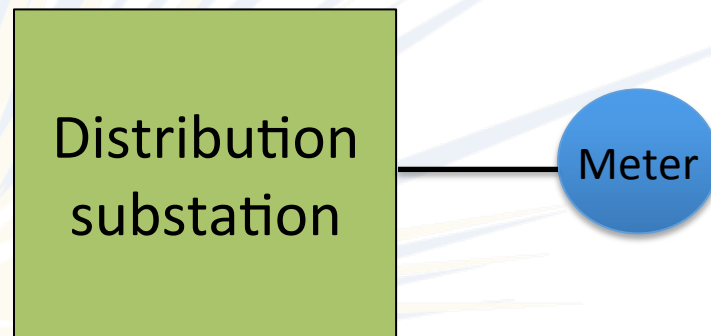
- **Inference:** Inferring the behavior of distributed energy resources with sparse measurements  
[Ledva, Balzano, & Mathieu *Allerton* 2015]
- **Control:** Controlling distributed electric loads to provide power system services with sparse measurements and input/measurement delays  
[Ledva, Vrettos, Mastellone, Andersson & Mathieu *HICSS* 2015]  
[Ledva & Mathieu *CDC (in review)* 2016]

# Disaggregating substation load data

Power consumption of all the  
loads/generators we care about



Power consumption of all the  
loads/generators we DON'T care about



## Why do we want to disaggregate resources at the substation?

- Energy efficiency via conservation voltage reduction
- Contingency planning
- Optimal reserve contracting
- Demand response event signaling
- Demand response bidding
- Load coordination feedback

# Disaggregation methods

e.g., [Berges et al. 2009; Kolter et al. 2010;  
Dong et al. 2013]

- State estimation
  - Linear techniques require LTI system models
  - Nonlinear techniques can be computationally demanding
- Online learning
  - Optimization formulations
  - Model-free
- Hybrid approach: Dynamic Mirror Descent [Hall & Willet 2015]
  - Admits dynamic models of arbitrary forms
  - Optimization-based method to choose a weighted combination of the estimates of a collection of models

# Outline: Part 1

- Dynamic Mirror Descent
- Problem setting: Plant data/models
- Algorithm Models
- Results
- Next steps

# Dynamic Mirror Descent

- Mirror Descent: online algorithm to estimate a fixed state
- Dynamic Mirror Descent: online algorithm to estimate a dynamic state using a *collection of models* [Hall & Willet 2015]
  1. Compute the error between the model predictions and the measured data (i.e., loss function)
  2. Update the state in the direction of the negative gradient of the loss function

$$\tilde{\theta}_t^i = \arg \min_{\theta \in \Theta} \eta_t \left\langle \nabla \ell_t(\hat{\theta}_t^i, y_t), \theta \right\rangle + D \left( \theta \parallel \hat{\theta}_t^i \right)$$

# Dynamic Mirror Descent

3. Use the estimated states to evaluate the models for the next time step

$$\hat{\theta}_{t+1}^i = \Phi_t^i(\tilde{\theta}_t^i)$$

4. Compute a weighted version of the estimates

$$\hat{\theta}_{t+1} = \sum_{i=1}^{N^{\text{mdl}}} w_{t+1}^i \hat{\theta}_{t+1}^i.$$

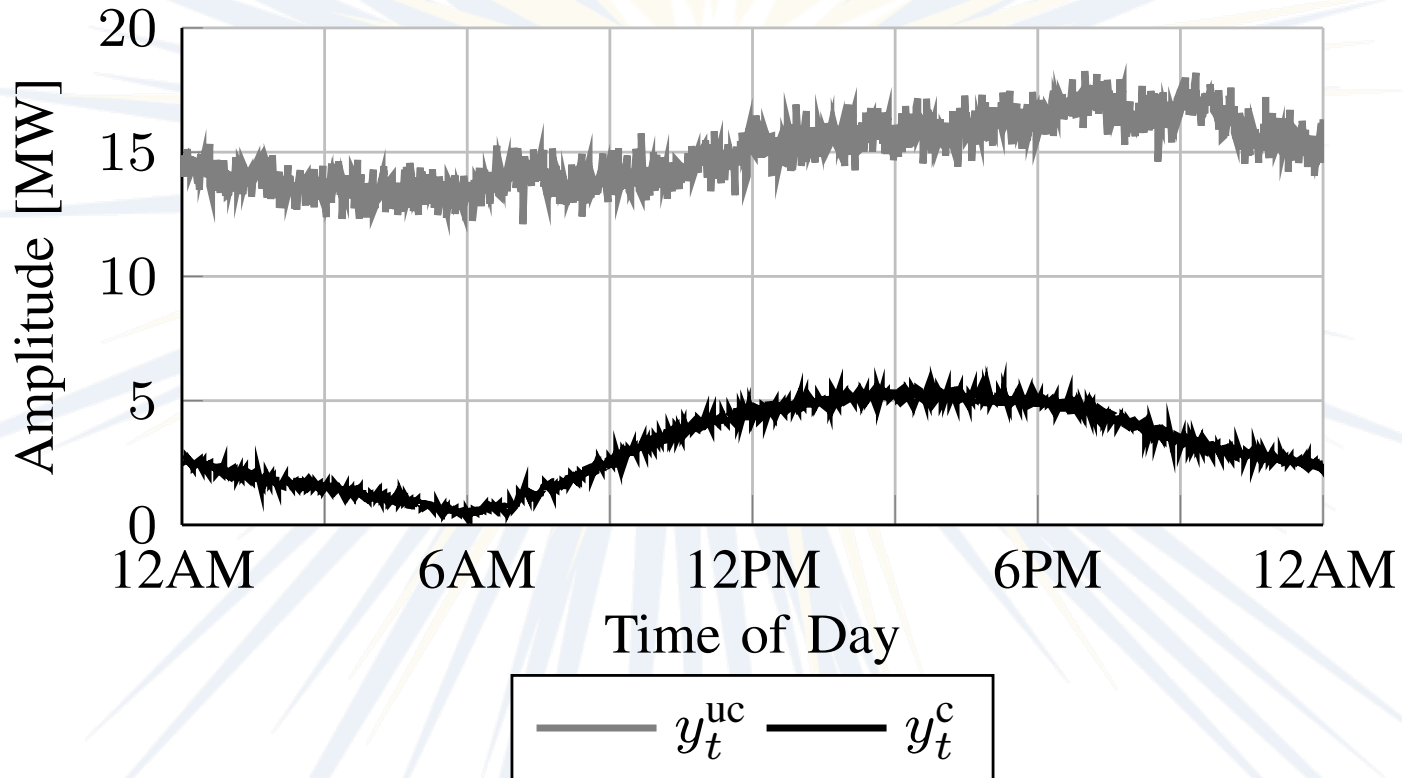
5. Update the model weights

$$w_{t+1}^i = \frac{\lambda}{N^{\text{mdl}}} + (1 - \lambda) \frac{w_t^i \exp\left(-\eta^r \ell_t\left(\hat{\theta}_t^i, y_t\right)\right)}{\sum_{j=1}^{N^{\text{mdl}}} w_t^j \exp\left(-\eta^r \ell_t\left(\hat{\theta}_t^j, y_t\right)\right)}$$

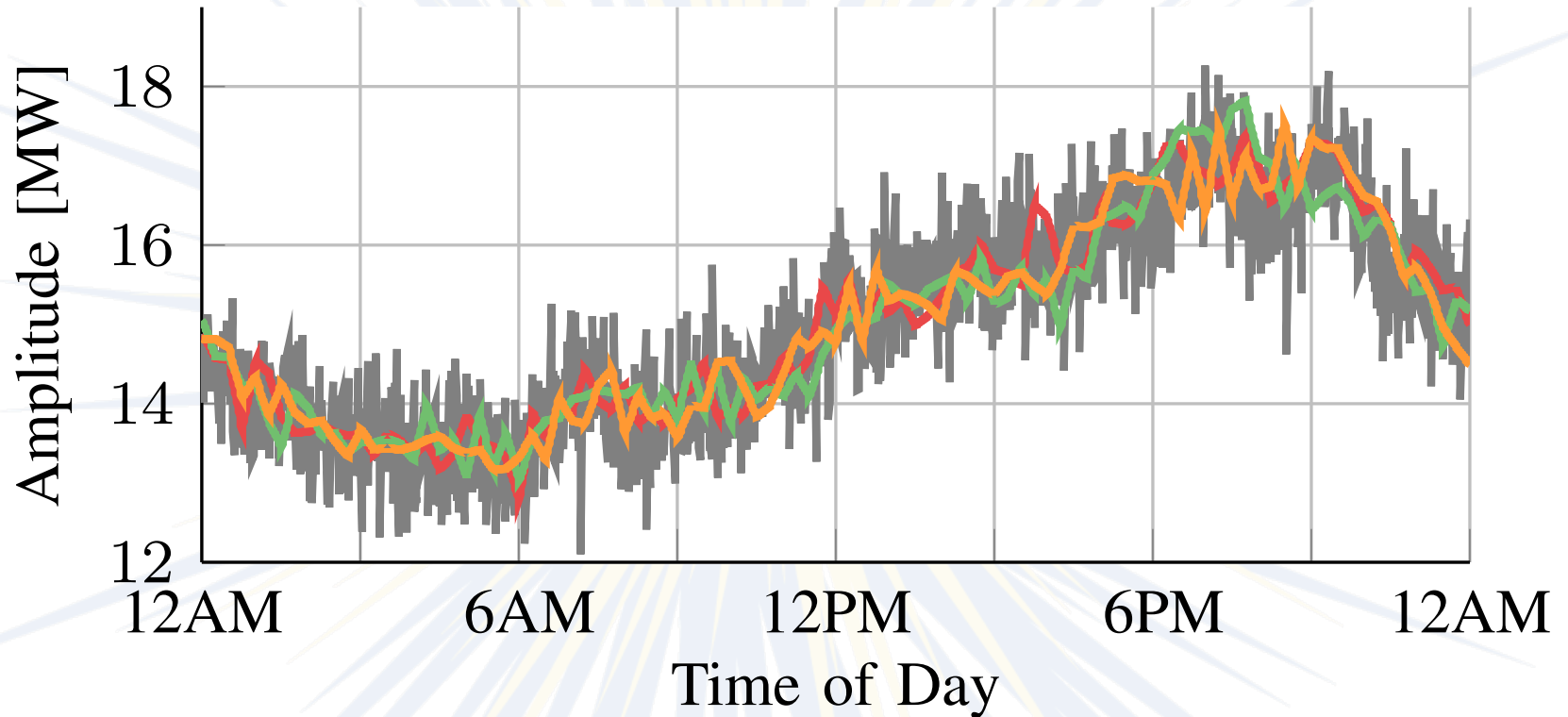


# Problem Setting: Plant Data/Models

- Uncontrollable loads: data from Pecan Street Inc. Dataport
- Controllable loads: three-state hybrid models of air conditioners [Sonderegger 1978]



# Algorithm Models: Uncontrollable loads



# Algorithm Models: Controllable loads

- Two-state hybrid models of air conditioners [Mortensen & Haggerty 1988]
  - Temperature and ON/OFF mode
- Sets of Linear Time Invariant (LTI) aggregate system models [Mathieu et al. 2013]

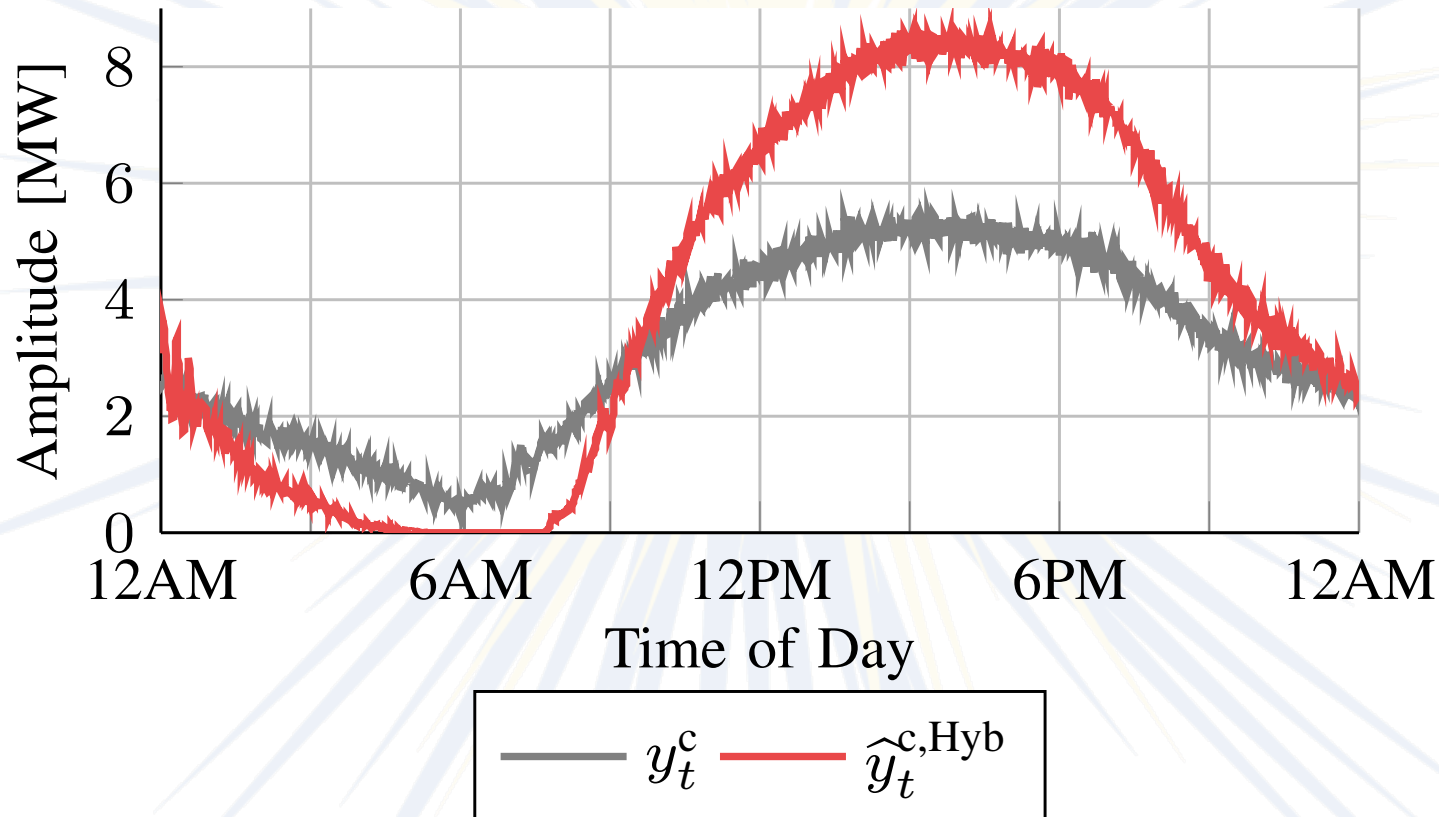
$$\begin{aligned}x_{t+1}^i &= A^i x_t^i & i \in \mathbb{N}^{\text{temps}} \\ \widehat{y}_t^{\text{c,LTI},i} &= C^i x_t^i & i \in \mathbb{N}^{\text{temps}}.\end{aligned}$$

- Sets of Linear Time Varying (LTV) aggregate system models

$$\begin{aligned}x_{t+1} &= A_t x_t \\ \widehat{y}_t^{\text{c,LTV}} &= C_t x_t.\end{aligned}$$

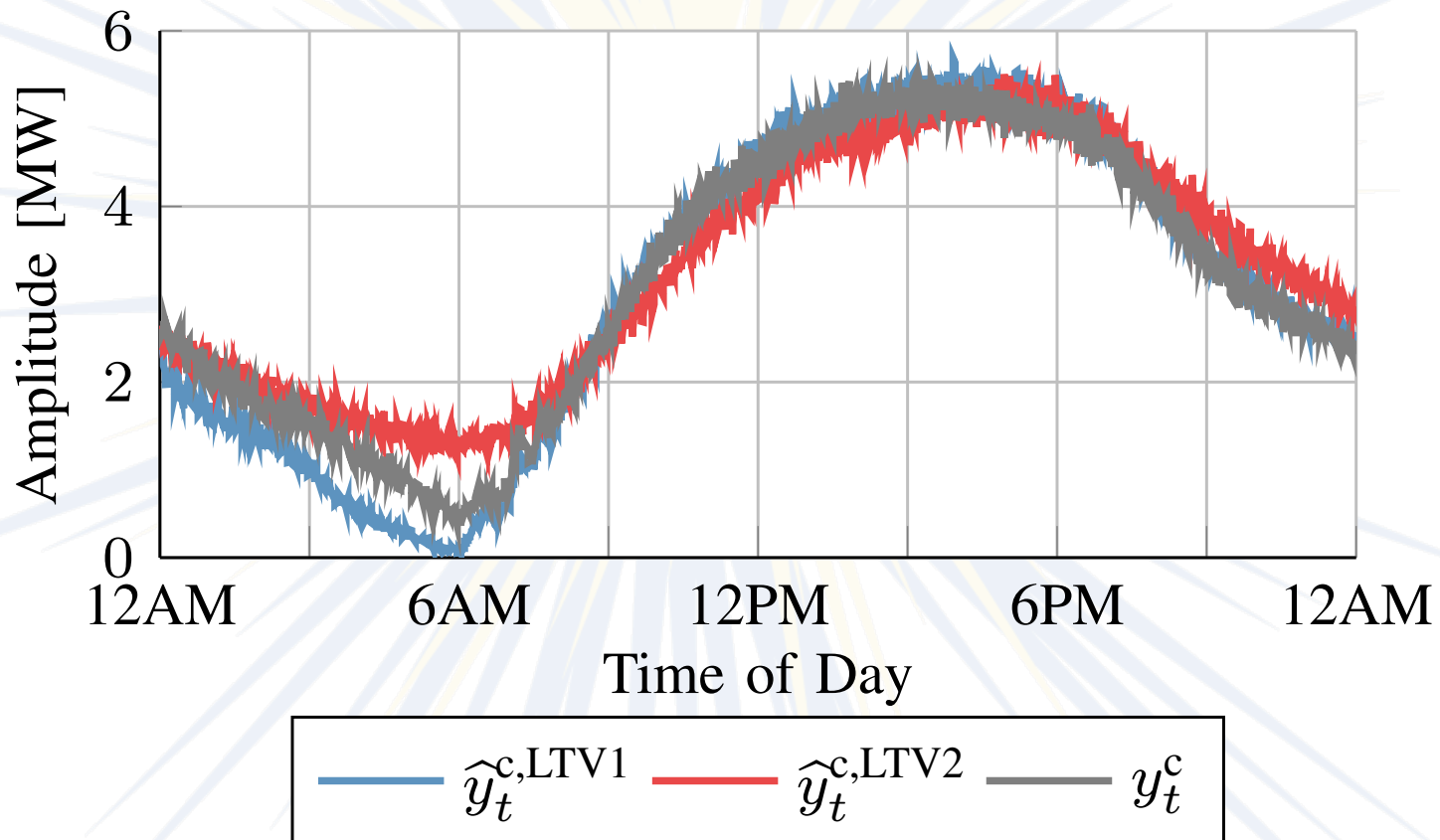
# Algorithm Models: Controllable loads

- Two-state hybrid AC models do not work well.



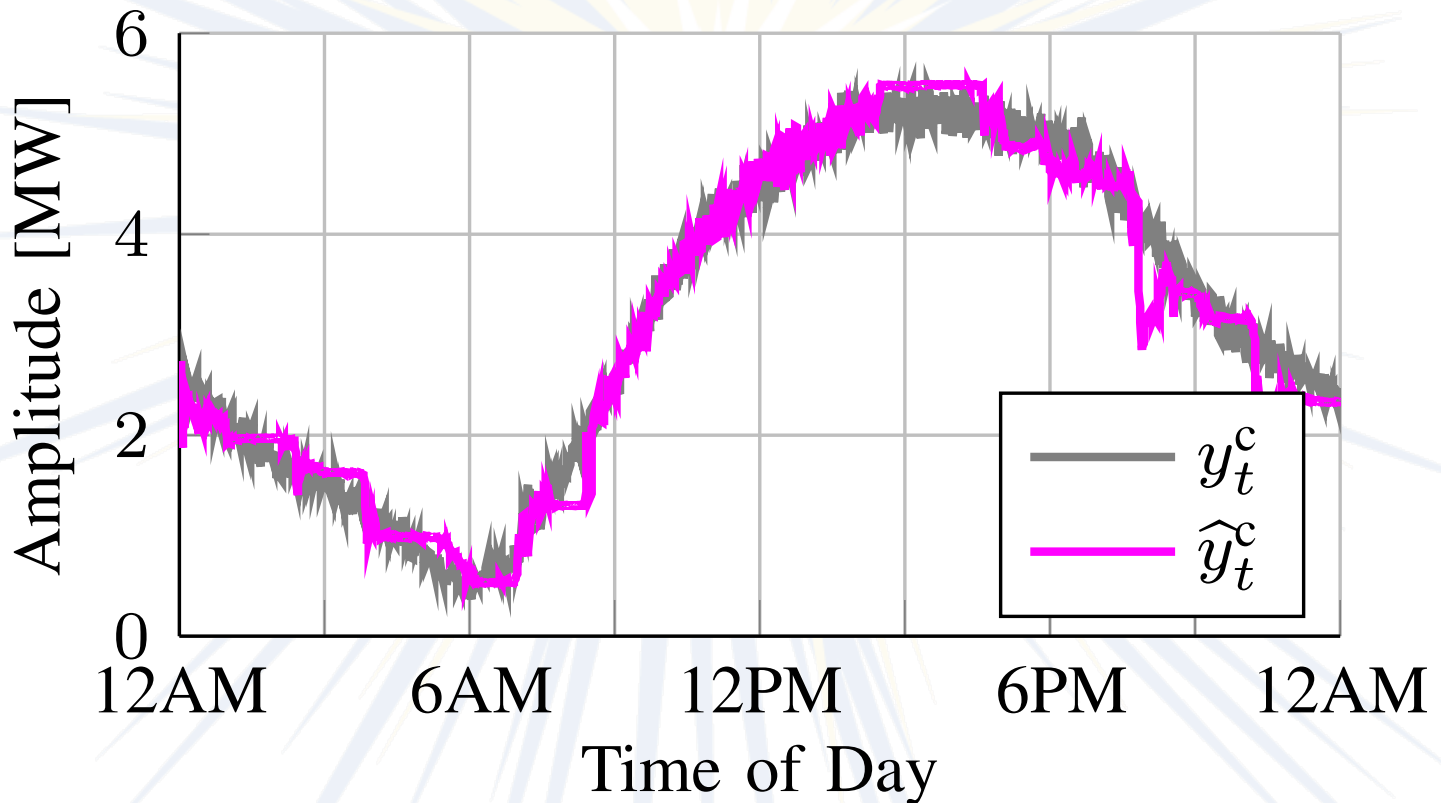
# Algorithm Models: Controllable loads

- LTV models work better.

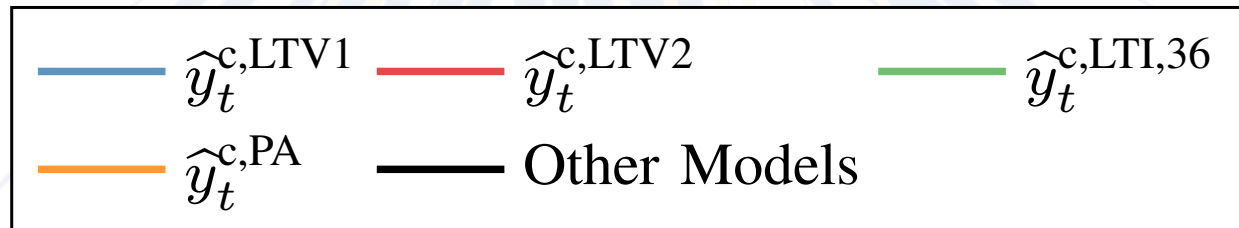
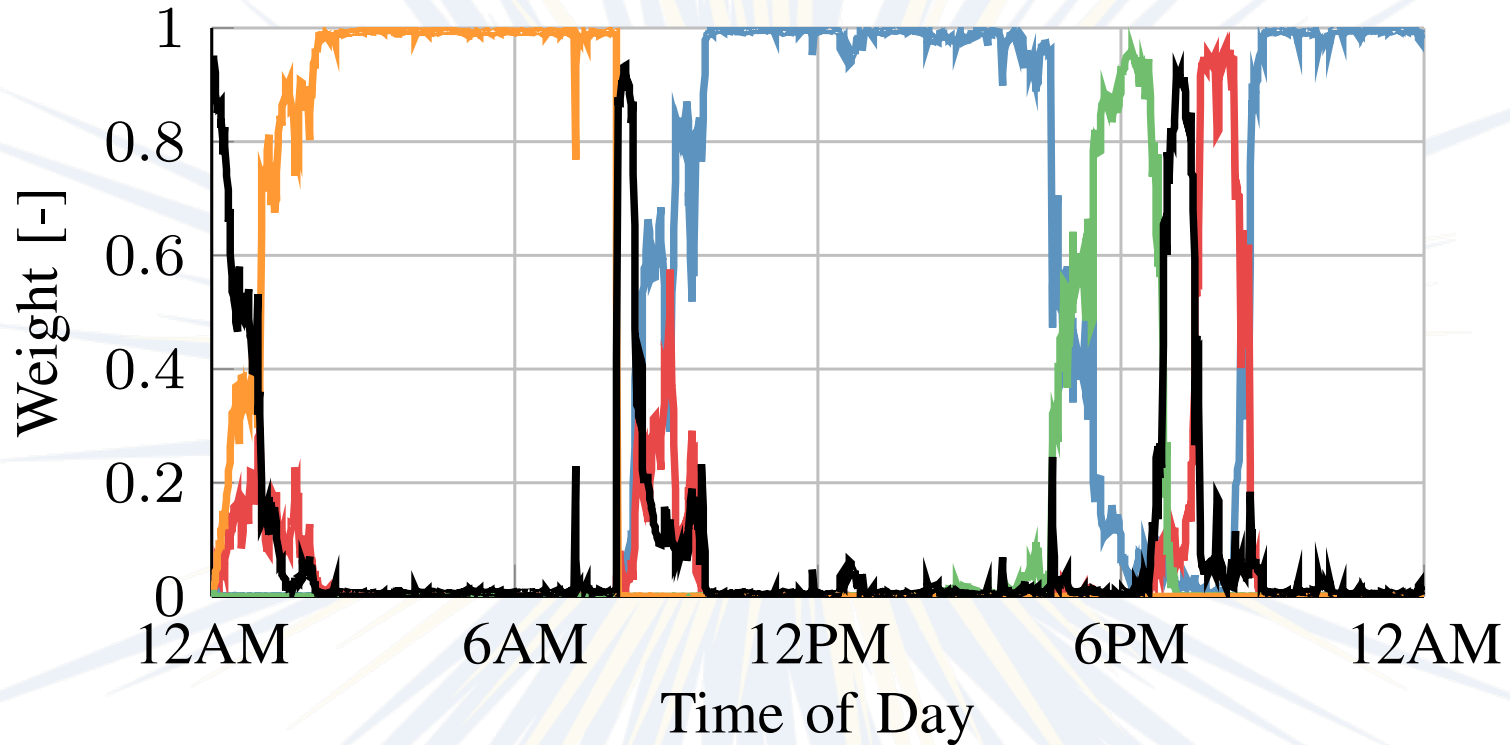


# Results

Model bank is all combinations of uncontrollable and controllable load models (57 models)

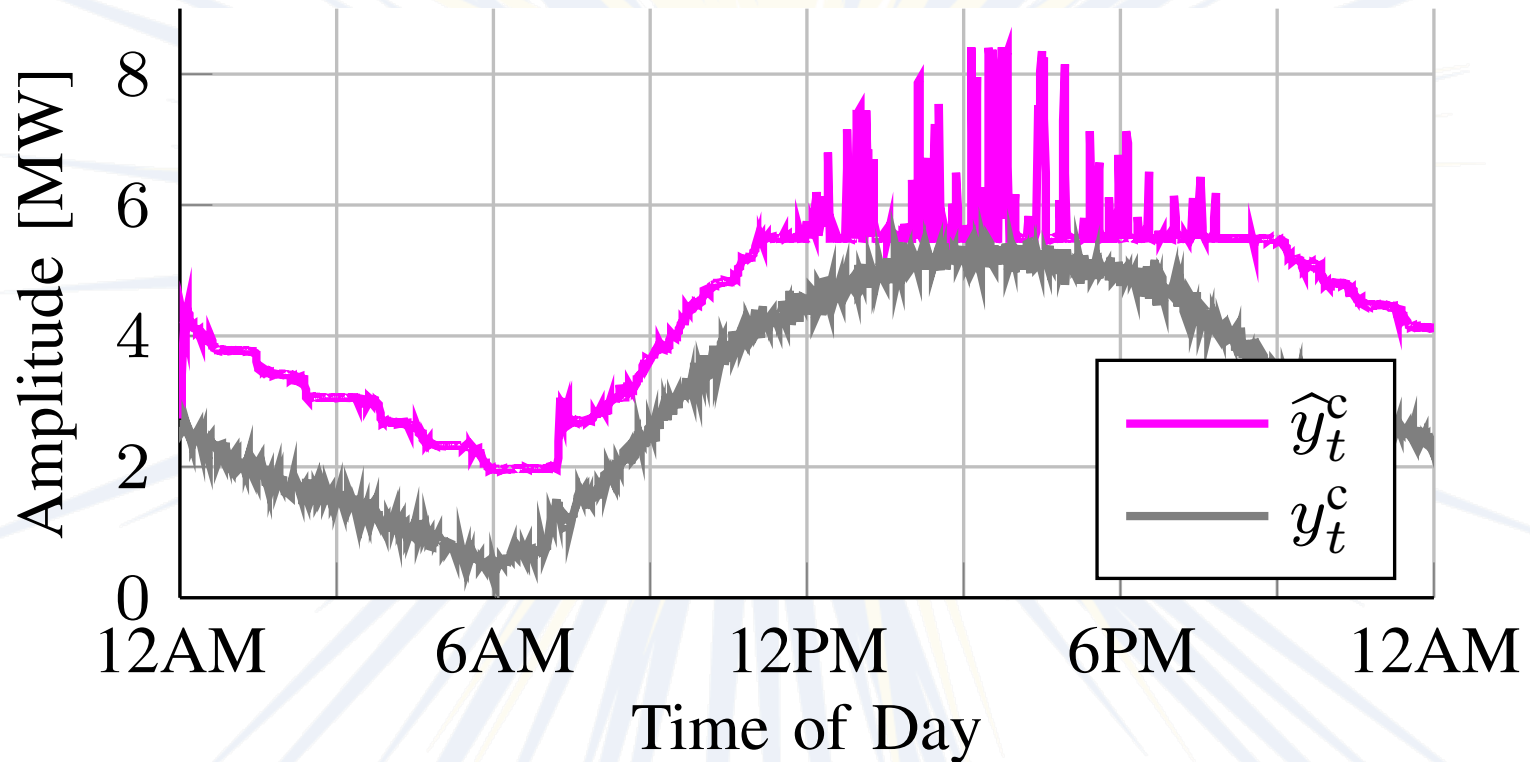


# Results: Weightings



# Results: Bad Models

- All uncontrollable load models are too low.



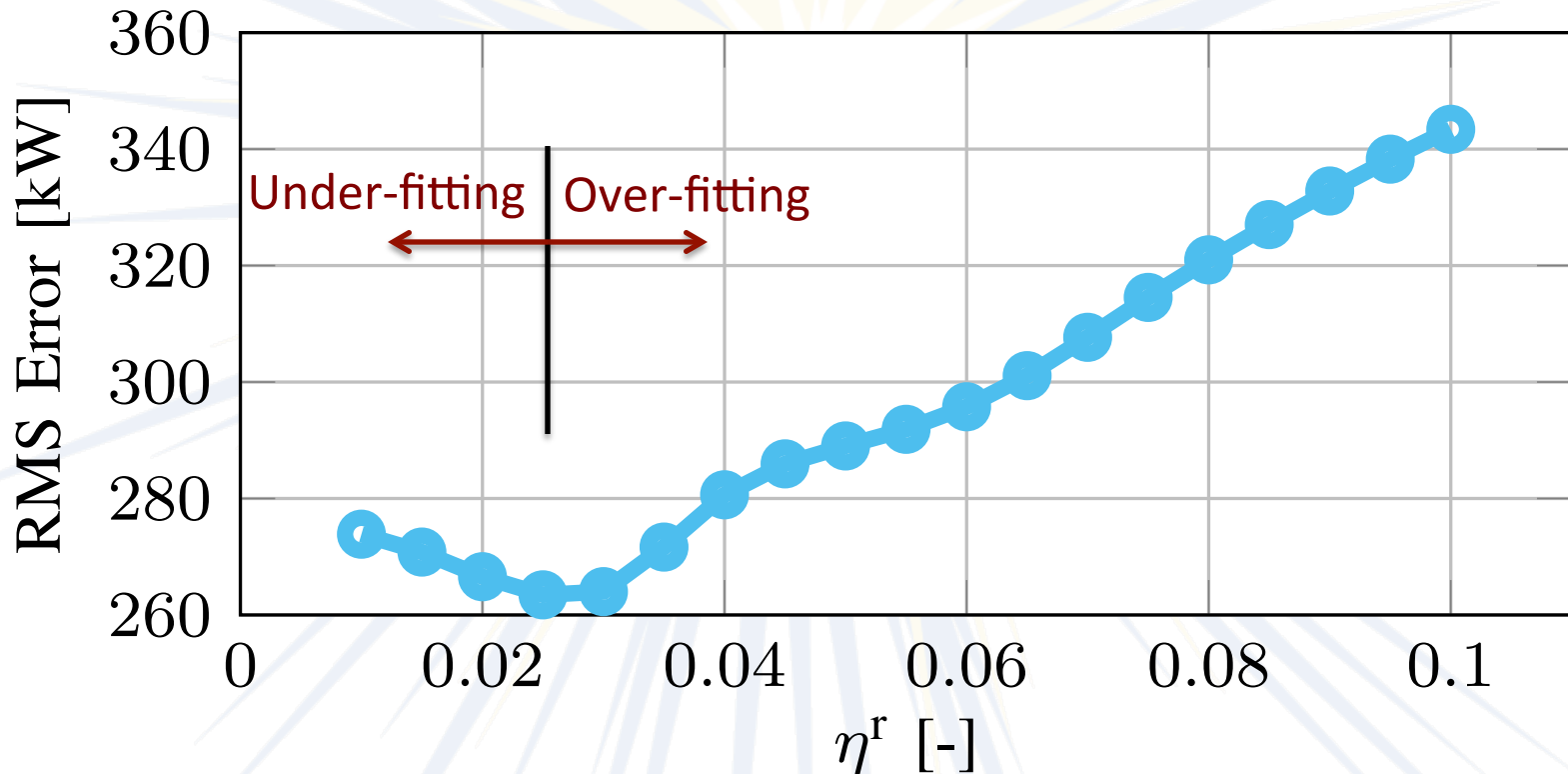


# Results: Summary

Case	RMS Error (kW)
<b>Benchmark:</b> Use current outdoor temperature to evaluate simple controllable load model	738
<b>DMD Case 1:</b> Includes every combination of uncontrollable and controllable models	264
<b>DMD Case 2:</b> Case 1 models plus a smoothed version of the actual uncontrollable load	211
<b>DMD Case 3:</b> Case 2 models plus more accurate models of the controllable load over time periods where the other models are less accurate	175
<b>DMD Case 4:</b> Includes uncontrollable load models that underestimate the uncontrollable load	1392

# Results: Varying Algorithm Parameters

Recall: 
$$w_{t+1}^i = \frac{\lambda}{N^{\text{mdl}}} + (1 - \lambda) \frac{w_t^i \exp\left(-\eta^r l_t\left(\hat{\theta}_t^i, y_t\right)\right)}{\sum_{j=1}^{N^{\text{mdl}}} w_t^j \exp\left(-\eta^r l_t\left(\hat{\theta}_t^j, y_t\right)\right)}$$



# Next steps

- Investigate more realistic settings (using more real data)
- Develop better load models
- Improve the algorithm, e.g., alternative weighting functions
- Investigate identifiability
- Incorporate additional measurements (reactive power, voltage) into the approach

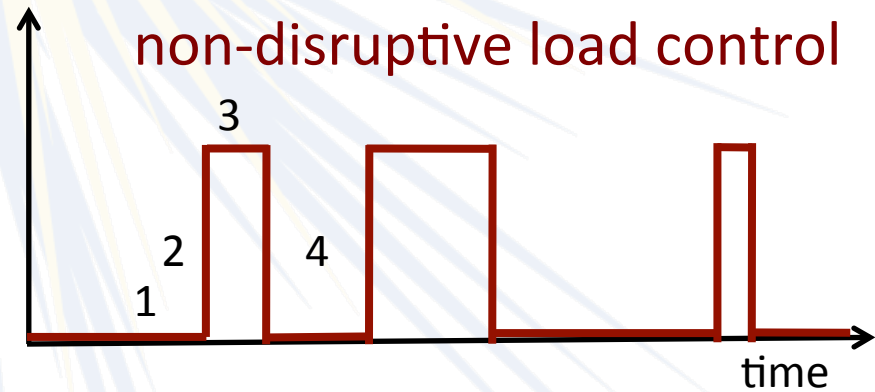
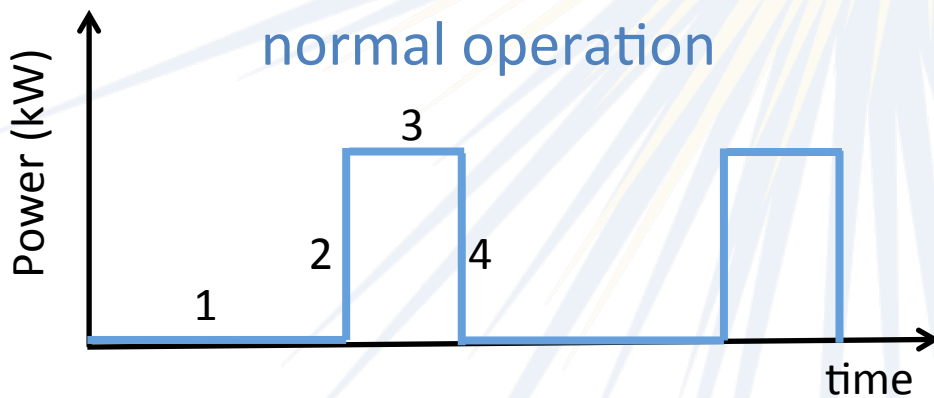
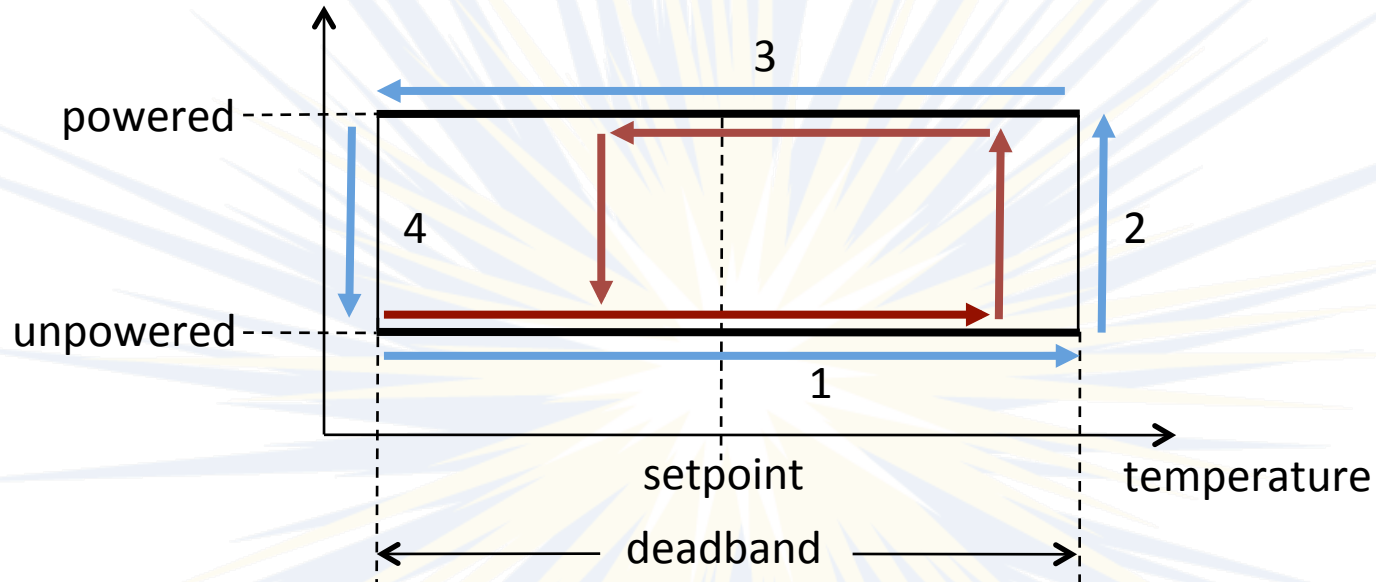
# Key findings

- Dynamic Mirror Descent (DMD) enables us to solve the substation disaggregation problem leveraging dynamical models of arbitrary form
- DMD can work well (on simple examples); however, it is easy to find instances where it does not work well

- **Inference:** Inferring the behavior of distributed energy resources with sparse measurements  
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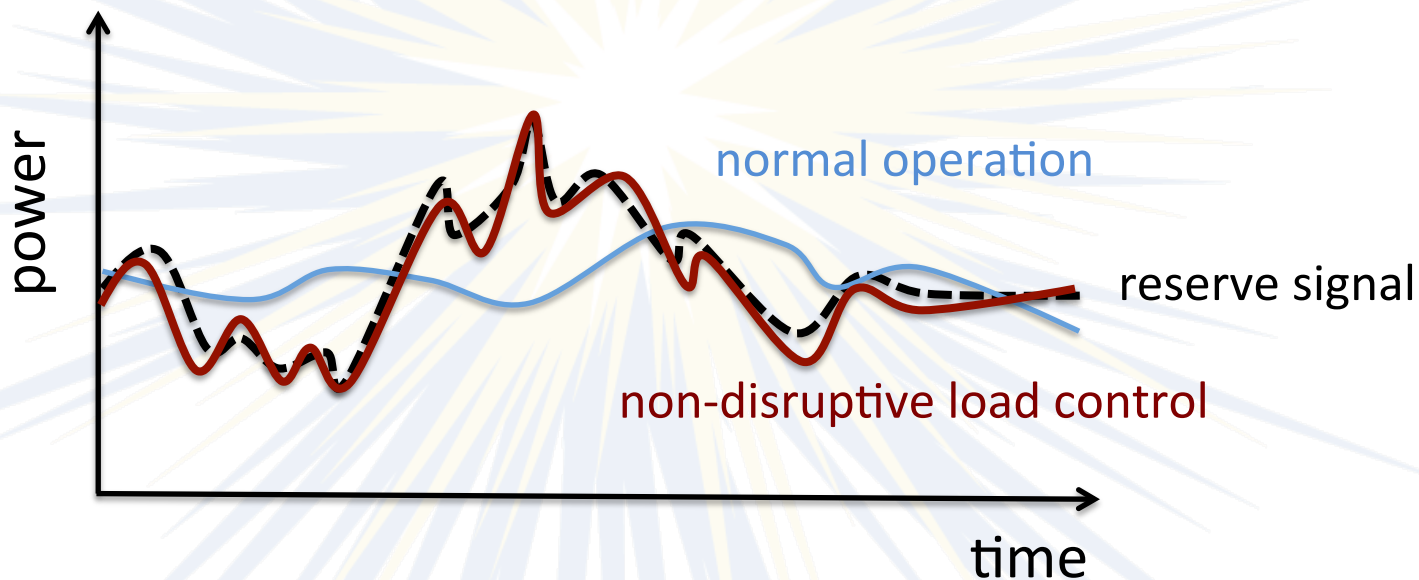
# How can loads provide reserves?

→ your refrigerator is already flexible



# Thousands of thermostatically controlled loads (TCLs) can track signals and provide reserves

TCLs: air conditioners, heat pumps, space heaters, electric water heaters, refrigerators

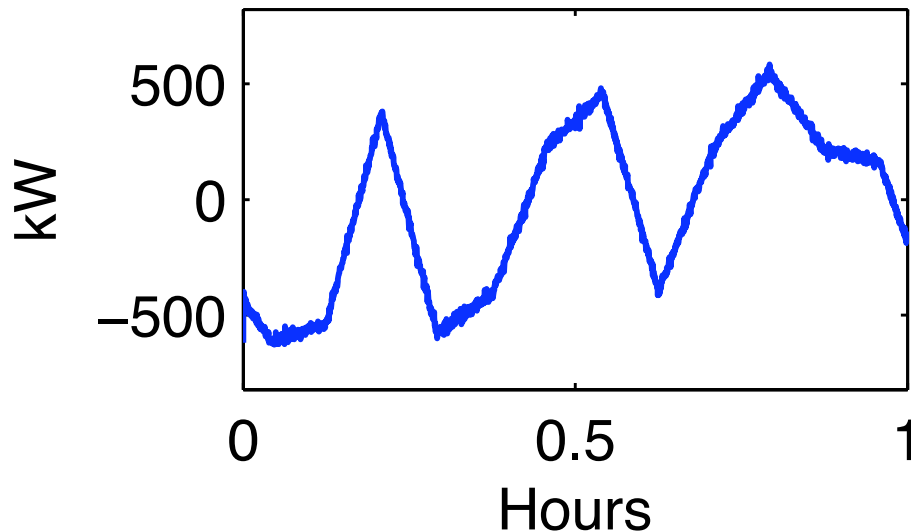


[Mathieu, Koch, and Callaway *IEEE Transactions on Power Systems* 2013]

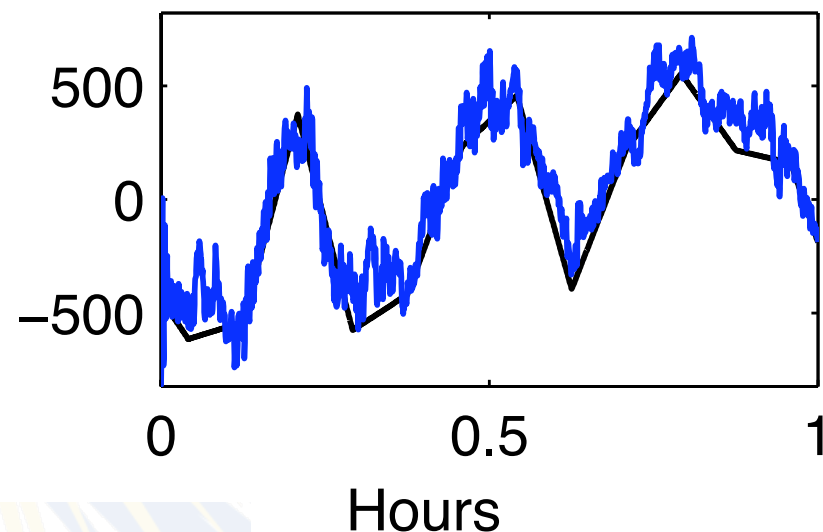
# Simulation results:

1000 ACs tracking 5-minute market signal

Controller gets temperature/state of each load every 2 seconds



Controller infers TCL behavior from power measurements at the substation



→ The more the controller knows about the loads, the better it can track a signal

[Mathieu, Koch, and Callaway *IEEE Transactions on Power Systems* 2013]



# Data from loads

- Parameters

- the make/model of the load?
- its temperature setpoint/dead-band width?
- some information about the household?

→ Modeling

- Real-time data

- Measurements of the on/off state and/or internal temperature?
- Household smart meter data?
- Power measurements from the distribution network?

→ Feedback control

→ High quality, infrequent

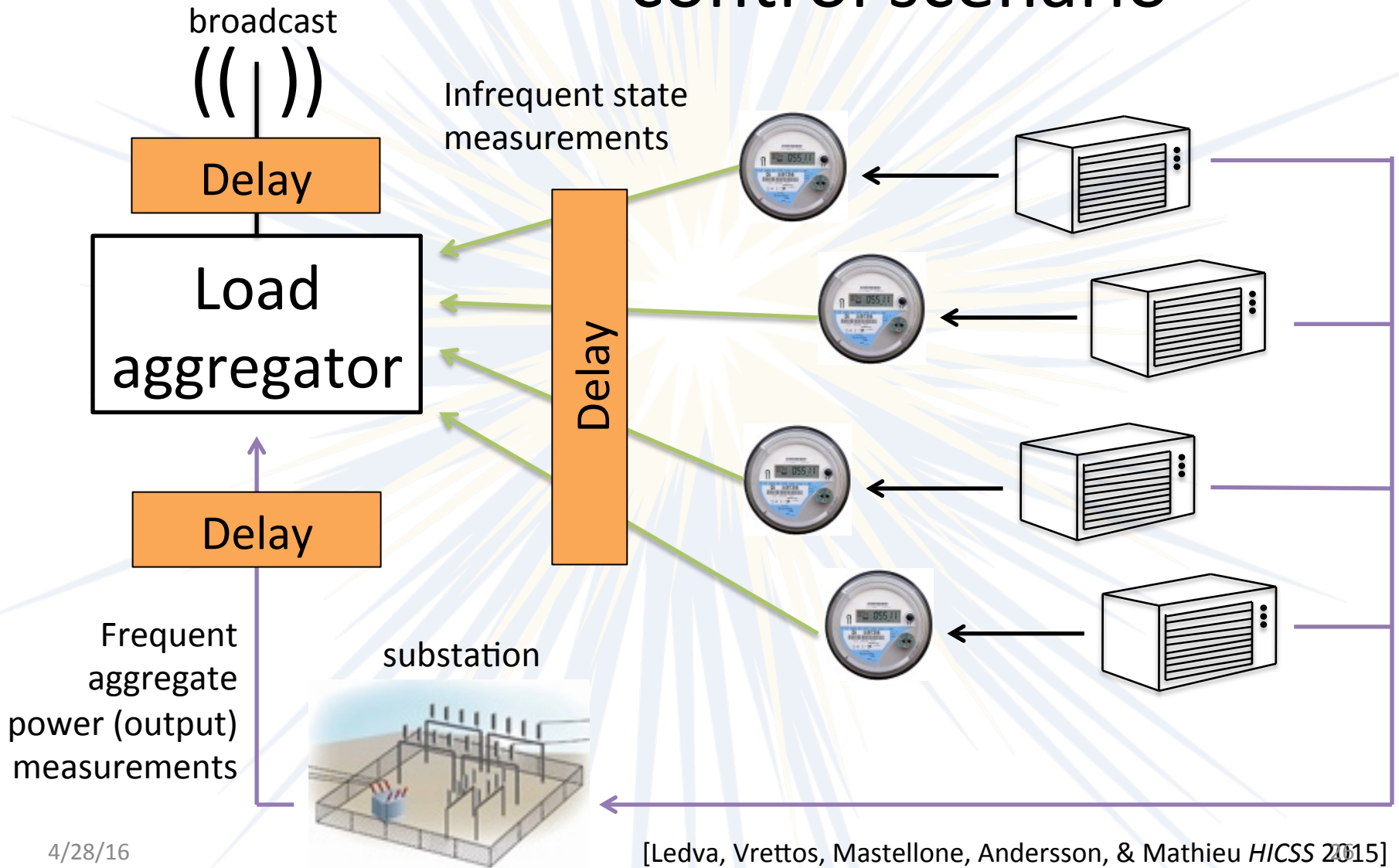
→ Low quality, frequent

- Recorded data

- high resolution power measurements of each load?

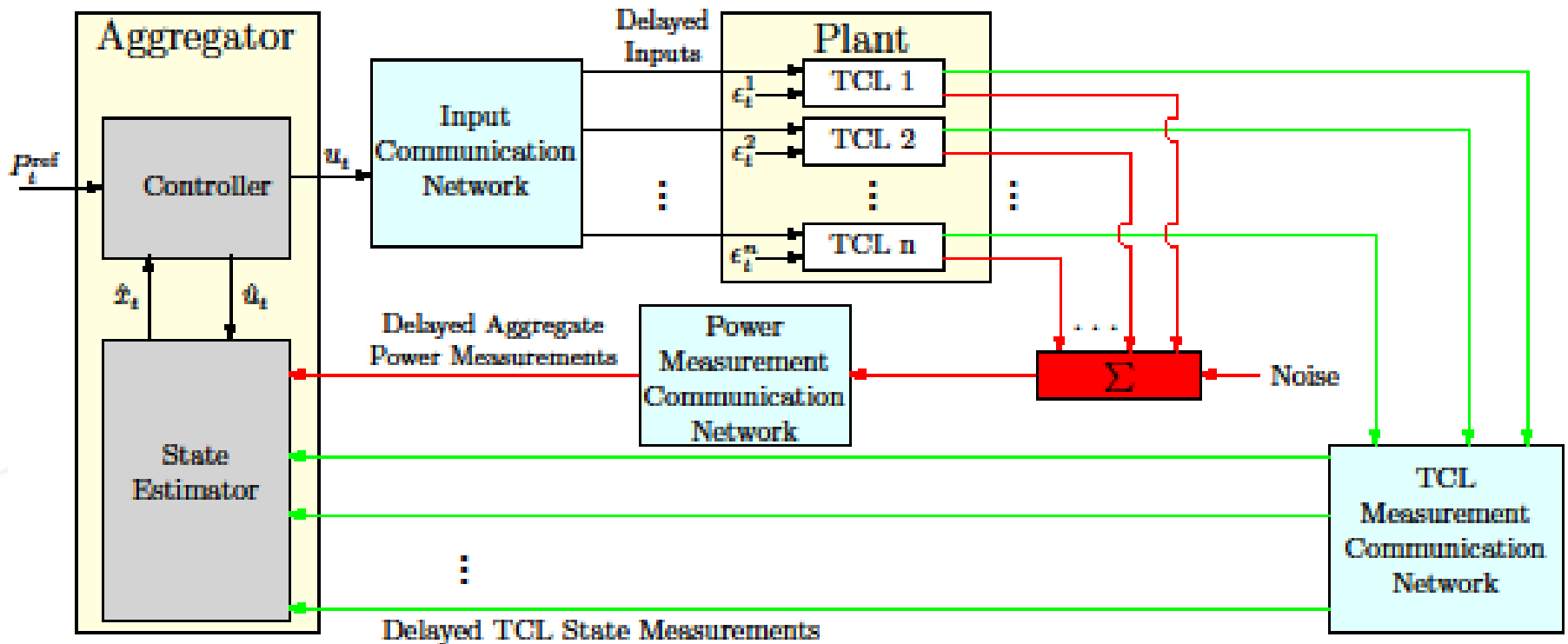
→ Auditing

# Communication and control scenario



# System block diagram

Delays cause unsynchronized arrivals of inputs at the loads and measurements at the controller



# The challenge

- Design an **estimator** and **controller** to enable loads to track a signal *despite delays*
- Assuming...
  - Control inputs & measurements are time-stamped
  - Delay statistics are known
  - State measurements are taken frequently; measurement *histories* are transmitted infrequently
  - Aggregate power measurements are *very* noisy (though the noise is normally distributed, zero-mean, and the standard deviation is known)

# Two-state TCL model

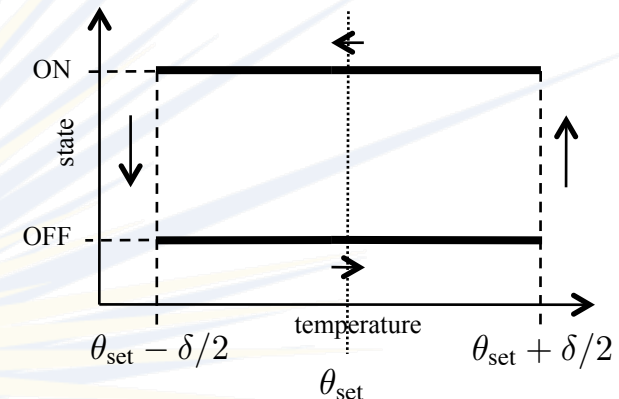
Each TCL  $i$  is modeled with a stochastic hybrid difference equation:

Temperature of the space

$$\theta_i(k+1) = a_i \theta_i(k) + (1 - a_i)(\theta_{a,i} - m_i(k)\theta_{g,i}) + \epsilon_i(k)$$

On/off state

$$m_i(k+1) = \begin{cases} 0, & \theta_i(k+1) < \theta_{\text{set},i} - \delta_i/2 \\ 1, & \theta_i(k+1) > \theta_{\text{set},i} + \delta_i/2 \\ m_i(k), & \text{otherwise} \end{cases}$$

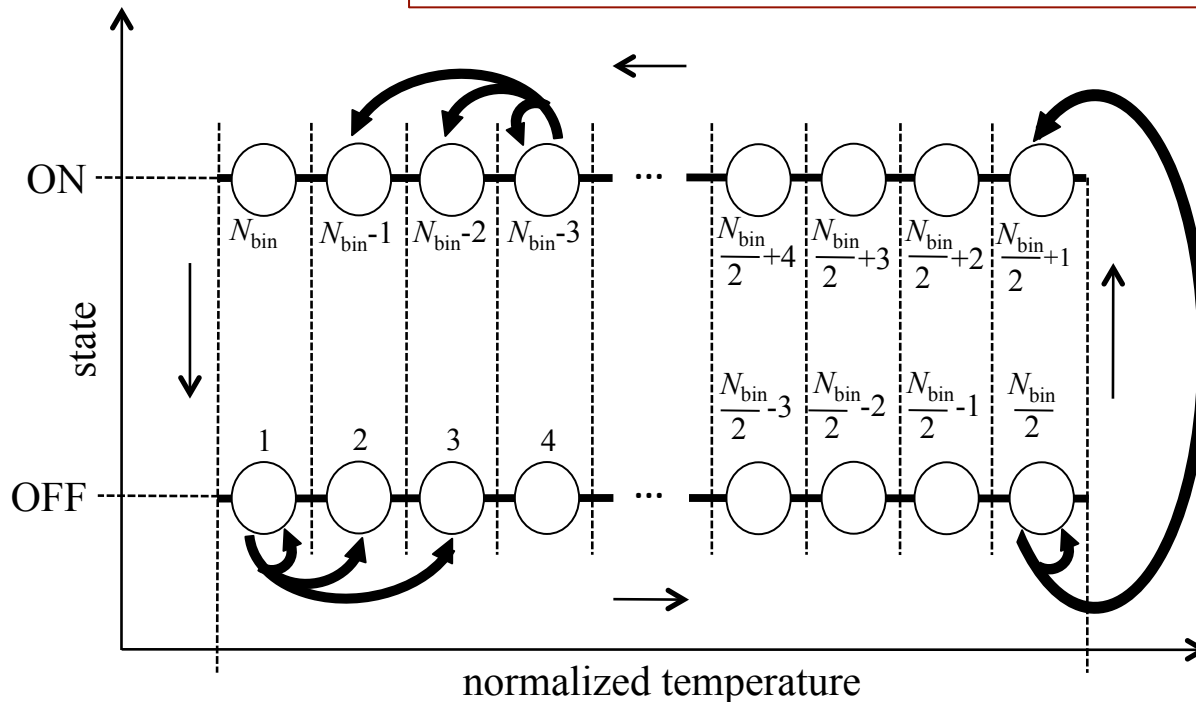


$a$ , thermal parameter  
 $\theta_g$ , temperature gain  
 $\theta_a$ , ambient temperature  
 $\epsilon$ , noise  
 $\theta_{\text{set}}$ , set point  
 $\delta$ , dead-band width

[Ihara & Schweppe 1981, Mortensen & Haggerty 1990, Uçak & Çağlar 1998]

# Aggregate System Model

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{B}_\omega\boldsymbol{\omega}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \boldsymbol{\nu}(k) \end{aligned}$$



Similar models in the literature:

- Lu & Chassin 2004/2005
- Bashash & Fathy 2011/2013
- Kundu & Hiskens 2011
- Zhang et al. 2013

[Mathieu, Koch, and Callaway *IEEE Transactions on Power Systems* 2013]

# Estimator Designs

- Based on Kalman Filtering
  - Estimator 1: Parallel filter estimator
    - One Kalman Filter per load
    - Each time a measurement arrives, filter it
    - Synthesize aggregate estimate from individual estimates
  - Estimator 2: Single Kalman Filter Using Aggregate State Predictions
    - Use state measurement histories to estimate \*individual\* load parameters (two-state model)
    - Use individual load models to predict current state
    - Use predictions as “measurements” in Kalman Filter

# Controller Design

- Based on Model Predictive Control
  - Use knowledge of delay distributions and past control inputs

First control sequence:

$$u_1, u_2, u_3, \dots, u_n$$

Second control sequence:

$$u_2, u_3, \dots, u_{n+1}$$

Third control sequence:

$$u_3, u_4, \dots, u_{n+1}$$

$$u_k$$

Input estimate:  $\hat{u}_k = u_k \mathcal{P}$



# Control Formulation

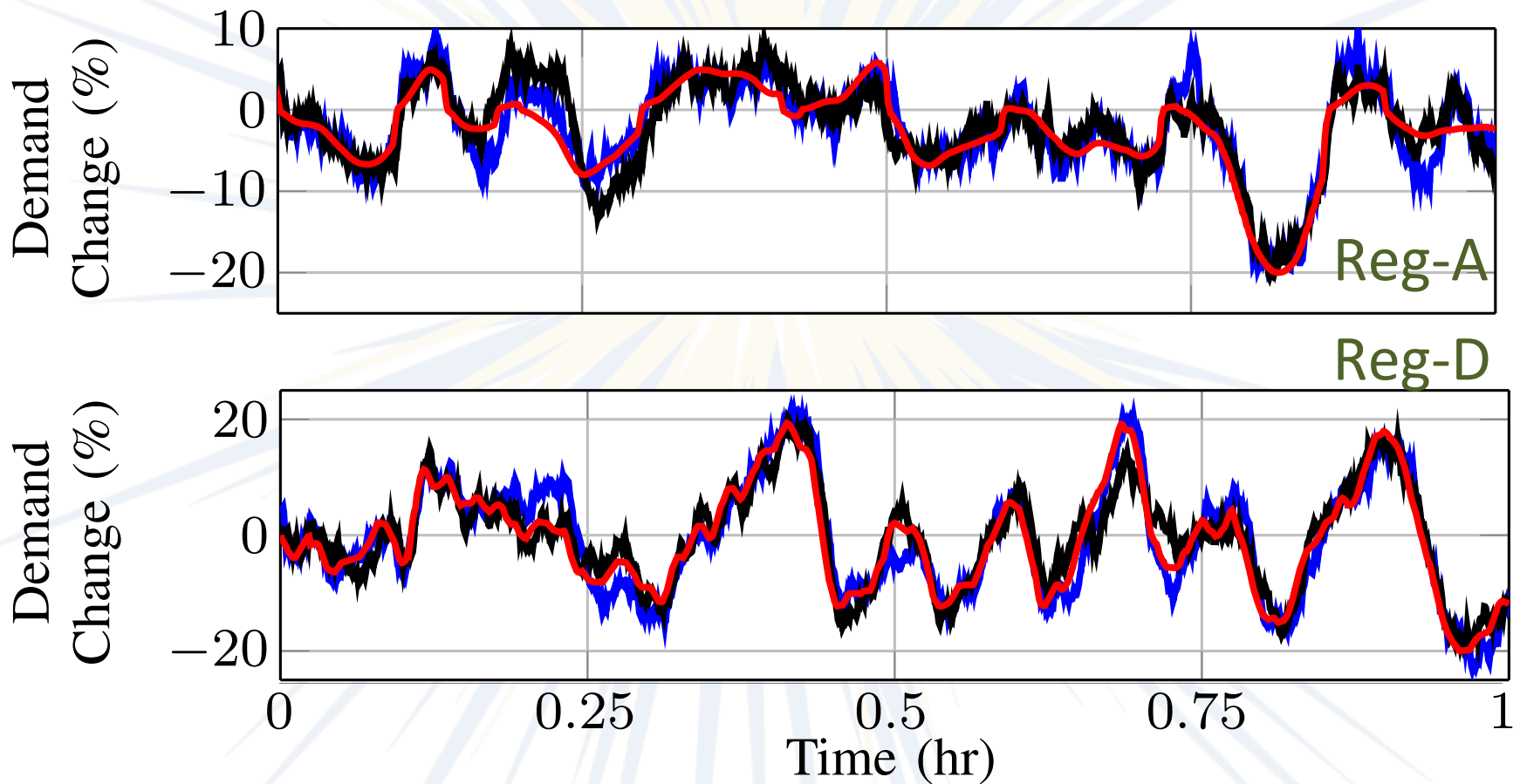
$$\text{minimize}_{u, \delta} \sum_{k=t}^{t+N^{\text{mpc}}-1} \left[ \underbrace{c^y (y_k^{\text{err}})^2}_{\text{Tracking error}} + \underbrace{c^\delta (\delta_k^- + \delta_k^+)}_{\text{State constraint deviations}} + \underbrace{\sum_{m=k-N^{\text{mpc}}+1}^k c^u (u_{k|m}^\top u_{k|m})}_{\text{Control effort}} \right]$$

$$\text{s.t.} \quad \begin{aligned} x_{k+1} &= A x_k + B \hat{u}_k \\ \hat{u}_k &= U_k \mathcal{P} \\ y_k^{\text{err}} &= y_k^{P, \text{ref}} - C^P x_k \\ u_{k|m}^i &\leq x_k^i & i \in \{1, \dots, N^x/2\} \\ -u_{k|m}^i &\leq x_k^{N^x+1-i} & i \in \{1, \dots, N^x/2\} \\ 0 - \delta_k^- &\leq x_k \leq 1 + \delta_k^+ \\ 0 &\leq \delta_k^-, \delta_k^+. \end{aligned}$$

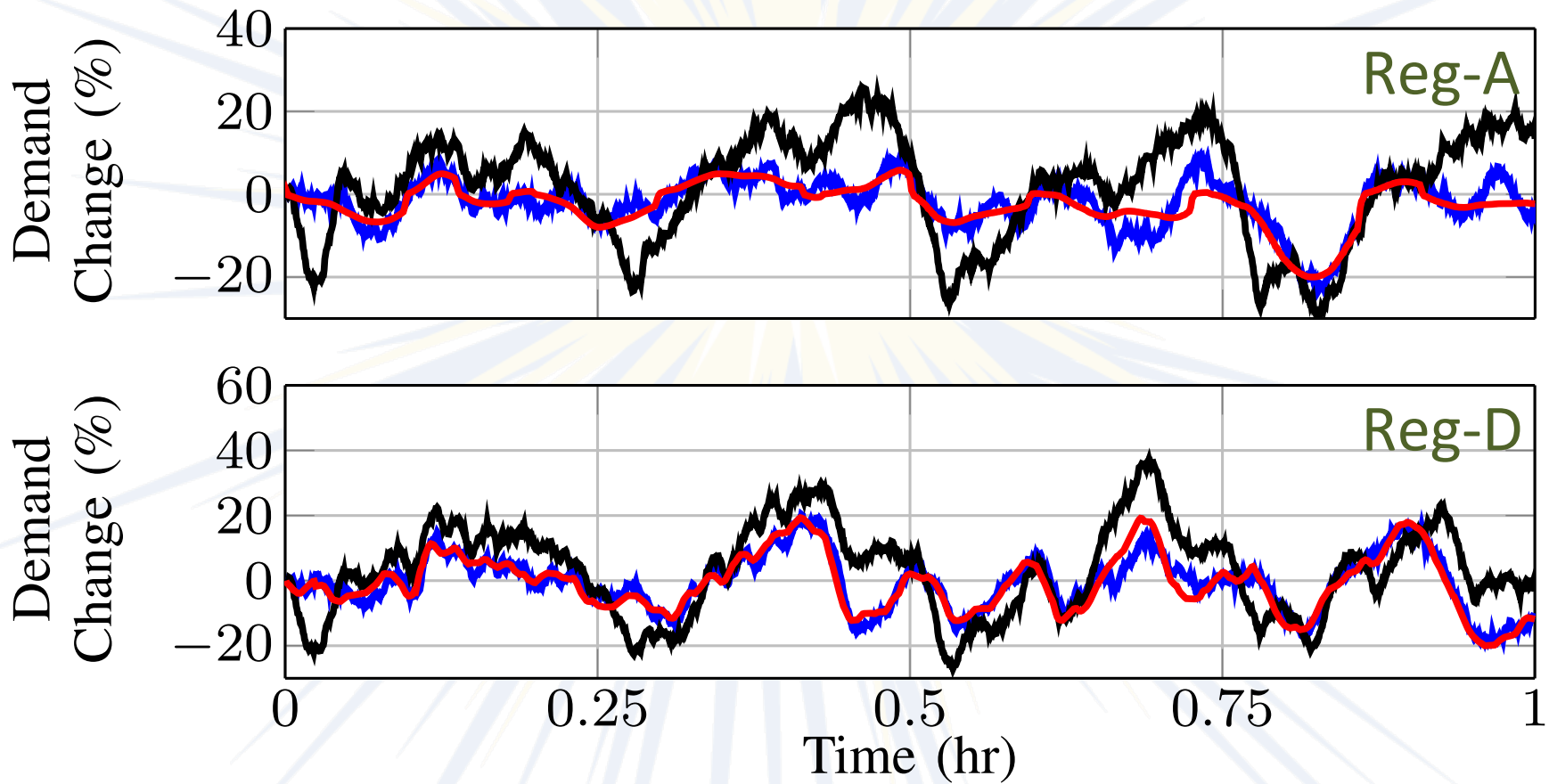
# Case Studies

- PJM Regulation Signals, Reg-A & Reg-D
- Average input delay of 20 seconds
- (Delayed) state histories arrive every 15 minutes

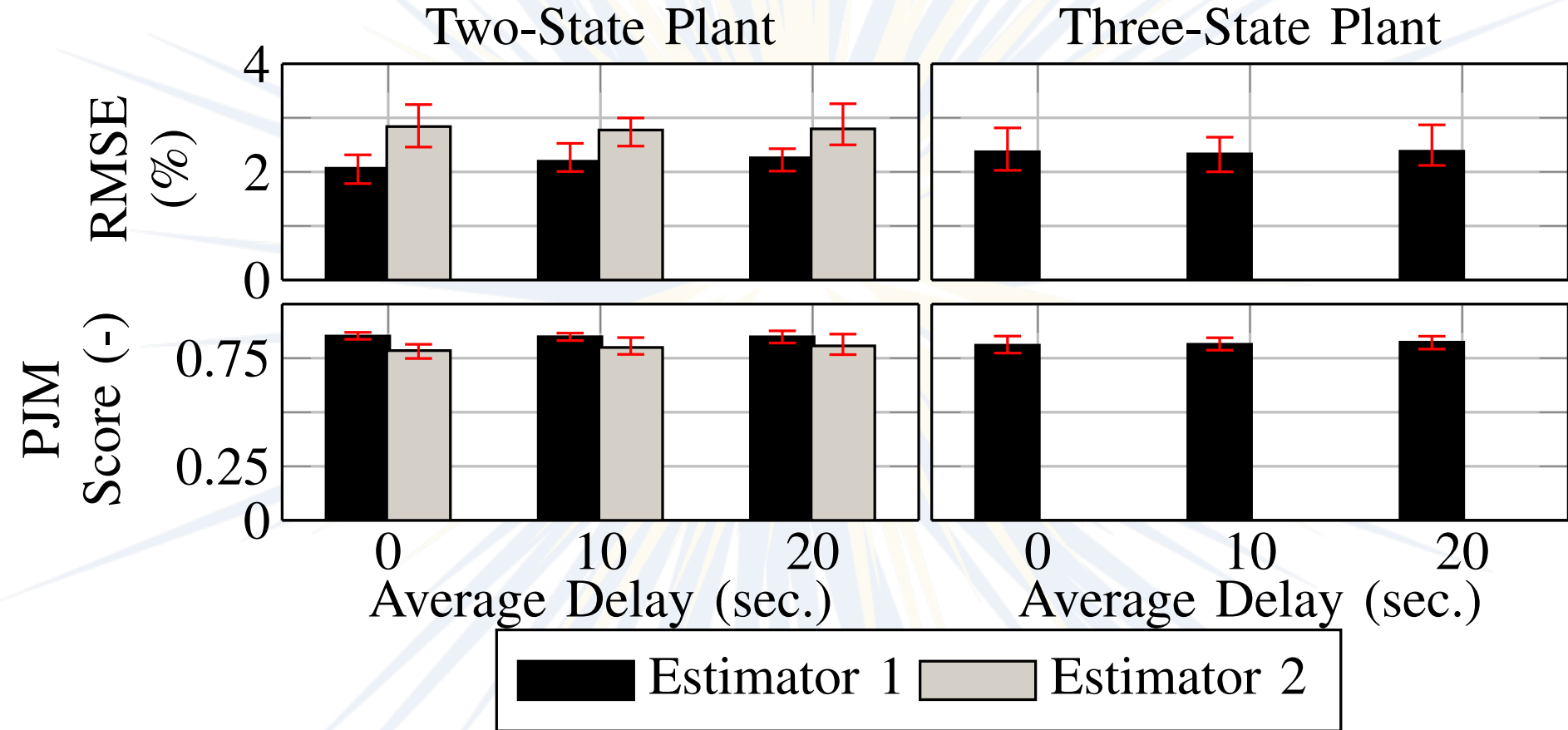
# Results



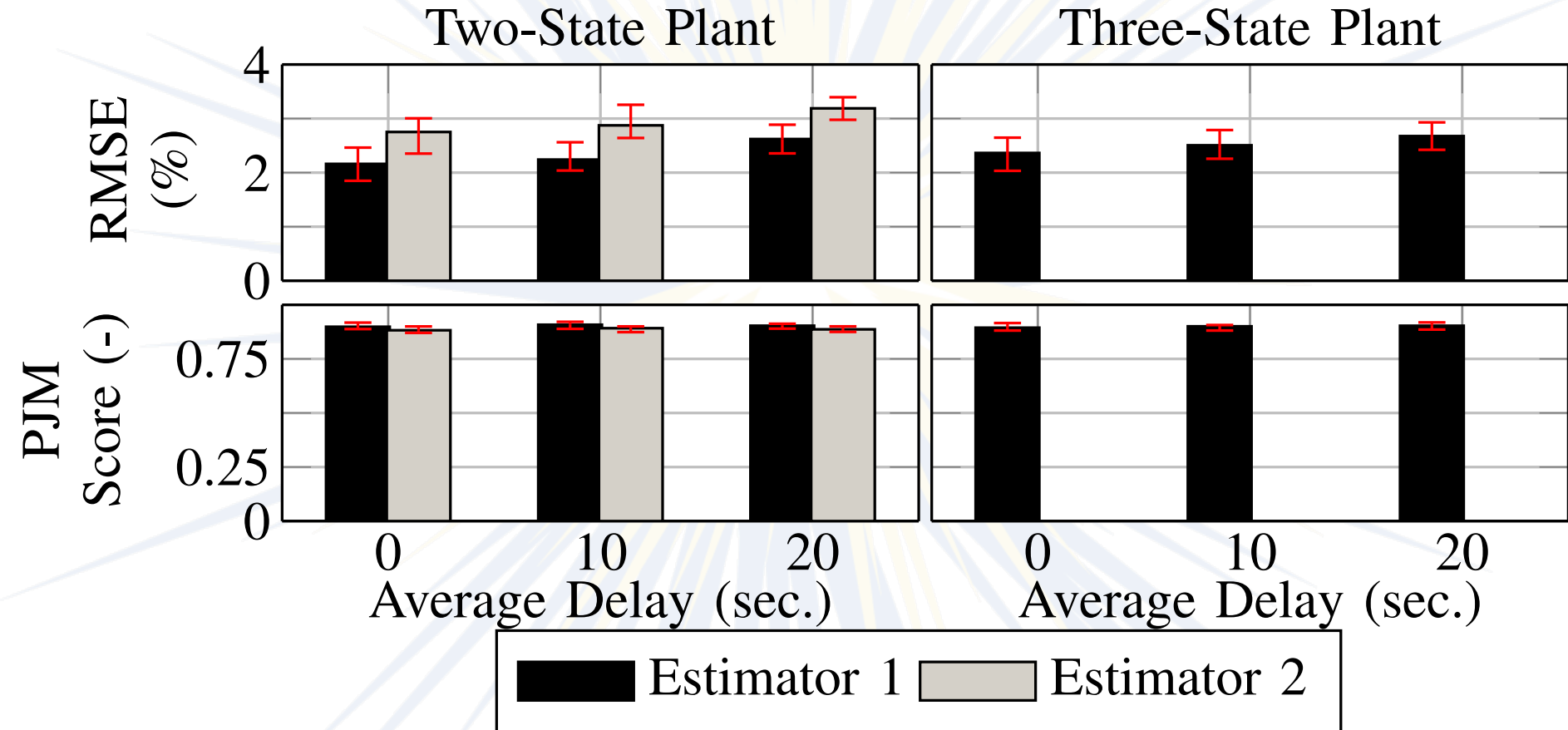
# Results: Model Mismatch



# Control Results: Reg-A



# Control Results: Reg-D



# Controller Reformulation

Original Model

$$x_{k+1} = A x_k + B u_k$$

$$y_k = C x_k.$$

Modal Model

$$\begin{bmatrix} 1 \\ \tilde{x}_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \tilde{A} \end{bmatrix}}_{A^*} \underbrace{\begin{bmatrix} 1 \\ \tilde{x}_k \end{bmatrix}}_{x_k^*} + \underbrace{\begin{bmatrix} 0 \\ \tilde{B} \end{bmatrix}}_{B^*} u_k$$

$$y_k = \underbrace{\begin{bmatrix} y_{ss} & \tilde{C} \end{bmatrix}}_{C^*} \begin{bmatrix} 1 \\ \tilde{x}_k \end{bmatrix}$$

Reduced-Order Model

$$\tilde{x}_{k+1} = \tilde{A} \tilde{x}_k + \tilde{B} u_k$$

$$\tilde{y}_k = \tilde{C} \tilde{x}_k.$$

# Controller Reformulation

The linear controller uses constant gains generated from an output-regulating Linear Quadratic Regulator (LQR) with reference feedforward.

Linear Controller  $u_t^{\text{seq}} = -K_{\infty}^x \bar{x}_t - K_{\infty}^w w_t + K_{\infty}^y y_t^{\text{des}}$

LQR Formulation  $\min_u \sum_{k=t}^{\infty} \begin{bmatrix} \tilde{x}_k \\ w_k \end{bmatrix}^T \begin{bmatrix} \tilde{C}^T & q^y \tilde{C} & 0 \\ 0 & 0 & q^w \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ w_k \end{bmatrix} + (u_k^{\text{seq}})^T R u_k^{\text{seq}}$

s.t.  $\begin{bmatrix} \tilde{x}_{k+1} \\ w_{k+1} \end{bmatrix} = \begin{bmatrix} \tilde{A} & 0 \\ \tilde{C} & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ w_k \end{bmatrix} + \begin{bmatrix} \tilde{B} \\ 0 \end{bmatrix} u_k^{\text{seq}}$

Feedforward Gain  $K_{\infty}^y = \left( \tilde{C} \{ zI - \tilde{A} + \tilde{B} \tilde{K}_{\infty}^x \}^{-1} \tilde{B} \right)^{-\dagger}$

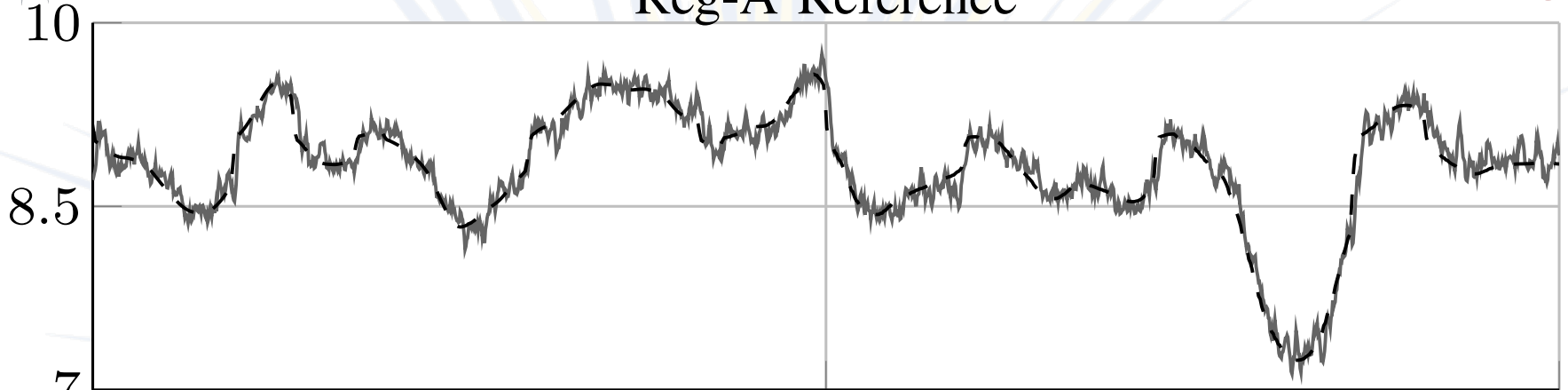


# Case Studies

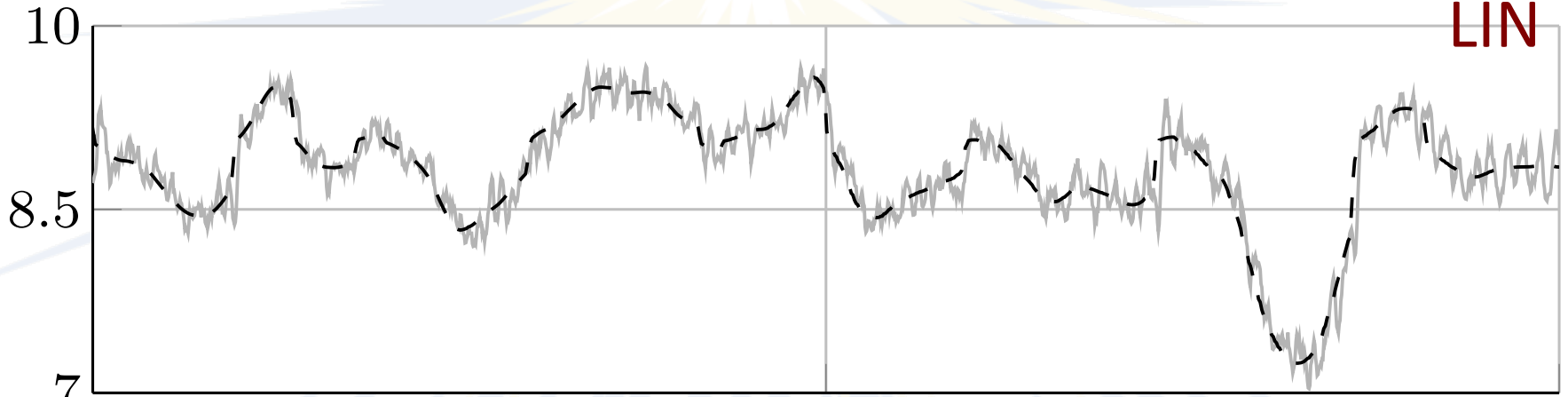
- PJM Regulation Signals, Reg-A & Reg-D
- Average input delay of 20 seconds
- Full state feedback, no measurement delay
- Three-state models used for the plant

# Reg-A Reference

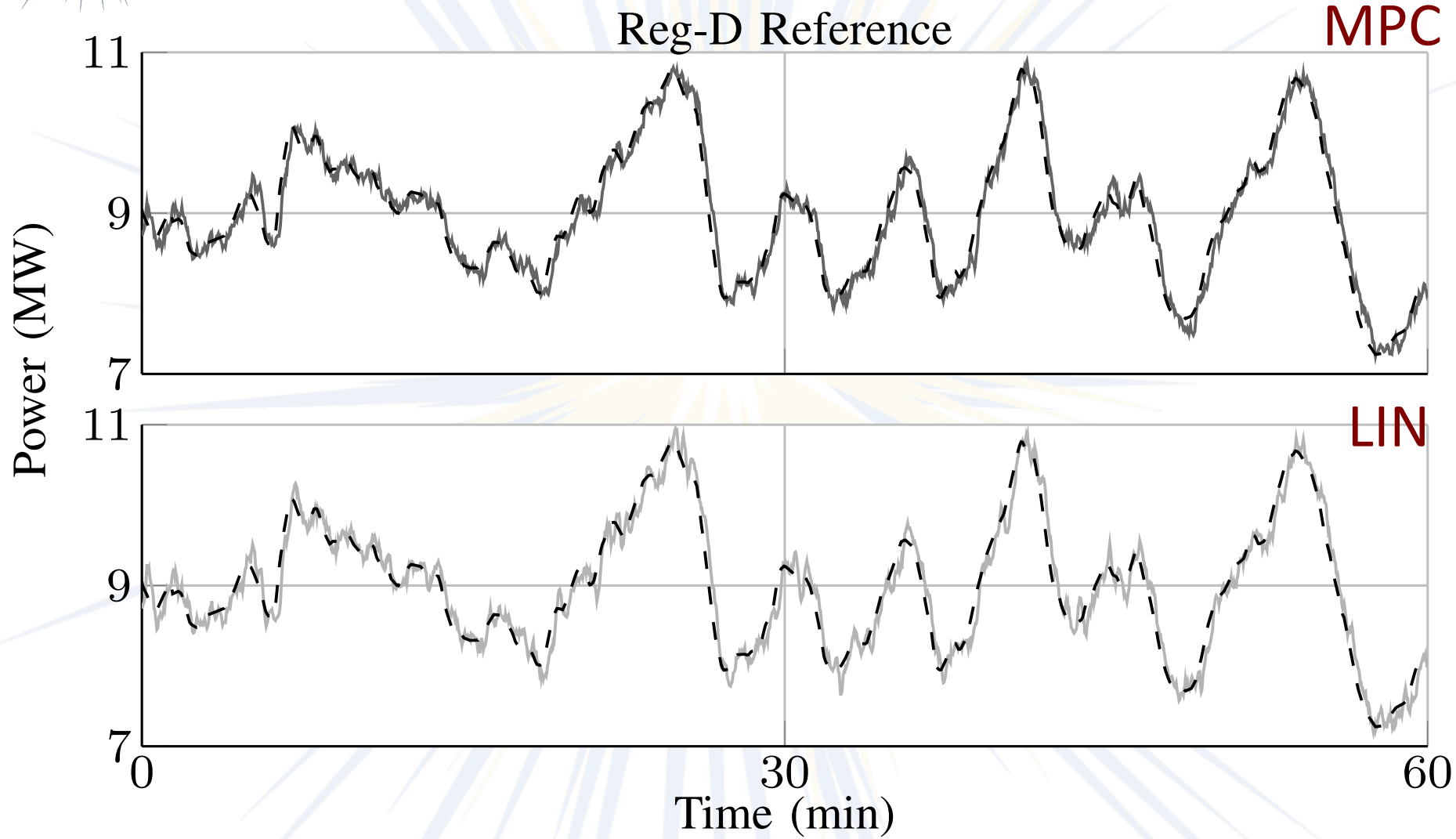
MPC



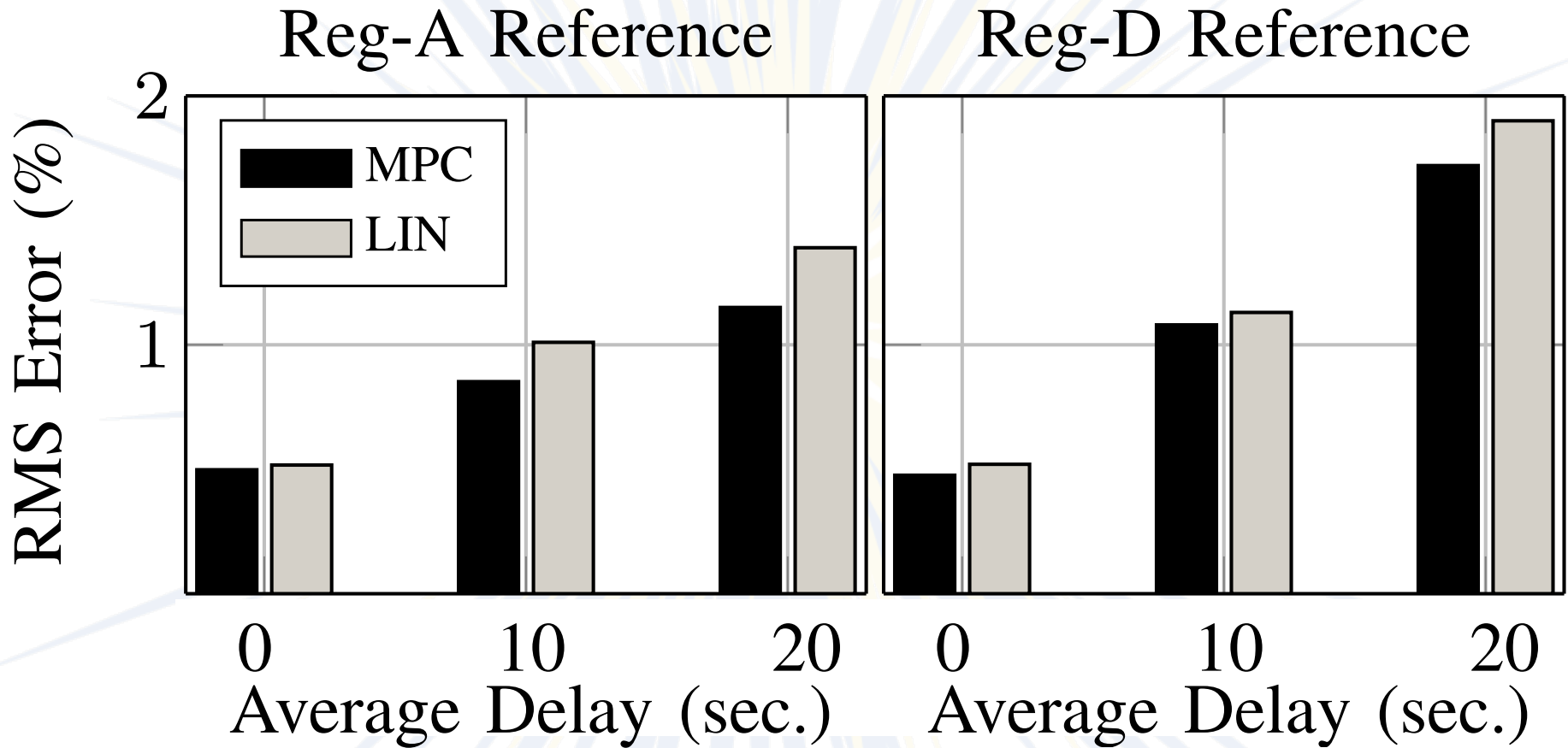
LIN



Time (min)



# Results Summary



→ LIN is 100 times faster than MPC

# Key takeaways

- Communication network limitations necessitate controller/estimator designs that cope with delays, bandwidth limitations, etc.
- Delays make loads less capable of providing fast services, but we can mitigate these impacts through delay-aware control and estimation.

# Conclusions

- Need methods to infer electric load behavior from existing measurements
  - Dynamic mirror descent applied to distribution substation measurements
- Need new sources of power system reserves
  - Coordination of distributed electric loads using delay-aware control/estimation

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