Michigan Power & Energy Laboratory

A Linear Approach to Manage Input Delays While Supplying Frequency Regulation Using Residential Loads

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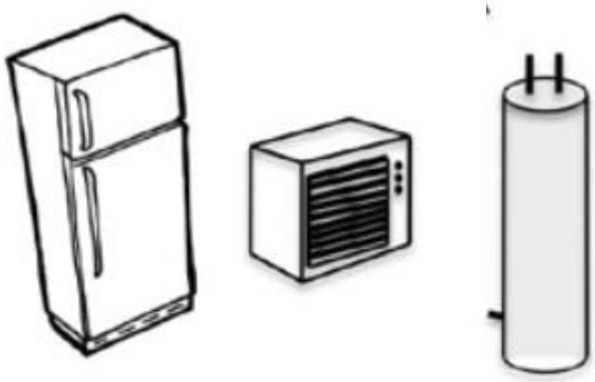
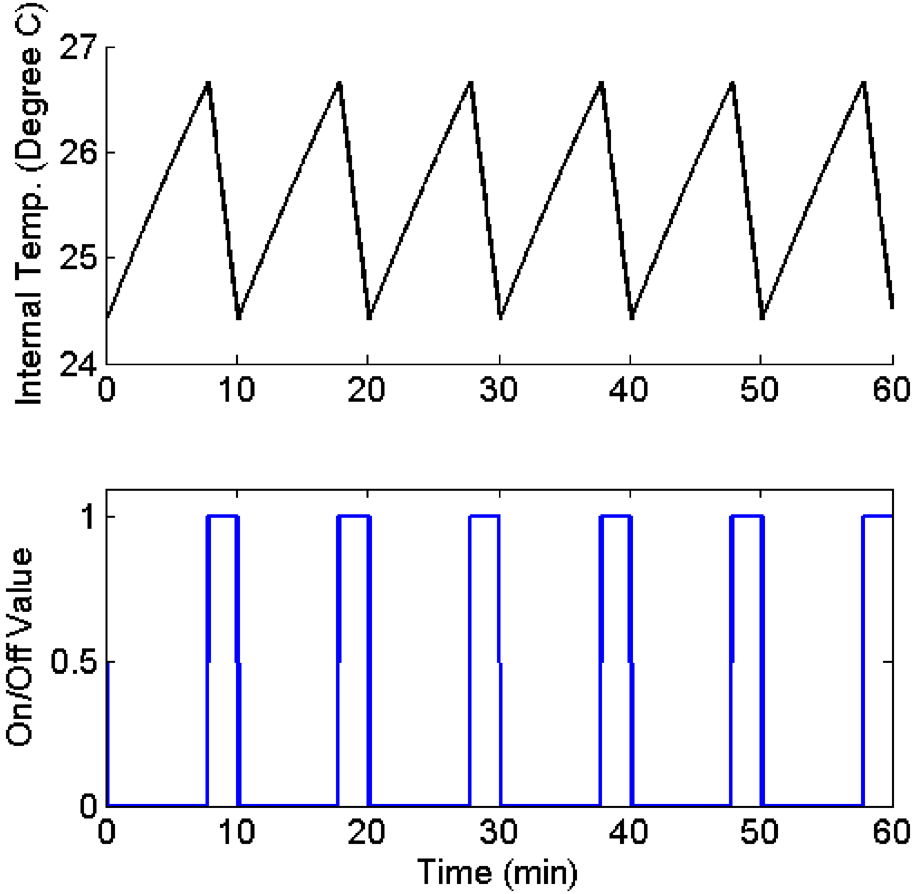
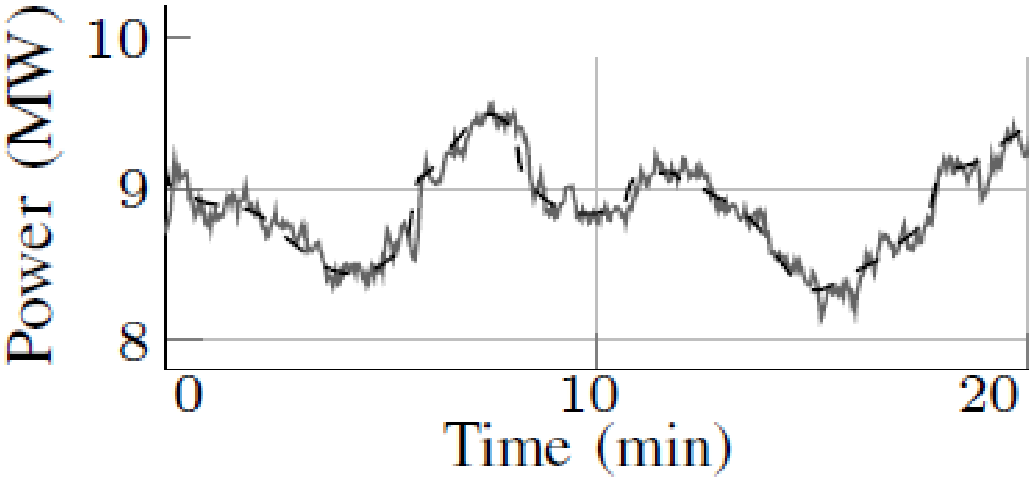
Residential thermostatically controlled loads (TCLs) store thermal energy and use hysteresis control.

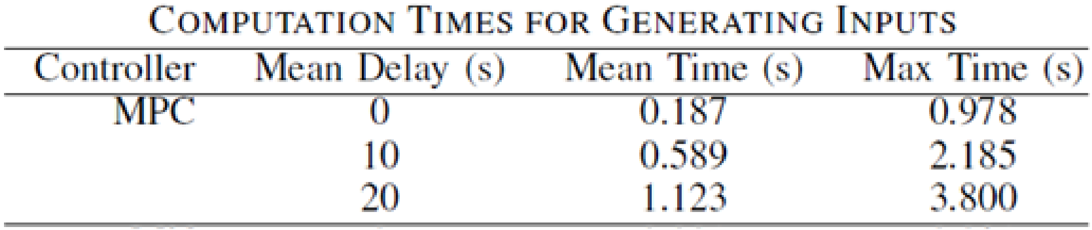




Background

We want to influence the power consumption of loads on time-scales of seconds, which can require accounting for communication delays.





COMPUTATION TIMES FOR GENERATING INPUTS

Controller	Mean Delay (s)	Mean Time (s)	Max Time (s)
MPC	0	0.187	0.978
	10	0.589	2.185
	20	1.123	3.800

[Ledva et al. 2015]

- **GOAL: account for delays in a simplified controller** with a closed-form control law to reduce online computation

Contents

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- Linear Controller (LIN)
- Results
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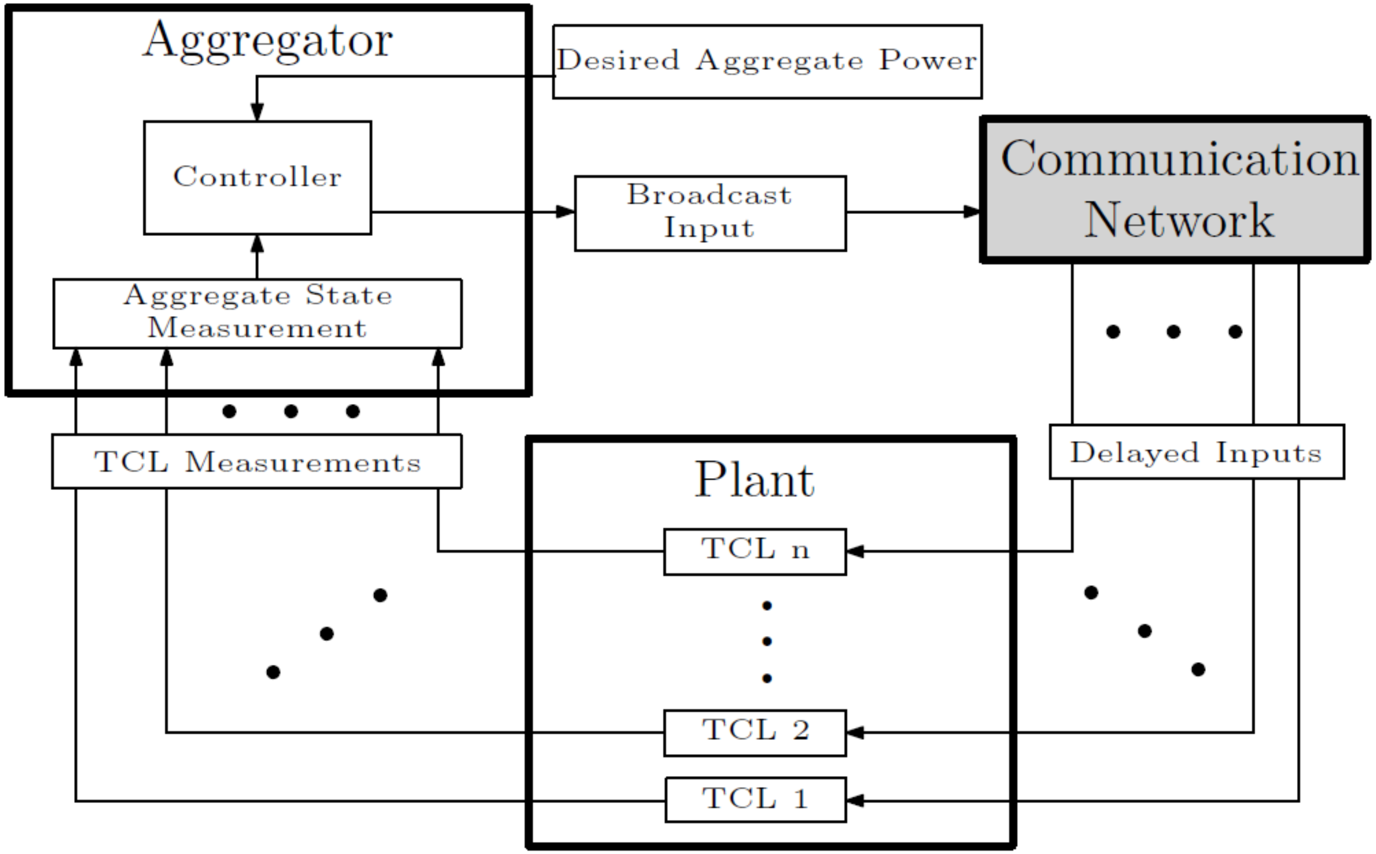
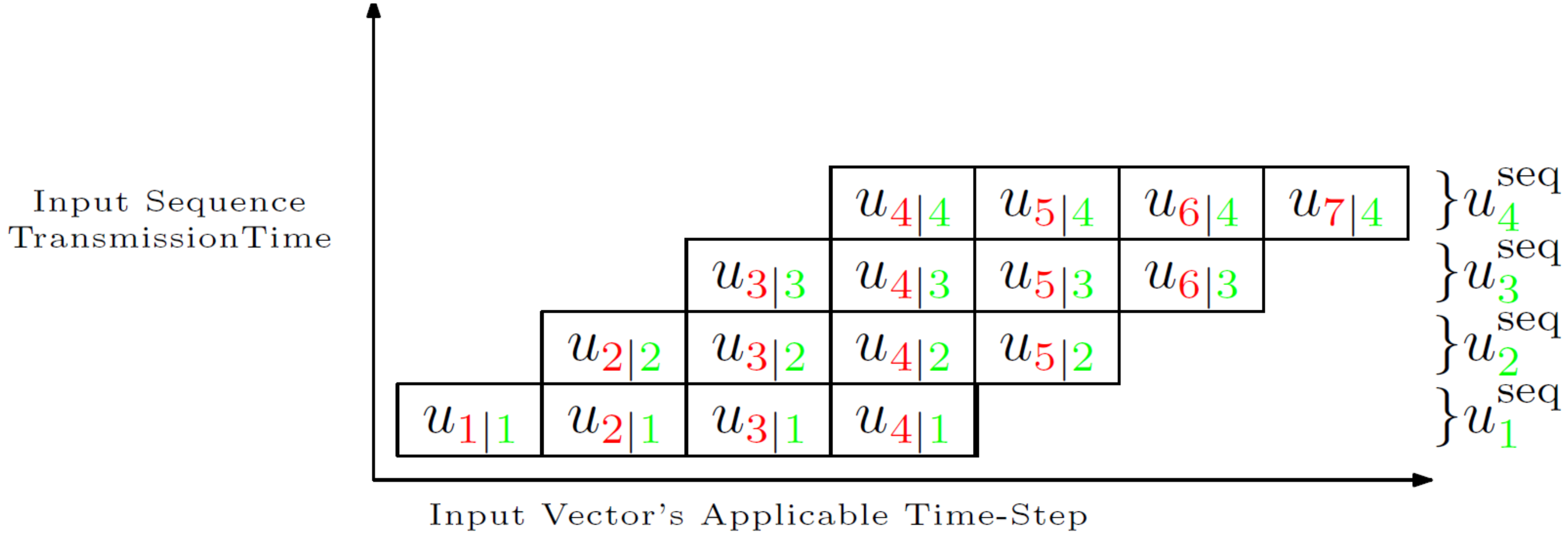
An aggregator attempts to provide frequency regulation despite input delays while using state measurements



Ledva and Mathieu. "A Linear Approach ...". ACC. 2017.

Problem Overview (2/2)

At each time-step, we transmit an input sequence and allow TCLs to select an input based on the realized delays.



			$u_{4\|4}$	$u_{5\|4}$	$u_{6\|4}$	$u_{7\|4}$	$\}u_{4}^{\text{seq}}$
		$u_{3\|3}$	$u_{4\|3}$	$u_{5\|3}$	$u_{6\|3}$		$\}u_{3}^{\text{seq}}$
	$u_{2\|2}$	$u_{3\|2}$	$u_{4\|2}$	$u_{5\|2}$			$\}u_{2}^{\text{seq}}$
$u_{1\|1}$	$u_{2\|1}$	$u_{3\|1}$	$u_{4\|1}$				$\}u_{1}^{\text{seq}}$

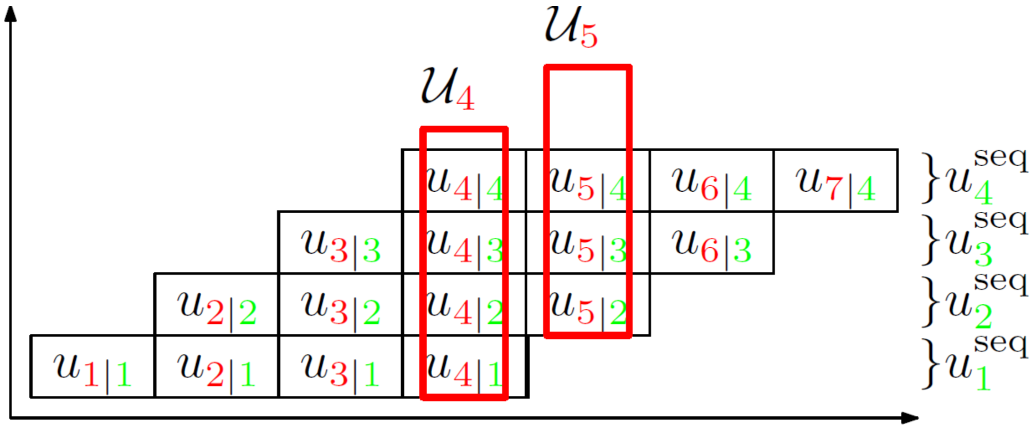
MPC is formulated as a quadratic program similar to a finite-horizon, tracking LQR with state and input constraints.

$$\min_{u} \sum_{k=t}^{t+N} \left[c^y \left(\text{tracking error}\right)^2 + c^u \left(\text{input effort}\right)^2 \right]$$

$$\begin{aligned} \text{s.t. } x_{k+1} &= A\, x_k + B\, \widehat{u}_k \\ y_k &= C\, x_k \\ \widehat{u}_k &= \mathcal{U}_k \mathcal{P} \\ & \text{input constraints} \\ & \text{state constraints} \end{aligned}$$

Ledva and Mathieu. "A Linear Approach ...". ACC. 2017.

We calculate an expected input using knowledge of the input delay distribution and the previously transmitted inputs.



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MPC (2/3)

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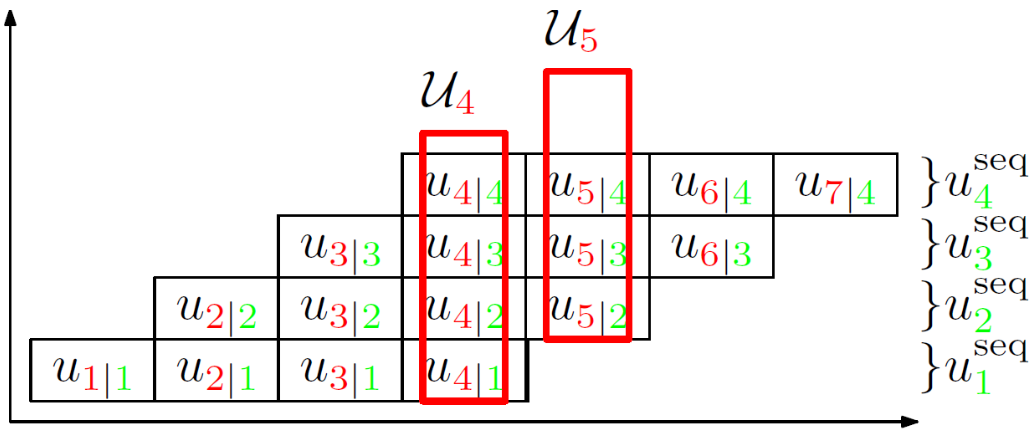
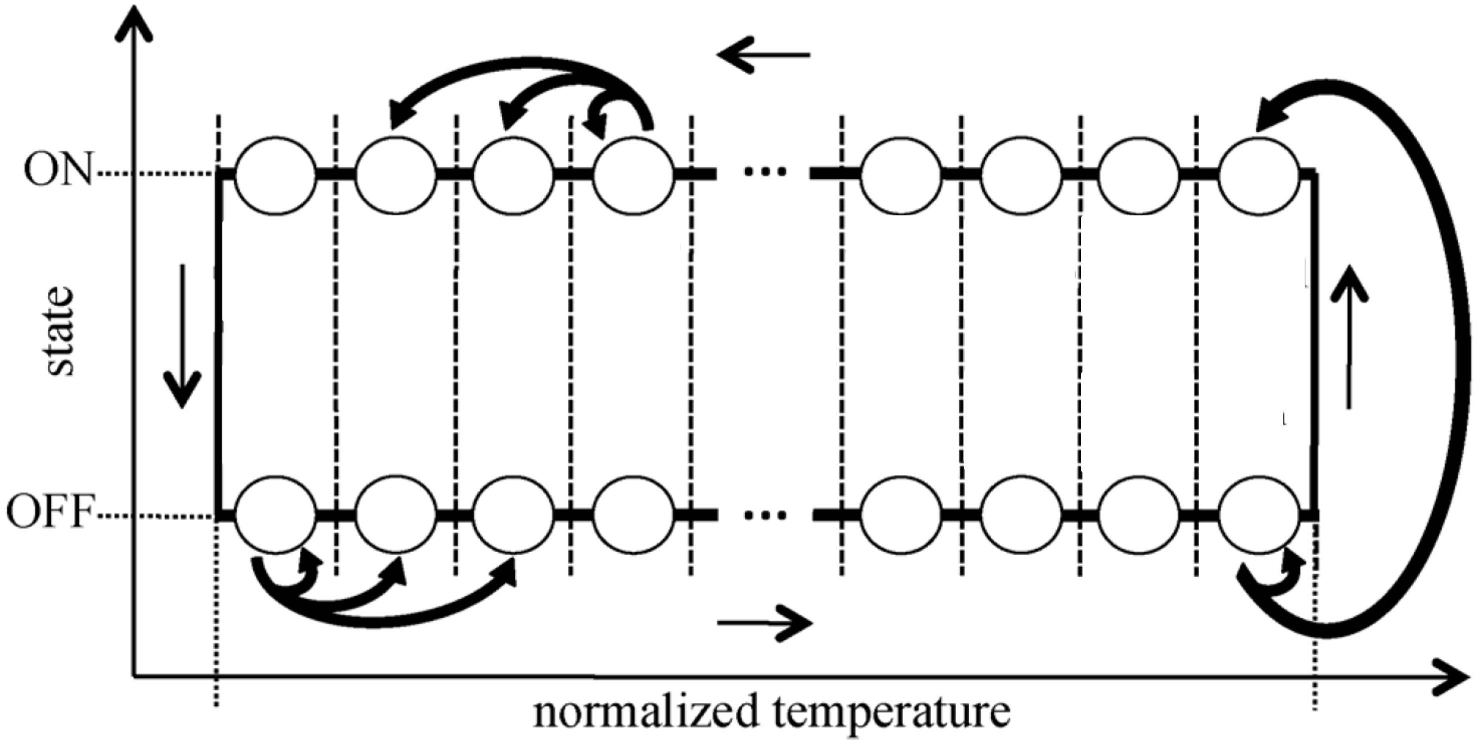


$$\widehat{u}_k = \overbrace{\begin{bmatrix} u_{k|k} & u_{k|k-1} & u_{k|k-2} & \cdots & u_{k|k-N} \end{bmatrix}}^{\mathcal{U}_k} \overbrace{\begin{bmatrix} p_k \\ p_{k-1} \\ p_{k-2} \\ \vdots \\ p_{k-N} \end{bmatrix}}^{\mathcal{P}}$$

Ledva and Mathieu. "A Linear Approach ...". ACC. 2017.

We model the dynamics of the TCLs using a Markov model that models state transitions probabilistically.



[Mathieu et al. 2013]

LIN (1/2)

To formulate LIN, we neglect the constraints, form a reduced-order model, and augment the system to include input data.

$$\min_{u} \quad \sum_{k=t}^{t+N} \left[c^y \left(\text{tracking error}\right)^2 + c^u \left(\text{input effort}\right)^2 \right]$$

$$\begin{aligned} \text{s.t. } x_{k+1} &= A\, x_k + B\, \widehat{u}_k \\ y_k &= C\, x_k \\ \widehat{u}_k &= \mathcal{U}_k \mathcal{P} \\ &\text{input constraints} \\ &\text{state constraints} \end{aligned}$$

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$$\left.\begin{array}{l} \text{input constraints} \\ \text{state constraints} \end{array}\right\} \textbf{Neglect}$$

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LIN (1/2)

To formulate LIN, we neglect the constraints, form a reduced-order model, and augment the system to include input data.

$$\min_{u} \quad \sum_{k=t}^{t+N} \left[c^y \left(\text{tracking error}\right)^2 + c^u \left(\text{input effort}\right)^2 \right]$$

$$\left.\begin{aligned} \text{s.t. } x_{k+1} &= A\,x_k + B\,\widehat{u}_k \\ y_k &= C\,x_k \end{aligned}\right\} \textbf{Reduce order}$$

$$\widehat{u}_k = \mathcal{U}_k \mathcal{P}$$

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LIN (1/2)

To formulate LIN, we neglect the constraints, form a reduced-order model, and augment the system to include input data.

$$\min_{u} \quad \sum_{k=t}^{t+N} \left[c^y \, (\text{tracking error})^2 + c^u (\text{input effort})^2 \right]$$

$$\begin{aligned} &\left.\begin{aligned} \text{s.t. } x_{k+1} &= A\, x_k + B\, \widehat{u}_k \\ y_k &= C\, x_k \end{aligned}\right\} \textbf{Reduce order} \\ &\left.\begin{aligned} \widehat{u}_k &= \mathcal{U}_k \mathcal{P} \end{aligned}\right\} \textbf{Augment system} \\ &\left.\begin{aligned} &\text{input constraints} \\ &\text{state constraints} \end{aligned}\right\} \textbf{Neglect} \end{aligned}$$

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After these modifications, we can use standard linear control tools (i.e., LQR, integrator, feedforward).

Linear Controller

$$u_t^{\mathrm{seq}} = -K_\infty^{\mathrm{x}}\,\overline{x}_t - K_\infty^{\mathrm{w}}\,w_t + K_\infty^{\mathrm{y}}\,y_t^{\mathrm{des}}$$

LQR Formulation

$$\min_u \sum_{k=t}^{\infty} \begin{bmatrix} y_k \\ w_k \end{bmatrix}^T \begin{bmatrix} q^{\mathrm{y}} & 0 \\ 0 & q^{\mathrm{w}} \end{bmatrix} \begin{bmatrix} y_k \\ w_k \end{bmatrix} + (u_k^{\mathrm{seq}})^T\, R\, u_k^{\mathrm{seq}}$$

$$\text{s.t. } \text{Reduced-order, augmented dynamics plus an integrator}$$

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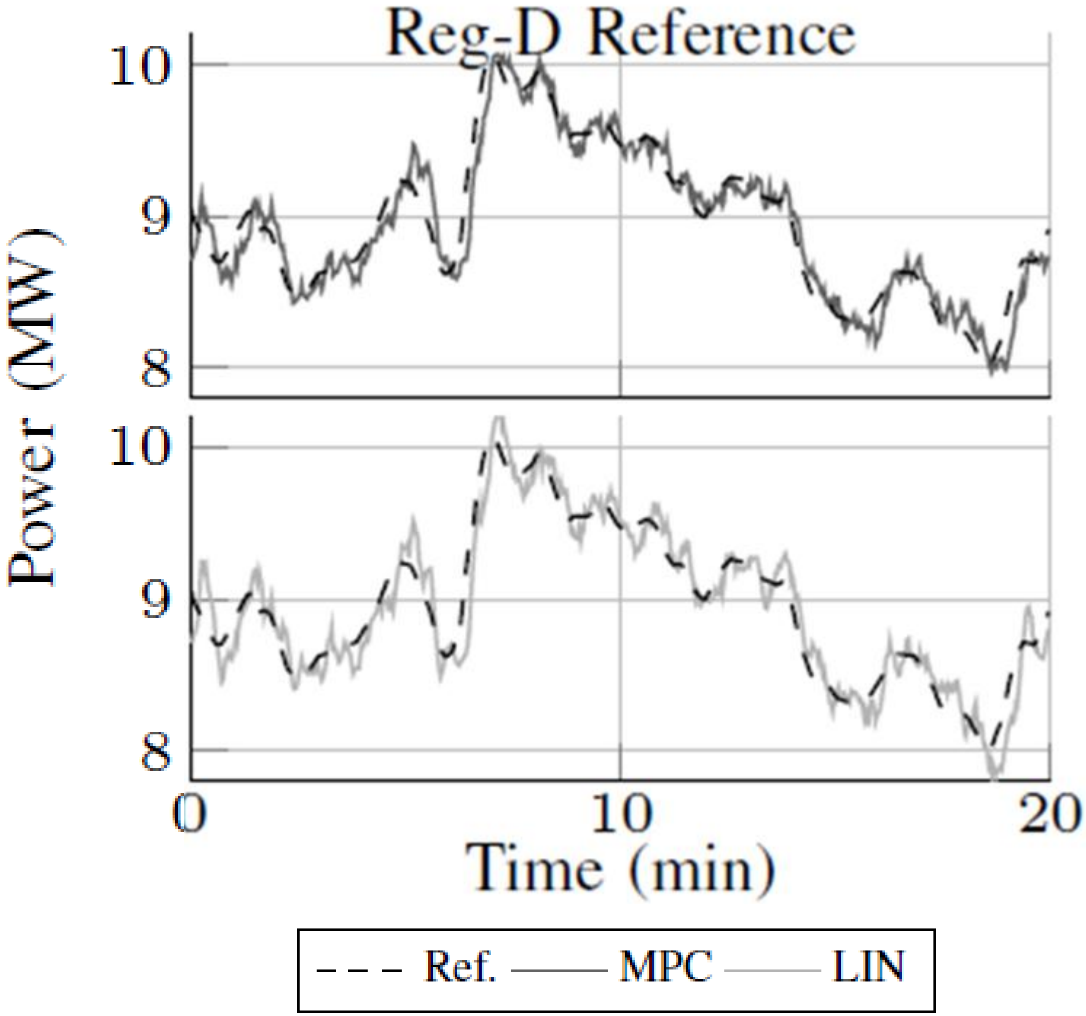
Using PJM frequency regulation signals and average delays of 20 seconds, both methods achieve accurate tracking.



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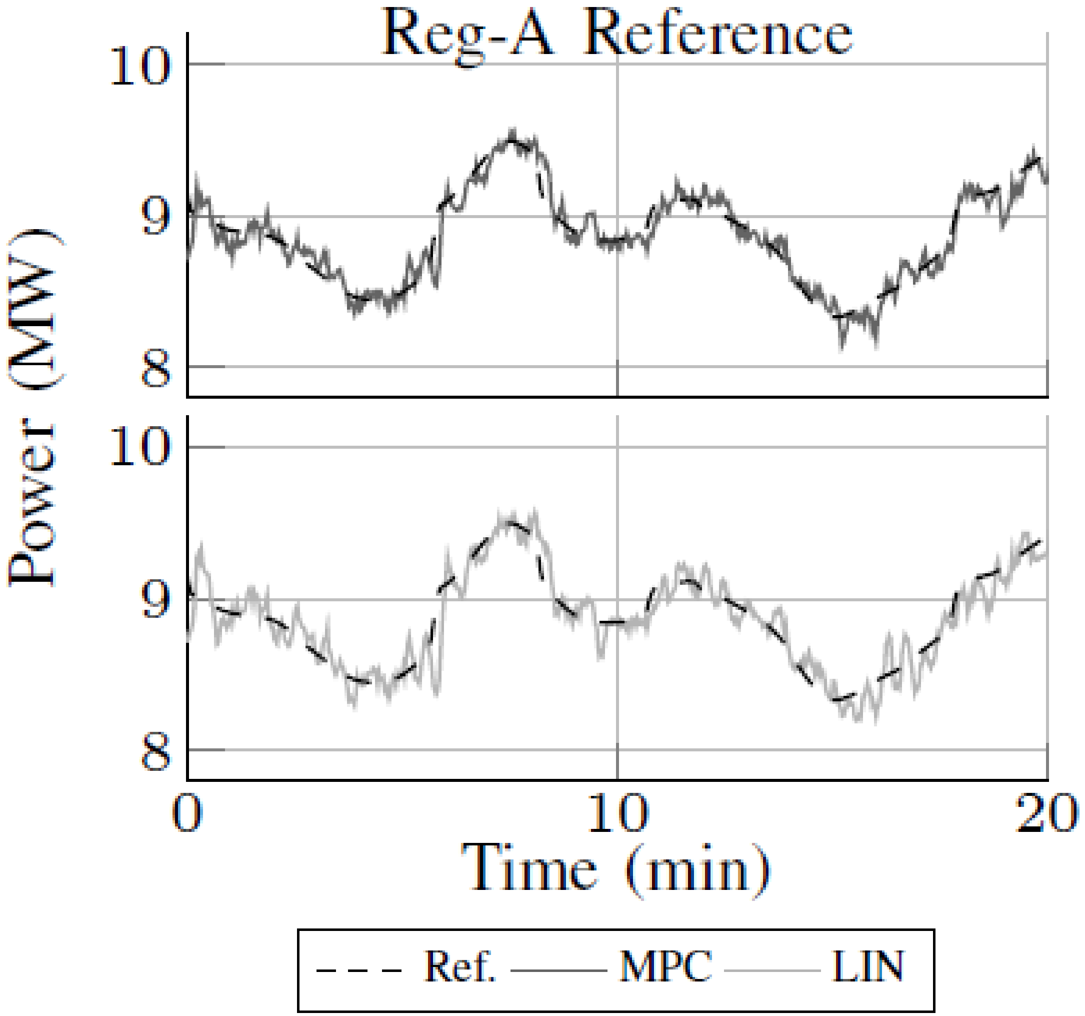

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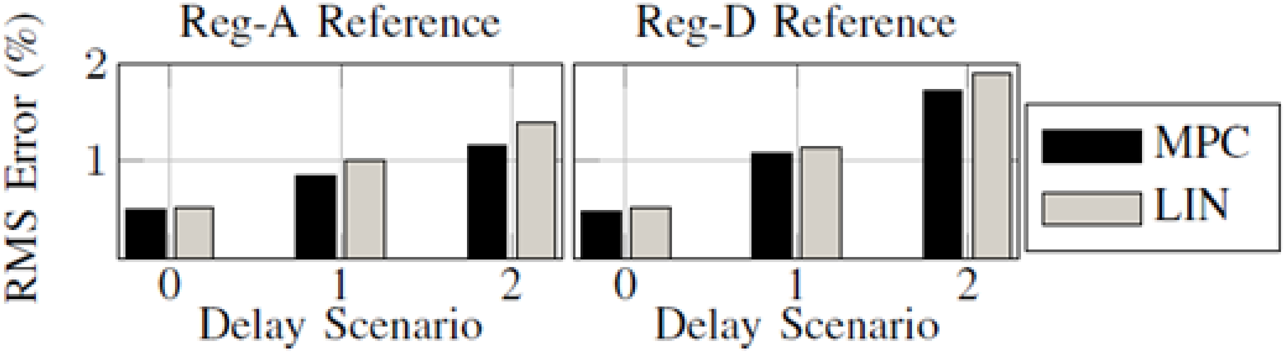
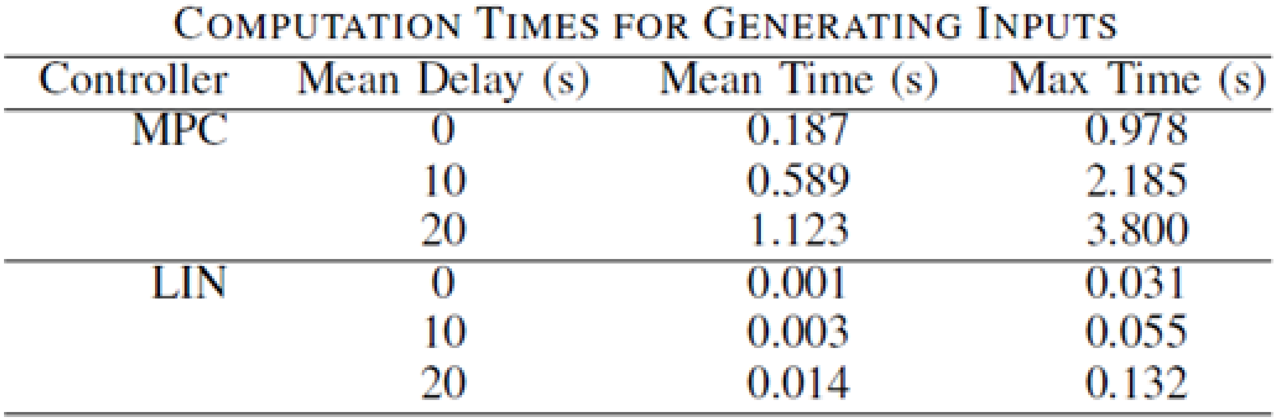
Results (3/3)

LIN slightly degrades tracking performance in this scenario but simplifies the controller to a closed-form control law.

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Controller	Mean Delay (s)	Mean Time (s)	Max Time (s)
MPC	0	0.187	0.978
	10	0.589	2.185
	20	1.123	3.800
LIN	0	0.001	0.031
	10	0.003	0.055
	20	0.014	0.132

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Conclusions

We developed two controllers, LIN and MPC, that account for communication delays and found that both perform well.

Future Work

- Investigate behavior when state feedback is not available
- Investigate inclusion of integrator into MPC controller

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