Exploring Connections Between a Multiple Model Kalman Filter and Dynamic Fixed Share with Applications to Demand Response

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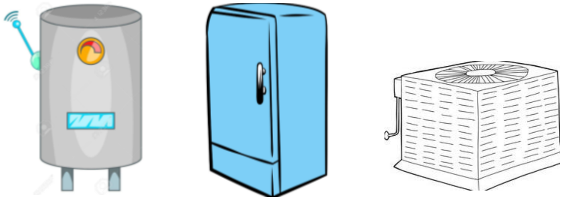
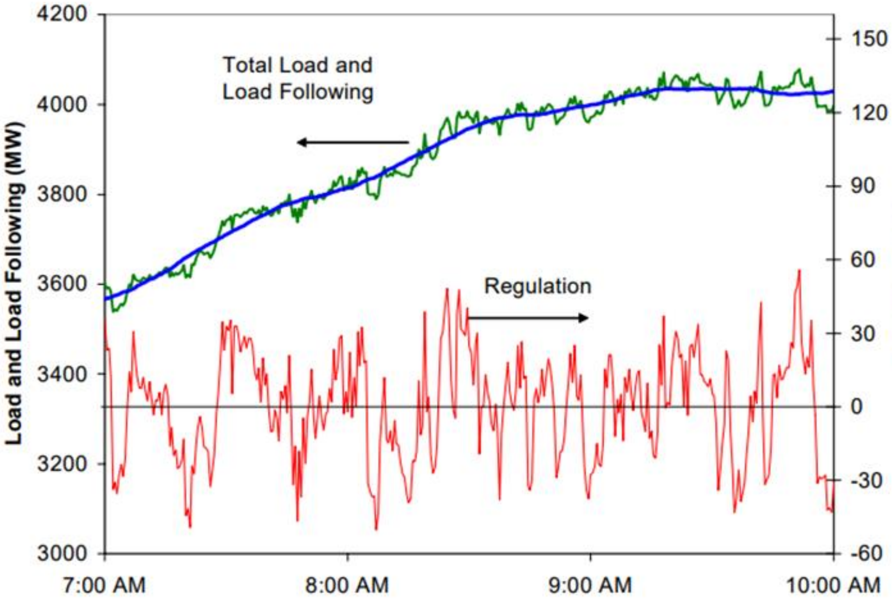
The power consumption of TCLs can be coordinated to help the electric power grid balance supply and demand of electricity

- Large populations of TCLs



- Often residential air conditioners (ACs)
- Can provide frequency regulation
- Common to assume an aggregator receives a desired power signal and controls an aggregation of loads to match that signal
- Rely on control and estimation algorithms
- Estimates of the aggregate AC demand can be used as a feedback signal for control and estimation algorithms



In this work, we apply Kalman filter and online learning algorithms to estimate the aggregate AC demand.

- **Contribution 1:** We show that Dynamic Fixed Share (DFS) can be constructed to produce identical estimates to those produced by a multiple model Kalman filter (MMKF)

- **Contribution 2:** We incorporate three heuristics used within MMKFs into DFS

- **Contribution 3:** We compare the performance of DFS and a MMKF within a demand response simulation

Contents

A Kalman filter relies on a linear model of the system and normally-distributed random variables to estimate a state.

- **System model:** $x_{k+1} = A_k \, x_k + \omega_k$

$$y_k = C_k \, x_k + v_k$$

- **Model-based update:** $\widehat{x}_{k+1} = A_k \, \widetilde{x}_k$

- **Measurement-based update:** $\widetilde{x}_k = \widehat{x}_k + \widehat{P}_k C_k^T \left[C_k \widehat{P}_k C_k^T + R_k \right]^{-1} (y_k - C_k \widehat{x}_k)$

- **Convex optimization formulation [Mattingley 2010]:**

$$\begin{array}{ll} \min\limits_{x, v_k} & v_k^{\mathrm{T}} R_k^{-1} v_k \; + \; (x - \widehat{x}_k)^{\mathrm{T}} \, \widehat{P}_k^{-1} \, (x - \widehat{x}_k) \\ \text{s.t.} & y_k = C_k x + v_k . \end{array}$$

[Simon 2006]

MPEL Michigan Power & Energy Laboratory

Similar to a Kalman filter, DMD uses a system model and a convex optimization formulation to update the predictions

- **System model:** Assumed below

- **Model-based update:** $\widehat{x}_{k+1} = \Phi(\widetilde{x}_k)$

- **Measurement-based update:** Depends on the user construction of the convex optimization function below

- **Convex optimization formulation:** $\widetilde{x}_k = \underset{x \in \mathcal{X}}{\operatorname{argmin}} \ \eta^s \left(\nabla \ell_k(\widehat{x}_k)\right)^T x + D\left(x \| \widehat{x}_k\right)$

[Hall 2015]

DMD can produce identical updates to a Kalman filter if the user-defined functions and parameters are chosen appropriately.

- **Desired update:** $\widetilde{x}_k = \widehat{x}_k + \widehat{P}_k C_k^T \left[C_k \widehat{P}_k C_k^T + R_k \right]^{-1} (y_k - C_k \widehat{x}_k)$

1. **Choose the Bregman divergence:** $D\left(x \| \widehat{x}_k\right) = \frac{1}{2}\left(x - \widehat{x}_k\right)^T \widehat{P}_k^{-1} \left(x - \widehat{x}_k\right)$
2. **Set step-size parameter:** $\eta^{\mathrm{s}} = 1$
3. **Solve the convex optimization formulation:** $\widetilde{x}_k = \widehat{x}_k + \widehat{P}_k \left(-\nabla \ell_k(\widehat{x}_k)\right)$
4. **Select the loss function to match the desired update:**

$$\ell_k(\widehat{x}_k) = \frac{1}{2} \left(C_k \widehat{x}_k - y_k\right)^T (\widehat{P}_k^{\mathrm{y}})^{-1} \left(C_k \widehat{x}_k - y_k\right)$$

$$-\nabla \ell_k(\widehat{x}_k) = C_k^T (\widehat{P}_k^{\mathrm{y}})^{-1} \left(y_k - C_k \widehat{x}_k\right)$$

[Ledva 2018]

An MMKF combines estimates from separate underlying Kalman filters using a Gaussian likelihood function.

- **Likelihood function:**

$$h(y_k|m^i) = \left[(2\pi)^{q/2}\sqrt{|\widehat{P}_k^{\mathrm{y},i}|}\right]^{-1} \exp\left(-\frac{1}{2}d_k^{\mathrm{y}}(\widehat{y}_k^i)\right)$$

$$d_k^{\mathrm{y}}(\widehat{y}_k^i) = (\widehat{y}_k^i - y_k)^T(\widehat{P}_k^{\mathrm{y},i})^{-1}(\widehat{y}_k^i - y_k)$$

- **Weighting function:**

$$w_{k+1}^i = \frac{h(y_k|m^i)\ w_k^i}{\sum_{j\in\mathcal{M}^{\mathrm{mdl}}} h(y_k|m^j)\ w_k^j}$$

- **Overall prediction:**

$$\widehat{x}_{k+1} = \sum_{i\in\mathcal{M}^{\mathrm{mdl}}} w_{k+1}^i\ \widehat{x}_{k+1}^i$$

[Welch 2001]

DFS combines estimates for separate underlying DMD algorithms, which each incorporate different models.

- **Weighting Function:** $$w_{k+1}^{i} = \frac{\lambda}{N^{\mathrm{mdl}}} + (1-\lambda)\,\frac{w_k^i\,\exp\left(-\eta^r\,\ell_k\left(\widehat{x}_k^i\right)\right)}{\sum_{j=1}^{N^{\mathrm{mdl}}} w_k^j\,\exp\left(-\eta^r\,\ell_k\left(\widehat{x}_k^j\right)\right)}$$

- **Overall estimate:** $$\widehat{x}_{k+1} = \sum_{i\in\mathcal{M}^{\mathrm{mdl}}} w_{k+1}^i\,\widehat{x}_{k+1}^i$$

[Hall 2015]

We can make the weighting functions equivalent between the MMKF and DFS by scaling parameters within the functions.

- **Set parameters in DFS:** $w_{k+1}^{i} = \frac{\lambda}{N^{\text{mdl}}} + (1-\lambda)\frac{w_k^i \exp\left(-\eta^r \ell_k\left(\widehat{x}_k^i\right)\right)}{\sum_{j=1}^{N^{\text{mdl}}} w_k^j \exp\left(-\eta^r \ell_k\left(\widehat{x}_k^j\right)\right)}$

- **Both weight updates now have the form:** $w_{k+1}^{i} = \frac{h(y_k|m^i)\, w_k^i}{\sum_{j\in\mathcal{M}^{\text{mdl}}} h(y_k|m^j)\, w_k^j}$

- **Scale output covariance:** $h(y_k|m^i) = \left[(2\pi)^{q/2}\sqrt{|\widehat{P}_k^{y,i}|}\right]^{-1}\exp\left(-\frac{1}{2}d_k^y(\widehat{y}_k^i)\right)$

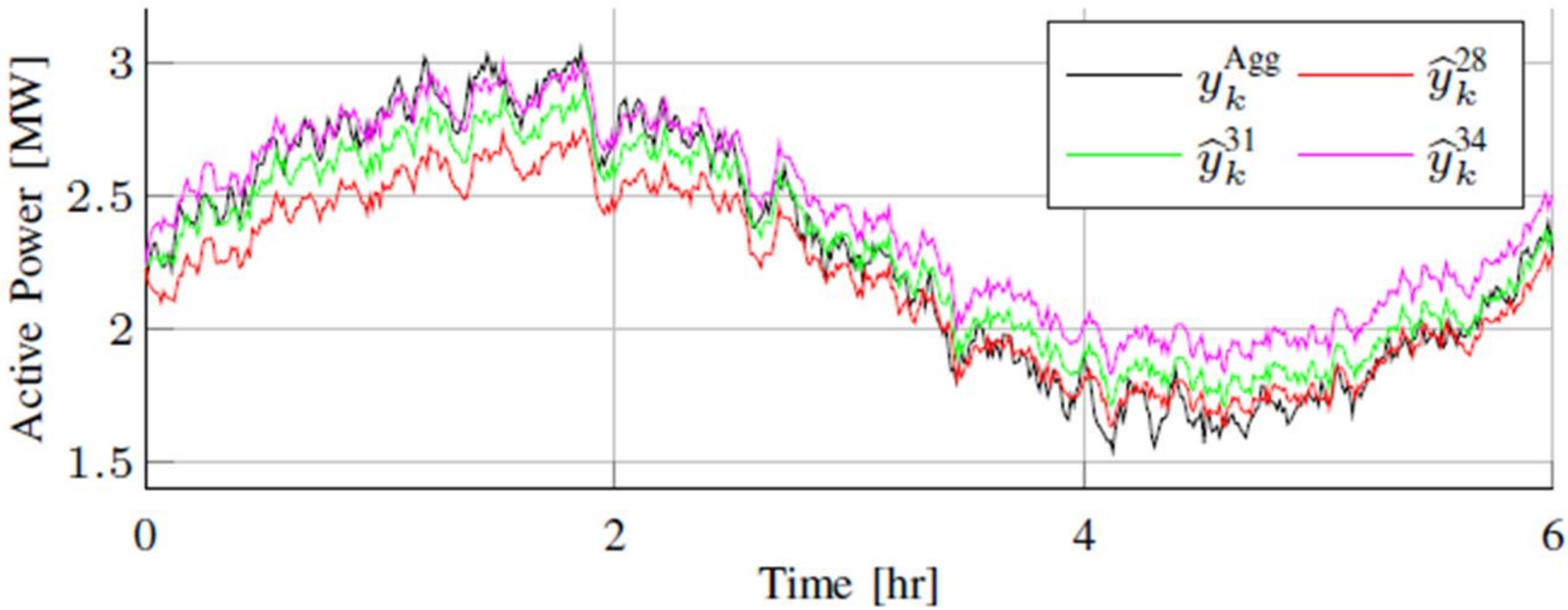
We show that several heuristic adjustments that are used within MMKFs can be incorporated into DFS by modifying the weight update.

- **A minimum weight:** $$w_{k+1}^{i} = \frac{\lambda}{N^{\text{mdl}}} + (1-\lambda)\,\frac{w_k^i \exp\left(-\eta^r\,\ell_k\left(\widehat{x}_k^i\right)\right)}{\sum_{j=1}^{N^{\text{mdl}}} w_k^j \exp\left(-\eta^r\,\ell_k\left(\widehat{x}_k^j\right)\right)}$$

- **Exponential decay:** $$w_{k+1}^{i} = \frac{\lambda}{N^{\text{mdl}}} + (1-\lambda)\,\frac{(w_k^i)^{\gamma} \exp\left(-\eta^r\,\ell_k(\widehat{x}_k^i)\right)}{\sum_{j=1}^{N^{\text{mdl}}} (w_k^j)^{\gamma} \exp\left(-\eta^r\,\ell_k(\widehat{x}_k^j)\right)}$$

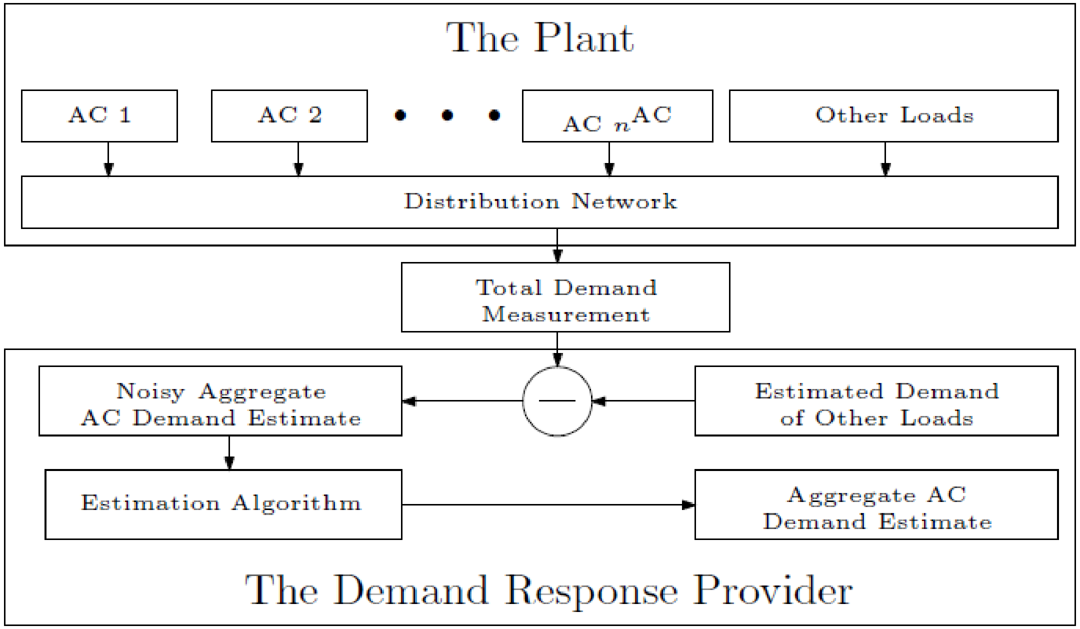
- **Sliding Window:** $$w_{k+1}^{i} = \frac{\lambda}{N^{\text{mdl}}} + (1-\lambda)\,\frac{\prod_{t=k-N^{\ell}}^{k} \exp\left(-\eta^r\,\ell_t\left(\widehat{x}_t^i\right)\right)}{\sum_{j=1}^{N^{\text{mdl}}} \prod_{t=k-N^{\ell}}^{k} \exp\left(-\eta^r\,\ell_t\left(\widehat{x}_t^j\right)\right)}$$

We simulate a demand response scenario to evaluate the ability of different algorithm implementations to estimate the aggregate demand.



- **Each algorithm uses three underlying models to form its overall prediction**
- **Identical individual predictions across the multiple model algorithm implementations**

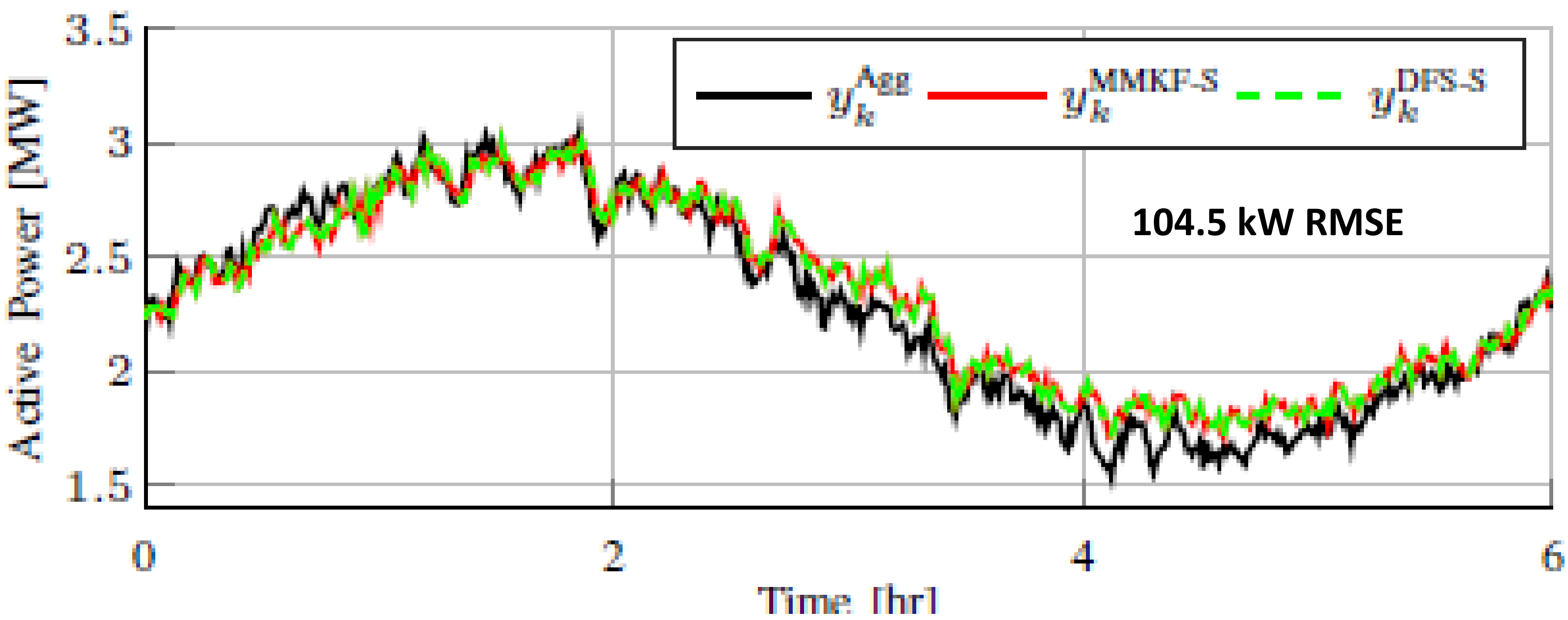
We simulate a population of 1,000 ACs over six hours using 4 second time-steps and a sinusoidal outdoor temperature.



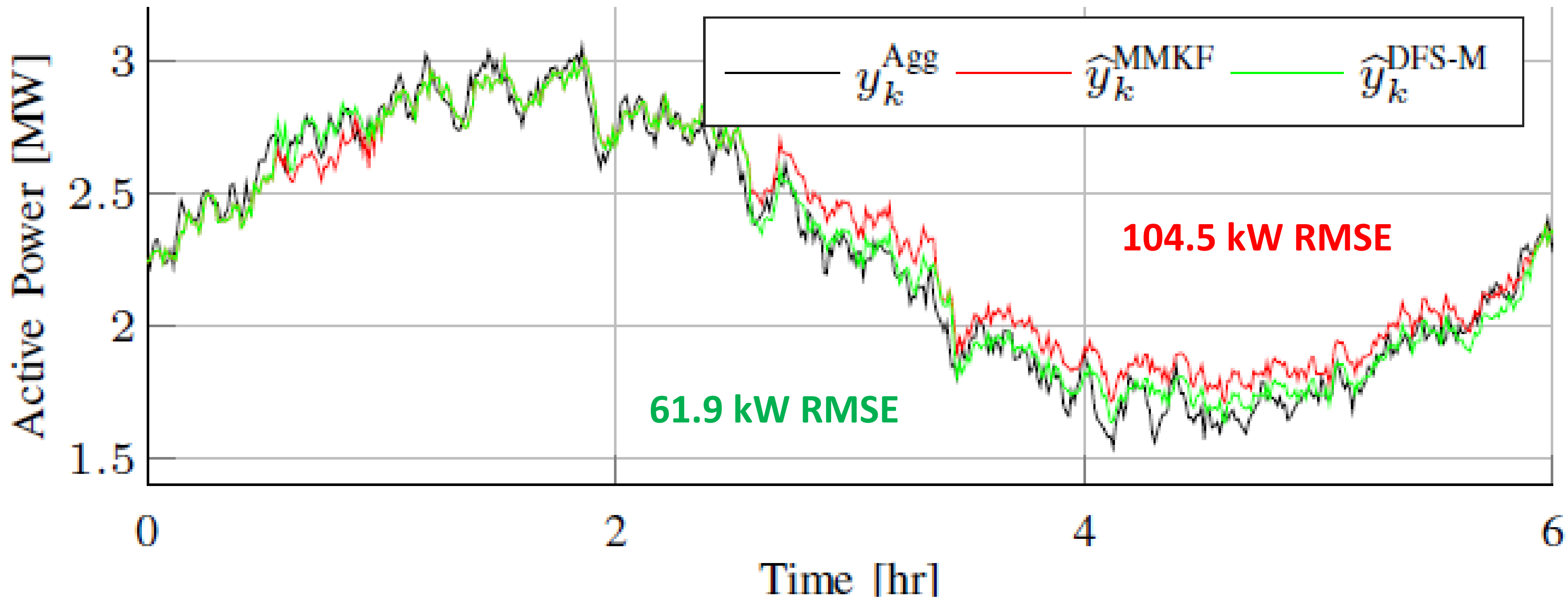
$$\theta_{k+1} = a\,\theta_k + (1-a)\,(\theta_k^{o} - m_k \Lambda P)$$

$$m_{k+1} = \begin{cases} 0 & \text{if } \theta_{k+1} < \theta^{\text{set}} - \frac{\theta^{\text{db}}}{2} \\ 1 & \text{if } \theta_{k+1} > \theta^{\text{set}} + \frac{\theta^{\text{db}}}{2} \\ m_k & \text{otherwise,} \end{cases}$$

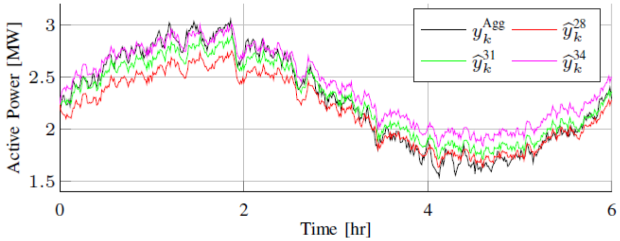
Performing the scaling of the MMKF weight update produces the same estimates for both the MMKF and DFS

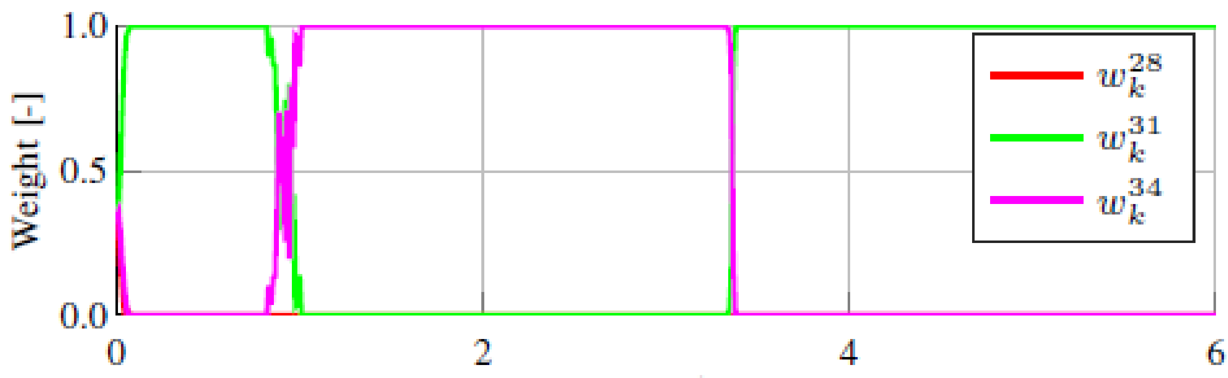


Including a minimum weight into DFS and a MMKF significantly improves the estimation accuracy



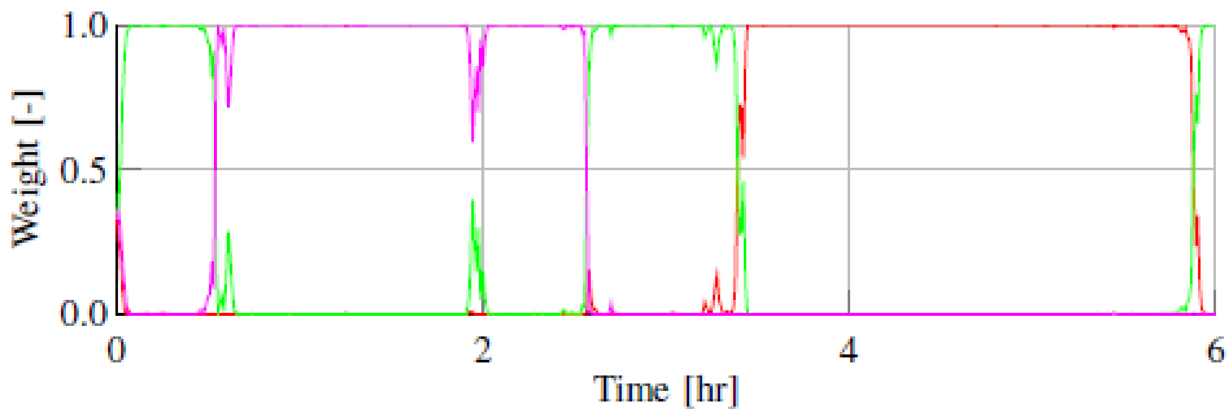
A minimum weight in MMKF allows the algorithm to shift weight to the most accurate model near the end of the simulation.





(a) MMKF Weights

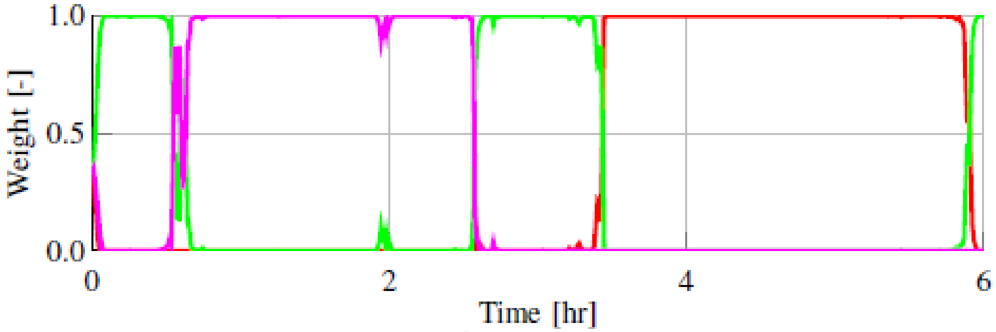
104.5 kW RMSE



(b) MMKF-M Weights

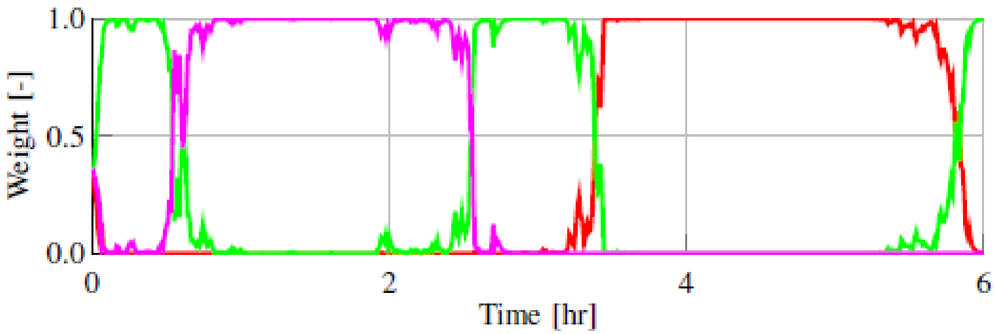
63.4 kW RMSE

Incorporating a sliding window and exponential decay allow the weights to remain dynamic as time progresses.



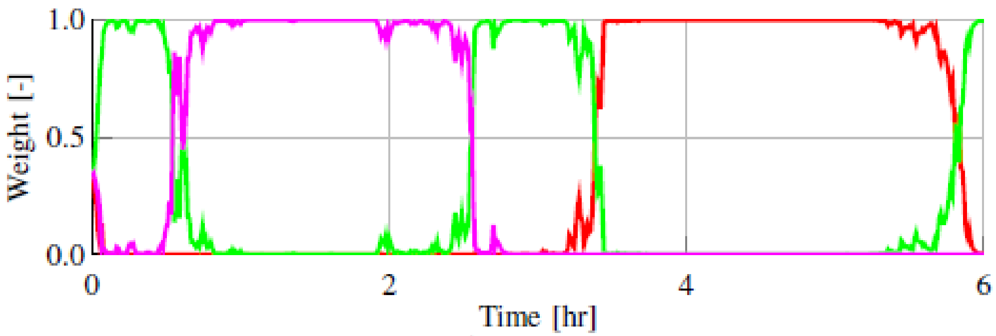
(c) DFS-M Weights

61.9 kW RMSE



(d) DFS-W Weights

61.4 kW RMSE



(e) DFS-E Weights

60.9 kW RMSE

There are close similarities between online learning methods and Kalman filter approaches

- Manipulating the weighting equations in the MMKF results in an identical weight update as in DFS
- DFS can be modified to incorporate heuristics that are commonly used within a MMKF
- Including a minimum weight threshold improves the performance of DFS and a MMKF
- Including exponential decay or a sliding window in the DFS weight update allows more consistent, responsive behavior in the weights.
- The DFS and MMKF algorithms effectively estimate the aggregate AC demand within the demand response simulation.

References

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