

# Chance-Constrained Optimal Power Flow with Uncertain Load Control

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#### Introduction

Demand response (DR) refers to changing electric load consumption patterns that help the grid operate more reliably and efficiently. With high penetration of renewable resources in power grid, more ancillary services are required to continuously balance the supply and demand due to the generation uncertainty. Previous researches have stated that DR could be potential resources with **faster responses**, **lower costs** and **less environmental impacts**. In this research, we tried to analyze the economic dispatch with both renewable generations and uncertain demand response.

#### **Problem Formulation**

In this problem, we want to determine the day-ahead optimal dispatch with uncertain load control and renewable resources by co-optimizing reserves and energy. We use chance-constrained optimization formulation to consider various uncertainties from controllable loads and renewable resources.

To model controllable loads, we use a thermal battery model developed from previous work. A linear charging/discharging mechanism is used to represent the energy change. The baseline power, power capacity and energy capacity are all related to ambient temperature. The aggregated power consumption (set point) are used as design variables.

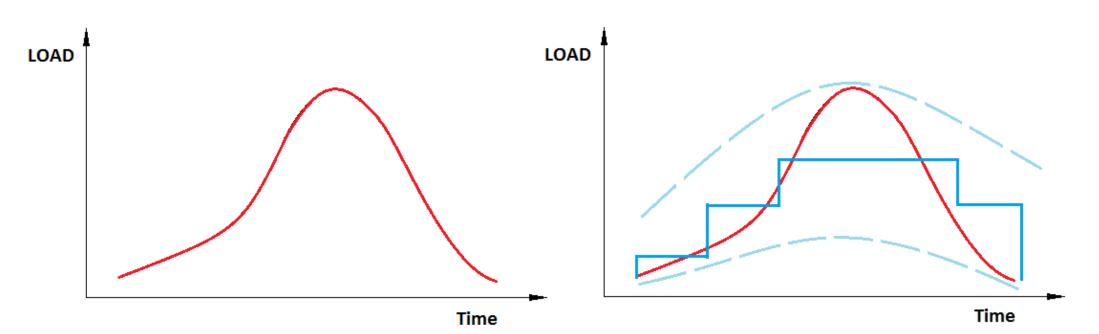


Figure 1: Baseline Power, Set Points and Power Capacity in Controllable Load

## Methodologies

To reformulate the stochastic problem, we use 2 different methodologies.

- 1. Probabilistically Robust Design Solve a robust optimization problem over a hyper-rectangular set constructed based on the confidence level
- 2. Analytical Reformulation

Reformulate constraints deterministically assuming specific uncertainty distribution and correlation

Convex approximations are performed for nonlinear chance constraints and Cutting-Plane Algorithm is used to solve the problem

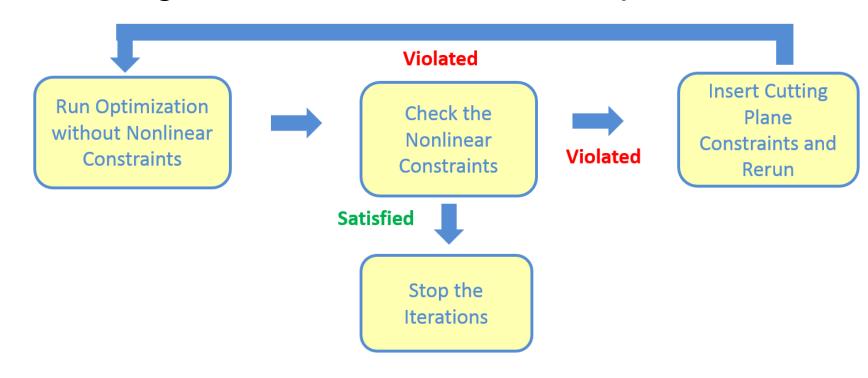


Figure 2: Cutting-Plane Algorithm

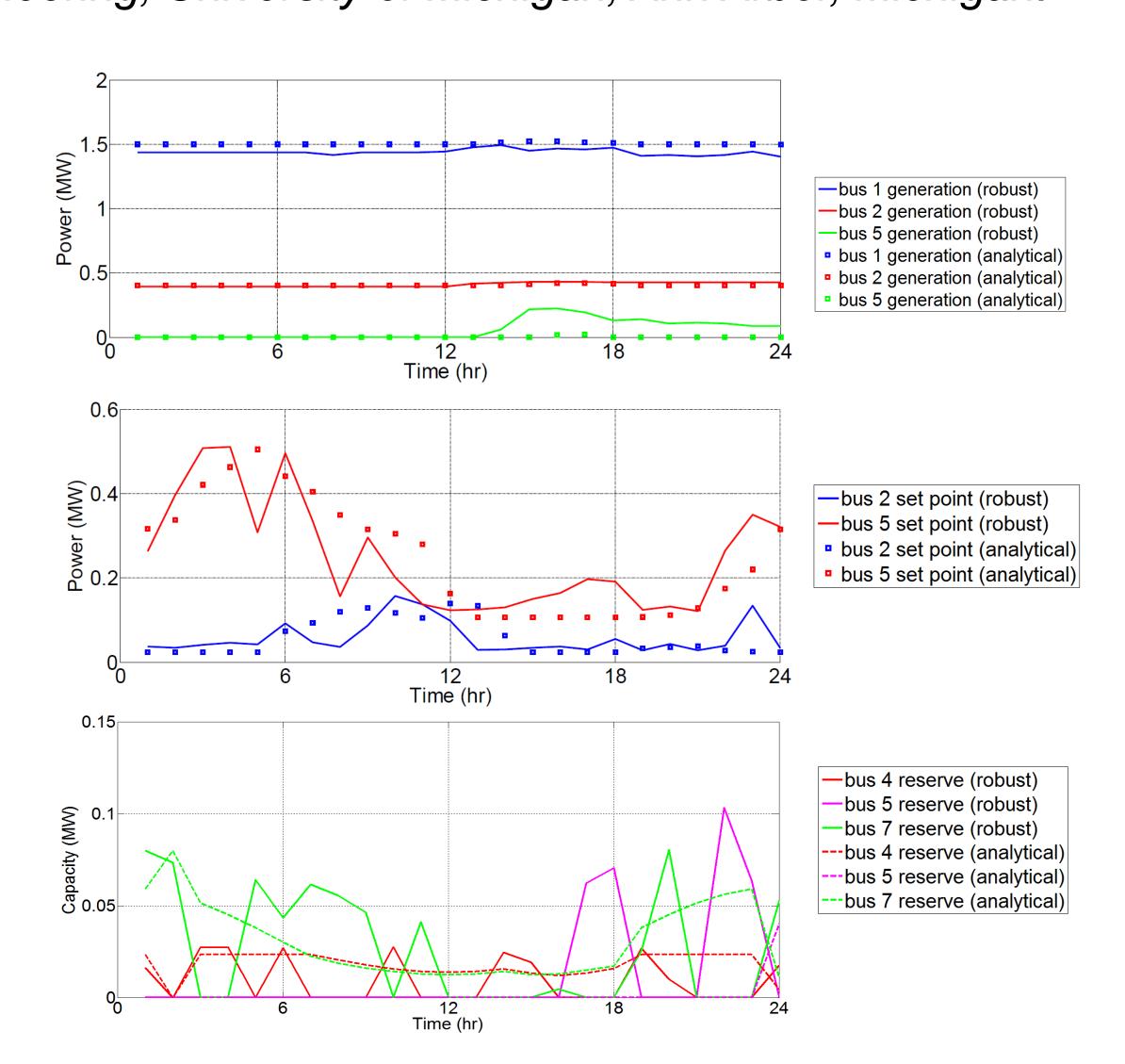
### **Results and Discussion**

The idea was evaluated on IEEE 30-bus system with single wind generator for both congested and uncongested cases.

For objective performance, analytical reformulation gives better objective values compared with robust method. This can be seen from effective load shifting, more load reserve and no usage of peak plant.

For reliability, robust methods provide both higher individual and joint reliability than analytical approach.

For computational effort, robust method is resilient to congestion and cutting-plane algorithm works better than standard nonlinear solver.



**Figure 3**: Optimal Solutions in Congested Case: Generation Schedule, Controllable Load Set Points and Reserve Preparation.

 Table 1: Computational Time.

	Robust	Analytical	
		Nonlinear	Cutting
uncongested	11.57	12.79	5.94
congested	12 32	42.34	15 21

Table 2: Cost Distribution.

		Scenario	Analytical
	generation	121010	120850
uncongested	secondary	1821	1443
	redispatch	52696	42127
congested	generation	123960	122460
	secondary	2134	1529
	redispatch	52487	42085

 Table 3: Reliability Check.

$1 - \epsilon = 0.99$	Robust	Analytical
Joint	0.995/0.994	0.925/0.900
Individual	0.998/0.997	0.975/0.967
Joint	0.985/0.978	0.941/0.921
Individual	0.996/0.983	0.988/0.973
Joint	0.997/0.991	0.897/0.881
Individual	0.999/0.997	0.968/0.960
	Joint Individual Joint Individual Joint	Joint         0.995/0.994           Individual         0.998/0.997           Joint         0.985/0.978           Individual         0.996/0.983           Joint         0.997/0.991

#### Conclusions

In this research, we reformulated the chance-constrained problem robustly and analytically. We compared the objective performance, computational effort and reliability between 2 formulations and different solving algorithms for both congested and uncongested case.

Our results demonstrated that analytical approach provides less conservative results with better objective values and more effective responses to peak load and congestion. Cutting-Plane algorithm could lead to faster convergence. Robust method is resilient to congestion and provide better individual and joint reliability.

# Acknowledgements

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#### References

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