Michigan Power & Energy Laboratory

Analytical Reformulation of Chance-Constrained Optimal Power Flow with Uncertain Load Control

Bowen Li
Johanna L. Mathieu
Electrical & Computer Engineering
University of Michigan
July 2015

Motivation

- Background
  - More renewable resources require more reserves (i.e. ancillary services)
  - Aggregations of controllable loads can provide reserves
  - Reserves provided by load control can be inexpensive and fast-responding
- Uncertain Reserves
  - Compared to existing reserves, load-based reserve capacity is uncertain and time varying
  - Load flexibility is affected by ambient conditions and human behavior
- Our Solution
  - Using stochastic optimal power flow (OPF) formulation to consider various uncertainties from loads, load-based reserves, and renewable resources
  - Solving with different methodologies to find the trade-offs among objective cost, computational effort and reliability.

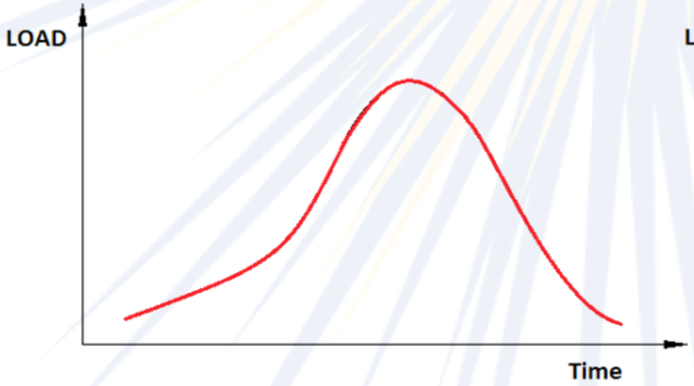
Load Model

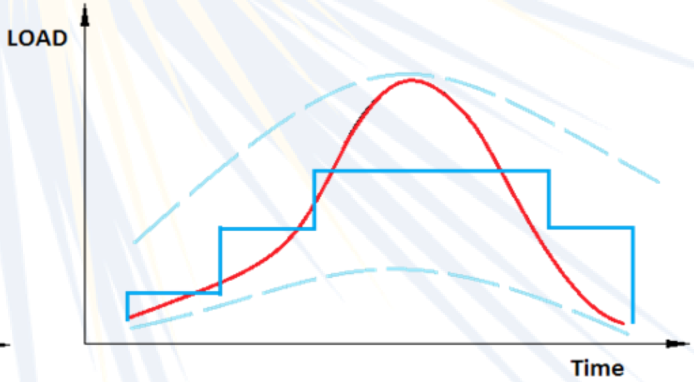
- Aggregation of residential loads
  - Thermostatically controlled loads (i.e. air conditioners) with temperature deadband
  - Nondisruptive control
  - On/Off signals from aggregator to individual loads
- Thermal battery model (Mathieu, et al. 2015)
  - Baseline power consumption P_T
  - Aggregated power consumption (set point) $P_{C,t}$
  - Real time energy state S_t
  - Energy Storage: Charging/Discharging

$$S_{t+\Delta\tau} = S_t + (P_{C,t} - P_T(T_t))\Delta\tau$$

$$\underline{P}_C(T_t) \le P_{C,t} \le \overline{P}_C(T_t)$$

$$\underline{S}(T_t) \le S_t \le \overline{S}(T_t)$$





Stochastic OPF Problem

- Optimization for day-ahead planning
  - Objective: To determine the optimal dispatch with uncertain load control and renewable resources by co-optimizing reserves and energy.
  - Uncertainties: Wind Power Production and Outdoor Temperature
  - Chance-constrained DC OPF based on Vrakopoulou, et al. 2014
- Design Variables
  - Generation schedule and load set points
  - Distribution vectors and reserve capacity (generator/load)
- Constraints
  - Deterministic/Probabilistic
  - Generation limits/Load limits/Line limits/Reserve limits
- Simulation Setup
  - Modified IEEE 30 bus system with single wind bus
  - Uncongested/Congested
  - All loads are 50% controllable with adequate capacities

Stochastic OPF Problem

minimize Generation costs + Reserve costs

x

subject to Power Flow Equations

Generation Constraints

Line Constraints

Controllable Load Constraints

…….

Wind and Temperature Uncertainty

$$S_{t+\Delta\tau} = S_t + (P_{C,t} - P_T(T_t))\Delta\tau$$

$$\underline{P}_C(T_t) \le P_{C,t} \le \overline{P}_C(T_t)$$

$$\underline{S}(T_t) \le S_t \le \overline{S}(T_t)$$

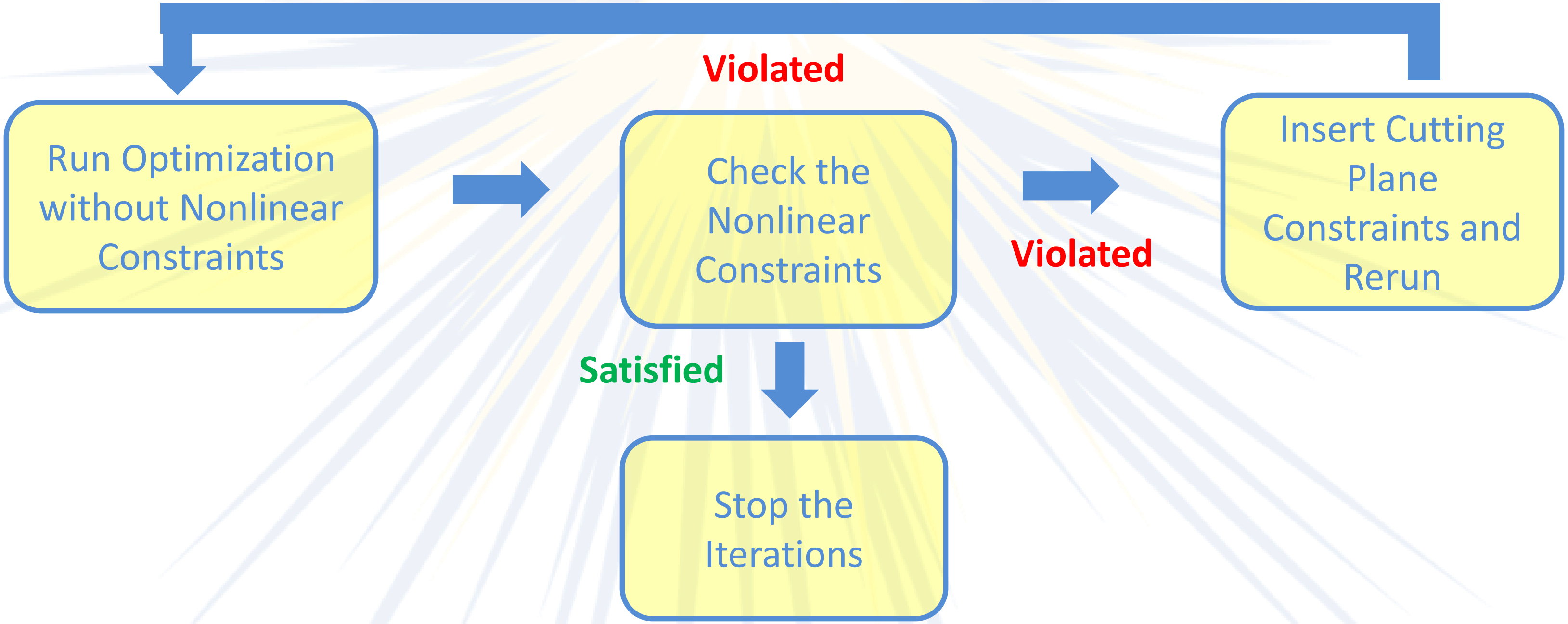
X $\in$ [Generation Schedule, Load Set Points, Distribution Vectors, Reserve Schedule]

Methodologies

- Scenario Approach (Campi, et al. 2009)
    - Probabilistic constraints transformed into deterministic constraints
    - Sufficient scenarios ensure a-priori guarantee at a specified confidence level
    - Conservative results and large computational effort
- Probabilistically Robust Design (Margellos, et al. 2014)
    - Uncertainty bounded by hyper-rectangular set based on the confidence level
    - Solve a robust optimization problem over the set
    - More conservative than scenario approach but less computational effort
- Analytical Reformulation (Roald, et al. 2013 and Bienstock, et al. 2014)
    - Reformulate constraints deterministically assuming specific uncertainty distributions
    - Less conservative result and less computation effort with efficient algorithm
    - Worse at satisfying multiple chance constraints (i.e., worse “joint reliability”)

Methodologies

- Convex Approximation
  - Use CDF of uncertainty distribution to find confidence bound given the specified constraint violation probability
  - Represent confidence bound with piecewise linear convex approximation
- Cutting-plane Algorithm (Bienstock, et al. 2014)



Run Optimization without Nonlinear Constraints

Check the Nonlinear Constraints

Violated

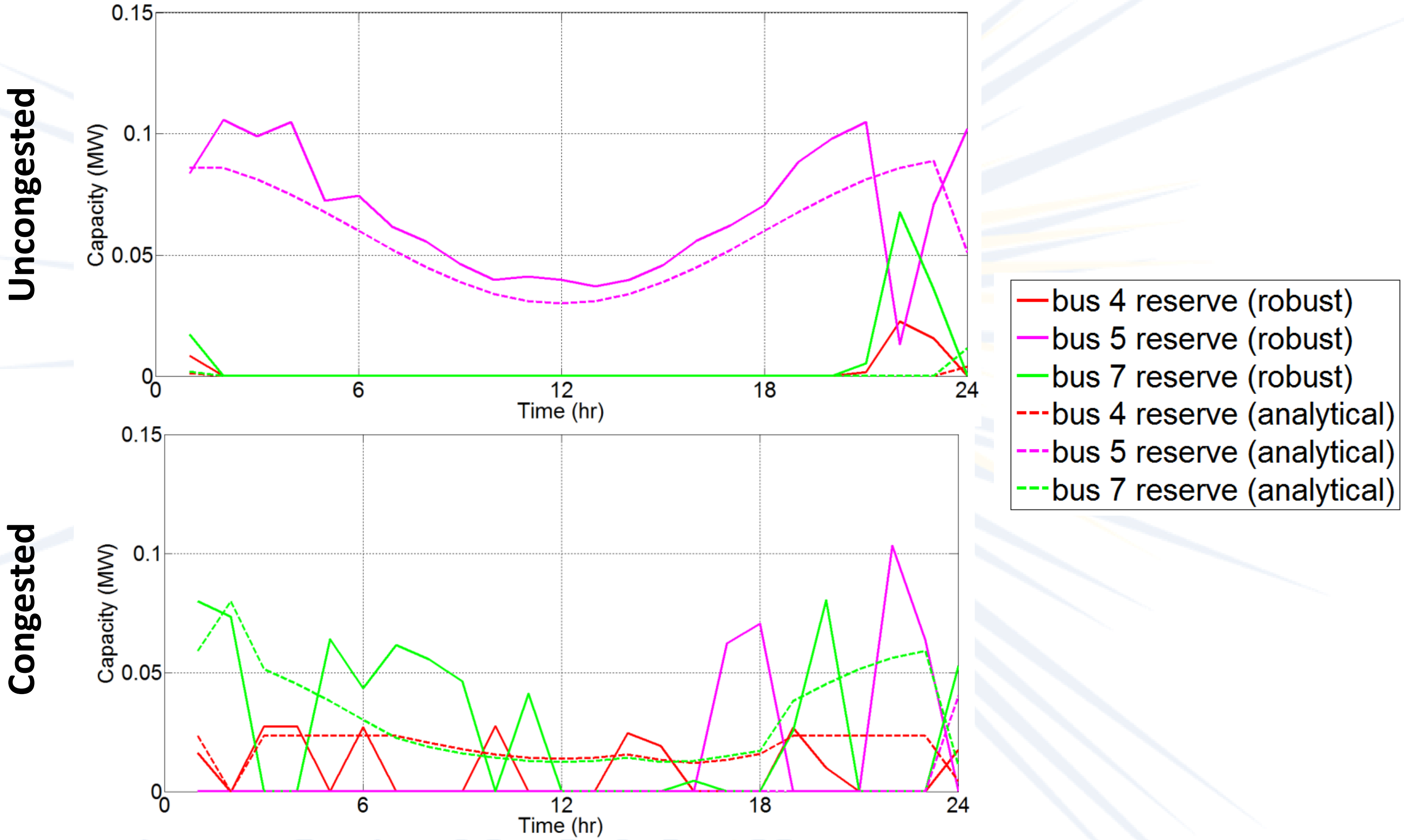
Insert Cutting Plane Constraints and Rerun

Violated

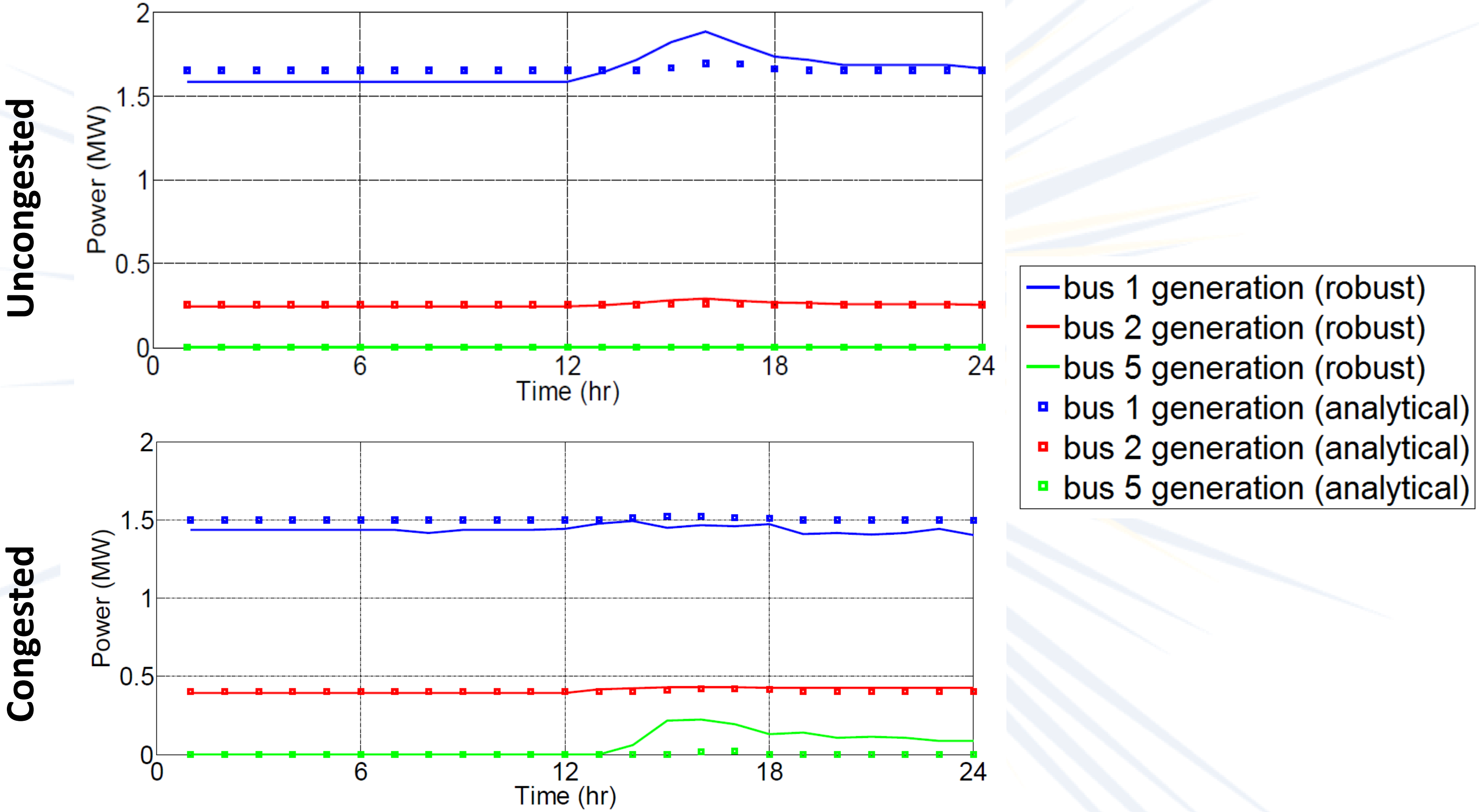
Satisfied

Stop the Iterations

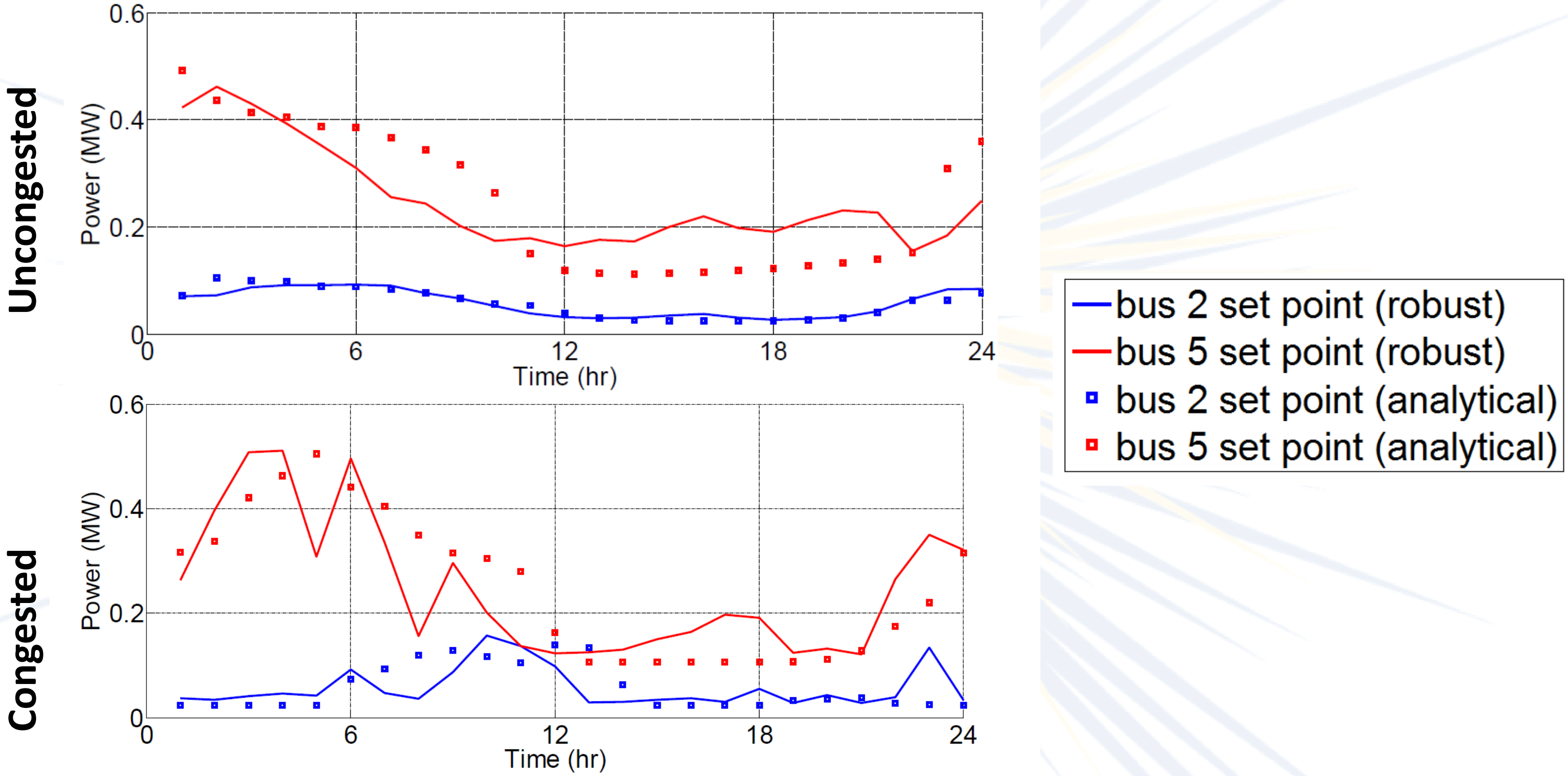
Simulation Results



Simulation Results

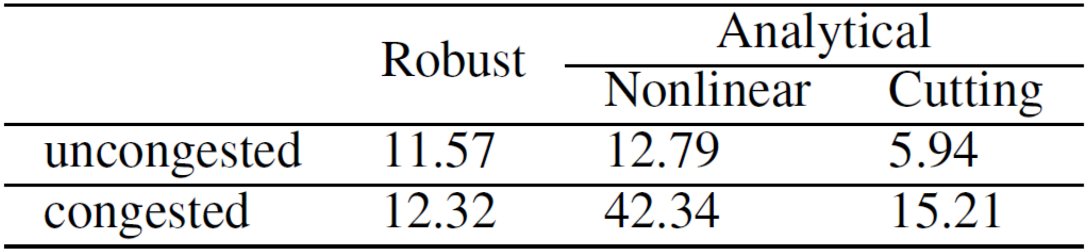


Simulation Results



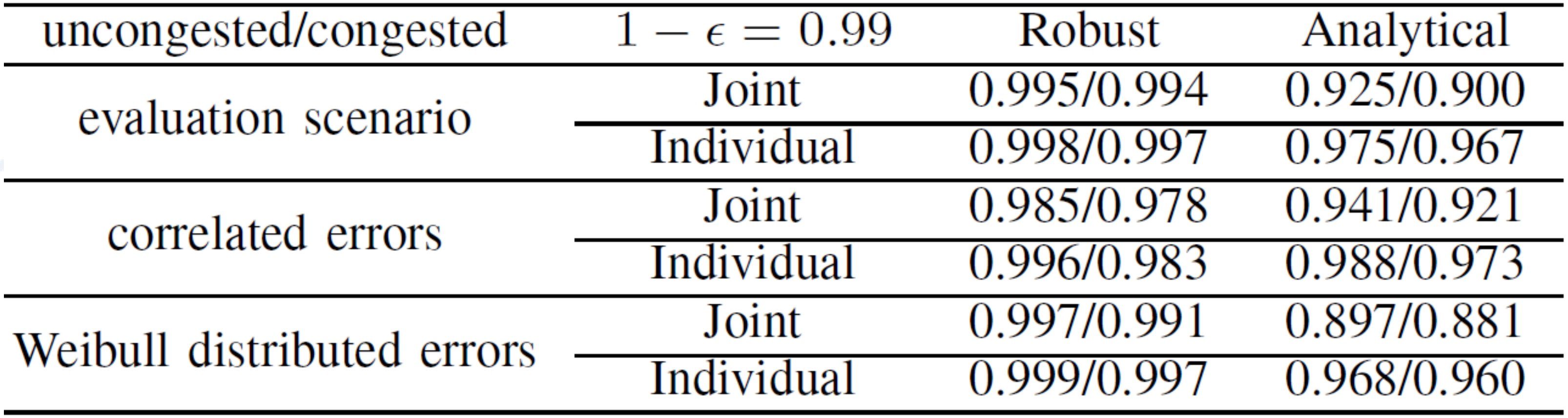
Simulation Results

TABLE I: Computational Time



	Robust	Analytical	
		Nonlinear	Cutting
uncongested	11.57	12.79	5.94
congested	12.32	42.34	15.21

TABLE III: Reliability Check



uncongested/congested	$1-\epsilon=0.99$	Robust	Analytical
evaluation scenario	Joint	0.995/0.994	0.925/0.900
	Individual	0.998/0.997	0.975/0.967
correlated errors	Joint	0.985/0.978	0.941/0.921
	Individual	0.996/0.983	0.988/0.973
Weibull distributed errors	Joint	0.997/0.991	0.897/0.881
	Individual	0.999/0.997	0.968/0.960

Conclusions

- Analytical Reformulation
  - Less conservative with lower cost
  - Results in more load shifting, more effectively managing peak load and congestion
  - Worse joint reliability
  - Computational effort comparable to probabilistically robust design for small uncertainty dimensions
- Probabilistically Robust Design
  - Conservative and so results in less load shifting
  - With congestion, peaking generator might be required
  - High reliability regardless of uncertainty distributions
  - Computational effort not related to congestion
- Future Work
  - Smaller demand response capacity
  - Larger uncertainty dimension including uncertainty correlation
  - More complicated stochastic load model

Michigan Power & Energy Laboratory

Key References

- “Benefits of Demand Response in Electricity Markets and Recommendations for Achieving them,” U.S. Department of Energy, 2006
- J.L. Mathieu, M. Kamgarpour, J. Lygeros, G. Andersson, and D. Callaway, “Arbitraging intraday wholesale energy market prices with aggregations of thermostatic loads,” *IEEE Trans Power Systems*, 30(2): 763-772, 2015.
- M. Vrakopoulou, J.L. Mathieu, and G. Andersson, “Stochastic Optimal Power Flow with Uncertain Reserves from Demand Response,” *Proceedings of HICSS*, pp. 2353-2362, 2014.
- M. Campi, S. Garatti, M. Prandini, “The scenario approach for systems and control design,” *Annual Reviews in Control*, 33(2): 149-157, 2009.
- K. Margellos, P. Goulart, and J. Lygeros, “On the Road Between Robust Optimization and the Scenario Approach for Chance Constrained Optimization Problems,” *IEEE Transactions on Automatic Control*, 59(8): 2258-2263, 2014.
- L. Roald, F. Oldewurtel, T. Krause, and G. Andersson, “Analytical reformulation of security constrained optimal power flow with probabilistic constraints,” PowerTech, 2013.
- D. Bienstock, M. Chertkov, and S. Harnett, ”Chance-Constrained Optimal Power Flow: Risk-Aware Network Control under Uncertainty,” *SIAM Review*, 56(3): 461-495, 2014.

Questions

Thanks!

Bowen Li, University of Michigan

libowen@umich.edu

This work supported by the U.S. National Science Foundation Grant #CCF-1442495 (CyberSEES).