Data-driven Optimization Approaches for Optimal Powe Flow with Uncertain Reserves from Load Control

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The Challenge

- Aggregations of electric loads can provide power systems reserves via load control
- But loads are stochastic…
  - We don't know the *future load* exactly
  - We don't know the *future load flexibility* exactly

- Two options:
  - Be conservative in how much load-based reserve we schedule (how it's done today)
  - Plan for *load control uncertainty* within the optimal power flow problem …

Planning for load control uncertainty

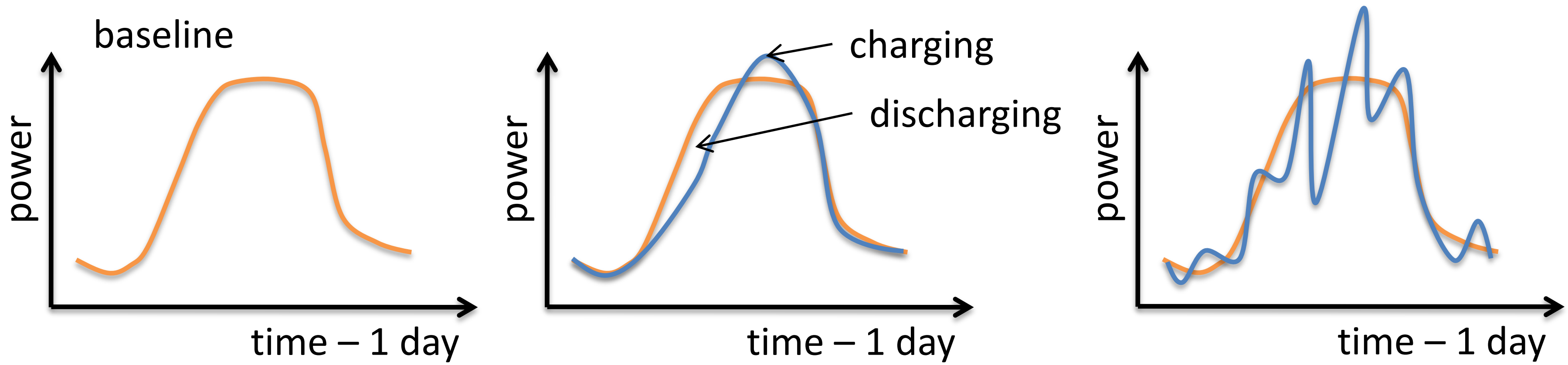
- Stochastic optimal power flow (OPF) including…
  - uncertain renewable energy production
  - uncertain load control (i.e., reserves provided by loads where feasible reserve capacities aren't known exactly)

- Chance-constrained formulation…
  - Ensure constraints with stochastic variables are not violated *with certain probabilities*

Outline

- Load modeling
- Problem formulation
- Solution approaches
- Computational results

Load Aggregations Modeled as Time-Varying & Uncertain Energy Storage



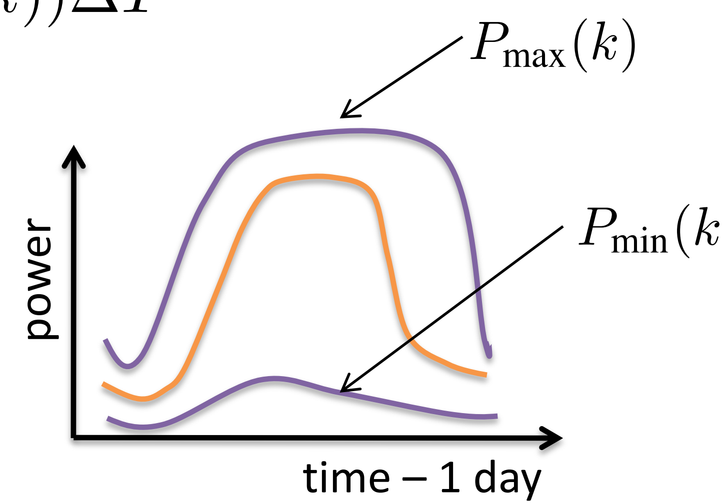
$$S(k+1) = S(k) + (P(k) - P_{\text{baseline}}(k))\Delta T$$

Mean power over an interval

$$P_{\min}(k) \leqslant P(k) \leqslant P_{\max}(k)$$

State of charge

$$S_{\min}(k) \leqslant S(k) \leqslant S_{\max}(k)$$



[Mathieu, Kamgarpour, Lygeros, Andersson, & Callaway TPWRS 2015]

Formulation: Assumptions

- DC-OPF
- Single-period problem
- Load-based reserves should be able to provide full power capacity for 15-minutes

  → Power capacity offered to market is a function of $P_{\min}(k)$, $P_{\max}(k)$, $S_{\min}(k)$, and $P_{\max}(k)$

Formulation: Notation

- Decision variables
  - Generator energy production, P_G
  - Generator up- and down reserve capacity, $\overline{R}_G, \underline{R}_G$
  - Load up- and down reserve capacity, $\overline{R}_L, \underline{R}_L$
  - Distribution vectors, $\overline{d}_G, \underline{d}_G, \overline{d}_L, \underline{d}_L$
- Random variables
  - Wind power production, $\tilde{P}_W$
  - Load, $\tilde{P}_L$
  - Maximum and minimum load, $\overline{\tilde{P}}_L, \underline{\tilde{P}}_L$

Formulation:
Joint Chance Constrained OPF

$$\min \quad c^{\mathsf{T}}[1, P_G, P_G^2, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L]$$

Energy+Reserve Cost

$$\text{s.t.} \quad P_m = \sum_{i=1}^{N_W} (\widetilde{P}_{W,i} - P_{W,i}^f) - \sum_{i=1}^{N_L} (\widetilde{P}_{L,i} - P_{L,i}^f)$$

Power mismatch

$$\left.\begin{aligned} &\sum_{i=1}^{N_G} \underline{d}_{G,i} + \sum_{i=1}^{N_L} \overline{d}_{L,i} = 1 \\ &\sum_{i=1}^{N_G} \overline{d}_{G,i} + \sum_{i=1}^{N_L} \underline{d}_{L,i} = 1 \end{aligned}\right\}$$

Distribution vectors

$$\left.\begin{aligned} &R_G = \overline{d}_G \max\{-P_m, 0\} - \underline{d}_G \max\{P_m, 0\} \\ &R_L = \overline{d}_L \max\{P_m, 0\} - \underline{d}_L \max\{-P_m, 0\} \end{aligned}\right\}$$

Reserves

$$\mathbb{P}\left(\widetilde{A}x \geq \widetilde{b}\right) \geq 1 - \epsilon$$

Chance constraints

$$x = [P_G, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L, \underline{d}_G, \overline{d}_G, \underline{d}_L, \overline{d}_L] \geq \mathbf{0}.$$

Decision vector

Formulation:
Joint Chance Constrained OPF

Expanding the chance constraints…

$$
\begin{aligned}
\widetilde{A}x \geq \widetilde{b} = \{\underline{P}_G \leq P_G + R_G \leq \overline{P}_G, &\qquad \longleftarrow \text{Generation limits}\\
\underline{\widetilde{P}}_L \leq \widetilde{P}_L + R_L \leq \widetilde{\overline{P}}_L, &\qquad \longleftarrow \text{Load limits}\\
\left.\begin{aligned} -\underline{R}_G \leq R_G \leq \overline{R}_G, \\ -\underline{R}_L \leq R_L \leq \overline{R}_L, \end{aligned}\right\} &\qquad \longleftarrow \text{Reserve limits}\\
-P_{\text{line}} \leq B_{\text{flow}} \begin{bmatrix} 0 \\ B_{\text{bus}}^{-1} \hat{P}_{\text{inj}} \end{bmatrix} \leq P_{\text{line}}\}. &\qquad \longleftarrow \text{DC Power Flow}
\end{aligned}
$$

Formulation:
Individual Chance Constrained OPF

$$
\begin{aligned}
\min \quad & c^{\mathsf{T}}[1, P_G, P_G^2, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L] \\
\text{s.t.} \quad & P_m = \sum_{i=1}^{N_W} (\widetilde{P}_{W,i} - P_{W,i}^f) - \sum_{i=1}^{N_L} (\widetilde{P}_{L,i} - P_{L,i}^f) \\
& \sum_{i=1}^{N_G} \underline{d}_{G,i} + \sum_{i=1}^{N_L} \overline{d}_{L,i} = 1 \\
& \sum_{i=1}^{N_G} \overline{d}_{G,i} + \sum_{i=1}^{N_L} \underline{d}_{L,i} = 1 \\
& R_G = \overline{d}_G \max\{-P_m, 0\} - \underline{d}_G \max\{P_m, 0\} \\
& R_L = \overline{d}_L \max\{P_m, 0\} - \underline{d}_L \max\{-P_m, 0\} \\
& \mathbb{P}\left(\widetilde{A}_i x \geq \widetilde{b}_i\right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m. \\
& x = [P_G, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L, \underline{d}_G, \overline{d}_G, \underline{d}_L, \overline{d}_L] \geq \mathbf{0}.
\end{aligned}
$$

Chance constraints

Solution Approaches

- A1: Sample Average Approximation
  - [Luedtke and Ahmed SIAM Opt 2008]
- A2: Gaussian Approximation
  - Used by [Roald et al. PowerTech 2013] and [Bienstock et al. SIAM Review 2014] for OPF with uncertain wind
  - Used by [Li and Mathieu PowerTech 2015] for OPF with uncertain wind *and load control*
- A3: Scenario Approximation
  - [Calafiore and Campi TAC 2006]
  - Used by [Vrakopoulou et al. TPWRS 2013] for OPF with uncertain wind
  - A variant based on [Margellos et al. TAC 2014] used by [Vrakopoulou et al. HICSS 2014] for OPF with uncertain wind *and load control*
- A4: Distributionally Robust Optimization
  - [Delage and Ye OR 2010]

A1: Sample Average Approximation

- Reformulate individual chance constraints as

$$A_i^s x \geq b_i^s - M y_s^i \; \forall s \in \Omega, \; i = 1, \ldots, m$$
$$\sum_{s \in \Omega} p^s y_s^i \leq \epsilon_i, \; \forall i, \; \text{and } y_s^i \in \{0, 1\} \; \forall s, \; i,$$

  where M is a large number each sample s is associated with a binary logic variable y_s
- This is a mixed integer quadratic program (MIQP).

A2: Gaussian Approximation

- Re-write the individual chance constraint

$$\mathbb{P}\left(\widetilde{A}_i'\bar{x} \le b_i'\right) \ge 1 - \epsilon_i \quad i = 1, \ldots, m,$$

- Assume the uncertainty is Gaussian

$$\widetilde{A}_i' \sim N(\mu_i, \Sigma_i).$$

- Then,

$$\widetilde{A}_i'\bar{x} - b_i' \sim N(\mu_i^{\mathsf{T}}\bar{x} - b', \bar{x}^{\mathsf{T}}\Sigma_i\bar{x}).$$

and the constraint can be rewritten as

$$b_i' - \mu_i^{\mathsf{T}}\bar{x} \ge \Phi^{-1}(1 - \epsilon_i)\sqrt{\bar{x}^{\mathsf{T}}\Sigma_i\bar{x}} \quad i = 1, \ldots, m.$$

- This is a second-order cone program (SOCP) if the probability of constraint violation is less than 50%.

A3: Scenario Approximation

- Replace each chance constraint with

$$A_i^s x \geq b_i^s \;\; \forall s \in \Omega_{\mathrm{ap}}.$$

- Use at least $\frac{2}{\varepsilon}\left(\ln\left(\frac{1}{\beta}\right)+n\right)$ samples to guarantee performance [Califiore and Campi TAC 2006], where ε is the probability of constraint violation, $1\text{-}\beta$ is the confidence level, n is the dimension of x.
- This is a quadratic program (QP).

A4: Distributionally Robust Optimization

- The distributionally robust variant of the individual chance constraint is

$$\inf_{f(\xi)\in\mathcal{D}} \mathbb{P}_\xi(\widetilde{A}_i^\xi x \geq \widetilde{b}_i^\xi) \geq 1-\epsilon_i \ \forall i = 1,\ldots,m.$$

- Given samples of the uncertainty, calculate the empirical mean μ_0 and covariance Σ_0, and build a confidence set

$$\mathcal{D} = \left\{ f(\xi) : \quad \begin{array}{l} \int_{\xi\in\mathcal{S}} f(\xi) d\xi = 1 \\ (\mathbb{E}[\xi]-\mu_0)^\mathsf{T}(\Sigma_0)^{-1}(\mathbb{E}[\xi]-\mu_0) \leq \gamma_1 \\ \mathbb{E}[(\xi-\mu_0)(\xi-\mu_0)^\mathsf{T}] \preceq \gamma_2\Sigma_0 \end{array} \right\}.$$

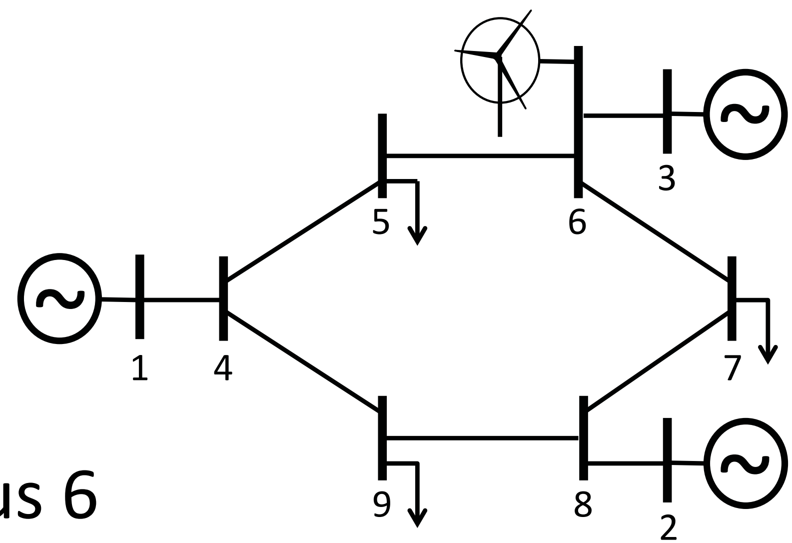
A4: Distributionally Robust Optimization

- Let r_i, $\begin{bmatrix} H_i & p_i \\ p_i^\top & q_i \end{bmatrix}$, and G_i be the dual variables associated with the three constraints within the confidence set.
- The chance constraints are equivalent to

$$\gamma_2 \Sigma_0 \cdot G_i + 1 - r_i + \Sigma_0 \cdot H_i + \gamma_1 q_i \leq \epsilon_i y_i$$
$$\begin{bmatrix} G_i & -p_i \\ -p_i^\top & 1 - r_i \end{bmatrix} \succeq \begin{bmatrix} 0 & \frac{1}{2}\bar{A}_i^x \\ \frac{1}{2}(\bar{A}_i^x)^\top & y_i + (\bar{A}_i^x)^\top \mu_0 - \bar{b}_i^x \end{bmatrix}$$
$$\begin{bmatrix} G_i & -p_i \\ -p_i^\top & 1 - r_i \end{bmatrix} \succeq 0, \begin{bmatrix} H_i & p_i \\ p_i^\top & q_i \end{bmatrix} \succeq 0, y_i \geq 0, \ i = 1, \ldots, m,$$

resulting in a semi-definite program (SDP).

Computational experiments



- IEEE 9-bus test system
  - Added one wind farm to bus 6
  - All loads assumed partially controllable
  - Wind forecast uncertainty (modeled with real data)
  - Load control uncertainty assumed a function of temperature forecast uncertainty

Computational Results: Comparison

Desired probability of constraint violation: 5% (95% Reliability)

	Obj.			Rel(%)			CPU		
	avg	min	max	avg	min	max	avg	min	max
SAA: Joint	1349	1328	1363	77	8	95	2	1	4
SAA: Individual	1346	1336	1357	72	46	90	5876	131	32817
Gaussian	1349	1340	1358	82	65	94	1	1	1
Scenario	1408	1371	1525	100	99	100	55	54	57
Dist. Robust	1393	1365	1458	100	98	100	5	4	6
	Cost			Performance			Computation		

Distributionally robust (empirically) requires 20 data points;
the scenario approach (theoretically) requires 900!

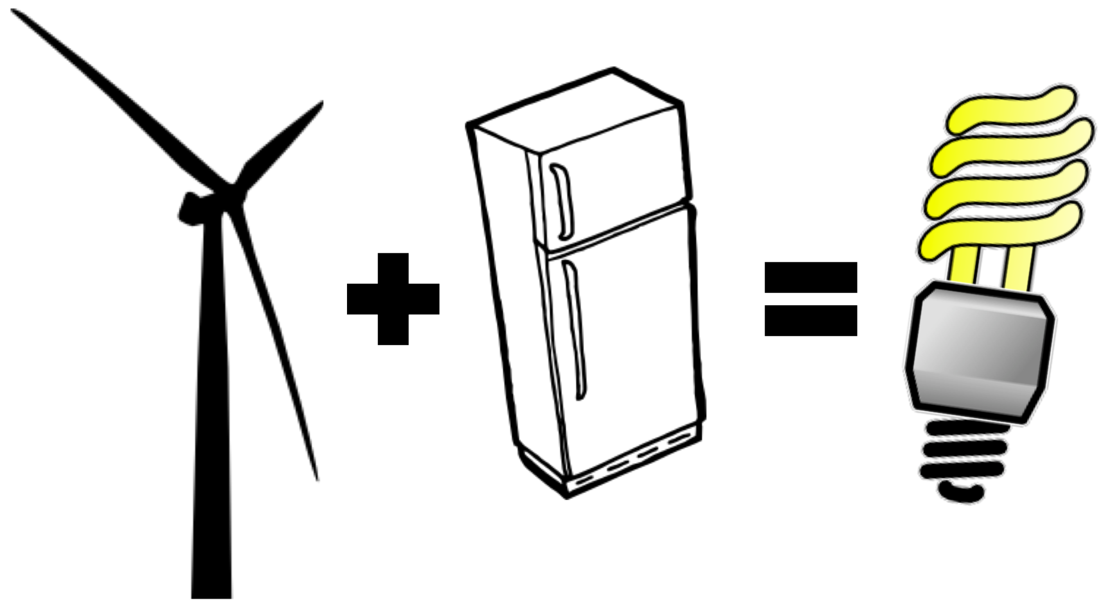
Computational Results: Distributionally Robust Optimization

		$1-\epsilon_i =$	95.00%	90.00%	85.00%
Individual	Objective cost	avg	1392.64	1369.23	1359.97
		min	1352.46	1346.62	1346.62
		max	1457.81	1385.24	1372.75
	Reliability (%)	avg	99.50	97.97	94.51
		min	91.40	91.40	83.29
		max	99.96	99.70	99.18
	CPU seconds	avg	6.63	6.98	6.95
		min	6.13	4.73	6.27
		max	8.19	8.44	7.83

Findings & Conclusions

- Distributionally robust optimization provides a good trade-off…
  - Less computationally-intensive than scenario-based methods
  - Requires less data than scenario-based methods
  - Better performance than Gaussian approximation or sample average approximation
- …but the semidefinite program doesn’t scale very well to larger systems.
- Next steps: more realistic problem formulation, development of scalable approximations.

THANK YOU! QUESTIONS?



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