Data-driven Optimization Approaches for Optimal Power Flow with Uncertain Reserves from Load Control

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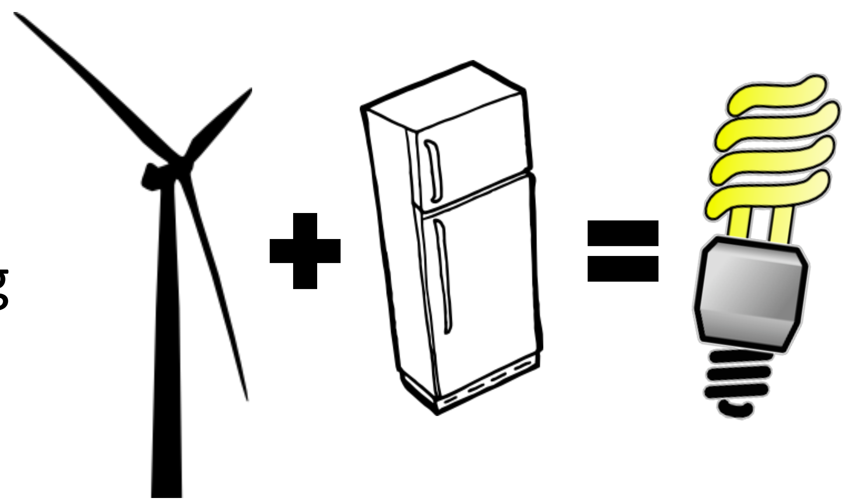


Joint work with Prof. Siqian Shen, Yiling Zhang, and Bowen Li

The Challenge

- Aggregations of loads can provide reserves via load control
- But loads are stochastic…
  - We don't know the *future load* exactly
  - We don't know the *future load flexibility* exactly

- Two options:
  - Be conservative in how much load-based reserve we schedule (how it's done today)
  - Plan for *load control uncertainty* within the optimal power flow problem …

The Solution:
Planning for load control uncertainty

- Stochastic optimal power flow including…
  - Uncertain renewable energy production
  - Uncertain load control (i.e., load-based reserves where feasible reserve capacities aren't known exactly)
- Chance-constrained formulation…
  - Ensure probabilities of constraints with stochastic variables are met with certain probabilities

Solution Approaches

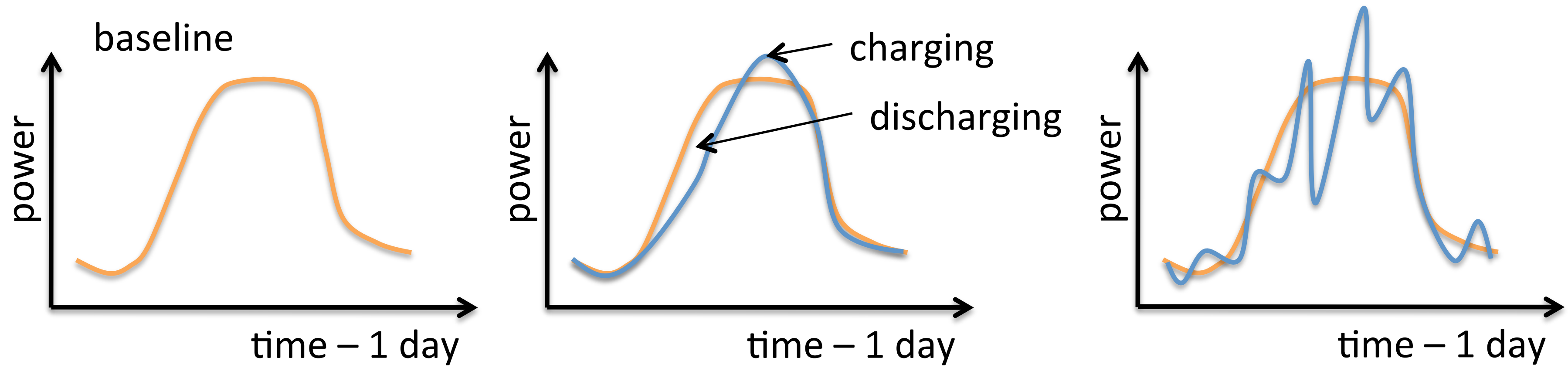
- Last year: scenario-based approach
  - Calafiore and Campi TAC 2006 – "The scenario approach" – provides probabilistic guarantees if certain number of scenarios are used
  - Margellos, Goulart, and Lygeros TAC 2012 – "Probabilistically Robust Design" – robustifies the scenario approach to reduce data needs/computation
  - *More information*: Vrakopoulou et al. HICSS 2014

- This year: distributionally robust optimization
  - Data-driven, but can cope with limited data availability
  - *More information*: Zhang et al. ACC 2015

Outline

- Load modeling
- Problem formulation
- Solution approaches
- IEEE 9-bus results
- IEEE 39-bus results
- Next steps

Load Aggregations Modeled as Time-Varying & Uncertain Energy Storage



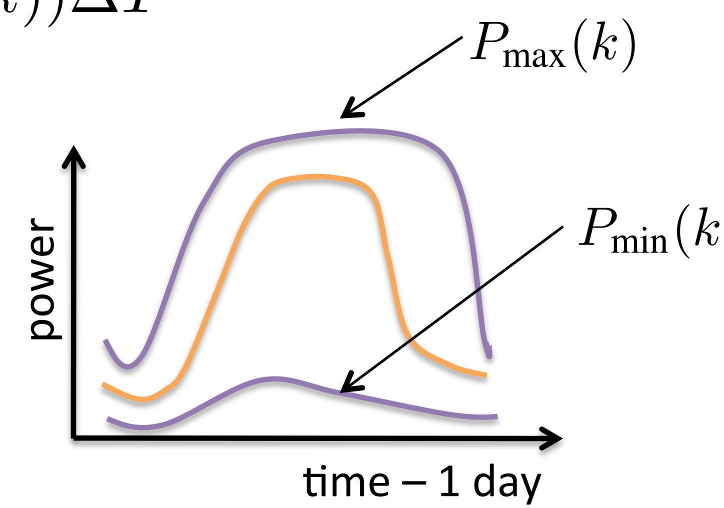
$$S(k+1) = S(k) + (P(k) - P_{\text{baseline}}(k))\Delta T$$

Mean power over an interval

$$P_{\text{min}}(k) \leqslant P(k) \leqslant P_{\text{max}}(k)$$

State of charge

$$S_{\text{min}}(k) \leqslant S(k) \leqslant S_{\text{max}}(k)$$



[Mathieu, Kamgarpour, Lygeros, Andersson, & Callaway TPWRS 2015]

Formulation: Assumptions

- DC-OPF
- Single-period problem
- Load-based reserves should be able to provide full power capacity for 15-minutes

  → Power capacity offered to market is a function of

  $P_{\min}(k)$, $P_{\max}(k)$, $S_{\min}(k)$, and $S_{\max}(k)$

Formulation: Notation

- Decision variables:
  - energy production at generators P_G
  - generators' up- and down-reserve capacities $\overline{R}_G$, $\underline{R}_G$
  - loads' up- and down-reserve capacities $\overline{R}_L, \underline{R}_L$
  - "distribution vectors" $\overline{d}_G, \underline{d}_G$ and $\overline{d}_L, \underline{d}_L$
- Other variables:
  - actual generator reserves R_G and load reserves R_L
  - real-time supply/demand mismatch P_m
- Cost parameters:
  - $c = [c_0, c_1, c_2, \overline{c}_G, \underline{c}_G, \overline{c}_L, \underline{c}_L]^\mathsf{T}$
- Given data:
  - loads forecast P_L^{f} and wind forecast P_W^{f}
  - actual wind power $\widetilde{P}_W$, actual load $\widetilde{P}_L$
  - actual minimum and maximum load $[\underline{\widetilde{P}}_L, \overline{\widetilde{P}}_L]$
  - min/max generator production $\underline{P}_G, \overline{P}_G$

Formulation: Joint and Individual Chance Constrained OPF

- **[J-CC-OPF]:**

$$\min \quad c^{\mathsf{T}}[1, P_G, P_G^2, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L] \tag{1}$$

$$\text{s.t.} \quad P_m = \sum_{i=1}^{N_W}(\widetilde{P}_{W,i} - P^f_{W,i}) - \sum_{i=1}^{N_L}(\widetilde{P}_{L,i} - P^f_{L,i}) \tag{2}$$

$$\sum_{i=1}^{N_G} \underline{d}_{G,i} + \sum_{i=1}^{N_L} \overline{d}_{L,i} = 1 \tag{3}$$

$$\sum_{i=1}^{N_G} \overline{d}_{G,i} + \sum_{i=1}^{N_L} \underline{d}_{L,i} = 1 \tag{4}$$

$$R_G = \overline{d}_G \max\{-P_m, 0\} - \underline{d}_G \max\{P_m, 0\} \tag{5}$$

$$R_L = \overline{d}_L \max\{P_m, 0\} - \underline{d}_L \max\{-P_m, 0\} \tag{6}$$

$$\mathbb{P}\left(\widetilde{A}x \geq \widetilde{b}\right) \geq 1 - \epsilon \tag{7}$$

$$x = [P_G, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L, \underline{d}_G, \overline{d}_G, \underline{d}_L, \overline{d}_L] \geq \mathbf{0}. \tag{8}$$

Formulation: Joint and Individual Chance Constrained OPF

- Constraints inside (7)

$$\begin{aligned}\widetilde{A}x \geq \widetilde{b} = \{&\underline{P}_G \leq P_G + R_G \leq \overline{P}_G,\\ &\underline{\widetilde{P}}_L \leq \widetilde{P}_L + R_L \leq \overline{\widetilde{P}}_L,\\ -&\underline{R}_G \leq R_G \leq \overline{R}_G,\\ -&\underline{R}_L \leq R_L \leq \overline{R}_L,\\ -&P_{\text{line}} \leq B_{\text{flow}} \begin{bmatrix} 0 \\ B_{\text{bus}}^{-1}\hat{P}_{\text{inj}} \end{bmatrix} \leq P_{\text{line}}\}.\end{aligned} \tag{9}$$

- **[I-CC-OPF]:**

$$\begin{aligned} \min \quad & (1) \\ \text{s.t.} \quad & (2)\text{–}(6),\ (8) \\ & \mathbb{P}\left(\widetilde{A}_i x \geq \widetilde{b}_i\right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m. \end{aligned} \tag{10}$$

Solution Approaches

- A1: Sample Average Approximation
- A2: Gaussian Approximation
- A3: Scenario Approximation
- A4: Distributionally Robust Optimization

A1: Sample Average Approximation
(solved with MILP)

- Reformulate individual chance constraints (10) $\mathbb{P}\left(\widetilde{A}_i x \geq \widetilde{b}_i\right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m$ as

$$A_i^s x \geq b_i^s - M y_s^i \ \forall s \in \Omega,\ i = 1, \ldots, m \tag{11}$$

$$\sum\nolimits_{s \in \Omega} p^s y_s^i \leq \epsilon_i,\ \forall i,\ \text{and } y_s^i \in \{0, 1\}\ \forall s,\ i, \tag{12}$$

  where M is a large scalar coefficient.

- Associate each $s \in \Omega$ with a binary logic variable y_s^i such that
  - $y_s^i = 0$ indicates that $A_i^s x \geq b_i^s$.
  - $y_s^i = 1$ indicates that $A_i^s x < b_i^s$.

A2: Gaussian Approximation

- Consider an equivalent of individual chance constraints (10) $\mathbb{P}\left(\widetilde{A}_i x \geq \widetilde{b}_i\right) \geq 1-\epsilon_i \quad i=1,\ldots,m$

$$\mathbb{P}\left(\widetilde{A}_i' \bar{x} \leq b_i'\right) \geq 1-\epsilon_i \quad i=1,\ldots,m, \tag{13}$$

- Assume the uncertainty is Gaussian distributed:

$$\widetilde{A}_i' \sim N(\mu_i, \Sigma_i).$$

Then,

$$\widetilde{A}_i' \bar{x} - b_i' \sim N(\mu_i^\top \bar{x} - b', \bar{x}^\top \Sigma_i \bar{x}).$$

- We rewrite (13) as

$$b_i' - \mu_i^\top \bar{x} \geq \Phi^{-1}(1-\epsilon_i)\sqrt{\bar{x}^\top \Sigma_i \bar{x}} \quad i=1,\ldots,m. \tag{14}$$

The above are second-order cone constraints if $\Phi^{-1}(1-\epsilon_i) \geq 0$, i.e., $1-\epsilon_i \geq 0.5$.

A3: Scenario Approximation

- Replace each chance constraint in (10)
  $\mathbb{P}\left(\widetilde{A}_i x \geq \widetilde{b}_i\right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m$ with

$$A_i^s x \geq b_i^s \ \forall s \in \Omega_{\text{ap}}. \tag{15}$$

- Both A1 and A2 require full distributional knowledge, while A3 requires large sample sizes and significant computation.

A4: Distributionally Robust (DR) Optimization

- The DR variant of (10):

$$\inf_{f(\xi)\in\mathcal{D}} \mathbb{P}_{\xi}(\widetilde{A}_i^{\xi} x \geq \widetilde{b}_i^{\xi}) \geq 1 - \epsilon_i \;\; \forall i = 1, \ldots, m. \tag{16}$$

- Given samples $\{\xi^i\}_{i=1}^N$ of ξ, we first calculate the empirical mean and covariance matrix as $\mu_0 = \frac{1}{N}\sum_{i=1}^N \xi^i$ and $\Sigma_0 = \frac{1}{N}\sum_{i=1}^N (\xi - \mu_0^i)(\xi - \mu_0^i)^{\mathsf{T}}$, and then build a confidence set

$$\mathcal{D} = \left\{ f(\xi) : \quad \begin{array}{l} \int_{\xi\in\mathcal{S}} f(\xi) d\xi = 1 \\ (\mathbb{E}[\xi] - \mu_0)^{\mathsf{T}} (\Sigma_0)^{-1} (\mathbb{E}[\xi] - \mu_0) \leq \gamma_1 \\ \mathbb{E}[(\xi - \mu_0)(\xi - \mu_0)^{\mathsf{T}}] \preceq \gamma_2 \Sigma_0 \end{array} \right\}.$$

A4: Distributionally Robust (DR) Optimization

- (Duality theory) Let r_i, $\begin{bmatrix} H_i & p_i \\ p_i^\mathsf{T} & q_i \end{bmatrix}$, and G_i be the dual variables associated with the three constraints in the above confidence set $\mathcal{D}$, respectively. The individual chance constraints (16) are equivalent to

$$\gamma_2 \Sigma_0 \cdot G_i + 1 - r_i + \Sigma_0 \cdot H_i + \gamma_1 q_i \leq \epsilon_i y_i \tag{17}$$

$$\begin{bmatrix} G_i & -p_i \\ -p_i^\mathsf{T} & 1 - r_i \end{bmatrix} \succeq \begin{bmatrix} 0 & \frac{1}{2}\bar{A}_i^x \\ \frac{1}{2}(\bar{A}_i^x)^\mathsf{T} & y_i + (\bar{A}_i^x)^\mathsf{T}\mu_0 - \bar{b}_i^x \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} G_i & -p_i \\ -p_i^\mathsf{T} & 1 - r_i \end{bmatrix} \succeq 0, \begin{bmatrix} H_i & p_i \\ p_i^\mathsf{T} & q_i \end{bmatrix} \succeq 0, y_i \geq 0,\ i = 1, \ldots, m, \tag{19}$$

where operator "$\cdot$" in constraint (17) represents Frobenius inner product of two matrices (i.e., $A \cdot B = \mathrm{tr}(A^\mathsf{T} B)$). This is a semi-definite program and can be solved by commercial solvers.

Computational experiments

- IEEE 9-bus test system
  - Added one wind farm to bus 6
  - All loads assumed partially controllable
  - Wind forecast uncertainty (modeled with real data)
  - Load control uncertainty assumed a function of temperature forecast uncertainty
- IEEE 39-bus test system
  - Similar assumptions…

Results: IEEE 9 Bus Test System

Table : Results for IEEE 9-Bus system with $1 - \epsilon_i = 95\%$

		Obj.			Rel(%)			CPU		
		avg	min	max	avg	min	max	avg	min	max
A1	J-CC-OPF	1349	1328	1363	77	8	95	2	1	4
	I-CC-OPF	1346	1336	1357	72	46	90	5876	131	32817
A2	I-CC-OPF	1349	1340	1358	82	65	94	1	1	1
A3	I-CC-OPF	1408	1371	1525	100	99	100	55	54	57
A4	I-CC-OPF	1393	1365	1458	100	98	100	5	4	6
		Cost			Performance			Computation		

A4 (empirically) requires 20 data points, A3 (theoretically) requires 900!

Results: IEEE 9 Bus Test System

Table : Results of I-CC-OPF solved by the DR approach A4

		$1-\epsilon_i =$	95.00%	90.00%	85.00%
		avg	1392.64	1369.23	1359.97
	Objective cost	min	1352.46	1346.62	1346.62
		max	1457.81	1385.24	1372.75
		avg	99.50	97.97	94.51
Individual	Reliability (%)	min	91.40	91.40	83.29
		max	99.96	99.70	99.18
		avg	6.63	6.98	6.95
	CPU seconds	min	6.13	4.73	6.27
		max	8.19	8.44	7.83

Results: IEEE 9 Bus Test System

Table : Solutions from A1–A4 of I-CC-OPF with $1 - \epsilon_i = 95\%$

	$(P_G)_1$	$(P_G)_2$	$(P_G)_3$	$(\overline{R}_G)_1$	$(\overline{R}_G)_2$	$(\overline{R}_G)_3$	$(\underline{R}_G)_1$	$(\underline{R}_G)_2$	$(\underline{R}_G)_3$	$(\overline{R}_L)_1$	$(\overline{R}_L)_2$
A1	10.00	28.84	20.94	0.00	0.00	0.00	0.00	0.00	0.00	**4.44**	**1.21**
A2	10.00	28.89	20.97	0.00	0.00	0.00	0.00	0.00	0.00	3.88	**1.88**
A3	10.03	29.32	21.27	0.03	**2.35**	0.00	0.03	**2.79**	0.00	10.49	9.73
A4	10.00	29.22	21.20	0.00	**0.25**	0.00	0.00	**0.34**	0.00	10.97	7.34

	$(\overline{R}_L)_3$	$(\underline{R}_L)_1$	$(\underline{R}_L)_2$	$(\underline{R}_L)_3$	$(d_G)_1$	$(d_G)_2$	$(d_G)_3$	$(d_L)_1$	$(d_L)_2$	$(d_L)_3$
A1	8.05	**1.86**	**0.63**	3.41	0.00	0.00	0.00	**0.32**	**0.09**	0.58
A2	9.45	2.03	**1.08**	4.21	0.00	0.00	0.00	0.25	**0.12**	0.62
A3	4.74	8.55	7.85	4.00	0.00	0.10	0.00	0.38	0.35	0.17
A4	15.17	8.46	5.68	11.59	0.00	0.01	0.00	0.32	0.21	0.46

Results: IEEE-39 Bus Test System

Table : Average performance (out of 37 Constraints) to IEEE 39-Bus system with $1 - \epsilon_i = 95\%$

	CPU seconds	Objective cost	Reliability (%)
A5	3015.98	25670.07	96.47
A2	4.10	25632.72	93.79
A3	6893.96	26129.16	99.99

A5: A hybrid between A4 (Distributionally Robust Optimization) and A2 (Gaussian Approximation)

Findings

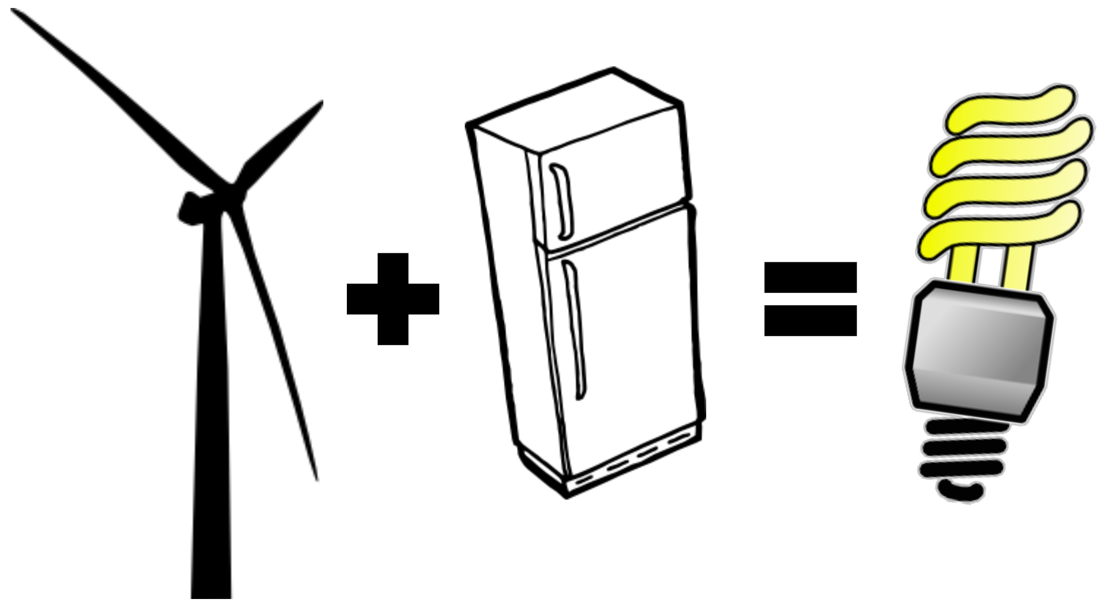
- Distributionally robust optimization provides a good trade-off…
  - Less computationally-intensive than scenario-based methods
  - Requires less data than scenario-based methods
  - Better performance than Gaussian approximation or sample average approximation (MILP formulation)

- …but the semidefinite program resulting from the IEEE 39 bus test system is already too big…

Next steps

- Apply distributionally robust optimization to the multi-period problem…
  - Solved with scenario approach: Vrakopoulou et al. HICSS 2014
  - Solved with Gaussian approximation: Li and Mathieu PowerTech 2015
- Alternatives to moment-matching
- Alternative approximation techniques

THANK YOU!   QUESTIONS?



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