Chance-constrained optimal power flow with uncertain reserves

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The Challenge

- Aggregations of electric loads can provide power systems reserves via load control
- But loads are stochastic…
  - We don't know the *future load* exactly
  - We don't know the *future load flexibility* exactly

- Two options:
  - Be conservative in how much load-based reserve we schedule (how it's done today)
  - Plan for *load control uncertainty* within the optimal power flow problem …

Planning for load control uncertainty

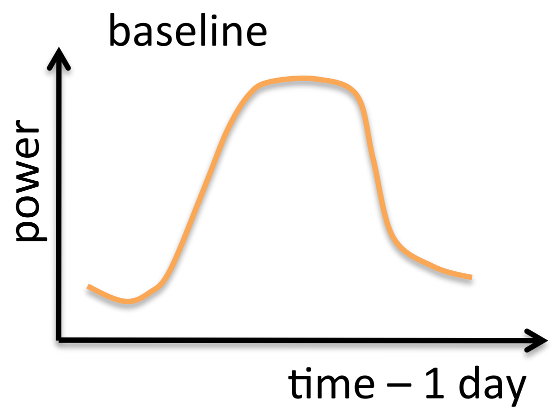
- Stochastic optimal power flow (OPF) including…
  - uncertain renewable energy production
  - uncertain load control (i.e., reserves provided by loads where feasible reserve capacities aren't known exactly)

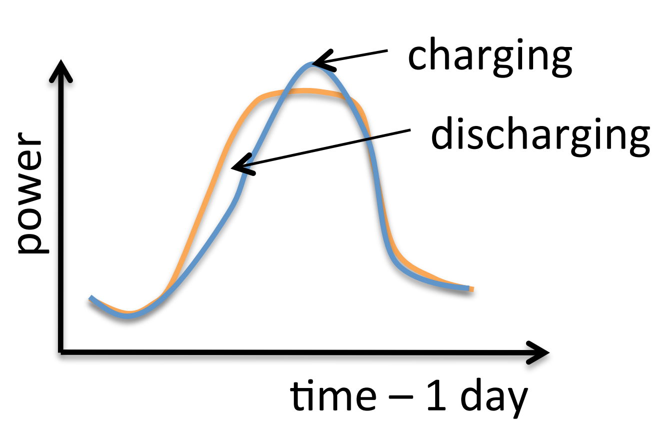
- Chance-constrained formulation…
  - Ensure constraints with stochastic variables are not violated *with certain probabilities*

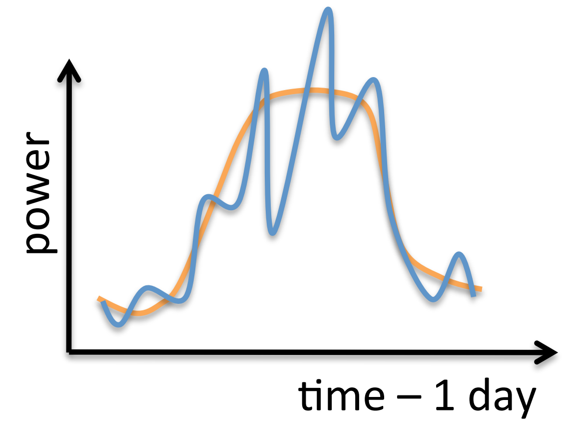
Outline

- Load modeling
- Problem formulation
- Solution approaches
- Computational results

Load Aggregations Modeled as Time-Varying & Uncertain Energy Storage







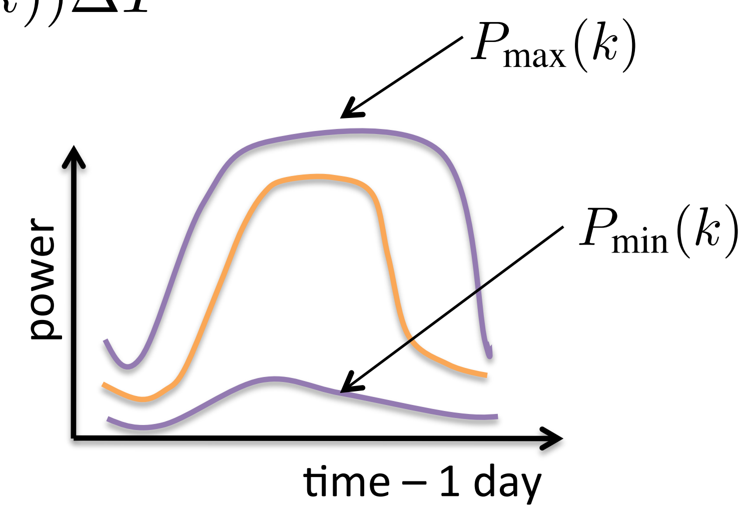
$$S(k+1) = S(k) + (P(k) - P_{\text{baseline}}(k))\Delta T$$

Mean power over an interval

$$P_{\text{min}}(k) \leqslant P(k) \leqslant P_{\text{max}}(k)$$

State of charge

$$S_{\text{min}}(k) \leqslant S(k) \leqslant S_{\text{max}}(k)$$



[Mathieu, Kamgarpour, Lygeros, Andersson, & Callaway TPWRS 2015]

Formulation: Assumptions

- DC-OPF
- Multi-period problem
  - [Vrakopoulou, Mathieu, & Andersson HICSS 2014]
  - [Li & Mathieu PowerTech 2015]
  - Energy dynamics included as constraint
  - Generator-based reserves manage the energy state of flexible loads

- Single-period problem
  - [Zhang, Shen, & Mathieu ACC 2015]
  - Load-based reserves should be able to provide full power capacity for 15-minutes

    → Power capacity offered to market is a function of

    $P_{\min}(k), P_{\max}(k), S_{\min}(k),$ and $P_{\max}(k)$

Formulation: Notation

- Decision variables
  - Generator energy production, P_G
  - Generator up- and down reserve capacity, $\overline{R}_G$, $\underline{R}_G$
  - Load up- and down reserve capacity, $\overline{R}_L$, $\underline{R}_L$
  - Distribution vectors, $\overline{d}_G$, $\underline{d}_G$, $\overline{d}_L$, $\underline{d}_L$
- Random variables
  - Wind power production, $\tilde{P}_W$
  - Load, $\tilde{P}_L$
  - Maximum and minimum load, $\overline{\tilde{P}}_L$, $\underline{\tilde{P}}_L$

Formulation:
Joint Chance Constrained OPF

$$\min \quad c^{\mathsf{T}}[1, P_G, P_G^2, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L]$$ ← Energy+Reserve Cost

$$\text{s.t.} \quad P_m = \sum_{i=1}^{N_W} (\widetilde{P}_{W,i} - P_{W,i}^f) - \sum_{i=1}^{N_L} (\widetilde{P}_{L,i} - P_{L,i}^f)$$ ← Power mismatch

$$\sum_{i=1}^{N_G} P_{G,i} = \sum_{i=1}^{N_L} P_{L,i}^f - \sum_{i=1}^{N_W} P_{W,i}^f$$ ← Power balance

$$\sum_{i=1}^{N_G} \underline{d}_{G,i} + \sum_{i=1}^{N_L} \overline{d}_{L,i} = 1$$

$$\sum_{i=1}^{N_G} \overline{d}_{G,i} + \sum_{i=1}^{N_L} \underline{d}_{L,i} = 1$$

← Distribution vectors

$$R_G = \overline{d}_G \max\{-P_m, 0\} - \underline{d}_G \max\{P_m, 0\}$$

$$R_L = \overline{d}_L \max\{P_m, 0\} - \underline{d}_L \max\{-P_m, 0\}$$

← Reserves

$$\mathbb{P}\left(\widetilde{A}x \geq \widetilde{b}\right) \geq 1 - \epsilon$$ ← Chance constraints

$$x = [P_G, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L, \underline{d}_G, \overline{d}_G, \underline{d}_L, \overline{d}_L] \geq \mathbf{0},$$ ← Decision vector

Formulation: Joint Chance Constrained OPF

Expanding the chance constraints…

$$
\begin{aligned}
\widetilde{A}x \geq \widetilde{b} = \{&\underline{P}_G \leq P_G + R_G \leq \overline{P}_G, && \leftarrow \text{Generation limits}\\
&\underline{\widetilde{P}}_L \leq \widetilde{P}_L + R_L \leq \overline{\widetilde{P}}_L, && \leftarrow \text{Load limits}\\
&\left.\begin{aligned}-\underline{R}_G \leq R_G \leq \overline{R}_G,\\ -\underline{R}_L \leq R_L \leq \overline{R}_L,\end{aligned}\right\} && \leftarrow \text{Reserve limits}\\
&-P_{\text{line}} \leq B_{\text{flow}} \begin{bmatrix} 0 \\ B_{\text{bus}}^{-1}\hat{P}_{\text{inj}} \end{bmatrix} \leq P_{\text{line}}\}. && \leftarrow \text{DC Power Flow}
\end{aligned}
$$

Formulation: Individual Chance Constrained OPF

$$
\begin{aligned}
\min \quad & c^{\mathsf{T}}[1, P_G, P_G^2, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L] \\
\text{s.t.} \quad & P_m = \sum_{i=1}^{N_W} (\widetilde{P}_{W,i} - P_{W,i}^f) - \sum_{i=1}^{N_L} (\widetilde{P}_{L,i} - P_{L,i}^f) \\
& \sum_{i=1}^{N_G} \underline{d}_{G,i} + \sum_{i=1}^{N_L} \overline{d}_{L,i} = 1 \\
& \sum_{i=1}^{N_G} \overline{d}_{G,i} + \sum_{i=1}^{N_L} \underline{d}_{L,i} = 1 \\
& R_G = \overline{d}_G \max\{-P_m, 0\} - \underline{d}_G \max\{P_m, 0\} \\
& R_L = \overline{d}_L \max\{P_m, 0\} - \underline{d}_L \max\{-P_m, 0\} \\
& \mathbb{P}\left(\widetilde{A}_i x \geq \widetilde{b}_i\right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m. \\
& x = [P_G, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L, \underline{d}_G, \overline{d}_G, \underline{d}_L, \overline{d}_L] \geq \mathbf{0}.
\end{aligned}
$$

← Chance constraints

Solution Approaches

- A1: Sample Average Approximation
  - [Luedtke and Ahmed SIAM Opt 2008]
- A2: Gaussian Approximation
  - Used by [Roald et al. PowerTech 2013] and [Bienstock et al. SIAM Review 2014] for OPF with uncertain wind
  - Used by [Li and Mathieu PowerTech 2015] for OPF with uncertain wind *and load control*
- A3: Scenario Approximation
  - [Calafiore and Campi TAC 2006]
  - Used by [Vrakopoulou et al. TPWRS 2013] for OPF with uncertain wind
  - A variant based on [Margellos et al. TAC 2014] used by [Vrakopoulou et al. HICSS 2014] for OPF with uncertain wind *and load control*
- A4: Distributionally Robust Optimization
  - [Delage and Ye OR 2010]
  - Used by [Zhang et al. ACC 2015] for OPF with uncertain wind *and load control*

A1: Sample Average Approximation

- Reformulate individual chance constraints as

$$A_i^s x \geq b_i^s - M y_s^i \;\forall s \in \Omega,\; i = 1, \ldots, m$$
$$\sum\nolimits_{s \in \Omega} p^s y_s^i \leq \epsilon_i,\; \forall i,\; \text{and } y_s^i \in \{0,1\} \;\forall s,\; i,$$

  where M is a large number and each sample s is associated with a binary logic variable y_s
- This is a mixed integer quadratic program (MIQP).

A2: Gaussian Approximation

- Re-write the individual chance constraint

$$\mathbb{P}\left(\widetilde{A}_i'\bar{x} \le b_i'\right) \ge 1 - \epsilon_i \quad i = 1, \ldots, m,$$

- Assume the uncertainty is Gaussian

$$\widetilde{A}_i' \sim N(\mu_i, \Sigma_i).$$

- Then,

$$\widetilde{A}_i'\bar{x} - b_i' \sim N(\mu_i^\mathsf{T}\bar{x} - b', \bar{x}^\mathsf{T}\Sigma_i\bar{x}).$$

and the constraint can be rewritten as

$$b_i' - \mu_i^\mathsf{T}\bar{x} \ge \Phi^{-1}(1 - \epsilon_i)\sqrt{\bar{x}^\mathsf{T}\Sigma_i\bar{x}} \quad i = 1, \ldots, m.$$

- This is a second-order cone program (SOCP) if the probability of constraint violation is less than 50%.

A3: Scenario Approximation

- Replace each chance constraint with

$$A_i^s x \geq b_i^s \;\; \forall s \in \Omega_{\mathrm{ap}}.$$

- Use at least $\frac{2}{\varepsilon}\left(\ln\left(\frac{1}{\beta}\right)+n\right)$ samples to guarantee performance [Califiore and Campi TAC 2006], where ε is the probability of constraint violation, 1-β is the confidence level, n is the dimension of x.
- This is a quadratic program (QP).

A4: Distributionally Robust Optimization

- The distributionally robust variant of the individual chance constraint is

$$\inf_{f(\xi)\in\mathcal{D}} \mathbb{P}_\xi(\widetilde{A}_i^\xi x \geq \widetilde{b}_i^\xi) \geq 1-\epsilon_i \ \forall i = 1,\ldots,m.$$

- Given samples of the uncertainty, calculate the empirical mean μ_0 and covariance Σ_0, and build a confidence set

$$\mathcal{D} = \left\{ f(\xi) : \quad \begin{array}{l} \int_{\xi\in\mathcal{S}} f(\xi)d\xi = 1 \\ (\mathbb{E}[\xi]-\mu_0)^\mathsf{T}(\Sigma_0)^{-1}(\mathbb{E}[\xi]-\mu_0) \leq \gamma_1 \\ \mathbb{E}[(\xi-\mu_0)(\xi-\mu_0)^\mathsf{T}] \preceq \gamma_2\Sigma_0 \end{array} \right\}.$$

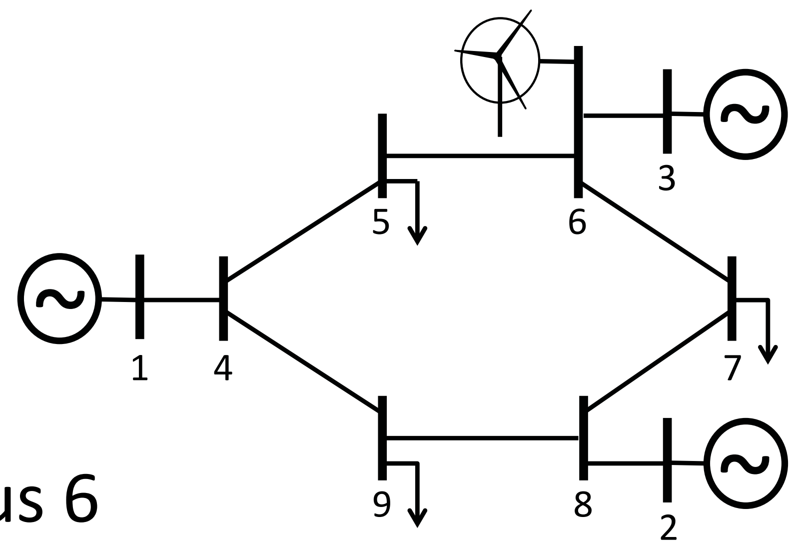
A4: Distributionally Robust Optimization

- Let $r_i, \begin{bmatrix} H_i & p_i \\ p_i^\top & q_i \end{bmatrix}$, and G_i be the dual variables associated with the three constraints within the confidence set.
- The chance constraints are equivalent to

$$\gamma_2 \Sigma_0 \cdot G_i + 1 - r_i + \Sigma_0 \cdot H_i + \gamma_1 q_i \leq \epsilon_i y_i$$
$$\begin{bmatrix} G_i & -p_i \\ -p_i^\top & 1 - r_i \end{bmatrix} \succeq \begin{bmatrix} 0 & \frac{1}{2}\bar{A}_i^x \\ \frac{1}{2}(\bar{A}_i^x)^\top & y_i + (\bar{A}_i^x)^\top \mu_0 - \bar{b}_i^x \end{bmatrix}$$
$$\begin{bmatrix} G_i & -p_i \\ -p_i^\top & 1 - r_i \end{bmatrix} \succeq 0, \begin{bmatrix} H_i & p_i \\ p_i^\top & q_i \end{bmatrix} \succeq 0, y_i \geq 0, \; i = 1, \ldots, m,$$

resulting in a semi-definite program (SDP).

Computational experiments



- IEEE 9-bus test system
  - Added one wind farm to bus 6
  - All loads assumed partially controllable
  - Wind forecast uncertainty (modeled with real data)
  - Load control uncertainty assumed a function of temperature forecast uncertainty

Computational Results: Comparison

Desired probability of constraint violation: 5% (95% Reliability)

	Cost			Performance			Computation		
	avg	min	max	avg	min	max	avg	min	max
SAA: Joint	4120	4112	4129	83.46	62.52	92.86	0.49	0.27	0.64
SAA: Individ.	4119	4111	4124	81.37	60.60	91.92	1981	731	5472
Gaussian	4123	4116	4128	88.70	74.92	95.22	0.03	0.02	0.05
Scenario	4173	4143	4229	99.69	99.42	99.88	17.06	16.82	17.38
Dist. Robust	4162	4141	4177	99.63	99.06	99.79	0.42	0.37	0.47

Distributionally robust (empirically) requires 20 data points;
the scenario approach (theoretically) requires 900!

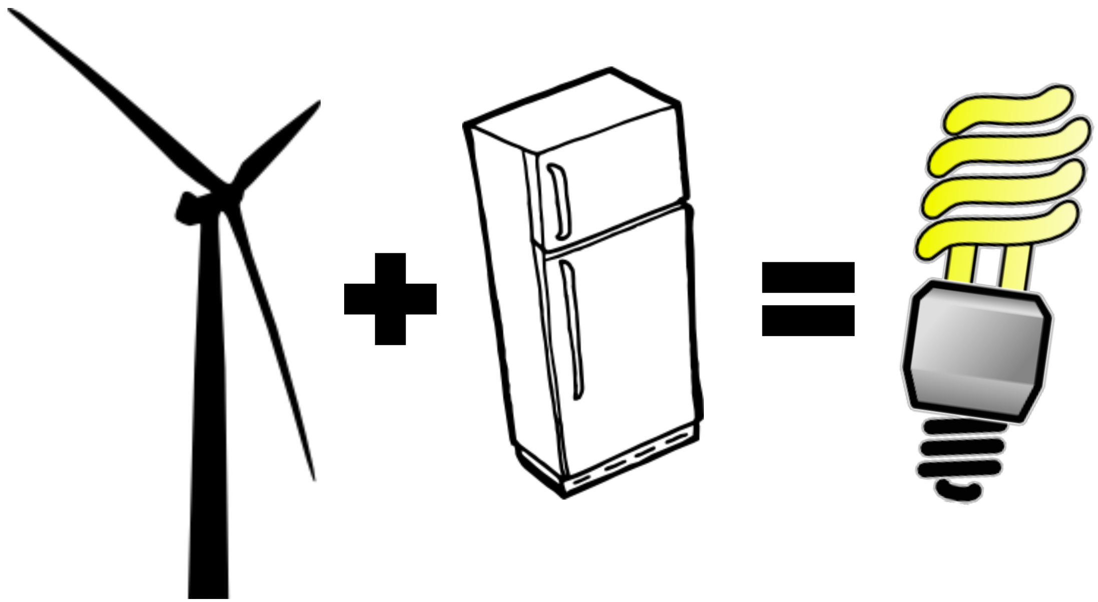
Computational Results: Distributionally Robust Optimization

		$1-\epsilon_i =$	95.00%	90.00%	85.00%
Individual	Objective cost	average	4161.91	4141.90	4133.24
		max	4177.33	4151.37	4140.76
		min	4141.33	4128.44	4122.56
	Reliability (%)	average	99.63	98.95	97.01
		max	99.79	99.73	99.28
		min	99.06	95.54	89.60
	CPU seconds	average	0.42	0.41	0.41
		max	0.47	0.50	0.45
		min	0.37	0.37	0.37

Findings & Conclusions

- Distributionally robust optimization provides a good trade-off…
  - Less computationally-intensive than scenario-based methods
  - Requires less data than scenario-based methods
  - Better performance than Gaussian approximation or sample average approximation
- …but the semidefinite program doesn't scale very well to larger systems.
- Next steps: more realistic problem formulation, development of scalable approximations.

THANK YOU!   QUESTIONS?



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