Chance-constrained optimal power flow with uncertain reserves

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The Challenge

• Aggregations of electric loads can provide power systems reserves via load control

• But loads are stochastic...
  – We don’t know the *future load* exactly
  – We don’t know the *future load flexibility* exactly

• Two options:
  – Be conservative in how much load-based reserve we schedule (how it’s done today)
  – Plan for *load control uncertainty* within the optimal power flow problem ...
Planning for load control uncertainty

- Stochastic optimal power flow (OPF) including...
  - uncertain renewable energy production
  - uncertain load control (i.e., reserves provided by loads where feasible reserve capacities aren’t known exactly)

- Chance-constrained formulation...
  - Ensure constraints with stochastic variables are not violated \textit{with certain probabilities}
Outline

• Load modeling
• Problem formulation
• Solution approaches
• Computational results
Load Aggregations Modeled as Time-Varying & Uncertain Energy Storage

\[ S(k+1) = S(k) + (P(k) - P_{baseline}(k)) \Delta T \]

Mean power over an interval
\[ P_{min}(k) \leq P(k) \leq P_{max}(k) \]

State of charge
\[ S_{min}(k) \leq S(k) \leq S_{max}(k) \]

[Mathieu, Kamgarpour, Lygeros, Andersson, & Callaway TPWRS 2015]
Formulation: Assumptions

• DC-OPF
• Multi-period problem
  – [Vrakopoulou, Mathieu, & Andersson HICSS 2014]
  – [Li & Mathieu PowerTech 2015]
  – Energy dynamics included as constraint
  – Generator-based reserves manage the energy state of flexible loads

• Single-period problem
  – [Zhang, Shen, & Mathieu ACC 2015]
  – Load-based reserves should be able to provide full power capacity for 15-minutes
    → Power capacity offered to market is a function of
      \[ P_{\text{min}}(k), P_{\text{max}}(k), S_{\text{min}}(k), \text{ and } P_{\text{max}}(k) \]
Formulation: Notation

• Decision variables
  – Generator energy production, $P_G$
  – Generator up- and down reserve capacity, $\bar{R}_G, R_G$
  – Load up- and down reserve capacity, $\bar{R}_L, R_L$
  – Distribution vectors, $\bar{d}_G, d_G, \bar{d}_L, d_L$

• Random variables
  – Wind power production, $\tilde{P}_W$
  – Load, $\tilde{P}_L$
  – Maximum and minimum load, $\tilde{P}_L, \tilde{P}_L$
Formulation: Joint Chance Constrained OPF

\[ \begin{align*}
\text{min} & \quad c^T [1, P_G, P_G^2, R_G, R_G, R_L, R_L] \\
\text{s.t.} & \quad P_m = \sum_{i=1}^{N_W} (\tilde{P}_{W,i} - P_{W,i}^f) - \sum_{i=1}^{N_L} (\tilde{P}_{L,i} - P_{L,i}^f) \\
& \quad \sum_{i=1}^{N_G} P_{G,i} = \sum_{i=1}^{N_L} P_{L,i}^f - \sum_{i=1}^{N_W} P_{W,i}^f \\
& \quad \sum_{i=1}^{N_G} d_{G,i} + \sum_{i=1}^{N_L} \bar{d}_{L,i} = 1 \\
& \quad \sum_{i=1}^{N_G} \bar{d}_{G,i} + \sum_{i=1}^{N_L} d_{L,i} = 1 \\
& \quad R_G = \bar{d}_G \max\{-P_m, 0\} - d_G \max\{P_m, 0\} \\
& \quad R_L = \bar{d}_L \max\{P_m, 0\} - d_L \max\{-P_m, 0\} \\
& \quad \mathbb{P} \left( \tilde{A} x \geq \tilde{b} \right) \geq 1 - \epsilon \\
x & = [P_G, R_G, R_G, R_L, R_L, d_G, \bar{d}_G, d_L, \bar{d}_L] \geq 0,
\end{align*} \]

Energy+Reserve Cost
Power mismatch
Power balance
Distribution vectors
Reserves
Chance constraints
Decision vector
Formulation: Joint Chance Constrained OPF

Expanding the chance constraints...

\[ \tilde{A} x \geq \tilde{b} = \{ P_G \leq P_G + R_G \leq \bar{P}_G, \]
\[ \tilde{P}_L \leq \tilde{P}_L + R_L \leq \bar{P}_L, \]
\[ -R_G \leq R_G \leq \bar{R}_G, \]
\[ -R_L \leq R_L \leq \bar{R}_L, \]
\[ -P_{\text{line}} \leq B_{\text{flow}} \begin{bmatrix} 0 \\ B_{\text{bus}}^{-1} \hat{P}_{\text{inj}} \end{bmatrix} \leq P_{\text{line}} \}. \]

- Generation limits
- Load limits
- Reserve limits
- DC Power Flow
Joint and Individual CC-OPF Models

**Formula:**

**Individual Chance Constrained OPF**

\[
\min \quad c^T [1, P_G, P_G^2, R_G, R_G, R_L, R_L] \\
\text{s.t.} \quad P_m = \sum_{i=1}^{NW} (\bar{P}_W,i - P_{W,i}^f) - \sum_{i=1}^{NL} (\bar{P}_L,i - P_{L,i}^f) \\
\sum_{i=1}^{NG} d_{G,i} + \sum_{i=1}^{NL} \bar{d}_{L,i} = 1 \\
\sum_{i=1}^{NG} \bar{d}_{G,i} + \sum_{i=1}^{NL} d_{L,i} = 1 \\
R_G = \bar{d}_G \max\{-P_m, 0\} - \bar{d}_G \max\{P_m, 0\} \\
R_L = \bar{d}_L \max\{P_m, 0\} - \bar{d}_L \max\{-P_m, 0\} \\
\mathbb{P}\left(\tilde{A}_i x \geq \tilde{b}_i\right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m. \\
x = [P_G, R_G, R_G, R_L, R_L, \bar{d}_G, \bar{d}_G, d_L, \bar{d}_L, \bar{d}_L] \geq 0.
\]

**Chance constraints**
Solution Approaches

• A1: Sample Average Approximation
  – [Luedtke and Ahmed SIAM Opt 2008]

• A2: Gaussian Approximation
  – Used by [Li and Mathieu PowerTech 2015] for OPF with uncertain wind and load control

• A3: Scenario Approximation
  – [Calafiore and Campi TAC 2006]
  – Used by [Vrakopoulou et al. TPWRS 2013] for OPF with uncertain wind
  – A variant based on [Margellos et al. TAC 2014] used by [Vrakopoulou et al. HICSS 2014] for OPF with uncertain wind and load control

• A4: Distributionally Robust Optimization
  – [Delage and Ye OR 2010]
  – Used by [Zhang et al. ACC 2015] for OPF with uncertain wind and load control
A1: Sample Average Approximation

• Reformulate individual chance constraints as

\[ A^s_i x \geq b^s_i - M y^i_s \quad \forall s \in \Omega, \ i = 1, \ldots, m \]

\[ \sum_{s \in \Omega} p^s y^i_s \leq \epsilon_i, \ \forall i, \ \text{and} \ y^i_s \in \{0, 1\} \ \forall s, \ i, \]

where \( M \) is a large number and each sample \( s \) is associated with a binary logic variable \( y^i_s \).

• This is a mixed integer quadratic program (MIQP).
A2: Gaussian Approximation

- Re-write the individual chance constraint

\[ \mathbb{P} \left( \tilde{A}_i^T \bar{x} \leq b_i' \right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m, \]

- Assume the uncertainty is Gaussian

\[ \tilde{A}_i \sim N(\mu_i, \Sigma_i). \]

- Then,

\[ \tilde{A}_i^T \bar{x} - b_i' \sim N(\mu_i^T \bar{x} - b', \bar{x}^T \Sigma_i \bar{x}). \]

and the constraint can be rewritten as

\[ b_i' - \mu_i^T \bar{x} \geq \Phi^{-1}(1 - \epsilon_i) \sqrt{\bar{x}^T \Sigma_i \bar{x}} \quad i = 1, \ldots, m. \]

- This is a second-order cone program (SOCP) if the probability of constraint violation is less than 50%.
A3: Scenario Approximation

- Replace each chance constraint with

$$A_i^s x \geq b_i^s \forall s \in \Omega_{ap}.$$ 

- Use at least \( \frac{2}{\epsilon} \left( \ln \left( \frac{1}{\beta} \right) + n \right) \) samples to guarantee performance [Calafiore and Campi TAC 2006], where \( \epsilon \) is the probability of constraint violation, \( 1 - \beta \) is the confidence level, \( n \) is the dimension of \( x \).

- This is a quadratic program (QP).
A4: Distributionally Robust Optimization

- The distributionally robust variant of the individual chance constraint is
  \[
  \inf_{f(\xi) \in D} \mathbb{P}_{\xi}(\tilde{A}_i^\xi x \geq \tilde{b}_i^\xi) \geq 1 - \epsilon_i \quad \forall i = 1, \ldots, m.
  \]

- Given samples of the uncertainty, calculate the empirical mean \( \mu_0 \) and covariance \( \Sigma_0 \), and build a confidence set
  \[
  D = \left\{ f(\xi) : \int_{\xi \in S} f(\xi) d\xi = 1 \right. \\
  \left. \begin{align*}
  (\mathbb{E}[\xi] - \mu_0)^T (\Sigma_0)^{-1} (\mathbb{E}[\xi] - \mu_0) &\leq \gamma_1 \\
  \mathbb{E}[(\xi - \mu_0)(\xi - \mu_0)^T] &\leq \gamma_2 \Sigma_0
  \end{align*} \right\}.
  \]
A4: Distributionally Robust Optimization

- Let \( r_i, \begin{bmatrix} H_i & p_i \\ p_i^T & q_i \end{bmatrix}, \) and \( G_i \) be the dual variables associated with the three constraints within the confidence set.

- The chance constraints are equivalent to

\[
\begin{align*}
\gamma_2 \Sigma_0 \cdot G_i + 1 - r_i + \Sigma_0 \cdot H_i + \gamma_1 q_i & \leq \epsilon_i y_i \\
\begin{bmatrix} G_i & -p_i \\ -p_i^T & 1 - r_i \end{bmatrix} & \succeq \begin{bmatrix} 0 \\ \frac{1}{2} (\bar{A}_i^x)^T y_i + (\bar{A}_i^x)^T \mu_0 - \bar{b}_i^x \end{bmatrix} \\
\begin{bmatrix} G_i & -p_i \\ -p_i^T & 1 - r_i \end{bmatrix} & \succeq 0, \begin{bmatrix} H_i & p_i \\ p_i^T & q_i \end{bmatrix} \succeq 0, y_i \geq 0, i = 1, \ldots, m,
\end{align*}
\]

resulting in a semi-definite program (SDP).
Computational experiments

• IEEE 9-bus test system
  – Added one wind farm to bus 6
  – All loads assumed partially controllable
  – Wind forecast uncertainty (modeled with real data)
  – Load control uncertainty assumed a function of temperature forecast uncertainty
Computational Results: Comparison

Desired probability of constraint violation: 5% (95% Reliability)

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<th>Cost</th>
<th>Performance</th>
<th>Computation</th>
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Distributionally robust (empirically) requires 20 data points; the scenario approach (theoretically) requires 900!
Computational Results: Distributionally Robust Optimization

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<tr>
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<th>$1 - \epsilon_i = 95.00%$</th>
<th>$90.00%$</th>
<th>$85.00%$</th>
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<td><strong>Individual Reliability (%)</strong></td>
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<tr>
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Findings & Conclusions

• Distributionally robust optimization provides a good trade-off...
  – Less computationally-intensive than scenario-based methods
  – Requires less data than scenario-based methods
  – Better performance than Gaussian approximation or sample average approximation

• ...but the semidefinite program doesn’t scale very well to larger systems.

• Next steps: more realistic problem formulation, development of scalable approximations.
THANK YOU! QUESTIONS?

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