Optimal Power Flow with Stochastic Reserves

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Outline

- What are “stochastic reserves”?
- Chance–constrained optimal power flow with stochastic reserves
- Exploring conventional solution approaches
- Distributionally robust optimization (DRO)
- Making DRO less conservative
- Concluding remarks

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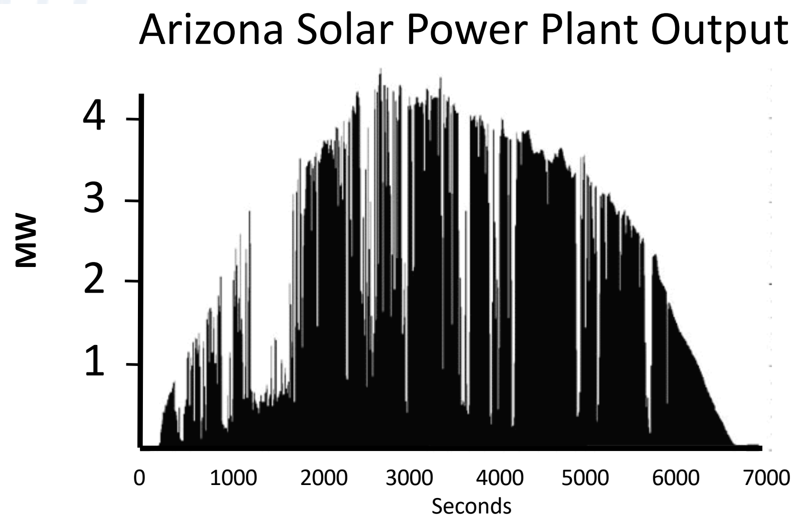
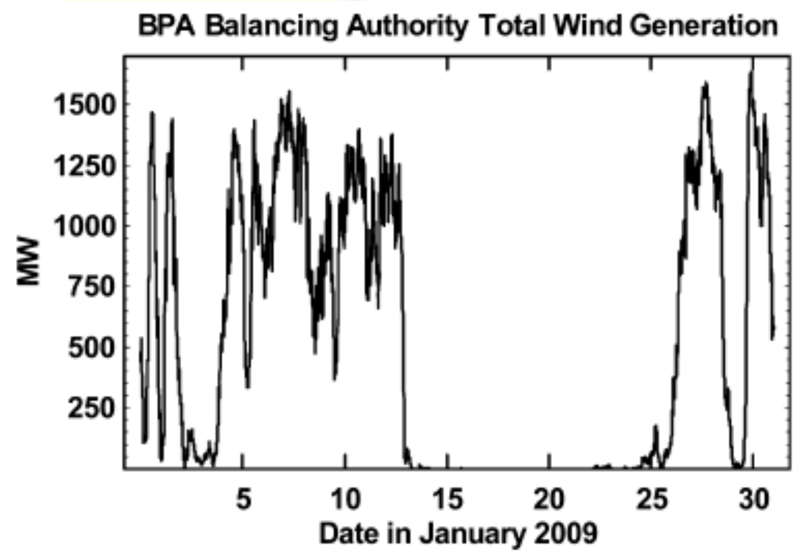
Increased need for balancing reserves

- Intermittent renewable energy generation
- Timescales of seconds to hours



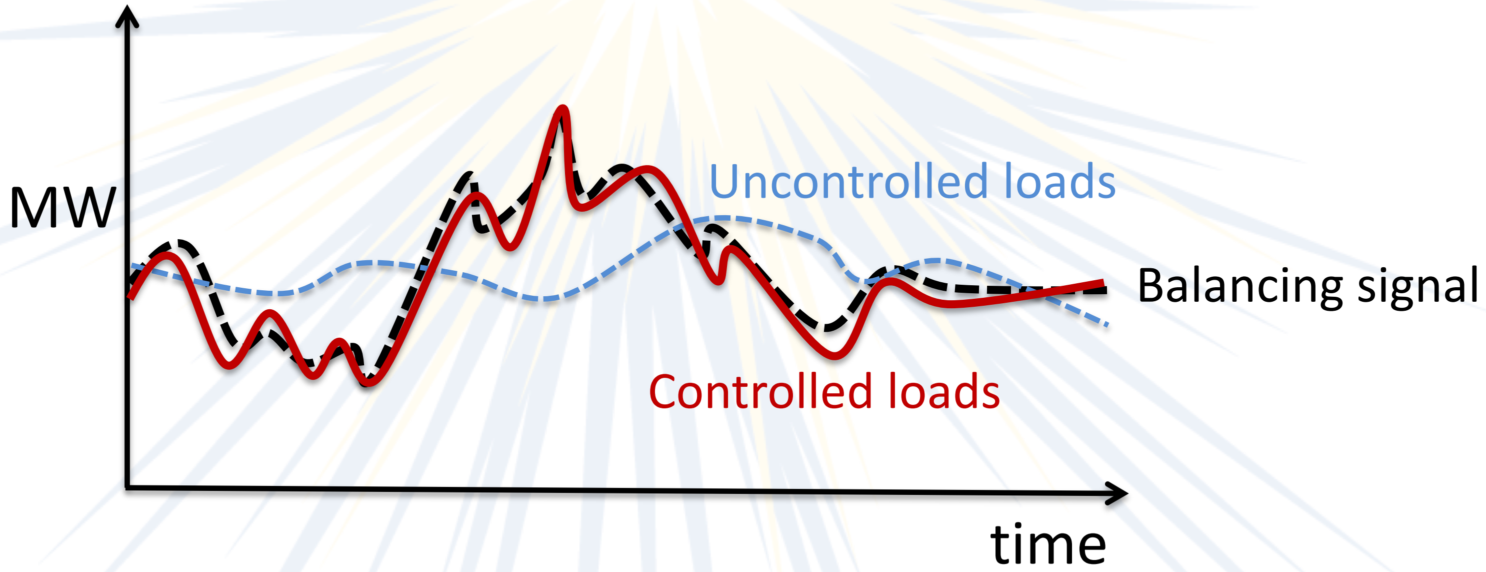
[Apt and Curtright 2007]



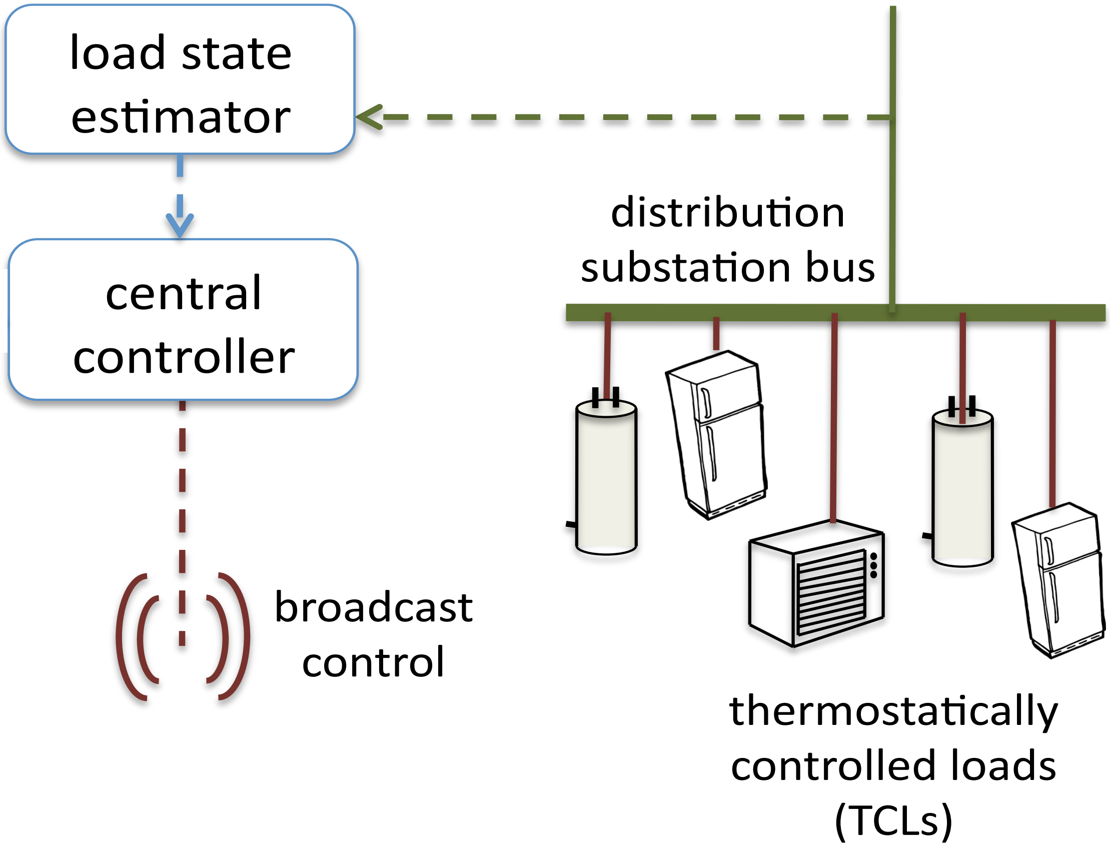
[Katzenstein and Apt 2009]

Loads (and Distributed Storage) Providing Reserves

- Thousands of small (a few kW) loads collectively tracking a balancing signal
- Energy consumption is unchanged

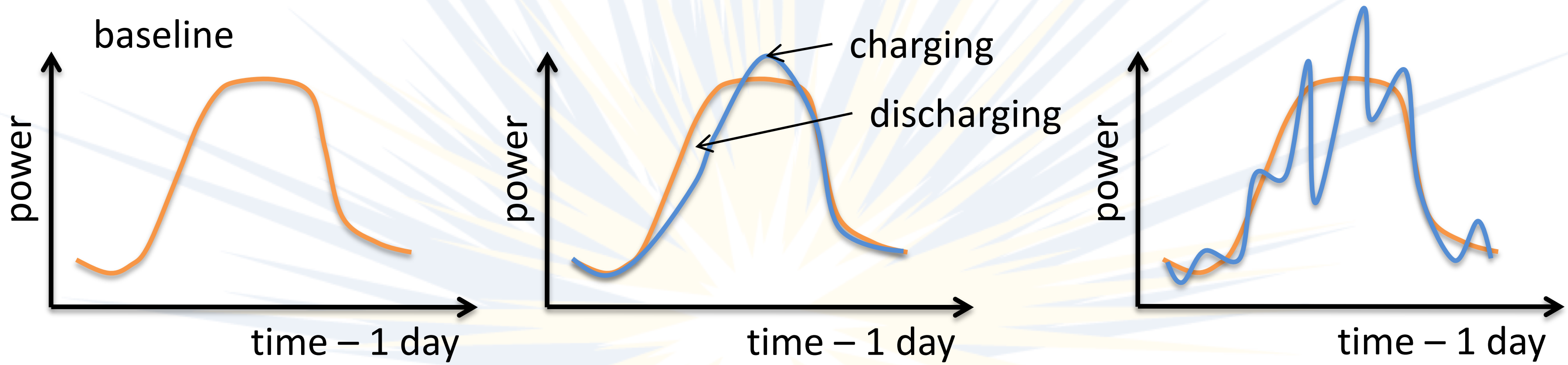


Load Coordination



[Mathieu, Koch, Callaway IEEE TPWRS 2013]
Many approaches: [Lu and Chassin 2005; Zhang et al. 2013; Bashash and Fathy 2013; Meyn et al 2015; Dall'Anese et al. 2018; Almassalkhi et al. 2018; Busic et al. 2018; and many many others]

Modeling Load Aggregations as Storage



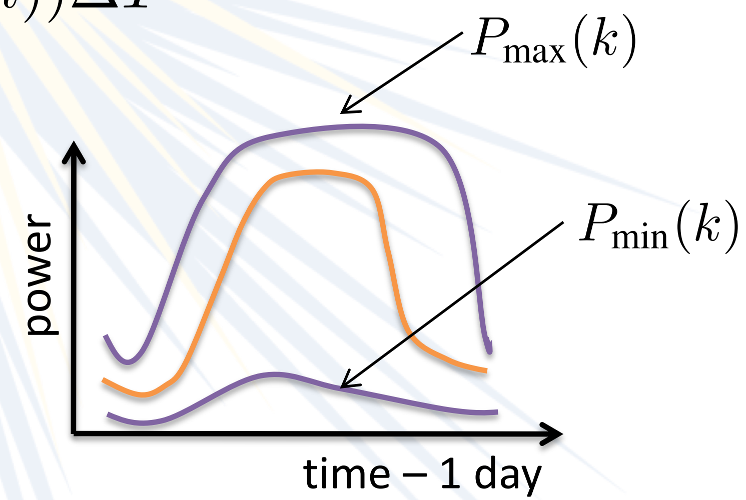
$$S(k+1) = S(k) + (P(k) - P_{\text{baseline}}(k))\Delta T$$

Mean power over an interval

$$P_{\text{min}}(k) \leqslant P(k) \leqslant P_{\text{max}}(k)$$

State of charge

$$S_{\text{min}}(k) \leqslant S(k) \leqslant S_{\text{max}}(k)$$

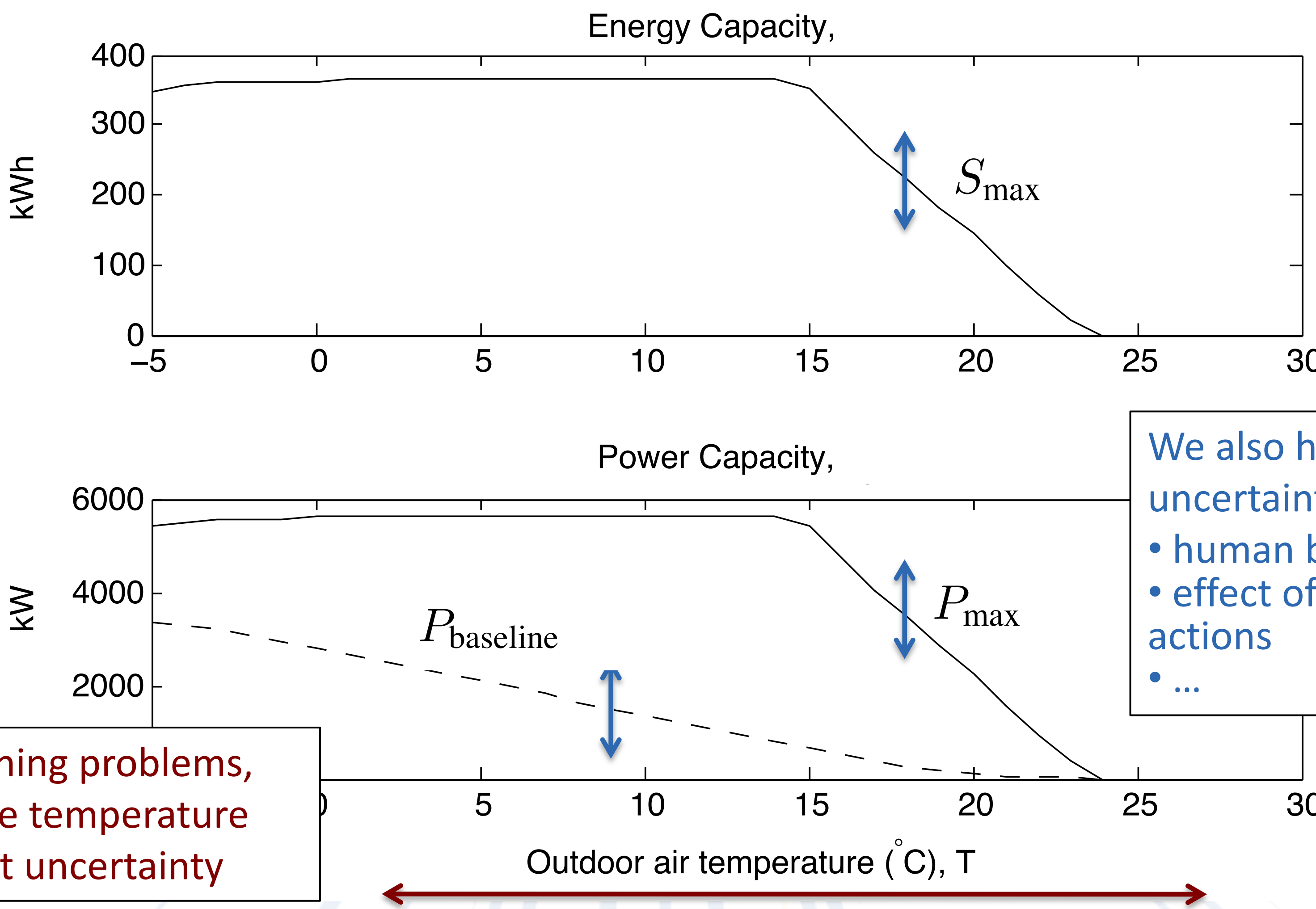


Storage Models

- [Mathieu, Dyson, Callaway ACEEE 2012; Mathieu, Kamgarpour, Lygeros, Andersson, Callaway IEEE TPWRS 2015]
- Other approaches: [Hao et al. 2015; Sanandaji et al. 2014; Trovato et al. 2016; and many others]

- The models are imperfect.
- Storage capacities aren't perfectly known.
- Storage capacities can vary over time as a function of stochastic phenomena.
- Imperfect control responses.

Example: Aggregation of 1000 electric space heaters



We also have uncertainty in
- human behavior
- effect of past DR actions
- …

In planning problems, we have temperature forecast uncertainty

How should we accommodate stochastic reserves?

- The load aggregator manages the uncertainty
- The system operator manages the uncertainty
- A hybrid approach

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Chance-Constrained Optimal Power Flow

- Many approaches: [Zhang and Li 2011; Jabr 2013; Vrakopoulou et al. 2013; Roald et al. 2013; Bienstock et al. 2014; and many others]

- Wind power uncertainty (assume 2 wind farms)
- DC Power Flow Equations
- Generators use an affine control policy to compensate wind power forecast error

$$R = -d(W_1 + W_2)$$

- Decision variables are generator power outputs, reserve capacities, and participation factors

Chance-Constrained Optimal Power Flow

$$\min \; P_G^T[C_1]P_G + C_2^T P_G + C_R^T(R^{up} + R^{dn})$$ Generation and reserve cost

$$\text{s.t.} \; -P_l \le A_s P_{\text{inj}} \le P_l$$ Bound power flows

$$P_{\text{inj}} = C_G(P_G + R) + C_W(P_W^f + \widetilde{W}) - C_L P_L$$ Compute power injections

$$R = -d(W_1 + W_2)$$

$$\underline{P}_G \le P_G + R \le \overline{P}_G$$

$$-R^{dn} \le R \le R^{up}$$ Bound reserve capacity

$$\mathbb{P}_\xi(f(x,\xi) \le 0) \ge 1 - \epsilon$$

$$\mathbf{1}_{1\times N_G} d = 1$$ Normalize participation factors

$$\mathbf{1}_{1\times N_B}(C_G P_G + C_W P_W^f - C_L P_L) = 0$$ Schedule generation to meet forecast

$$P_G \ge \mathbf{0}_{N_G\times 1}, \; d \ge \mathbf{0}_{N_G\times 1}$$

$$R^{up} \ge \mathbf{0}_{N_G\times 1}, \; R^{dn} \ge \mathbf{0}_{N_G\times 1}$$

where $\widetilde{W} = [W_1, W_2]^T$

Similar to [Vrakopoulou, Margellos, Lygeros, Andersson 2013; Bienstock, Chertkov, Harnett 2014].

Incorporating Stochastic Reserves

- Single period problem [Zhang, Shen, Mathieu TPWRS 2017]
  - New decision variables: flexible load power consumption, reserve capacities, and participation factors
  - Modify the constraints: load-based reserve actions affect power injections and power flows
  - Add constraints: Load consumption bounds based on the (uncertain) power capacity and (uncertain) load forecast

$$\underline{\widetilde{P}}_L \leq \widetilde{P}_L + R_L \leq \overline{\widetilde{P}}_L$$

Multiperiod Problem

- Two-Part Paper in IEEE TPWRS: [Vrakopoulou, Li, Mathieu 2019; Li, Vrakopoulou, Mathieu 2019]
- Add energy capacity constraints
- Reserves provided by aggregations of thermostatically controlled loads where baseline power consumption, energy and power capacities are a function of outdoor temperature
- Assume wind and temperature forecast error
- Reserve designs to mitigate uncertainty propagation

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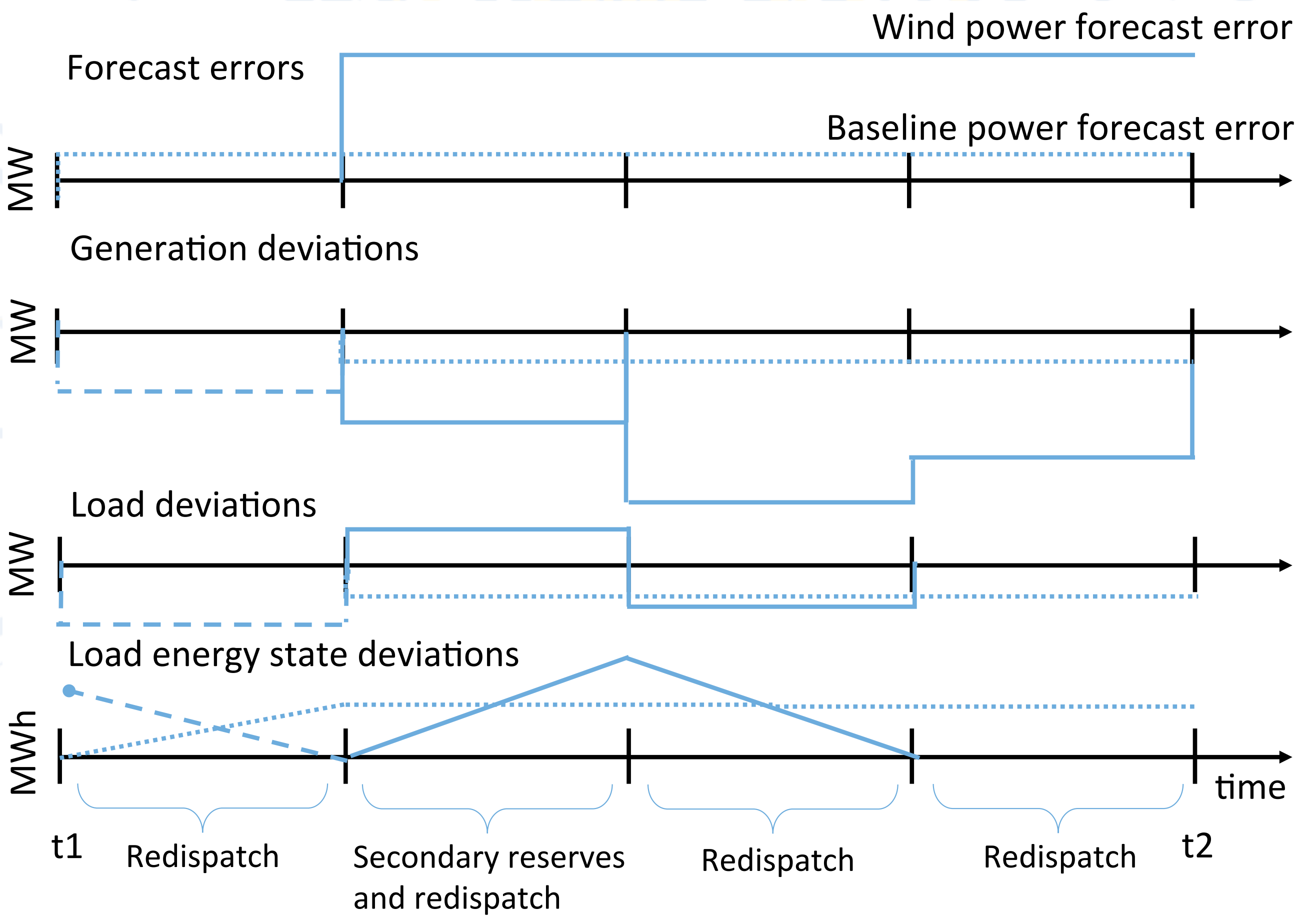
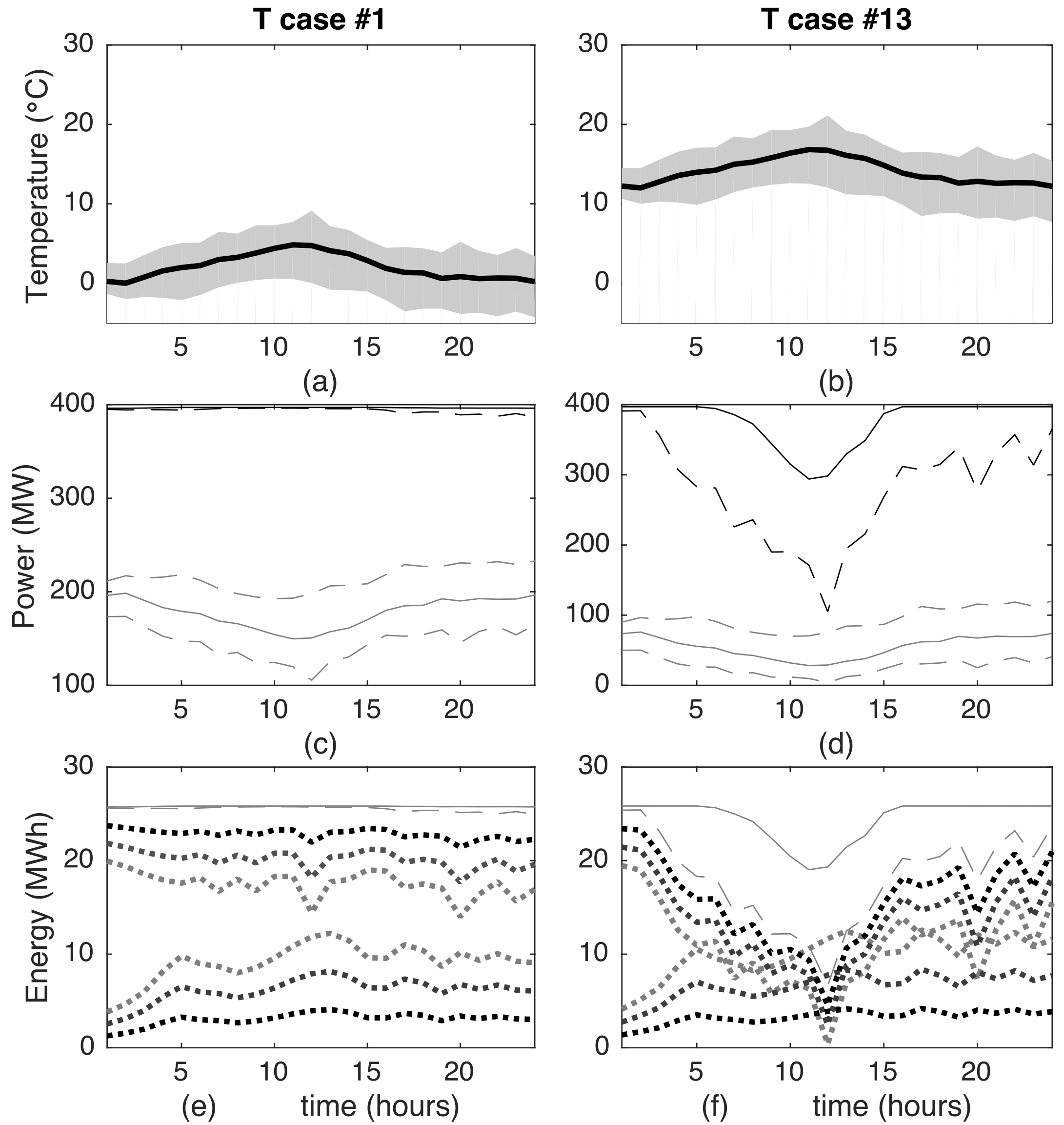
Reserve Design



Influence of temperature forecast error



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Approaches

- Probabilistically robust optimization [Margellos et al. 2014], a hybrid of robust optimization and the scenario approach [Campi et al. 2009]
  - Quadratic programming problem
- Deterministic reformulation assuming multivariate normal uncertainty distributions
  - Nonlinear convex optimization problem
  - Solved via
    - Nonlinear solver
    - Polyhedral or second-order cone (SOC) approximation of nonlinear constraints and SOC programming with a cutting plane algorithm from [Bienstock et al. 2014]

IEEE 30-Bus Network: Cost Comparison

COST DISTRIBUTION AND RESERVE ALLOCATION, $1 - \varepsilon = 99\%$

		Scenario 1	Scenario 2	Analytical 1 & 2
Cost	Total	19657	19721	13474
	Dispatch	11978	11978	11876
	GS	2863	2938	0
	LS	97	90	87
	GD	4719	4716	1510
Capacity (MW)	GS	572	588	0
	LS	194	179	175
	GD	3162	3158	1370

IEEE 30-Bus Network: Reliability Comparison

AVERAGE EMPIRICAL JOINT AND INDIVIDUAL RELIABILITY (%)

	$1-\varepsilon$	Scenario 1	Scenario 2	Analytical 1 & 2
Joint	90%	99.66	99.67	80.42
Individual	90%	100.00	100.00	99.54
Joint	99%	99.98	99.98	95.47
Individual	99%	100.00	100.00	99.89

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Distributionally robust chance constraints

$$\inf_{\mathbb{P}_\xi \in \mathcal{D}_\xi} \mathbb{P}_\xi(f(x,\xi) \leq 0) \geq 1 - \epsilon$$

- Chance constraints satisfied for all distributions within a data-driven ambiguity set [Delage and Ye 2010]. Can incorporate:
  - Moment-based information: mean, covariance, higher-order moments
  - Density-based information: distance between empirical distribution and real data-generating distribution
  - Distribution structure: support, unimodality, symmetry, etc.
- Distributionally Robust Optimal Power Flow [Roald et al. 2015; Zhang et al. 2017; Xie and Ahmed 2017; Summers et al. 2015; Lubin et al. 2016; Guo et al. 2018; Louca and Bitar 2018]

Example: Moment-based ambiguity set

- [Zhang, Shen, Mathieu TPWRS 2017]
- Ambiguity set from [Jiang and Guan 2015]

$$\mathcal{D} = \left\{ f(\xi) : \begin{array}{c} \int_{\xi \in \mathbb{R}^K} f(\xi) d\xi = 1 \\ (\mathbb{E}[\xi] - \mu_0)^\mathsf{T} \Sigma_0^{-1} (\mathbb{E}[\xi] - \mu_0) \leq \gamma_1 \\ \mathbb{E}[(\xi - \mu_0)(\xi - \mu_0)^\mathsf{T}] \preceq \gamma_2 \Sigma_0 \end{array} \right\}$$

- Reformulated problem is a semidefinite program (SDP)
- We also test an ambiguity set that matches the mean and covariance to the empirical ones; the reformulated problem is a SOC program (SOCP)

Comparison of Approaches IEEE 9-bus System

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COST, RELIABILITY, AND CPU TIME OF A1–A4 FOR THE IEEE 9-BUS SYSTEM WITH NO CONGESTION

		A1: Gaussian		A2: Scenario		A3: DR (SDP)		A4: DR (SOCP)	
	$1 - \epsilon_i =$	95%	90%	95%	90%	95%	90%	95%	90%
Objective cost	avg	4392.63	4330.41	4758.32	4738.73	4875.35	4633.57	4875.41	4633.61
	max	4478.08	4394.57	4895.40	4812.65	5102.61	4789.59	5102.65	4789.62
	min	4308.60	4262.52	4678.17	4649.48	4652.84	4480.48	4652.92	4480.59
Reliability (%)	avg	84.47	75.63	99.65	99.57	99.43	97.45	99.65	97.95
	max	94.07	86.69	99.87	99.83	99.83	99.56	99.83	99.74
	min	65.40	61.98	99.36	99.26	97.60	90.99	98.80	91.94
CPU Time (s)	avg	0.03	0.03	15.21	3.51	0.47	0.46	0.44	0.37
	max	0.05	0.06	15.41	3.85	0.55	0.53	0.36	0.41
	min	0.03	0.02	14.88	3.34	0.34	0.41	0.39	0.34

DR approaches even more costly than scenario-based approach!

IEEE 39-bus System

Cost, Reliability, and CPU Time of A1– A4 for the IEEE 39-Bus System With Congestion ($1 - \epsilon_i = 95\%$).

		Gaussian	Scenario	DR (SDP)	DR (SOCP)
Objective cost	avg	42350.49	45489.17	44765.94	44787.62
	max	42788.23	45871.69	45676.41	45744.09
	min	41888.16	45187.54	43996.11	44070.00
Reliability (%)	avg	72.38	99.79	93.25	99.10
	max	86.90	99.87	98.23	99.77
	min	60.47	99.70	85.74	96.60
CPU Time (s)	avg	0.10	1003.45	502.69	0.72
	max	0.12	1040.64	666.80	0.78
	min	0.09	978.80	395.95	0.63

SDP solver starting to have trouble.

Some thoughts…

- Is this really worth it?
- Can we do better?
  - Less conservative, sufficiently reliable
  - Scalable to large networks

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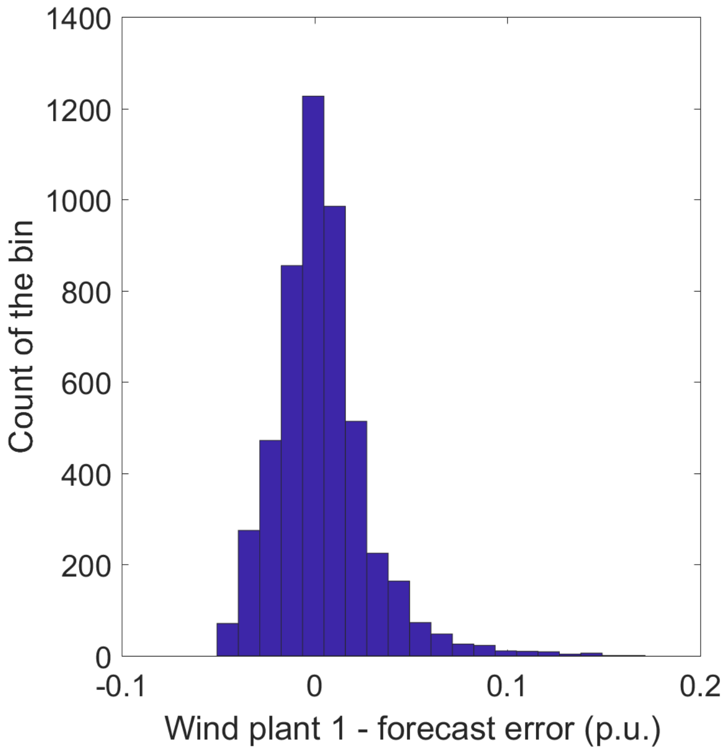
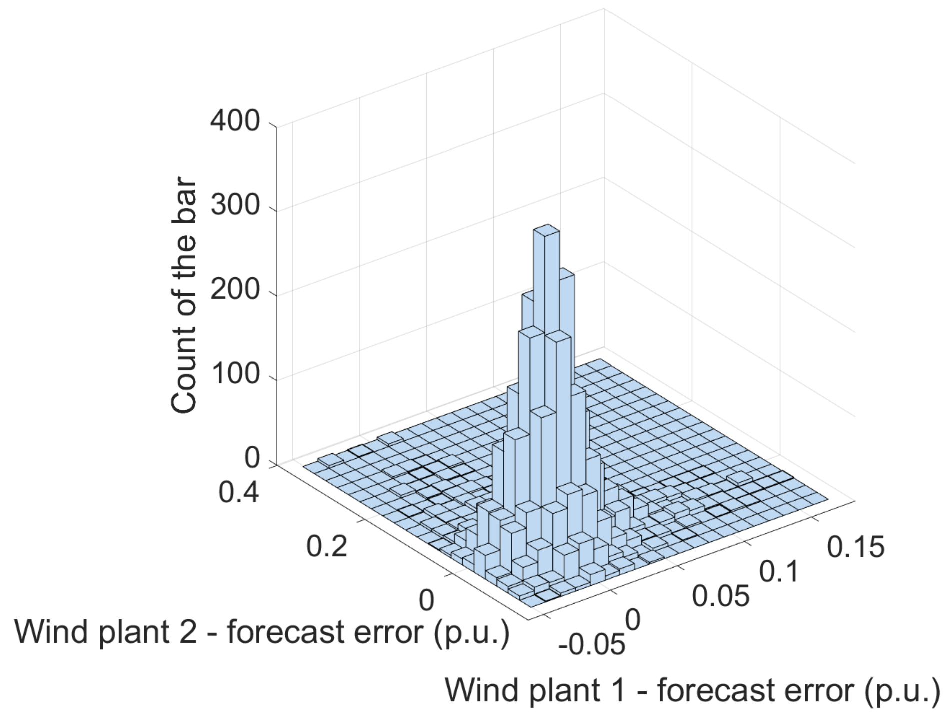
Unimodality of Wind Forecast Error





Data from [Jensen and Pinson 2017]

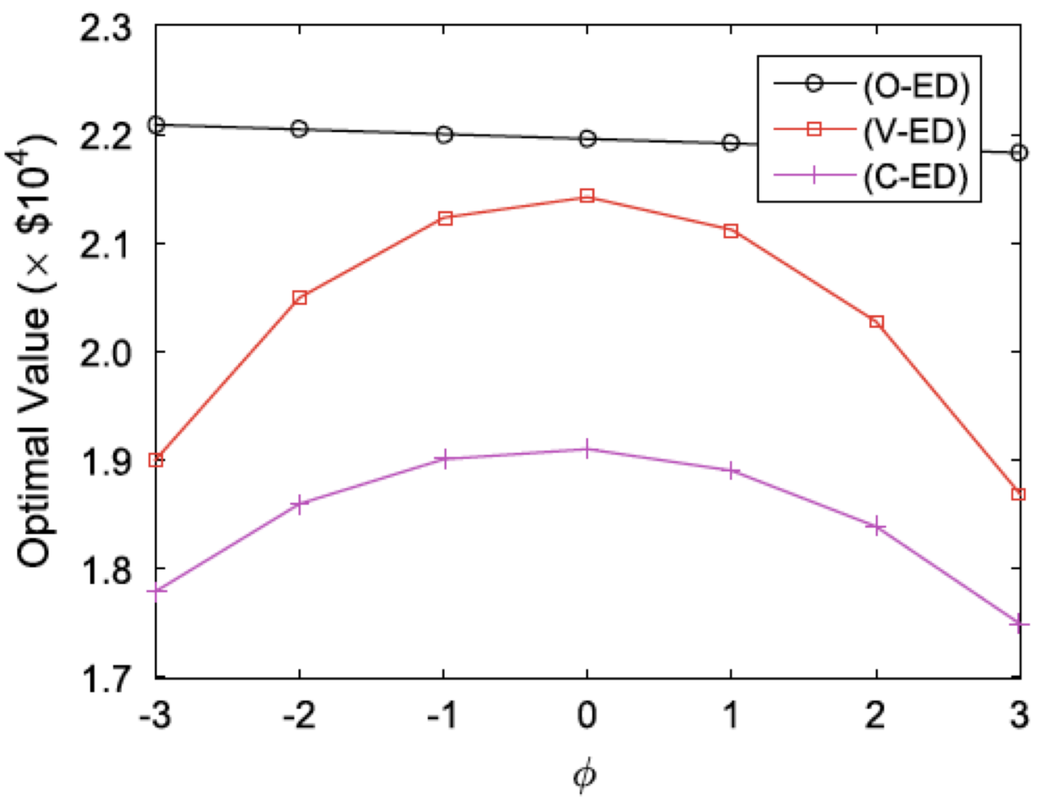
Incorporating Unimodality Information

- Ambiguity set in [Li, Jiang, Mathieu Mathematical Progamming 2019]:

$$\mathcal{D}_\xi(\mu, \Sigma, \alpha) := \left\{ \mathbb{P}_\xi \in \mathcal{M}_T : \mathbb{E}_{\mathbb{P}_\xi}[\xi] = \mu,\ \mathbb{E}_{\mathbb{P}_\xi}[\xi\xi^\top] = \Sigma,\ \mathbb{P}_\xi \text{ is } \alpha\text{-unimodal about } 0 \right\}$$

- α-unimodality [Dharmadhikari and Joag-Dev 1988
- Exact and approximate reformulations of distributionally robust chance and CVaR constraints using this ambiguity set
- Efficient solution algorithms

Reduction in Conservatism

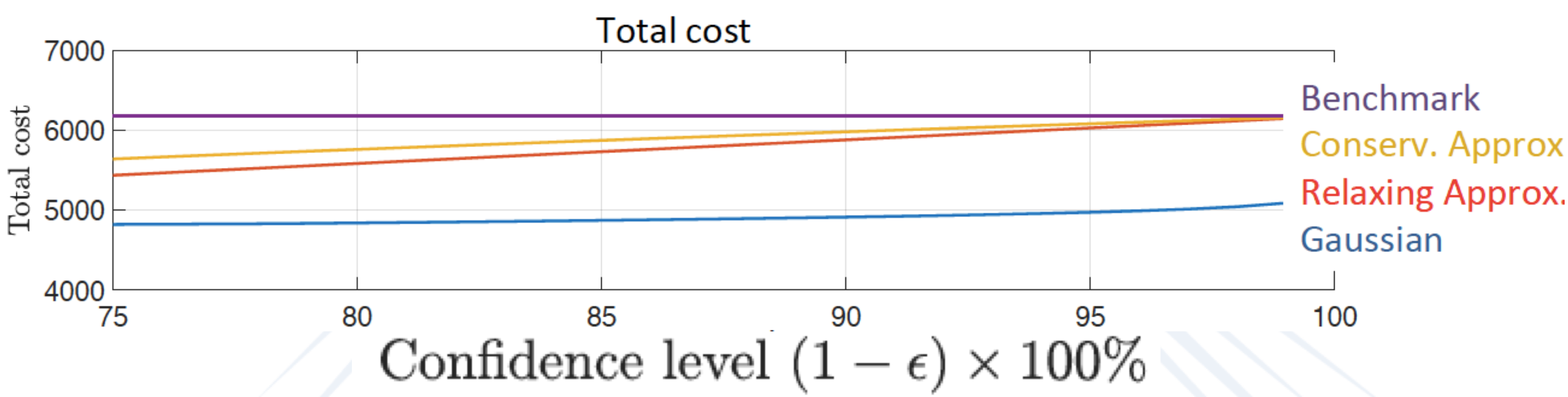


Original
CVaR constraints
Chance constraints

Mean of uncertainty distributions
(Mode at 0)

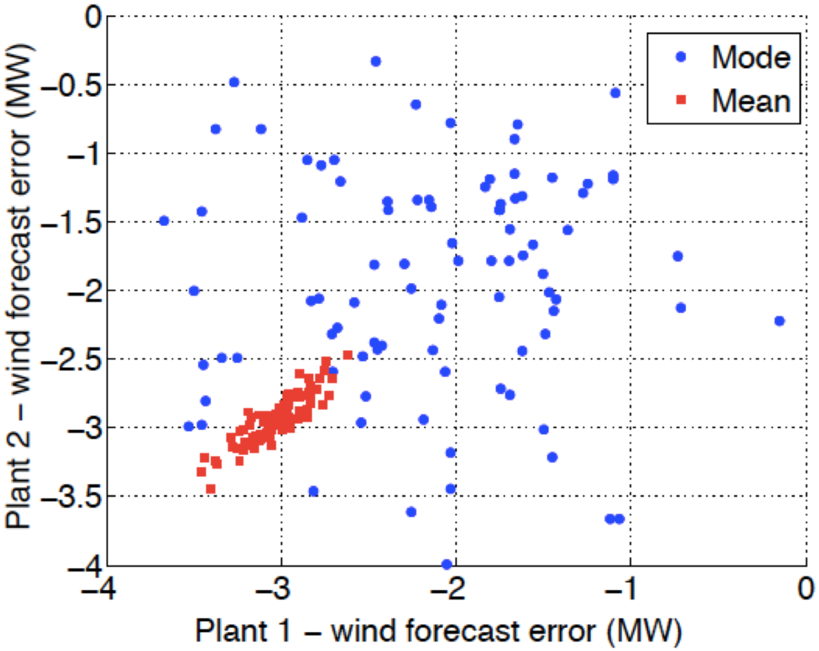
Other Directions We've Explored: Log-concavity

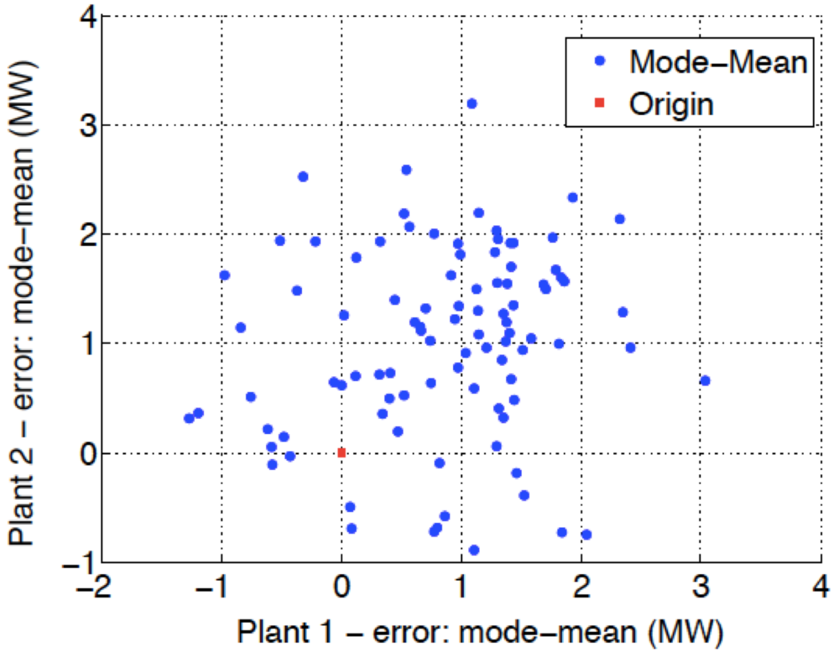
- Assume the distribution is log-concave, support is ellipsoidal
  - We don't have an exact reformulation
  - Sandwich approximation [Li, Jiang, Mathieu PSCC 2018]



Other Directions We've Explored: Misspecified Modes

- Extend the unimodality formulation to consider inaccurately estimated modes
- Modes are harder to estimate accurately than means





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Concluding thoughts

- Load aggregations can provide reserves, but their capacities are uncertain.
- We can plan for this uncertainty in the OPF problem… but who should really be managing this uncertainty?
- Distributionally robust optimization *might* hold promise for chance constrained OPF problems like this one, but still not yet clear if it leads to the right cost/reliability trade-off and reasonable computational requirements