2015 American Control Conference
Palmer House Hilton
July 1-3, 2015. Chicago, IL, USA

Data-driven Optimization Approaches for Optimal Power Flow with Uncertain Reserves from Load Control

Yiling Zhang, Siqian Shen, and Johanna L. Mathieu

***Abstract*— Aggregations of electric loads, like heating and cooling systems, can be controlled to help the power grid balance supply and demand, but the amount of balancing reserves available from these resources is uncertain. In this paper, we investigate data-driven optimization methods that are suited to dispatching power systems with uncertain balancing reserves provided by load control. Specifically, we consider a chance-constrained optimal power flow problem in which we aim to satisfy constraints that include random variables either jointly with a specified probability or individually with different risk tolerance levels. We focus on the realistic case in which we do not have full knowledge of the uncertainty distributions and compare distribution-free approaches with several stochastic optimization methods. We conduct experimental studies on the IEEE 9-bus test system assuming uncertainty in load, load-control reserve capacities, and renewable energy generation. The results show the computational efficacy of the distributionally robust approach and its flexibility in trading off between cost and robustness of solutions driven by data.**

I. Introduction

In U.S. competitive electricity markets, system operators dispatch generators to provide energy and reserves by solving an optimal power flow (OPF) problem. The objective of the OPF problem is to minimize system-wide energy and reserve costs subject to the physical constraints of the system. Traditionally, the OPF problem is solved as a deterministic optimization problem, using forecasted demand/supply values. The amount of reserves procured is commonly a heuristic function of the expected demand/supply uncertainty. Large amounts of uncertainty in power systems motivates stochastic optimization approaches for determining reserve capacities and scheduling generation and reserves. Past work, e.g., [1]–[3], has focused on managing uncertainty stemming from renewable energy production and load consumption.

Flexible electric loads, such as heating and cooling systems, coordinated via load control algorithms, are also capable of providing reserves [4]. Scheduling load-based reserves is an especially challenging task because the amount of available reserves is itself uncertain; specifically, it is a function of stochastic factors including weather and load usage patterns [5]. One option for offering these *uncertain reserves* to the system is to be conservative and offer only a fraction of the expected amount. Another option is to offer the expected amount but explicitly consider reserve uncertainty within a stochastic OPF formulation. Ref. [6] formulated a chance-constrained optimal power flow (CC-OPF) problem to handle uncertainty in load control reserve capacities, load consumption, and renewable energy production, and solved it with probabilistically robust design [7]. The method is a robust reformulation of the scenario approach [8], which requires no assumptions on or knowledge of uncertainty distributions but does require significant numbers of "scenarios," and therefore data. In practice, such data may not be available, and robust reformulations, which require less data, are often overly conservative.

Our goal is to investigate the performance of a variety of methods to solve CC-OPF problems *given limited information about uncertainty distributions*. We first investigate solution approaches that require knowledge of uncertainty distributions and/or significant data: mixed-integer linear programming (MILP), reformulation via Gaussian approximation, and scenario approximation. Then, assuming we do not know the uncertainty distributions or their forms a-priori, and do not have sufficient data for scenario approximation, we apply distributionally robust optimization [9], to demonstrate "the value of data" for managing uncertainty.

The contributions of this paper are twofold. First, we apply distributionally robust optimization to CC-OPF problems with uncertain reserves. Second, we conduct computational experiments to compare the distributionally robust solutions to solutions generated with more standard stochastic optimization approaches. Specifically, we compare objective function values, reliability, and CPU time. Our results demonstrate that distributionally robust optimization provides a better trade off between solution cost and robustness.

The remainder of the paper is organized as follows. In Section II, we describe the CC-OPF models with joint and individual chance constraints, which are solved via mixed-integer linear programming reformulations. Section III considers an equivalent conic program of the individual CC-OPF model by assuming Gaussian distributed uncertainty. When the distribution is not Gaussian, this convex program is an approximation of the original CC-OPF model. Section IV introduces a distributionally robust optimization approach that uses the first two moments to construct a confidence set of possible distributions. Using this set, we reformulate the robust chance constraints as equivalent conic duals and the CC-OPF problem as a convex program. Section V describes the simulation setup and Section VI compares the performance of the different approaches. We summarize the paper and state future research directions in Section VII.

Y. Zhang and S. Shen are with the Department of Industrial and Operations Engineering, University of Michigan at Ann Arbor {zyiling, siqian}@umich.edu; J.L. Mathieu is with the Department of Electrical Engineering and Computer Science, University of Michigan at Ann Arbor, jlmath@umich.edu

This work was partially supported by Grant #CCF-1442495 from the National Science Foundation, Cyber-Innovation for Sustainability Science and Engineering (CyberSEES) program.

II. Modeling CC-OPF Variants

We consider a power system with N_B buses, N_G generators providing energy and reserves, N_W wind farms producing random energy, and N_L load aggregations consuming random energy. Additionally, load aggregations can provide reserves via load control, but the available reserve capacity is also random. The goal is to minimize the expected costs of producing energy and providing generator/load reserves subject to power balance, generator/load constraints, and line limits. In line with most work on stochastic OPF (e.g., [3], [10], [11]), we use the DC power flow approximation. Also, for ease of exposition and results interpretation, we formulate single-period problems, unlike [6] which considered multi-period problems with dynamic constraints.

A. Joint and Individual CC-OPF Models

We present two types of CC-OPF models, where the risk of undesirable outcomes is bounded i) jointly or ii) individually. Decision variables include energy production at generators $P_G \in \mathbb{R}^{N_G}$, generators' up- and down-reserve capacities $\overline{R}_G,\ \underline{R}_G \in \mathbb{R}^{N_G}$, and loads' up- and down-reserve capacities $\overline{R}_L,\ \underline{R}_L \in \mathbb{R}^{N_L}$. The actual generator reserves $R_G \in \mathbb{R}^{N_G}$ and load reserves $R_L \in \mathbb{R}^{N_L}$ depend upon actual energy production and consumption. Following the techniques used in [3], [10], we define additional decision variables $\overline{d}_G, \underline{d}_G, \in \mathbb{R}^{N_G}$ and $\overline{d}_L, \underline{d}_L \in \mathbb{R}^{N_L}$ named "distribution vectors" to distribute real-time supply/demand mismatch $P_m \in \mathbb{R}$ to generators and loads. The generators' production P_G is scheduled to cover the difference between the sum of forecasted loads $P_L^{\text{f}} \in \mathbb{R}^{N_L}$ and the wind forecast $P_W^{\text{f}} \in \mathbb{R}^{N_W}$, i.e., $P_G = \sum_{i=1}^{N_L} P_{L,i}^f - \sum_{i=1}^{N_W} P_{W,i}^f$, where i denotes the i^{th} element of the associated vector. The actual wind power $\widetilde{P}_W$, the actual load $\widetilde{P}_L$, and the actual minimum and maximum load $[\widetilde{\underline{P}}_L, \widetilde{\overline{P}}_L]$ are random variables. Let $c = [c_0, c_1, c_2, \overline{c}_G, \underline{c}_G, \overline{c}_L, \underline{c}_L]^{\mathsf{T}}$ be the cost vector and $[\underline{P}_G, \overline{P}_G]$ be the min/max generator production. We formulate a joint CC-OPF model as:

[Joint-CC-OPF]:

$$\min \quad c^{\mathsf{T}}[1, P_G, P_G^2, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L] \tag{1}$$

$$\text{s.t.} \quad P_m = \sum_{i=1}^{N_W} (\widetilde{P}_{W,i} - P_{W,i}^f) - \sum_{i=1}^{N_L} (\widetilde{P}_{L,i} - P_{L,i}^f) \tag{2}$$

$$\sum_{i=1}^{N_G} \underline{d}_{G,i} + \sum_{i=1}^{N_L} \overline{d}_{L,i} = 1 \tag{3}$$

$$\sum_{i=1}^{N_G} \overline{d}_{G,i} + \sum_{i=1}^{N_L} \underline{d}_{L,i} = 1 \tag{4}$$

$$R_G = \overline{d}_G \max\{-P_m, 0\} - \underline{d}_G \max\{P_m, 0\} \tag{5}$$

$$R_L = \overline{d}_L \max\{P_m, 0\} - \underline{d}_L \max\{-P_m, 0\} \tag{6}$$

$$\mathbb{P}\left(\widetilde{A}x \geq \widetilde{b}\right) \geq 1 - \epsilon \tag{7}$$

$$x = [P_G, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L, \underline{d}_G, \overline{d}_G, \underline{d}_L, \overline{d}_L] \geq \mathbf{0}, \tag{8}$$

where (2) calculates the real-time supply/demand mismatch. Constraints (3)–(4) normalize the distribution vectors, and (5)–(6) compute the reserve capacity from generators and from loads, respectively. The joint chance constraint (7) guarantees that $\widetilde{A}x \geq \widetilde{b}$ holds with probability $1 - \epsilon$ where $\widetilde{A}x \geq \widetilde{b}$ is a set of linear constraints:

$$\begin{aligned} \widetilde{A}x \geq \widetilde{b} = \{&\underline{P}_G \leq P_G + R_G \leq \overline{P}_G, \\ &\widetilde{\underline{P}}_L \leq \widetilde{P}_L + R_L \leq \widetilde{\overline{P}}_L, \\ &-\underline{R}_G \leq R_G \leq \overline{R}_G, \\ &-\underline{R}_L \leq R_L \leq \overline{R}_L, \\ &-P_{\text{line}} \leq B_{\text{flow}} \begin{bmatrix} 0 \\ B_{\text{bus}}^{-1} \hat{P}_{\text{inj}} \end{bmatrix} \leq P_{\text{line}}\}, \end{aligned} \tag{9}$$

where $P_{\text{inj}} = C_G(P_G + R_G) + C_W \widetilde{P}_w - C_L(\widetilde{P}_L + R_L) \in \mathbb{R}^{N_B}$ is a vector with the net power injection each bus; $\hat{P}_{\text{inj}} \in \mathbb{R}^{N_B - 1}$ is the last $N_B - 1$ rows of P_{inj}; $C_G,\ C_W$, and C_L are matrices that map generators/loads to buses; B_{flow} is the flow admittance matrix; B_{bus} is the bus admittance matrix; and P_{line} is a vector containing the line flow limits.

Alternatively, we analyze another CC-OPF model where the constraints in $\widetilde{A}x \geq \widetilde{b}$ are guaranteed individually with pre-specified probability levels. We formulate it as:

[Individual-CC-OPF]:

$$\begin{aligned} \min \quad & (1) \\ \text{s.t.} \quad & (2)\text{–}(6),\ (8) \\ & \mathbb{P}\left(\widetilde{A}_i x \geq \widetilde{b}_i\right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m, \end{aligned} \tag{10}$$

where m is the number of constraints in $\widetilde{A}x \geq \widetilde{b}$. We assume that $\widetilde{A}_i$ represents the i^{th} row of matrix $\widetilde{A}$ and $\widetilde{b}_i$ is the i^{th} entry of vector $\widetilde{b}$. Therefore, each constraint i in (7) has exactly one row with probability $1 - \epsilon_i$. In practice, decision makers can choose values of ϵ_i according to the importance of satisfying each constraint.

B. Solution Approaches

To solve either the joint or individual CC-OPF models, we first assume that full distributional information of all random variables is available. Assuming the reserve is symmetric, i.e., $\overline{d}_G = \underline{d}_G = d_G$ and $\overline{d}_L = \underline{d}_L = d_L$, we can reformulate the models as mixed-integer linear programs, and obtain optimal solutions using off-the-shelf optimization solvers.

A1. Mixed-integer linear programming (MILP) approach

To solve the Joint-CC-OPF, we use the Monte-Carlo sampling method to generate finite realizations (samples) of the uncertainty. We enforce $\widetilde{A}x \geq \widetilde{b}$ in sufficient numbers of samples. This is known as the Sample Average Approximation (SAA) approach [12] for deriving objective bounds and feasible solutions to general chance-constrained programs.

Denote Ω as the set of samples, p^s as the probability of realizing sample $s \in \Omega$, and (A^s, b^s) as the realization of $(\widetilde{A}, \widetilde{b})$ in sample s. For each $s \in \Omega$, define a binary logic variable z_s such that $z_s = 0$ indicates that $A^s x \geq b^s$ and $z_s = 1$ indicates that $A^s x < b^s$. We reformulate the joint chance constraint (7) as

$$A^s x \geq b^s - M z_s \;\forall s \in \Omega, \tag{11}$$

$$\sum\nolimits_{s\in\Omega} p^s z_s \leq \epsilon, \text{ and } z_s \in \{0,1\} \;\forall s \in \Omega, \tag{12}$$

where M is a large scalar coefficient. Constraints (11) use big-M coefficients to enforce $A^s x \geq b^s$ when $z_s = 0$ in a subset of scenarios s in Ω. Constraints (12) ensure that the probability of violating $A^s x \geq b^s,\ \forall s \in \Omega$ is no more than ϵ, and all z_s variables are binary valued. We reformulate the Joint-CC-OPF by replacing (7) with (11)–(12).

Similarly, corresponding to individual constraints (10), define binary variables y_s^i to indicate the satisfaction status of each constraint $\widetilde{A}_i x \geq \widetilde{b}_i$ in scenario s, such that $y_s^i = 0$ if $\widetilde{A}_i x \geq \widetilde{b}_i$ and $y_s^i = 1$ otherwise. As a result, constraints (10) are equivalent to

$$A_i^s x \geq b_i^s - M^i y_s^i \;\forall s \in \Omega,\; i = 1, \ldots, m \tag{13}$$

$$\sum\nolimits_{s\in\Omega} p^s y_s^i \leq \epsilon_i,\; \forall i, \text{ and } y_s^i \in \{0,1\} \;\forall s,\; i, \tag{14}$$

where M^i is a sufficiently large coefficient for each $i = 1, \ldots, m$.

III. Special Cases and Approximations

In this section, we study special cases of the Individual-CC-OPF in which we can derive convex programs to compute optimal results. We also use the results to derive convex approximations for general cases.

A2. Reformulation via Gaussian approximation

Assuming Gaussian distributed uncertainty, we reformulate the Individual-CC-OPF as a convex program, similar to [10], [11], which, unlike our formulation, do not consider load-based reserves.

Without loss of generality, we consider constraints (10) in an equivalent form

$$\mathbb{P}\left(\widetilde{A}_i' \widetilde{x} \leq b_i'\right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m, \tag{15}$$

where only the constraint vector $\widetilde{A}_i'$ is uncertain and the right-hand side scalar b_i' is deterministic. Given a chance constraint i of the form (10) where both $\widetilde{A}_i$ and $\widetilde{b}_i$ are random, we define an artificial variable $x_b \in \mathbb{R}$ and rewrite $\widetilde{A}_i x \geq \widetilde{b}_i$ as

$$-\widetilde{A}_i x + \widetilde{b}_i x_b \leq 0 \;\Leftrightarrow\; (-\widetilde{A}_i, \widetilde{b}_i)^\mathsf{T}(x, x_b) \leq 0,$$

for which we enforce $x_b = 1$. Correspondingly, we have $\widetilde{A}_i' = (-\widetilde{A}_i, \widetilde{b}_i)$, $\widetilde{x} = (x, x_b)$, and $b_i' = 0, \forall i = 1, \ldots, m$ in the above constraint (15).

When $\widetilde{A}_i'$ follows a multivariate Gaussian distribution, denoted by $\mathcal{N} \sim (\mu_i, \Sigma_i)$ with μ_i and Σ_i representing the mean and covariance of $\widetilde{A}_i'$, respectively, the random outcome $\widetilde{A}_i' x' - b_i'$ follows a multivariate Gaussian distribution $\mathcal{N} \sim (\mu_i^\mathsf{T} \bar{x} - b', \bar{x}^\mathsf{T} \Sigma_i \bar{x})$. Therefore,

$$\mathbb{P}(\widetilde{A}_i' \bar{x} \leq b_i') = \Phi\left(\frac{b_i' - \mu_i^\mathsf{T} \bar{x}}{\bar{x}^\mathsf{T} \Sigma_i \bar{x}}\right) \quad i = 1, \ldots, m, \tag{16}$$

following which, we replace (10) by deterministic constraints

$$b_i' - \mu_i^\mathsf{T} \bar{x} + \Phi^{-1}(\epsilon_i)\sqrt{\bar{x}^\mathsf{T} \Sigma_i \bar{x}} \geq 0 \quad i = 1, \ldots, m, \tag{17}$$

where $\Phi^{-1}(\epsilon_i)$ is the ϵ_i-quantile of the Gaussian distribution. We rewrite (17) as

$$b_i' - \mu_i^\mathsf{T} \bar{x} \geq \Phi^{-1}(1 - \epsilon_i)\sqrt{\bar{x}^\mathsf{T} \Sigma_i \bar{x}} \quad i = 1, \ldots, m, \tag{18}$$

The above are second-order cone constraints if $\Phi^{-1}(1-\epsilon_i) \geq 0$, i.e., $1 - \epsilon_i \geq 0.5$, which is a mild assumption since a chance constraint is normally closely related to the quality of service and thus needs to be satisfied with probability higher than 0.5.

A3. Scenario approximation

Ref. [8] proposed scenario approximations for solving individual chance constraints by enforcing $A_i^s x \geq b_i^s,\ \forall i$ in all scenarios s of an approximate sample set Ω_{ap}. The scenarios in Ω_{ap} could be i) data observations or ii) generated from known distributions using the Monte Carlo sampling approach. The advantage of this approach is that it does not require binary variables z_s^i as in the SAA method in A1. However, to ensure solution quality, it usually requires a large sample size $|\Omega_{\text{ap}}|$ to guarantee the reliability $1 - \epsilon$ with high confidence. To use this approach for this problem, we replace each chance constraint in (10) with

$$A_i^s x \geq b_i^s \;\forall s \in \Omega_{\text{ap}}. \tag{19}$$

Remark Both A1 and A2 require full distributional knowledge, while A3 requires large sample sizes and significant computation. (It is shown in [8] that $|\Omega_{\text{ap}}| \geq \frac{2}{\epsilon}(\ln\frac{1}{\beta} + n)$ where ϵ is the risk parameter, $1 - \beta$ is the confidence level, and n is the dimension of x.) The restrictions of these approaches motivate the search for new data-driven methods.

IV. Distributionally Robust Optimization

When full distributional information of the uncertain parameters is not available, we can use methods that rely on historical data, statistical inference, and optimization, such as distributionally robust optimization [9].

A4. Data-driven distributionally robust optimization

This approach builds a confidence set to bound the probability density function (pdf) of the uncertainty distribution. Solutions are reliable with respect to a family of unknown distributions, and the quality of the result is correlated to the quantity of data. Additionally, solutions are less conservative than those produced by robust optimization methods and require less computation than those produced by scenario-based methods.

We describe the approach using a generic stochastic program formulated as $\min_{a\in A} \mathbb{E}_\xi[c(a,\xi)]$ where a is a decision vector and $\xi \in \mathbb{R}^K$ is a random vector, both affecting the random outcome of an objective cost function $c(a,\xi)$. The goal is to minimize the expected cost given the distribution of ξ with pdf $f(\xi)$. When the exact $f(\xi)$ is not known we can build a confidence set $\mathcal{D}$ of possible pdfs for describing the uncertainty ξ. The distributionally robust variant of the stochastic program, which minimizes the worst-case expected cost over the choice of $f(\xi) \in \mathcal{D}$, is

$$\min_{a\in A} \max_{f(\xi)\in\mathcal{D}} \mathbb{E}_\xi[c(a,\xi)]. \tag{20}$$

For the chance constraints (10), suppose that $(\widetilde{A}_i, \widetilde{b}_i) = (\widetilde{A}_i^\xi, \widetilde{b}_i^\xi)$ is determined by the uncertainty ξ. The distributionally robust variant requires that

$$\min_{f(\xi)\in\mathcal{D}} \mathbb{P}_\xi(\widetilde{A}_i^\xi x \geq \widetilde{b}_i^\xi) \geq 1-\epsilon_i,\ \forall i=1,\ldots,m. \tag{21}$$

Without loss generality, assume that $\widetilde{A}_i^\xi$ and $\widetilde{b}_i^\xi$ are affine combinations of random variable ξ, i.e.,

$$\widetilde{A}_i^\xi = \widetilde{A}_{i0}^\xi + \sum_{k=1}^K \widetilde{A}_{ik}^\xi \xi_k, \quad \widetilde{b}_i^\xi = \widetilde{b}_{i0}^\xi + \sum_{k=1}^K \widetilde{b}_{ik}^\xi \xi_k, \tag{22}$$

where $\widetilde{A}_{i0}^\xi$ and $\widetilde{b}_{i0}^\xi$ represent the deterministic part of $\widetilde{A}_i^\xi$ and $\widetilde{b}_i^\xi$, and $\widetilde{A}_{ik}^\xi$ and $\widetilde{b}_{ik}^\xi$ for $k=1,\ldots,K$ are the affine coefficients. We reformulate (21) as:

$$\begin{aligned}
&\left(\widetilde{A}_{i0}^\xi + \sum_{k=1}^K \widetilde{A}_{ik}^\xi \xi_k\right) x \geq \widetilde{b}_{i0}^\xi + \sum_{k=1}^K \widetilde{b}_{ik}^\xi \xi_k \\
\Rightarrow\quad & \widetilde{A}_{i0}^\xi x + \sum_{k=1}^K \left(\widetilde{A}_{ik}^\xi x\right)\xi_k \geq \widetilde{b}_{i0}^\xi + \sum_{k=1}^K \widetilde{b}_{ik}^\xi \xi_k \\
\Rightarrow\quad & \bar{A}_i^x \xi \geq \bar{b}_i^x,
\end{aligned} \tag{23}$$

with $\bar{A}_i^x = [\widetilde{A}_{i1}^\xi x - \widetilde{b}_{i1}^\xi, \ldots, \widetilde{A}_{iK}^\xi x - \widetilde{b}_{iK}^\xi]$ and $\bar{b}_i^x = -\widetilde{A}_{i0}^\xi x + \widetilde{b}_{i0}^\xi$. We use the first and second moments of ξ to build the confidence set $\mathcal{D}$ (see, e.g., [13]). The details are as follows.

Given samples $\{\xi^i\}_{i=1}^N$ of ξ, we calculate the empirical mean and covariance matrix as $\mu_0 = \frac{1}{N}\sum_{i=1}^N \xi^i$ and $\Sigma_0 = \frac{1}{N}\sum_{i=1}^N (\xi^i - \mu_0)(\xi^i-\mu_0)^\mathsf{T}$. We build a confidence set

$$\mathcal{D} = \left\{ f(\xi): \begin{array}{c} \int_{\xi\in\mathcal{S}} f(\xi)d\xi = 1 \\ (\mathbb{E}[\xi]-\mu_0)^\mathsf{T}\Sigma_0^{-1}(\mathbb{E}[\xi]-\mu_0) \leq \gamma_1 \\ \mathbb{E}[(\xi-\mu_0)(\xi-\mu_0)^\mathsf{T}] \preceq \gamma_2\Sigma_0 \end{array} \right\},$$

where $\mathcal{S}$ represents a convex support set of random ξ. The set $\mathcal{D}$ is determined by the estimated mean μ_0, covariance matrix Σ_0 and by the values of γ_1 and γ_2, which reflect how likely ξ is close to the empirical mean μ_0 in terms of the correlations given by Σ_0. The three constraints in $\mathcal{D}$ ensure that i) values of $f(\xi)$ sum to 1 over the support set $\mathcal{S}$; ii) the mean of ξ lies in an ellipsoid of size γ_1 centered at the empirical μ_0; iii) the true covariance matrix lies in a positive semi-definite cone bounded by a matrix inequality $\gamma_2\Sigma_0$.

For individual constraints in (21), let r_i, $\begin{bmatrix} H_i & p_i \\ p_i^\mathsf{T} & q_i \end{bmatrix}$, and G_i be the dual variables associated with the three constraints in $\mathcal{D}$, respectively. The individual chance constraints (21) are equivalent to

$$\gamma_2\Sigma_0 \cdot G_i + 1 - r_i + \Sigma_0 \cdot H_i + \gamma_1 q_i \leq \epsilon_i y_i, \tag{24}$$

$$\begin{bmatrix} G_i & -p_i \\ -p_i^\mathsf{T} & 1-r_i \end{bmatrix} \succeq \begin{bmatrix} 0 & \frac{1}{2}\bar{A}_i^x \\ \frac{1}{2}(\bar{A}_i^x)^\mathsf{T} & y_i + (\bar{A}_i^x)^\mathsf{T}\mu_0 - \bar{b}_i^x \end{bmatrix}, \tag{25}$$

$$\begin{bmatrix} G_i & -p_i \\ -p_i^\mathsf{T} & 1-r_i \end{bmatrix} \succeq 0, \begin{bmatrix} H_i & p_i \\ p_i^\mathsf{T} & q_i \end{bmatrix} \succeq 0, \tag{26}$$

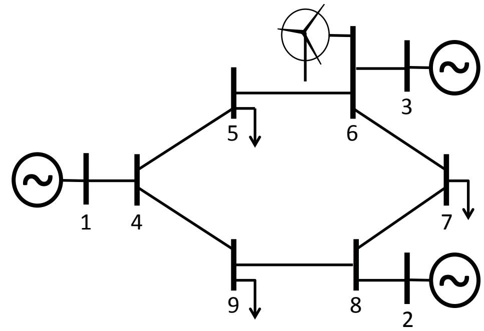

Fig. 1. IEEE 9-bus network, with added wind generation.

and $y_i \geq 0, \forall i = 1,\ldots,m$, where the operator "$\cdot$" in (24) represents the Frobenius inner product of two matrices (i.e., $A \cdot B = \text{tr}(A^\mathsf{T}B)$). Note that the above approaches for bounding the unknown $f(\xi)$ are general and allow the uncertainty ξ to be time-varying, correlated, and endogenous.

V. Simulation Setup

For our computational experiments, we use the IEEE 9-bus system, shown in Fig. 1, with reserve costs $\overline{\mathbf{c}}_G = \underline{\mathbf{c}}_G = [5,6,7]$ (corresponding to buses 1, 2, and 3) and $\overline{\mathbf{c}}_L = \underline{\mathbf{c}}_L = [1,1,1]$ (corresponding to buses 5, 7, and 9). Parameters relevant to the DC power flow approximation were calculated with data obtained from MATPOWER [14]. We add one wind generator at bus 6 with rated capacity 75 MW and forecasted output 50 MW. We use load forecasts of $P_L^f = [90, 100, 125]^\mathsf{T}$ MW and assume 80% of the load at each bus is perfectly forecastable and uncontrollable. We assume the remaining load at each bus is controllable but uncertain (in terms of both consumption and min/max load).

We assume the controllable loads are aggregations of electric heating loads, which can be modeled as energy storage units with power and energy capacities that vary with outdoor air temperature [15]. While, in practice, load uncertainty is a function of a variety of complex and correlated factors [5], for simplicity, we assume that, given a temperature forecast or realization, $\widetilde{P}_L$, $\widetilde{\underline{P}}_L$, and $\widetilde{\overline{P}}_L$ can be determined from a look-up table. Therefore, we consider only two types of uncertain variables: temperature and wind power infeed.

We use the same look-up table as was used in [6] to determine load consumption (P_L), power capacities, and energy capacities of aggregations of residential heat pump heating systems as a function of outdoor air temperature. We scale these values to ensure that our assumed temperature forecasts, i.e., $T^f = [13, 10, 14]^\mathsf{T}$ °C, produce the assumed load forecasts. To determine the minimum and maximum load ($\underline{P}_L$, $\overline{P}_L$), we assume a load aggregation providing reserves must be able to operate at full capacity for 15-minutes and must operate within its power capacity. Specifically,

$$\underline{P}_L = \max(P_L - 4(E_{\text{cap}}/2), 0),$$
$$\overline{P}_L = \min(4(E_{\text{cap}}/2) + P_L, P_{\text{cap}}),$$

where E_{cap} is the energy capacity (i.e., maximum energy increase plus maximum energy decrease from the baseline energy consumption) and P_{cap} is the power capacity (i.e., maximum power consumption).

We use forecasted and actual outdoor air temperatures from eleven weather stations in Switzerland to compute temperature errors, and then add these errors to the assumed temperature forecasts to create temperature samples. Note that this method assumes the error is independent of the magnitude of the temperature forecast, which is not particularly realistic. In future research, a model could be used to generate more realistic temperature error samples. However, in practice, we would need to collect significantly more data to be able to apply scenario approximation (which points to the key drawback of this approach).

We use the same wind power scenarios as in [6]. In that paper, forecasted and actual hourly wind power data from Germany from 2006-2011 were used together with a Markov chain mechanism [16], to generate the appropriate number of scenarios (again, we were unable to obtain sufficient amounts of real data). Here, since we assume a wind forecast of two-thirds of rated capacity, we extract wind power errors associated with a real wind power forecast of approximately two-thirds of rated capacity, and we scale these errors as a function of our assumed rated capacity.

We follow the above procedures to randomly generate $10,000$ samples of sets of parameters P_W, P_L, $\underline{P}_L$, $\overline{P}_L$, denoted by Ω. We use $\epsilon = \epsilon_i = 5\%, 10\%, 15\%$. Recall that ϵ is the risk tolerance in the joint chance constraint (7) in the Joint-CC-OPF and ϵ_i is the risk tolerance for violating the i^{th} chance constraint (10) in the Individual-CC-OPF.

We randomly select data samples with appropriate sizes from set Ω, and test all four approaches (A1-A4). For the MILP approach (A1), we solve the MILP reformulations of the Joint-CC-OPF and Individual-CC-OPF. For other approaches (A2-A4), we solve the Individual-CC-OPF models. For approaches A1, A2, and A4, we pick 20 arbitrary samples from Ω. When $\epsilon_i = 5\%$, 10%, 15% under the same confidence level $1 - \beta_i = 0.95$, the scenario approximation approach (A3) requires $\approx$ 900, 500, 300 random samples, respectively, for approximating each individual chance constraint i (note $n = 21$ in our problem). For the distributionally robust approach (A4), we construct the confidence set of the unknown pdf of ξ by matching the first and second moments of the 20 samples, while the Gaussian approximation approach (A2) uses the same moment information for constructing the conic constraint (18).

The computations are performed on a Windows 7 machine with Intel(R) Core(TM) i7-2600 CPU 3.40 GHz and 8GB memory. All models are solved by CVX implemented in Matlab with MOSEK as the optimization solver.

VI. Computational Results

To compare the solutions of the approaches, we compare the CPU time, objective values, and solution reliability. To assess the reliability of a particular solution, we test the optimal solution x on all 10,000 samples in set Ω, and determine the percentage of the samples in which the constraints are satisfied jointly.

TABLE I. Comparison of the results of Joint-CC-OPF and Individual-CC-OPF solved by A1

		$1-\epsilon$ or $1-\epsilon_i =$	95.00%	90.00%	85.00%
Joint	Objective cost	average	1348.79	1347.57	1345.16
		min	1327.77	1327.33	1327.33
		max	1363.47	1363.47	1362.11
	Reliability (%)	average	77.01	73.44	68.38
		min	7.69	1.11	1.11
		max	94.83	94.83	91.66
	CPU seconds	average	2.04	2.20	2.53
		min	1.39	1.50	1.54
		max	3.84	3.71	4.70
Individual	Objective cost	average	1345.60	1343.09	1340.72
		min	1336.43	1335.47	1332.26
		max	1356.52	1358.39	1358.18
	Reliability (%)	average	72.13	67.77	60.35
		min	45.75	39.30	29.31
		max	89.57	91.21	88.27
	CPU seconds	average	5876.22	4737.06	2308.63
		min	130.96	3.06	3.29
		max	32816.60	131206.17	127438.50

First, we compare the results of the Joint-CC-OPF and Individual-CC-OPF given by A1. For $\epsilon = \epsilon_i = 5\%, 10\%, 15\%$, we perform 100 repetitions and report results in Table I, which presents the average, minimum, and maximum values of the objective cost, reliability, and the CPU time for solving instances of the two models. We observe that the Individual-CC-OPF solutions have lower objective costs since we can be more flexible in satisfying constraints individually at different risk tolerance levels compared with satisfying them jointly. However, the cost benefit is not obvious since it takes significantly longer to solve the Individual-CC-OPF since its MILP reformulation has many more binary variables and constraints. Both models can achieve sufficiently high reliability (i.e., the max reliability values can be higher than the required $1 - \epsilon$), but do not on average. Solutions of the Joint-CC-OPF yield more diverse reliability results, and can perform poorly (i.e., the min reliability values can be much less than the required $1 - \epsilon$).

Next, we present the results of 100 repetitions of solving different variants of Individual-CC-OPF with approaches A2, A3, and A4. The average, minimum, and maximum values of the objective values, reliability, and CPU time are given in Tables II–IV for each approach and each risk tolerance level. Comparing the results, the distributionally robust approach A4 outperforms the other two in terms of the objective cost and reliability. All the three approaches require less CPU time to solve the Individual-CC-OPF as compared to approach A1. A3 takes the longest time amongst the three approaches due to its large sample sizes. Moreover, the CPU time of A3 depends on the number of samples we select, while the CPU time of approaches A2 and A4 are independent of the sample size. The objective cost values of A4 are lower on average than the ones yielded by A3 because A4 is less conservative, employing moment information and bounding the worst case probability for any distribution that matches the moments. As expected, A3 yields the highest objective cost and the highest reliability since it requires no constraint violation for a large subset of samples in set

TABLE II. Results of Individual-CC-OPF solved by the Gaussian approximation approach A2

		$1-\epsilon_i =$	95.00%	90.00%	85.00%
Individual	Objective cost	average	1348.97	1343.53	1339.87
		min	1339.61	1336.14	1333.80
		max	1357.95	1350.66	1345.73
	Reliability (%)	average	82.15	70.03	58.87
		min	64.79	51.12	39.90
		max	94.40	86.32	76.68
	CPU seconds	average	0.63	0.61	0.62
		min	0.59	0.59	0.59
		max	0.72	0.64	0.72

TABLE III. Results of Individual-CC-OPF solved by the scenario approximation approach A3

		$1-\epsilon_i =$	95.00%	90.00%	85.00%
Individual	Objective cost	average	1407.72	1391.90	1383.31
		min	1370.76	1364.91	1360.71
		max	1525.17	1487.74	1521.52
	Reliability (%)	average	99.59	99.27	98.85
		min	98.76	97.82	96.30
		max	99.91	99.89	99.83
	CPU seconds	average	54.96	17.69	8.75
		min	53.74	15.93	8.53
		max	56.91	19.45	9.22

Ω. Approach A4 generally yields higher objective cost than A2, but much better reliability since A2 assumes Gaussian-distributed uncertainty and does not yield sufficiently high rates of reliability for any risk tolerance level (i.e., even the max reliability values in Table II are much lower than the required $1-\epsilon_i,\ \forall i = 1,\ldots,m$.) Therefore, A4 is a much more reliable optimization approach than A1 and A2 for solving the Individual-CC-OPF and also has significantly shorter CPU time than both A1 and A3.

VII. Concluding Remarks

In this paper, we studied CC-OPF problems in which the risk of undesirable outcomes is bounded by joint or individual chance constraints. We modeled uncertainty from several sources including renewable energy production, load consumption, and, importantly, load-based reserve capacities. We compared the results of four approaches, in terms of the objective value, reliability, and CPU time. Under the realistic assumption that the distribution of uncertain variables is not known, we showed that the distributionally robust optimization reformulation yields high reliability at relatively low cost and CPU time (especially as compared to scenario approximation approaches). This provides decision makers a nonparametric distribution-free method for solving CC-OPF problems under ambiguous distributional information.

Future research includes more computational tests of each approach using more diverse reference samples and sample sizes. We are also interested in quantifying the relationship between result quality and the amount of data we use to construct the confidence set of the probability distribution function. Additionally, we aim to test the distributionally robust approach on multi-period problems that capture the full complex and correlated uncertainty associated with reserves provided by loads.

TABLE IV. Results of Individual-CC-OPF solved by the distributionally robust approach A4

		$1-\epsilon_i =$	95.00%	90.00%	85.00%
Individual	Objective cost	average	1392.64	1369.23	1359.97
		min	1352.46	1346.62	1346.62
		max	1457.81	1385.24	1372.75
	Reliability (%)	average	99.50	97.97	94.51
		min	91.40	91.40	83.29
		max	99.96	99.70	99.18
	CPU seconds	average	6.63	6.98	6.95
		min	6.13	4.73	6.27
		max	8.19	8.44	7.83

Acknowledgment

Special thanks to M. Vrakopoulou for providing the wind power scenarios.

References

[1] F. Bouffard and F.D. Galiana. Stochastic security for operations planning with significant wind power generation. *IEEE Transactions on Power Systems*, 23(2):306–316, May 2008.

[2] A. Papavasiliou, S. Oren, and R O'Neill. Reserve requirements for wind power integration: a scenario-based stochastic programming framework. *IEEE Transactions on Power Systems*, 26(4):2197–2206, 2011.

[3] M. Vrakopoulou, K. Margellos, J. Lygeros, and G. Andersson. A probabilistic framework for reserve scheduling and $n-1$ security assessment of systems with high wind power penetration. *IEEE Transactions on Power Systems*, 28(4), 2013.

[4] D.S. Callaway and I.A. Hiskens. Achieving controllability of electric loads. *Proceedings of the IEEE*, 99(1):184–199, 2011.

[5] J.L. Mathieu, M. González Vayá, and G. Andersson. Uncertainty in the flexibility of aggregations of demand response resources. In *Proceedings of the IEEE Industrial Electronics Society Annual Conference*, Vienna, Austria, November 2013.

[6] M. Vrakopoulou, J. L. Mathieu, and G. Andersson. Stochastic optimal power flow with uncertain reserves from demand response. In *Proceedings of Hawaii International Conference on System Sciences*, pages 2353–2362, Jan 2014.

[7] K. Margellos, P. Goulart, and J. Lygeros. On the road between robust optimization and the scenario approach for chance constrained optimization problems. *IEEE Transactions on Automatic Control*, 59(8), 2014.

[8] M. Campi, G. Calafiore, and Maria Prandini. The scenario approach for systems and control design. *Annual Reviews in Control*, 33(2):149–157, 2009.

[9] E. Delage and Y. Ye. Distributionally robust optimization under moment uncertainty with application to data-driven problems. *Operations Research*, 58(3):595–612, 2010.

[10] D. Bienstock, M. Chertkov, and S. Harnett. Chance constrained optimal power flow: Risk-aware network control under uncertainty. *SIAM Review*, 56(3):461–495, 2014.

[11] L. Roald, F. Oldewurtel, T. Krause, and G. Andersson. Analytical reformulation of security constrained optimal power flow with probabilistic constraints. In *Proceedings of IEEE PowerTech*, June 2013.

[12] J. Luedtke and S. Ahmed. A sample approximation approach for optimization with probabilistic constraints. *SIAM Journal on Optimization*, 19(2):674–699, 2008.

[13] R. Jiang and Y. Guan. Data-driven chance constrained stochastic program. Available: `http://www.optimization-online.org/DB_FILE/2013/09/4044.pdf`, 2013.

[14] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas. Matpower: Steady-state operations, planning, and analysis tools for power systems research and education. *IEEE Transactions on Power Systems*, 26(1):12–19, 2011.

[15] J.L. Mathieu, M. Kamgarpour, J. Lygeros, G. Andersson, and D.S. Callaway. Arbitraging intraday wholesale energy market prices with aggregations of thermostatic loads. *IEEE Transactions on Power Systems (in press)*, 2014.

[16] G. Papaefthymiou and B. Klockl. Mcmc for wind power simulation. *IEEE Transactions on Energy Conversion*, 23(1):234–240, 2008.