Data-driven Optimization Approaches for Optimal Power Flow with Uncertain Reserves

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Introduction

Wind, solar, and other uncertain power generations have increased to reduce the environmental impact of the electric grid. Power system operators thus will have to purchase more reserves to balance real-time supply-demand imbalances stemming from the large amount of uncertainty. Scheduling load-based reserves is an especially challenging task because the amount of available reserves is itself uncertain; specially, it is a function of stochastic factors including weather and load usage patterns. One option is to offer the expected amount but explicitly consider reserve uncertainty within a stochastic Optimal power flow (OPF) formulation. We formulated a chance constraints to handle the uncertainty in load control reserve capcities. Previously, a chance-constrained optimal power flow (CC-OPF) is reformulated by the scenario approach (Margellos et al., 2014), which requires no assumptions of uncertain distributions but does require significant numbers of uncertain scenarios, therefore, data. In practice, such data may not be available. While, robust reformulations require less data and are more conservative.

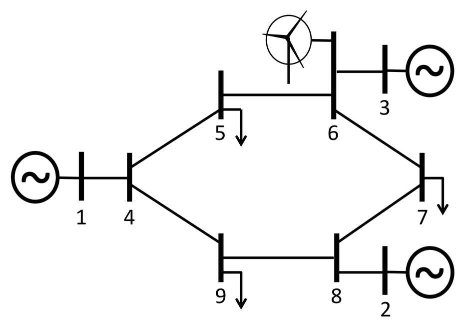

Figure 1: IEEE 9-bus system with one wind farm.

Objectives

The challenging part in CC-OPF problems (J-CC-OPF and I-CC-OPF) is how to reformulate chance constraints (1) so that our problems are tractable.

$$\mathbb{P}\left(\widetilde{A}_i x \geq \widetilde{b}_i\right) \geq 1-\epsilon_i \quad i=1,\ldots,m. \tag{1}$$

We provide a variety of methods to handle the chance constraints and investigate the performance of methods given limited information about uncertainty distributions, which is specified as follows

1. Investigate solution approaches that require knowledge of uncertainty distributions and/or significant data: mixed-integer linear programming, reformulation via Gaussian approximation, and scenario approximation.
2. Assume that we do not know the uncertainty distributions or their forms a-priori, and do not have sufficient data for scenario approximation. We apply the distributionally robust (DR) optimization approach to use "the value of data" to manage uncertainty.

Materials and Methods

We provide the key parts of reformulations in each method we are going to investigate as follows

1. refomulate (1)

(a) A1: Mixed Integer Linear Programming (MILP) (Luedtke and Ahmed, 2008)

$$A_i^s x \geq b_i^s - M y_s^i \; \forall s \in \Omega, \; i=1,\ldots,m \tag{2}$$

$$\sum\nolimits_{s\in\Omega} p^s y_s^i \leq \epsilon_i, \; \forall i, \text{ and } y_s^i \in \{0,1\} \; \forall s, \; i, \tag{3}$$

where M is a large scalar coefficient.

(b) A2: Gaussian Approximation

Assume the uncertainty is Gaussian distributed.

$$b_i' - \mu_i^{\mathsf{T}} \bar{x} \geq \Phi^{-1}(1-\epsilon_i)\sqrt{\bar{x}^{\mathsf{T}} \Sigma_i \bar{x}} \quad i=1,\ldots,m. \tag{4}$$

(c) A3: Scenario Approximation (Campi et al., 2009)

$$A_i^s x \geq b_i^s \; \forall s \in \Omega_{\text{ap}}. \tag{5}$$

2. reformulate the DR variant

$$\inf_{f(\xi)\in\mathcal{D}} \mathbb{P}_\xi(\widetilde{A}_i^\xi x \geq \widetilde{b}_i^\xi) \geq 1-\epsilon_i \; \forall i=1,\ldots,m. \tag{6}$$

(a) A4: Distributionally Robust Optimization

Given samples $\{\xi^i\}_{i=1}^N$ of ξ, we first calculate the empirical mean and covariance matrix as $\mu_0 = \frac{1}{N}\sum_{i=1}^N \xi^i$ and $\Sigma_0 = \frac{1}{N}\sum_{i=1}^N (\xi-\mu_0^i)(\xi-\mu_0^i)^{\mathsf{T}}$, and then build a confidence set (Delage and Ye, 2010)

$$\mathcal{D} = \left\{ f(\xi) : \begin{array}{l} \int_{\xi\in\mathcal{S}} f(\xi)d\xi = 1 \\ (\mathbb{E}[\xi]-\mu_0)^{\mathsf{T}}(\Sigma_0)^{-1}(\mathbb{E}[\xi]-\mu_0) \leq \gamma_1 \\ \mathbb{E}[(\xi-\mu_0)(\xi-\mu_0)^{\mathsf{T}}] \preceq \gamma_2\Sigma_0 \end{array} \right\}.$$

(Jiang and Guan, 2013) Let r_i, $\begin{bmatrix} H_i & p_i \\ p_i^{\mathsf{T}} & q_i \end{bmatrix}$, and G_i be the dual variables associated with the three constraints in the above confidence set $\mathcal{D}$, respectively. The individual chance constraints (6) are equivalent to

$$\gamma_2\Sigma_0 \cdot G_i + 1 - r_i + \Sigma_0 \cdot H_i + \gamma_1 q_i \leq \epsilon_i y_i \tag{7}$$

$$\begin{bmatrix} G_i & -p_i \\ -p_i^{\mathsf{T}} & 1-r_i \end{bmatrix} \succeq \begin{bmatrix} 0 & \frac{1}{2}\bar{A}_i^x \\ \frac{1}{2}(\bar{A}_i^x)^{\mathsf{T}} & y_i + (\bar{A}_i^x)^{\mathsf{T}}\mu_0 - \bar{b}_i^x \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} G_i & -p_i \\ -p_i^{\mathsf{T}} & 1-r_i \end{bmatrix} \succeq 0, \begin{bmatrix} H_i & p_i \\ p_i^{\mathsf{T}} & q_i \end{bmatrix} \succeq 0, y_i \geq 0, \; i=1,\ldots,m, \tag{9}$$

where operator "·" in constraint (7) represents Frobenius inner product of two matrices (i.e., $A \cdot B = \text{tr}(A^{\mathsf{T}}B)$). This is a semi-definite program and can be solved by commercial solvers.

Results and Discussion

We present the results of J-CC-OPF/I-CC-OPF that correspond to approaches A1/A1–A4 on the IEEE 9-bus system (Figure 1). We use the same randomly selected 20 samples as A1 to derive the first and second moments that are needed by A2 and A4; for A3, we randomly select 900, 500, 300 samples with $1-\epsilon_i=$ 95%, 90%, 85%, respectively. The average, minimum, and maximum values of the objective values (i.e., Obj.), reliability (i.e., Rel(%)), and CPU time (i.e., CPU) are given for each approach in Table 1. Since the results under different risk levels for each approach are similar in their pattern. We only give the results of I-CC-OPF from A4 under different risk levels in Table 2.

Table 1: Results for IEEE 9-Bus System with $1-\epsilon_i = 95\%$

	Obj.			Rel(%)			CPU		
	avg	min	max	avg	min	max	avg	min	max
A1 J-CC-OPF	1349	1328	1363	77	8	95	2	1	4
I-CC-OPF	1346	1336	1357	72	46	90	5876	131	32817
A2 I-CC-OPF	1349	1340	1358	82	65	94	1	1	1
A3 I-CC-OPF	1408	1371	1525	100	99	100	55	54	57
A4 I-CC-OPF	1393	1365	1458	100	98	100	5	4	6
	Cost			Performance			Computation		

Table 2: Results of I-CC-OPF solved by the DR approach A4

		$1-\epsilon_i =$	95.00%	90.00%	85.00%
Individual	Objective cost	avg	1392.64	1369.23	1359.97
		min	1352.46	1346.62	1346.62
		max	1457.81	1385.24	1372.75
	Reliability (%)	avg	99.50	97.97	94.51
		min	91.40	91.40	83.29
		max	99.96	99.70	99.18
	CPU seconds	avg	6.63	6.98	6.95
		min	6.13	4.73	6.27
		max	8.19	8.44	7.83

A2–A4 use much shorter time to compute I-CC-OPF. Approach A3 takes the longest time due to the large sample sizes it requires. Moreover, the solution time of A3 depends on the number of samples we select, while those of A2 and A4 are independent of samples sizes. The objective cost of A4 are averagely lower than that of A3, which is because A4 is less conservative and involved only with the moment information. As a trade-off, the lowest reliability of A4 is less than that of A3.

Conclusions

The DR approach provides decision makers a nonparametric distribution-free method for solving CC-OPF problems under ambiguous distributional information. It is less computationally-intensive and requires less data than scenario-based methods. While the DR approaches perform better than the Gaussian approxiamtion or sample average approximation (MILP formulation).

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