Two-stage Distributionally Robust Optimal Power Flow with Flexible Loads

MPEL MICHIGAN POWER & ENERGY LABORATORY UNIVERSITY OF MICHIGAN

Yiling Zhang, Bowen Li*, Siqian Shen, and Johanna L. Mathieu
*Corresponding author <libowen@umich.edu>

Motivation

- Aggregations of electric loads can provide reserves but their capacities are usually uncertain and affected by usage patterns and ambient conditions.
- To manage uncertainties from **renewables, loads and load-based reserves**, stochastic optimal power flow problems have been formulated and solved, e.g., [2].
- ***Contribution:*** **We develop a two-stage distributionally robust optimal power flow (DR-OPF) model to optimize energy and reserve dispatch under these uncertainties and derive a quadratic program using the method in [1]. We further compare the performance of this model with a distributionally robust chance constrained optimal power flow model (DR CC-OPF) [2].**

Formulation

1. Distributional Ambiguity Set and Lifting Transformation

$$\mathcal{F} = \left\{ \mathbb{P} \in \mathcal{P}_0\left(\mathbb{R}^{I_1}\right) : \begin{array}{l} \mathbb{E}_{\mathbb{P}}[\tilde{z}] = \mu \\ \mathbb{E}_{\mathbb{P}}[|\tilde{z}_i - \mu_i|] \le \sigma_i, \ \forall i \in [I_1] \\ \mathbb{P}(\tilde{z} \in \mathcal{V}) = 1 \end{array} \right\}$$

$$\mathcal{G} = \left\{ \mathbb{Q} \in \mathcal{P}_0\left(\mathbb{R}^{I_1} \times \mathbb{R}^{I_2}\right) : \begin{array}{l} \mathbb{E}_{\mathbb{Q}}[\tilde{z}_i] = \mu_i, \ \forall i \in [I_1] \\ \mathbb{E}_{\mathbb{Q}}[\tilde{u}_i] \le \sigma_i, \ \forall i \in [I_2] \\ \mathbb{Q}((\tilde{z}, \tilde{u}) \in \bar{\mathcal{V}}) = 1 \end{array} \right\}$$

2. Two-stage Formulation

Objective:

$$\min_{x \in X} \left\{ c_x^\top \langle 1, P_G, P_G^2, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L \rangle + \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[Q(x, \tilde{z})\right] \right\}$$

First Stage (deterministic)

$$\sum_{i=1}^{N_G} P_{G,i} = \sum_{i=1}^{N_L} P_{L,i}^f - \sum_{i=1}^{N_W} P_{W,i}^f,$$

$$\sum_{i=1}^{N_G} d_{G,i} + \sum_{i=1}^{N_L} d_{L,i} = 1,$$

$$x \ge \mathbf{0},$$

Second Stage (Stochastic)

$$Q(x, \tilde{z}) = \min_{y \in Y} c_y^\top y,$$

Some example constraints

$$\underline{P}_G \le P_G + R_G + y_{\text{Gl}},$$

$$P_G + R_G - y_{\text{Gu}} \le \overline{P}_G,$$

$$\underline{\widetilde{P}}_L \le \widetilde{P}_L + R_L + y_{\text{Ll}},$$

$$\widetilde{P}_L + R_L - y_{\text{Lu}} \le \overline{\widetilde{P}}_L,$$

3. Enhanced Linear Decision Rule

$$Q(x, \tilde{z}) = \min_y \left\{ c_y^\top y : \ A(\tilde{z})x + By \ge b(\tilde{z}) \right\}$$

$$y(\tilde{z}, \tilde{u}) = y^0 + \sum_{i \in W} y_i^1 \tilde{z}_i + \sum_{j \in U} y_j^2 \tilde{u}_j,$$

$$\min_{y^0, y^1, y^2} \sup_{\mathbb{Q} \in \mathcal{G}} \mathbb{E}_{\mathbb{Q}}\left[c_y^\top y(\tilde{z}, \tilde{u})\right]$$

$$A(\tilde{z})x + By(\tilde{z}, \tilde{u}) \ge b(\tilde{z}),$$

4. Combining 1 to 3, we derive a quadratic program, following the steps in [1].

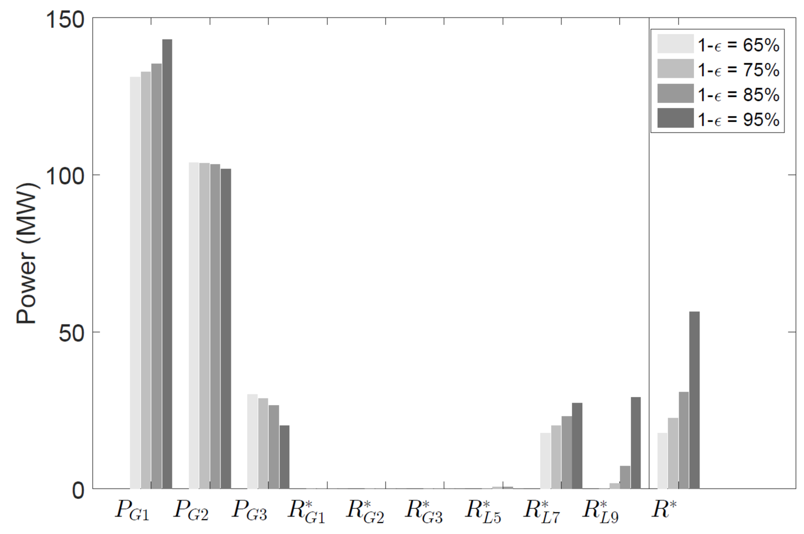
References

[1] D. Bertsimas, M. Sim, and M. Zhang, "Distributionally adaptive optimization," Working paper; available at Optimization-Online, 2016.
[2] Y. Zhang, S. Shen, and J. Mathieu, "Distributionally robust chance-constrained optimal power flow with uncertain renewables and uncertain reserves provided by loads," IEEE Transactions on Power Systems, 32(2), 1378-1388, 2017.

Results

1. Energy and reserve dispatch differences

Two-stage Distributionally Robust OPF (DR-OPF)



Distributionally Robust Chance-constrained OPF (DR CC-OPF)

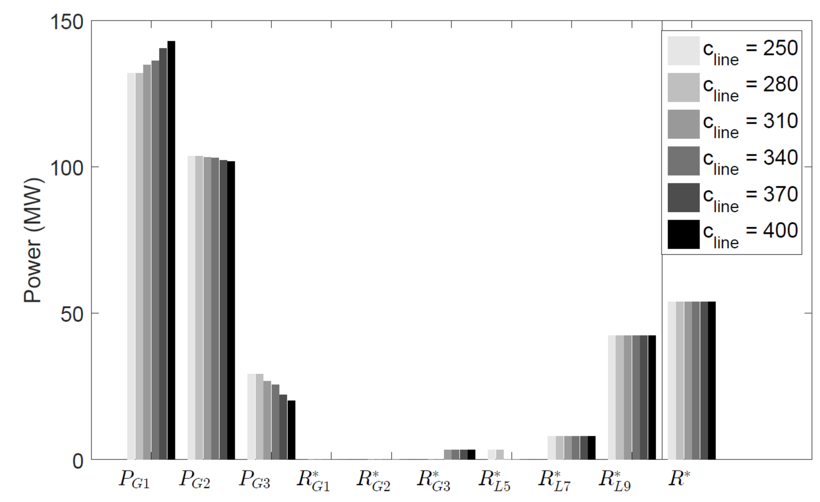


Figure 1. Solution pattern differences between DR-OPF and DR CC-OPF under different penalty costs C_{line} and confidence levels 1-ε.

2. Comparison of cost and reliability (high reliability cases)

Congested Line	Model	Cost	Reliability	Reserve Capacity
1-4	DR-OPF	**4388.16**	100.00%	53.83
	DR CC-OPF	4401.08	100.00%	56.42
4-5	DR-OPF	**4369.12**	100.00%	53.83
	DR CC-OPF	4382.05	100.00%	56.42
5-6	DR-OPF	**4651.04**	**100.00%**	53.83
	DR CC-OPF	4655.00	99.95%	56.42
3-6	DR-OPF	**4437.25**	100.00%	53.83
	DR CC-OPF	4450.17	100.00%	56.42
6-7	DR-OPF	**4388.73**	**99.89%**	53.83
	DR CC-OPF	4396.97	99.78%	56.42
7-8	DR-OPF	**4369.12**	100.00%	53.83
	DR CC-OPF	4382.05	100.00%	56.42
8-2	DR-OPF	**4787.64**	100.00%	53.83
	DR CC-OPF	4800.56	100.00%	56.42
8-9	DR-OPF	**4375.74**	**99.99%**	53.83
	DR CC-OPF	4382.06	98.45%	56.42
9-4	DR-OPF	**4371.73**	**99.55%**	53.83
	DR CC-OPF	4382.07	98.45%	56.42

3. Comparison of cost and reliability (equivalent cost)

$c_{\text{line}}/1-\epsilon$	Model	Cost	Reliability	Reserve Capacity
320	DR-OPF	5007.29	92.31%	30.74
79.9%	DR CC-OPF	5007.51	**94.20%**	25.80
338	DR-OPF	5048.54	95.44%	30.74
83.4%	DR CC-OPF	5048.60	**95.93%**	29.01
350	DR-OPF	5098.47	**97.72%**	30.74
86.6%	DR CC-OPF	5098.50	97.66%	32.90

Findings

- **Under high reliability (close to 100%), out of sample tests show that DR-OPF yields slightly better reliability and lower cost than DR CC-OPF.** When reliability requirement is low, DR-OPF could perform worse than DR CC-OPF**.**
- Enhanced linear decision rule is justified through empirical tests. Inclusion of auxiliary variables strengthens its explanatory power.
- Future work will focus on developing a multi-stage distributionally robust optimal power flow formulation using similar techniques.

This work was supported by the National Science Foundation under Grant No. CCF-1442495.