Examining Institutional Racism Within Mathematics Instruction

by

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Abstract

In this work I investigate the connection between teaching practices and institutional racism. I combine concepts from critical realism and critical race theory to develop a theory to better describe how local social interactions that occur in a mathematics classroom can disrupt common interactions that lead to the reproduction of the racial structure that permeates contemporary U. S. society. Drawing primarily on the concept of norm circles, I discuss how specific mathematical instructional practices supported the creation of a conflictive normative spaces inside of a classroom in which local disruption of racism are more likely to occur.

I apply the theory in an empirical experiment to refine and improve it. I analyzed episodes of instruction from an elementary mathematics laboratory classroom. The application of the theoretical framework consisted first of identifying instances in which the teacher enacted a teaching practice that counter an expected action. The expected action was guided by the literature review on teaching Black children and positioning Black girls in a classroom. Then I checked the normativity of the teacher action to confirm it as a regular instructional practice in this classroom. I also checked the norm circle the endorsed such practice by identifying the members of the circle and how the teaching practice was reinforced.

I identified four instructional practices that locally disrupted racism: (1) regulating student seating; (2) keeping the focus on mathematics; (3) regulating speaker and audience participation; and (4) responding to student's thinking. All these practices, in connection with the conception of mathematics endorsed in this classroom, supported the creation of intersectional normative spaces in which Black children were more likely to engage in doing mathematics and to expect and be expected to do so. In these spaces, they were also less likely to be disciplined or have their thinking immediately evaluated and corrected. In these spaces, Black children, in particular Black girls, were often actively and deliberately being positioned as academically and mathematically

smart. A second set of findings from this work center on the methodological operationalization of the framework. The strength of the framework rests in the normativity of the teaching practice, therefore the necessity of verifying the norm circle that locally endorses each instructional practice.

This dissertation contributes to theory and method related to the study of how racism permeates teaching practice. Connecting analyses of institutional racism to classroom micro-interactions, this study tests and articulates a theoretical framework that also offers practical leverage for developing approaches to disrupting racist patterns in instruction.

Chapter 1. The research problem

1.1. Situating the theoretical intentionality of this work

Issues of power and oppression have been studied and theorized for a long time in social sciences and a variety of theories have been developed to both understand how oppression is perpetuated in society and explore avenues for change. Different forms of oppression may require different theoretical approaches to capture particular nuances. Racial inequality in the United States is a perennial problem that is currently perpetuated in complex and covert ways (Bonilla-Silva, 2018). Critical race theory (CRT) is the more prominent theory to capture the specificities of how race operates in current U. S. society, but still portraits a stationary[1] picture of racism in society and does not explicitly unpack mechanisms of reproduction of racism between individual actions and social structure, or, in other words, how micro-level interactions can shape social structure, and how macro-structures influence human behavior. The main goal of this study is to incorporate the critical realist constructs of emergence and norm circles to CRT to capture a dynamic society that changes[2] based on individual actions that simultaneously shape and are influenced by social structures. Because education and schooling comprehend such an integral part of contemporary society and, therefore, can

1 In this case I use the term stationary in an analogy to mathematical stationary models that reflect a static non-time dependent description of a phenomena, such as a distribution of a particle in a media.
2 In this case, I use the term changes in some sort of mathematical interpretation of it in which not-changing is still some kind of change, like a zero.

reproduce major social structures (Freire, 2015; Giroux, 1983), I will focus my analyses in educational context, in particular in interactions at the level of classrooms between teacher and students.

Critical theory originated at the Frankfurt School around 1930 (Horkheimer, 2002) can be viewed as one of the more, if not the most, significant theory that attempts to capture how power permeates human relationships and interactions in society. With the development of the social sciences until now, however, the same critical theory can be seen as situated in a time and space, reflecting the relations of a German population in a context of implantation and establishment of contemporary capitalism amidst World Wars. This was certainly a context that could not provide reality about other forms of oppression occurring in other places and times. The United States is an economic power in a global capitalist world, so, in many, ways, the “classical critical theory” applies to this context. The U. S., however, has also a history of colonization and slavery that generated forms of oppression not existing in the context of the original development of critical theory. The influence of critical theory, however, is salient in the rise of critical legal studies in the U. S., which, on one hand, opened the way to the rise of critical race theory. CRT, then, is originated from critical legal studies, but focuses squarely on effects of race and racism in the interpretation and application of law (DeCuir & Dixson, 2004). It was later brought to the field of education by Ladson-Billings and Tate (1995) to investigate inequalities with respect to educational opportunities to people positioned in different racial groups (DeCuir & Dixson, 2004). CRT is currently better described as a broad range of theories that attend to issues of race and can represent a variety of communities of people of color and has provided foundational ideas to outgrown branches that address specific communities of color such as LatCrit (Bernal, 2002) and TribalCrit (Brayboy, 2005). In spite these differences that characterize such different branches and interpretations of concepts about race and racism, there are some agreement about the salience of race in current U. S. society and the importance to address issues of race in ways that reveal patterns of racial privilege and that value the experiences people of color face in their lives. In the context of education, CRT has revealed some important patterns of racial inequality.

School segregation, in particular, has been extensively documented by critical race scholars. *Brown vs. Board of Education* (1954) is the landmark judicial case about ending school segregation in the United States. The immediate consequence of this court decision is that children of color, particularly African Americans, should be allowed to attend schools attended by White children. The expectations with respect to this law is that African American students would then receive education with the same quality of White students. Derrick Bell (1980) denounced that this court decision was actually influenced by other motives that ultimately benefited Whites and perpetuated White privilege in education. Critical race scholars then documented a variety of ways in which schools are still segregated by race, including housing policies and student placements. In the case of student placements that have been documented for more than 30 years, the usual tracking practices of school tend to underrepresent students of color in advanced classes and overrepresent them in lower and remedial tracks (Oakes, 1995). Although studies have documented that such disproportional representation is, in part, due to subjective orientation by school authorities that are usually White, it is still unclear how exactly their individual actions are shaped by institutional racism and how their actions actually shape such institutions.

Moreover, the interactions that occur inside of classrooms are still not well explored with lenses that foreground race and that view classroom interactions as mediated by racialized dispositions and practices. In this study I develop and test the usefulness of the construct of norm circles as a tool to investigate classroom interactions with respect to race and racist norms. The theory I am putting forward in this paper should support the identification of actions that occur inside of classrooms and locating them with respect to racialized social norms. This theory should support the explanation of actions in light of social norms and also how these same actions may shape and change norms. I intend to expand CRT by incorporating a critical realist framework to develop a more dynamic interpretation of how race operates in contemporary society.

1.2. Situating the mathematical aspect of the work

It is important to consider that I have a strong mathematical background, my teaching experiences are from mathematics classrooms, as well as my research experiences. So, although the theory articulated here is not mathematics specific by design, this study will be *mathematically oriented*. This means that I will examine mathematics instruction rather than generic instruction and, thus, will be attentive to specificities of mathematics to carry out this work. I see instruction as “interactions among teachers and students around content, in environments.” (Cohen, Raudenbush, & Ball, 2003), and the specificity of the content greatly impacts the relationships that constitute instruction. There are however, “different mathematics” to be considered. I will be particularly attentive to the specificity of the mathematics that is taught at schools and how it resembles (or not) mathematics as a field of knowledge.

As a field of study, mathematics often enjoys a distinctive status among the sciences. Such high status comes from the idea accepted by most practicing mathematicians that mathematics is universal, objective, and certain (Ernest, 1999), or, in other words, mathematics comprises *a priori* knowledge, that deals with truths that are true by virtue of necessity, and with objects that are abstract (Linnebo, 2017). These ideas, however, are not unanimously shared, and disagreement about such assumptions can be traced back to the work of Plato and Aristoteles (Machado, 2013). Whereas refined arguments were developed to support one or another claims, the debate is still open. The work of Kitcher (1984) and Ernest (1998) are examples of relatively recent work supporting an *a posteriori* view of mathematics that is intrinsically dependent of human activity in society, but introductory texts on philosophy of mathematics such as George and Velleman (2002) still presuppose that mathematical knowledge can be philosophically investigated disregarding its social and historical construction.

In the context of education, researchers concerned with socio-political dimensions of mathematics education have been interested in these philosophical disputes. Such scholars have a long-term commitment to understanding the relationships between mathematics education and society, particularly relationships involved in social inequality and injustice. To these scholars it is important to understand if mathematics inherently carries fragments of its socio-political-historical development or if mathematics itself is value-free and only reflects political dimensions by how it is used in society. It is clear for socio-political researchers that mathematics and mathematics education are deeply connected with our contemporary social structure that privileges a group of people over others. Less clear is whether this connection comes from pure mathematical knowledge itself or not.

Narrowing the gaze to possible relationships between mathematics and race, the teaching and learning of mathematics that occur in schools is embedded in the broader society, hence a place where racist practices are normalized. In current U.S. society that is highly dependent on production of technology, mathematics as often perceived as a highly specialized, complex domain of knowledge predominantly mastered by Whites. Thus, through a CRT lens, mathematics is often inequitably constructed and leveraged as "White property" in school classrooms, and in society at large (Moses & Cobb, 2001; Ladson-Billings, 1999). In other words, the ability to learn and excel in mathematics is a function of being White. Mathematics is a means to sustain control over what is developed in technology and who can develop such technology (Apple, 1992). Of course any school will offer mathematics courses, but the kind of mathematics offered will be different to different groups of students (Anyon, 1981). The highly valued mathematics is the mathematics that promotes reasoning and conceptual understanding, the kind that will, eventually, lead to technological innovations. This is the kind of mathematics that is usually not accessible to socially marginalized students of color (Powell & Brantlinger, 2008). So, racism permeates mathematics classrooms is by selecting who gets access to what mathematics.

Such selection does not, however, occur in one single way. One way mathematics can be restricted to students of color is by tracking placements (Oakes, 1995) and course offerings (Solórzano & Ornelas, 2002), but what I am interested in this study is how interactions that occur within instruction can interfere with the distribution of mathematics. Students of color are often exposed to classes in which mathematics is taught as a set of rules and its learning can be achieved only by drill and repetition. For example, Ladson-Billings (1997) in p. 701 briefly elaborates on the pedagogy of poverty from Haberman (1991) to call out how a set of normal teaching practices such as “giving information, asking questions, giving directions, making assignments, monitoring seatwork, reviewing assignments, giving tests, reviewing tests, assigning homework, reviewing homework, settling disputes, punishing noncompliance marking papers, and giving grades” can be appealing to those who fear people of color and have low expectations from students of color and supports the teaching of an impoverished version of mathematics, based on drill, rules, and repetition.

In summary, I will investigate mathematics instruction and its relationships with racial structures in U. S. contemporary society while being particularly attentive to the specificity of philosophical foundations of mathematics.

1.3. The layered aspect of this work

This work starts from a theoretical purpose of combining critical race theory and critical realism to better understand institutional racism in U.S. While this is the central and crucial point of this work, a social theory is only good when it can be applied to actually investigate social problems, so I intend to add one more layer to account for the applicability of the theory. This layer is methodological and refers to questions about study design and analytical approaches to operationalize the theory with actual empirical data. To answer such question I conduct an empirical experiment in which I study a case of mathematics instruction in a mathematics laboratory classroom.

The research question I am seeking to answer are:

1. *How can we better understand the mechanisms through which institutional racism is perpetuated through social micro-level interactions that occur in classrooms? Conversely, how can we envision ways to challenge institutional racism through social micro-level interactions?*
2. *How can the concept of norm circles help us better understand the relationship between micro interactions that occur in a classroom and the institutional racism in which they are situated?*
3. *What considerations are there in operationalizing the concept of norm circles methodologically?*

1.4. Organization of this dissertation

This dissertation is organized in four chapters. Chapter 1 introduces the research problem of how to investigate institutional racism that permeates micro interaction that occur through mathematical instruction. It presents what is the theoretical problem of looking to micro-level interactions in light of macro-level institutions that constitute contemporary racism in U. S. It also presents how mathematics, in particular, may also shape such micro-level interactions. Chapter 2 introduces critical realism (CR). The foundational concepts of what makes the world, causation, and emergence are discussed to introduce the concept of norm circles, which are central to this work. Social stability and change are then discussed in light of norm circles. Then critical race theory is framed with a critical realist lens by interpreting each tenet using critical realist concepts. Finally, I present how I view instruction as a relational activity between teacher, students, and content mediated by social institutions that are, in particular, racist. Chapter 3 brings an empirical exploration of the framework to investigate methodological implications of the theory discussed in chapter two. In this chapter I apply the framework to study a mathematics elementary laboratory classroom. Chapter 4 discusses contributions of this work as well as its limitations. It also suggests possible future directions for research.

Chapter 2. Theory

2.1. Critical Realism

Critical realism is a philosophy of science that starts from a realist conception of the world, which Elder-Vass (2012) concisely summarizes as “the belief that there are features of the world that are the way they are independent of how we think about them” (p. 6). Moreover, these are *the real things* that make the world and are viewed as structures and mechanisms, or, in other words, causal laws (Bhaskar, 2008). The world, however, according to Bhaskar (2008) has different ontological dimensions: empirical, actual, and real. The empirical world contains events that actually occur, the actual is our material world that contains things and events caused by those things, and the real world contains not only things and events, but also structures and mechanisms underlying events that occurred or can potentially occur. Elder-Vass (2010) highlights how, in Bhaskar’s conception, the empirical world is a subset of the actual, which is a subset of the real. He also argues that although the power to bring about an outcome depends of a material thing from the actual domain, the power itself lies in the real domain. The example he cites is that a bird, by its corporeal structure, has the power to fly. The power, even though is attached to a bird, is not of the bird, but of its physical structure, so, in fact, anything with the same structure could fly (Elder-Vass, 2010, p. 46). One way of making sense of such interaction between worlds is that a particular power may exist in the real world, but we can only see its effect one it is activated in the actual or empirical domain. In a critical realist perspective, science looks for structure and mechanisms in the real world but can only observe events that actually occur. Table 1 brings Bhaskar's summary of the three domains of the world and figure 1 brings a Venn diagram representing the same domains.

Table 2.1: Bhaskar's three domains. (Bhaskar, 2008, p. 56)

	Domain of the real	Domain of the actual	Domain of the empirical
Mechanisms	X		
Events	X	X	
Experiences	X	X	X

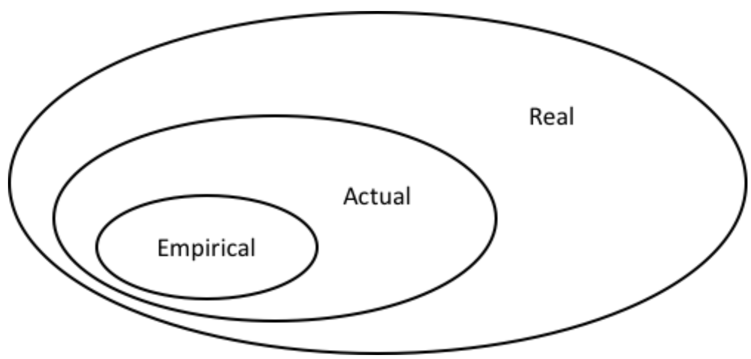


Figure 2.1: Bhaskar's three domains.

In CR, phenomena cannot be completely determined by scientific laws, they are only influenced by scientific laws. The main idea is that these laws impose constraints and prevent possibilities otherwise available, describing a tendency rather than a certain outcome. The example cited by Bhaskar (2008) in p. 95 is that the path of his pen does not violate any law of physics, nevertheless it is also not determined by such laws. There is a limitation of what a pen can do that is described by the laws of physics, yet such laws do not determine what is being traced by the pen. What is important in these basic ideas is that the world, especially the social world, is made by real things; and that scientific laws, social laws in the social world, describe tendencies rather than determination. When I say that race causes segregation within schools through tracking I am describing the tendency of over representing White and Asian students while underrepresenting African American and Latinx students in advanced classes, and over representing African American and Latinx while underrepresenting Whites and Asian in lower tracks (Oakes, 1995). Race influences but does not completely determine what is going to be a student placement.

One concept that is central for CR and that will be very relevant for this study is the concept of emergence. Here, I am particularly adopting the compositional version of emergence as described by Elder-Vass (2010). In this version, the real things in the world can be combined in a way that, because of their structure and not only its individual properties put together, a new thing emerges in the world. Elder-Vass (2010) also refers to this new thing as an “entity” or whole, and it possesses “properties or capabilities that are not possessed by its parts.” (p. 4) The idea is that the whole is not just the sum of its parts, but it is something else, with a new causal power that is, of course, derived from the individual properties of its parts, but not only this, the way the parts interact and relate with each other is also responsible for the emergence of the new thing. Water is a common example of emergence. It is composed of hydrogen and oxygen atoms, but it has properties not possessed by them. The boiling point of water is an emergent property because it is not possible to determine it based only in hydrogen and oxygen properties. The mass of water is not an emergent property because it can be determined by just adding the atomic masses of hydrogen and oxygen. The concept of emergence is what forms the layered or laminated view of the world under the critical realist perspective. A particular whole is said to be in a higher level or layer than its parts. The same whole, however, can be a part of another emergent structure; in this case the whole is in a lower level than the new emergent structure.

Before I apply the concept of emergence to interpret the social world, I will define some key terms and ideas. According to the Oxford Dictionary of Sociology (Scott & Marshall, 2009), *social norms* are shared expectations of appropriate behavior; *social institutions* are clusters of norms; and *social structure* is the organization of society with respect to norms and actions, or, in other words, the combination of institutionalized expectations and actual individual actions. Social norms about going to a restaurant include waiting for the host or hostess to be seated, ordering drinks first, then ordering food when the server brings the drinks, giving a tip of about 20 percent of the bill to the server. Norms in a classroom can include daily routines, such as delivering homework; norms about discourse, such as disagreeing with ideas, not people; sociomathematical norms (Yackel & Cobb, 1996), such as what is considered a different solution to a problem; among others.

With these basic concepts and the idea of emergence, I am viewing an individual in the lowest level of the social world, the whole society as the highest level, with many intermediate levels in between, such as social institutions. The immediate higher level to an individual is, according to Elder-Vass's (2010) definition, a *norm circle*. The norm circle is defined by the group of individuals who hold a normative belief of endorsing a social norm (Elder-Vass, 2010). By endorsing, he means that each individual in the norm circle acts to reinforce the norm and discourage behavior that does not conform to the norm. Elder-Vass (2010) argues that the shared endorsement of a norm

> when combined with these sorts of parts, provide a generative mechanism that gives the norm circle an emergent property or causal power: the tendency to increase conformity by its members to the norm. The property is the institution and the causal power is the capability that the group has to affect the behaviour of individuals. That causal power is implemented *through* the members of the group, although it is a power *of* the group, and when its members act in support of the norm, it is the group (as well as the member concerned) that acts. (p. 124)

With this argument, Elder-Vass (2010) is explaining why the norm circle is actually an emergent structure rather than only a group of people. He is explicitly pointing out what is the new causal power by showing the tendency it describes: to increase conformity to the norm. One usual norm in a mathematics classroom is assigning homework. Teachers develop the habit of assigning homework to their students from several influences, including their own school experience and the ways in which that normalizes the assumption that homework is necessary. Additionally, if a teacher were to fail to assign homework, there would be consequences, such as parents and school supervisors asking the teacher to assign homework. All of this mostly goes unquestioned and is "normal." Teachers, teacher educators, educational researchers, parents, and school supervisors are members of such circle acting to enforce the norm of assigning homework.

It is important to consider that the causal power in CR is in reference to tendencies, so the fact that a norm circle enforces compliance with a particular norm indicates that someone in this norm circle will have the tendency to act in conformity to such norm, but this is not determined. Individuals have agency to act in conformity to a norm, resisting the norm, or even in another way. Moreover, there may be norm circles enforcing different norms about a same situation and that such different norms might not be consistent at times. Individuals may be exposed to or even participate in these kinds of circles, and then they would have to decide what norm to follow every time they face a situation guided by this norm circles. Elder-Vass (2010) asserts that in this situation the final human behavior is difficult to anticipate:

> In contexts of complex normative intersectionality, skilled social performances depend upon the possession by the individual of a sophisticated practical consciousness of the diversity, applicability and extent of the normative circles in which they are embedded, and indeed of others to which they are exposed, even though they may not be parts of them. Whether or not they are able to articulate this consciousness discursively, members of such societies depend upon it whenever they act. (p. 133)

Particularly when individuals participate in conflicting norm circles –i.e., circles that enforce opposite norms—the outcome in terms of individual behavior can be very poorly predicted Elder-Vass (2012). The individual can decide in favor of either norm, or, they can create an innovative action to escape the ambiguous situation. For instance, given the fact that African American boys frequently experience low expectations in mathematics (Berry, 2008) and the deficit discourse toward minority populations (Gutiérrez, 2008), there must be a norm that a mathematics teacher should hold lower expectations for Black boys in a mathematics classroom. There are, however, teachers who hold high expectations from these students (Ladson-Billings, 1997). Stinson (2008) reports that the motivation for his study was the confrontation of this norm. He was exposed, through his experience as a teacher, to African Americans who excelled in mathematics, which contradicted research reproducing deficit discourses about these students. One outcome of such conflicting discourses was his study telling the counterstories of four African American males who were successful in mathematics.

Individual action in a critical realist emergentist account can be viewed as two different processes. In one hand, individual action is an emergent property of the biological human body composition. Because of the organization of brain cells, humans acquire mental properties that have decision-making causal power. Individual actions are not, however, solely determined by an individual's will. Norm circles increase the tendency of humans acting according a particular social norm, they causally influence human behavior. Elder-Vass (2010) draws on the work of Margaret Archer, that stresses conscious reflexive deliberation, and Pierre Bourdieu, with the notion of habitus, to elaborate his explanation of human actions. In his theory, human action tends to reproduce social structure, but, in some conditions, because of the reflexivity, individuals can modify their own dispositions. This explanation makes explicit that reflexivity may actually change a disposition to act, which can consequently change an outcome action, even if the action was made without any conscious decision about it.

Moreover, social structure is dynamically shaped by individual actions in a cycle that can either reproduce it or change it. In one moment, the structure, in the form of norm circles, constrain individual actions that, as a consequence, tend to act to reproduce it. Nevertheless, at the moment of action, the final outcome behavior, that depend on multiple actual factors, may not conform with the norm. Elder-Vass (2010) claim that it is not necessary that every action needs to be in conformity to the norm to the norm to be sustained, only that "such behaviour tends to predominate to the extent required to sustain the normative beliefs of the members of the norm circle." (p. 134) Social change would happen when the collective human action would be subverted enough to weaken the power of the circle to increase conformity with the norm, or even to completely disappear or change, society would "naturally" change when a particular norm becomes obsolete. Additionally, in Elder-Vass's (2010) theory, contexts of complex normative intersectionality may provide increased actual factors that will highly influence final behavior, and therefore, may change the norm altogether.

2.2. A critical realist take on critical race theory

Critical race theory is a theoretical framework for research that foregrounds race, racism, and racialized experiences. To critical race scholars, race is a complex social construct that goes beyond the color of skin and citizenship (Ladson-Billings, 1999), and brings real consequences to people once they are identified as member of a particular racial group (Bonilla-Silva, 2018). Critical race scholars often discuss how a social construct can actually bring real consequences to lives of people. In a critical realist perspective, there should be a set of racist norms being reinforced by norm circles that sustain such norms. These set of norms would constitute an institutional reality with respect to race that "it is so enmeshed in the fabric of our social order, it appears both normal and natural to people in this culture." (Ladson-Billings, 1999, p. 12). This idea is the gist of *permanence of racism,* the foremost premise of CRT (DeCuir & Dixson, 2004). In a critical realist perspective, this is viewed as a stabled phase of society characterized by the reproduction of racist norms through their endorsement by norm circles.

One example of norm being reinforced to construct this institutional reality is the one endorsing the normative deficit discourse that positions African Americans (and other minority groups) as academically less than Whites (Gutiérrez, 2008). This norm may be traced back to the colonial period as a way to justify the slavery system (Douglass, 1892). Once the norm circle reinforcing such discourse had emerged, it started to operate downwards, constraining individuals to act accordingly. The way the discourse is reinforced has changed throughout time though. For example, in the beginning of 20th century, IQ tests (Karier, 1986) helped to disseminate the idea that Blacks were less intelligent than Whites; and more recently, research reports such as Oakes (1995) reveals that African American students are less likely to be placed in higher track courses, in comparison to their White peers, even when their achievement is similar. The norm being reinforced in this example is that members of this circle can say (and believe) that African Americans are academically less in comparison to their Whites counterparts, or equivalently, they cannot say that African Americans and Whites are equally good.

Racial structure, as defined by Bonilla-Silva (2018), is “the totality of the social relations and practices that reinforce white privilege.” (p. 9) This racial structure can be seen as the complete set of social norms endorsed by circles committed in sustaining white privilege. These circles, through their members, have the causal power to increase conformity with such norms. Such causal power indicates the reality of racial structure in an emergentist critical realist sense. In short, the reality of race as a social construct is given by the causal power it has, or, in other words, race causes racial inequality through institutional racism, thus is real. Furthermore, CRT posits that people of color experience racial oppression differently based on their individual background and multiple identities and insists in a non-essentialist approach (Delgado & Stefanic, 2001). Critical race scholars particularly acknowledge the complexity generated by being in the intersection of multiple forms of subjugation such as race, gender, and social class. For example, Gholson and Martin (2014) discuss how the intersectionality of race, gender, and age plays out in the mathematics learning of a group of Black girls. They report how different Black girls enact Black girlhood differently and argue that, “Black girlhood in our view is not to be perfected or achieved in a universal or developmental sense, but rather, to be seen as an elastic, eclectic, and useful construction for understanding the life experiences of Black girls.” (p. 32)

Additionally, one of the underlying assumptions of this study is that each kind of social oppression operates in particular and specific ways. So, I am assuming that sexism operates differently than racism, for example. Critical race scholars understand that CRT is not an absolute theory, but there are different interpretations of its foundations and methodologies. There are, however, some common essence, some common ideals in revealing patterns of racial subjugation. So, I look to CRT tenets as they were first introduced to me and often cite DeCuir and Dixson (2004) when I discuss them. My studies in the field have dynamically shaped my understanding of the theory, and I will now describe how I am viewing some of these tenets and how they contribute to this work by offering specific elements to look to with respect to race and how this particular form of oppression operates. Table 1.1.1 brings a summary of my view of the operationalization of racism in light of critical realism which will be unpacked in the following sections.

Table 2.2: Summary of the connection between CRT and CR.

CRT theme	CR lens
Permanence of racism: race is the norm, not the aberrant.	Describes a stable phase of society characterized by institutional racism. There are norm circles sustaining each racist norm and practice.
Whiteness as property: set of social privileges conferred to Whites that operate as property.	These ideas are the materiality of institutional racism. They give shape and form to racist norm circles informing social interactions, liberal framings protect Whiteness as property.
Abstract liberalism: set of ideas with respect to individual liberties often used to mask and render invisible racial inequality. Equal opportunities for all, meritocracy, and freedom of speech can be seen as examples of such liberal ideas.	
Interest convergence: verification that many, if not all, civil rights people of color gain were achieved because Whites would also benefit from them.	The endorsement of one particular norm by racist and non-racist circles with the outcome of changing such particular norm without changing any of the other norms with respect to racial structure.
Double consciousness: ability to see normalized and invisible racial structure.	Outcome of participating or, at least, being exposed to conflicting norm circles with respect to race.

2.2.1. Whiteness as property and critique of liberalism

Whiteness as property refers to the idea that Whiteness can be viewed as a set of social (privileged) possessions that can operate similarly as property in a capitalist society (DeCuir & Dixson, 2004) and *critique of liberalism* is a direct critique to liberal ways of understanding and living in the world (DeCuir & Dixson, 2004). Critical race scholars argue that such liberal framings support colorblind racism (Bonilla-Silva, 2018). In light of the construct of norm circles, these two tenets speak about the materialization of institutional racism in daily social interactions among people, abstract liberal ideas fuel discursive norm circles to sustain White privilege. In her seminal work, Harris

(1993) already argued that the “protection of the property interest in whiteness is achieved by embracing the norm of colorblindness.” (p. 1768) One important discourse norm reinforced under a colorblind frame is the idea that race-is-something-that-should-not-be-named and facets of institutional racism are then masked under liberalist discourses and ideas. There are different ways in which race may not-be-named, it could be shifted to a matter of wealth difference (I have heard these kind of explanations countless times in Brazil: “it is not because their race, it is because they are poor”) or it could be framed under a meritocratic assumption (“all have the same opportunities, it is a matter of individual effort”), for example. The not-naming-race discursive rule renders racial structural inequality invisible and thus supports the idea that *all* are now “equal from start” and, consequentially, that racism does not exist in current U. S. society. The material outcome of such circles is that Whiteness as property is preserved.

In the context of education, Whites have some sort of control of what is considered valuable knowledge and who gets access to it, which can be interpreted as a kind of intellectual property. Narrowing it even more, to the context of mathematics education, mathematical ability is embedded in the construction of Whiteness in different ways that form a vicious circle in which Blacks and other marginalized populations are systematically excluded from mathematics and Whites are often seen as smarter in mathematics. The exclusionary practices are often justified on biased assumptions of White superiority in mathematics. In this context, meritocratic assumptions often support the idea that mathematics ability is an individual quality, that is contingent on natural ability and/or personal effort in addition of an implicit taken for granted assumption that all have the same opportunities with respect to mathematics access. Such liberal framings make systemic racial differences with respect to school resources, curriculum, and teacher qualification among other become invisible in a way that takes what “should be” for what “is.” (Delgado & Stefanic, 2013) In this work, I am looking to how the materiality of institutional racism framed as the protection of Whiteness as property under abstract liberal ideas plays out in the context of interactions among students and teacher in mathematics instruction.

2.2.2. Counter-storytelling

Counter-storytelling is the main methodological strategy used by critical race scholars to challenge inequality and White privilege by "coloring" colorblind discourse. It consists in "telling the stories of those people whose experiences are not often told" (Solórzano & Yosso, 2002, p. 32), usually telling the stories of people of color. The main purpose of counter-storytelling is to cast doubt in majoritarian stories about racial privilege and expose systemic hierarchies of power. They can also support people of color in making sense of oppressive situations they live by confirming such situations do exist and are unjust and that other people of color experience similar situations (Solórzano & Yosso, 2002).

One particular aspect explored by counter-storytelling is the double consciousness or angled vision. Du Bois (1996) was the first to use the term "double consciousness" to express how African Americans experience and make meaning of the world. He describes it as follow:

> It is a particular sensation, this double consciousness, this sense of always looking at one's self through the eyes of others, of measuring one's soul by the tape of a world that looks on in amused contempt and pity. One ever feels his twoness—an American, a Negro; two souls, two thoughts, two unreconciled strivings; two warring ideals in one dark body, whose dogged strength alone keeps it from being torn asunder. (p. 4)

Double consciousness can be attributed to oppressed people more broadly (Ladson-Billings & Donnor, 2005), however, I will focus on the experiences of people of color, which includes the intersection of race with other forms of subjugation such as gender. For example, Anzaldúa (1987) talked about her identity as mestiza to describe her Chicana multiple consciousness, and Collins (1986) used the term outsider within (p. S14) to describe the perspective of being an African American woman.

In DuBois' (1996) conception, the double consciousness granted African Americans "the gift of a second sight" to see racial privilege where Whites usually do not, to see a societal structure of racial inequality hidden in normalized social interactions. So, White teacher candidates may see student achievement as a result of individual effort (Solomon, Portelli, Daniel & Campbell, 2005), but may not recognize individual effort when carried out by African American students because it may look differently than normalized ways of doing so. Stinson (2008) reports how African American men participants in his study negotiated between identities to succeed in mathematics. In a critical realist perspective, I interpret the double consciousness as a product of participating in, or, at least being exposed to, conflicting norm circles with respect to race.

The methodological approach through counter-storytelling can be done by inviting people of color to tell their life experiences in interviews, in what is called naming their own reality (Ladson-Billings & Tate, 1995). That is not how I am viewing counter-storytelling in the framework I am putting forward though. I am viewing this approach as a space to tell experiences that challenge inequality, but from an observer perspective. Within a critical realist perspective, I understand that every human action is guided by social norms and the final outcome behavior will often be a result of non-conscious elaborations of such norms, that may then remain inexplicit in an interview. I can see that it is part of the job of the researcher to see hints of situations, combine with similar accounts, and make sense of what participants say in an interview in light of a theoretical approach. However, it is also important to call out that it is not necessary to experience oppression to perceive it. One of the goals of this work is to provide a framework that supports the understanding of mechanisms racial subjugation in contemporary U. S. society without requiring a first-person experience to identify racist situations. I view that studies that focuses on people of color naming their own reality provide a background necessary to move in the direction I am suggesting. It is important that we have some knowledge of what kind of social interaction we have to look to and how we have to look to, but what I am proposing now, is that we start actually looking to instructional interactions with a better set of eyes, that can now see race where we did not before. Moreover, I hope that we not only are able see race, but we have theoretical tools that help us to explain how some racial inequalities seems to persist, even when we claim they do not exist anymore. Or, what I am even more interested, how can we envision a disruption of these hierarchies of power and privilege that is actually possible. This is why I am focusing in telling counter narratives based on observation of instructional episodes.

2.2.3. Interest convergence

Another important tenet from CRT is *interest convergence*, which is the verification that many, if not all, civil rights people of color gain were achieved because Whites would also benefit from them (DeCuir & Dixson, 2004). Critical race scholars often argue that interest convergence prevents real disruption of racial inequality because Whites are still benefiting from changes, so the difference still persists. In the context of mathematics education, Danny Martin has recently argued against policies and slogans of "mathematics for *all*" and for a complete transformation of the educational system that would really serve Black children's educational needs (Martin, 2018). There are other scholars, such as Richard Milner, who defend the negotiation of intersectional spaces in favor of making social change actually happen (Milner, 2008). Such intersectional spaces can be interpreted as intersectional norm circles in which racist circles and non-racist circles may endorse the same norm even if they do not endorse the same set of norms altogether. Because the totality of racist norms is not challenged, the institution as a whole is maintained, even if not exactly in the same way.

2.3. Framing instruction

In this paper I am reporting the articulation of two different theories to investigate mechanisms of reproduction of social oppression with respect to race: critical realism and critical race theory. So far, the theory was described somewhat generically, and it could be applied to investigate a variety of social interactions. My work, however, is focused in educational interaction, particularly interactions that happen between teacher and students during instruction. Because this kind of interaction have specificities that make it different than other social interactions, I also need particular lenses to capture and interpret this specific type of social interaction.

I am mostly framing my view in two similar models of instruction, one from Lampert (2001) and one from Cohen, Raudenbush, and Ball (2003). Lampert (2001) starts delineating basic relationships that take place in instructional episodes. One set of relationships is between teacher and students and it is quite obvious that teacher and students must develop some kind of relationship to enable students' learning. Another relationship is between teacher and content and a third one is between students and content. Lampert (2001) focuses her representation in the actions the teacher does in these relationships, actions she views as teaching practice. In any of these relationships, students, content, and the teacher can serve as resources but also as constraints for teaching practice. Additionally, the relationship between students and content is not a teaching practice, but students' practice. Lampert, however, elaborates how a teacher has to directly work on this relationship so learning can occur: "As a teacher, I take action to make studying happen, and to make it happen in ways that are likely to result in learning." (p. 32) Moreover, the instruction representation gets even more complicated because these teaching practices are not independent; they must be coordinated by the teacher in the moment of instruction. Instruction is then a complex relational activity because it involves multiple relationships that occur simultaneously and that need to be coordinated by the teacher in the moment.

While Lampert (2001) elaborates more her model by adding layers of relationships from teaching across the year, across students, and across the curriculum, I will focus in one single piece of instruction, but with some aspects elaborated by Cohen, Raudenbush, and Ball (2003). The first aspect I am going to add in my representation for instruction is one extra relationship among students, understanding that students are interacting with one another. Even though I believe Lampert (2001) acknowledges this interaction, this is not her focus, nor does she mention it explicitly in the way Cohen, Raudenbush, and Ball (2003) do. A second aspect of their framework that I want to point out is the presence of what they call "environments." For Cohen, Raudenbush, and Ball (2003), instruction does not happen in a vacuum, it happens in social contexts that can also serve as resources or constraints for teaching and/or learning. They argue that instruction is situated in social contexts, and that teachers have to manage instruction within such social contexts. In my work I am using the concept of norm circles to better articulate how these social contexts actually serve as resource or constrain for teaching and/or learning, I am looking to classroom interaction as individual actions in light of norms enforced by norm circles. I view instruction as complex relationships among teacher, students, and content, permeated by social institutions, that are often oppressive. Figure 1 shows a representation of how I view instruction.

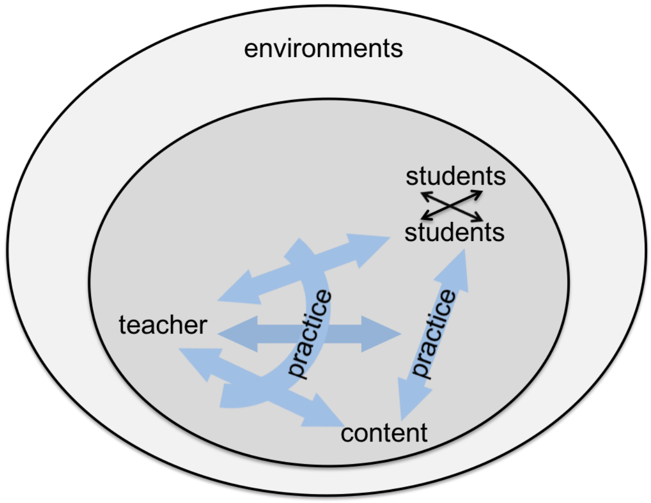


Figure 2.2: Representation of instruction adapted from Lampert (2001) and Cohen, Raudenbush, and Ball (2003) (p. 124).

Within this model for instruction I want to emphasize and that there is a great variety of norms that can influence individual action at a single instructional episode. There are broader social norms from outside the classroom and school environment that permeate instruction. These norms can be from society at large, or from the field of knowledge being taught to students. In the case of mathematics there are norms associated with what counts as a mathematical proof or what does it mean to make a conjecture for example. But there are norms that are created in the context of instruction and schooling. In some cases, the dynamics of interactions inside the classroom may create norms that contradicts norms from outside of classroom. Ball and Forzani's (2009) example about norms with respect of asking questions shows that in daily social interaction people ask questions to which they do not know the answers, but during instruction, teachers ask questions they actually know the answers as a part of their professional practice.

To investigate interactions that happen in the context of instruction requires an understanding that this context and space will bring these different layers of normativity. Moreover, the interpretation of individual actions in light of this representation of instruction requires analyzing the fine detail in each action. This may not be completely accounted for by focusing on teaching or instructional practices in the case of teacher actions, the grain size of a practice may be too large for this purpose. So I will add an expanded version of talk tools (Chapin, O'Connor, & Anderson, 2013) to refer to a specific piece of practice that is strategic to perform it well. I will call it the *instructional tool* and I will look to it within the different layers of normativity. When this piece of practice is characterized by discursive tools, such as asking questions and inviting participation in class, I may also use the term *instructional move.* Moreover, I use the term instructional practices to refer to teaching practices that occur during instruction time inside classroom. For example, I consider planning a lesson a teaching practice, but not an instructional practice.

Chapter 3. A study within a study

In this section I will apply the theory laid out in section 2 to study the mathematics instruction that occurs in an elementary mathematics laboratory. The main purpose is to investigate methodological implications of the theory, specifically point out considerations with respect to analytical approach.

In this methodological experiment, I first focus in institutional racism and the specific ways it is sustained in U. S. contemporary society. Moreover, I focus in closely on the microdynamics of classrooms, particularly to the microdynamics that occur during instruction, and how such micro interactions can reproduce or challenge institutional racism. In this setting, I am looking to teaching practices, or in other words, to actions a teacher does during instruction. As I discussed in section 1.2, my study is situated in a mathematics classroom and I am attentive to the specificities of the mathematical content. Thus, I look to the relationship between institutional racism in the United States and the mathematics instructional practices of a mathematics classroom during instructional time, such as leading a class discussion or managing small group work. Finally, I narrow my analysis to a group of Black girls to better address the complexity of race as a social construct. The classroom I am investigating is a laboratory classroom in which an experienced teacher teaches mathematics to a group of elementary students in the summer for two weeks. In some ways this classroom is unique for a variety of reasons such as: it is taught in the summer, in a university, there are observers and recording equipment. But in many other ways it is just like as any other classroom, which makes this in interesting environment to conduct research such as this study.

3.1. Situating the work in the field

3.1.1. Mathematics instruction and its practices in a critical mathematics perspective

Mathematics instruction is a persistent object of study in mathematics education. The process-product paradigm of research on teaching (Shulman, 1986) can be seen as the beginning of the recent history of the field. In this paradigm the processes of teaching were connected with products in terms of student achievement. The research of Good and Grouws (1977, 1979) is a good example of research about mathematics instruction in this perspective. In their work, the authors categorized and numerically measured teacher behavior and compared with student test scores to identify teacher behaviors that generated higher student achievement. Process-product research, however, leaves out some important aspects of human interaction, such as the interplay of individual thinking and social norms in influencing human behavior. After some other paradigms that consider student thinking and teacher thinking, research on mathematics instruction shifted primarily to perspectives derived from social studies and anthropology, embracing more the complexities of human interaction in social organizations, giving rise to the ecological paradigm (Shulman, 1986) and the sociocultural turn in mathematics education (Lerman, 2000). Classroom social interaction became one of the main foci of research on mathematics instruction. Sociocultural perspectives on learning and qualitative methodologies become more prominent in the field. The work of Doyle (1988) investigating the enactment of a task in classroom is often cited as representative of this shift of paradigm because it was one of the first studies to argue that live interactions in a classroom could drastically modify a mathematical task. A plurality of theoretical framings then promoted a variety of research attending to different themes such as classroom life and culture (Cobb, Wood, & Yackel, 1993; Cobb et al. 1997; Cobb et al., 2001), the enactment curriculum

(Remillard, 2005), teacher characteristics, including teacher knowledge (Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008) and teacher beliefs (Clark, et al. 2014; Ambrose, Clement, Philipp, & Chauvot, 2004; Stipek, Givvin, Salmon, & MacGyvers, 2001), classroom discourse (González & DeJarnette, 2012; Herbel-Eisenmann & Wagner, 2010), and the unfolding of task during instruction (Doyle, 1988; Henningsen & Stein, 1997; Hsu & Silver, 2014). However, few studies focused on what teachers actually *do* when they teach and how this impacts student learning (Ball, 2017; Ball & Forzani, 2009).

Given the need teacher educators have in preparing their students to become skilled teachers, at least skilled enough for novices, there are some studies about teaching practice within the teacher education literature. Recently, a growing body of researchers argue in favor of understanding teaching as a professional field, in which the work of teaching comprises a shared understanding by the professional community of what things teachers *do* in their practice that makes them a community of practitioners exercising the same profession, even though their work settings and contexts can be remarkably different (Lampert, 2010; Grossman, Hammerness, & McDonald, 2009; Ball & Forzani, 2009). Understanding teaching as a profession entails understanding that there are commonalities shared by the community of professionals, even when the actual practice looks strikingly different because of the local specificities and context in which it is been exercised. Additionally, becoming a teacher entails not only learning the teaching practice, but also becoming a member of a community of practitioners and identifying oneself and others as members of such community (Lampert, 2010).

Researchers and teacher educators often start from the assumption that, although learning can occur without teaching, that is not very likely, and one of the main goals of the teaching profession is actually to increase the likelihood of learning in formal school settings (Lampert, 2001; Ball, 2017; Ball & Forzani, 2009). In studying the work teachers do, such scholars insist in the complexity of teaching, although it often seems simple (Grossman, Hammerness, & McDonald, 2009). Teaching is relational and intricate work (Lampert, 2001; Ball, Raudenbush, & Ball, 2003, Ball & Forzani, 2009), in the sense that much cannot be controlled by the teacher. Teaching is highly dependent on how students respond to teachers and other students, in-the-moment judgements and decisions by the teacher, and the specificity of the content also enacted dynamically in a classroom. Furthermore, professional teaching is very different from the usual and informal teaching people do in their daily lives (Ball & Forzani, 2009) and most of what teachers do is invisible, in the sense that it hides in plain sight going unnoticed even to seasoned observers (Lewis, 2007).

To better understand the complexities of teaching, teacher educators and researchers have tried to unpack it. They attempted to understand what defines a professional practice and what are the professional teaching practices, often with a goal to teach novices to perform it well, but there is not a decisive consensus about this. Grossman, Hammerness, and McDonald (2009), however, claim that there are some commonalities in defining what constitute a teaching practice, including: practices that occur with high frequency in teaching; practices that preserve the integrity and complexity of teaching; and practices that are research-based and have the potential to improve student achievement.

I want to point out that so far, in this review of the literature on mathematics instruction I have not addressed equity or social justice, or gender and race inequalities. This is partly deliberate to show how it is possible to trace some history of the field contemplating the main paradigms used in research, the little that is known about teaching practice, without actually addressing pressing issues about social inequality and the struggle for social justice. It is true that the field of mathematics education has recently paid attention to how power dynamics significantly mediate human interaction in ways that are often hidden in most sociocultural perspectives. Some authors, such as Valero (2004) and Gutiérrez (2013) argue that the field of mathematics education is undergoing another shift, now to make such power dynamics visible. These authors claim that themes such as equity and social justice are more accepted in the field and that sociopolitical frameworks have been expanded and elaborated to capture different ways in which social oppression is (re)produced in mathematics education. Within the study of mathematics instruction, the incorporation of sociopolitical frameworks has suggested definitions for equitable instruction (Goffney, 2010; Hand, 2012), as well as alternative definitions to equity (Boaler, 2008), but it has also brought up how power dynamics may (re)create unequal participation in mathematics classrooms (Lubienski, 2000).

Gutiérrez (2013) argues that, although, there is an intense development of sociopolitical frameworks and the “talk of equity” is now more mainstream in the field of mathematics education, still many scholars rely in sociocultural perspectives and methods to address issues of social inequality in education and some research problems are yet to be addressed. The study of the interplay of race and mathematics instruction, particularly instructional practices, for example, is one such problem. There are studies about external factors that impact the opportunity to learn for students of color mostly focusing on curriculum and patterns of student placements (Oakes, 1985; Battey, 2013), there are also studies about the whole set of educational experiences of Black males (Berry, 2008; Stinson, 2008), and the experiences of Black parents with respect with respect to the schooling process their children go through (Martin, 2006), but there are no studies about how mathematics instructional practices are connected with race in ways that can sustain institutional racism. Ladson-Billings (1997) only superficially address the issue by elaborating on Haberman’s (1991) pedagogy of poverty.

In contrast, the particular area of critical mathematics has embraced sociopolitical positions and frameworks for a relatively long time. Critical mathematics as it was initially conceived, was partly influenced by critical theory present in the seminal works by Skovsmose (1994a, 1994b) and partly inspired by the work of Paulo Freire present in the seminal work by Frankenstein (1983). In any case, Paulo Freire’s ideas about *conscientização*, that means developing a social awareness to see and question oppressive sturctures, were very important in these initial studies. The application of these ideas to mathematics education led to many studies focused on the teaching of mathematics to question the distribution of income (Frankenstein, 1983; Skovsmose, 1994b; Gutstein, 2005). More recently, critical mathematics scholars have broadened their conception of critical mathematics to include a variety of perspectives that acknowledge the socio-political role of mathematics education in contemporary society (Skovsmose, 2008; Skovsmose, 2011; Powell & Brantlinger, 2008).

Within this more pluralistic view of mathematics education, my gaze in this research is on two significant perspectives on teaching mathematics, each of which seeks to challenge social oppression and promote social justice. The first one comes from the work of Rico Gutstein with Latinx students. Gutstein's work was originally influenced by Paulo Freire's ideas of *conscientização* but grew to a complex conception of teaching mathematics for social justice that acknowledges the importance of the interrelationship of community, critical, and classical knowledge (Gutstein, 2005). The second comes from the work of Gloria Ladson-Billings on culturally relevant pedagogy situated largely in her work with African American children (Ladson-Billings, 1995, 2014). Her writing is not mathematics specific, however, it still partly focuses on teaching and learning of mathematics because she studied mathematics classrooms. In her initial conception, culturally relevant pedagogy already espoused three domains aligned with Gutstein's perspective: academic success, cultural competence, and sociopolitical consciousness (Ladson-Billings, 1995). Although the two perspectives differ in how their constructs are defined, they share some important critical foundations and each of their three domains are not that far apart. First, it is important to notice that Gutstein's work is mathematics specific while Ladson-Billings' is not, but her work was developed partly based on mathematics and brings some specificity even if not as much as his. Gutstein defines his constructs based on the knowledge students must acquire from a curriculum, while Ladson-Billings thinks of them as parts of a pedagogy. Gutstein thinks students must learn academic mathematics that will help them to pass gatekeeping tests, while Ladson-Billings asserts students should be academically successful by learning academic skills to be active participants in a democracy. So both talk about the need students have to learn academic mathematics, but not exactly with the same goals. The most similar constructs are critical knowledge and sociopolitical consciousness that are both defined in terms of Paulo Freire's conscientização. The least similar are community knowledge and cultural competence. While Gutstein talks about the importance of valuing and taking as legitimate the knowledge of people from outside of academia, Ladson-Billings claims that "culturally relevant teaching requires that students maintain some cultural integrity as well as academic excellence" (Ladson-Billings, 1995, p. 160) and suggests building classical knowledge from community knowledge. Moreover, both perspectives envision a whole program for teaching mathematics, suggesting curriculum and activities to do with students, but are not focused specifically on the teaching practices of mathematics. So, for example, Gutstein discusses one activity he did with his students from the curriculum they were using about wealth distribution, what students learned during their work with the activity, but not what the teacher actually did.

This work is situated in this pluralistic view of critical mathematics, in which Powell and Brantlinger (2008) argue that

> An objective of critical mathematics ought to be to engage students, socially marginalized in their societies, in cognitively demanding mathematics in ways that help them succeed in learning that which dominant ideology and schooling practices position them to believe they are incapable. (p. 1-2)

This objective parallels Gutstein's classical knowledge and Ladson-Billings' academic competence. This objective of critical mathematics directly addresses the gatekeeping role mathematics plays in shaping citizenship in current U. S. society (Moses & Cobb, 2001). There are not, however, many studies in critical mathematics, if any, that specifically focus on the mechanisms through which mathematics instructional practices support the gatekeeping function of mathematics or, in another direction, how mathematics instructional practices can challenge it. In summary, there are many studies in mathematics instruction, but not in mathematics teaching practices and not on the link between teaching practices and student learning. There are many studies about critical mathematics, but not that focus on the link between teaching practices and disruption of oppression. This work is about the link between teaching practices and disruption of racism in a pluralistic critical mathematics perspective that encompasses the learning of academic mathematics.

3.1.2. Black girls in classrooms

Some scholars argue that Black females are often used as a comparative group to highlight experiences and characteristics of other demographic groups (Henry, 1998; Gholson, 2016), and are often "relegated to footnotes, occasional lines, a few meager paragraphs, or a couple of pages." (Henry, 1998, p. 154) However, the intersectional space occupied by Black females produces distinct identity and experiences. A few studies show that such intersectional space gets even more complex in the context of classroom activity, in which identities of Black females are also shaped by the context of schooling that include their relative position as learners and children (Gholson & Martin, 2014; Grant, 1984, 1994; Henry, 1998; Leander, 2002; Morris, 2007). This context supports positioning Black girls as subject to the authority of the teacher who occupies a position of knower and adult.

Identity, from sociopolitical perspectives, is frequently described as negotiated in the sense that an individual's identity depends not only on how the person acts, but also how their actions are perceived by other through social interaction (Gutiérrez, 2013). In the context of schooling, the identity of Black girls depends on how other students, teachers, and other school figures see them with respect to their own frames to make sense of the world. The creation of some sort of label or category to make sense of Black girls' actions have two consequences. First it narrows the view towards Black girls, making more difficult to see what they could do outside of it. For example, if Black girls are perceived as more socially developed (Grant, 1994), it would be more difficult to interpret their actions, particularly a "social fault", as a simple mistake because they are immature. Second it also supports the reproduction of the category characteristics, which promotes more people having such a narrow view towards Black girls. Grant (1984, 1994) shows that teachers not only had particular views for Black girls, but their actions also promoted the girls to act according such particular view. For example, while teachers thought Black girls' high social development hindered their academic development, their actions towards Black girls, such as in group assignments and feedback, continued to position them as more socially developed and not academically oriented. In a similar way, but with different power relationships in place, Leander (2002) showed how the actions of Shameen, a Black boy, supported positioning Latanya, a Black girl, as "being ghetto," a term that encapsulates a category of negatively perceived meanings. In particular, Latanya was perceived as losing her temper and not able to follow "appropriate" norms of participation in class. Moreover, even though the identities of Black girls are interpreted with respect to particular labels, such labels are not static, they are dynamically shaped by each social interaction and can occur inside or outside of the boundaries of classrooms (Gholson & Martin, 2014). In this work, I use the term “positioning” (Davies & Harré, 1990) to capture the dynamic negotiated nature of identity on which I focus. Finally, I am assuming, as Davies and Harré (1990), that positions are not always conscious or intentional.

I will turn now to a brief discussion about the labels attributed to Black girls in the context of education in classroom settings as reported by Grant (1984, 1994), Henry (1998), Leander (2002), Morris (2007), and Gholson, and Martin (2014). First of all, I want to point out that such studies are from a qualitative tradition and are more oriented to the description of how such labels were formed and sustained in a particular context, rather than trying to exhaustively map all categories and their members. Nevertheless, the (few) articles I am reviewing report studies from settings separated by time, space, culture, and theoretical perspectives, but their findings show remarkable similarity. I am not claiming the findings are the same in any way, but given the distinct background of each study, the emergence of some commonalities is worthy of notice.

These studies report, on the one hand, the positioning of Black girls as being “loud”, or aggressive, and describe this loudness as a consequence of femininity construction that is in opposition of a White femininity characterized by submission and fragility. The (re)production of the “loud” Black girl involved at times confronting teachers in subject matter (Morris, 2007), confronting boys both academically (Leander, 2002) or in response to borderline sexual harassment (Morris, 2007), and standing up to others to defend themselves and others in physical and verbal ways (Grant, 1984). The actions of the girls (re)producing their positioning were not a single product of their agency though, there were actions that other social actors did to motivate and feed the patterns, such as boys provoking and school authorities over disciplining Black girls (Morris, 2007). On the other hand, Black girls were usually seen as highly socially skilled. Grant (1984, 1994) is the work that describes their social skills in more details, describing how well the Black girls navigated complex social roles in classrooms to get positioned as go-betweens and coordinate communication between different groups in classroom. But the complex social network established by a community of Black girls in Gholson, and Martin (2014) also shows how Black girls can enact such positionings, and function as highly competent social actors.

In any case, Black girls were often not positioned as smart - Grant (1984, 1994), Henry (1998), and Morris (2007) all reported how teachers do not perceive Black girls as smart. Again, these studies used a variety of data sources and analytical methods, but their findings were incredibly consistent: teachers in all these studies did not position Black girls as academically competent. Moreover, the other positionings, as “loud” or socially developed, contributed to positioning the girls as not smart. For example, Grant (1994) mentions how one teacher praised a Black girl for helping her working mother at home take care of her younger siblings, but that such time allocation to her home support left her less time to focus on her academic development. Additionally, being seen as “loud” or socially developed seems to work as a “noise” in viewing Black girls, it seems to overshadow any other aspect of their personality and any other possible positioning. In both Grant (1994) and Morris (2007), teachers’ views of Black girls were consistently focused on their social skills and often related with Black girls not corresponding to White conceptions of femininity.

3.1.3. Teaching Black children (or regulating their behavior?)

One important aspect to be considered with respect to teaching Black children is that of disciplining children so they can learn. Research has extensively documented that Black children, especially boys, are over-punished in schools, being referred to the office, suspended, and expelled in higher rates than any other group (Monroe, 2005; Gregory, Skiba, & Noguera, 2010) in what has been called the discipline gap. While the problem is not new, it is persistent. A recent report showed that Black students represent 16% of the student population, but 32-42% of the students who are suspended or expelled (U. S. Department of Education Office for Civil Rights, 2014), with the discrepancy starting as early as preschool. Schools often punish students by excluding them from classroom in form of suspension in and out of school and expulsion (Arcia, 2006), mirroring punishment in society at large when individuals are removed from social interaction (Noguera, 2003b). Such punishments certainly impact student learning (Arcia, 2006) and the discipline gap can be seen as another aspect of racial structure in U. S. society (Skiba et al. 2002) that impairs Black children’s education.

While some other factors such as low SES and low achievement are often also associated with suspension and expulsion rates, the review conducted by Gregory, Skiba, and Noguera (2010) shows that these factors are not sufficient to explain the discipline gap, race is still a predictor when such factors are inexistent or controlled for. The authors also cast doubt on theories that claim that Black children actually “behave badly” more often than others and that over punishments are justified solely by their own behavior, in a form of naturalization of racism used to justify it (Bonilla-Silva, 2003). The major factor accounting for the difference in discipline rates between Blacks and Whites is then biased differential selection originated primarily at the classroom level (Skiba et al, 2002). Black children are referred to discipline for reasons that Whites are not, and reasons that are more subjective in comparison to Whites, meaning that Whites are punished for reasons that are more objectively observable, such as smoking and vandalism, but Blacks are punished for disrespect, for example. Additionally, Black children also suffer harsher punishment along the process of disciplining in comparison to Whites (Skiba et al, 2011).

Such differential selection and processes can be credited to cultural mismatches and race privilege (Monroe, 2005) in complex social interactions that also intersects with identity formation of Black children (Noguera, 2003a) and the social contract of school (Noguera, 2003b). On the one hand, Black children, particularly boys, may act in ways that resonate with Black male stereotypes associated with violence and criminalization either because it is part of the process of growing-into-a-man (Noguera, 2003a) or because they are resisting compliance with the norms of school as a response to how the educational system has consistently failed them (Noguera, 2003b). But the majority of schools’ authorities are White who frequently fail to acknowledge Black ways of participation in schools, deeming Black children’s behavior as inappropriate even when they are not (Monroe, 2005).

Finally, it is important to notice that most research about racial disparity in school discipline is about Black boys, but results are similar in a few studies targeting Black girls: they are suspended and expelled in higher rates in comparison with White girls (Crenshaw, Ocen, & Nanda, 2014; Blake et al. 2011). Blake et al. (2011) also noted that

Black girls were mostly punished for being defiant, a reason that is not only highly subjective, but also intersects with Black notions of femininity.

3.2. A pilot study

I analyzed data from this edition of this laboratory classroom in a pilot study (Salazar, 2019) designed to test the conceptual framework. In this pilot study I analyzed episodes of instruction that focused how instructional practices enacted by the teacher: (1) supported the participation of three Black girls that lead to challenging mathematics ability as a function of being White and (2) shifted knowledge authority in the classroom in ways that challenge normalized liberal distributions of knowledge. I am building on these analyses to further develop and refine the theory.

The three Black girls are Alex[3], La'rayne (*La-rain'*[4]), and Miah (*Maya*). These girls motivated the selection of episodes because they were doing mathematics in the summer program. By "doing mathematics" I mean that they were engaging in mathematical conversations, explaining their mathematical thinking, both verbally and in writing, and listening to others thinking and making sense of it. They were also observing classroom norms for participation, including sociomathematical norms (Yackel & Cobb, 1996). Moreover, they did not do mathematics in the same way, and their participation was perceived differently by the teacher and their peers at moments. It is important to report on these girls not because they illustrate cases of "extraordinary" Black girls that could do math, but instead to choose three different Black girls in order to focus on their modes of doing mathematics "while Black" (Martin, 2012). Showing how Black girls engage in doing mathematics builds on the idea of learning and doing mathematics while a Black girl (Martin 2012; Gholson & Martin, 2014) and explores what does it look like to do mathematics while being a Black girl. The analysis of these episodes started with watching and writing fieldnotes of this laboratory instruction.

[3] Children's real first names are used with research consent and IRB approval. Other identifying information (e.g., last names) are not used.
[4] Names of African Americans in U. S. often have unusual spellings. Additionally, their names are frequently misspelled or mispronounced. In this work, I am trying to use their first names with correct spelling and pronunciation as a sign of respect. When I cite a name with an unusual spelling, I will write the pronunciation in the first time I write the name in parenthesis, so La'rayne is pronounced La-rain'.

These fieldnotes were then used to generate initial broad assumptions about the instruction and the mathematics that occurred at this laboratory classroom, as well as a list of possible episodes to be further examined. Episodes were then selected from this list to check and refine the assumptions initially made. Additional episodes were searched, and selected when possible, when the initial list was not sufficient to provide enough data to check an assumption. Additional data, such as student interview, pre and post survey about mathematics identity, videos from pre and post class meetings with observers, lesson plans, and students' notebooks were also analyzed to provide further evidence of assumptions as well to look for alternative explanations.

Results of this study suggested that instructional practices can locally challenge institutional racism by supporting Black girls accessing mathematics content and practices. Moreover, the practice of assigning competence can challenge normalized assumptions of who can know and do mathematics embedded in the construction of Whiteness as property. Finally, the focus on Black girls also provided a window to see their modes of doing mathematics and how "doing mathematics" can be racialized.

3.3. Methods

3.3.1. The laboratory setting

The data source for this study is a mathematics elementary laboratory classroom. This classroom is the main part of a summer program held each summer at a large research university in the midwestern United States. In this program, an experienced White teacher teaches lessons to a group of rising fifth grade students[5] in public—that is, while over 70 other educators observe. This summer program serves different purposes: one is to be a site of learning for practicing teachers and teacher educators, because of the nature of the public teaching and the professional development sessions that follow it; another is to be a site of research for both student learning and teaching practices. The data were not collected specifically for the

[5] Students who were going to start fifth grade in the subsequent fall.

purposes of this study; nevertheless, it provided useful information to investigate how mathematics instruction can disrupt racism.

The student body of the summer class is composed by a sampling of students from one school district in the midwest United States. The group is constructed to represent the demographic distribution of this district. The majority are African American and most come from low-income households. Only a few of them are White. The students have different levels of English proficiency; and their mathematical performance in school is homogenously low. Some older students, who participated in other editions of this program, now serve as teaching assistants. They support the work of the teacher not only with logistical support, such as passing out handouts, but also answering students' questions in small group work and checking students' written work.

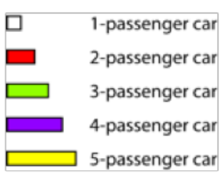
Because this summer program is a site of research for student learning and teaching practices, different types of data are collected by the research team. The data set includes video records of instruction, video records of pre-brief and debrief meetings with learning teachers, copies of students' notebooks, pictures of classroom records such as charts, lesson plans, class materials such as handouts for students, etc. In this particular edition of the program, the students answered a survey designed by one member of the research team for a specific project about mathematics identities. A small sample of the students were also interviewed three times by this same member of the research team. Because I am analyzing data that were already collected, I did not engage in a relationship with the participants. I sought to observe and respect their voices the best way I can by triangulating different sources of data. I focus on the data produced during instruction time, so video records of classroom interaction (approximately 2.5 hours per day across 10 days), detailed lesson plans for each class, copies of student work (notebooks, homework, and assessments), photos of every collective record produced in classroom (such as charts and white board records). I also used data from the survey and interviews. The high-quality documentation of the laboratory classes allows detailed observation of classroom interaction often difficult in regular school settings. Also, the composition of the student body, the qualification and experience of the teacher, and the laboratory setting provide a fruitful environment for the presence of multiple norm circles operating in this same space, which is important for this research.

3.3.2. Analytical approach

To expand the analysis from the pilot study, I rewatched all videos from the laboratory classroom that focused in the Train Problem (figure 2). This problem has some interesting mathematical components such as the need for particular organization strategies to list possible trains and the fact that the second part of the problem does not have a solution. In these cases, “solving” a problem means to logically and mathematically argue why the problem cannot be solved. Moreover, the resolution of this problem by the laboratory classroom spans the full two weeks of the problem, which allows capturing how consistently an instructional practice was enacted to indicate local normativity.

<u>The Train Problem</u>

The SP Train Company has five different-sized train cars: a 1-passenger car, a 2-passenger car, a 3-passenger car, a 4-passenger car, and a 5-passenger car. These cars can be connected to form trains that hold different numbers of people.



<u>Part 1</u>

You can use <u>only</u> these five types of cars to build trains, and you can use <u>at most one</u> of each type of car in each train. What are the different numbers of people that the SP Train Company can build trains to hold?

<u>Part 2</u>

Ms. McDuff wants to order a special 5-car train that uses exactly one of each of the different-sized cars. Ms. McDuff wants to be able to break apart the 5-car train to form smaller trains that hold exactly each number of people from 1 to 15. The customer wants to be able to build these smaller trains using cars that are next to each other in the 5-car train. Can the SP Train Company build Ms. McDuff’s order? Explain how you know.

Figure 3.1: Summer program train problem.

The analysis of the lessons in which the classroom engaged in solving the train problem also started with watching the video records of the lessons and writing fieldnotes. In this study, however, I focused on identifying consistent instructional practices and/or tools that could signal norms created in the context of this classroom. Guided by the literature on disciplining Black children and positioning Black girls in classrooms, I initially described in the fieldnotes situations in which I expected a particular teaching move to occur, but the teacher did something else instead. So, for example, when I expected the teacher would reprimand a Black student because they were laughing, or rolling their eyes, or acting in a way that is often considered disrespectful by typical White teachers (Skiba et al., 2002; Monroe, 2005) but the teacher asked a seriously mathematical question instead. Then, I analyzed how these practices and tools were connected with mechanisms of local disruption of racism reported in Salazar (2019). Instructional episodes that were related with such practices and tools were selected to be analyzed in more details to refine the initial analysis.

By looking across the episodes, I was able to identify how frequently and consistently an instructional move or tool was enacted by the teacher, which indicates a normative aspect of the practice. However, if a particular behavior is normative, then there must be a norm circle endorsing the norm (Elder-Vass, 2010), so I checked, in each episode, who were the members of the circle and how their actions endorsed the norm, or, in other words, how the members of the circles acted so the teacher would be compelled to reenact the practice in a similar situation.

It is important to notice that the relationship I had with the teacher of this class may have supported the identification of these normative instructional practices and tools. This teacher is not only an experienced teacher, but an experienced mathematics education researcher. As such, she was my mentor in this study. We also worked together in other teacher education projects that included working directly with practicing teachers in mathematics teaching practices with an equity orientation. From the orientation conversations we had and our interactions with other participants from the teacher education projects I may have become predisposed to see the patterns I saw. That does not mean such patterns did not occur in the ways I am reporting, only that I might have missed them if I was not discussing and reflecting about them with different people in different settings. This laboratory classroom is in many ways very similar to regular classrooms and, although it is easy to notice that something does not quit fit our normalized expectations for a classroom, the similarity makes it difficult to identify explicit elements worthy of careful examination. In this sense, the relationship I had with the teacher of this classroom provoked some specific lenses to look to this classroom and see instructional practices and tools significant to this study.

Furthermore, also building on the pilot study (Salazar, 2019), I sought to compose a brief narrative of each girl, so I could show how these teaching practices and tools supported counter narratives with respect of positioning of Black girls in classrooms. The main component from their narratives comes from the pilot study, but was refined by the analysis of the surveys, interviews, and episodes from this study.

3.4. The girls

3.4.1. La'rayne

La'rayne is an average-sized medium-dark skinned Black girl. She had her hair in a few small ponytails and usually wore colorful t-shirts, pants, and sneakers. She often had a sweatshirt tied around her waist. Her pre-survey and first interview indicate that she thought being good at math meant to get right answers, particularly to hard problems. She also thought she was not good at it. In class, however, she often answered questions correctly.

In her interactions with the teacher, both in one-to-one or in whole class, La'rayne spoke her mind freely. She used an assertive tone most of the times that she talked with the teacher. She did so both about mathematics, presenting her thinking out loud, or about other classroom related topics. She was not afraid to voice a concern about the students being called by the teacher to participate. She voiced such concern in a serious and clear way in a discussion about what the teacher should do so they could "learn a lot". When talking to the teacher about math, La'rayne shared her thinking without any hesitation, usually promptly after the teacher asked a question or added a comment. She seemed to always trust her answers and thinking, and she shared them freely if she was right or wrong. In one episode, in particular, in the second day of the program, the class was working in the train problem part 1. When the teacher asked what was the smallest train that could be built, La'rayne said wr (that holds three passengers), but given the blatant disagreement of her classmates, she immediately revised her thinking and said it was possible to build trains with only a white or a red car, unless no train at all was actually built.

It is easy to picture La'rayne being positioned as a "loud" Black girl because of how open she is with respect to voicing her thinking out loud. She could be seen as confronting the teacher academically when she defended a wrong answer so honestly and trustily. She could also be seen as defiant more broadly because of how she could so clearly demonstrate her ideas about regulation of student participation in class as well as her seating assignment. In this laboratory class, however, La'rayne was often respected by the teacher, both mathematically and as a full participant of the classroom. There were some moments in which the teacher regulated La'rayne's behavior that could be still seen as positioning La'rayne as "loud". In these moments, the teacher often did not only regulated behavior, the teacher did so with a purpose of supporting La'rayne doing mathematics and emphasizing she was not "in trouble."

3.4.2. Alex

Alex is a big dark-skinned Black girl, meaning that she is tall (in comparison to other girls in the class), heavy, and her breasts seem already developed. She uses her hair short, but not very short. She used a head band a few times. She uses some variety of types of clothes, wearing colorful and black alike, as well as simple shirts, more elaborate shirts, and even a dress once. Her pre-survey indicates a broad spectrum about what does it mean to be smart in mathematics, including knowing the answers, paying attention in class, and solving problems others cannot solve. She felt she was not smart in math though. Alex often ended her written work, and sometimes even work she shared orally, with the expression "the end."

She was in many ways similar to La'rayne in the sense that she spoke with confidence, but the way she did so looked very different than La'rayne. She did not voice her opinions as freely and openly, but she still defended her mathematical answers. The teacher often misunderstood her answers, but she argued and explained, and, in many cases, the teacher revised her first opinion saying things like "ah, I now see what you mean. You're right, that is a good point." Or "oh, I'm sorry, I didn't understand what you wrote, but now I do, and it is actually very good.” One example of such interaction occurred in the end of day 6 when the teacher approached Alex to see whether she had appropriately completed her end of class check. They discuss one question about a fraction problem. The teacher first says Alex was wrong:

> Teacher: I don’t think you follow the steps for number one. [the teacher was referring to the steps for naming a fraction they had worked in the first half of class that day]
>
> Alex “makes a face” and rolls her eyes, then the teacher changed her mind:
>
> Teacher: or maybe I can’t read it, can you read it for me?

The teacher realizes she indeed had wrongly read what Alex had written, but continues to talk to the student asking her if the 6 looked like a 6, Alex says she wrote too fast and the 6 actually looked like a 4. The teacher finished reviewing her end of class check and Alex asks is she is done. Teacher confirms, then Alex says an excited “YES!”. The record is not clear, but it seems that Alex was the first one to finish the end of class check that day.

In this study, because of the constraints imposed by video analysis, it is difficult to see more complex interactions that happen among students that could provide evidence to the kind of findings such as reported by Grant (1984). But Alex’s recorded interactions with the teacher reveal some kind of highly developed social skills. One such evidence come from interactions in which the teacher recognized Alex’s mathematical competence after an initial misevaluation. She was often determined to not let her ideas go down easily. During this kind of interaction, Alex faced the initial misunderstanding by the teacher, she continued arguing and laying down her explanations. Her rolling eyes betrayed her composed façade while she kept her arguments fiercely. She seemed genuinely pleased whenever the teacher acknowledged her mistakes and apologized though. Anytime that the teacher acknowledged Alex’s mathematical ideas, she was positioning Alex as a competent mathematics learner and doer.

Other evidence of Alex's highly developed social skills, in particular within a math class domain, was her use of authoritative role of the teacher and teaching assistants to check her work. In one episode, when checking a train for the train problem part 2, she asked Ms. Bria if she could break the big train in a particular way to make a particular small train. When the teacher checked her work, she told Alex she could not do that small train because she "was moving things around", in other words, she was not using cars next to each other to make a smaller train, which was not allowed by the conditions of the problem. Alex then immediately replied to the teacher that Ms. Bria told her she could do it, so her train was valid. Even though this is not a mathematical reason to defend her work, such deliberate use of a knowledge authority demonstrates how well Alex navigated the learning environment. This example illustrate another kind of social skill than that described by Grant (1984).

3.4.3. Miah

Miah is a big medium-dark skinned Black girl, meaning that she is tall (in comparison to other girls in the class), heavy, and her breasts seem already developed. She uses her hair long with cornrows to her right side, wears more adult looking clothes, and a variety of shoes. Of all three, Miah is the one that looks more womanish, with La'rayne looking more like a girl. Miah, however, still has many girly traces, and does not look like a grown woman completely. As Alex, Miah's pre-survey indicates a broad spectrum about what does it mean to be smart in mathematics, including knowing the answers, paying attention in class, raising your hand a lot, and asking questions when you don't know something. Different from La'rayne and Alex, Miah felt she was smart in math.

Her participation in class was remarkably different too. She frequently demonstrated interest in sharing her ideas, but she spoke softly, even hesitantly at times, and was often laughed at by other children when speaking to the whole class. The way she acts resonates better with White notions of femininity (Morris, 2007), she was more submissive than La'rayne or Alex, she did not fight for her ideas as fiercely. Her ideas about being good in mathematics include actively doing things such as raising her hand, she wrote notes about it in her notebook, end-of-class checks, pre and post surveys. But defending her ideas, on the other hand, could position her as “loud.”. She often set goals for herself, as notes in her notebooks shows, to not talk during class or in whole class discussions. While this dynamic between actively participating and trying not to talk in class was consistent throughout the length of the program, there is one participation on the third day that captures this conflicting position.

On this day, the class was discussing the teacher contract, which describes the commitments the teacher agrees to assume to support students in their learning. In this discussion, Miah comments that a good teacher should listen to what students say and explained:

> Miah: because sometimes when uh… like you’re asking your neighbor something and your teacher thinks that you’re talking during class and then you get in trouble. And then you tell them you’re just asking something, you still get into trouble.”

She seemed trapped by White narratives about academics and femininity. Moreover, her classmates, in whole class settings and discussions, often did not value her concerns nor her mathematical ideas. In this context it is easy to imagine two possible outcomes in terms of positioning Miah in this laboratory classroom. She could be positioned as “loud”, given her active participation, or she could become invisible, given her efforts in not be seen as “loud.” The work of the teacher in this class, however, was deliberate in trying to shift her position, frequently assigning competence to her. Towards the end of the program, Miah solved a complex warm-up problem that was later used to solve the triangle problem part 2. Her explanation was recognized as valid by the whole class, the mathematical result was named Miah’s Theorem, and children regularly used this theorem in building other explanations and gave Miah credit for it.

3.5. The specificity of mathematics: solving the train problem.

The class worked to find the solution of the train problem. The process started in class 2 and involved individual and small group work, as well as whole class discussions. To solve part 1, the strategy consisted in finding the smallest train that the

SP company could build, then the greatest, then find trains “in between”.[6] This is a common strategy in mathematics for solving problems that have multiple but finite solutions over the set of integers. It entails finding the smallest and greatest solution with logic mathematical reasoning explaining why these are actually the smallest and greatest solutions, then testing every number “in between”. Because the set of solutions is ordered and finite, then it is possible to find all solutions with this method.

In this initial discussion, La’rayne says wr[7] which holds 3 passengers is the smallest possible train because w=1 and r=2. Other students loudly say “no”, disagreeing. La’rayne immediately revise her thinking:

> La’rayne: You can use just the white or just the red.
>
> The teacher asks if there is a smaller than one.
>
> La’rayne: No, unless you don’t make any train at all.

In this brief interaction, it is proved that the smallest train holds one passenger because it is made with the smallest number of rods and uses the smallest rod. There is an interesting question posed by La’rayne that is not further explored in this class, but parallels with the mathematical definition about the smallest natural number that can be equivalently axiomatically defined as one or zero.

The class does not reach a consensus about the greatest train as quickly, so they work independently or in small groups in this question. During this work, the teacher approaches La’rayne, points to her train and asks if this is fifteen and if she is trying to make nineteen.

> Teacher: How many is this?
>
> La’rayne: Nineteen.
>
> Teacher: No, no. Add them. One…
>
> La’rayne: Three, six…
>
> Teacher: Okay, you go ahead, you add them.

---

[6] I am using the idea of size to talk about the number of passengers a train holds. So, a train of size 7 holds seven passengers, a train of size 7 is smaller than a train of size 11, and the greatest train is a term used to refer to the train who holds the greatest number of passengers.

[7] The letters used here are a reference to the first letter of the name of the color of the train cars. So the train wr is the train white-red which holds 1+2=3 passengers. Similarly, yrpwg is the train yellow-red-purple-white-green which holds 5+2+4+1+3=15 passengers.

La’rayne counts in her head.

La’rayne: Fifteen.

[This next piece of conversation is very dynamic, with questions and answers being said immediately on top of each other.]

Teacher: Okay. Now, is that the biggest train you can make?

La’rayne: Yes.

Teacher: Why?

La’rayne: Cause that’s all the numbers.

Teacher: Excellent. Did you just prove it?

La’rayne: Yes.

Teacher: So now, can you prove why you can’t make nineteen?

La’rayne: Because you don’t have anymore, you can’t use anymore.

In their collective work, the students concluded that the greatest train would use the maximum number of rods, which made a train that holds 15 passengers. The class, then, continue their collective work, trying to find trains for natural numbers greater than 1 and smallest than 15, and concluded that the company could build trains of size 1 to 15. Madison summarized their conclusion by the end of class 3:

Madison: You can make trains for the SP I think from one to fifteen passengers.

The second part of the problem is much more complex than the first and requires a solid mathematical interpretation of the logic conditions of the problem. The first condition tells to start with a train made with one of each car, so a train that holds 15 passengers with one white, one red, one green, one purple, and one yellow car, for example wrgpy and grwyp. Additionally, it is necessary that the required train can be broken to form smaller trains that hold 1 to 15 passengers, using only cars that are next to each other. So w and rgpy are trains that can be made from wrgpy, as well as wr, gp, and y, but wp cannot be made because w and p are not next to each other in the original train wrgpy. Moreover, from the train wrgpy it is possible to make trains to hold 1 (w), 2 (r), 3 (g), 4 (p), 5 (y), 6 (wrg), 7 (gp), 9 (py), 10 (wrgp), 12 (gpy), 14 (rgpy), and 15 (wrgpy) passengers, but not 8, 11, or 13 passengers, because all the possible trains that hold 8 (wgp, gy, wry), 11 (wrpy, rpy), or 13 (wgpy) passengers are not next to each other in the train wrgpy. It is worth of notice that in some cases more than one train can hold the same number of passengers, like wr and g that hold 3 passengers. However, just finding one train that holds a particular number of passengers is enough to support the claim that it is possible to build a train to hold such particular number. It is also worth noticing that there are a total of 5!=120 trains that can be made with w, r, g, p, y. They could be reduced to 60 by noticing that mirrored trains make equivalent smaller trains, but this means that there are many trains to test, and just listing all of them without missing one is already an challenging task.

The solution of the part 2 spanned across the remaining 7 classes. They started doing some exploratory work to make sense of the conditions and understand the problem. During this part of the work, on day 5, they build collectively one train that holds 15 passengers, namely yrpwg, and broke it apart in different ways, trying to understand how they could break it, how they could not, and what were smaller trains they could built from it. They agreed that this train did not work because they could not build 9, 13, or 14. Then they started testing out other trains of size 15 and began making records of all the trains they were testing that did not work. In this work on day 5, the teacher talks with the pair Layla and Olivia who said: "we're done," meaning they had found a train.

> Teacher: Do you find a train that you get all the numbers?
>
> When the girls said "yes" the teacher starts "I don't…" but interrupts herself and asks "let me see it" instead. Her follow up question is then about what is the train they found. The girls seemed confused with the question. They talk, the girls show the train: ypgrw. The teacher continues:
>
> Teacher: Okay. Let's put it down, then show me you can get all those numbers.
>
> The girls built their train with the Cuisinaire rods. The teacher said she did not see how they had made 13, they showed, the teacher then said they could not move things around. The girls insisted they were not moving things around. The teacher continued
>
> Teacher: I don't see how you can make 8 with this either. How did you make eight?
>
> Girls answer, but it is inaudible.
>
> Teacher: You said yellow, red, white, but yellow, red, white aren't next to each other. You see, they have to be next to each other. Remember?

> Girls laugh and make a few comments that indicate they understand their mistake. The teacher directs them to get another sheet and try again. The end of conversation seems friendly, in a good tone, indicating a good relationship between teacher and both girls.

Many students have similar confusion and interactions with the teacher. With practice and interactions with the teacher, the students seemed to overcome such difficulty with respect to the condition about cars having to be next to each other, and also progressed to make more organized records of their attempts. They continued the exploratory work of testing out trains for about three days.

During class 7 the children made some important progress with respect to understanding the mathematical work they were doing: they discussed why it is always possible to make trains for 1 to 5 and for 15, how to decide a train did not work, and started to think what are the numbers that are difficult to make and why it that so.

> Teacher: Have you found any trains that always work, for example? Are there any trains that work for every single one?
>
> Some students say yes and added something like one, two, three, four, five.
>
> Teacher: Why does one always work?
>
> Unidentified Student: Cause it's white
>
> Teacher: Are there any others that always work.
>
> Some students say one, two, three, four, five, some students say one through five, one student add six to the list. Teacher asks why one to six always work and calls on Luiz to answer.
>
> Luiz: Because, uh, they're, uh, they're all in one rod.
>
> Teacher revoices Luiz's ideas to class and asks if he, and class, if they agree with one to six and calls on Jeremiah to answer. He points to the chart with some trains in which six does not work. Teacher asks why six doesn't always work. Rayveion answers and the teacher revoices that you need two rods for six. She praises Rayveion for his good explanation.
>
> Teacher: Is there anything else that always work for any single train you build? Is there any other number of passengers that always work?
>
> Teacher calls on Helen who answers fifteen. Teacher probes Helen asking why fifteen always work. She answers:
>
> Helen: Because it always uses the, uh, every one of them.
>
> Teacher asks class what is the number they have to try first, students answers fourteen.

Teacher: And then what?

Class: [in chorus] thirteen, twelve, eleven, ten, nine, eight, seven, six.

Later on the same discussion, the teacher asks:

Teacher: How do you know when a train isn't good? How quickly can you decide if a train isn't good?

La'rayne's answer is inaudible. Teacher continues:

Teacher: Do you have to try all the numbers to know if a train isn't good? How do you know when a train isn't good?

Miah answers, inaudibly, teacher comments that her answer was excellent, Helen revoices Miah's idea and the teacher rephrase it to class

Teacher: When you get to any number that doesn't work, if you're sure, you might as well try a new train.

Because there are many trains to test, it is helpful to reduce the work of testing trains to solve the problem faster. So, deciding that some numbers are always possible is helpful because then there is no need to test these numbers. Mathematically, however, it is necessary to have a proof that there are trains for 1 to 5 and 15 passengers independently of what is the original five car train. In class 7, students worked collectively both on the proof and in understanding the efficiency of not needing to test these numbers. Mathematical efficiency is also the point of only needing to find one number that does not work to decide a five train car does not solve the problem. In the wrgpy example, it is sufficient to show that it is not possible to make a train to hold 8 passengers to decide that wrgpy is not a solution of the problem, there is no need to show that it is not possible to make trains for 11 or 13 passengers.

By the end of day 7, the class collectively produced a chart with a total of 35 trains listed, but 4 of them were listed twice. None of the listed trains could be a solution to the problem. The program was getting close to its end on day 8, and the students did not know how many more trains they need to test, nor how could they be sure they tested all possible trains. On day 8, possibly to support the class solving the problem by the end of the program, the students received the red and white clue: "If there is a train that can be made for Ms. McDuff, it will have to have the red car (2) on one end and the white car (1) on the other end." This "clue" is based in the idea that a smaller train that hold 13 passengers can only be built if the red car is in one end of the five car train, because 13=15-2 and the only way of “taking away” two passengers from the five car train is to remove the red car, so it has to be in one end so the remaining cars are all next to each other. Similarly, a smaller train to hold 14 passengers can only build if the white car is in one end and putting the two ideas together gets us at the red and white clue. This logic implication was not easy for the students to understand. Moreover, the clue was given to the students with the idea that it could be a “wrong” clue, so before they could use it, they would need to find out whether the clue was valid, which means understanding the validity of the logic implication. Students were confused in the beginning, they thought the clue was wrong because they were testing trains with the red and white cars in the ends of the trains and still could not find a train that works. It was only on class 9 that the students agreed the clue was right, meaning that if they were to find a train that works, they needed to build trains for 13 and 14, and that was only possible with red and white in the ends.

The clue then helps to reduce even more the process of solving the problem, because it is only necessary to check five car trains that have white and red in the ends, which is equivalent to trains that have green, purple, and yellow in the middle. In the first half of class 7 the students solved another problem that helped to finish the solution for the train problem part 2: Find all the ways to arrange the light green, purple, and yellow rods into three car trains, using exactly one of each color. Miah solved the problem and shared her solution with the class. The result stating that there are six possible trains, namely gpy, gyp, pgy, pyg, ygp, and ypg, become known as Miah’s theorem.

Still on class 9, using the clue and Miah’s theorem, they arrived at the conclusion that they only needed to test six trains: wgpyr, wgypr, wpgyr, wpygr, wygpr, and wypgr. On class 10, the students were divided into six groups, so each group were to test one of the six possible trains. After a final discussion in which each group reported their train did not work, the class seemed convinced there was no train that could fulfill Ms. McDuff’s request, which indicates they “solved” the problem.

3.6. Normative instructional tools and practices in the laboratory classroom

3.6.1. The regulation of student seating

One common instructional tool teachers do to regulate students' behavior is to control students' access to, movement in, and use of the classroom space and furniture. A traditional image of a classroom has students seated in in rows facing the board. This organization tends to constrain student-student interactions and to facilitate students keep looking to a teacher lecturing and writing at the board. In contemporary classrooms, specifically elementary classrooms, is more common to find different seating arrangements, such as students' desks organized in small groups and having an open space that students can all seat together on the floor. In the laboratory classroom, the seats are organized in a U shape with the open part facing the board. This organization supports student-student interaction in whole class discussion because each student can easily see all other students and the board, which may contain representations about what they are discussing. Desks are, however, fixed in this laboratory class due to recording equipment constraints, so students cannot move their desks to work in small groups, for example. Even so, such limitation does not impact this kind of work. Although students are usually required to go back to their seats at the U to have class discussions, they are usually allowed, and sometimes even encouraged to, to go around the U to work across their partners, and to go work on the floor or at the teacher desk during individual or small group work.

Sometimes the teacher in this class uses her authority to influence student seating to support student learning. Some of these are more permanent and registered in the class seating chart. A quick look at the four different charts used during the ten days of the program, however, reveals that the first three charts do not show students being moved individually, that is, students were placed in different seats by pairs, which allowed partner work to continue without much disruption, which may already counter a more typical pattern of using a seating arrangement to separate partners who may be considered loud or troublemakers. The last two days' seating chart brings an unusual arrangement in which two Black boys are to sit at the teacher desk and two Black girls are to sit in a desk separated from the U. What may look like a blatant discrimination action at first is actually a reflection of an instructional practice about work seating that had been happening for a few days before being registered in the chart. Figure 3 shows the second seating chart used in days 2 to 5 and the last used in days 9 and 10.

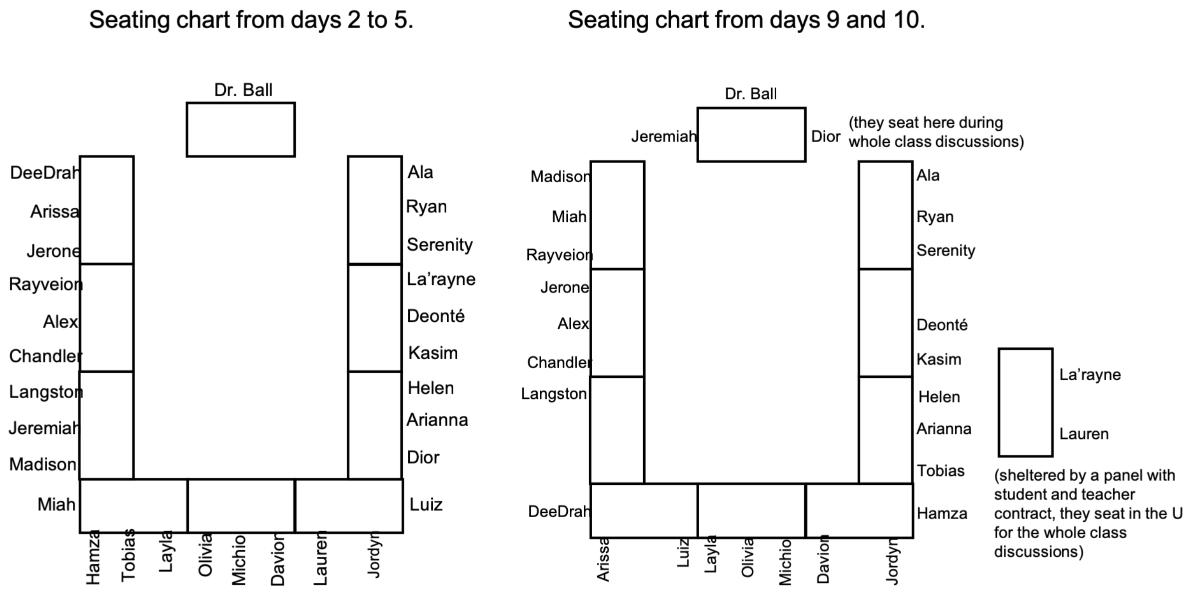


Figure 3.2: Seating charts from laboratory classroom.

In this instructional practice the teacher either approached a student during independent or small group work and asked the student to move from their original seat indicating somewhere else the student should do the work or the teacher allowed the student to choose a place they wanted to do the work. In both cases, the reallocation had the purpose of supporting the student to do the intended mathematical work. The first seating change was completely prompted by a change on the seating chart and there is not enough data to make claims about it. On day 5, however, when students were working independently or in pairs on the train problem part 2, the first change of seats occurred originating from a student request. Olivia asked to work with Layla somewhere away from them, indicating with her hands to her right side. The teacher allowed with the condition that they did the work and told the girls she was going to check if they were doing it along the way. After the girls went to sit somewhere else, the teacher talked to her teaching assistants to inform other students they could as well. Then other students started moving and finding different places to work too. After this day, the teacher always encouraged students to work wherever they wanted, as long as they did the work.

The first time the teacher asked a student to sit in a different place occurred in class 7. The teacher asked Jeremiah to sit at her desk. She told him she was "kinda tired of calling him" and he was helping with the work. She called him so he could continue helping with the work, saying that she "could get better help from him if he was not talking back there." In these cases, in which the teacher asked students to sit somewhere else, that occurred with the two boys that ended up moving their regular seat to the teacher desk, the teacher assured the students that they were not "in trouble", that she was only doing that so they could do mathematics. The teacher often verbally emphasized they were not "in trouble", used a caring and respectful tone of voice, and offered alternatives if they seem upset. It is hard to tell how the children felt about it though just from observing the videos. The intentionality of the teacher is clearer in this aspect. She also usually reinforced she knew they could do great work, but their seat was not supporting it somehow, so she, as the teacher, was making this change so they could do the work, thus learn.

In day 7, the teacher asked Dior to move from his seat at the U to work at the rug.

> Teacher: I wanna give you a quiet spot to concentrate, cause I know you can do really good work. Are you working by yourself right now? Where's your instruction sheet? Is it in your notebook?
>
> Teacher keeps asking about the sheet and flips pages of his notebook looking for it. While she is doing this, she adds:
>
> Teacher: You're not in trouble, I'm just trying to find a place you can concentrate.
>
> Dior was really upset. The teacher kept asking if he agreed with her request and tried to direct him to the mathematical work, focusing the conversation on the task. At one point she left him to work independently, but she continued to come back to check on his work. About 8 minutes later, when she checked on Dior a second time, she said:
>
> Teacher: Dior, I'm getting the impression that you don't like working here. Is that right? I was trying to help you, but it sounds that you don't like it.
>
> Dior: I need a partner.

It seems Dior suggests Ala, but conversation is difficult to understand.

Teacher: What?

Dior: I-need-a-part-ner.

At this point the conversation gets interrupted by interactions with other students. The teacher is talking to La'rayne, who is also not focused in the mathematical work, and Dior is talking about the train problem with Ala.

Ala arrives and sits with Dior on the rug. The teacher seems confused by the move, then sits with them and asks what was happening. When Ala explain they are going to work together, the teacher supports the work showing Ala the train Dior is working so they can work on that train together.

This interaction shows that the teacher wanted the students to work on the math and when she realized Dior was working, she supported how he was working even though it was not individually as she had initially suggested. During the entire episode, there were several moments in which the teacher could have reprimanded Dior by the way he was behaving. In a typical classroom, he could have been interpreted as disrespectful towards the teacher by not looking her in the eyes when she was talking to him, or by his tone of voice when he said "I-need-a-part-ner", leading the teacher to discipline him in some way. The teacher, however, continued talking with him respectfully, asking serious questions about mathematics and his seating, counteracting patterns of over disciplining Black children in classroom (Skiba et al., 2002; Monroe, 2005). The teacher explicitly saying Dior was not in trouble and her actions encouraging the partnership with Ala suggest the regulation of Dior's seating had the purpose of supporting him doing mathematics. Moreover, the instructional move was successful, in the sense that Dior actually engaged in doing mathematics. This success of the instructional move can also be interpreted as a reinforcement of the normative teaching practice. If Dior had not engaged in doing mathematics as an outcome of the instructional move, the teacher might have been discouraged to reenact it in the future.

When the teacher allowed students to work where they wanted, she monitored their work to see if their "seat" of choice was being supportive of their learning. In some cases, one of which occurred with one of the girls that ended up moving her regular seat to the separate desk, the teacher approached the student to negotiate that the student could only work on their “seat” of choice if they were doing the work they were supposed to. That occurred on day 8.

> Ms. Bria approaches the teacher to talk about checking on a pair of students, La’rayne and her partner, possibly Lauren. The teacher says she is going to check in them. The interactions that occur where the girls are working is only fully captured by audio from the teacher microphone. The video never capture the girls because they are working under a support for chart paper in form of a tent. The teacher approaches the pair saying:
>
> Teacher: Okay, you know- what- Lauren- La’rayne [pause]
>
> Teacher: This is a really special privilege to work in here an Ms. Bria doesn’t think you are getting good work done and I haven’t seen it. So-
>
> [Lauren interrupts the teacher.]
>
> Lauren: [inaudible]
>
> [Teacher interrupts Lauren.]
>
> Teacher: Listen.
>
> Girls talk at the same time for a little while, but it is inaudible. Then the teacher speaks alone.
>
> Teacher: If you want to work in here you gotta show me you deserve to be here. La’rayne, do you understand that?
>
> Girls talk at the same time, but it is inaudible. The teacher continues.
>
> Teacher: It’s a nice place to work, but you can’t fool around here. I’m gonna be back in a minute to check on you.
>
> The teacher says she will come back to check again and asks what she should be doing. La’rayne doesn’t answer, the teacher presses, then answers herself that La’rayne should be testing a train. She then replies she is doing it already. About 6 minutes later the teacher comes back to check on La’rayne and Lauren and then the conversation is completely focused on math:
>
> Teacher: La’rayne, can I see your paper? [pause] Can I see your paper, please?
>
> La’rayne: Yeah.
>
> [Long pause. Teacher is likely looking La’rayne’s paper.]
>
> Teacher: Why do you have yes written down all the way? You don’t know that yet?
>
> La’rayne: What?
>
> Teacher: You have all this yesses written down.
>
> La’rayne: [inaudible]

Teacher: La'rayne, let me see this again.

La'rayne: [inaudible]

Teacher: Oh, this is the one you are doing?

La'rayne: Yeah.

Teacher: But remember the clue is red has to be in one end and the white in the other.

Lauren: Told you! [emphasizing told]

La'rayne: You did! [emphasizing did]

Girls speak at the same time, but it is inaudible.

Teacher: Lauren,

Girls speak at the same time, but it is inaudible. Teacher raises her voice a little.

Teacher: It's okay. I want you to try the clue and see if it's helpful.

Teacher leaves the girls.

In this episode, Lauren and La'rayne were already working in a very unusual place under a support for chart paper in form of a tent. The teacher went to talk to Lauren and La'rayne because Ms. Bria, her teaching assistant, told her they were not making good use of their seat of choice. Even in this class, that is not typical, it could have been expected that the teacher asked the girls to move their seat, if not to their regular seating, at least to a more visible location, where it is easier to check on them and their work. The teacher, however, allowed the girls to work there as long as they were committed in doing the mathematical work. The girls reinforced the teacher's action by doing mathematics together there and showing their work when the teacher followed up. In this case, the teacher negatively approached the girls stating upfront that "Ms. Bria doesn't think you are getting good work done and I haven't seen it," but nevertheless allowed the girls to show they deserved to work where they wanted, always emphasizing the importance of doing the mathematical work as a condition for it.

There was some power inbalance in cases like this, in which the student could "choose" their seat, but not completely freely. The teacher still got to approve, not the choice, but whether the student deserved to have "the power of seat choice." But even in these cases, the teacher often emphasized that the students were not "in trouble".

Additionally, the teacher always followed up to make sure the student was ultimately engaged in doing mathematical work.

This kind of practice is, of course, regulative of bodies and behaviors, and yet is different than simply regulating behavior. The goal of having students doing mathematics already makes some difference and could be even connected to a goal of “teaching the dominant discourse”, in which doing challenging mathematics represents the dominant discourse. But what strikes the most is the effort the teacher puts in emphasizing with students they are not “in trouble.” Such effort is not only accomplished by the teacher verbally repeating to students they are not “in trouble”, but is also supported by a caring tone of voice, follow up checks to ensure the seat is working to support students in doing mathematics, and allowing originally not intended ways of working if such ways are actually supporting students in doing mathematic, as occurred with the partner work of Dior and Ala. It is subtle, but signals the teacher is aware of how behavior management may over-punish students, specially students of color, that not only may be punished by teacher’s reprimands, but are most likely to be punished by missing class worktime by being removed from class and sent to school supervisors (Monroe, 2005; Gregory, Skiba, & Noguera, 2010). In particular if they are routinely taken out of class and sent to school supervisors, then they are routinely deprived of class instructional time, which impacts their learning (Arcia, 2006). So, in this classroom, when the teacher moves a student’s seating, this is an instructional tool that disrupts normative practices about work seating that: (1) constrain collective work in favor of regulating behavior, (2) position students as troublemakers, (3) removes students from content work. This instructional tool challenges normalized theories about Black children behaving badly in ways that justify disproportionate discipline referrals (Skiba et al. 2002, Gregory, Skiba, & Noguera, 2010). Moreover, students were being positioned as competent mathematics doers through this tool, challenging ideas about who can know and do mathematics. For example, La’rayne was often “fooling around” and the teacher had to work with her so she could learn. The teacher allowed and sometimes encouraged La’rayne to do her work away from her regular seat. The teacher frequently only regulated her behavior asking her to stop “fooling around”, but she also frequently highlighted it was only to ensure she could concentrate in the work and always emphasized she was not “in trouble.”

It is important to call out the normative aspect of such instructional tool. The teacher consistently regulated student seating, seeking to reinforce the academic aspect of it in multiple ways and to discourage the common and likely association with purely disciplining. The teacher often engaged in more “traditional” ways of regulating student seating, and student behavior more broadly, but the specific tool described in this section was used often and consistently enough to be considered a norm, at least locally. The instructional tool of regulating student seating can be considered a normative practice in the context of this laboratory classroom. Teacher and students are members of the circle, and when students consistently engage in mathematical work whenever they had their seating moved, they endorse the practice supporting the teacher actions. While this was not the only way of regulating student seating and behavior, the normative aspect of this instructional tool created an intersectional space about norms with respect of disciplining Black children in classrooms that can, potentially, lead to local disruption of racism as observed in these episodes. In this intersectional space, Black children are then less likely to be disciplined and more likely to engage in doing mathematics.

3.6.2. The focus of individual and small group interactions with the teacher

Often in a classroom, during individual or small group work time, students are not focused, or do not seem to be focused, on the content work they should be doing. In these cases, teachers frequently reprimand students for not doing what they were supposed to, or for not “trying,” and even if teachers do not explicitly reprimand, the focus of the conversation with the students can still be on their behavior rather than the content they should be working on. In the laboratory classroom, the teacher often keeps the focus of the conversation on math. This means that, when the teacher is talking with a student or small group of students and they seem to be behaving in ways that would likely to be reprimanded in many classrooms, the teacher of this laboratory class consistently did not engaged in reprimanding student, continuing the mathematical questioning instead. In one meeting of a project I participated with the teacher of this laboratory classroom and practicing teachers, two of the participants, at different moments, commented on how they changed their practice as a result of their reflection about this instructional move. The two teachers said they experimented in their classrooms to “allow the mathematics to continue” without stopping to discipline students they perceived as disturbing the classroom. While they faced other dilemmas as a result, they both reported being surprised with how their students were doing mathematics, students they were not expecting. These conversations led me to notice how ingrained the instructional move of keeping the focus on mathematics was important to the laboratory classroom.

So, one of the typical interactions goes like this: (1) The teacher approaches a pair of students, who are talking and laughing seemingly about something other than the mathematics task they were supposed to be working on, and asks a question about the problem, such as “what is the train you are checking now?”. (2) Students are confused by the question and respond to the teacher with something that could be related to the problem or not. (3) The teacher continues asking about the problem and their strategy to solve the problem. (4) Students now are more likely to respond with a comment that has a relation to the task they are working on, but still seem not completely focused in the task. (5) The teacher continues to talk and ask questions about the task. (6) Students begin to show more engagement in the task, seem to pay more attention to the questions the teacher asks, and seem to think about it before answering. (7) Teacher continues asking questions about the task. (8) Students are then focused on the task. They understand the teacher’s questions and answer them appropriately, even if incorrectly. In some cases, students continue to laugh and fool around, but the content of their conversation shifts towards the mathematical task they should be working on. (9) The teacher leaves the pair of students productively working on the task.

The following example comes from day 6 when the teacher was talking to the pair Ryan and Deonté. While Ryan seems more focused in the conversation with the teacher, Deonté, a dark-skinned Black boy, does not stop playfully laughing. The teacher never asks him to stop laughing or to focus on the work. Instead, she keeps asking mathematical questions and inviting Deonté to answer. Ryan answers all the teacher's questions, but Deonté makes a comment at the end of the interaction that shows he is actually following the conversation and may have learned from it.

> Teacher: So what number are you up to with that train?
>
> Deonté is laughing, Ryan answers but it is inaudible.
>
> Teacher: You can make seven? How did you make seven?
>
> Ryan: Yellow and orange.
>
> Teacher: I don't see how you made seven.
>
> Deonté keeps laughing, Ryan and the teacher are looking to their work.
>
> Teacher: I don't think you're being careful right now boys. How can you make seven with this train?
>
> Deonté: *I* make it. (laughs again)
>
> Teacher: Deonté, do you see how to make seven?
>
> Deonté: No.
>
> Teacher: I see a way but it's not yellow plus red. Why would yellow plus red not work?
>
> Deonté now laughs harder. Ryan says in the midst of Deonté's laugh:
>
> Ryan: I told you (inaudible – pointing to their work)
>
> Teacher insists:
>
> Teacher: Why couldn't that work though?
>
> Ryan: Because they are not right by each other.
>
> Teacher: Exactly. But I see a way to make seven. Do you see it Deonté?
>
> Ryan: Oh, right here. (pointing to their train)
>
> Teacher: What is it?
>
> Ryan: uh… four plus three.
>
> Deonté: Oh, here? (seriously asking and pointing to their train)
>
> Teacher: Good. So, record that one.

In this sample interaction, the teacher never stopped the focus on mathematics to ask Deonté to stop laughing, even though he laughed almost the entire conversation. In a typical classroom, in which research shows how Black boys are systematically over disciplined (Skiba et al., 2002; Monroe, 2005), it is reasonable to expect that the teacher could have, at least, stopped asking mathematics questions to request Deonté to stop laughing and sit quietly in his seat. In this laboratory classroom, however, the teacher kept seriously asking questions about the mathematics they were doing, even when Deonté emphasized a seemingly deliberate wrong answer in form of a joke ("*I* make it.") In the end of the conversation, it is possible to notice that Deonté is actually engaged in the conversation when he points to the rods saying "Oh, here?" In this case, the simple persistence with the mathematics supported Deonté in doing mathematics with Ryan, no form of discipline was required so he engaged in doing mathematics.

In another episode, that occurred on day 9 when they were working on the red and white clue, the teacher approached Dior, who was working by himself at her desk beside Jeremiah, the teacher's chair between them. Dior was seating with his chair backwards.

> Teacher: What is the train you're trying right now, Dior? Can you make it?
>
> Dior hums a "no" musically.
>
> Teacher: Can you build it?
>
> Dior does not answer.
>
> Teacher: Where's the train? Can you build me a train so I can see the train you're trying?
>
> Dior start moving cars.
>
> Teacher: Remember white on one end and red on the other. No-
>
> Teacher interrupts Dior putting a hand over his.
>
> Dior then answers looking away:
>
> Dior: What was that for?
>
> Teacher: Because we are trying the ones that have the white and the red at the ends.
>
> Dior balances on his chair and turns his head up, facing the ceiling with his mouth open.
>
> Dior: [sigh]
>
> Then, Dior comes back to his original position, looking to the desk, and says to himself:
>
> Dior: You messed up.
>
> Teacher: Can you build it so it has white on one end and red in the other, okay?
>
> They continue their discussion, with Dior implying he wanted to test a different train, and the teacher insisting he tried one to test the red and

white clue. Dior ended up doing the train he wanted, which seems to be wgpyr, but data is not conclusive. Teacher and Dior were occasionally interrupted by Jeremiah in their conversation, and the teacher asked him to stop interrupting them and focusing on his work. At one point, Jeremiah sat back in his seat and Dior said to him:

Dior: You disgust me.

The teacher immediately said to Dior, but then continued talking with Dior about his work:

Teacher: That's not okay.

[Both boys smile. It is hard to tell if this interaction between the boys were just playful as the smile has implied, or if it was something more serious.]

Dior stands up.

Teacher: Now start with the big numbers, okay?

Dior seems to be thinking while standing up, seating, and looking to his train and charts on the wall. Teacher waits a little, then asks:

Dior, can you make fifteen with this one?

He vigorously, balances on his chair, holding its back with one hand and counting fingers with the other. He is looking away, possibly to the chart at the wall containing the number of passengers each car holds.

Hamza approaches the teacher to show his work. They briefly talk while Dior is looking away balancing on his chair. Then, Dior speaks while standing:

Dior: No. No, you cannot.

Teacher: How- Why not? How much is that right here? [pointing to the whole train]

They continue for a little while, but do not reach a conclusion. The teacher decides to interrupt the small group work to call a class discussion.

In this episode, the teacher could have reprimanded Dior on multiple occasions for different motives: seating backwards, not answering when she asked a question, or inappropriately responding ("What was that for?"), balancing on the chair, looking away and sighing. Every time she kept the focus on math, and so did Dior in response. Even when she said what he did was not okay ("You disgust me."), she did not fully stop the mathematical interaction. As with the other episode, at the end of the interaction, Dior is engaged in the mathematics the teacher is talking about, even if he is still moving in his chair, which reinforces the instructional move, or, in other words, because Dior appropriately responded to a mathematical question the teacher asked, the teacher will be inclined to enact the practice again. In both episodes, the boys engaged in doing mathematics as a consequence of the instructional practice which functions as an endorsement of the norm.

Even though the typical interaction described here does not capture the full range of interactions in this class, it illustrates the instructional move of keeping the focus on math, which is something the teacher in this classroom does often in many different situations. Similarly, as in the case of regulating student seating, the teacher also engaged in more “traditional” instructional moves, explicitly requesting students to focus on the work, such as the one she did with Jeremiah when he interrupted her interaction with Dior. But keeping the focus on math was a move the teacher used often and consistently enough to be considered normative in this classroom. By doing that, the teacher shifts students’ topic of conversation towards mathematics, and supports students engaging in challenging mathematics. Moreover, whenever she focuses on the math rather than reprimanding the student, the teacher refrains from using discourse moves that could reinforce the positioning of students as troublemakers. This instructional move also cast doubt on the need to discipline Black children so they can learn. The existence of a norm in which students do not need to be disciplined so they can learn creates an intersectional normative space in which Black children are, at least locally, less likely to be disciplined in classroom. Furthermore, these sample interactions challenges not only ideas about who can know and do mathematics, but also about *how* one can know and do mathematics. Deonté was playfully laughing during the entire interaction with the teacher, but the seriousness and accuracy of his last commentary indicates he was actually following the talk between Ryan and the teacher and was participating in it somehow. Dior was also seating differently, balancing on his chair, standing up, but doing mathematics with the teacher nevertheless. So, in the intersectional space created by this instructional move, Black children are not only less likely to be disciplined, but they are more likely to be allowed to do mathematics their own ways.

3.6.3. Regulating speaker and audience participation in class.

Teachers usually ask students to participate in class in ways that involve one student speaking to the whole class while other students are listening. This kind of classroom interaction may occur in instances in which a student goes to the board to represent and talk about their own solution of or strategy to solve a problem in a form that reproduces a traditional lecture with the speaker assuming the role of the teacher for a short time and the remaining students passively observing. It can also occur when students are engaged in a mathematics discussion, in which students respond to one another in order to collaboratively do mathematics. In both examples, there are, at moments, one student talking to the whole class while others are listening. Whenever this situation occurs, it is common that some students in the audience are not quietly paying attention to the speaker, which may lead the teacher to intervene and regulate students' behavior. Such regulation often occurs in the form of reprimands that sound like "be quiet, and pay attention", with only "being quietly" representing the expected response a student should comply.

The students in this laboratory classroom routinely engage in mathematics discussions, and in this context, the teacher frequently required the audience to pay attention to the speaker, but students were usually also required to agree or disagree, always mathematically justifying their position. Moreover, the audience were often oriented to ask clarifying questions, to add on, or to restate what the speaker just said. This kind of talk moves shifts common strategies teachers use into something that goes beyond managing behavior, something that also promotes the audience to engage in content discussion with the speaker. In addition, the teacher in this laboratory classroom often asked the speaker to speak loudly and/or face the audience when speaking so the audience could hear and understand what they were sharing. When done together, both asking the speaker to talk to the whole class, and the audience to actively listen, such moves causally influence how students will act during class discussion, and in the case of this laboratory it supports students collectively doing mathematics together.

> In day 9, students were working on the validity of the red and white clue and trying to make sense of it by exploring the train wygpr, which was built at the board with magnetic rods. Madison went to the board to explain her

thinking about why she thought the white and red had to be in the ends to build 13 and 14. The teacher asked the class to pay attention to Madison, saying that all eyes should be on her. She starts talking at the board by the train wygpr:

Madison: I think you have to have red (pause) white and red on the ends, because uh [pause] say you take white away then it would be uh]pause] fourteen. [Removes w while talking.]

Teacher: Why?

Madison: Because it's one. [Puts back white while talking.] Let's say that [Breaks the train apart and removes g, then makes wypr.]

Teacher interrupts Madison.

Teacher: Now, go back to the full train for a second, Madison. [She puts the train back.] So, how much is the full train, Madison?

Madison: The full train is fifteen.

Teacher: The full train is fifteen, so why when you take the white off do you get fourteen?

Madison: Because it's one.

Teacher: Who can explain what Madison is showing? Why does taking one off make fourteen automatically? Can someone explain what Madison is showing? Davion?

Davion: Because fifteen minus one is fourteen.

Teacher: Okay. What Davion mean by that? What does he mean when he says fifteen minus one fourteen? Why does that go with what she is showing us?

Davion answers himself.

Davion: Because she took away the white and the white is one.

In this example, the teacher asked students to pay attention to Madison, but she not only did that, she asked students to comment on what she had shared. Davion, a Black boy, had to be actively listening and looking at Madison and her interaction with the teacher to be able to answer that Madison had taken the white away. In a typical classroom, the teacher could not have asked such a follow up question, especially after all the probing questions she asked Madison. Madison's thinking was fully laid out to the class, asking Davion to explain it again did something more than only explain her thinking, it reinforced the norm of actively listening. It is important to notice that this example occurred in class 9, at almost the end of the whole Summer program, when many norms were already stablished. In this example, it is possible to see that the teacher, in the beginning, asked students to pay attention. The students knew that simply “being quiet” might not be sufficient in this class, the teacher might ask questions after Madison shared her thinking. By asking the question, the teacher reinforced the norm, and by answering appropriately, Davion also reinforced the norm. The result is that they did mathematics together, Davion building on Madison.

This instructional tool was particularly relevant to position Miah as a competent mathematics learner and doer. In this example, occurring on day 5, Miah was sharing her work in her notebook from the end of class check #4 (figure 4). The teacher first asked other students to listen to what Miah was saying and then asked for something that she did good. Just “being quiet” while Miah was explaining her thinking would not be sufficient to answer this follow up question.

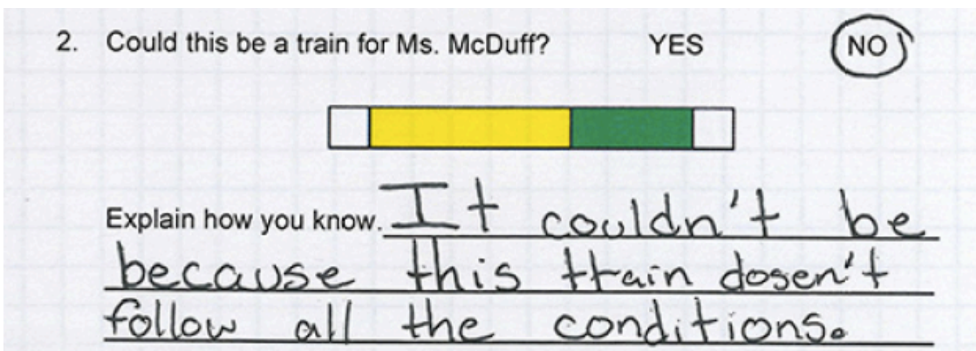
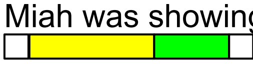


Figure 3.3: Miah’s end of class check excerpt.

> Miah was showing her answer to the question that asked if the train  could be a train for Ms. McDuff (figure 4). She was projecting her notebook so all students could see. When verbally explaining, she actually named which condition wasn’t being followed (using one cart of each, there are two whites):
>
> Miah: I thought it couldn’t be because we’re suppose to … uh … use only one of … uh … each cart … and use …
>
> Teacher: [to Miah] One second.
>
> Teacher: [to class] Could everyone stop for a moment. I don’t … I only see about half of the people looking at Miah right now. What Miah is showing you will help you your notebook and help you with the trains problem today.
>
> Teacher: [to Dior] Dior, I’d like you to look up at Miah’s notebook.
>
> Teacher: [to Jeremiah] Jeremiah, are you looking up here?

Teacher: [to class] Okay, so she was explaining why the second train doesn't fit the new train problem.

Teacher: [to Miah Why not?

Miah: Because … uh … the trains problem you're supposed to only use one of each color and there's two whites and …

Teacher: So what did you write?

Miah: I wrote [she reads her answer out loud fluently]

Teacher asked others to say something that was good about Miah's work on notebook. Ala says she used the word condition. Jerone says she wrote clearly and writing clear makes it easier to understand. Teacher added to Jerone that Miah wrote a complete sentence.

I argued in a pilot study (Salazar, 2019) that assigning competence can promote local disruption of racism by shifting positioning with respect to what counts as knowing mathematics and who knows it. In the case of Miah, she spoke softly and did not face the class when sharing her ideas to the whole group. She did not seem confident in her ideas by the way she presented them to the class. She was sometimes laughed at by her classmates, and they frequently did not pay attention when she was speaking. She was certainly not positioned as a competent mathematics learner and doer in the beginning of the laboratory class. The teacher, however, worked throughout the program to shift her position. The instructional move described in this episode is one example of this work. By asking Miah to share her thinking, speak loudly, and face her classmates, she opened up the possibility for Miah to demonstrate her competence. By orienting other students to Miah's ideas, she was creating the possibility so others could see Miah's thinking. Then, by calling out something important that Miah did, she assigned competence to Miah, which could lead to shift her position as a competent mathematics learner and doer. In this case, it was the instructional move of regulating participation in class that enabled the practice of assigning competence. By assigning competence to Miah, the teacher locally disrupted a likely position of academically invisibility for Miah (Grant, 1984, 1994; Henry, 1998; Morris, 2007).

One more time, it is important to notice the relevance of the normative aspect of the instructional move of regulating student participation in class. Even if sometimes the teacher only asked students to be quiet or pay attention when someone was sharing an idea, her consistency in asking follow up questions and supporting speakers in speaking to the whole class was enough to create shared expectations about it. As noticed in the episode in which Madison and Davion did mathematics together, towards the end of the program, students knew they were expected to actively listen when someone was sharing their thinking, and they could be asked to comment, add on, or agree/disagree. So, when the teacher asked that all eyes should be on Madison, students already knew this meant they had to be actively listening. Furthermore, it is exactly the normativity of this instructional move that makes it significant in locally disrupting racism. It creates an intersectional space in which Black children, Black girls in particular, are more likely to be positioned as mathematically competent and their ways of doing mathematics acknowledged as legitimate.

3.6.4. Teacher on-the-fly responses to students' thinking.

Teachers often engage in conversations with students to gauge students' thinking about a particular topic they are working in class. Although this kind of interaction may happen in a whole class discussion setting, interactions in which teacher and students talk so that the teacher can gather information about what the student knows or does not know about the content are more frequent in one-to-one interaction. Moreover, teachers usually engage in this kind of interaction with the purpose of assessing what the student does not know, so they can (re)teach it. However, when teachers engage in this kind of conversations, what they can learn from students is informed by their own background and experiences. Assessing whether a student understands a mathematics concept entails knowing what are legitimate mathematics standards or, in other words, assessing whether the mathematics produced by their student would be accepted as mathematics by other mathematics authorities. Because of their willingness to help their students learn, teachers often stop a student immediately when they listen to something that does not resonate with their own expected answers, even if such answers are right or have an interesting reasoning if not quite right. I expect that such interruption of student thinking is more likely to occur with students of color because school tends to reproduce normal White middle class realities (Delpit, 1988).

In this laboratory class, the teacher did not typically interrupt her students. On the contrary, the teacher consistently did not judge or evaluate students' responses too quickly, keeping a "poker face" whenever she heard something that did not fit usual mathematics standards, and asking follow-up questions to elicit more complete accounts of students' ideas. The teacher did not narrow her interaction of this kind to find out what were possible students' misconceptions, she asked students for their thinking in order to build on this either individually or collectively. By doing that, the teacher opened up the space for new ways of doing mathematics and created possibilities to assign competence to students. One example very common in this classroom occurred when students were working on the train problem part 2 and claimed they had found a train for Ms. McDuff. The teacher knew from the start that there were no possible train for Ms. McDuff, but she kept her "poker face" and asked the children to show her what was the train. Then the teacher asked the students to make a particular small train she knew was not possible. When the students showed her the small train, the teacher indicated it was not actually possible because of the conditions, often the condition about the cars having to be next to each other. The teacher typically did not say there were no other possibilities to find the small train, only that the particular one the students had found could not be.

This kind of interaction occurred with the pair Olivia and Layla on day 5, as described in section 3.5. In this interaction, the teacher asked the girls to let her see the train they had found, even though she knew in advance there was no possible train. She continued asking the girls to build their train and show all the small trains they made. Every time the teacher asked follow up questions, she kept a positive acceptance of the girls' responses, not indicating by her immediate actions whether they were answering right or wrong, just attentively listening to their responses. The interaction above, as is often the case with human interactions, is not as clearly aligned with the described practice though. When the girls first say they "are done", meaning they had found a train, the teacher starts with a negative "I don't…", but quickly interrupts herself and changes to a positive tone to ask what was the train they had found. The girls did not seem to be disturbed by this slight change made by the teacher and continued responding as if the teacher had not done it. Additionally, the teacher also said to the girls during the interactions that they might have been “moving thins around”, meaning they were not following the condition of the problem that the cars need to be next to each other. The girls, however, always pushed back, and the teacher did not insist on it, and continued her follow up questions about the trains the girls had found. In spite of such little discrepancies, the way the teacher kept asking follow up questions about the girls’ thinking can be considered an illustration of the instructional move of responding to students on-the-fly, because the teacher did not correct or re-directed the girls insistently, or dismiss their ideas as not relevant. The girls’ responses seemed to reinforce this as well, they continued showing their thinking regardless, which indicates they might have perceived showing their thinking as their expected behavior and certainly reinforced the teacher to act accordingly. Moreover, with this instructional move, by listening carefully to their thinking, the teacher acknowledged Olivia and Layla knew something about mathematics, even if their answer was not quite right. They built a correct 15 passenger train, and they were breaking it up to make smaller trains to hold from 1 to 15 passengers. Olivia and Layla were acknowledged by something they knew, not only what they did not, which positions them as knowers and doers of mathematics, challenging the usual position of Black girls not being academically competent in mathematics (Grant, 1984, 1994; Morris, 2007).

While the teacher in this laboratory classroom endorsed the norm of asking students about their thinking, her interactions with Alex reveal that there are conflicting norms about this type of interactions. The norm of correcting students’ mistakes is so deeply ingrained in typical teaching that, even in this class, with the teacher clearly engaging in not judging students’ thinking too quickly, she often did so with Alex. The consistent revising of evaluation that frequently occurred in interactions with Alex show that the teacher is in a conflicting normative space and the outcome actions are not exactly in accordance with any of the norms.

One example of such interaction occurred at the end of day 6 when the teacher approached Alex to see whether she had appropriately completed her end of class check. They discuss one question about naming a fraction (figure 3.4) on which they had worked earlier that day following a sequence of steps. The teacher first says Alex is wrong:

Teacher: I don’t think you follow the steps for number one.

Alex “makes a face” and rolls her eyes, then the teacher changed her mind:

Teacher: or maybe I can’t read it, can you read it for me?

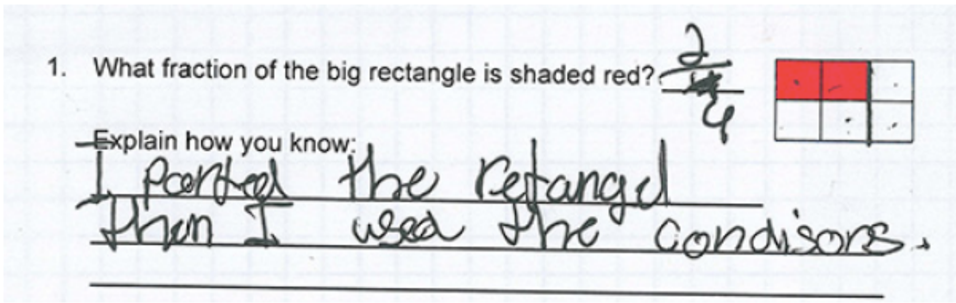


Figure 3.4 Alex’s end of class check excerpt.

The teacher realizes she indeed had wrongly read what Alex had written, but continues to talk to the student asking her if the 6 looked like a 6, Alex says she wrote too fast and the 6 actually looks like a 4. Alex rewrites her answer so it looks more like a 6, as can be seen in her work in figure 3.4. The teacher finished reviewing her end of class check and Alex asks is she is done. The teacher confirms, then Alex says an excited “YES!”.

This interaction shows local disruption of racism in two different aspects. First, when Alex confronts the teacher by “making a face” and rolling her eyes, the teacher could have followed a normative behavior of interrupting the conversation and disciplining Alex by such behavior, which is an expected action given the kinds of discipline Black children face in regular classroom settings (Skiba et al., 2002; Monroe, 2005). By revising her initial response, the teacher supported Alex in showing her appropriate mathematical answer and promoting a second form of local disruption of racism. The teacher recognized Alex’s answer as appropriate and her excited ‘yes’ reveals how she noticed the teacher validating her work. Alex was positioned as a competent doer of mathematics, one more time challenging ideas about the academic competence of Black girls in typical classroom settings (Grant, 1984, 1994; Morris, 2007).

Part of the work of teachers is to teach content that is deemed correct to their students, so it is common for teachers to correct students' errors and misconceptions. This part of the work is conducted in a variety of ways, including formal assessments and informal in-the-moment interactions. In many examples in this laboratory classroom, the teacher simply told students "they could not move things around" to direct their work towards a correct use of the conditions of the problem. It is reasonable to expect that the teacher and students have a shared expectation about teacher evaluation following a student response, but in the case of the instructional practice of responding on-the-fly to students' thinking, the teacher acted differently, the teacher asked follow up questions without letting any evaluation be noticed by the student. Sometimes students did not responded appropriately, as can be noticed in the interaction between the teacher and Kasim described in the next section, but students engaged in answering her probing questions appropriately often and consistently enough to reinforce the practice.

It is important to call out the normative aspect of this move of responding on-the-fly to students to elicit their thinking without immediate evaluation and correction. In all episodes, the observed move was not enacted in isolation of conflicting moves, or in other words, the interactions showed an overlapping of different kinds of responses by the teacher, at times judging, and at times redirecting the student. But responding to students on-the-fly, in the way reported in this section occurred consistently enough to be considered normative. Moreover, often when the teacher enacted such instructional moves, students responded offering their genuine thinking, which, then, led the teacher to act similarly in a future similar situation. By asking students to share their thinking more openly, without any immediate evaluation, the teacher supported students in making their thinking visible, also valuing and respecting their ideas. Moreover, the normativity of the move creates a space in which students ideas are not only valued and respected, but expected as a norm by the students themselves who are members of the circle.

3.7. When school norms intersect with mathematical norms

There were often school norms and mathematical norms shaping the interactions observed in this laboratory classroom, in some instances such norms were conflicting, meaning they assigned different behaviors for a same situation. For example, in this interaction between the teacher and Kasim, the boy tried to guess what would be the answer the teacher was expecting. He answered, waited for the teacher's response to his question, and was able to quickly change his first answer if the teacher's reaction was not confirming. This behavior can support school success given the role teachers have in assigning grades often based on students correct responses to questions. In this classroom, however, the teacher tried to promote mathematical reasoning. Thus frequently her responses to students' answers was a follow up question asking for more explanation. In the end, to answer with a "good" answer, Kasim would need to provide a mathematical explanation, and so he did. This interaction occurred after the brief discussion described above on day 7 about what numbers need to be checked in each train.

> Kasim: Am I supposed to start working on this? [Pointing to his notebook.]
>
> Teacher: Yeah, but is that one of the ones we've already tried? Did you check? Purple, white, yellow, green, red. Is that up there?
>
> Kasim: Oh, no. [Disappointedly.]
>
> Teacher: What? Where? I don't see it. No, I don't see that one, so that's a new one. That's good. And now you start checking, do you have to check one, two, three, four, five?
>
> Kasim: Uh?
>
> Teacher: Do you have to check one, two, three, four, five?
>
> Kasim: What you mean?
>
> Teacher: Do you have to check for a car, train that holds one?
>
> Kasim: Yeah. [Hesitantly.]
>
> Teacher: Do you need to do that?
>
> Kasim: Yeah. [Assertively.]
>
> Teacher instantly replies:
>
> Teacher: No, remember we said it always works?

He nods negatively.

Teacher: For every train?

Kasim: I must have been somewhere else.

Teacher: But, look at the list on the board. What numbers always work?

Kasim: Oh, one, two, three, four, five

Teacher: Why do they always work?

Kasim: Because you make, yeah.

Teacher: How do you make white, one always work?

Kasim: Uh?

Teacher: How do you make the train for one?

Kasim: With white. [Pointing to the white rod]

Teacher says “uhum” and adds that this is the reason he doesn’t need to check this number and asks where does he need to start. He automatically says “one”. She says one more time:

Teacher: One is the number that always work, so you don’t need to check. What is the first one you need to check?

Kasim: Ah, seven,.., six.

Teacher confirms six and asks if he needs to check 15.

Kasim: No.

Teacher asks why not, which is interpreted by Kasim this is the wrong answer, so he changes:

Kasim: Oh, yes.

Teacher again immediately asks why.

Then Kasim presents a full explanation:

Kasim: No, because all of these is equal to fifteen. [Pointing to the rods.]

In this end of the interaction, the teacher also engaged in the last practice described in the previous section, Kasim revised his first answer after the teacher asked why, but revised it again when the teacher continued asking why even after he changed his initial answer. In this case, the routine follow up question asked by the teacher caused Kasim to reason mathematically in order to answer it. The conflict of norms, in this case, was caused by the practice of asking questions to students without immediate evaluation.

Another issue generated by the conflict between school norms and mathematical norms arises when students are “solving” a problem that has no solution. It is common in school, even more at this grade level, that mathematical problems always have a solution, often a unique one. Problems with no solutions, or with multiple solutions will most likely only appear in algebra in secondary school mathematics. It is difficult for students, especially at the elementary level, to understand what it means to “solve” a problem that actually does not have solutions. On day 7, Luiz expresses the frustration of not finding a solution:

> Luiz: I was so close, I needed two more, 13 and 10.

Moreover, even when students deal with this kind of problem in algebra, it is not always discussed how such problems are “solved” by mathematicians. In the field of mathematics, “solving” a problem with no solutions means to prove the problem is unsolvable. In this laboratory class, the students had to negotiate between a school norm, in which they had to find a solution to the problem, and a mathematical norm, in which they had to prove there were no solutions to be found. On day 10, the class receives a visit from Ms. McDuff and they have to report how her request cannot be fulfilled. They discuss before she arrives how they are going to report it to her. They are all sure there is no solution, and they discuss how they are going to talk about the red and white clue, how they tested all the “possible trains” and none works. But when she arrives, they struggle to say it. Even Olivia who said before very clearly “there is no train” was speaking more quietly, and seemed ashamed of not having a train to report, saying “we couldn’t figure out a train.” The discussion proceeds and the teacher supports students in reporting what they actually discovered. Eventually, Miah said very clearly and initiates this interaction:

> Miah: Your train is impossible.
>
> Ms. McDuff: My train is impossible?
>
> Chandler: Your train is impossible, we can’t make it. [Loudly speaks, raising her hand to her front with the palm upwards.]
>
> Some students now agree, saying confirmation speeches at the same time. The teacher then asks Jerone to comment.
>
> Jerone: So what we’re saying is that your train could never be made.

> Ms. McDuff replies that she will have to find another company and students respond loudly at the same time “no no no”.
>
> They continue a brief discussion after, and students convince Ms. McDuff her train could not be made and she did not need to keep requesting it to any other company.

This episode shows how difficult it was to negotiate between the two norms, the school norm in which solving a problem means to find the unique solution for it, and the mathematics norm in which solving a problem might mean proving it does not have any solutions. When the students finished solving the problem, they were very convinced that finding the solution was to actually reason about why it was not possible to find the train Ms. McDuff requested. However, when they had to report it to her, it was they struggled to articulate that she was not going to have her train, and they still seemed concerned they could not fill her order.

Chapter 4. Synthesizing the work, presenting implications, and suggesting directions

The main goal of this project was to develop a theory that makes the connection between mathematics instructional practices and institutional racism more visible and that can help us understand how these teaching practices can challenge some institutional norms and practices that systematically disengage children of color from mathematics. CRT is a useful theory to attend to issues of race in contemporary U. S. society, but its first and foremost principle, permanence of racism, already brings a stationary picture of society. CRT is also insufficient to describe the interaction between the macro level racial structures and the micro interactions that people experience in their daily lives. I am building on CRT by incorporating critical realist concepts to capture a more dynamic society with respect to racial structure. The construct of norm circles helped me to see how teaching practices can create a normative intersectional space in which institutional racism is more likely to be challenged and/or disrupted.

While teaching is embedded in a racist society, hence inherently carries racist practices, it is possible to enact teaching practices that create intersectional normative spaces in classrooms in which institutional racism is more likely to be challenged and/or disrupted. The normativity of the such practices makes members of the circle, namely teacher and students, have shared expectations about their participation and learning in class that conflict with typical norms that systematically disadvantage students of color. In these conflicting spaces, Black children can be less likely to be disciplined for reasons that their White peers are not, less likely to be removed from classroom learning, more likely to engage in meaningful mathematical work, and more likely to have their ways of engaging in mathematics acknowledged.

When developing the theory, I also tested it empirically to refine it and to propose methodological considerations with respect to empirical applications of the theoretical ideas I am putting forward in this project. I reported four instructional tools or moves that I observed in this empirical experiment that supported local disruption of racism. In each case, I attended to the creation of a norm circle that locally supported disruption of institutional racism. To operationalize the theoretical construct of norm circles methodologically with respect to racialized patterns that emerged in the context of this laboratory classroom, I had to first notice teaching practices that did not followed an expected norm. So, for example, when the teacher kept asking mathematics questions rather than stopping the interaction to discipline Black boys, this is an action that does not follow an expected norm. Then it required checking whether the action could be characterized as normative or not, by looking across episodes for frequency and consistency of the practice, by checking the members of the circle, and how the norm was reinforced by them. In this case of looking to teacher's practices, it is relevant that, although teacher and students are the typical members of the circle in a classroom and act to reinforce the norm, ultimately only students' actions endorse teacher's actions. The teacher initiates the action in the form of an instructional practice, tool, or move, which already reinforces the norm if the teacher does that often and consistently enough, but students' responses to teacher actions also function as reinforcements for the teacher actions. Whenever students engage in doing mathematics as a consequence of the teacher's instructional move or tool, the teacher is more likely to reenact the move or tool in a similar situation. Conversely, if the students did not engage in doing mathematics, the teacher would likely rethink her teaching strategies and whether they were supporting student learning. So, when students responded to a mathematical question the teacher posed when keeping the focus on math, the teacher might interpret this, consciously or not, as a successful or effective teaching move and become inclined to reenact it. Moreover, the fact that the teacher actually reenacted it consistently over the course of this laboratory classroom, shows the teacher had not revised her disposition to act in such way.

This process revealed that the rigor of the framework rests on normativity, so examining snip shots of particular social interactions is not sufficient to account for norms, given the necessity of regularity and consistency for something to be accounted as normative. The observation of recurrent patterns over time, or across multiple episodes, supported a more accurate description of local norms of interactions. Moreover, the confirmation of the norm was also checked by looking to the members of the circle and how their actions reinforced the norms. A second observation from the process, is that all captured teaching practices were initially captured because they did not follow an expected norm. That means that there are still questions about how to capture teaching actions that reproduce racism, especially when most teaching practice is already invisible (Lewis, 2007). Such questions could be addressed in future work.

A third contribution of this work is that the conception of mathematics endorsed in this classroom also mediates instructional practices, and therefore interactions between teacher and students. By regulating student seating focusing on getting the student to actually work rather than just regulating behavior, the teacher supported students of color in engaging in mathematical practices and content. By sustaining the focus of individual and small group conversations in mathematics, the teacher took students seriously, showing she believed they could do mathematics, and they responded accordingly. By regulating speaker and audience participation in classroom discourse, the teacher oriented students to one another's thinking and created opportunities for students to share their thinking in ways that the sharer could share thinking and listeners could learn from it. By often asking follow up questions without demonstrating any sign of evaluation in the questioning, the teacher showed, on the one hand, that students' thinking was valued, and on the other hand, she was reinforcing the mathematical practice of explaining your thinking.

As an outcome from instructional practices carried out by the teacher in this classroom, students engaged in doing mathematics in similar ways mathematicians do. They explained their reasoning mathematically, why some trains always worked, so they did not need to test them; why they did not need to find all small trains they could not build from the whole train to be sure the whole train did not work; why the red and white clue was right and how to use it to solve the problem. They used strategies to solve problems frequently used in mathematics, such as finding the smallest and greatest possible train for part 1, then looked for trains “in between.” They engaged in common and authentic problems of mathematics. When La’rayne considered the possibility of “no train at all” actually being a “train made with zero rods”, she engaged in an authentic mathematical problem that parallels the first or smallest natural number. When engaging in these practices of doing mathematics, the students took an active role in producing mathematical knowledge in their community. The active enactment of mathematics by these students was crucial to support local disruption of racism. When the students described in this report, mostly African Americans, and one Latino, boys and girls, performed mathematics they countered the idea that mathematics intelligence is a function of being White (men). The key element supporting local disruption of racism in this classroom does not question Western mathematics as a means to promote social oppression, rather it lies on viewing mathematics as something you *do*, rather than *know*, and, in this class, they could all do it.

4.1. A few more thoughts on the critical aspect of this work

When presenting this work I was asked questions that made me reflect on how this work contributed to seeking a society more just and how I could make it more explicit to my readers. So, in this final section, I am adding answers to some of those questions hoping they will contribute to a better understanding of this work.

4.1.1. What does this work challenge (in terms of racial structural oppression)?

It is necessary to understand that this work is not seeking to present a big plan to completely dismantle racial structure. I believe that our social structures are deeply ingrained and very hard to break. My position is to seek micro level alternatives that can make a difference quite immediately. When I discuss teaching practices, these are things teachers do in their daily lives, and if they know are the kind of practices that can support Black children in understanding a form of mathematics they would not otherwise, they could enact them. Teacher educators can also teach them to novices.

In this perspective, this work challenges assumptions about teaching practices that are typically observed in classrooms, such as disciplining Black children so they can learn and seeing Black girls as not academically smart. Such typical practices not only function as lenses to view Black children as undisciplined or not smart, but also to actually enforce these positions to them (Grant, 1984, 1994; Morris, 2007; Gregory, Skiba, and Noguera, 2010). The mathematics instructional practices I described in this work function as examples that counter these more typical practices. Furthermore, the normativity creates different expectations to these children. In a typical classroom, a Black girl might not expect her mathematical ideas will be valued, but if she experiences a normative space in which her ideas are often valued, she might become inclined to expect it, she might question when her ideas are not valued.

This work also challenges some assumptions about mathematics itself, but not what is commonly challenged, about Western mathematics as an instrument for social oppression (Apple, 1992; D'Ambrosio, 1993; Valero, 2004), but at a more foundational and philosophical position. One of the foci of this study is the gatekeeping function of mathematics in contemporary society. I am suggesting that a key element of this function are actually mathematical practices rather than knowledge. In all reported episodes students were engaged in doing mathematics in ways that are similar to what mathematicians do. It matters that in this class mathematics is something you *do*, rather than *know*, and all students could do it.

4.1.2. Does this work claim to be disruptive while not challenging major systematic problems? Is it ultimately complicit with the system?

Yes and no. Yes, in the sense answered in the previous question. The gatekeeping function of mathematics is an important point of this work, but some scholars, such as D'Ambrosio (1993), argue that this function only exists because mathematics was already constructed within our contemporary society to mediate oppressive relationships. In this perspective, addressing the gatekeeping function of mathematics addresses the symptoms without addressing the causes of the problem.

There are, however, many ways to interpret disruption. I believe that supporting people in facing their daily live struggles are disruptive. This work helps teacher and teacher educators to see how mathematical instructional practices directly impacts the racialized experiences children of color.

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